

# Group Hug: Platform Competition with User-groups\*

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## Abstract

We consider platform competition in the presence of small users and a user-group. One platform enjoys a quality advantage and the other benefits from favorable beliefs. We study whether the group mitigates the users' coordination problem –i.e., joining a low-quality platform because they believe that other users would do the same. We find that when the group is sufficiently large to facilitate coordination on the high-quality platform, the group may choose to join the low-quality one. When the group joins the more efficient platform it does not necessarily increase consumer surplus. Specifically, a non-group user benefits from a group with an intermediate size, and prefers a small group over a large group. The utility of a group user is also non-monotonic in the size of the group.

**JEL Classification:** L1

**Keywords:** network extremities, coordination

## 1 Introduction

In markets with network effects, platforms aim at attracting a large number of users as a large consumer base allows for more interactions across users and thus gives users higher benefits. Users would like to join the same platform that other users adopt in order to benefit from network effects. This may create a coordination problem and inefficiencies where users join the “wrong” platform, e.g., a low-quality platform, simply

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because they expect that other users will do the same. In such a case, each user is too small to affect the decisions of other users, and as market expectations are for all other users to join the low-quality platform, all users do the same.

Yet, in many markets, some users join a platform as a group rather than each user joining individually. Moreover, platforms compete on small users as well as large users, where the latter can affect the former's decisions as to which platform to join. This raises the question of whether the presence of a user-group or a large user can mitigate users' coordination failure. This paper studies platform competition between a high and a low-quality platform. Because of customers' inertia or incumbency advantage, users expect that other users join the low-quality platform. A subset of users belong to a group that makes a collective decision about which platform to join. We ask when the presence of a user-group enhances efficiency, and how the size of the group affects profits and users' surplus of users inside and outside the group.

User-groups are common in many markets for platforms. For example, when launching iTunes, Steve Jobs first approached Warner Music, and other big labels, like Universal and Sony. Each of these big labels brought with it contracts with a large number of artists that joined the iTunes platform as a group.<sup>1</sup> Other examples include marketplace lenders who aim at attracting both private and large, institutional investors. These platforms connect investors and borrowers. Institutional investors are organizations that pool capital from many smaller investors and choose how to, collectively, invest on behalf of these investors. Investor composition on lending platforms like LendingClub and Prosper has significantly evolved since the platforms' inception in late 2000s. While, initially, investors were composed of only small private investors that were looking for new investment opportunities, nowadays institutional investors represent a large share of investors on these platforms. In the early days of marketplace lending, private investors were browsing borrowers and individually choosing the loans they want to fund and the amount they want to invest in each and every loan they fund. Institutional investors, in contrast, choose borrowers for the investors that are part of their fund and typically fund the entire loan. Likewise, a large restaurant chain like McDonalds would likely make a collective decision to all its franchisees whether to join the Apple Pay platform. In contrast, small merchants, such as family-owned restaurants, make an individual decision on whether to join Apple Pay. In the mar-

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<sup>1</sup><https://www.rollingstone.com/culture/culture-news/itunes-10th-anniversary-how-steve-jobs-turned-the-industry-upside-down-68985/>

ket for mobile operating systems, Apple and Google are competing on both small and large application developers. For example, EA Sports holds a high market share in the market for online sports gaming.<sup>2</sup> Likewise, GetTaxi is attempting to democratize the taxi rides market and may attract individual drivers or contract with large taxi providing companies. Note that user-groups are not necessarily very large. For example, a student club may choose to join a peer-to-peer (P2P) payment network (e.g., Venmo) in order to simplify money transfer across the club members. Furthermore, the same type of effects may arise with a large user, as opposed to a user group. For example, accredited investors (investors with net worth above \$1M) may invest large amounts on a lending platform, facilitating the same type of network effects that institutional investors generate.

In general, user-groups create high value to a platform as the group creates network effects not only to its members but also to users outside of the group. The group's choice of which platform to join then may affect the platform's ability to attract other users that do not belong to the group. For example, a popular artist who is part of the Warner Music group attracts users to iTunes and thereby creates value for other artists that are part of the Warner Music group as well as for other independent artists that join the iTunes platform. As such, iTunes' contract with Warner Music affected individual artists' and listeners' decision to join iTunes. Similarly, institutional investors bring with them large amount of capital which is highly desirable for the lending platform. An institutional investor's decision to join a specific marketplace lending platform makes it more likely that small individual investors as well as borrowers would join that same lending platform. It is, therefore, likely that platforms would compete over attracting user-groups to join their network. The student club decision to join Venmo creates value to all club members who can now easily transfer money to one another but also creates value to other users who can now easily transfer and/or receive payments from this group of college students.

A network structure where some users join as part of a group while others join individually brings up some interesting questions. In particular, does the student club's decision to join Venmo as opposed to Zelle, for example, affect students that are not part of the club choice of which P2P payment network to join? While users want to join the platforms other users join, if the student club is relatively small and chooses the non-focal platform, other students may opt to join the other platform they believe

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<sup>2</sup>For data on EA's market share, see: [https://csimarket.com/stocks/compet\\_glance.php?code=EA](https://csimarket.com/stocks/compet_glance.php?code=EA).

most other users would adopt. Furthermore, does the presence of user-groups affect the likelihood that the higher quality, yet maybe not focal platform, becomes the dominant platform? That is, does an institutional investor choice of a lending platform makes it easier for the market to coordinate on the “better” platform? Does the presence of large application developers make it easier for the market to coordinate on the better mobile operating system? Intuitively, if the group size is small, it might not have any effect on other users’ decisions. Yet, a large group may affect the decisions of individual users. In the latter case, the question is whether, when the group anticipates that its choice would affect the choice of individual users, it chooses the platform with the better quality. This question is important for policy towards the size of such large users. In particular, should antitrust authorities allow mergers between users – such as application developers or institutional investors – into a large user-group with market power to affect the identity of the dominant platform? In markets for networks, mergers between users may have the welfare-enhancing effect of facilitating coordination on the right platform. At the same time, one must consider the effects of a large user-group on small users outside the group, as well as on individual users inside the group.

Another interesting question is the incentives to be part of the user-group. While the user-group may achieve better prices from the platforms that try to attract it, as the size of the group increases the value the group gets is split over a larger number of users—decreasing the value of being part of the group as well as decreasing the group incentives to accept additional users. Finally, it is not clear how the answers to these questions change once users can multihome and join more than one platform.

To study these questions, we develop a model of platform competition in a market with network effects with two types of users: a set of users that join the platform as a group—a user-group, and individual users. We assume a game where platforms first compete over attracting the group and then set prices to attract the non-group users. Competition in prices implies that a platform may be willing to pay the user-group to become part of its network. In this setting, we examine the effect of the proportion of the user-group, out of the total size of the market, on market structure, profits, and market efficiency.

In order to capture the effect of network effects and users’ beliefs, we assume a market with two competing platforms where one has an advantage in terms of the value it offers at the outset while the other platform enjoys focality—meaning, users believe that it would be able to attract other users and become the dominant platform

in the market. As such, the two platforms differ in terms of the overall value they create and thus in terms of the value they are able and willing to share with the user-group. Note that since the platforms compete first over the user-group and then over the rest of the users, beliefs about focality affect only the decision of the non-group users.

We find that there is a threshold group size where groups that are smaller than the threshold cannot affect the equilibrium winning platform. The intuition is simple. As long as the group is a small proportion in comparison to the size of the market, it does not create large enough network effects to attract the non-group users and thus to affect their decision. Thus, the focal platform always wins the non-group users. If, however, the user group is larger than the threshold, the group creates large network effects that affect the decision of the non-group users. Specifically, if the non-focal platform is able to attract the group, it also wins the non-group users. In this latter case, we say that the group is *pivotal*. The cutoff above which the group is pivotal decreases with the quality advantage that platform enjoys—the larger the quality gap across the platform, the smaller the group size needed to make the group pivotal.

These results imply that a proportionally large user-group has the ability to facilitate coordination on the more efficient platform and thereby solve the coordination failure. This, however, does not necessarily imply that a pivotal group joins the high-quality platform. We find that under some market conditions, a pivotal group chooses to join the low-quality focal platform, dragging non-group users to join the low-quality platform as well. Only when the pivotal group is sufficiently large, it facilitates coordination on the high-quality platform. The intuition for this result is that the focal platform can extract the network effects that non-group users gain from both group users and non-group users. However, the non-focal platform can only extract the former value. Hence, when the group is not too large, the focal platform can transfer this benefit to the group, making it more beneficial for the group to join the low-quality platform, on the expense of the non-group users. When size of the group is large enough, the group places a higher emphasis on the superior quality of the non-focal platform, and a lower emphasis to the network effects that the focal platform extracts from non-group users. In this case, the group joins the more efficient platform.

A large group may not necessarily increase consumer surplus. Specifically, we find that the utility of a non-group user increases when an increase in the size of the group motivates the group to switch from the low-quality to the high-quality platform. Then, a further increase in the size of the group decreases the utility of a non-group user,

because the platform extracts from non-group users the network effects that they gain from meeting the group. While they can still get the network effects they gain from meeting other non-group users, the share of non-group users is now smaller. In total, a non-group user prefers a small group over a large group, and prefers the most an intermediate group size.

As for the utility of a group user, this too is non-monotonic in the size of the group. When the group is small yet not pivotal, an increase in the size of the group increases consumer surplus as the group-users get the network effects they generate to one another. Once the group becomes large enough to be pivotal, the two platforms compete more aggressively to attract it—allowing the group-users to get more of the overall value created in the market. Yet, when most consumers belong to the group, platforms do not place large emphasis on winning the non-group and the utility of each group member decreases as the groups’ gains from attracting the non-group users decreases. Under some parameter values, an increase in the size of the group may have a steeper negative effect on group users than on non-group users. As a result, group users may gain lower utility than non-group users.

Combining the utility of group and non-group users, an increase in the size of the group has conflicting effects on total consumers’ surplus and the profits of the two platforms. Yet, total welfare increases discontinuously with the proportion of the group, at the point where the group switches from the low-quality to the high-quality platform. The share of group-users required for the market to get to this point decreases with the gap in qualities across the two platforms: the larger the gap, the smaller the group needs to be in order for the market to achieve the maximum value in the vertical chain.

For policy, these results suggest that user mergers, or large users, can indeed mitigate users’ coordination problem. However, antitrust authorities should not adopt a too lenient approach towards user-merger for two reasons. First, a large user group may not facilitate coordination, even when it has the ability to do so. Second, a large user group can indirectly extract utility from non-group users, through the subsidy offered by the platform, which in total can harm consumers. Qualitatively, intermediate sized user-groups seem to have positive effects on both non-group and group users. However, large user groups can be harmful, sometimes to both types of users.

Previous literature on platform competition focused mainly on small buyers. Cailaud and Jullien (2001; 2003) consider competition between homogeneous platforms—an incumbent and an entrant—in a two-sided market. The incumbent platform benefits

from a focality advantage: if there is an equilibrium in which consumers join it, then consumers play this equilibrium, even if there is a second equilibrium in which consumers join the entrant.<sup>3</sup> They find that the non-focal platform adopts a “divide-and-conquer” strategy, in order to overcome its non-focal position, where it attracts one of the sides by subsidizing it, and charges a high price from the other side. In turn, the focal platform does the same, resulting in an uneven competition on the two sides. Still, the focal platform earns positive profits, even though at the outset the platforms are homogeneous. Hagiu (2006) extends the focality approach to a setting where platforms first compete on one of the sides and then compete on the other side. When platforms compete on the first side, each user on this side expects that other users on the same side join the focal platform. The paper studies whether platforms may want to commit to prices for the second side at the same time they announces prices to the first side. Jullien (2011a) extends the focality approach to a multi-sided market, when one of the platforms offers a superior base quality. He finds that a focal platform can dominate the market even when competing against a higher-quality platform. This result occurs when focality advantage outweighs the platform’s quality advantage. Halaburda and Yehezkel (2013) consider focality advantage when users are ex-ante uninformed about their benefits from joining a platform, and become privately informed once they join. They find that the combination of focality advantage and asymmetric information result in a market failure, where the presence of competition motivates the focal platform to distort the level of trade between users downward. Hałaburda and Yehezkel (2016) extends the concept of focality to a partial degree of focality. Accordingly, users adjust their beliefs concerning the winning platform depending on the price gap between the two platforms. They show how the degree of focality affects the platforms’ divide-and-conquer strategies and how focality enables a low-quality platform to win the market.

In the context of a dynamic game, Halaburda et al. (2017) consider a repeated platform competition between a high and a low quality platform. Users take the winning platform in the previous period to be focal in the current period. They find that dynamic considerations may increase the ability of a low-quality platform to maintain its focal position. Biglaiser and Crémer (2018) consider dynamic platform competition on two groups of consumers that differ in their network effects. At the beginning of each period, users are attached to the platform that they joined in the previous period, and engage

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<sup>3</sup>In the terminology of Caillaud and Jullien, the incumbent benefits from “favorable beliefs”: consumers expect other consumers to join it, whenever it is rational for them to do so.

in a sequential, non-cooperative migration path to the winning platform in the current period.

A common feature of all of the above papers is that users are too small to affect the winning platform. Each user takes the focal position, and hence the decisions of other users, as given. We contribute to this literature by considering platform competition in a market with both a user-group (or a large user) and a set of small users. Two qualitative differences between our paper and the previous literature mentioned above are that (i) the user-group makes a coordinated collective decision about which platform to join for all of its members, and (ii) the user group takes into account the effect of its decision on the decision of the non-group users. We show how the presence and size of such strategic user-group help mitigate market inefficiencies the focality of a low quality platform creates.

Biglaiser et al. (forthcoming) survey reasons for incumbency advantage in platform competition, and speculate that a pivotal buyer may be a mitigating factor on a platform's incumbency advantage. Our paper contributes to this idea by identifying how a pivotal buyer may, in fact, preserve the incumbency advantage of the low-quality platform. Akerlof et al. (2018) consider a market with network effects, allowing both for a focality advantage and product differentiation, such that a non-focal platform gains a small, yet positive share of the market. In the context of a monopolistic platform, they show that an "influencer" buyer, that can affect the decisions of other buyers, can be in a pivotal position. We contribute to this paper by considering competition for the pivotal buyer, who internalizes the ability to affect the winning platform.

A second strand of relevant literature to our paper concerns entry deterrence and naked exclusion. Rasmusen et al. (1991) and Segal and Whinston (2000) consider competition between an incumbent and a more efficient entrant. The latter needs a sufficient scale (i.e., number of buyers) in order to enter the market. This creates a buyers' coordination problem: when buyers expect other buyers to join the incumbent, the entrant cannot profitably enter the market. In a closely related paper to ours, Karlinger and Massimo (2012) study naked exclusion when buyers differ in size. The paper focuses mainly on small buyers, such that no individual buyer is large enough to provide the entrant with the sufficient scale needed for entry. In this case, there are exclusionary equilibria in which the inefficient incumbent dominates the market. Yet, the paper finds that when there is a sufficiently large buyer that can meet the entrant's scale requirement, or when a group of buyers can take a joint decision, such inefficient

exclusionary equilibria do not arise. Consequently, in the context of naked exclusion, buyer-groups have pro-competitive effects. Our paper contributes to these findings by showing that in the alternative context of platform competition, a pivotal user-group may not choose the efficient platform. Moreover, an increase in the proportion of the user-group may decrease the utilities of users in and outside the group. The intuition behind the difference in results is that in the context of naked exclusion there are no direct network effects. Buyers' utility then depends on the decisions of other buyers only through the effect of other buyers' decision on the identity of the winning platform. In contrast, under platform competition with network effects, users gain direct positive network effects as more users join the platform. This enables the focal platform to win the group and non-group users even when the group is pivotal. More specifically, in our settings, the group internalizes the effect its decision has on non-group users as it can extract some of this value through the price it pays. In the case of naked inclusion, there are no incentives for the group to internalize the effect of its decision on other buyers.

## 2 The model

Consider two platforms, platform  $A$  and platform  $B$ , and a mass 1 of identical users.<sup>4</sup> The utility of a consumer from joining platform  $i$  ( $i = A, B$ ) is  $V_i(n_i) - p_i$ , where  $p_i$  is the price of platform  $i$  and  $n_i$  is the number of users in platform  $i$ .  $V_A(n)$  and  $V_B(n)$  are two (different) continuous and twice differentiable functions with the following features:

**Assumption 1:**  $V_i'(n) > 0$

**Assumption 2:**  $V_B(n) > V_A(n), \forall n \in [0, 1]$

**Assumption 3:**  $V_{A1} \equiv V_A(1) > V_{B0} \equiv V_B(0)$

Assumption 1 indicates that there are positive network effects: a consumer benefits the more other users are joining the same platform as the consumer does. For now, we allow for  $V_i(n)$  to be concave or convex. Later on, we show that our main results hold as long as  $V_i(n)$  is not too concave. Assumption 2 means that given the same number of users on each platform, platform  $B$  offers higher value than platform  $A$ .

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<sup>4</sup>Alternatively, one could assume a market with  $n_G$  group members and  $n_{NG}$  non-group users where  $n_G + n_{NG} \leq 1$ . In this case, however, changes in the size of the group are confounded with changes in network effects. Since we are interested in separating these two effects, we assume that  $n_G + n_{NG} = 1$ .

This higher value can be due to superior quality, such that  $V'_B(n) = V'_A(n)$ , and/or from platform's  $B$  superior ability to connect between users (i.e.,  $V'_B(n) > V'_A(n)$ ). Yet, by assumption 3, a consumer prefers to join platform  $A$  when all other users are joining it, over joining an empty platform  $B$ . That is, network effects (meeting other members) are more important to users than the quality gap between the two platforms. Finally, we normalize  $V_A(0) = 0$ .

Out of the mass of 1 users, a fraction  $x$  ( $0 \leq x \leq 1$ ) belong to a user-group. We can interpret  $x$  as a group of small users that makes a collective decision, such as institutional investors in marketplace lending. Alternatively,  $x$  can measure the relative size of a large user, such as a large application developer in the market for mobile operating systems. In order to focus on the effect of users making a collective rather than an individual decision, we assume that the per-user value of joining a platform is the same for users in the user-group and non-group users; i.e.,  $V_i(n)$ .<sup>5</sup> Hence, the utility of the entire group from joining platform  $i$  is  $xV(n_i) - p_i^G$ , where  $p_i^G$  is the price that platform  $i$  charges the group. The remaining users,  $1 - x$ , are “non-grouped”. We assume that  $x$  is exogenous, that users (both the group and non-group) can only join one platform (i.e., “single-home”), and that the group cannot divide its members between the two platforms.

The timing of the model is as follows. In the first stage, the two platforms compete by simultaneously setting prices to the group,  $p_A^G$  and  $p_B^G$ , and the group chooses a platform. We denote the group's decision by  $J = \{A, B\}$ . In the second stage, the two platforms compete by setting prices to the non-group users,  $p_A$  and  $p_B$ . In the third stage, non-group users observe  $J$ ,  $p_A$  and  $p_B$ , and decide simultaneously and non-cooperatively which platform to join. Notice that we assume that the platforms have market power on both the group and non-group users. Our qualitative results in sections 3 and 4 hold when the group has the market power to make take-it-or-leave-it offers to the two platforms (or any Nash-bargaining combination).

As is typically the case when markets exhibit network effects, expectations play an important role. Consequently, given some values of  $J$ ,  $p_A$  and  $p_B$ , the third stage may have two equilibria: one in which each non-group user expects that all other users

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<sup>5</sup>In our motivating examples, members of the group may have higher utility than regular users. For example, in marketplace lending, institutional investors may enjoy higher returns as compared to independent investors as they are more diversified or may have better information on the loans. Still, we assume that the platform offers identical value to the group and non-group users in order to focus on the net strategic effect of the size of the group.

join  $A$ , in which case everyone joins  $A$ . In the second equilibrium, for the same values of  $J$ ,  $p_A$  and  $p_B$ , all non-group users join platform  $B$ , expecting that other users will do the same. Users play one of these equilibria, based on their beliefs concerning the platforms' ability to attract other users. An incumbent platform, or a platform that dominated the market in the past, may benefit from favorable beliefs, as users may expect it to maintain its dominance and thus for other users to join it. These beliefs may make it difficult for a new, yet of higher quality, platform to gain a foothold in the market.<sup>6</sup> In order to model such beliefs advantage, in what follows, we assume that platform  $A$  is *focal*: whenever both outcomes are possible, non-group users join platform  $A$ , expecting that other users will do the same. Users join platform  $B$  only if it is a dominant strategy for them to do so. We assume that platform  $A$ 's focal position is independent of whether the group joined  $A$  or  $B$ . That is, the group is too small to affect users' beliefs.

To illustrate the role of focality, consider a benchmark case in which all users are non-group:  $x = 0$ . In stage 3, when users decide which platform to join given  $p_A$  and  $p_B$ , there is an outcome in which all users join platform  $A$  if:

$$V_{A1} - p_A \geq V_{B0} - p_B \iff V_{A1} - V_{B0} \geq p_A - p_B. \quad (1)$$

Likewise, there is an outcome in which all users join platform  $B$  if

$$V_{B1} - p_B \geq V_{A0} - p_A \iff p_A - p_B \geq V_{A0} - V_{B1}. \quad (2)$$

Since  $V_{A1} > V_{A0} = 0$  and  $V_{B0} < V_{B1}$ ,  $V_{A1} - V_{B0} > V_{A0} - V_{B1}$ , implying that for  $V_{A1} - V_{B0} > p_A - p_B > V_{A0} - V_{B1}$ , both outcomes are possible. By the assumption that platform  $A$  is focal, users join  $A$  if  $V_{A1} - p_A \geq V_{B0} - p_B$ , and join platform  $B$  if  $V_{A1} - p_A < V_{B0} - p_B$ , expecting that others will do the same. Focality can emerge because of incumbency advantage and users' inertia. If platform  $A$  was the first in the market, users may expect that other users will continue to join the old platform, even though there are better, new alternatives. These beliefs can be rational, given that high network effects keep users on platform  $A$ .<sup>7</sup>

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<sup>6</sup>We focus on outcomes in which all users join the same platform. All equilibria in which some users join platform  $A$  while others join platform  $B$  are not stable. To see why, notice that in such an equilibrium all users have to be indifferent between joining  $A$  or  $B$ . Hence, if a consumer of mass  $\varepsilon$  switches from platform  $i$  to  $j$ , then now all users gain a higher utility in platform  $j$  than in  $i$  and all users will switch.

<sup>7</sup>For a formal definition of focality, and its potential sources, see Halaburda and Yehezkel (2018).

We demonstrate the coordination problem by looking at the case where platform  $A$  is focal and  $x = 0$ . In this case, platform  $A$  always wins the market. To see why, platform  $A$  charges  $p_A$  such that equation (1) holds in equality, while the losing platform  $B$  charges the lowest price possible,  $p_B = 0$ . Substituting  $p_B = 0$  in (1), platform  $A$  charges  $p_A = V_{A1} - V_{B0} > 0$  and earns positive profit, where the inequality holds by Assumption 3. In a putative equilibrium in which the non-focal platform  $B$  wins, if such an equilibrium were to exist, platform  $B$  needs to charge  $p_B$  such that (1) holds in equality given  $p_A = 0$ , but then  $p_B = -(V_{A1} - V_{B0}) < 0$ , again from Assumption 3, implying that platform  $B$  cannot profitably win the market. We summarize these results in the following lemma:

**Lemma 1.** *(with no group, the inefficient platform always wins) When all users are non-group ( $x = 0$ ), the low-quality but focal platform  $A$  wins the market and earns  $V_{A1} - V_{B0}$ .*

Intuitively, focality means that platform  $A$  can collect the users' network effects because users expect that other users join  $A$ . Platform  $B$  can only collect its quality advantage. Yet, network effects are more important to users than the quality advantage, by Assumption 3, resulting in an equilibrium in which platform  $A$  wins. That is, the inability of users to coordinate their choices creates a miss-coordination in which they all join the inefficient platform. This raises the question of when and how the group can correct this market failure.

### 3 Competition on the non-group

We start by solving the second and third stages: platforms' competition on the non-group users, given that the group already joined a platform. The main result of this section is that a large group can determine which platform wins the non-group users. Yet, when the group is relatively small, the focal platform  $A$  always wins the non-group users. All proofs can be found in the appendix.

We study the two cases where the group joins platform  $A$  and platform  $B$  in turn. Suppose first that  $J = A$ . An equilibrium in which platform  $A$  wins the non-group users satisfies the following conditions. First, prices for the non-group users are :

$$V_{A1} - p_A \geq V_{B0} - p_B, \quad p_B = 0 \quad \implies \quad p_A = V_{A1} - V_{B0}. \quad (3)$$

That is, platform  $A$  charges the highest price that ensures that non-group users prefer joining the focal platform  $A$  over joining platform  $B$ , given that all other non-group users (and all the group users) are on platform  $A$ . Platform  $B$  charges the lowest price that ensures non-negative profits. The second condition is that platform  $A$  earns positive profit from attracting the non-group users. Let  $\pi_i(x; J) = p_i(1 - x)$  denote the profit of platform  $i$  from the non-group users given  $x$  and the decision of the group,  $J$ . Using (3), we have:

$$\pi_A(x; A) = (1 - x)p_A = (1 - x)(V_{A1} - V_{B0}) > 0, \quad (4)$$

where the inequality follows from Assumption 3. Hence, given  $J = A$ , there is an equilibrium in which platform  $A$  wins the non-group. To see that given  $J = A$  there is no equilibrium in which platform  $B$  wins the non-group, note that if such equilibrium were to exist,  $p_A = 0$  and  $V_{A1} - p_A = V_{B0} - p_B$ , implying that  $p_B = V_{B0} - V_{A1}$  and platform  $B$  earns:  $\pi_B(x; A) = (1 - x)(V_{B0} - V_{A1}) < 0$ . Hence, when  $J = A$ , platform  $A$  always wins the non-group.

Suppose now that the group joins platform  $B$ . An equilibrium in which platform  $A$  wins the non-group users satisfies the following conditions. First, platform  $A$  charges the highest price possible that induces the non-group users to join it, given that they know that the group joined platform  $B$ , yet still expect that all other non-group users join platform  $A$ . Unable to attract the non-group users, platform  $B$  again charges 0. Hence:

$$V_A(1 - x) - p_A \geq V_B(x) - p_B, \quad p_B = 0 \implies p_A = V_A(1 - x) - V_B(x). \quad (5)$$

As before, the equilibrium requires that platform  $A$  earns positive profit from the non-group users:  $\pi_A(x, B) = (1 - x)p_A > 0$ , where using (5):

$$\pi_A(x, B) = (1 - x)(V_A(1 - x) - V_B(x)). \quad (6)$$

Likewise, in an equilibrium in which platform  $B$  wins, it charges and earns, respectively,

$$p_B = V_B(1 - x) - V_A(x), \quad \pi_B(x, B) = (1 - x)(V_B(x) - V_A(1 - x)). \quad (7)$$

Hence, platform  $A$  wins iff  $V_A(1 - x) \geq V_B(x)$ . The following lemma summarizes the results in this section:

**Lemma 2. (The effect of  $x$  on the winner of the non-group)** When  $J = A$ , there is a unique equilibrium where platform  $A$  always wins the non-group users and earns from the non-group  $\pi_A(x; A) = (1 - x)(V_{A1} - V_{B0})$  while platform  $B$  earns from the non-group  $\pi_B(x; A) = 0$ .

When  $J = B$ , there is a threshold,  $\hat{x}$ , where  $\hat{x}$  is the solution to  $V_A(1 - \hat{x}) = V_B(\hat{x})$ , and  $0 < \hat{x} < \frac{1}{2}$  such that:

1. when  $x \in [0, \hat{x}]$ , platform  $A$  wins the non-group and earns from the non-group:  $\pi_A(x; B) = (1 - x)(V_A(1 - x) - V_B(x))$ ;
2. when  $x \in [\hat{x}, 1]$ , platform  $B$  wins the non-group and earns from the non-group:  $\pi_B(x; B) = (1 - x)(V_B(x) - V_A(1 - x))$  while platform  $A$  earns  $\pi_A(x; B) = 0$ .

Lemma 2 shows that a small group has no effect on the winning platform: platform  $A$  wins the non-group users due to its focal position, regardless of whether the group joins it or not. Yet, once the group is sufficiently large, it becomes *pivotal* in the sense that the group determines the winning platform. By choosing to join the non-focal but more efficient platform  $B$ , the group provides platform  $B$  with a network effect advantage to win the non-group users. That is, the group creates large enough network effects such that the value for a single non-group user from joining platform  $B$  is larger than the value from platform  $A$  when only the non-group users join it— $V_B(x) > V_A(1 - x)$ .

The result that, depending on its size, the group may solve the inefficiency created by platform  $A$ 's focal position raises the question: under what market conditions the group makes the efficient choice of joining the higher quality platform  $B$ . We study this below.

## 4 Competition on the group

Consider now the first stage, where platforms compete on attracting the group. The group joins the platform that provides it with the highest benefit as a group,  $xV_i(n_i) - p_i^G$ . Hence, when making a decision, the group takes into account the platforms' qualities, prices, and how the group's decision affect the decision of the non-group. The latter case depends on whether the group's size is smaller or larger than  $\hat{x}$ . We study each possibility in turn.

The main results of this section is that when the group is not pivotal—i.e., it cannot affect the winning platform—it always chooses the inefficient outcome and joins platform  $A$ . Nevertheless, being large enough to become pivotal does not guarantee an efficient outcome. Specifically, an intermediate sized group may still choose to join the inefficient platform  $A$ . It is only when the group is large enough that the group makes the efficient decision

#### 4.1 The group is not pivotal: $x < \hat{x}$

Suppose first that the group is too small to affect the winning platform: platform  $A$  always wins the non-group. An equilibrium in which platform  $A$  wins the group has to satisfy the following conditions. First, platform  $B$  charges the lowest price that ensures non-negative profit and platform  $A$  charges the highest price possible that still compels the group to join it, given that it also wins the non-group:

$$xV_{A1} - p_A^G \geq xV_B(x) - p_B^G, p_B^G = 0 \implies p_A^G = x(V_{A1} - V_B(x)). \quad (8)$$

Second, it is advantageous for platform  $A$  to win the group:  $\pi_A(x; A) + p_A^G \geq \pi_A(x; B)$ . Denoting platform  $i$ 's total profit when  $J = i$  by  $\Pi_i(x; i) \equiv \pi_i(x; i) + p_i^G$  and substituting (4) and (8) into  $\Pi_A(x; A) = \pi_A(x; A) + p_A^G$ , we have:

$$\Pi_A(x; A) = V_{A1} - (1 - x)V_{B0} - xV_B(x), \quad (9)$$

In this case, there is an equilibrium in which platform  $A$  wins the group iff:  $\pi_A(x; A) + p_A^G - \pi_A(x; B) = V_{A1} - (1 - x)(V_{B0} + V_A(1 - x)) + (1 - 2x)V_B(x) \geq 0$ . The following proposition shows that this is always the case:

**Proposition 1.** *(When not pivotal, the group always chooses the inefficient platform) Suppose that  $x < \hat{x}$ . Then, there is a unique equilibrium in which platform  $A$  wins the group and non-group users. Platform  $A$  charges the group and non-group users:  $p_A = V_{A1} - V_{B0}$  and  $p_A^G = x(V_{A1} - V_B(x))$ , respectively, and earns  $\Pi_A(x; A) = V_{A1} - (1 - x)V_{B0} - xV_B(x)$  while platform  $B$  earns 0.*

Intuitively, since focality implies that non-group users expect other non-group users to join platform  $A$ , platform  $A$  can offer the group a higher value than platform  $B$ . The group does not create large enough network effects and thus cannot help platform

$B$  win the non-group. These network effects are also not large enough for its group members to prefer choosing the non-focal platform, and is therefore better off joining  $A$ .

## 4.2 The group is pivotal: $x \geq \hat{x}$

Suppose now that the group is large enough to determine the winning platform:  $x \geq \hat{x}$ . In this case, the group gains a utility of  $xV_{i1} - p_i^G$  from joining platform  $i$ .

Consider an equilibrium in which platform  $A$  wins the group (and consequently the non-group). The lowest price that platform  $B$  is willing to charge the group is its profit from winning the non-group. Platform  $A$  charges the highest price possible that induces the group to join it, given that the non-group users will follow. Hence:

$$xV_{A1} - p_A^G \geq xV_{B1} - p_B^G, \quad p_B^G = -\pi_B(x, B). \quad (10)$$

Notice that the group gains a higher gross utility from joining platform  $B$ :  $xV_{B1} > xV_{A1}$ . However, the group would join platform  $A$  if platform  $A$  sets a sufficiently low price. Substituting (7) into (10),

$$p_A^G = -(1-x)(V_B(x) - V_A(1-x)) - x(V_{B1} - V_{A1}). \quad (11)$$

Note that both platforms set negative prices for the group. The logic is similar to the “divide-and-conquer” strategy (Caillaud and Jullien 2001; 2003), where platforms compete in subsidizing one set of users in order to attract another set. Here, platforms compete on attracting the group because the group determines the platform that wins the non-group.

The second condition for an equilibrium in which platform  $A$  wins is that platform  $A$  earns positive total profit. Substituting (4) and (11) into  $\Pi_A(x; A) = \pi_A(x; A) + p_A^G$ ,

$$\Pi_A(x; A) = (V_{A1} - V_{B0}) - x(V_{B1} - V_{B0}) - (1-x)(V_B(x) - V_A(1-x)). \quad (12)$$

Using similar logic, the equilibrium in which platform  $B$  wins the group and the non-group satisfies  $p_A^G = -\pi_A(x; A)$  and  $xV_{A1} - p_A^G = xV_{B1} - p_B^G$ , hence:

$$p_B^G = x(V_{B1} - V_{B0}) - (V_{A1} - V_{B0}). \quad (13)$$

Notice that while  $p_A^G < 0$ ,  $p_B^G$  can be negative (if  $x$  is close to  $\hat{x}$ ) or positive (if  $x$  is sufficiently close to 1). Intuitively, once the group size is close to 1, the superior utility that platform  $B$  offers the group,  $xV_{B1}$ , is sufficiently high to enable platform  $B$  to attract the group with a *positive* price, even though platform  $A$  charges the group a negative price. An equilibrium in which platform  $B$  wins exists if  $\Pi_B(x; B) = \pi_B(x; B) + p_B^G > 0$ . Substituting (7) and (13) into  $\Pi_B(x; B) = \pi_B(x; B) + p_B^G$ , we have:

$$\Pi_B(x; B) = x(V_{B1} - V_{B0}) + (1 - x)(V_B(x) - V_A(1 - x)) - (V_{A1} - V_{B0}). \quad (14)$$

The first two terms in (14) are positive and represent the advantage of platform  $B$ . In particular, the first term is the network effects that the group induces on all users. Platform  $B$  internalizes this network effect because when the non-group users join platform  $B$ , they already know that  $J = B$ . The second term is platform  $B$ 's total revenue from serving the non-group, when the non-group expect that only the group joins  $B$  (which result from platform  $B$ 's non-focal position). The third term in (14) is negative, and represents platform  $A$ 's positive competitive advantage from being focal: the degree to which network effects are more important to users than platform  $B$ 's superior quality advantage. Recall from (1) that when  $x = 0$ , indeed platform  $A$  always wins and earn  $V_{A1} - V_{B0}$ .

Note that:  $\Pi_B(x; B) = -\Pi_A(x; A)$ . Hence, when the first two effects are stronger than the third one, such that (14) is positive, there is a unique equilibrium in which platform  $B$  wins the group and the non-group users. Otherwise, (12) is positive, and there is a unique equilibrium in which platform  $A$  wins. To evaluate the signs of (12) and (14), we start with the two extreme cases of  $x = \hat{x}$  and  $x = 1$ :

**Proposition 2. (*The group is pivotal*)** *Suppose that  $x > \hat{x}$ , then*

1. *if  $x$  is close to  $\hat{x}$ ,  $V_A(n)$  is convex or linear in  $n$ , and  $V_{B1} - V_{B0} \leq V_{A1}$ , there is a unique equilibrium in which platform  $A$  wins the group and the non group .*
2. *if  $x$  is close to 1, there is a unique equilibrium in which platform  $B$  wins the entire market.*

Proposition 2 shows that when the group is large enough to determine the identity of the winning platform but not too large, the outcome may be inefficient as the group may choose to adopt the low-quality platform,  $A$ . This result holds when  $V_A(n)$  is convex or linear in  $n$  and  $V_{B1} - V_{B0} \leq V_{A1}$ , and may also hold when  $V_B(n)$  is concave in

$n$  as long as  $V_B(n)$  is not too concave, and  $V_{B1} - V_{B0} > V_{A1}$  as long as the gap between  $V_{B1} - V_{B0}$  and  $V_{A1}$  is not too large. The market ends up with the efficient outcome only when the group is substantially large, in which case it chooses the high-quality platform  $B$ .

The intuition behind this result comes from platform  $A$ 's focal position. The group's base utility is always higher when the group joins platform  $B$ :  $xV_{B1} > xV_{A1}$ . Yet, platform  $A$  offers the group a larger subsidy:  $-p_A^G > -p_B^G$ . This is because platform  $A$ 's focal position enables it to collect the utility of each non-group user from meeting other non-group users as well as from meeting group users. In contrast, the non-focality of platform  $B$  implies that it can only collect the non-group's utility from meeting the group. This makes winning the non-group more profitable for platform  $A$  than for platform  $B$ . Since winning the non-group requires winning the group, platform  $A$  is willing to offer the group a lower price than platform  $B$ . When the number of non-group users is high enough (i.e.,  $x$  is close to  $\hat{x}$ ), this focality advantage of platform  $A$  dominates platform  $B$ 's superior quality and platform  $A$  wins the group. When there are almost no non-group users (i.e.,  $x$  is close to 1), focality advantage is small and platform  $B$  can profitably attract the group.

The intuition for the two conditions in Proposition 2 are the following. First, a convex  $V_A(n)$  implies that marginal network effects are increasing with  $n$ . Consequently, given that at least  $x$  users – the group – join platform  $A$ , the marginal network effects from an additional  $1 - x$  users – the non-group – should be sufficiently large to provide platform  $A$  with a sufficient network effect advantage. In contrast, when  $V_B(n)$  is highly concave, the marginal network effect from the non-group may be too small to sustain platform  $A$ 's advantage. The importance of the second condition,  $V_{B1} - V_{B0} \leq V_{A1}$ , has similar intuition. This condition implies that total network effect on platform  $A$ ,  $V - V_A(0)$  (recall that we set  $V_A(0) = 0$ ), should not be much lower than the the total network effects on platform  $B$ ,  $V_{B1} - B_{B0}$ . Otherwise, platform  $A$ 's advantage, which emerges from its focal position through the network effects, may be too small for it to win.

Proposition 2 details the equilibrium in the extremes: when  $x$  is close to  $\hat{x}$  and when  $x$  is close to 1. The switch from an equilibrium where platform  $A$  wins on one end of the range to an equilibrium where platform  $B$  wins on the other end, raises the question of whether the range of  $[\hat{x}, 1]$  contains a unique threshold value of  $x$  such that platform  $A$  wins when  $x$  is below the threshold and platform  $B$  wins otherwise. Alternatively, it

is possible that there are multiple cutoffs such that there are several ranges within this range where each platform wins.

The following proposition provides a condition for a unique cutoff:

**Proposition 3. (*Minimum group size for efficient platform to win*)** *Suppose that  $x > \hat{x}$  and the conditions in Proposition 2 hold. Then, there is a unique cutoff,  $\hat{x} < \tilde{x} < 1$ , such that, if  $\pi_B(x; B)$  is concave in the number of non-group users :*

$$-2(V'_B(x) + V'_A(1-x)) + (1-x)(V''_B(x) - V''_A(1-x)) < 0, \quad (15)$$

*platform A wins the group and non-group when  $x < \tilde{x}$  and platform B wins when  $\tilde{x} < x$ .*

Proposition 3 shows that when platform  $B$ 's revenue function from serving the non-group (given  $J = B$ ) has the standard concavity feature, the model has a unique cutoff in the size of group (or non-group) users such that platform  $B$  wins if the size of the group (non-group) is larger (smaller) than this cutoff. From (7), an increase in the number of non-group users (a decrease in  $x$ ) decreases the price that platform  $B$  charges them and therefore has a conflicting effect on total revenues. as a result, firm  $B$ 's revenues are concave in the number of non-group users where the increase in the number of non-group users - quantity effect - is stronger (weaker) than the decreased in price- price effect when the number of non-group members is large (small).<sup>8</sup> The first term in (15) is ne gative because  $V'_i(n) > 0$ . Therefore, (15) always holds when  $V_i(n)$  are linear and also holds when  $V_A(n)$  ( $V_B(n)$ ) is convex (concave), or not too concave (convex).<sup>9</sup>

In what follows, we assume that the conditions in Propositions 2 and 3 hold. To conclude this section, we find that for  $x \in [0, \hat{x}]$ , the group cannot affect the winning platform and platform  $A$  wins. For  $x \in [\hat{x}, \tilde{x})$ , the group is pivotal, yet, joins platform  $A$ . When  $x \in [\tilde{x}, 1]$ , the group is still pivotal and chooses platform  $B$ .

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<sup>8</sup>To see how this condition translates to the standard concavity assumption, let  $Q \equiv 1 - x$  denote the number of non-group users and  $P(Q) \equiv V_B(1 - Q) - V_A(Q)$  denote the price charged by platform  $B$  from the non-group, by replacing  $Q$  and  $x = 1 - Q$  into (2). Then, platform  $B$ 's revenues as a function of  $Q$  are  $QP(Q)$ , and they are concave when  $2P'(Q) + QP''(Q) < 0$ , which is consistent with (15).

<sup>9</sup>Using numerical simulations, we find that condition (15) holds, for example, when  $V_A(n) = \lambda n^\alpha$  and  $V_B(n) = V + \lambda n^\alpha$ , at least when  $0 < V < \lambda < 1$  and  $0 < \alpha \leq 2$ .

## 5 How the size of the group affects profits and users' surplus

This section studies how the ratio of the group,  $x$ , affects the platforms' profits and users' utility.<sup>10</sup> An increase in  $x$  implies that the size of the group increases and the size of the non-group decreases. That is, a marginal user switches from the non-group to the group. We model the size of the group in proportional terms because an increase in the absolute size of the group increases the total network effect that each user gains. By keeping the number of users constant, we can evaluate the net effect of a user group.

The main conclusion of this section is that a large group may not always be beneficial to users. In particular, the utilities of both a non-group and a group user are non-monotonic in the proportion of the group.

The qualitative results in Section 4 do not depend on the platforms' bargaining power over the group. That is, we obtain the same cutoff values of  $\hat{x}$  and  $\tilde{x}$  if we assume that the group has the bargaining power or the platforms are the ones with bargaining power. Yet, the welfare analysis is sensitive to this assumption. We focus on the case where the platforms make offers to the group in order to give the non-group users the same level of market power as the group users. This allows us to disentangle the different effects and focus on the net effect of the group's collective decision on group and non-group users. To do that, we first look at the effect of  $x$  on total profits and total users' utility and then at the per-user utility.

### 5.1 The effect of the size of the group on total profits and total users' surplus

Recall that when  $x \in [0, \hat{x})$  and  $x \in [\hat{x}, \tilde{x})$ , platform  $A$  wins and earns (9) and (12) respectively. When  $x \in [\tilde{x}, 1]$ , platform  $B$  wins and earns (14). Therefore, total profit as a function of  $x$  is:

$$\Pi(x) = \begin{cases} V_{A1} - (1-x)V_{B0} - xV_B(x), & \text{if } x \in [0, \hat{x}), \\ (V_{A1} - V_{B0}) - x(V_{B1} - V_{B0}) - (1-x)(V_B(x) - V_A(1-x)), & \text{if } x \in [\hat{x}, \tilde{x}), \\ x(V_{B1} - V_{B0}) + (1-x)(V_B(x) - V_A(1-x)) - (V_{A1} - V_{B0}), & \text{if } x \in [\tilde{x}, 1]. \end{cases}$$

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<sup>10</sup>Recall that  $x$  measures the ratio of group users out of the total number of users, and not their absolute size

Let  $CS(x) = V_{i1} - (1 - x)p_i - p_i^G$  denote users' surplus – total users' utility – given the winning platform  $i$ . Notice that  $CS(x) = V_{i1} - \Pi(x)$ . Total welfare is  $W(x) = \Pi(x) + CS(x) = V_{i1}$ . The following proposition describes how  $x$  affects  $\Pi(x)$  and  $CS(x)$ :

**Proposition 4. (A large group may harm users)**

1. When  $x \in [0, \tilde{x}]$ ,  $\Pi(x)$  is decreasing with  $x$  with a discontinuous drop at  $x = \hat{x}$ . When  $x \in [\tilde{x}, 1]$ ,  $\Pi(x)$  is an inverse U-shape function of  $x$ .
2. When  $x \in [0, \tilde{x}]$ ,  $CS(x)$  is increasing with  $x$  with a discontinuous jumps at  $x = \hat{x}$  and  $x = \tilde{x}$ .  $CS(x)$  is a U-shape function of  $x$  when  $x \in [\tilde{x}, 1]$ . Moreover, when  $x$  is close to 1,  $CS(x)$  is lower than  $CS(x)$  when  $x$  is close (from below) to  $\tilde{x}$ .

Figure 5.1 illustrates the results in Proposition 4. The bold line represents  $CS(x)$ , the double line represents  $W(x)$ , and  $\Pi(x)$  is the gap between the two.<sup>11</sup> The figure shows that an increase in  $x$  is not always beneficial to users. For  $x \in [0, \tilde{x}]$ , users' surplus is increasing in  $x$  and jumps at  $x = \tilde{x}$  where it reaches its maximal level. Then, for  $x > \tilde{x}$ , users' surplus first decreases with  $x$  and then slightly increases with it. This last result implies that users may be better off under the inefficient outcome where the group chooses to join the low-quality platform as compared to the efficient case where the group chooses the high-quality platform. The size of the group affects the platforms' profits in the opposite direction. Platform  $A$ 's profit is always decreasing with  $x$ , while platform  $B$ 's profit is first increasing and then decreasing with  $x$ .

To see the intuition for these results, below we break total users' surplus into the utility of a single group and a single non-group user.

## 5.2 The effect of the size of the group on group and non-group users

In order to be able to generate further insight on the effect of the size of the group on users' utility, in the analysis below we assume a specific parametric form where  $V_A(n_A) = \lambda n_A$ ,  $V_B(n_B) = V + \lambda n_B$ . The parameter  $\lambda$  represents the network effect and  $V$  the relative quality advantage platform  $B$  offers. We assume that  $0 < V < \lambda$  such that Assumptions 1 – 3 hold.

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<sup>11</sup>The first part of  $CS(x)$  (when  $x \in [0, \hat{x})$ ) is concave (convex) in  $x$  when  $V_B''(x) < 0$  ( $V_B''(x) > 0$ ).

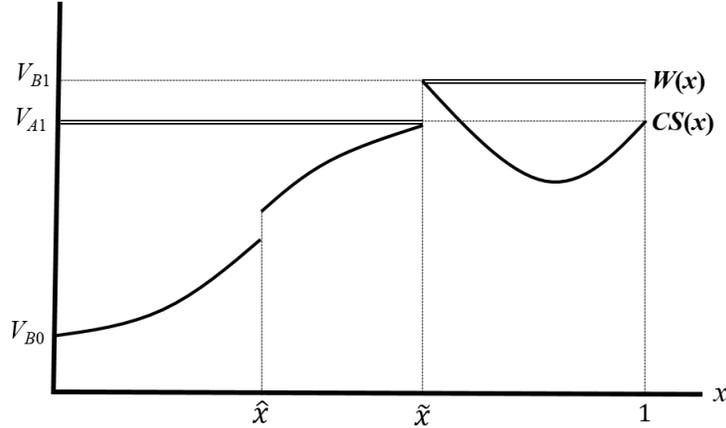


Figure 1: Consumers' surplus ( $CS(x)$ ) and welfare ( $W(x)$ ) as a function of  $x$

### 5.2.1 Comparative static on $\hat{x}$ and $\tilde{x}$

Given this functional form,  $\hat{x} = \frac{1}{2} - \frac{V}{2\lambda}$  and  $\tilde{x} = 1 - \frac{1}{4\lambda} \left( V + \sqrt{(8\lambda + V)V} \right)$ . We start by looking at how platform  $B$ 's quality advantage affects the thresholds  $\hat{x}$  and  $\tilde{x}$  (see Figure 2). If  $V = 0$ ,  $\hat{x} = \frac{1}{2}$ . In this case, both platforms offer the same quality. Platform  $A$ 's focality then implies that as long as the group is smaller than half the mass of users, non-group users will always prefer platform  $A$  over platform  $B$ . Once the group is larger than half the mass of users, it is large enough to “override” platform  $A$ 's focality and become pivotal. As platform  $B$ 's quality advantage increases ( $V \uparrow$ ),  $\hat{x}$  decreases and reaches zero when  $V = \lambda$ . It is easy to show that  $\hat{x}$  increases with  $\lambda$ —as the focality advantage increases, a larger group is needed to make the group pivotal.

The threshold  $\tilde{x}$  starts at 1 when  $V = 0$  and reaches 0 when  $V = \lambda$ . Moreover,  $\tilde{x}$  decreases in  $V$ —as platform  $B$ 's advantage increases, the threshold required for  $B$  to win the market decreases. In contrast, as the network effect becomes more important ( $\lambda \uparrow$ ), it becomes harder for  $B$  to win the market so  $\tilde{x}$  increases in  $\lambda$ .

As Figure 2 shows, as the quality gap between the platforms increases, the range within which the group is pivotal yet chooses the inefficient platform,  $[\hat{x}, \tilde{x}]$ , becomes smaller and the range of group sizes that result in an efficient choice of platform  $B$  increases.

### 5.2.2 Individual utility

We first look at consumer surplus for each non-group user and then for group users.

Given our functional form, the utility of each non-group user is  $u(x) = \lambda - p_A$  if  $A$

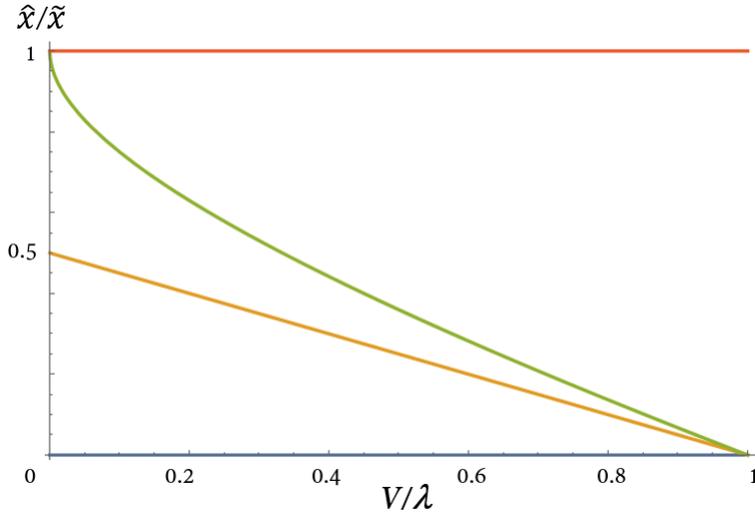


Figure 2:  $\hat{x}$  and  $\tilde{x}$  as a function of  $V$

wins, and  $u(x) = V + \lambda - p_B$  if  $B$  wins, where  $p_A$  is given by (3) and  $p_B$  is given by (7). Putting this together, we get the following utility function for the non-group users:

$$u(x) = \begin{cases} V, & \text{if } x \in [0, \tilde{x}), \\ 2\lambda(1 - x), & \text{if } x \in [\tilde{x}, 1]. \end{cases} \quad (16)$$

Figure 3 illustrates  $u(x)$ . The figure shows the effect of the size of the group on the utility of non-group users is non-monotonic. In the range of group size such that platform  $A$  wins the market ( $x < \tilde{x}$ ), the individual utility of non-group users is not affected by the size of the group and remains constant at  $V$ —the alternative value from being a single user on platform  $B$ . Note that this implies that the utility of a non-group user is not directly affected by whether the group is pivotal or not, but only by the identity of the winning platform. At  $x = \tilde{x}$ , there is a discontinuous climb in  $u(x)$ . Then, a non-group user is hurt by a further increase in the size of the group. The optimal size of the group from the view point of an individual non-group user is therefore slightly higher than  $\tilde{x}$ . Moreover, a non-group user would rather have a small group (i.e.,  $x < \hat{x}$ ) over a very large group (i.e.,  $x \rightarrow 1$ ).

In order to understand the intuition behind these results, it is useful to first look at the case where  $x < \hat{x}$ . In this case, the group is too small to affect the winning platform and thus, given its focality, platform  $A$  needs only give non-group users their alternative value from independently joining the higher quality platform. Once the group becomes

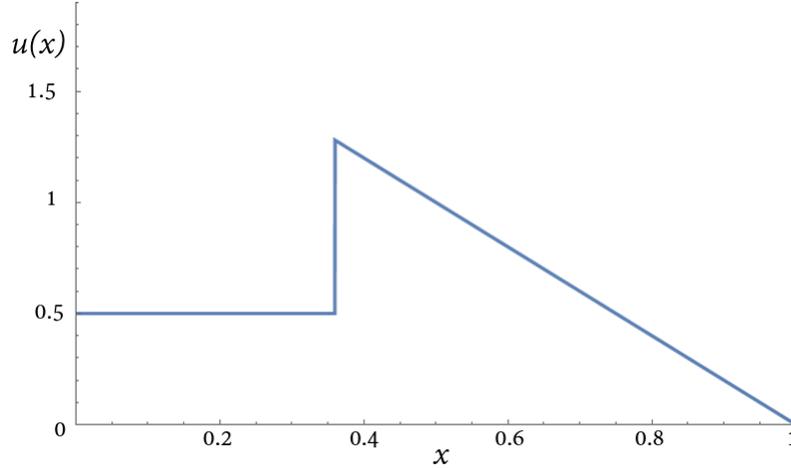


Figure 3: Non-group users' utility as a function of  $x$   
*Non-group users' utility when  $\frac{V}{\lambda} = 0.5$*

pivotal, for  $\hat{x} < x < \tilde{x}$ , the group still joins platform  $A$  so platform  $A$  can set prices such that non-group users still only get  $V$  as surplus as this is still their alternative value from choosing platform  $B$  over  $A$ . Note that this means, that when  $x < \tilde{x}$  platform  $A$  can extract all of the network effect value created by the non-group users and enjoy it as profit.

Once the size of the group is larger such that  $\tilde{x} < x$ , platform  $B$  wins the market and the non-group users' utility is  $2\lambda(1-x)$ —the alternative value from joining platform  $A$ . Evaluating the non-group users' utility at  $x = \tilde{x}$ , we see that there is a large jump in utility at this value. This point is the threshold where the market switches from the case where platform  $A$  wins the market to the case where platform  $B$  wins. Given  $A$ 's focality, platform  $B$  cannot extract the non-group's network effects they create to each other; resulting in the discontinuous jump. As the size of the group increases beyond  $\tilde{x}$ , the non-group users' utility decreases as the increase in the size of the group implies that the size of the non-group decreases and thus that the network effect they generate to each other decreases as well. This also explains why as  $x$  approaches 1, the utility of a non-group users approaches 0. Specifically, in this range, most network effects are generated by the group, which platform  $B$  can extract from the non-group users.

We move now to the utility of the group users. Suppose that the group equally

divides the price  $p_i^G$  among its members. The utility of a group user is thus  $V_{i1} - \frac{p_i^G}{x}$ , where  $p_i^G$  is given by (8) when  $x < \hat{x}$ , (11) when  $\hat{x} < x < \tilde{x}$ , and (13) otherwise. The utility for each group-user can then be written as:

$$u^G(x) = \begin{cases} V + \lambda x, & \text{if } x \in [0, \hat{x}), \\ V + \lambda + \frac{(1-x)}{x} (V - \lambda(1 - 2x)), & \text{if } x \in [\hat{x}, \tilde{x}), \\ \lambda + \frac{(1-x)(\lambda-V)}{x}, & \text{if } x \in [\tilde{x}, 1]. \end{cases} \quad (17)$$

The three panels in Figure 4 illustrate the effect of  $x$  on the utility of a group user, for small, intermediate, and large quality gaps across the platforms. For comparison, the figure also show the utility of a group-user. The figure reveals that for  $x \in [0, \hat{x})$ ,  $u^G(x)$  is increasing in  $x$ , with a jump at  $x = \hat{x}$ . Then, for  $x \in [\hat{x}, \tilde{x})$ ,  $u^G(x)$  is an inverse U-shape function of  $x$  when  $V$  is small, and increasing with  $x$  otherwise. Finally, for  $x \in [\tilde{x}, 1]$ ,  $u^G(x)$  is decreasing with  $x$ . This last result indicates that not only that a too large group may hurt non-group users, it may also hurt group users. Moreover, the right panel in Figure 4 shows that for a large quality gap, the decrease in the utility of a non-group user when  $x \in [\tilde{x}, 1]$  can be rather steep, such that a group-user may gain a lower utility than a non-group user.

The intuition behind these patterns is as follows. In the range where the group does not affect the non-group users' decision ( $x < \hat{x}$ ), the group-users' utility is linear and increasing in  $x$ . In this range, the difference between the utility to non-group users and group-users is  $\lambda x$  which is essentially the network effect the group users create for each other. Unlike with the non-group users, platform  $A$  cannot extract the network effect value the group users create and the group is the one that internalizes this value. That is, regardless of the non-group users' decision, the group-users always create  $\lambda x$  for its group users and thus the alternative value of the group users from platform  $B$  is not only the higher quality the platform offers but also the network effect the group would generate to its group members.

At  $x = \hat{x}$  there is a jump in the group-users' utility as, at this point, the group becomes pivotal. As a consequence, the group can internalize its effect on the *entire* market (i.e.,  $\lambda$  rather than  $x\lambda$ ) as platform  $A$  must compensate the group for joining its platform and as a result attracting the non-group users to join platform  $A$  as well. Furthermore, in the range where  $\hat{x} \leq x < \tilde{x}$ , the group is also able to, indirectly, extract the network effect they generate to the non-group users as were the group choosing the

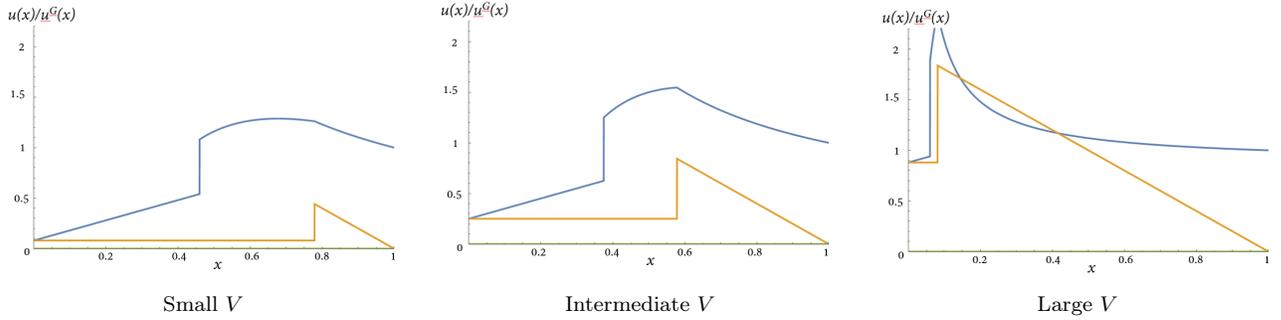
losing platform  $B$ , platform  $B$  would have been willing to transfer to the group its entire revenues from the non-group users (represented by the last term in equation (17)). This forces platform  $A$  to do the same. As the size of the group increases, there are two conflicting effects on the utility of a member in the group. First, the group generates larger network effects to its members –increasing the group-users’ utility in this range. Second, recall that platform  $B$ ’s revenues from non-group users is an inverse U-shape function of the size of the non-group, hence the transfer that platform  $B$  is willing to provide the group is first increasing and then decreasing in  $x$ . When  $V$  is small, the second effect may dominate and  $u^G(x)$  is an inverse U-shape function of  $x$ . In contrast, for intermediate and high values of  $V$ , the first effect always dominates.

In order to understand the difference between a small and an intermediate  $V$ , it is useful to look back at Figure 2. As shown in the figure, when the quality gap between the two platforms is small, the range  $[\hat{x}, \tilde{x}]$  is quite large and  $\tilde{x}$  is close to 1. Hence, the range  $[\hat{x}, \tilde{x}]$  can be large enough for  $u^G(x)$  to be an inverse U-shape function of  $x$ . When the quality gap is small, the range  $[\hat{x}, \tilde{x}]$  is small and accommodates only the increasing part of  $u^G(x)$ .

When  $x > \tilde{x}$ , the group-users’ utility decreases with the size of the group. The intuition here goes back to the the network effect that the group generates for the non-group users. As we note above, once platform  $B$  wins, the non-group users are able to extract the network effect they create for each other but not the network effect they create for the group. It is the group that enjoys the value of the network effect that is generated from the interaction of the group and non-group users. As the size of the group increases, the size of the non-group users decreases and thus the network effect the non-group users generate decreases. Since the group users are the ones that capture this value, their utility decreases as the size of the group increases beyond  $\tilde{x}$ .

## 6 Conclusion

This paper considers platform competition when some users belong to a user group. The group makes a collective decision which platform to join, while small, non-group users make individual decision. One of the platforms is of low-quality, but benefits from a focality advantage: non-group users expect that other non-group users will join it. Such focality can emerge from incumbency advantage or users’ inertia towards a platform that users joined in the past.



The figure shows the utility of group users (blue) and non-group users (orange) as a function of the size of the group.

Figure 4: Group users' and non-group users' utility

The model reveals two main results. First, a proportionally large group can solve users' coordination problem by joining the high-quality platform. Yet, such a pivotal group will choose to join the low-quality platform unless its proportion is large enough. This happens because the low-quality focal platform is better positioned to subsidize the group, as the focal platform can extract the network effects the non-group users create to each other. When the proportion of the group is large, the pivotal group joins the high-quality platform. In this case, the quality advantage is more important to the group than earning the focal platform's revenues from serving the non-users.

The second main result is that an increase in the proportion of the group has a non-monotonic effect on the individual utility of group and non-group users. When the size of the group is small, the group is not pivotal and an increase in the proportion of the group increases the utility of each of its members, while keeping the utility of non-group members fixed. When the group becomes pivotal, each of its members earns additional payoff, even when it continues to choose the low-quality platform. When the group chooses the high-quality platform, a further increase in its proportion decreases the utilities of each member inside and outside the group. For group users, the alternative of joining the low-quality platform becomes less attractive, making it possible for the high-quality platform to attract it with a higher price. The utility of a non-group user decreases with the proportion of group users because the non-focal platform can extract the network effects that the group provides to the non-group users.

These results suggest that mergers between users may not always be beneficial for users. Both group and non-group users would rather have a group of intermediate size. Interestingly, non-group users prefer a small group over a large group, while the opposite holds for group users.

Our model makes two simplifying assumptions. First, users are homogeneous. This

assumption combined with the presence of network effects imply that in equilibrium all users join the same platform and the competing platform remains with no users. In contrast, some of our motivating examples exhibit markets with more than one active platform. Intuitively, multiple platforms can emerge because users may differ in their subjective preferences over the competing platforms. Armstrong (2006a) shows that when users have heterogeneous preferences that are stronger than network effects, the equilibrium involves two active platforms. In such markets, coordination problems and beliefs do not play a significant role. Since our paper focuses on beliefs, we assume that network effects are more important than heterogeneous preferences, and we take it to the extreme by assuming that users are homogeneous. In the background of our model, the losing platform may still be active and serve only users that have strong preferences towards it, or operate in a different market.<sup>12</sup>

Our second simplifying assumption is that we focus on markets with direct network effects. However, some of our motivating examples concern two-sided markets. As Caillaud and Jullien (2001; 2003) and Armstrong (2006a) show, in a two-sided market, platform competition is typically asymmetric: platforms compete more aggressively on one of the sides. This raises the question of how the presence of a user-group affects such asymmetric competition. Since the focus of this paper is on coordination and focality, we leave this question for future research.

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<sup>12</sup>Halaburda and Yehezkel (2016) consider platform competition when some consumers are “loyal” to each platform, while other consumers are non-loyal. They apply the concept of focality to the non-loyal consumers and show that qualitative features of focality are not affected by the presence of loyal consumers.

# Appendix

We prove Lemma 1 in the text. Below are the proofs of all other Lemmas and propositions in the text.

## Proof of Lemma 2:

To complete the analysis preceding Lemma 2, we only need to prove that  $V_A(1 - x) > V_B(x)$  iff  $x < \hat{x}$  where  $0 < \hat{x} < \frac{1}{2}$ . Evaluating  $V_A(1 - x) - V_B(x)$  at  $x = 0$ ,  $V_A(1 - 0) - V_B(0) = V_{A1} - V_{B0} > 0$ , where the last inequality follows from Assumption 3. Evaluating  $V_A(1 - x) - V_B(x)$  at  $x = \frac{1}{2}$ ,  $V_A(1 - \frac{1}{2}) - V_B(\frac{1}{2}) = V_A(\frac{1}{2}) - V_B(\frac{1}{2}) < V_B(\frac{1}{2}) - V_B(\frac{1}{2}) = 0$ , where the last inequality follows from Assumption 2. Since by Assumption 1,  $V_A(1 - x)$  is decreasing with  $x$  and  $V_B(x)$  is increasing with  $x$ , there is a unique  $x < \frac{1}{2}$  such that  $V_A(1 - x) > V_B(x)$  iff  $x < \hat{x}$ . ■

## Proof of Proposition 1:

We first show that, when  $x < \hat{x}$ , there is always an equilibrium in which platform  $A$  wins the group. Then, we show that it is unique. To do this, following the analysis preceding the proposition, we need to show that  $\pi_A(x; A) + p_A^G - \pi_A(x; B) \geq 0$ . Plugging in equations (8) and (9), we have:

$$\begin{aligned}
 & \pi_A(x; A) + p_A^G - \pi_A(x; B) \\
 &= V_{A1} - (1 - x)(V_{B0} + V_A(1 - x)) + (1 - 2x)V_B(x) \\
 &> V_{A1} - (1 - x)(V_{B0} + V_A(1 - x)) + (1 - 2x)V_{B0} \\
 &> V_{A1} - (1 - x)(V_{B0} + V_{A1}) + (1 - 2x)V_{B0} \\
 &= x(V_{A1} - V_{B0}) \\
 &> 0,
 \end{aligned}$$

where the first inequality follows because  $V_B(x) > V_{B0}$  and from Lemma 1,  $x < \hat{x} < \frac{1}{2}$ . The second inequality follows because  $V_A(1 - x) < V_{A1}$ , and the last inequality follows from Assumption 3. Hence, there is an equilibrium in which platform  $A$  wins the group.

Next, consider a putative equilibrium in which platform  $B$  wins the group (and platform  $A$  wins the non-group). In such an equilibrium, if it were to exist, platform  $A$  charges the highest price that makes it indifferent between winning and not winning the group. When platform  $A$  wins, it earns  $\Pi_A(x; A) = (1 - x)(V_{A1} - V_{B0}) + p_A^G$ .

When platform  $A$  loses, platform  $A$  charges the non-group  $p_A = V_A(1 - x) - V_B(x)$  and earns  $\pi_A(x; B) = (1 - x)(V_A(1 - x) - V_B(x))$ . Hence, a losing platform  $A$  charges  $p_A^G = (1 - x)(V_A(1 - x) - V_B(x) - (V_{A1} - V_{B0}))$ . The highest price that platform  $B$  can charge the group solves

$$xV_B(x) - p_B^G \geq xV_{A1} - p_A^G,$$

hence,  $p_B^G = -V_{A1} + (1 - x)(V_{B0} + V_A(1 - x)) - (1 - 2x)V_B(x)$ . Platform  $B$  earns  $\Pi_B(x; B) = p_B^G$ , because  $B$  cannot win the non-group even when  $J = B$ . Yet, notice that  $\Pi_B(x; B) = -(\Pi_A(x; A) - \Pi_A(x; B))$ . Since  $\Pi_A(x; A) - \Pi_A(x; B) > 0$ ,  $\Pi_B(x; B) < 0$ , implying that there is no equilibrium in which platform  $B$  wins. ■

### Proof of Proposition 2:

We prove the proposition by showing that at top extreme ( $x = 1$ ), platform  $A$ 's profits are negative while at the bottom ( $x = \hat{x}$ ), platform  $A$  enjoys positive profits. Evaluating (12) at  $x = 1$ :

$$\begin{aligned} \Pi_A(1; A) &= (V_{A1} - V_{B0}) - (V_{B1} - V_{B0}) \\ &= V_{A1} - V_{B1} \\ &< 0, \end{aligned}$$

where the inequality follows from Assumption 3. Since  $\Pi_B(x; B) = -\Pi_A(x; A)$ , it follows that  $\Pi_B(1; B) > 0$ .

Next, evaluating (12) at  $x = \hat{x}$ :

$$\begin{aligned} \Pi_A(\hat{x}; A) &= (V_{A1} - V_{B0}) - (1 - \hat{x})(V_B(\hat{x}) - V_A(1 - \hat{x})) - \hat{x}(V_{B1} - V_{B0}) \\ &= (V_{A1} - V_{B0}) - \hat{x}(V_{B1} - V_{B0}) \\ &\geq (1 - \hat{x})V_{A1} - V_{B0} \\ &\geq V_A(1 - \hat{x}) - V_{B0} \\ &= V_B(\hat{x}) - V_{B0} \\ &> 0, \end{aligned}$$

where the equality in the second line follows because by definition,  $V_B(\hat{x}) = V_A(1 -$

$\hat{x}$ ), the inequality in the third line follows since  $V_{B1} - V_{B0} \leq V_{A1}$  (and rearranging), the inequality in the fourth line follows because the convexity of  $V_A(n)$  together with  $V_{A0} = 0$  imply that  $(1 - \hat{x})V_{A1} = \hat{x}V_A(0) + (1 - \hat{x})V_A(1) \geq V_A(\hat{x}0 + (1 - \hat{x})1) = V_A(1 - \hat{x})$ , the equality in the fifth line follows again because  $V_B(\hat{x}) = V_A(1 - \hat{x})$  and the inequality in the last line follows from Assumption 1. Since the last inequality is strong,  $\Pi_A(\hat{x}; A) > 0$  also holds when  $V_B(n)$  is linear or concave in  $n$ , as long as it is not too concave, and when  $V_{B1} - V_{B0} > V_{A1}$  as long as the gap is not too large. ■

### Proof of Proposition 3:

The plan of the proof is the following. First, we show that  $\frac{d\Pi_A(x; A)}{dx}\big|_{x=\hat{x}} < 0$  and  $\frac{d\Pi_A(x; A)}{dx}\big|_{x=1} > 0$ . Second, recalling that from Proposition 2,  $\Pi_A(\hat{x}; A) > 0$  and  $\Pi_A(1; A) < 0$ , there is a cutoff,  $\tilde{x}$ , where  $\Pi_A(\tilde{x}; A) = 0$ , such that  $\Pi_A(x; A) > 0$  iff  $x < \tilde{x}$  when  $\frac{d^2\Pi_A(x; A)}{d^2x} < 0$ . We therefore show that  $\frac{d^2\Pi_A(x; A)}{d^2x} < 0$  whenever condition (15) holds. Finally, since  $\Pi_B(x; B) = -\Pi_A(x; A)$ , we have that the same cutoff satisfies that  $\Pi_B(x; B) > 0$  iff  $x > \hat{x}$ .

We start with  $\frac{d\Pi_A(x; A)}{dx}$ :

$$\frac{d\Pi_A(x; A)}{dx} = -(V_{B1} - V_{B0}) + (V_B(x) - V_A(1 - x)) - (1 - x)(V'_B(x) + V'_A(1 - x)). \quad (18)$$

Evaluated at  $x = \hat{x}$ , the term in the second large brackets in (18) disappears because by definition,  $V_B(\hat{x}) - V_A(1 - \hat{x}) = 0$ , we get

$$\frac{d\Pi_A(x; A)}{dx}\bigg|_{x=\hat{x}} = -(V_{B1} - V_{B0}) - (1 - \hat{x})(V'_B(\hat{x}) + V'_A(1 - \hat{x})) < 0,$$

where the inequality follows because  $V'_i(n) > 0$ . Evaluated at  $x = 1$ :

$$\frac{d\Pi_A(x; A)}{dx}\bigg|_{x=1} = -(V_{B1} - V_{B0}) + (V_{B1} - 0) = V_{B0} > 0.$$

Next, differentiating (18) with respect to  $x$  yields condition (15). ■

### Proof of Proposition 4:

Part (i): We start with the first line of  $\Pi(x)$ :  $x \in [0, \hat{x})$ . We have  $\Pi(0) = V_{A1} - V_{B0}$ . Moreover,

$$\Pi'(x) = V_{B0} - V_B(x) - xV'_B(x) < 0,$$

where the inequality follows because for all  $x > 0$ ,  $V_B(x) > V_{B0}$ . Since  $V_B(x)$  increases

with  $x$ , when  $x \in [0, \hat{x}]$ ,  $\Pi(x)$  is decreasing with  $x$  (notice that  $\Pi''(x) = -V_B'(x) - xV_B''(x)$ ). Evaluating  $\Pi(x)$  at  $x \rightarrow \hat{x}^-$  (first line in  $\Pi(x)$ ) and  $x \rightarrow \hat{x}^+$  (second line in  $\Pi(x)$ ), we have  $\Pi(x \rightarrow \hat{x}^-) = V_{A1} - V_{B0} - \hat{x}(V_B(\hat{x}) - B_{B0})$  and  $\Pi(x \rightarrow \hat{x}^+) = (V_{A1} - V_{B0}) - \hat{x}(V_{B1} - V_{B0})$  (where recall that  $V_A(1 - \hat{x}) = V_B(\hat{x})$ ). The gap:

$$\Pi(x \rightarrow \hat{x}^-) - \Pi(x \rightarrow \hat{x}^+) = \hat{x}(V_{B1} - V_B(\hat{x})) > 0.$$

Hence, there is a discontinuous drop in  $\Pi(\hat{x})$ .

Next, consider the second line of  $\Pi(x)$ :  $x \in [\hat{x}, \tilde{x}]$ . From the proof of Proposition 3,  $\Pi(x) = \Pi_A(x; A)$  is decreasing and convex with  $x$  and  $\Pi(\tilde{x}) = 0$ .

Next, consider the third line of  $\Pi(x)$ :  $x \in [\tilde{x}, 1]$ . Again from the proof of Proposition 3,  $\Pi(\tilde{x}) = \Pi_B(\tilde{x}; B) = 0$ , hence,  $\Pi(\tilde{x})$  is continuous. Moreover,  $\Pi(x) = \Pi_B(x; B)$  is concave with  $x$ . Since  $\frac{d\Pi_B(x; B)}{dx}|_{x=\tilde{x}} > 0$  and  $\frac{d\Pi_B(x; B)}{dx}|_{x=1} < 0$ , is  $\Pi(x) = \Pi_B(x; B)$  is an inverse U-shape function of  $x$ .

Part (ii): Starting with  $x \in [0, \hat{x})$ , we have  $CS(0) = V_{A1} - \Pi(0) = V_{B0}$ . Because  $\Pi(x)$  is decreasing with  $x$ , and has discontinuous decline at  $x = \hat{x}$ ,  $CS(x)$  is increasing with  $x$  and has a discontinuous climb at  $x = \hat{x}$ . At  $x = \tilde{x}$ ,  $\Pi(\tilde{x}) = 0$ , but total utility increases from  $V_{A1}$  to  $V_{B1}$ , hence, there is a discontinuous climb in  $CS(\tilde{x})$ . Finally, at  $x \in [\tilde{x}, 1]$ ,  $\Pi(x) = \Pi_B(x; B)$  is an inverse U-shape function of  $x$ , hence  $CS(x)$  is a U-shape function of  $x$ . Finally,  $CS(1) = V_{B1} - \Pi_B(1, B) = V_{A1}$ . ■

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