

Private Information, Price Discrimination, and Collusion*

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Abstract

We analyze firms' ability to sustain collusion in a setting in which horizontally differentiated firms can price-discriminate based on private information regarding consumers' preferences. In particular, firms receive private signals which can be noisy (e.g., big data predictions). We find that there is a non-monotone relationship between signal quality and sustainability of collusion. Starting from a low level, an increase in signal precision first facilitates collusion. There is a watershed, however, from which any further increase renders collusion less sustainable. Our analysis provides important insights for competition policy. In particular, a ban on price discrimination can help to prevent collusive behavior as long as signals are sufficiently noisy.

JEL classification: L13, D43, L41.

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1. Introduction

In this paper, we analyze sustainability of tacit collusion in a setting in which horizontally differentiated firms can price-discriminate based on private information about consumer preferences. In particular, prices for different consumer groups can be based on private and imperfect signals. For example, firms may price-discriminate on (possibly imprecise) big data predictions. These aspects are of high relevance in industries like traditional brick-and-mortar and online retailing.

In order to price-discriminate between different consumer segments, most firms in these industries collect data on their own customers through different channels (e.g., loyalty programs, cookies) or buy data from data-collection firms. In the US, for example, the second-largest discount store retailer Target uses a data-mining program to assign many different predictors to customers.¹ The quality of data and the precision of predictions, however, can crucially affect firms' pricing decisions.² In particular, data quality is rarely perfect. In the example of Target, their "pregnancy prediction" was flawed. Pregnancy-related mailers were sent out to women for months after a miscarriage.³

At the same time, antitrust policy is highly concerned with collusive behavior in these industries, especially with tacit collusion in online retailing. The acuteness of this issue can be seen by the recent stern warning of the Competition and Market Authority (CMA) in the UK. The warning was issued after the CMA had found signs of price coordination among retailers in different markets on platforms such as Amazon.⁴

We find that the critical discount factor necessary to sustain tacit collusion is non-monotone in signal precision. In particular, an increase in precision reduces the critical discount factor whenever the level of signal precision lies below a certain threshold. From levels above this threshold, an increase in precision leads to a higher critical discount factor. The intuition behind this finding is as follows. In our model, collusive profits are independent of signals,

¹See <http://www.nytimes.com/2012/02/19/magazine/shopping-habits.html> (accessed on June 2, 2017). See also Esteves (2014) for these and other examples.

²For discussions of this issue in different contexts, see Liu and Serfes (2004), Esteves (2004, ch. 2), and Colombo (2016) among others.

³As Charles Duhigg, a journalist with the New York Times, puts it: "I can't tell you what one shopper is going to do, but I can tell you with 90 percent accuracy what one shopper is going to do if he or she looks exactly like one million other shoppers. You expect that there is some spillage there, and as a result that you will give the wrong message to a certain number of people." See, <https://6thfloor.blogs.nytimes.com/2012/02/21/behind-the-cover-story-how-much-does-target-know/> (accessed on June 2, 2017).

⁴For more details see, for instance, <https://www.theguardian.com/business/2016/nov/07/online-sellers-price-fixing-competition-and-markets-authority-amazon> (accessed on June 2, 2017).

whereas both deviation profits and competitive profits, which serve as punishment for deviations, depend on how precise signals are. Deviation profits are weakly increasing in signal quality, as price discrimination allows to target consumers more effectively. At the same time, competitive profits are falling in signal quality. Competition gets fiercer, as both firms can price-discriminate more effectively. Hence, improvements in signal precision have opposing effects on the critical discount factor, as both the gains from deviation and the losses from punishment increase. We show that below the threshold, the gain from defecting outweighs the loss from punishment. Intuitively, potential misrecognition of consumers renders deviation from collusive prices relatively unprofitable. Above the threshold, the reverse turns out to be true. As consumers can be targeted effectively, deviation becomes relatively tempting.

This paper adds to the combination of two strands of theoretical industrial organization literature: third-degree price discrimination and collusion, both among horizontally differentiated firms. In the first strand, Bester and Petrakis (1996) show that third-degree price discrimination by using coupons intensifies competition in markets that are segmented exogenously by consumer preferences. In a similar setup, Shaffer and Zhang (1995) illustrate that the possibility of third-degree price discrimination leads to a prisoner's dilemma. Corts (1998) then generalizes these findings. Under best-response asymmetries, that is, firms find different groups of consumers most valuable, third-degree price discrimination leads to profits lower than under uniform pricing. Fudenberg and Tirole (2000) analyze the impact of third-degree price discrimination in a dynamic context in which learning about consumer preferences is endogenous from the purchasing history. After the first period, firms learn about the preferences of their own customers. In the second period, poaching can take place through price discrimination. They also find third-degree price discrimination, which they refer to as behavior-based price discrimination, results in more intense competition and hence lower profits. Villas-Boas (1999) extends their setup to long-lived firms and overlapping consumer generations and finds that competition is intensified if firms and consumers are patient.

While in the previous contributions, firms have or obtain perfect information about consumer preferences, Esteves (2009, 2014) analyzes the impact of imperfect information. She shows that improving the quality of information also results in lower competitive profits under third-degree price discrimination. If information is imperfect, potential misrecognition of consumers dampens competition. As information becomes more accurate, firms can better target different consumer groups, which results in more intense competition. She argues that imperfect information can also be understood as a reduced-form of imperfect learning. Colombo (2016) explicitly investigates the impact of imperfect information in the dynamic context of Fudenberg and Tirole (2000).

First-period learning is noisy, as firms cannot recognize every first-period consumers and hence only learn the preference of a proportion. He finds that there is an inverse U-shaped relationship between quality of information and competitive profits, whereby the result of Fudenberg and Tirole (2000) is nested. Following Stole (2007), fiercer competition due to third-degree price discrimination creates incentives to commit to uniform pricing, that is, firms may seek to collude. In our paper, we focus on how potential misrecognition as in Esteves (2009, 2014) affects the scope for tacit collusion.

Combining the two strands, Liu and Serfes (2007) consider the impact of information on collusion. In their setup, however, information is publicly available and its quality is defined by the number of market segments. Then, an increase in information quality is equivalent to an increase of the number of perfectly distinguishable segments and hence the number of segment-specific prices. The authors analyze different collusive schemes. Their main finding is that collusion becomes harder to sustain as the number of market segments increases. Helfrich and Herweg (2016), which is closest to our work, consider two settings with perfect information in which price discrimination leads to either best-response symmetries or best-response asymmetries. Compared to the situation in which there is a ban on price discrimination, the authors show that third-degree price discrimination helps to fight collusion under both best-response symmetries and best-response asymmetries.

The findings from the theoretical literature on the relationship between collusion and third-degree price discrimination can thus be summarized as follows: When price discrimination is based on perfect information, theory predicts that third-degree price discrimination renders collusion less likely. Then, the implication for antitrust policy is that a legal ban on price third-degree price discrimination helps to fight tacit collusion. In most markets, however, information is not perfect. We contribute to this literature by relaxing the assumption of third-degree price discrimination under perfect information. By generalizing parts of the results in Helfrich and Herweg (2016), our analysis provides an important insight, namely that the outcomes can be fundamentally different when firms' information about consumer preferences is private and imperfect.

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, we derive the relevant payoffs for the case that firms can price-discriminate as well as for the case of uniform pricing and determine the critical discount factors. Then, we compare sustainability of collusion in the two pricing regimes. In Section 4, we discuss the robustness of our results. Section 5 concludes.

2. Model

In this section, we first introduce the stage game, which is a static Bertrand pricing game of incomplete information. Thereafter, we describe the supergame, which is an infinite repetition of the stage game.

Stage Game

We consider a model of incomplete information developed in Armstrong (2006), which is a variant of Esteves (2014, 2004, chap. 2). Consider a linear city à la Hotelling (1929) with two symmetric firms, A and B , which are located at $\ell_A = 0$ and $\ell_B = 1$, respectively. Firms' marginal and fixed costs are normalized to zero. They compete in prices p_i with $i \in \{A, B\}$. We analyze two different pricing schemes: (i) third-degree price discrimination and (ii) uniform pricing.

Consumers of mass one are uniformly distributed along the line and derive a gross utility from buying the product, which is normalized to one. Additionally, they incur linear transport costs τ per unit of distance. Hence, when buying from firm i and paying price p_i , a consumer located at x derives net utility

$$U(x; p_i) = 1 - p_i - \tau|\ell_i - x|.$$

Consumers' outside option is normalized to zero.⁵

In our setup, there are two groups of consumers, L and R , consisting of the left and right half of the linear city, respectively. Given equal prices, consumers in group L (group R) prefer firm A (firm B). Synonymously, we can call consumers in group L (group R) loyal to firm A (firm B).⁶

Consumer types are private information. When facing a consumer, each firm i receives a noisy private signal $s_i \in \{s_L, s_R\}$ indicating the consumer's preference. Signal precision is measured by probability σ and drawn independently for each firm.⁷ In other words, with probability σ , information about a consumer's preference is correctly passed on to a firm through the signal. With probability $1 - \sigma$, the preference is misrecognized. We assume that the signal is weakly informative, that is, $\sigma \in [1/2, 1]$. Thereby, our setup nests the following two extreme cases: (i) the signal does not convey any information,

⁵Our results hold qualitatively if the outside option is located at each end node of the line as in Bénabou and Tirole (2016) (see the discussion below).

⁶As in Esteves (2014), the definition of brand loyalty is similar to Raju et al. (1990, p. 279), where “the degree of brand loyalty is defined to be the minimum difference between the prices of the two competing brands necessary to induce the loyal consumers of one brand to switch to the competing brand”.

⁷In Section 4, we relax both the assumptions of independent and symmetric signals.

that is, $\sigma = 1/2$, and (ii) market segments are perfectly distinguishable, that is, $\sigma = 1$.⁸ The timing of the game is summarized below in detail.

1. Firms independently receive a private signal for any consumer along the linear city. If a consumer is located at $x \in [0, 1/2]$, each firm receives signal s_L with probability σ and signal s_R with probability $1 - \sigma$. If a consumer is located at $x \in (1/2, 1]$, each firm receives signal s_R with probability σ and signal s_L with probability $1 - \sigma$.
2. Firms simultaneously set prices. Under price discrimination, firms can condition their prices on their private signal, whereas they set a single price under uniform pricing.
3. Consumers decide from which firm to buy, and payoffs are realized.

As signals are private, firms do not know their competitor's payoff function. Hence, we consider a game of incomplete information. In order to solve the stage game, we use the notion of Bayesian Nash equilibrium. Our tie-breaking rule is the following: whenever a consumer values the outside option and a firm equally, she chooses the firm, and in case she is indifferent between the firms, she chooses randomly.⁹

Dynamic Game

In order to study the scope for collusive behavior, we extend our setup to a game of infinite horizon. In the infinitely repeated game, the stage game described above is played in each period $t = 0, \dots, \infty$. Firms are long-lived, that is, they play over the entire sequence of the infinitely repeated game. Expected payoff in period t is defined as the stage game payoff plus the discounted value of the stream of future payoffs determined by the continuation game strategy profile. Firms' common discount factor is $\delta \in (0, 1)$. Consumers are short-lived, that is, they only play for a single period and are replaced by a new cohort of consumers in the subsequent period.¹⁰ As a consequence, intertemporal price discrimination is not possible. Their payoff is given by their net utility in the respective period. All players are payoff-maximizing. Consumers are perfectly informed. Hence, their payoff is deterministic.

As the stage game is Bayesian, we use the notion of perfect Bayesian equilibrium when analyzing the dynamic game. We refrain from explicitly stating

⁸The first case hence represents the classic model, whereas the second case corresponds to a segmented market with two distinguishable segments (see Liu and Serfes, 2007 and Helfrich and Herweg, 2016).

⁹Ties are not outcome-relevant as the distribution of consumers is atomless.

¹⁰As argued in Section 4.1, asymmetric signal quality can be interpreted as relaxing the assumption of short-lived consumers.

the set of players' beliefs as part of the equilibrium description. In addition, as consumers are short-lived, firms cannot learn their preferences over time. The same holds true for beliefs regarding the signals of the competitor, as these are independent across periods.

Further, we assume that firms use grim-trigger strategies as defined in Friedman (1971) to support collusive outcomes. Thereby, we follow the related literature and can compare results. On the other hand, we want to focus on the impact of signal quality on the following trade-off for a firm: (i) long-term gains from collusive behavior compared to competitive outcomes against (ii) short-term gains by deviating unilaterally from collusive behavior. This seems plausible to us especially when thinking about tacit collusion without a certain punishment mechanism, where defection might lead to competition for an undetermined time horizon. Grim-trigger strategies generate exactly this trade-off, as punishment coincides with competitive outcomes. If, instead, optimal penal codes as in Lambson (1987) and Abreu (1988) are employed in our setup, punishment payoffs become deterministic and finite. Then, firms still trade off gains from deviation against losses from punishment, but do not take into account competitive outcomes at all by construction.¹¹ The stationary strategy profile can be summarized as follows:

- In the starting period $t = 0$, each firm charges the collusive price. In any subsequent period $t = 1, \dots, \infty$, each firm
 - charges the collusive price as long as it does not observe any other price in period $t - 1$ and
 - plays Bayesian Nash equilibrium strategies else.
- Consumers buy from the firm providing the highest net utility if it weakly exceeds the value of the outside option. If a consumer is indifferent between the two firms, she chooses randomly.

In order to verify whether the suggested strategy profile constitutes a perfect Bayesian equilibrium, we need to verify that the one-shot-deviation principle (OSDP) is satisfied (for a formal argument, see Hendon et al., 1996). Given firms' strategies and beliefs over consumers' preferences and the respective competitors private information, this is true if and only if the following inequality is satisfied in any period t :

$$\frac{\pi^c}{1 - \delta} \geq \pi^d + \frac{\delta\pi^*}{1 - \delta}, \quad (1)$$

where π^* , π^c , and π^d denote competitive (punishment) payoffs, collusive payoffs, and deviation payoffs, respectively. From this, it follows that the critical

¹¹See Appendix B for a characterization of an optimal penal code in our game.

discount factor is defined by

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^*} =: \bar{\delta}. \quad (2)$$

All things equal, a lower (higher) punishment or deviation payoffs facilitates collusion (makes collusion harder to sustain), that is, the set of discount factors which satisfy OSDP becomes larger (smaller). The opposite is true for the respective change in the collusive payoffs. Put differently, lower (higher) gains from defecting (i.e., $\pi^d - \pi^c$) and higher (lower) losses from punishment (i.e., $\pi^c - \pi^*$) make collusion easier (harder) to sustain.

Throughout the analysis, we focus on equilibria in which the market is covered, that is, all consumers along the line buy from one of the two firms. For this purpose, we impose the following assumption on consumers' transport costs:

Assumption 1. $\tau \in [0, 2/3]$.

Assumption 1, which is common in the related literature, guarantees that the market is fully served under uniform pricing.^{12,13}

3. Analysis and Results

In this section, we derive the critical discount factors for the case of price discrimination and the uniform pricing case. From the comparison of these two cases, we provide policy implications for a ban on price discrimination.

3.1. Price Discrimination

In order to evaluate firms' ability to sustain collusion under price discrimination, we need to derive the payoffs under competition, deviation, and collusion. Firms want to condition their prices on the signal they receive as long as it is informative: After observing signal s_L , firm i charges $p_{i,L}$, and after observing s_R , it charges $p_{i,R}$. For demand under competition to be well-defined, suppose for now that given prices of firm B , it has to hold for firm A that $p_{B,L} \leq p_{A,R} \leq p_{A,L} \leq p_{B,R}$ and given prices of firm A , it has to hold for firm B

¹²For the case of price discrimination, the market is covered for larger values of the transport costs as prices tend to be lower. To ensure better comparability, we use the more restrictive upper bound on the transport-cost parameter.

¹³Instead, one could follow Bénabou and Tirole (2016) by assuming that the outside option is costly, that is, it is located at either end of the linear city. Then, Assumption 1 would not be needed. This, however, would not change our results qualitatively but make the comparison to the above mentioned literature less clean.

that $p_{A,R} \leq p_{B,L} \leq p_{B,R} \leq p_{A,L}$. The intuition behind the restrictions is that, on the one hand, a firm does not find it profitable to charge lower prices from its loyal consumers than its rival. Neither it finds it profitable to charge lower prices from consumers that prefer the firm than from consumers that prefer its competitor. On the other hand, a firm cannot attract any consumer that is loyal to its competitor by charging a higher price. The remaining conditions are without loss of generality and can be specified differently. It will be shown later in this subsection that equilibrium prices indeed satisfy all conditions.

In order to derive expected demand of a firm conditional on its private signal, we need to distinguish all possible outcomes, where an outcome is characterized by a tuple (s_j, s_k) with $j, k \in \{L, R\}$, and where the first (second) element is the signal of firm A (firm B). Since signals are independently drawn for each consumer, firms can either receive identical signals ($j = k$) or different signals ($j \neq k$), that is, the set of possible signal realizations is given by $S := \{(s_L, s_L), (s_L, s_R), (s_R, s_L), (s_R, s_R)\}$. As firms condition their prices on their private signal, they have to take into account four different indifferent consumers, which determine the probability of winning a certain consumer or, equivalently, the market share in a segment, for any signal tuple. To this end, \tilde{x}_1 denotes the indifferent consumer for tuple (s_L, s_L) , \tilde{x}_2 for (s_R, s_L) , \tilde{x}_3 for (s_L, s_R) , and \tilde{x}_4 for (s_R, s_R) . Solving for each, we get

$$1 - p_{A,L} - \tau \tilde{x}_1 = 1 - p_{B,L} - \tau (1 - \tilde{x}_1) \Leftrightarrow \tilde{x}_1 = \frac{1}{2} - \frac{p_{A,L} - p_{B,L}}{2\tau},$$

$$1 - p_{A,L} - \tau \tilde{x}_2 = 1 - p_{B,R} - \tau (1 - \tilde{x}_2) \Leftrightarrow \tilde{x}_2 = \frac{1}{2} - \frac{p_{A,L} - p_{B,R}}{2\tau},$$

$$1 - p_{A,R} - \tau \tilde{x}_3 = 1 - p_{B,L} - \tau (1 - \tilde{x}_3) \Leftrightarrow \tilde{x}_3 = \frac{1}{2} - \frac{p_{A,R} - p_{B,L}}{2\tau},$$

and

$$1 - p_{A,R} - \tau \tilde{x}_4 = 1 - p_{B,R} - \tau (1 - \tilde{x}_4) \Leftrightarrow \tilde{x}_4 = \frac{1}{2} - \frac{p_{A,R} - p_{B,R}}{2\tau}.$$

Due to the restriction of the set of feasible prices above, it holds true that $\tilde{x}_1, \tilde{x}_3 \in [0, 1/2]$ and $\tilde{x}_2, \tilde{x}_4 \in [1/2, 1]$. For firm A , the probability of winning consumer $x \in L$ given (s_L, s_L) is equal to $2x_1$. In the same firm and segment, the probability of winning the consumer given (s_L, s_R) is equal to $2(x_2 - 1/2)$. In both cases, the winning probability is equivalent to the firm's expected market share in segment L . The remaining cases can be derived analogously.

The notion of Bayesian Nash equilibrium requires that firm i —after receiving a signal—updates its beliefs regarding the respective consumer's actual preference and regarding the signal of its competitor. As signal realizations are independent across firms and periods, the updating process is independent

in each stage game. A firm's posterior belief that a consumer prefers firm A given signal s_L is

$$\Pr(L|s_L) = \frac{\Pr(s_L|L) \Pr(L)}{\Pr(s_L|L) \Pr(L) + \Pr(s_L|R) \Pr(R)} = \sigma,$$

which is equal to the precision of the signal due to symmetry. Conditional on this, firm i 's posterior belief that firm j has received signal s_L is equal to the conditional probability of this event, namely σ . In the remaining cases, beliefs are updated analogously.

Then, firm A 's expected demand conditional on receiving signal s_L can be derived as

$$\begin{aligned} D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_L) &= \sigma(2\sigma\tilde{x}_1 + 1 - \sigma) + 2\sigma(1 - \sigma)\left(\tilde{x}_2 - \frac{1}{2}\right) \\ &= \sigma\left(1 - \frac{p_{A,L} - \sigma p_{B,L} - (1 - \sigma)p_{B,R}}{\tau}\right). \end{aligned} \quad (3)$$

Similarly, conditional on receiving signal s_R , firm A 's expected demand can be derived as

$$\begin{aligned} D_A(p_{A,R}, p_{B,L}, p_{B,R}|s_R) &= (1 - \sigma)(2\sigma\tilde{x}_3 + 1 - \sigma) + 2\sigma^2\left(\tilde{x}_4 - \frac{1}{2}\right) \\ &= 1 - \sigma - \frac{\sigma p_{A,R} - \sigma(1 - \sigma)p_{B,L} - \sigma^2 p_{B,R}}{\tau}. \end{aligned} \quad (4)$$

Expected demand for firm B conditional on its signal realization can be derived analogously. In the following, we solve for the different

Competition

We start by analyzing the competitive payoffs, that is, the static Bayesian Nash equilibrium payoff of the stage game as defined in Section 2. These are used as punishment payoffs in the dynamic game.¹⁴ The maximization problem of firm i is given as

$$\begin{aligned} \max_{p_{i,L}, p_{i,R}} \mathbb{E}[\pi_i] &= p_{i,L} \Pr(s_i = s_L) D_{i,L}(p_{A,L}, p_{B,L}, p_{B,R}|s_L) \\ &\quad + p_{i,R} \Pr(s_i = s_R) D_{i,R}(p_{A,R}, p_{B,L}, p_{B,R}|s_R), \end{aligned} \quad (5)$$

where $\Pr(s_i = s_L) = \Pr(s_i = s_R) = 1/2$. Differentiating with respect to prices and solving the system of first-order conditions gives symmetric equilibrium

¹⁴The results from this part are equivalent to Armstrong (2006).

prices of

$$p_{A,L}^* = p_{B,R}^* = \frac{2\tau}{1+2\sigma} \quad \text{and} \quad p_{A,R}^* = p_{B,L}^* = \frac{\tau}{\sigma(1+2\sigma)},$$

where $p_{A,R}^* < p_{A,L}^*$ and $p_{B,L}^* < p_{B,R}^*$ hold as long as the signal is informative. Then, price discrimination allows firms to set higher prices for those consumers who are signaled to be located more closely to their own location, that is, consumers with a higher willingness to pay for their product. Thereby, the market is segmented into four as in Fudenberg and Tirole (2000) under informative signals, although firms can only distinguish between two consumer groups. In each half, there are consumers served by their preferred firm, and consumers poached by the less preferred firm, i.e., $0 < \tilde{x}_1^* < \tilde{x}_2^* = \tilde{x}_3^* = 1/2 < \tilde{x}_4^* < 1 \forall \sigma \in (1/2, 1]$. The equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau(1+4\sigma^2)}{2\sigma(1+2\sigma)^2}.$$

We observe that firms' payoffs are decreasing in the signal precision. As Esteves (2014) points out, an increase in signal precision has two opposing effects: On the one hand, misrecognition of consumers decreases, which means that a firm can charge more from its loyal consumers, while reducing the price to those consumers who are loyal to its rival. In other words, a firm can poach more effectively. On the other hand, since the rival behaves more aggressively as well when poaching loyal consumers, a firm optimally responds by reducing its prices. In this setup, it turns out that the latter effect (competition) outweighs the increase in prices due to reduction in misrecognition (information). Hence, competition is intensified with a rise in signal precision. As a result, for any $\sigma \in (1/2, 1]$, payoffs are strictly lower than static Bayesian Nash equilibrium payoffs under uniform pricing, as we have best-response asymmetries.

Collusion

Under full collusion, firms maximize industry profits by minimizing total transport costs, that is, firms divide the market in two and each firm serves its own turf. In our game, this allocation can only be induced by charging symmetric prices. As firms try to extract the maximal surplus from consumers net of transport costs, it is not optimal to attract consumers in the competitor's turf. Put differently, firms will not price-discriminate based on private information about consumers' preferences. Instead, they will set a single price for all consumers such that the marginal consumer located at $1/2$ is indifferent between buying and not buying, that is, $1 - p^c - \tau|\ell_i - 1/2| = 0$. We summarize these considerations in the following lemma:

Lemma 1. *Collusive prices and payoffs are given by*

$$p^c = 1 - \frac{\tau}{2}$$

and

$$\pi^c = \frac{1}{2} - \frac{\tau}{4}.$$

We observe that price discrimination cannot lead to higher payoffs compared to uniform pricing, as firms can only distinguish two consumer groups.¹⁵

Deviation

In order to characterize the optimal deviation strategy, we need to define the following thresholds for τ :¹⁶

$$\tau_1 := \frac{2(1-\sigma)}{5-3\sigma}, \quad \tau_2 := \frac{2\sigma}{2+3\sigma}, \quad \text{and} \quad \tau_3 := \frac{2(1-\sigma)}{1+\sigma}.$$

It is easily checked that $\tau_1, \tau_2, \tau_3 \in [0, 2/3]$ for any $\sigma \in [1/2, 1]$ and that $\tau_2 \leq \tau_3$ for $\sigma \leq 1/\sqrt{2}$. The following lemma characterizes optimal deviation behavior:

Lemma 2. *The optimal deviation from collusive prices yields the following prices and payoffs, which are continuous and differentiable in both σ and τ :*

$$p_{A,L}^d = p_{B,R}^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } \tau \in [0, \tau_1] \\ \frac{1}{2} + \frac{\tau(3\sigma-1)}{4(1-\sigma)} & \text{if } \tau \in (\tau_1, \tau_3], \\ 1 - \frac{\tau}{2} & \text{if } \tau \in (\tau_3, \frac{2}{3}], \end{cases}$$

$$p_{A,R}^d = p_{B,L}^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } 0 \leq \tau \leq \tau_2, \\ \frac{1}{2} - \frac{\tau(3\sigma-2)}{4\sigma} & \text{if } \tau_2 < \tau \leq \frac{2}{3}, \end{cases}$$

¹⁵In this setup, firms do not price-discriminate under collusion, which is also the case in Helfrich and Herweg (2016) and Liu and Serfes (2007) (with two segments). This is due to the fact that we only allow for a left and a right market, i.e., two signals. The present model could easily be extended to more signals, which would yield price discrimination also under collusion. At the same time, results would not change qualitatively (in particular, see the deviation incentives for low values of signal precision and transport costs below). For tractability reasons, we restrict our attention to two signals.

¹⁶The derivation of these thresholds is part of the proof of Lemma 2 in Appendix A.

and,

$$\pi^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } \tau \in [0, \tau_1], \\ \frac{(3\tau(1+\sigma)+2(1-\sigma))^2-32\tau^2}{32\tau(1-\sigma)} & \text{if } \tau \in (\tau_1, \min\{\tau_2, \tau_3\}], \\ \frac{\tau}{8\sigma(1-\sigma)} + \frac{4(\tau+1)-15\tau^2}{32\tau} & \text{if } \sigma \in \left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right] \wedge \tau \in (\tau_2, \tau_3], \\ \frac{2-3\tau+\sigma(2-\tau)}{4} & \text{if } \sigma \in \left(\frac{1}{\sqrt{2}}, 1\right] \wedge \tau \in (\tau_3, \tau_2], \\ \frac{\tau^2(2-\sigma)^2+4\sigma^2(\tau+1)+8\tau\sigma(1-\tau)}{32\tau\sigma} & \text{if } \tau \in \left(\max\{\tau_2, \tau_3\}, \frac{2}{3}\right]. \end{cases}$$

Proof. See Appendix A.

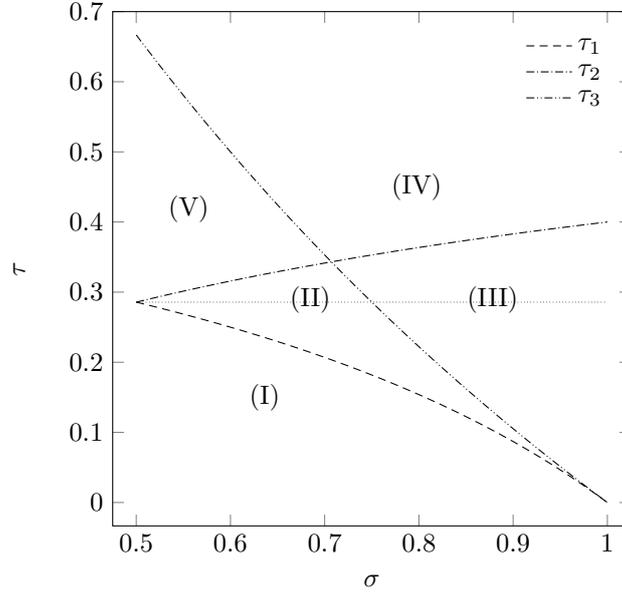


Figure 1: Characterization of deviation strategies.

Note: the dotted horizontal line at $2/7$ gives the threshold below which a deviating firm wants to serve the whole market in the case of uniform pricing.

Corresponding to the cases in Lemma 2, Figure 1 divides all combinations of parameter values of σ and τ into regions I–V. Intuitively, the regions are divided corresponding to the following considerations: (i) does a deviating firm want to serve all consumers in its competitor’s turf (I–III) or not (IV, V), and (ii) does it want to charge uniform prices (I), price-discriminate cautiously (II, V), or aggressively (III, IV). The first consideration is well known from the related literature on uniform pricing (see, e.g., Chang, 1991). The second consideration is unambiguous if information is perfect (see Helfrich and Herweg,

2016 and Liu and Serfes, 2007). Then, a deviator price-discriminates aggressively by charging the collusive price from its loyal consumers while poaching its competitor's loyal consumers with low prices. If information is imperfect, however, the quality of information plays an additional role. If signals are relatively noisy, the firm might misrecognize consumers' preferences, which can be costly. Then, it prefers to charge rather similar prices conditional on the signal received. Thereby, it avoids losing infra-margins by offering a relatively low price to a loyal customer as well as foregoing demand when offering a relatively high price to a disloyal consumer. If the signal is sufficiently reliable, however, misrecognition becomes less likely, and hence the firm prefers to act more aggressively by charging the collusive price to consumers it expects to be loyal and rather low poaching prices to consumers it expects to be loyal to its competitor. The behavior of a deviating firm in each region is explained in detail below.

In region I, transport costs are very low, and hence a deviating firm captures the entire market by setting a uniform price independent of σ . In region II, transport costs are still sufficiently low such that the deviator wants to capture all consumers, whereas it prefers to price-discriminate between the different groups depending on the signal it receives. To be precise, it still wants to charge a price independent of σ to its competitor's consumers, while the price it wants to charge to its own consumers rises in σ . This results in an increasing price difference between signals. In region III, the deviator still captures all consumers and wants to price its competitor's consumers as before. The relatively precise signal, however, makes it profitable for the deviator to charge the collusive price to consumers it expects to be loyal. In region IV, transport costs are high such that the firm finds it too costly to capture all of its competitor's consumers. As it can price-discriminate between signals, and signals are relatively precise, it still wants to charge the collusive price to its own consumers. The price difference, however, increases in σ , as the price it wants to charge to its competitor's consumers decreases in σ . In region V, transport costs are again high such that it is too costly for the deviating firm to serve all consumers whose signal indicates a preference for its competitor. It is too costly as well to charge the collusive price to consumers it expects to be loyal, as the signal is relatively noisy. This price, however, increases in σ and hence the price difference between the signals increases as well.

Critical Discount Factor

Using the payoffs derived in the three above scenarios, we can determine the critical discount factor $\bar{\delta} := \bar{\delta}(\sigma, \tau)$ as defined in Condition (2) as characterized in the following proposition:

Proposition 1. *When firms can price-discriminate, the critical discount factor $\bar{\delta}$ is a continuous and differentiable function of σ and τ with the following properties:*

- (i) $\bar{\delta}$ is non-monotone in the signal quality such that $\partial\bar{\delta}/\partial\sigma < 0$ (> 0) holds for low (high) values of σ .
- (ii) $\bar{\delta}$ is non-monotone in the transport costs such that $\partial\bar{\delta}/\partial\tau < 0$ (> 0) holds for low (high) values of τ .

Proof. See Appendix A.

Let us have a closer look at the intuition behind these findings. By construction, the collusive payoff is independent of signal quality, whereas the deviation payoff and the Bayesian Nash equilibrium payoff depend on it, as we can see from the analysis above. More precisely, for a given value of the transport-cost parameter, the deviation payoffs are weakly increasing in signal quality, as targeting consumers becomes easier. At the same time, Bayesian Nash equilibrium payoffs are falling in signal quality, as competition gets fiercer. Hence, increasing signal quality has opposing effects on the critical discount factor, as both the gains from deviation and the losses from punishment increase. For perfect signal quality, Helfrich and Herweg (2016) and Liu and Serfes (2007) find that the destabilizing effect dominates. As a consequence, collusion is harder to sustain under price discrimination than under uniform pricing, that is, $\bar{\delta}(1/2, \tau) < \bar{\delta}(1, \tau)$.

Now consider the case in which signal quality is imperfect. From a low level of signal precision, as precision increases the gain from defecting increases relatively slower than the loss from punishment. For the case with relatively low transport costs, this is intuitive. A deviating firm finds it profitable to capture the entire market irrespective of the signal precision (see region I). Meanwhile, competition is intensified as σ increases, and hence the loss from punishment increases.

The case in which transport costs are relatively high is more involved. On the one hand, punishment payoffs are decreasing as before. On the other hand, deviation payoffs increase in σ (see regions II, IV, V). As signal quality is relatively low, a deviating firm expects to misrecognize consumers often and hence price-discriminates cautiously, that is, the difference in prices conditional on signals is rather low. In our model, this misrecognition effect slows down the increase in deviation payoff relative to the increase in loss from punishment. As a result, collusion is facilitated.

From a high level of signal precision, as precision increases the misrecognition effect becomes less pronounced. The deviating firm price-discriminates more aggressively, that is, the difference in prices conditional on signals is rather

high. Thereby, the increase in deviation payoffs outweighs the increase in loss from punishment impeding collusion. We thus shed light on the intermediate cases between uniform pricing ($\sigma = 1/2$) and price discrimination conditional on perfect information ($\sigma = 1$) and show that sustainability of collusion is non-monotonic in signal quality. Moreover, it turns out that there is a non-monotonic relationship between sustainability of collusion and transportation cost. The logic behind this result can be derived from Figure 1 similarly.

Proposition 1 provides new insights for competition policy. In our setup, an increase in signal precision leads to lower consumer prices under competition due to best-response asymmetries. If signals are perfect, both competitive prices and the likelihood of collusive behavior are lowest. Either effect benefits consumers. We know from the analysis above that an increase in signal quality from a relatively low level facilitates collusion. In this area, any policy that deregulates access to or usage of consumer data resulting in a gain in predictive power of firms' algorithms¹⁷ can also support collusive behavior. In particular, regulators should be alarmed if such deregulation is demanded by the industry. While a single firm always gains from an increase in its predictive power, an increase of all firms' predictive power drives down competitive payoffs. Deregulation, however, might enable firms to coordinate their prices. From a relatively high level, an increase in predictive power impede collusive behavior. In this area, any policy concerned with consumer privacy that restricts predictive power of firms can come at the cost of collusive behavior.

3.2. Uniform Prices

The above case nests the scenario in which firms are not allowed to price-discriminate, because outcomes are the same as in the situation in which signals are uninformative (i.e., $\sigma = 1/2$). Hence, punishment payoffs reduce to

$$\pi_u^* = \frac{\tau}{2},$$

collusive payoffs to

$$\pi_u^c = \pi^c = \frac{1}{2} - \frac{\tau}{4},$$

¹⁷To a certain extent, it seems natural to assume a positive relation between the amount and variety of available data and predictive power. Yoganarasimhan (2017) provides evidence for this relation in the context of search queries. She finds that personalized search, especially long-term and across-session, helps to improve accuracy of suggested results significantly.

and deviation payoffs to

$$\pi_u^d = \begin{cases} 1 - \frac{3\tau}{2} & \text{if } 0 \leq \tau \leq \frac{2}{7}, \\ \frac{1}{8} + \frac{\tau}{32} + \frac{1}{8\tau} & \text{if } \frac{2}{7} < \tau \leq \frac{2}{3}. \end{cases}$$

Given these payoffs and Condition (2), it immediately follows that the critical discount factor is given as

$$\bar{\delta}_u = \begin{cases} \frac{2-5\tau}{4(1-2\tau)} & \text{if } 0 \leq \tau \leq \frac{2}{7}, \\ \frac{2-3\tau}{2+5\tau} & \text{if } \frac{2}{7} < \tau \leq \frac{2}{3}. \end{cases}$$

By construction, $\bar{\delta}_u$ is independent of σ . It decreases in the transport-cost parameter, that is, $\partial\bar{\delta}_u/\partial\tau < 0$, as established in Chang (1991).

3.3. Comparison

We can now compare the critical discount factors in the two scenarios, namely price discrimination and uniform prices. Profits are to a large extent affected differently by the possibility to price-discriminate. Figure 2 illustrates for all permissible parameter values of signal quality and transport costs when the two critical discount factors coincide.

When signal quality does not provide any information (i.e., $\sigma = 1/2$), price discrimination is not feasible. Hence, the critical discount factors are equal. When signal quality is perfect (i.e., $\sigma = 1$), we know from Liu and Serfes (2007) and Helfrich and Herweg (2016) that the linear city is divided into two distinguishable markets. Then, collusion is harder to sustain under price discrimination than uniform prices.

For $\sigma \in (1/2, 1)$, there is a non-monotonic relationship of the critical discount factor in signal quality under price-discrimination as stated in Proposition 1. In particular, starting from $\sigma = 1/2$, the critical discount factor first decreases and then after a cut-off increases in signal quality. At the same time, the critical discount factor under uniform pricing remains unchanged. The corollary below immediately follows.

Corollary 1. *For any $\tau \in (0, 2/3)$, there exists a threshold $\tilde{\sigma}(\tau) \in (1/2, 1)$ such that for $\sigma = \tilde{\sigma}(\tau)$, we have $\bar{\delta} = \bar{\delta}_u$. Moreover, for any $\sigma \leq \tilde{\sigma}(\tau)$, it holds true that $\bar{\delta} \leq \bar{\delta}_u$.*

From the above corollary, we can derive the following policy implications. A ban on price discrimination facilitates collusion as in Liu and Serfes (2007) and Helfrich and Herweg (2016) as long as signal quality is relatively high. Else, we find that a ban on price discrimination hinders collusion.

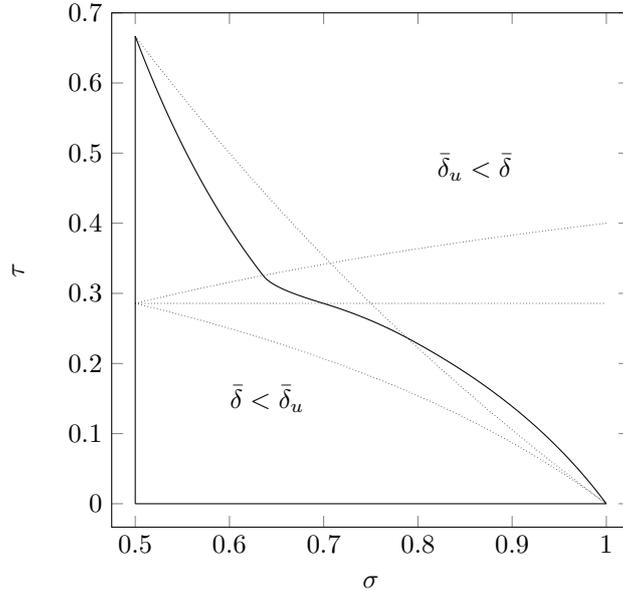


Figure 2: Comparison of the critical discount factors with and without price discrimination for all permissible parameter values.

Note: For those parameter combinations represented by the solid lines, the two critical discount factors coincide, that is, $\bar{\delta} = \bar{\delta}_u$. The dotted lines separate the different regions with respect to the deviation strategies for the cases with and without price discrimination.

4. Robustness

In this section, we test the robustness of our main results by relaxing some of the assumptions imposed on the signal structure. In particular, we consider the cases of asymmetric signal quality and correlated signals.

4.1. Asymmetric Signal Quality

In this subsection, we relax the assumption of symmetric information accuracy. Similar to Esteves (2014), we assume that the signal a firm receives is a function of the respective consumer's preference. We consider the following case: The signal a firm receives when facing a loyal consumer is weakly more precise than the signal it receives when facing a disloyal consumer. Let us denote the probability that the signal is correct if the consumer is loyal (disloyal) by σ_1 (σ_2) and assume that $1/2 \leq \sigma_2 \leq \sigma_1$. Thereby, we address the concern that a firm might know most about the characteristics of its loyal consumers and hence should be able to identify these with higher probability, which can

also be interpreted as a short-cut approach to modeling consumers who live for more than a single period and firms which have access to an imperfect tracking technology similar to the one defined in Colombo (2016).

Consider the set S , which contains all possible signal tuples (s_j, s_k) , and let $f(s_j, s_k|x \in l)$ denote the joint probability density function conditional on consumer x 's preference $l \in \{L, R\}$. We impose the following assumption on the functional form of $f(\cdot)$:

Assumption 2.

$$f(s_j, s_k|x \in L) = \begin{cases} \sigma_1\sigma_2 & \text{for } (s_L, s_L), \\ \sigma_1(1 - \sigma_2) & \text{for } (s_L, s_R), \\ (1 - \sigma_1)\sigma_2 & \text{for } (s_R, s_L), \\ (1 - \sigma_1)(1 - \sigma_2) & \text{for } (s_R, s_R), \end{cases}$$

and

$$f(s_j, s_k|x \in R) = \begin{cases} (1 - \sigma_2)(1 - \sigma_1) & \text{for } (s_L, s_L), \\ (1 - \sigma_2)\sigma_1 & \text{for } (s_L, s_R), \\ \sigma_2(1 - \sigma_1) & \text{for } (s_R, s_L), \\ \sigma_2\sigma_1 & \text{for } (s_R, s_R). \end{cases}$$

The density function under Assumption 2 is well-defined and nests the extreme case of symmetric signals (for $\sigma_1 = \sigma_2 = \sigma$). As before, after observing signal s_i , firm i has to infer on the consumer's actual preference and on the signal s_j received by its competitor. Suppose firm i receives signal s_L . Applying Bayes' rule, its updated belief that a consumer prefers firm A , and its competitor has received the same signal is

$$\Pr(s_L, L|s_L) = \frac{f(s_L, s_L|L) \Pr(L)}{f_{s_i}(s_L|L) \Pr(L) + f_{s_i}(s_L|R) \Pr(R)} = \frac{\sigma_1\sigma_2}{1 + \sigma_1 - \sigma_2},$$

where f_{s_i} denotes the marginal distribution of s_i . In the remaining cases, beliefs are updated analogously. Given beliefs, we can specify each firm's maximization problem and determine mutual best responses similarly to the main analysis (see the Appendix). Firms optimally set prices equal to

$$p_{A,L}^* = p_{B,R}^* = \frac{2\tau\sigma_1}{\sigma_2 + 2\sigma_1\sigma_2} \quad \text{and} \quad p_{A,R}^* = p_{B,L}^* = \frac{\tau}{\sigma_2 + 2\sigma_1\sigma_2},$$

where $p_{A,R}^* < p_{A,L}^*$ and $p_{B,L}^* < p_{B,R}^*$ as long as the signal is informative. The

resulting equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau(1 + 4\sigma_1^2)}{2\sigma_2(1 + 2\sigma_1)^2}.$$

These payoffs serve as punishment payoffs in the dynamic game as defined in Section 2 and equal those derived in Section 3 for $\sigma_1 = \sigma_2 = \sigma$ by construction. The intuition from the symmetric case can be misleading here by suggesting a similar relation between punishment payoffs and average signal quality. In fact, we observe that the more asymmetric the signal quality is, the higher the punishment payoffs are—namely, they rise in σ_1 and fall in σ_2 . When σ_1 increases, firms can better identify their loyal consumers allowing for an increase of their price. On the other hand, when σ_2 decreases, firms more often misrecognize their disloyal consumers leading to less aggressive poaching, as costly mistakes become more likely. Overall, signal asymmetry softens competition. As deviation payoffs are affected in the same way (see the proof of Proposition 2), it is not clear from an ex-ante perspective how signal asymmetry translates into the critical discount factor $\bar{\delta}_{\text{asy}}$. The following proposition summarizes our result:

Proposition 2. *For any $\sigma_2 < \sigma_1$, the critical discount factor $\bar{\delta}_{\text{asy}}$ is strictly larger compared to both cases of symmetric signal quality $\sigma = \sigma_1$ and $\sigma = \sigma_2$. In addition, $\bar{\delta}_{\text{asy}}$ is non-monotone in σ_1 and σ_2 .*

Proof. See Appendix A.

By construction, the critical discount factors in the symmetric and asymmetric case are equivalent for $\sigma_1 = \sigma_2 = \sigma \in [1/2, 1]$. Starting from $\sigma_1 = \sigma_2 = 1/2$, we can see from the proof of Proposition 2 that a marginal increase in both dimensions leads to a marginal reduction of $\bar{\delta}_{\text{asy}}$. From continuity and Proposition 1, it immediately follows that we can always find $1/2 \leq \sigma_2 < \sigma_1$, such that $\bar{\delta}_{\text{asy}} < \bar{\delta}$. Then, collusion is more likely in terms of set inclusion if price discrimination is permitted compared to the case of no price discrimination. The corollary below summarizes this argument:

Corollary 2. *For $\sigma_2 < \sigma_1$, a ban on price discrimination helps to fight collusion if signals are sufficiently noisy.*

4.2. Correlated Signals

In this subsection, we relax the assumption of independent signal realizations by allowing for positive correlation of the private signals received by the firms. This is natural, as firms might, for instance, use similar algorithms in order

to infer on consumer types from available data or obtain consumer data from similar sources.

Consider the set of all signal tuples S and let $g(s_j, s_k|x \in l)$ denote the joint probability density function conditional on consumer x 's preference $l \in \{L, R\}$. We assume the following functional form of $g(\cdot)$:

Assumption 3.

$$g(s_j, s_k|x \in L) = \begin{cases} \sigma^2 + \gamma & \text{for } (s_L, s_L), \\ \sigma(1 - \sigma) - \gamma & \text{for } (s_L, s_R), (s_R, s_L), \\ (1 - \sigma)^2 + \gamma & \text{for } (s_R, s_R), \end{cases}$$

and

$$g(s_j, s_k|x \in R) = \begin{cases} (1 - \sigma)^2 + \gamma & \text{for } (s_L, s_L), \\ \sigma(1 - \sigma) - \gamma & \text{for } (s_L, s_R), (s_R, s_L), \\ \sigma^2 + \gamma & \text{for } (s_R, s_R), \end{cases}$$

where $\gamma \in [0, \sigma(1 - \sigma)]$ measures the degree of correlation.

The density function under Assumption 3 is well-defined and nests the two extreme cases: (i) independent signals (for $\gamma = 0$) and (ii) perfectly correlated signals (for $\gamma = \sigma(1 - \sigma)$). The second case is equivalent to a model with imperfect public information about consumer preferences. It is easily checked that the interval $[0, \sigma(1 - \sigma)]$ is non-empty for $\sigma \in [1/2, 1)$. In the following, we solve for the Bayesian Nash equilibrium of the stage game. As before, after observing signal s_i , firm i has to infer on the consumer's actual preference and on the signal of its competitor. For illustration, suppose that firm i receives signal s_L . Applying Bayes' rule, its posterior belief that a consumer prefers firm A , and firm j receives the same signal is

$$\Pr(s_L, L|s_L) = \frac{g(s_L, s_L|L) \Pr(L)}{g_{s_i}(s_L|L) \Pr(L) + g_{s_i}(s_L|R) \Pr(R)} = \sigma^2 + \gamma,$$

where g_{s_i} denotes the marginal distribution of s_i . In the remaining cases, beliefs are updated similarly. Given beliefs, we can specify each firm's maximization problem and determine mutual best responses analogously to the main analysis (see the Appendix). Firms optimally set prices equal to

$$p_{A,L}^* = p_{B,R}^* = \frac{\tau(\gamma + 2\sigma^2)}{\sigma(2\gamma + \sigma + 2\sigma^2)} \quad \text{and} \quad p_{A,R}^* = p_{B,L}^* = \frac{\tau(\gamma + \sigma)}{\sigma(2\gamma + \sigma + 2\sigma^2)},$$

where $p_{A,L}^* < p_{A,R}^*$ when the signal is informative. Resulting equilibrium pay-

offs for each firm are

$$\pi^* = \frac{\tau (2\gamma^2 + 2\gamma\sigma(2\sigma + 1) + 4\sigma^4 + \sigma^2)}{4\sigma (2\gamma + 2\sigma^2 + \sigma)^2}.$$

These payoffs are the punishment payoffs in the dynamic game as defined in Section 2. By construction, punishment payoffs are equal to those derived in Section 3 for $\gamma = 0$. Furthermore, we observe that these payoffs fall as γ is rising, that is, gains from collusion are higher. As collusive prices are set uniformly and hence optimal deviation only depends on a firm's private signal, collusive and deviating payoffs remain unchanged compared to the symmetric-signal case. We therefore arrive at the following proposition:

Proposition 3. *For any $\gamma > 0$, the critical discount factor $\bar{\delta}_{\text{cor}}$ is strictly lower compared to the case of independent signal quality σ . In addition, $\bar{\delta}_{\text{cor}}$ is non-monotone in σ .*

Proof. See Appendix A.

At the lower and upper bound of σ , the cases of correlated and independent signals are equivalent by construction and hence the critical discount factors are equal. The following corollary directly results from Propositions 1 and 3:

Corollary 3. *For any $\gamma > 0$, the probability that a ban on price discrimination facilitates collusion is strictly lower compared to the case of independent signal quality σ . Furthermore, the difference strictly increases in γ .*

5. Conclusion

The use of big data—especially consumer data—for pricing strategies has substantially increased in recent times. Big data predictions of consumer preferences have been improving tremendously. Imprecision, however, is still an important factor when firms make their pricing decisions.

In this paper, we focus on the impact of data-driven price-discrimination strategies on the scope for tacit collusion. We find enhanced prediction of consumer preferences results in a U-shaped effect on firms' ability to sustain collusion. Compared to uniform pricing, we find that for low levels of predictive capabilities, collusion is easier to sustain under price discrimination. For sufficiently high levels, we find that collusion is harder to sustain under a discriminatory pricing than under uniform pricing. Thereby, potential misrecognition of consumers plays a crucial role.

Thereby, we provide the following policy implications. Data regulation should take into account adverse effects on competition. In particular, deregulation of access to or usage of consumer data facilitates coordinated behavior of firms as long as initial predictions of consumer preferences are weak. In contrast, for relatively strong predictions, policies intending to restrict access to and usage of consumer data facilitate coordinated behavior among firms. Moreover, the effect of a legal ban on price discrimination on firms' ability to collude crucially depends on the quality of predictions. On a more general note and related to the above-mentioned aspect, one may argue that when the exchange of consumer data leads to higher signal precision towards perfect information, competition authorities should be less concerned with regard to collusive activity than in the case in which firms exchange data on prices, demands, etc. At the same time, the model we employ does not allow to draw conclusions with regard to welfare, as we do not take into account consumer preferences for privacy or other adverse effects due to discrimination of consumers.

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Appendix A

Proof of Lemma 2. Without loss of generality, suppose that firm B sets the collusive price and firm A deviates unilaterally. As firm B charges p^c regardless of its signal, we have both $\tilde{x}_1 = \tilde{x}_2$ and $\tilde{x}_3 = \tilde{x}_4$. Substituting this into Equations (3) and (4), firm A expects its demand conditional on receiving signal s_L to be

$$D_{A,L} = \sigma + 2(1 - \sigma) \left(\tilde{x}_1 - \frac{1}{2} \right), \quad (6)$$

and its demand conditional on receiving signal s_R to be

$$D_{A,R} = (1 - \sigma) + 2\sigma \left(\tilde{x}_3 - \frac{1}{2} \right). \quad (7)$$

Then, the maximization problem of firm A is given as

$$\max_{p_{A,L}^d, p_{A,R}^d} \mathbb{E} [\pi_A^d] = \frac{1}{2} (p_{A,L}^d D_{A,L} + p_{A,R}^d D_{A,R}),$$

with $p_{B,L} = p_{B,R} = p^c$. Taking first order conditions with respect to firm A 's deviation prices, we get inner solutions

$$p_{A,L}^{d*} = \frac{1}{2} + \frac{\tau(3\sigma - 1)}{4(1 - \sigma)} \quad \text{and} \quad p_{A,R}^{d*} = \frac{1}{2} - \frac{\tau(3\sigma - 2)}{4\sigma}.$$

Using these, we make the following observations:

- $\tau > \frac{2(1 - \sigma)}{5 - 3\sigma} =: \tau_1 \implies \tilde{x}_1 < 1,$
- $\tau > \frac{2\sigma}{2 + 3\sigma} =: \tau_2 \implies \tilde{x}_3 < 1,$
- $\tau < \frac{2(1 - \sigma)}{1 + \sigma} =: \tau_3 \implies p_{A,L}^{d*} < p^c,$

where $p^c = 1 - \tau/2$. Thereby, it holds that $\tau_3 > \tau_2$ if and only if $\sigma < 1/\sqrt{2}$. Consequently, for $\sigma < 1/\sqrt{2}$, we obtain the order of parameters $0 < \tau_1 < \tau_2 < \tau_3 < 2/3$. On the other hand, for $\sigma > 1/\sqrt{2}$, we obtain the order of

parameters $\tau_1 < \tau_3 < \tau_2 < 2/3$. In the following, we determine the optimal deviation behavior of firm A conditional on τ by distinguishing the following five cases:

Case (i): For $\tau \leq \tau_1$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 = \tilde{x}_3 = 1$ in order to take over the whole market, that is, $p_{A,L}^d = p_{A,R}^d = 1 - 3\tau/2$. Thereby, its conditional expected demand as defined in Equations (6) and (7) is equal to $1/2$ regardless of the signal. Then, the expected payoff from deviating is given by

$$\pi_A^d = 1 - \frac{3\tau}{2}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $1/2 < \tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L}^d = p_{A,L}^{d*}$ and $p_{A,R}^d = 1 - 3\tau/2$. Substituting this into Equations (6) and (7), we get an expected deviation payoff of

$$\pi_A^d = \frac{(3\tau(1 + \sigma) + 2(1 - \sigma))^2 - 32\tau^2}{32\tau(1 - \sigma)}.$$

Case (iii): Suppose $\sigma \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L}^d = p_{A,L}^{d*}$ and $p_{A,R}^d = p_{A,R}^{d*}$. Substituting this into Equations (6) and (7), we get an expected deviation payoff of

$$\pi_A^d = \frac{\tau}{8\sigma(1 - \sigma)} + \frac{4(\tau + 1) - 15\tau^2}{32\tau}.$$

Case (iv): For now suppose $\sigma > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L} = p^c$ and $p_{A,R}^d = 1 - 3\tau/2$. By Assumption 1, firm A does not find it profitable to charge more than p^c from its loyal consumers as long as firm B uniformly charges p^c . Substituting this into Equations (6) and (7), we get an expected deviation payoff of

$$\pi_A^d = \frac{2 - 3\tau + \sigma(2 - \tau)}{4}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L} = p^c$ and $p_{A,R}^d = p_{A,L}^{d*}$. For the same reason as before, by Assumption 1, firm A charges p^c from its loyal consumers as long as firm B uniformly charges p^c . Substituting this into

Equations (6) and (7), we get an expected deviation payoff of

$$\pi_A^d = \frac{\tau^2(2 - \sigma)^2 + 4\sigma^2(\tau + 1) + 8\tau\sigma(1 - \tau)}{32\tau\sigma}.$$

Now, it is straightforward to check that for both prices and deviation payoffs their respective left-hand and right-hand limits for σ and τ approaching the bounds of Case (i)–(iv) from above are equal. Hence, they are continuous.

Further, it is straightforward to check that there are no kinks in both prices and deviation payoffs since the respective left-hand and right-hand limits of their derivatives for σ and τ approaching the bounds of Cases (i)–(iv) are equal. Hence, they are differentiable. \square

Proof of Proposition 1. Taking the collusive payoffs from Lemma 1, the deviation payoffs from Lemma 2 and the punishment payoffs as given in Section 3.1, we can solve for the critical discount factor as defined in Condition (2). As only the functional form of the deviation payoff is changing with τ , we distinguish the five cases as defined in the proof of Lemma 2, that is:

Case (i): For $\tau \leq \tau_1$, we get

$$\bar{\delta} = \frac{\sigma(2\sigma + 1)^2(2 - 5\tau)}{(2\sigma + 1)^2(4\sigma - 6\sigma\tau) - 2(4\sigma^2 + 1)\tau}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we get

$$\bar{\delta} = \frac{2\sigma(2\sigma + 1)^2((\sigma(3\tau - 2) + 3\tau + 2)^2 + (8\tau - 16)(\tau - \sigma\tau) - 32\tau^2)}{2\sigma(2\sigma + 1)^2((\sigma(3\tau - 2) + 3\tau + 2)^2 - 32\tau^2) - 32(4\sigma^2 + 1)\tau(\tau - \sigma\tau)}.$$

Case (iii): Suppose $\sigma \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we get

$$\bar{\delta} = \frac{((\sigma(3\tau - 2) + 3\tau + 2)^2 + (8\tau - 16)(\tau - \sigma\tau) - 32\tau^2)}{((\sigma(3\tau - 2) + 3\tau + 2)^2 - 32\tau^2) - 32(4\sigma^2 + 1)\tau(\tau - \sigma\tau)}.$$

Case (iv): Now suppose $\sigma > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we get

$$\bar{\delta} = \frac{\sigma(2\sigma + 1)^2(\sigma(\tau - 2) + 2\tau)}{(\sigma(\sigma(4\sigma(\sigma + 4) + 21) + 3) + 2)\tau - 2\sigma(\sigma + 1)(2\sigma + 1)^2}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we get

$$\bar{\delta} = \frac{(2\sigma + 1)^2(\sigma(\tau + 2) - 2\tau)^2}{C},$$

where $C := 4\tau(\sigma^2(2\sigma + 3)^2 + \sigma\tau) + (\sigma^2(4(\sigma - 11)\sigma - 95) - 12)\tau^2 + 4\sigma^2(2\sigma +$

$1)^2 + 8\sigma\tau$. From continuity and differentiability of all payoff functions entering Condition (2)—namely π^*, π^c, π^d —continuity and differentiability of $\bar{\delta}$ with respect to σ and τ immediately follows.

In order to do comparative statics, we take the derivative of $\bar{\delta}$ as defined in Condition (2) with respect to σ , that is

$$\frac{\partial \bar{\delta}}{\partial \sigma} = \frac{\frac{\partial \pi^d}{\partial \sigma} (\pi^c - \pi^*) + \frac{\partial \pi^*}{\partial \sigma} (\pi^d - \pi^c)}{(\pi^d - \pi^*)^2}.$$

We observe that

$$\frac{\partial \bar{\delta}}{\partial \sigma} \geq 0 \Leftrightarrow \frac{\partial \pi^d}{\partial \sigma} (\pi^c - \pi^*) + \frac{\partial \pi^*}{\partial \sigma} (\pi^d - \pi^c) \geq 0.$$

Exploiting this, we show that $\partial \bar{\delta} / \partial \sigma|_{\sigma=1/2} < 0$ and $\partial \bar{\delta} / \partial \sigma|_{\sigma=1} > 0$ in all relevant cases. Figure 1 nicely illustrates which parameter ranges of τ have to be considered for the respective extreme value of σ . For $\sigma = 1/2$, we have $\tau_1 = \tau_2 = 2/7 < \tau_3 = 2/3$. For $\sigma = 1$, we have $\tau_1 = \tau_3 = 0 < \tau_2 = 2/5$. We obtain the following:

- If $\sigma = 1/2$, we observe that
 - for $\tau \in \left(0, \frac{2}{7}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1/2} = \frac{4\tau(5\tau-2)}{(4-8\tau)^2} < 0$,
 - for $\tau \in \left(\frac{2}{7}, \frac{2}{3}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1/2} = -\frac{32\tau^2}{(5\tau+2)^2} < 0$.
- If $\sigma = 1$, we observe that
 - for $\tau \in \left(0, \frac{2}{5}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1} = \frac{9(3\tau-2)}{46\tau-36} > 0$,
 - for $\tau \in \left(\frac{2}{5}, \frac{2}{3}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma}|_{\sigma=1} = \frac{24(\tau-2)\tau(\tau(149\tau-4)-108)}{(\tau(143\tau-108)-36)^2} > 0$.

Hence, $\bar{\delta}$ is non-monotonic with respect to σ .

In order to do comparative statics of $\bar{\delta}$ with respect to τ , we apply the implicit function theorem to the binding case of Inequality (1). We get

$$\frac{\partial \bar{\delta}}{\partial \tau} = \frac{\frac{\partial}{\partial \tau} (\pi^d - \pi^c) + \frac{\partial}{\partial \tau} (\pi^* - \pi^d) \bar{\delta}}{\pi^d - \pi^*}.$$

Exploiting that $\pi^d > \pi^*$, the sign of the above expression only depends on the sign of the numerator. It is straightforward to verify that the numerator is strictly negative in Case (i)–(iv) as defined in the proof of Lemma 2. Only in Case (v) the sign of the numerator can change. Solving for τ , we get

$$\left(\frac{\partial \pi^d}{\partial \tau} - \frac{\partial \pi^c}{\partial \tau}\right) (1 - \bar{\delta}) + \bar{\delta} \frac{\partial (\pi^* - \pi^c)}{\partial \tau} < 0 \Leftrightarrow \tau < \frac{2\sigma(2\sigma + 1)^2}{\sigma(4\sigma(3\sigma + 5) - 5) + 2} =: \tilde{\tau}.$$

We observe that $\tilde{\tau} \in (\max\{\tau_2, \tau_3\}, 2/3)$. Given this, we conclude that for $\tau \in (\max\{\tau_2, \tau_3\}, \tilde{\tau})$, the numerator is negative and hence it holds true that $\partial\bar{\delta}/\partial\tau < 0$. For $\tau \in (\tilde{\tau}, 2/3]$, the numerator is positive and hence it holds true that $\partial\bar{\delta}/\partial\tau > 0$. Finally, the numerator is zero at $\tau = \tilde{\tau}$ and hence it holds true that $\partial\bar{\delta}/\partial\tau = 0$. \square

Proof of Proposition 2. As payoffs under collusion remain unchanged, we are left with determining punishment and deviation payoffs. Then, we compute the critical discount factor $\bar{\delta}_{\text{asy}}$. Finally, we show that the critical discount factor is always increasing in signal asymmetry compared to the symmetric case.

Lets first determine punishment payoffs. Given beliefs as derived in Section 4.1, we obtain expected demand of firm A conditional on receiving signals s_L and s_R , respectively, of

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_L) = \frac{1}{\sigma_1 + 1 - \sigma_2} \times \left(2\sigma_1\sigma_2\tilde{x}_1 + \sigma_1(1 - \sigma_2) + 2(1 - \sigma_1)\sigma_2 \left(\tilde{x}_2 - \frac{1}{2} \right) \right) \quad (8)$$

and

$$D_A(p_{A,L}, p_{B,L}, p_{B,R}|s_R) = \frac{1}{\sigma_2 + 1 - \sigma_1} \times \left(2(1 - \sigma_1)\sigma_2\tilde{x}_3 + (1 - \sigma_1)(1 - \sigma_2) + 2\sigma_2\sigma_1 \left(\tilde{x}_4 - \frac{1}{2} \right) \right), \quad (9)$$

with \tilde{x}_1 – \tilde{x}_4 referring to the indifferent consumers as defined in the main analysis. Firm A 's maximization problem is then defined as in Equation (5). Firm B 's maximization problem is determined analogously. Solving first-order conditions with respect to prices simultaneously, we get optimal prices

$$p_{A,L}^* = p_{B,R}^* = \frac{2\tau\sigma_1}{\sigma_2 + 2\sigma_1\sigma_2} \quad \text{and} \quad p_{A,R}^* = p_{B,L}^* = \frac{\tau}{\sigma_2 + 2\sigma_1\sigma_2},$$

where $p_{A,R}^* < p_{A,L}^*$ and $p_{B,L}^* < p_{B,R}^*$ as long as the signal is informative. The resulting equilibrium payoff for each firm amounts to

$$\pi^* = \frac{\tau(1 + 4\sigma_1^2)}{2\sigma_2(1 + 2\sigma_1)^2}.$$

Next, lets determine deviation payoffs. Without loss of generality, suppose that firm B sets the collusive price and firm A deviates unilaterally. As firm B charges p^c regardless of its signal, we have both $\tilde{x}_1 = \tilde{x}_2$ and $\tilde{x}_3 = \tilde{x}_4$. Substi-

tuting this into Equations (8) and (9), firm A expects its demand conditional on receiving signal s_L to be

$$D_{A,L} = \frac{1}{\sigma_1 + 1 - \sigma_2} \left(\sigma_1 + 2(1 - \sigma_1) \left(\tilde{x}_1 - \frac{1}{2} \right) \right), \quad (10)$$

and its demand conditional on receiving signal s_R to be

$$D_{A,R} = \frac{1}{\sigma_2 + 1 - \sigma_1} \left((1 - \sigma_2) + 2\sigma_2 \left(\tilde{x}_3 - \frac{1}{2} \right) \right). \quad (11)$$

Then, the maximization problem of firm A is given as

$$\max_{p_{A,L}^d, p_{A,R}^d} \mathbb{E} [\pi_A^d] = \frac{1}{2} \left(\frac{p_{A,L}^d D_{A,L}}{\sigma_1 + 1 - \sigma_2} + \frac{p_{A,R}^d D_{A,R}}{\sigma_2 + 1 - \sigma_1} \right),$$

with $p_{B,L} = p_{B,R} = p^c$. Taking first order conditions with respect to firm A 's deviation prices, we get inner solutions

$$p_{A,L}^{d*} = \frac{1}{2} + \frac{\tau(3\sigma_1 - 1)}{4(1 - \sigma_1)} \quad \text{and} \quad p_{A,R}^{d*} = \frac{1}{2} - \frac{\tau(3\sigma_2 - 2)}{4\sigma_2}.$$

Using these, we make the following observations:

- $\tau > \frac{2(1 - \sigma_1)}{5 - 3\sigma_1} =: \tau_1 \implies \tilde{x}_1 < 1,$
- $\tau > \frac{2\sigma_2}{2 + 3\sigma_2} =: \tau_2 \implies \tilde{x}_3 < 1,$
- $\tau < \frac{2(1 - \sigma_1)}{1 + \sigma_1} =: \tau_3 \implies p_{A,L}^{d*} < p^c,$

where $p^c = 1 - \tau/2$. The thresholds are ordered as $\tau_1 < \tau_2 < \tau_3$ if $\sigma_1 < (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 < 1/\sqrt{2}$. Else, thresholds are ordered as $\tau_1 < \tau_3 < \tau_2$. In the following, we determine the optimal deviation behavior of firm A conditional on τ by distinguishing the following five cases:

Case (i): For $\tau \leq \tau_1$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 = \tilde{x}_3 = 1$ in order to take over the whole market, that is, $p_{A,L}^d = p_{A,R}^d = 1 - 3\tau/2$. Thereby, its conditional expected demand as defined in Equations (10) and (11) is equal to $1/2$ regardless of the signal. Then, the expected payoff from deviating is given by

$$\pi_A^d = 1 - \frac{3\tau}{2}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L}^d = p_{A,L}^{d*}$ and $p_{A,R}^d = 1 - 3\tau/2$. Substituting this into Equations (10) and (11), we get an expected deviation payoff of

$$\pi_A^d = \frac{(3\tau(1 + \sigma_1) + 2(1 - \sigma_1))^2 - 32\tau^2}{32\tau(1 - \sigma_1)}.$$

Case (iii): Suppose $\sigma_1 < (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L}^d = p_{A,L}^{d*}$ and $p_{A,R}^d = p_{A,L}^{d*}$. Substituting this into Equations (10) and (11), we get an expected deviation payoff of

$$\pi_A^d = \frac{D}{32(\sigma_1 - 1)\sigma_2\tau},$$

where $D := 4(\sigma_1 - 1)(\tau(3\sigma_1 - 3\sigma_2 + 1)\sigma_2(-\sigma_1 + \sigma_2 + 1)) + \tau^2(9(\sigma_1 - 1)\sigma_2^2 - 3\sigma_1(3\sigma_1 + 2)\sigma_2 + 4\sigma_1 + 11\sigma_2 - 4)\sigma_2$.

Case (iv): Suppose $\sigma_1 \geq (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1 < \tilde{x}_3 = 1$, that is, $p_{A,L} = p^c$ and $p_{A,R}^d = 1 - 3\tau/2$. By Assumption 1, firm A does not find it profitable to charge more than p^c from its loyal consumers as long as firm B uniformly charges p^c . Substituting this into Equations (10) and (11), we get an expected deviation payoff of

$$\pi_A^d = \frac{2 - 3\tau + \sigma_1(2 - \tau)}{4}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we infer from our observations above that firm A optimally sets prices such that $\tilde{x}_1, \tilde{x}_3 < 1$, that is, $p_{A,L} = p^c$ and $p_{A,R}^d = p_{A,L}^{d*}$. For the same reason as before, by Assumption 1, firm A charges p^c from its loyal consumers as long as firm B uniformly charges p^c . Substituting this into Equations (10) and (11), we get an expected deviation payoff of

$$\pi_A^d = \frac{1}{32}(16\sigma_1 - 4(2\sigma_1 + 3)\tau + \frac{\sigma_2(2 - 3\tau)^2}{\tau} + \frac{4\tau}{\sigma_2} + 8).$$

Now, it is straightforward to check that for both prices and deviation payoffs their respective left-hand and right-hand limits for σ_1, σ_2 and τ approaching the bounds of Case (i)–(iv) from above are equal. Hence, they are continuous.

Further, it is straightforward to check that there are no kinks in both prices and deviation payoffs since the respective left-hand and right-hand limits of their derivatives for σ_1, σ_2 and τ approaching the bounds of Case (i)–(iv) are

equal. Hence, they are differentiable.

Taking the collusive payoffs from Lemma 1, the deviation payoffs from the above analysis and the punishment payoffs as given in Section 4.1, we can solve for the critical discount factor as defined in Equation (2). As only the functional form of the deviation payoff is changing with τ , we distinguish the five cases as defined in the proof of Lemma 2, that is:

Case (i): For $\tau \leq \tau_1$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1 + 1)^2 \sigma_2 (5\tau - 2)}{2(4\sigma_1^2 + 1)\tau + 2(2\sigma_1 + 1)^2 \sigma_2 (3\tau - 2)}.$$

Case (ii): For $\tau_1 < \tau \leq \min\{\tau_2, \tau_3\}$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1 + 1)^2 \sigma_2 (-15\tau^2 + \sigma_1^2(2 - 3\tau)^2 + 2\sigma_1(\tau + 2)(5\tau - 2) - 4\tau + 4)}{E},$$

where $E := (2\sigma_1 + 1)^2 \sigma_2 (4(\sigma_1 - 1)^2 + (9\sigma_1(\sigma_1 + 2) - 23)\tau^2 - 12(\sigma_1^2 - 1)\tau) + 16(\sigma_1 - 1)(4\sigma_1^2 + 1)\tau^2$.

Case (iii): Suppose $\sigma_1 < (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 \leq 1/\sqrt{2}$. For $\tau_2 < \tau \leq \tau_3$, we get

$$\bar{\delta}_{asy} = \frac{F}{G}$$

where $F := (2\sigma_1 + 1)^2 (\sigma_2 (4(\sigma_1 - 1)^2 + (\sigma_1(9\sigma_1 - 2) - 3)t^2 - 12(\sigma_1 - 1)^2\tau) - 4(\sigma_1 - 1)\tau^2 - (\sigma_1 - 1)\sigma_2^2(2 - 3\tau)^2)$, and $G := (2\sigma_1 + 1)^2 \sigma_2 (-11\tau^2 + \sigma_1^2(2 - 3\tau)^2 + 2\sigma_1(\tau + 2)(3\tau - 2) + 4\tau + 4 - (\sigma_1 - 1)\sigma_2(2 - 3\tau)^2) + 4(\sigma_1 - 1)(4\sigma_1(3\sigma_1 - 1) + 3)\tau^2$.

Case (iv): Suppose $\sigma_1 \geq (1 + \sigma_2)/(1 + 2\sigma_2)$ and $\sigma_2 > 1/\sqrt{2}$. For $\tau_3 < \tau \leq \tau_2$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1 + 1)^2 \sigma_2 ((\sigma_1 + 2)\tau - 2\sigma_1)}{2(4\sigma_1^2 + 1)\tau + (2\sigma_1 + 1)^2 \sigma_2 ((\sigma_1 + 3)\tau - 2(\sigma_1 + 1))}.$$

Case (v): For $\tau > \max\{\tau_2, \tau_3\}$, we get

$$\bar{\delta}_{asy} = \frac{(2\sigma_1 + 1)^2 (4\tau^2 - 4\sigma_2\tau(2\sigma_1(\tau - 2) + \tau + 2) + \sigma_2^2(2 - 3\tau)^2)}{H},$$

where $H := (2\sigma_1 + 1)^2 (\sigma_2^2(2 - 3\tau)^2 + \sigma_2\tau(16\sigma_1 - 4(2\sigma_1 + 3)\tau + 8)) - 16\sigma_1(3\sigma_1 - 1) + 3)\tau^2$. From continuity and differentiability of all payoff functions entering Condition (2)—namely π^* , π^c , π^d —continuity of $\bar{\delta}$ with respect to σ_1 , σ_2 and τ immediately follows.

Using this, we show that $\bar{\delta}_{asy} > \max\{\bar{\delta}(\sigma = \sigma_1), \bar{\delta}(\sigma = \sigma_2)\}$. For this to hold, it is sufficient that $\partial\bar{\delta}_{asy}/\partial\sigma_1|_{\sigma_1=\sigma_2=\sigma} > 0$ and $\partial\bar{\delta}_{asy}/\partial\sigma_2|_{\sigma_1=\sigma_2=\sigma} < 0$. Why is this? Starting from the symmetric case, asymmetry can be created by either

$\sigma_1 > \sigma$ or $\sigma_2 < \sigma$. Straightforward calculations immediately verify that the stepwise derivatives of $\bar{\delta}_{\text{asy}}$ actually satisfy the sufficient conditions.

Finally, we show that $\bar{\delta}_{\text{asy}}$ is non-monotonic in σ_1 and σ_2 . At $\sigma_1 = \sigma_2 = \frac{1}{2}$ and $\sigma_1 = \sigma_2 = 1$, we have $\bar{\delta}_{\text{asy}} = \bar{\delta}$ by construction. Hence, by exploiting Proposition 1, it is sufficient to show that $\bar{\delta}_{\text{asy}}$ is decreasing around the lower bound of its support. By evaluating the relevant cases, we obtain the following:

- for $\tau \in \left[0, \frac{2}{7}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma_1} \Big|_{\sigma_1=\sigma_2=\frac{1}{2}} + \frac{\partial \bar{\delta}}{\partial \sigma_2} \Big|_{\sigma_1=\sigma_2=\frac{1}{2}} = \frac{\tau(5\tau-2)}{4(1-2\tau)^2} < 0$,
- for $\tau \in \left(\frac{2}{7}, \frac{2}{3}\right]$, $\frac{\partial \bar{\delta}}{\partial \sigma_1} \Big|_{\sigma_1=\sigma_2=\frac{1}{2}} + \frac{\partial \bar{\delta}}{\partial \sigma_2} \Big|_{\sigma_1=\sigma_2=\frac{1}{2}} = -\frac{32t^2}{(5t+2)^2} < 0$,

where $\tau_1 = \tau_2 = 2/7$ and $\tau_3 = 2/3$ for $\sigma = 1/2$. By continuity of $\bar{\delta}_{\text{asy}}$, there exist $\sigma_1 > \sigma_2 \geq 1/2$, such that the above signs of the derivative continue to hold. \square

Proof of Proposition 3. As collusion and deviation payoffs remain unchanged, we are left with determining punishment payoffs. Then, we argue how δ_{asy} is affected.

Lets determine punishment payoffs. Given beliefs as derived in Section 4.2, we obtain expected demand of firm A conditional on receiving signals s_L and s_R , respectively, of

$$D_A(p_{A,L}, p_{B,L}, p_{B,R} | s_L) = 2(\sigma^2 + \gamma)\tilde{x}_1 + (\sigma(1 - \sigma) - \gamma) + 2((1 - \sigma)\sigma - \gamma) \left(\tilde{x}_2 - \frac{1}{2} \right)$$

and

$$D_A(p_{A,L}, p_{B,L}, p_{B,R} | s_R) = 2(\sigma^2 + \gamma) \left(\tilde{x}_4 - \frac{1}{2} \right) + 2((1 - \sigma)\sigma - \gamma)\tilde{x}_3 + ((1 - \sigma)^2 + \gamma),$$

with $\tilde{x}_1 - \tilde{x}_4$ referring to the indifferent consumers from above. Expected payoffs of firm A are then defined as in (5), and the decision problem of firm B is derived analogously. Solving first-order conditions with respect to prices simultaneously, we get optimal prices

$$\frac{\partial \pi^*}{\partial \gamma} = -\frac{(1 - 2\sigma)^2 \sigma \tau}{2(2\gamma + 2\sigma^2 + \sigma)^3} < 0 \quad \forall \gamma \in [0, \sigma(1 - \sigma)], \sigma \in \left(0, \frac{1}{2}\right), \tau \in \left(0, \frac{2}{3}\right].$$

We further observe, that collusion and deviation payoffs only depend on a firm's private signal and hence are defined as in Section 3. It follows immediately from the definition of the critical discount factor in 2 that the lower is the

punishment payoff, the less patient players have to be in order to sustain collusion. Hence, we conclude that for any σ and $\gamma > 0$, we have $\bar{\delta}_{\text{cor}} < \bar{\delta}$. In addition, $\bar{\delta}_{\text{cor}}$ is continuous in σ .

Finally, we show that $\bar{\delta}_{\text{cor}}$ is non-monotonic in σ . At $\sigma = 1/2$ and $\sigma = 1$, we have $\bar{\delta}_{\text{cor}} = \bar{\delta}$ by construction. Hence, from the above observations and Proposition 1, the non-monotonicity immediately follows. \square

Appendix B

In this section, we characterize an optimal penal code. The game and strategy profile is as described above except for punishment. In order to derive optimal penal codes, we first need to determine the *minmax* payoff of firm $i = 1, 2$ —the stick. Due to positive transport costs and strategic complementarity, the worst firm $j \neq i$ can do to i is charging $p^o := 0$ irrespectively of its private signal s_j . Given this, we can specify beliefs over consumers' preferences and the relevant indifferent consumers analogously to Section 3. Firm i faces the following optimization problem:

$$\max_{p_{i,L}, p_{i,R}} \mathbb{E}[\pi_i] = \frac{1}{2} \left(\sigma p_{i,L} \left(1 - \frac{p_{A,L}}{\tau} \right) + (1 - \sigma) p_{i,R} \left(1 - \frac{p_{A,R}}{\tau} \right) \right).$$

As the objective function is concave, the optimal solution is $p_{i,L} = p_{i,R} = \tau/2 =: p^{mx}$. Then, firm i 's *minmax* payoff is given by

$$\pi^{mx} = \frac{\tau}{8}.$$

Next, we have to make sure that it is incentive compatible for firm j to punish firm i after observing a deviation from charging the collusive price—the carrot. As punishment is costly for firm j , it has to be compensated after charging a zero price for T periods. In our game, the most efficient compensation is reversion to collusive behavior as defined in Lemma 1, which provides each firm with payoff π^c . First, we need to find the minimum amount of punishment periods T^* such that punishment is incentive compatible for any discount factor δ . Observing that punishment is most efficient if the deviator charges p^o as well throughout the respective T periods,¹⁸ we define the following punishment strategy profile:

- If firm j observes an unexpected deviation of firm i from p^c in any period t , both firms charge p^o in periods $t + 1$ to $t + T^*$. Then,

¹⁸One can easily verify, that the critical discount factor is strictly larger when allowing the deviator to receive *minmax* payoffs during punishment phase.

- if a firm deviates from p^o in any period $t' \in \{t+1, \dots, t+T^*\}$, both firms charge p^o in periods $t'+1$ to $t'+T^*$, and
- if there is no deviation from p^o throughout T^* periods, both firms charge p^c again.

To see why this is optimal, let's define T^* such that a firm is indifferent between the following scenarios: (i) receiving zero payoffs for T periods and afterwards receiving π^c for the rest of the game; and (ii) deviating to p^{mx} in period t , receiving zero payoffs for T periods and afterwards receiving π^c for the rest of the game. Hence, T^* solves

$$V^p = \pi^{mx} + \delta V^p,$$

where $V^p := 0 + \delta 0 + \dots + \delta^{T-1} 0 + \delta^T \pi^c$. The implicit solution is given by

$$\delta^{T^*} = \frac{\pi^{mx}}{\pi^c}.$$

As $\delta \in (0, 1)$, π^{mx} is bounded from above, and π^c is bounded away from zero, T^* is finite for any $\tau > 0$. We observe that the larger δ , the larger T^* . The intuition behind this trade-off is that the more patient firms are, the more tempted they are to trade π^{mx} in period t against delaying the future stream of π^c by a single period.

Finally, we substitute for the payoff stream from optimal penal codes V^p in Inequality (1) to get the following condition for OSDP to hold:

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta (\delta^{T^*} \pi^c)}{1-\delta}.$$

Substituting for the implicit characterization of T^* , we obtain

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^{mx}} =: \bar{\delta}_{mx}.$$

It is easily verified that $\bar{\delta}_{mx} < \bar{\delta}$ as $\pi^{mx} < \pi^*$ for all σ and τ .¹⁹ Since V^p is independent of signal quality, $\bar{\delta}_{mx}$ always rises in σ .

¹⁹Moreover, $\bar{\delta}_{mx}$ is lower compared to the critical discount factors in case of asymmetric signal quality and correlated signals.