Endogenous Product *Design* and *Quality* with Rationally Inattentive Consumers*

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July 2018

Abstract

In some markets, consumers do not know the attributes of all the products that are available in the market, or the prices at which they are offered. To overcome this uncertainty consumers may, at a cost, gather and process information about the attributes and prices of the different products. We present a theoretical framework that couples endogenous firms' decisions on the multi-attribute dimensions and pricing of products with endogenous consumers' decisions on what and how much information to gather and process, and which product to purchase. We find a number of interesting results. First, consumers may rationally select information strategies that do not fully eliminate their uncertainty and so, rationally select to be inattentive. Second, firms do have an incentive to respond to lower information costs by increasing differentiation, as established by the standard search literature, but solely *if the proportion of "informed" consumers in the market is small* and *along the least-costly attribute dimension.* This implies that equilibrium prices may, as the unit cost of gathering and processing information decreases, increase in some markets and decrease in others. Further, it implies also that when the cost of quality improvement in a market changes, there can be radical shifts in product attributes, as those observed in the U.S. coffee market after the entry of Starbucks.

JEL Classification: D43; D83; L13; L15

Keywords: Rational Inattention, Information Frictions, Product Differentiation, Pricing.

*We would like to thank Ganesh Iyer, Dmitri Kuksov, Carlos D. Santos, André Silva, Susana Silva, Juanjuan Zhang and seminars participants at Nova School of Business & Economics, Católica Porto Business School and Faculdade de Economia da Universidade do Porto, as well as numerous participants at the 40th annual ISMS Marketing Science Conference, for helpful comments and suggestions. All remaining errors are of course our own.

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1 Introduction

In some markets, consumers are imperfectly informed. They do not know the attributes of all the products in the market, or the prices at which they are available (Stiglitz, 1989). To overcome this uncertainty consumers may gather and process information about the attributes and prices of the different products (e.g., contact the different sellers, examine the products, ask questions and expert advice, read internet sites or forums, among many other). However, to do so, consumers must incur a cost, since gathering and processing information takes money, time and effort. These costs affect the amount of information gathered and processed and, therefore, the uncertainty that will persist at the time of purchase, which, in turn, affects the competition in the market, via prices (in the short-run) and product attributes (in the long-run).

The *internet*, by decreasing the cost of gathering and processing information on product attributes and prices, has dramatically changed this process.[1] The consequences of this change on firms' product attribute and pricing decisions has been the object of a significant and growing literature (Kuksov, 2004; Bar-Isaac, Caruana and Cuñat, 2012; Larson, 2013; Fishman and Levy, 2015). This literature typically (*i*) endogeneizes firms' product attribute decisions on an one-dimensional attribute space, and (*ii*) assumes that, before entering the choice situation, *consumers are not in contact with any product*, use a sequential step-wise *search* procedure to gather and process information about the different products, and learn *perfectly* all the attributes of the searched products. The main conclusion of this strand of the literature is that lower consumer search costs may lead to higher equilibrium prices, due to the fact that firms may respond to lower costs by changing product attributes in order to increase product differentiation (and thus decrease price competition).

In this paper, we take up the same issue: the consequences of consumer information costs on firms' product attribute and pricing decisions. However, we depart from the

[1] However, even though the internet has been facilitating this process, gathering and processing information on product attributes and prices remains costly for consumers: see, for example, Lach (2002), Lewis (2008), and Dubois and Perrone (2015). Lach (2002) examines the Israeli refrigerator, chicken, coffee, and flour markets. Lewis (2008) examines the San Diego gasoline market. Dubois and Perrone (2015) examine the French food retail market for beer, cola, coffee, and whisky. They all find evidence of price dispersion, even after controlling for observed and unobserved product characteristics, which suggests that even information on price remains not freely and readily available.

existing search literature in two aspects. First, we endogenize firms' decisions on a *multi-attribute* space. We do so because the literature that examines firms' product attribute and pricing decisions under perfect information shows that the standard differentiation results no longer hold when firms compete in a multi-attribute space (Neven and Thisse, 1987, 1990; Heeb, 2001). Second, we follow the rational inattention literature in assuming that, before entering the choice situation, *consumers are in contact with all products*, may have a prior rough idea about their attributes and prices (which may be extremely incomplete, imprecise or even completely wrong), and *are completely free to gather information in a non-random fashion about any (sub)set of products, with any precision about their attributes and prices.* We do so because in a post-internet era: (i) most consumers are aware of and use product comparison websites (e.g., Moneysupermarket, uSwitch and Confused.com) to gather information about the products available in the market (Bailey, 2005; Laffey and Gandy, 2009) - this implies that, in contrast with the existing search literature, consumers typically do not gather information sequentially about each product (Honka and Chintagunta, 2017); (ii) gathering and processing information is clearly much easier for some attributes (e.g., price, since it is typically the most communicated aspect of a product and does not involve any subjective or personal evaluation) than for others (e.g., quality) - this implies that, in contrast with the existing search literature, when consumers gather and process information about a product, they may typically focus on certain attributes and, as a consequence, may not learn *perfectly* all the attributes of the product (Kivetz and Simonson, 2000).

The two contributing aspects are incorporated into a discrete-choice framework in which two firms, manufacturing a single product characterized by two attributes, *design* (which we portray as horizontal differentiation) and *quality* (which we portray as vertical differentiation), compete over a continuum of consumers. Consumers are assumed to engage into a two-stage decision problem. In the *first stage*, each consumer selects at a cost (which may vary from consumer to consumer) an information gathering and processing strategy. She is completely free to choose *any* information gathering and processing strategy, i.e., to choose what and how much information to gather and process. This strategy, following the rational inattention literature, generates signals that are then used to update her prior beliefs about the attributes and prices of the different products. In the *second stage*, each consumer makes use of her updated beliefs to select (and purchase) the product whose perceived attributes and price yield the highest utility. The solution to this two-stage decision problem yields an interesting (and intuitive) result. Consumers may rationally select information strategies that do not fully eliminate their uncertainty about the attributes and prices of the different products.

Firms are also assumed to engage into a two-stage decision problem that precedes the consumer decision problem described above, given correct expectations about the aggregate demand it will face for any given set of firms' product attributes and pricing decisions. In the *first stage*, each firm (simultaneously) selects the attributes of its own product (design and quality) that yield the highest own profit. This stage can be viewed as the long-term when strategic decisions to determine the positions in the attribute space are taken. In the *second stage*, given the product attributes chosen in the first stage, each firm (simultaneously) sets the price of its own product that yields the highest individual profit. This stage can be interpreted as the short-run where only prices are flexible. The solution to this problem yields that the equilibria is inherently different from the standard search model. Firms do have an incentive to respond to lower information costs by increasing differentiation, as established by the standard search literature, but solely *if the proportion of those consumers that can gather and process information at no cost ("informed" consumers) in the market is small* and *along the least-costly attribute dimension*. This implies that equilibrium prices may, as the unit cost of gathering and processing information decreases, increase in some markets and decrease in others. Further, it implies also that when the cost of quality improvement in a market changes, there can be radical shifts in equilibrium product attributes, as those observed in the U.S. coffee market after the entry of Starbucks, providing an alternative explanation to the salience model of Bordalo, Gennaioli and Shleifer (2016).

The remainder of the paper is organized as follows: Section 2 reviews the literature, Section 3 describes the consumer and firm behaviour, Section 4 addresses the timing and equilibrium of the model, Section 5 offers relevant managerial and policy implications, and Section 6 concludes.

2 Literature Review

The decisions of firms on product attribute and pricing is examined by the literature on product positioning and it can be divided into two strands: product positioning under perfect information and product positioning under consumer information frictions.

2.1 Perfect Information

The theory of product positioning under perfect information begins with Hotelling (1929), who introduces the idea that products compete on more than just price. Price is an important aspect, but it is definitely not the sole one. To do so, Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979) incorporate competition, in addition to price, over an one-dimensional horizontal differentiation attribute, which we will address as *design* (different consumers may rank the same design, differently). Shaked and Sutton (1982), Tirole (1988) and Moorthy (1988) establish a different strand that models the same idea by incorporating competition over, instead, an one-dimensional vertical differentiation attribute in the lines of Spence (1975) and Mussa and Rosen (1978), which we will address as *quality* (all consumers prefer high quality to low quality). The two strands establish a *differentiation principle*, which yields that firms should differentiate their one-dimensional attribute to relax price competition (see the online Appendix A for a more detailed description of this literature).[2]

However, the attributes of most products cannot, however, be sorted out into either horizontal or vertical alone. Rather, most products combine horizontal and vertical attributes. In order to capture this feature, Neven and Thisse (1987, 1990) incorporate competition over *three* aspects: price, design, and quality. The solution to the firms'

[2] The only exception is Hotelling (1929), which establishes a principle of no differentiation. However, d'Aspremont, Gabszewicz and Thisse (1979) asserts Hotelling (1929)'s result to be invalid. Due to a flawed key calculation in Hotelling (1929), who neglects to consider strategies through which a firm undercuts the price of the rival to capture the whole market.

strategic problem in this setting involves, as in one-dimensional cases, trading-off two opposing forces. On the one hand, firms have a tendency, for any given prices, to supply the same attributes, designs (near the "center" of the market) and quality (high-quality), in order to increase demand. On the other hand, they have a tendency to supply different designs and qualities in order to relax price competition. In order to examine this trade-off, Neven and Thisse (1987, 1990) assume (*i*) a continuum of consumers that, after observing the available designs, qualities and prices, make indivisible and mutually exclusive purchase decisions involving the two products (i.e., implicitly assuming that the two products serve the whole market), (*ii*) that consumers' ideal designs are uniformly distributed along a compact linear real space and that the utility loss is quadratic in the distance between the actual design of the product and the ideal design of the consumer, (*iii*) that consumers' valuation for quality is uniformly distributed on some positive support, and (*iv*) that quality is costless. Under these assumptions, they conclude that the interplay between horizontal and vertical attributes leads to a surprising result: firms do not need to differentiate their products along all attribute dimensions. *Differentiation along one dimension* is more than enough to relax price competition. In particular, they establish that the most effective product positioning strategy consists in *maximizing* differentiation along one attribute, while *minimizing* differentiation along the other dimension.[3] Given the two-dimensional attribute space assumed, this yields (*i*) a *max-min* equilibrium, in which product differentiation is maximized along the horizontal attribute and minimized along the vertical attribute, (*ii*) a *min-max* equilibrium, in which product differentiation is minimized along the horizontal attribute and maximized along the vertical attribute, or (*iii*) both, depending on the range of potential qualities. Heeb (2001) examines a slightly different strategic problem, that (among other changes) allows consumers the option of not purchasing any of the two products and incorporates quadratic marginal costs (but no fixed costs) of supplying quality.[4] He concludes that Neven and Thisse (1987, 1990)'s

[3]This justifies why the product positioning strategy that consists in maximizing differentiation along all attribute dimensions is not an equilibrium.

[4]We point out two other main changes. First, he allows for a distribution of preferences (for designs and qualities) that is not only uniform, but also normal and asymmetric. Second, he allows for a three stages strategic problem. In the first stage, each firm chooses the design of its product. In the second stage, each firm chooses the quality of its product. In the third stage, each firm sets prices.

differentiation result still holds, although not abiding the *max-min* or *min-max* principle.

2.2 Consumer Information Frictions

The literature on product positioning described above assumes perfect information on all sides, both firms and consumers. We now address the literature that relaxes the perfect information assumption on the consumers' side. This literature can be sub-divided into two smaller strands: product positioning under consumer search and product positioning under rational inattention.[5]

2.2.1 Consumer Search

The theory of product positioning under consumer search begins by examining the effect of consumer imperfect information (and consequently, of the costs consumers have to incur to search for information) on equilibrium prices. Diamond (1971), Wolinsky (1986), and Anderson and Renault (1999) consider this question under different product settings (see the online Appendix A for a more detailed description of this strand of the literature) and conclude that (*i*) consumers' imperfect information relaxes price competition and creates market power for firms, and (*ii*) lower search costs do not necessarily decrease equilibrium prices.[6]

A recent strand of the literature extends the consumer search framework to examine product positioning also in terms of attributes: Kuksov (2004), Bar-Isaac, Caruana and Cuñat (2012), Larson (2013), and Fishman and Levy (2015). Kuksov (2004) assumes a setting in which products are differentiated by design, consumers are imperfectly informed about prices, but may engage in costly search to reduce their uncertainty. The strategic problem of each firm is to select its design and price so to maximize its own profit, recognizing that the other firm is doing exactly the same. Kuksov (2004) results confirm that (*i*) consumers' imperfect information relaxes price competition and provides an

[5] There is also (less relevant to our problem) literature on product positioning under firm information frictions, which begins with de Palma et al. (1985) and Rhee et al. (1992), who model firms to have imperfect information regarding consumer preferences.

[6] Diamond (1971) considers a homogeneous product setting and finds that lower search costs do not decrease equilibrium prices. In contrast, Wolinsky (1986) and Anderson and Renault (1999) consider an exogenous differentiated product setting and find that lower search costs decrease equilibrium prices.

additional source of market power to firms, and (*ii*) lower search costs may lead to higher endogenous product differentiation, which - by relaxing the otherwise more intense price competition - imply higher equilibrium prices. Larson (2013) confirms these results under the more general setting in which, in addition to prices, consumers are also imperfectly informed about the designs of the products available in the market.[7]

Bar-Isaac, Caruana and Cuñat (2012) extend Kuksov (2004) and Larson (2013)' settings by considering that products are differentiated over a multi-attribute setting (design *and* quality). They assumed consumers are imperfectly informed about designs and prices, and may engage in costly search to reduce their uncertainty. The strategic problem of each firm is to select its designs and price (taking quality as exogenous) so to maximize its own profit, recognizing that the other firm is doing exactly the same. The solution to this problem confirms Kuksov (2004)'s two results above.[8] Fishman and Levy (2015) extend Bar-Isaac, Caruana and Cuñat (2012)' setting by considering that, in addition to designs and prices, consumers are also imperfectly informed about qualities. Under Fishman and Levy (2015)'s formulation, the strategic problem of each firm is to select its quality and price (taking design as exogenous) so to maximize its own profit, recognizing that the other firm is doing exactly the same. Similarly to Kuksov (2004) and Bar-Isaac, Caruana and Cuñat (2012), they find that lower search costs lead to higher endogenous product differentiation.[9]

2.2.2 Rational Inattention

The literature on rational inattention dates back to Sims (1998), who argued that the stickiness observed in most prices, wages and other macroeconomic aggregates could be modeled as arising from the limited capacity of decision-makers (that have many things to think about and limited time) to gather and process information flows about uncertain decision situations. This idea has since been applied to a variety of different problems (see

[7] He finds, similarly to Kuksov (2004) that, in a general sense, firms respond to lower search costs by endogenously increasing product differentiation (by choosing niche designs for their products).

[8] Further, they show that the increased price competition from lower search costs adversely affects low-quality firms more than high-quality firms, yielding that, as search costs decrease, the former increase horizontal product differentiation (by choosing niche designs for their products) before the latter.

[9] This increased differentiation depends on the initial quality distribution in the market. If the initial proportion of high quality firms is high, lower search costs lead to lower endogenous quality, whereas if the initial proportion of high quality firms is small, lower search costs leads to higher endogenous quality.

the online Appendix A for a more detailed description of this strand of the literature), including - as in Matějka and McKay (2012, 2015) - problems of differentiated product choice decisions by consumers that are uncertain - and as a consequence must gather and process information - about the attributes and prices of the products in the market.[10] Matějka and McKay (2015) consider a discrete product choice problem and show that the optimal strategy of a rationally inattentive consumer leads to probabilistic product choices that follow a generalized multinomial logit formula. This generalized formula depends on three elements: (*i*) the *true* attributes and prices of the products, (*ii*) the consumer's *prior beliefs* about those attributes and prices, and (*iii*) the consumer's unit cost of gathering and processing information. This result establishes a foundation for the multinomial logit demand model, traditional to the product differentiation literature, and makes the consumer decision problem, under inattention, tractable. Matějka and McKay (2012) draw on the results from Matějka and McKay (2015) to examine the effect of rational inattention on equilibrium prices.[11] The strategic problem of each firm is to select its price so to maximize its own profit, recognizing that consumers are rationally inattentive and other firms are doing exactly the same. They conclude that the solution to this problem, even if products are homogeneous, yields that (*i*) consumers' imperfect information relaxes price competition and creates market power (otherwise inexistent) for firms, and, in contrast with Diamond (1971), (*ii*) lower unit costs of gathering and processing information decrease equilibrium prices. This implies that modeling information frictions via the rational inattention framework generates equilibrium prices that, in contrast with the original search framework, are continuous in the degree of information frictions.

3 Theoretical Model

We contribute to the literature of product positioning under consumer information frictions (Kuksov, 2004; Bar-Isaac, Caruana and Cuñat, 2012; Larson, 2013; Fishman and

[10] Although Matějka and McKay (2015) was published after Matějka and McKay (2012), it was developed first. In fact, Matějka and McKay (2012) draws heavily on the results from Matějka and McKay (2015).

[11] Martin (2015) and Matějka (2015) examine a similar question, but for a monopoly setting.

Levy, 2015) in two dimensions. First, we examine product positioning on a *multi-attribute space* as in Neven and Thisse (1987, 1990). Second, we model the process of gathering and processing information according to the *rational inattention framework*, so to address the two stylized facts discussed above. This section details our consumer and firm behavioral assumptions to do so.

3.1 The Setup

We consider a continuum of heterogeneous consumers of measure 1, indexed by i, each of which, following the discrete-choice framework, is assumed to choose one of the $j = 1, 2$ products available in the market. Each product j is characterized by its position in a two-dimensional attribute space, a setting similar to Neven and Thisse (1987, 1990)' seminal work. The first attribute, which we denote by x_j, represents the design characteristics of the product. The range of potential designs is, without loss of generality, represented by the $[0, 1]$ interval. The second attribute, which we denote by δ_j, represents the level of quality of the product. The range of potential qualities is represented by the interval $\left[\underline{\delta}, \overline{\delta}\right]$. The lower bound level of quality can be interpreted as a minimum standard legal requirement or as being inherent to the production process, following Motta (1993). Without loss of generality, we define $\underline{\delta} = 1$. The upper bound level of quality can be interpreted as the maximum quality level that is sustained by a market with a finite measure, following Berry and Waldfogel (2010). Without loss of generality, and solely for comparison purposes, we define $\overline{\delta} = 4$, such that $\overline{\delta} - \underline{\delta}$ falls inside the nondegenerate segment in which the two product positioning equilibria (the *max-min* equilibrium and the *min-max* equilibrium), established by Neven and Thisse (1987, 1990), coexist.

3.2 Consumer Behaviour

We model consumer preferences using a characteristics approach in the lines of Lancaster (1966) and model consumer information frictions using the rational inattention framework in the lines of Matějka and McKay (2015).

3.2.1 Consumer Preferences

The preferences of each consumer are, in a characteristics-based approach (Lancaster, 1966), defined directly over the attribute dimensions of the available products. We consider that consumers do not rank designs in the same way, which portrays horizontal differentiation, following Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979). However, we consider that all consumers prefer a high quality to a low quality, which portrays vertical differentiation, following Spence (1975) and Mussa and Rosen (1978).

We allow consumer preferences over the two attribute dimensions to be heterogeneous. First, each consumer i has a most preferred design, denoted by $v_i \in [0, 1]$, and incurs in an utility loss whenever purchasing a product with a design that differs from her ideal preference point. The utility loss is quadratic with respect to the distance between the two points. This implies that the flow utility loss derived by this consumer from the design attribute of product j is given by $-(v_i - x_j)^2$. Second, each consumer i has a specific valuation per unit of quality, which we denote by $\theta_i \in [0, 1]$. This implies that the flow utility derived by this consumer from the quality dimension of product j is given by $\theta_i \delta_j$.[12]

The conditional indirect utility derived by each consumer i from purchasing a unit of product j aggregates the flow utilities associated to the product's attributes with the flow utilities associated to the consumption of goods from other markets. We assume a linear functional form for this aggregation, as follows:

$$u_{ij} = (y_i - p_j) - (v_i - x_j)^2 + \theta_i \delta_j, \tag{1}$$

where y_i denotes the income of consumer i, p_j denotes the price of product j and $(y_i - p_j)$ denotes the flow utility from consuming all other goods, which we treat as a composite commodity. We follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough

[12] The quadratic utility loss assumption above avoids, as discussed in the literature review, the discontinuities in the firms profit functions that may cause a problem for the existence of a pure-strategy price equilibrium. However, it introduces a functional form distinction between the marginal flow utility associated to the two attribute dimensions. The marginal flow utility of design is given by $2(v_i - x_j)$, which is product-specific and decreases with the design position, whereas the marginal flow utility of quality for consumer i is given by θ_i, which is constant with respect to the identity of the product and the level of quality. This functional form distinction has implications (although very slight) for the equilibrium designs and qualities, an issue we address in Section 4.

for all consumers to find a product that generates a positive utility in equilibrium.

The conditional indirect utility function above makes use of the common assumption in the discrete-choice framework literature that income and prices are additive separable, i.e., that income effects from price changes are negligible (see, e.g., Martin, 2015). This implies that income can be omitted from the indirect utility specification, since it does not vary across products:

$$u_{ij} = -p_j - (v_i - x_j)^2 + \theta_i \delta_j. \tag{2}$$

Exploring the implications of relaxing the additive separability assumption seems a very interesting area of future research.

3.2.2 Consumer Information Frictions

We consider information frictions to be an important part of the consumers' product choice environment. We do so by assuming that consumers have imperfect information in the following lines. Before entering the choice situation, consumers know the number of available products, but lack specific knowledge about their attributes and prices. However, they do hold a prior belief about the probability distribution of the unknown attributes and prices, which we denote by $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, with $\mathbf{x} = (x_1, x_2)'$, $\boldsymbol{\delta} = (\delta_1, \delta_2)'$ and $\mathbf{p} = (p_1, p_2)'$ representing the vector of designs, qualities and prices, respectively, of the different products.

In order to counteract the lack of specific knowledge, each consumer i can engage in an information gathering and processing strategy that refines (updates) her knowledge. For example, she can contact the firms, examine the products, ask questions or read internet forums. Such strategies generate signals that consumers can use to update their beliefs about the attributes and prices of the different products. Let $\mathbf{sg}_i = (\mathbf{sg}_{i1}, \mathbf{sg}_{i2})'$ denote the vector of signals (about the attributes and prices of all the products in the market) obtained from consumer i's information gathering and processing strategy, where $\mathbf{sg}_{ij} = \left(x_j^{s_i}, \delta_j^{s_i}, p_j^{s_i}\right)'$ represents the subvector of signals associated to the design, quality and price of product j: $x_j^{s_i}$, $\delta_j^{s_i}$ and $p_j^{s_i}$, respectively.

We follow Matějka and McKay (2015) and allow consumers to choose any information gathering and processing strategy. They are completely free in deciding *what* and *how much* information to gather and process, i.e., in deciding, for example, what and how many questions to ask or posts to read. However, since different information gathering and processing strategies generate different signals (asking questions to a shop assistant is inherently different from reading internet forums, reading five forum posts is inherently different from reading fifty), the choice of an information gathering and processing strategy is implicitly a choice of the (distribution of) signals that are generated. As a consequence, and for simplicity, we model consumer i's information strategy choice as a decision involving the joint distribution of signals, attributes, and prices, i.e., $\mathbf{sg}_i$ and $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, that are implicitly generated (in the lines of Caplin and Dean, 2013, and Matějka and McKay, 2015). Let $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ denote this joint distribution. Having chosen an information strategy (or equivalently, a joint distribution of signals, attributes, and prices), consumers use the signals received to update their beliefs. Let $F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i)$ denote the updated beliefs of consumer i.

Consumers have, as discussed above, complete freedom to choose their information gathering and processing strategy. Nevertheless, they must consider that all such strategies are costly. For example, examining the products, asking questions or reading internet forums takes money, time and effort. We follow Caplin and Dean (2013), de Oliveira (2014), and Matějka and McKay (2015) and assume the cost of an information gathering and processing strategy to be proportional to the *amount* of information gathered and processed. We capture the latter by the reduction in the expected uncertainty involving the attributes and prices of the different products, where uncertainty (following Shannon, 1948) is measured by entropy. This reduction (even in cases associated a multivariate distributions like ours) is summarized in a single number, the mutual information between the prior and the updated (posterior) beliefs of consumers about $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$. The cost of any information gathering and processing strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ chosen by consumer i

can then be expressed as:

$$c\left(F\left(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right); \gamma_i\right) \equiv \gamma_i \left(H\left(G\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right)\right) - \int\limits_{\mathbf{sg}_i} H\left(F\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p} | \mathbf{sg}_i\right)\right) F\left(d\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right) \right), \tag{3}$$

where $c\left(F\left(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right); \gamma_i\right)$ denotes the cost of strategy $F\left(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right)$, $\gamma_i > 0$ denotes consumer i's unit cost of gathering and processing information, $H\left(\cdot\right)$ denotes Shannon (1948)'s entropy function, $H\left(G\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right)\right)$ denotes the uncertainty associated with the prior belief and, finally, $\int_{\mathbf{sg}_i} H\left(F\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p} | \mathbf{sg}_i\right)\right) F\left(d\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right)$ denotes the expected uncertainty associated with the posterior belief. We allow the unit cost of gathering and processing information to be consumer-specific, since the money, time and effort required to, for example, examine the products, ask questions or read internet forums may vary extensively from consumer to consumer.

To sum up, consumers face a trade-off. Strategies that gather and process more information are more informative, in the sense that generate more precise signals about $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, but are also more costly. Due to this trade-off, it may happen that strategies that could generate signals precise enough to fully eliminate the uncertainty about $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ are, from the consumer perspective, too costly. This implies that some uncertainty about the attributes of the different products may *rationally* persist when consumers make a purchase decision, leading consumers to select a product that may not be the one that yields the highest conditional indirect utility (inattention). In other words, incorporating consumer information frictions into the model introduces errors, and therefore, randomness, in the purchase decisions of consumers.

3.3 Firm Behaviour

We consider that there are two single-product risk-neutral firms in the industry, each of which producing one of the $j = 1, 2$ products available in the market. The production technology of each firm is characterized by a marginal cost function that has both a quality-independent and a quality-dependent component:

$$mc(\delta_j) = \eta + \frac{\varphi}{2}\delta_j^2, \tag{4}$$

with $\eta, \varphi \geq 0$. We assume the production technology to be identical for both firms so to rule out the trivial case in which product differentiation arises from technological differences between firms (Moorthy, 1988). We assume the quality-dependent component to be essentially a variable cost, reflecting the cases where firms must engage in more skilled labour or more expensive raw materials and inputs to improve quality, following Motta (1993). The parameters η and φ, which drive the production technology of each firm j, are exogenously drawn from probability distribution $T(\eta, \varphi)$.

4 Game, Timing and Equilibrium

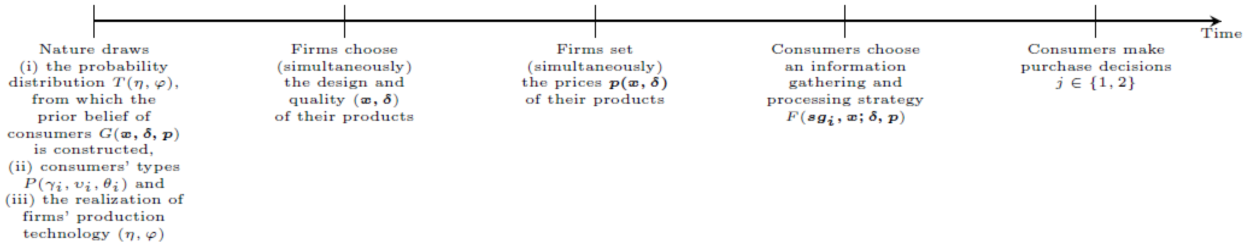
We consider that consumers and firms play the following game, timed as depicted in Figure 1. At the beginning of the game, nature draws (i) the probability distribution of the probution technology parameters $T(\eta, \varphi)$, from which the prior belief of consumers about the probability distribution of product attributes and prices, $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ is constructed, (ii) the probability distribution of consumer types (associated with consumers' unit costs of gathering and processing information and preferences regarding product attributes), which we denote $P(\gamma_i, \upsilon_i, \theta_i)$ and (iii) the realization of the production technology of the firms (η, φ). We assume that the probability distributions $T(\eta, \varphi)$ - and, consequently, $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ - and $P(\gamma_i, \upsilon_i, \theta_i)$ are common knowledge among firms and consumers while the realization (η, φ) is known only to firms.

Next, firms address a two-stage decision problem so to maximize own-profit. In the first stage, each firm (simultaneously) chooses the design and the quality of its single product.[13] In the second-stage, each firm (simultaneously) sets prices. The intuition behind the firms' two-stage structure assumption is borrowed from Hotelling (1929) and lies on the fact that prices are more flexible than design or quality in the short run. Thus,

[13]Having firms choose quality is entirely equivalent to having firms choose vertical innovation rates, given identical initial qualities (Heeb, 2001).

FIGURE 1

Timing of the Game



as discussed above, the second stage can be interpreted as the short-run where only prices are flexible, while the first stage can be viewed as the long-term when strategic decisions to determine the positions in the attribute space are taken. We model the decisions about design and quality as being simultaneous because production will often require the joint specification of these attributes.

Finally, consumers also address a two-stage decision problem so to maximize their expected utility. In the first stage, each consumer chooses an information gathering and processing strategy, which generates signals that the consumer uses to refine her prior beliefs about the probability distribution of the unknown product attributes and prices. In the second stage, each consumer selects the product that provides the highest expected conditional indirect utility, given her updated beliefs.

We follow Bakos (1997) and Kuksov (2004) in assuming that the game is played in a single period setting. This assumption is illustrative and is presented for simplicity. It can be relaxed by incorporating into consumers' prior beliefs the eventual reputation effects that could result from the repeated interaction of consumers in the industry. This extension to the analysis seems a very interesting area of future research.

We focus on the sub-game perfect Nash equilibrium of the game. We begin by addressing the consumers decision problem.

4.1 Consumers Decision Problem

We model, as discussed above, the decision problem of each consumer i in two stages. In the second stage, each consumer i is assumed to select the product which provides the highest expected conditional indirect utility, given her posterior belief $F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i)$:

$$U\left(F\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i\right)\right) \equiv \max_{j\in\{1,2\}} \int\limits_{(\boldsymbol{\delta},\mathbf{x},\mathbf{p})} u_{ij} F\left(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}|\mathbf{sg}_i\right), \tag{5}$$

where $U(F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i))$ denotes the highest expected utility induced by the information gathering and processing strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ chosen in the first stage.

We assume that the choice of strategy $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, in the first stage, is driven by the desire to maximize the *ex-ante* expectation over $U(F(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i))$ deducted of the cost of engaging in such strategy:

$$\max_{F(\mathbf{sg}_i,\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \int\limits_{\mathbf{sg}_i} \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} U\left(F\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}|\mathbf{sg}_i\right)\right) F\left(d\mathbf{sg}_i|\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right) G\left(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}\right) - c\left(F\left(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right); \gamma_i\right) \tag{6}$$

$$\text{such that} \int\limits_{\mathbf{sg}_i} F\left(d\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right) = G\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}\right),$$

where $F(\mathbf{sg}_i|\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ and $\int_{\mathbf{sg}_i} F(d\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ ensures that the posterior beliefs about $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ are consistent with the prior.

Proposition 1 *The solution to consumer i's decision problem involves a first stage choice of information gathering and processing strategy,* $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*, that implies a second stage purchase of product j, conditional on the realization* $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*, with probability:*

$$\Pr_{ij}\left(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i\right) = \frac{\Pr^0_{ij} e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} \Pr^0_{ik} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} \quad \textit{almost surely,} \tag{7}$$

where $\Pr^0_{ij} = \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ *denotes the unconditional probability (i.e. before engaging in any information gathering and processing strategy) that the consumer purchases product j, which is computed across the different realizations of* $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ *according to the prior belief* $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*.*

Proof. See online Appendix B. ■

Proposition 1 establishes that consumer i's conditional probability of purchasing product j has three drivers. First, consumer i's indirect utilities u_{ik} for $k \in \{1, 2\}$, whose impact follows the lines of the discrete-choice literature: the probability that the consumer purchases product j increases with the utility derived from product j and decreases with the utility derived by the competing product $k \neq j$. Second, consumer i's *a priori* unconditional choice probabilities $\Pr_{ik}^0$ for $k \in \{1, 2\}$, whose impact follows the rational inattention literature: when the consumer has a high *a priori* unconditional probability of purchasing product j, the conditional probability can be high even if the product gives the consumer a *true* low indirect utility. Third, consumer i's unit cost of gathering and processing information γ_i, which weights the importance of the above two drivers: when γ_i is high, the consumer rationally gathers and processes less information and so a higher degree of uncertainty about the attributes (and therefore about the induced indirect utilities) of the different products will persist at the time she makes the purchase decision. In such case, the consumer bases her decision more on prior beliefs. This result is consistent with several recent empirical studies documenting that consumers process relatively little information in car insurance (Honka, 2014), S&P 500 index funds (Hortaçsu and Syverson, 2004), and automobiles (Moorthy, Ratchford and Talukdar, 1997; Morton, Silva-Risso and Zettelmeyer, 2011), industries associated (for different reasons) with high unit costs of gathering and processing information.

The computation of the conditional choice probabilities of each consumer i established in Proposition 1 requires the ex-ante computation of the unconditional probabilities $\Pr_{ik}^0$ of the consumer for $k \in \{1, 2\}$, across the different realizations of $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ according to the prior belief $G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$. To do so, we make the following assumption about $G(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p})$.

Assumption 1 *Consumers have no prior knowledge about the attributes of the different products before entering the choice situation.*

This assumption, in line with the search literature (see, e.g., Bakos, 1997), implies that products are exchangeable in the prior $G(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p})$ and therefore, from a consumer perspective, *a priori* homogeneous. As a result, the unconditional probability that consumer i chooses to purchase product 1 matches the corresponding probability for product 2: $\Pr_{i1}^0 = \Pr_{i2}^0 = 1/2$.

Corollary 1 *Under Assumption* 1*, the solution of consumer* i*'s decision problem involves* *a first stage choice of information gathering and processing strategy,* $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*, that implies a second stage purchase of product* j*, conditional on the realization* $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$*, with probability:*

$$\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i) = \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} \quad \textit{almost surely.} \tag{8}$$

Having computed the conditional purchase probabilities of each consumer i for the two products, we derive the aggregate demand for each product by integrating the corresponding consumer-specific probabilities over the probability distribution of consumer types $P(\gamma_i, \theta_i, v_i)$. The aggregate demand, $D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, for each product j is thereby given by:

$$D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = \int_{\gamma_i} \int_{v_i} \int_{\theta_i} \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} P(d\gamma_i, dv_i, d\theta_i) \quad \textit{almost surely.} \tag{9}$$

We make the following assumptions about the probability distribution of consumer types $P(\gamma_i, v_i, \theta_i)$.

Assumption 2 *Consumer types over the unit cost of gathering and processing information and the different product attributes are independently distributed:* $P(\gamma_i, v_i, \theta_i,) = P_\gamma(\gamma_i) P_v(v_i) P_\theta(\theta_i)$.

Assumption 3 *Consumer types for each product attribute are uniformly distributed.*

Assumption 2 allows us to rule out the trivial case in which product differentiation arises from correlation between consumer types, whereas Assumption 3 allows us to eliminate the effect of nonuniformity of preferences over attributes as a possible explanation of equilibrium product positioning (Moorthy, 1988). Both regularities, correlation between consumer types and non-uniform preference distribution (e.g., unimodal or bimodal), may lead to trivial standardization or differentiation (Neven, 1986), and confounds the effect of information frictions, which is what we wish to analyze.

Assumptions 2 and 3 imply that the aggregate demand for product j is given by:

$$D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = \int_{\gamma_i}\int_0^1\int_0^1 \frac{e^{\left(-p_j-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} e^{\left(-p_k-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}} P_\gamma(d\gamma_i)\, P_v(dv_i)\, P_\theta(d\theta_i) \qquad \textit{almost surely.} \tag{10}$$

4.2 Firms Decision Problem

We model, as discussed above, the decision problem of each single-product firm j in two stages. The sub-game perfect Nash equilibrium of the game involving the decision problems of the two firms is obtained by backward induction. In the second stage, each firm is assumed to (simultaneously) set the prices that provide the highest expected profit, taking as fixed the set of first stage product designs and qualities, $(\mathbf{x}, \boldsymbol{\delta})$:

$$\max_{p_j} \Pi_j(\boldsymbol{\delta}, \mathbf{x}, \mathbf{p}; \varphi) = p_j D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) - C_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \varphi), \tag{11}$$

where $C_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \varphi) = mc(\delta_j; \varphi) D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) = \left(\varphi\delta_j^2/2\right) D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ denotes the cost function of firm j, that yields the total cost incurred by firm j in supplying a product of quality δ_j to aggregate demand $D_j(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$.

A Nash equilibrium $\mathbf{p}^*$ in the second stage sub-game is a pair of prices p_j^* and p_{-j}^* such that, for any pair of product designs, $\bar{\mathbf{x}} = (\bar{x}_j, \bar{x}_{-j})'$, and qualities, $\bar{\boldsymbol{\delta}} = \left(\bar{\delta}_j, \bar{\delta}_{-j}\right)'$, we have that:

$$\Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*; \varphi\right) \geq \Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j, p_{-j}^*; \varphi\right), \ \forall p_j \geq 0, \ j = 1, 2. \tag{12}$$

The following results characterize the price equilibrium $\mathbf{p}^*$ in pure strategies.

Lemma 1 *If $P_\gamma(\gamma_i)$ is a log concave function, there exists almost surely an unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$.*

Proof. See online Appendix B. ■

Lemma 2 *If $P_\gamma(\gamma_i)$ is a log concave function, the price vector $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ that supports the almost surely unique Nash equilibrium in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, is strictly positive.*

Proof. See online Appendix B. ■

Lemmas 1 and 2 imply that the almost surely unique Nash equilibrium $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, must satisfy the following system of first-order equations for all $j \in \{1, 2\}$:

$$\frac{\partial \Pi_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*; \varphi\right)}{\partial p_j} = D_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*\right) + \left(p_j^* - \varphi\bar{\delta}_j^2/2\right) \frac{\partial D_j\left(\bar{\mathbf{x}}, \bar{\boldsymbol{\delta}}, p_j^*, p_{-j}^*\right)}{\partial p_j} = 0, \quad (13)$$

which must have a unique solution $\mathbf{p}^*$, since any solution $\mathbf{p}^*$ must be almost surely a Nash equilibrium in pure strategies, and Lemma 1 establishes that $\mathbf{p}^*$ is almost surely unique. This almost surely unique Nash equilibrium defines prices to be functions of the pair of product designs and qualities in the market: $p_j^*(\mathbf{x}, \boldsymbol{\delta})$ and $p_{-j}^*(\mathbf{x}, \boldsymbol{\delta})$, which establish an equilibrium mapping from the vector of product attributes $(\mathbf{x}, \boldsymbol{\delta})$ to the vector of prices chosen by firms $\mathbf{p}$.

Having established that, if $P_\gamma^*(\gamma_i)$ is a log concave function, there exists almost surely a unique Nash price equilibrium in pure strategies in the second-stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\boldsymbol{\delta}}$, we now address the first stage sub-game. If we substitute $p_j^*(\mathbf{x}, \boldsymbol{\delta})$ and $p_{-j}^*(\mathbf{x}, \boldsymbol{\delta})$ in firm $j \in \{1, 2\}$'s profits, we have:

$$\Pi_j\left(\mathbf{x}, \boldsymbol{\delta}, p_j^*(\mathbf{x}, \boldsymbol{\delta}), p_{-j}^*(\mathbf{x}, \boldsymbol{\delta}); \varphi\right) = \Pi_j^*(\mathbf{x}, \boldsymbol{\delta}; \varphi). \quad (14)$$

The first stage Nash equilibrium in designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$ is a pair of designs x_j^* and x_{-j}^*, and a pair of qualities δ_j^* and δ_{-j}^*, such that, for all $j \in \{1, 2\}$:

$$\Pi_j^*\left(x_j^*, x_{-j}^*, \delta_j^*, \delta_{-j}^*; \varphi\right) \geq \Pi_j^*\left(x_j, x_{-j}^*, \delta_j, \delta_{-j}^*; \varphi\right), \ \forall x_j \in [0, 1], \delta_j \in \left[\underline{\delta}, \bar{\delta}\right]. \quad (15)$$

The complexity of this problem makes it difficult to find an analytical solution. We therefore resort to a numerical procedure in the lines of Rhee *et al.* (1992), Heeb (2001) and Matějka and McKay (2012). This procedure makes use of a grid of product designs (x_j, x_{-j}) and qualities (δ_j, δ_{-j}) so to cope with eventual multiple equilibria.[14] The details are as follows. *For each pair* of product designs and qualities $(\mathbf{x}, \boldsymbol{\delta})$ in the grid, we derive the almost surely unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage subgame using the system of first-order equations (13). We then use $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}^*)$ to compute the corresponding profits for the two firms. Next, we use these profits to find the best response function of each firm $j \in \{1, 2\}$ in terms of product design and quality: $(x_j, \delta_j) = f(x_{-j}, \delta_{-j})$. Finally, we identify the intersection (intersections) that characterize the almost surely Nash equilibrium (equilibria) in designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$. The vectors of product designs and qualities $(\mathbf{x}^*, \boldsymbol{\delta}^*)$ and prices $\mathbf{p}^*$ constitute almost surely a subgame perfect Nash equilibrium. We examine this equilibrium for a realization of the production technology in which - following Bar-Isaac, Caruana and Cuñat (2012) - there are no costs associated with choosing different product designs: $\eta = 0$, while - following Mussa and Rosen (1978) - there may exist costs associated with choosing different product qualities: $\varphi \geq 0$. In doing so, we consider that (i) information costs can be homogenous or heterogenous across consumers and (ii) quality improvement may or may not be costly.

4.2.1 Homogenous Information Costs

Given the second stage almost surely Nash equilibrium in prices, we begin by examining the almost surely first stage Nash equilibrium in designs and qualities for the case in which consumers are homogeneous in their unit costs of gathering and processing information.

Assumption 4a $\gamma_i = \gamma$ *for all* i.

This implies that the probability distribution of the unit cost of information across consumers is a $0-1$ indicator function over a convex set, as follows:

$$P_\gamma^*(\gamma_i) = \begin{cases} 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \\ 0 & \text{otherwise} \end{cases}, \tag{16}$$

[14] We define the grid with an initial size of 5×10^{-2}, which we decrease whenever necessary to narrow our results.

which constitutes a classical example of a log concave function, as required by Lemma 1.

Costless Quality We first examine the implications of Assumption $4a$ for the case in which the costs of quality improvement are null, following Shaked and Sutton (1982) and Neven and Thisse (1987, 1990). Such case corresponds to the following assumption.

Assumption 5a $\varphi = 0$.

The following result establishes the first stage almost surely Nash equilibria in designs and qualities for the setting described above.

Proposition 2 *Under Assumptions* $4a$ *and* $5a$*:*

(a) If $\gamma \geq 0.51$*, there exists almost surely a single Nash equilibrium in designs and qualities: a* *min-min* *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(b) If $0.43 \leq \gamma < 0.51$*, there exist almost surely two Nash equilibria in designs and qualities: (1) an* *intermediate-min* *equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as* γ *decreases), and minimize quality differentiation, given by:* $x_j < 1$, $x_{-j} = 1 - x_j > 0$ *and* $\delta_j = \delta_{-j} = 4$*, and (2) a* *min-min* *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(c) If $0.39 \leq \gamma < 0.43$*, there exist almost surely two Nash equilibria in designs and qualities: (1) a* *max-min* *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$*, and (2) a* *min-min* *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 4$.

(d) If $\gamma < 0.39$*, there exist almost surely two Nash equilibria in designs and qualities: (1) a* *min-max* *equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by:* $x_j = x_{-j} = 1/2$*, and* $\delta_j = 4$, $\delta_{-j} = 1$*, and (2) a*

<u>max-min</u> *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$*,* $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$*.*

Proposition 2 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists almost surely a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \overline{\delta} = 4$. The reason for this *min-min* differentiation equilibrium is that given the high costs of gathering and processing information, consumers rationally choose to gather and process a low level of information. As a result, a high degree of uncertainty about the attributes of the products in the market will rationally persist at the time consumers make a purchase decision. As a consequence, consumers base the purchase decision mostly on prior beliefs. This implies that they are not too sensitive to actual prices and, thus, attribute differentiation is not required to relax price competition.

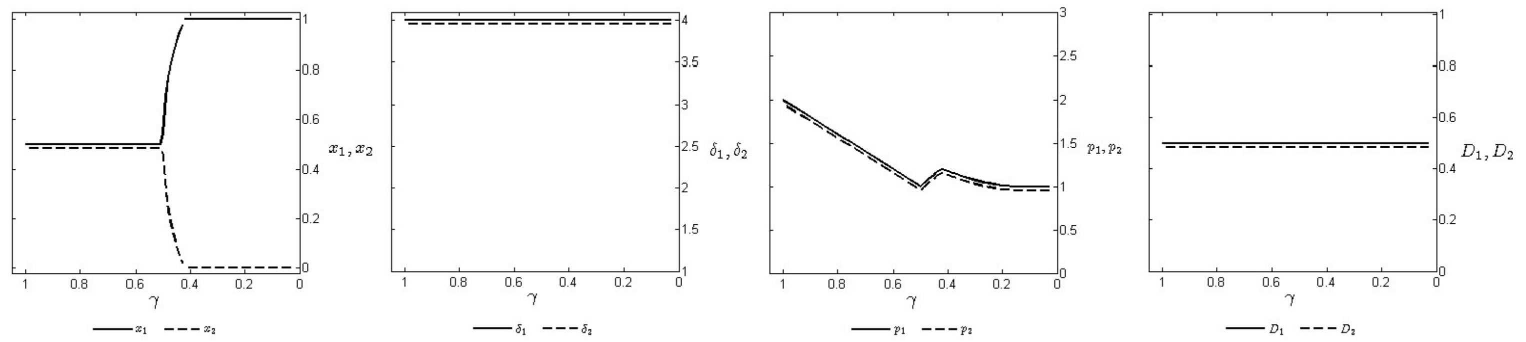
Proposition 2 also implies that, as the unit cost of gathering and processing information decreases, product attributes become instrumental in relaxing price competition. In order to see why, note that, as that cost decreases, consumers rationally gather and process relatively more information, which generates more precise signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes at the time consumers make a purchase decision, decreases, increasing price competition between the two firms and decreasing the equilibrium price level. Firms respond, so to relax the increasing price competition, by engaging in attribute differentiation strategies. *Three* equilibrium strategy paths (depicted in Figure 2) emerge from Proposition 2, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$.[15]

[15] The difference among the three equilibrium strategy paths presented is due to the functional form distinction between the marginal flow utility of design and quality, discussed in section 3.2. First, the primary attribute dimension to be differentiated, as the unit cost of gathering and processing information decreases, is design. The reason being that as that cost decreases to levels below $\gamma = 0.51$, the incentives to deviate from the min-min equilibrium (in which firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \overline{\delta} = 4$) by differentiating the

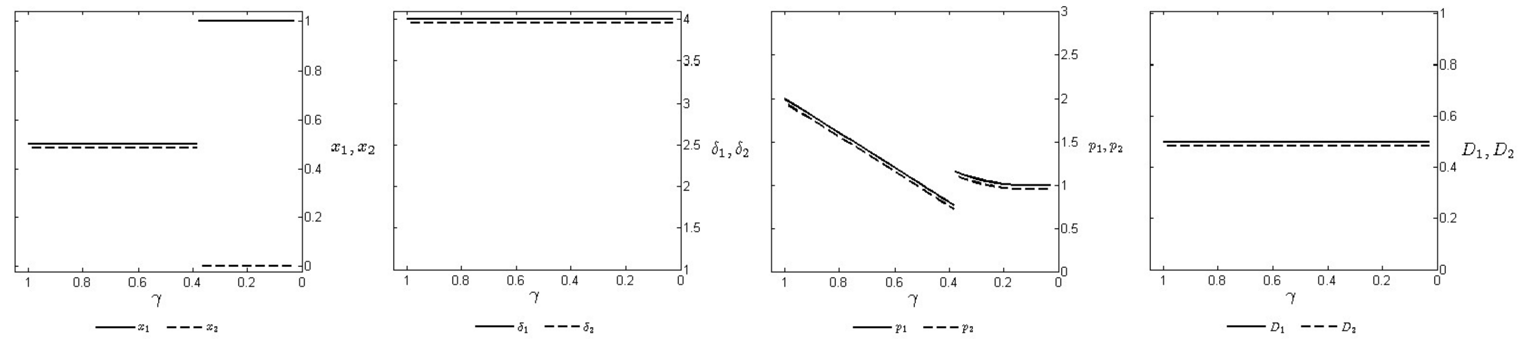
FIGURE 2

Equilibrium Designs, Qualities, Prices and Aggregate Demands under Assumptions 4a and 5a

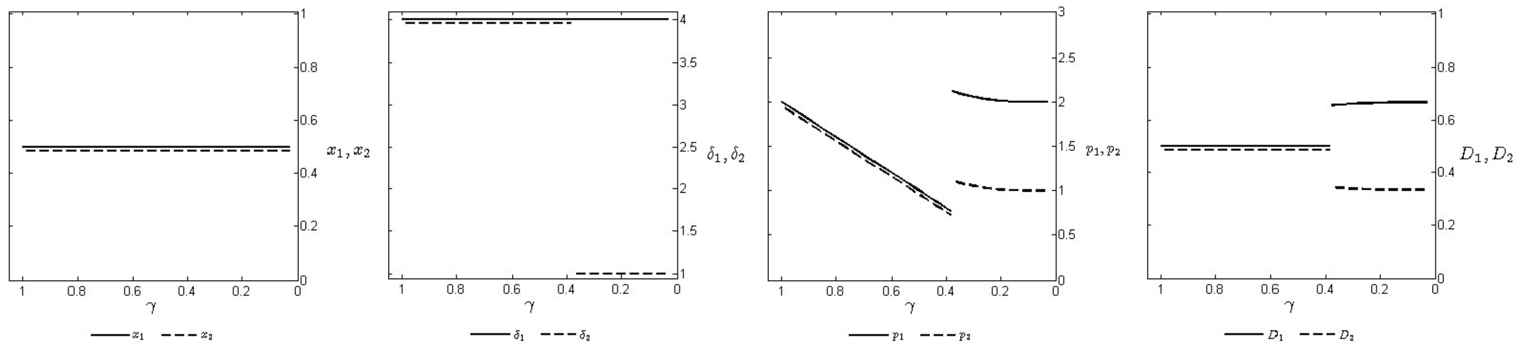
1. *min-min >>> intermediate-min >>> max-min path*



2. *min-min >>> max-min path*



3. *min-min >>> min-max path*



A *min-min >>> intermediate-min >>> max-min path*, characterized by a continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards the *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). This convergence occurs through a series of *intermediate-min* equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases. Both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

A *min-min >>> max-min path*, characterized by a discrete shift, that occurs at $\gamma = 0.43$, from the *min-min* equilibrium towards the *max-min* equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

A *min-min >>> min-max path*, characterized by a discrete shift, that occurs at $\gamma = 0.39$, from the *min-min* equilibrium towards the *min-max* equilibrium, in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$). The *min-max* equilibrium segments the market according to the valuation of consumers for quality: high-valuation consumers are targeted by the high-quality (hence, high-price) product, whereas low-valuation consumers are targeted by the low-quality (hence, low-price) product. This yields an asymmetric outcome in terms of price, aggregate demand,

design attribute are higher than the incentives to deviate by differentiating the quality attribute. In order to see why this is the case, note that the expectation of the marginal flow utility due to a decrease in the quality of a given product across consumers is given by $-E(\theta_i) = -0.5$, whereas the expectation of the marginal utility due to an increase in the design of a given product across consumers is given by $E[2(v_i - 0.5)] = 0$. Second, differentiation in quality always exhibits a discrete path (in γ), in contrast with differentiation in design, which also exhibits a continuous and gradual path (in γ). The reason being that the marginal utility for design is product-specific and decreases with the level of design, as discussed in Section 3.2, which penalizes high magnitude deviations in the design level.

and profit, which favours the high-quality product.

The three equilibrium strategy paths above imply that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.39$, no *min-min* equilibrium is sustainable, establishing a *differentiation principle.* In such situations, the *min-max* and *max-min* equilibria coexist, in the lines of Neven and Thisse (1987, 1990),[16] establishing that the almost surely Nash equilibrium is robust to small deviations in the unit cost of information (as it is continuous in the degree of information frictions). Interestingly, firms are not indifferent between the two equilibria. The asymmetric outcome of the *min-max* strategy is favoured by the high-quality firm (but not by the low-quality firm) when compared to the symmetric outcome of the *max-min* strategy.

Costly Quality We now examine the impact of incorporating the (more realistic) assumption that firms must incur in costs of quality improvement, following Moorthy (1988). In order to do so, we make the following assumption.

Assumption 5b $\varphi = 1 > 0$.

The following result establishes the first stage almost surely Nash equilibria in designs and qualities for the setting described by Assumptions $4a$ and $5b$.

Proposition 3 *Under Assumptions* $4a$ *and* $5b$*:*

(a) If $\gamma \geq 0.51$*, there exists almost surely a single Nash equilibrium in designs and qualities: a <u>min-min</u> equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 1$.

(b) If $0.43 \leq \gamma < 0.51$*, there exist almost surely two Nash equilibria in designs and qualities: (1) an <u>intermediate-min</u> equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as* γ

[16] This implies that the equilibria of our rational inattention model converges to the equilibria established in Neven and Thisse (1987, 1990)'s information frictionless model, as the unit cost of gathering and processing information becomes negligenciable. In other words, the introduction of information frictions does not change per se the nature of the attribute differentiation equilibria, which remains valid as long as that unit cost is negligenciable.

decreases), and minimize quality differentiation, given by: $x_j < 1$, $x_{-j} = 1 - x_j > 0$ *and* $\delta_j = \delta_{-j} = 1$, *and (2) a* *min-min* *equilibrium, in which firms minimize design and quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 1$.

(c) *If* $0.27 \leq \gamma < 0.43$, *there exist almost surely two Nash equilibria in designs and qualities: (1) a* *max-min* *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 1$, *and (2) a* *min-min* *equilibrium, in which firms minimize quality and design differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = \delta_{-j} = 1$.

(d) *If* $\gamma < 0.26$, *there exists almost surely a single Nash equilibrium in designs and qualities: a* *max-min* *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 1$.

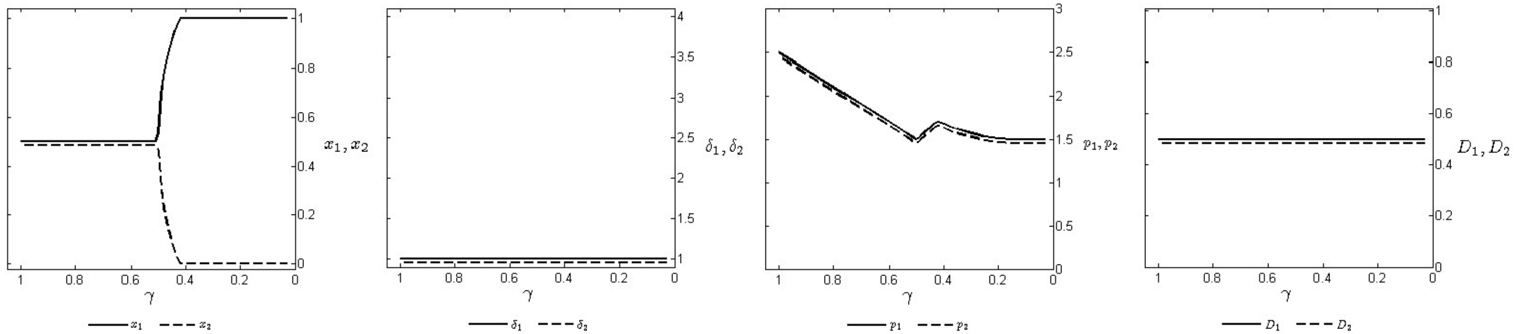
Proposition 3 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists, as in the costless quality case, a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, as in the costless quality case, but select, instead, the bottom (and not the top) quality, $\delta_j = \delta_{-j} = \overline{\delta} = 1$. The reason is as follows. In face of high information costs, consumers are highly uncertain about the attributes of the products at the time they make a purchase decision (since they rationally gather and process a low level of information and base their purchase decision mostly on prior beliefs), giving firms an incentive to deviate from specifications that incorporate costly attributes on the products.

Proposition 3 also implies, as in the costless quality case, that, as the unit cost of gathering and processing information decreases, attribute differentiation strategies become instrumental in relaxing price competition. *Two* equilibrium strategy paths (depicted in Figure 3) emerge from Proposition 3, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$.

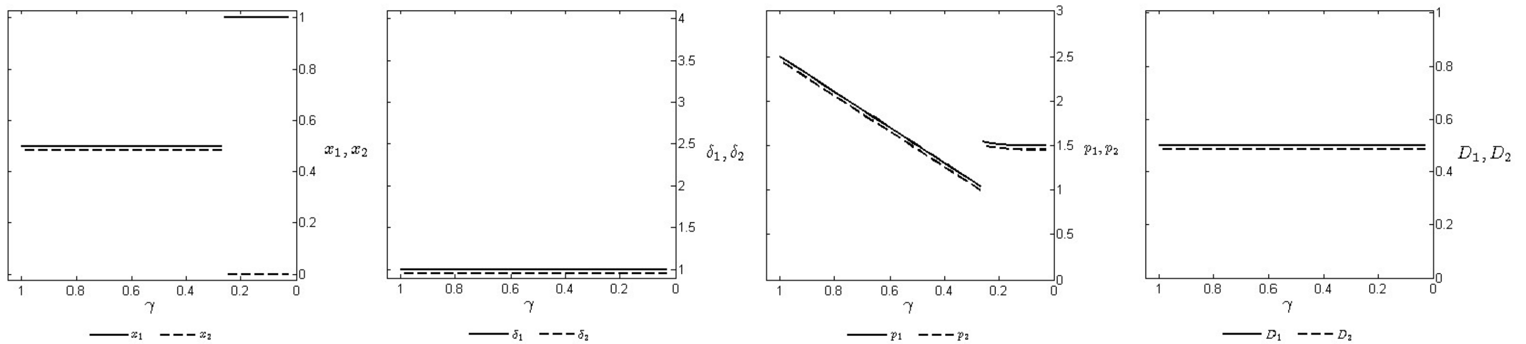
Figure 3

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4a and 5b

1. *min-min >>> intermediate-min >>> max-min path*



2. *min-min >>> max-min path*



A *min-min >>> intermediate-min >>> max-min path*, characterized by a continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards the *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). This convergence occurs through a series of *intermediate-min* equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). As in the costless quality case, both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

A *min-min >>> max-min path*, characterized by a discrete shift, that occurs at $\gamma = 0.27$, from the *min-min* equilibrium towards the *max-min* equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

The two equilibrium strategy paths above imply that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.27$, no *min-min* equilibrium is sustainable, establishing, as in the costless quality case, a *differentiation principle*. However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation equilibrium exists.[17] The reason is as follows. Differentiation along one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation

[17] This implies that although the introduction of information frictions does not change per se the nature of Neven and Thisse (1987, 1990)'s attribute differentiation equilibria, the introduction of costs of quality improvement does change it.

strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

4.2.2 Heterogenous Information Costs

Given the second stage almost surely Nash equilibrium in prices, we now re-examine the almost surely first stage Nash equilibria in designs and qualities for the case in which consumers are heterogeneous in their costs of gathering and processing information. To do so, we follow Salop and Stiglitz (1977) in assuming that there are only two groups of consumers. The "informed" consumers (that can gather and process information at no cost) and the "uninformed" consumers (that must incur a cost to gather and process information). As in Salop and Stiglitz (1977), this assumption is made for analytic convenience only, it is not crucial to any of the results obtained. Finally, in order to illustrate the differential impact towards the homogeneous information costs' case, we make the simplest assumption that the proportion of "informed" consumers is of a sizeable dimension, as in Assumption 4*b* below. The equilibrium for cases in which the proportion of "informed" consumers is smaller converges gradually from the ones established in the previous section towards the ones established in this section.

Assumption 4b *There are two equally-sized groups of consumers. An "informed" group* Γ_a *with* $\gamma_i \to 0$ *for all* $i \in \Gamma_a$ *and an "uninformed" group* Γ_b *with* $\gamma_i = \gamma > 0$ *for all* $i \in \Gamma_b$*.*

This implies that the probability distribution of the unit cost of information across consumers, within each group, is a $0-1$ indicator function over a convex set, as follows:

$$P_{\gamma}^{*}\left(\gamma_i\right) = \begin{cases} 1 & \text{if } \gamma_i \to 0 \text{ for all } i \in \Gamma_a \\ 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \in \Gamma_b \\ 0 & \text{otherwise} \end{cases}, \tag{17}$$

which constitutes a classical example of a log concave function, as required by Lemma 1.

Costless Quality We first examine the implications of Assumption $4b$ for the case in which the costs of quality improvement are null, i.e., under Assumption $5a$. The following result establishes the corresponding first stage almost surely Nash equilibrium in designs and qualities.

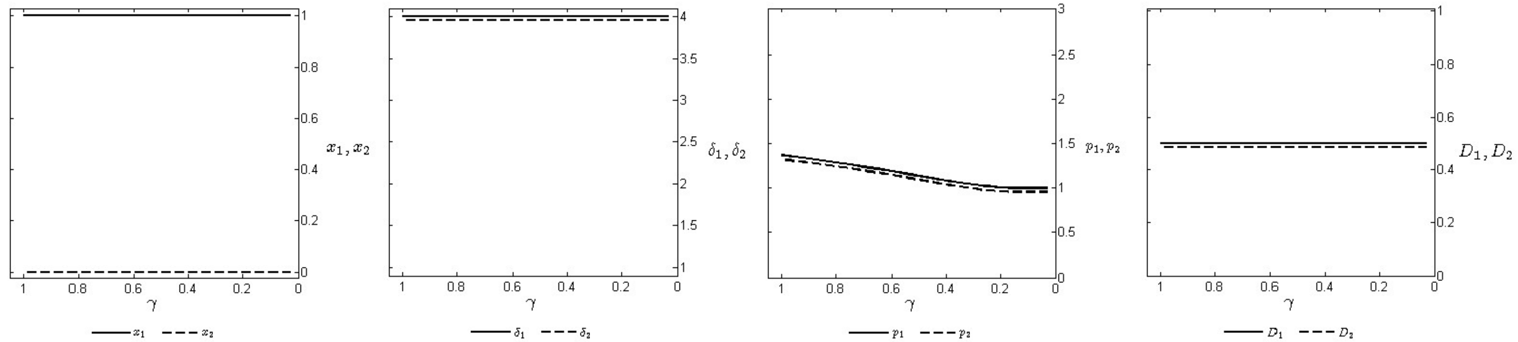
Proposition 4 *Under Assumptions* $4b$ *and* $5a$*, there exist almost surely two Nash equilibria in designs and qualities: (1) a* *min-max* *equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = 4$, $\delta_{-j} = 1$*, and (2) a* *max-min* *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$.

Proposition 4 implies that, in face of two equally-sized groups of consumers, one "informed" and another "uninformed", product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the high cost consumers. The reason is as follows. The group of "informed" consumers rationally gathers and processes information that generates accurate signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes, at the time those consumers make a purchase decision, is null. If this group of consumers is of a sizeable dimension (as in Assumption $4b$), the competing firms must engage in attribute differentiation strategies, in order to relax the otherwise fierce price competition (required to attract those "informed" consumers). In particular, Proposition 4 establishes that two equilibrium strategies coexist, as depicted in Figure 4. A *min-max* equilibrium, in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$), and a *max-min* differentiation equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). The latter yields, as discussed above, a symmetric outcome in terms of price, aggregate demand, and profit, whereas the former yields an asymmetric outcome in terms of price, aggregate demand, and profit, which favours the high-quality firm. Interestingly, also as discussed above, firms are not indifferent between the two equilibria. The

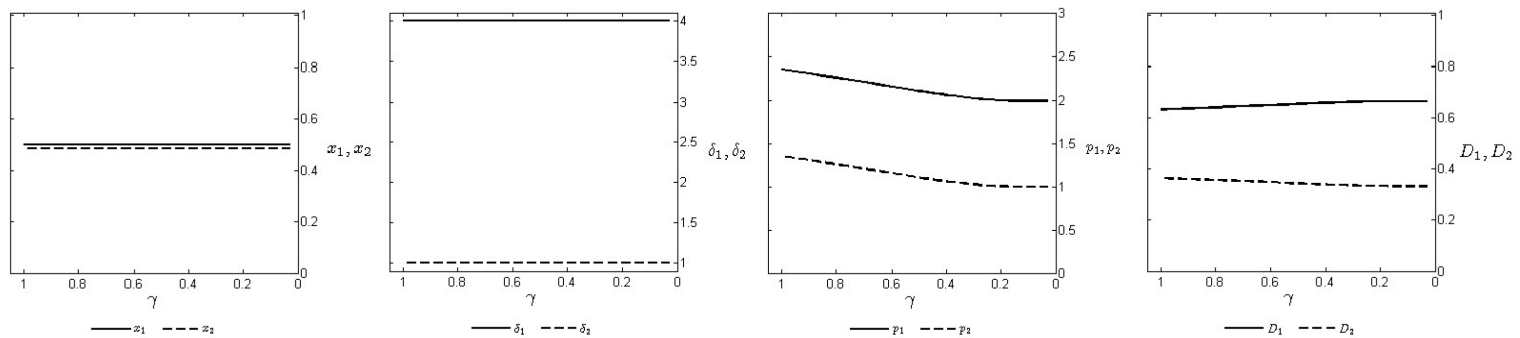
Figure 4

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4b and 5a

1. *max-min equilibrium*



2. *min-max equilibrium*



asymmetric outcome of the *min-max* strategy is favoured by the high-quality firm (but not by the low-quality firm) when compared to the symmetric outcome of the *max-min* strategy.

Costly Quality We now examine the implications of Assumption 4b for the case in which firms must incur in costs of quality improvement, i.e., under Assumption 5b. The following result establishes the corresponding first stage almost surely Nash equilibrium in designs and qualities.

Proposition 5 *Under Assumptions* 4b *and* 5b*, there exists almost surely a single Nash equilibrium in designs and qualities: a* <u>*max-min*</u> *equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$*,* $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 1$.

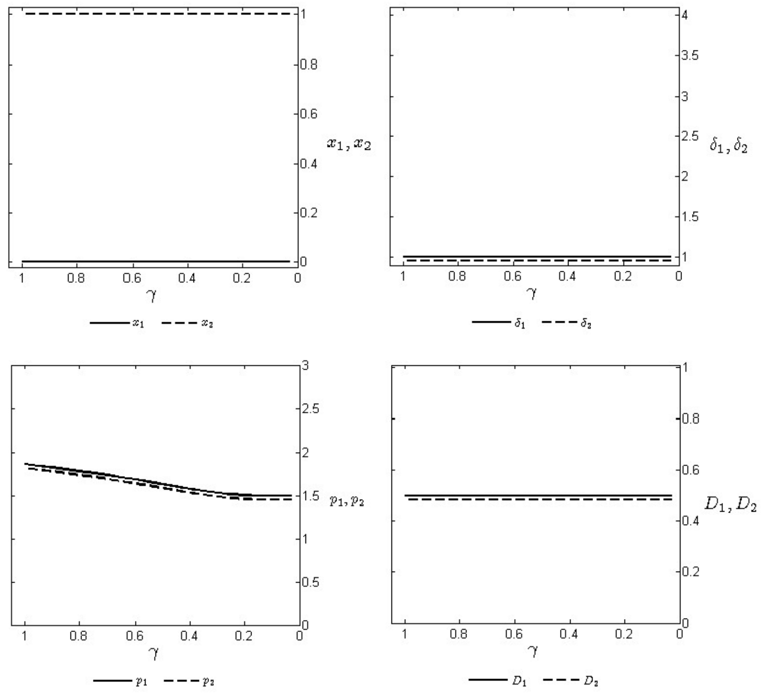
Proposition 5 implies, as in the costless quality case, that, in face of two equally-sized groups of consumers, one "informed" and another "uninformed", product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the "uninformed" consumers - for exactly the same reason as described above. However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation strategy exists in equilibrium, as depicted in Figure 5. The reason is as follows. Differentiation along one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

5 Managerial and Policy Implications

This section summarizes the implications of our results. We begin by addressing the managerial implications. We focus on three. First, Propositions 5 and 6 imply that managers of firms that face a single homogeneous group of "uninformed" consumers in

Figure 5

Equilibrium Product Designs, Qualities, Prices and Aggregate Demands under Assumptions 4b and 5b



their information costs *do not have an incentive to differentiate their products when information costs are high*, but should *increase differentiation as those information costs fall*, so to relax the otherwise increasing price competition. Independently of whether quality improvement is costly or not. This suggests that the unit cost of information and product differentiation are *substitutes*. Further, since product differentiation countervails the negative impact on prices, Propositions 5 and 6 also imply, as depicted in Figures 2 and 3, that equilibrium price levels *may increase* as the unit cost of gathering and processing information decreases. These implications are consistent with the standard search literature. See, for example, as discussed above, Kuksov (2004), Bar-Isaac, Caruana and Cuñat (2012), Larson (2013), and Fishman and Levy (2015). Furthermore, they are also consistent with the empirical evidence in the literature. See, for example, Lynch and Ariely (2000) who find (in an experiment with an homogeneous group of MBA and Ph.D. students) that wine retailers have incentives to respond to lower information costs by carrying more differentiated products.

Second, Propositions 7 and 8 imply that managers of firms that face two equally-sized and heterogeneous groups of consumers in terms of their information costs, one "informed" (that can gather and process information at no cost) and another "uninformed" (that must incur a cost to gather and process information), should *always differentiate their products as maximum as possible*, and independently of whether quality improvement is costly or not. This suggests that the firms' incentive (identified above) to respond to lower information costs by increasing differentiation *only holds* if the proportion of "informed" consumers in the market is small. Further, Propositions 7 and 8 also imply, as depicted in Figures 4 and 5, that equilibrium price levels *do not increase* (and, in fact, tend to decrease) as the unit cost of gathering and processing information of the "uninformed" consumers decreases. In order to see why note that, as this unit cost decreases, "uninformed" consumers rationally gather and process more information, which generates more precise signals about the attributes of the products in the market. As a consequence, the degree of uncertainty that rationally persists about product attributes at the time those consumers make a purchase decision, decreases, increasing price competition (to attract not only the "informed" consumers, but also the "uninformed" ones). This result seems to suggest that the incentives of firms, identified by the standard search literature, to respond to lower information costs by increasing differentiation *depend critically* on the heterogeneity of those costs across consumers. This implication is consistent with, for example, Brown and Goolsbee (2002)'s finding that the rise of the Internet from 1995 to 1997 appears to have reduced the prices of term life insurance (typically purchased by an heterogenous group of consumers) by about 8-15 percent.

Third, Propositions 5 to 8 imply that, in the two cases above, firms *do not need to differentiate their products along all attribute dimensions.* Differentiation along one attribute dimension is more than enough to relax price competition. In a costless quality setting, firms may, in equilibrium, differentiate along *any* attribute dimension, while in a costly quality setting, firms should, in equilibrium, differentiate along the *least-costly* attribute dimension. This extends Neven and Thisse (1987, 1990)'s result to imperfect information settings, making clear that the equilibrium outcomes derived from our theoretical framework are (unlike most standard search literature) robust to small deviations in information costs (as they are continuous in the degree of information frictions). Further, it also makes clear that the differentiation principle identified by the standard search literature does not hold for all attributes when firms compete in a multi-attribute space. Finally, it also makes clear that when the cost of quality improvement in a market changes, there can be radical shifts in product attributes and prices. These implications are consistent, for example, with the shift observed in the U.S. coffee market after the entry of Starbucks, which shifted from a low-quality equilibrium to a high-quality equilibrium after a decrease in the cost of quality. Bordalo, Gennaioli and Shleifer (2016) explain this shift with the introduction, by Starbucks, of a technology which drastically reduced the cost of quality improvement. This cost reduction induces an increase in quality and, in turn, an increase in the attention that consumers assign to quality. We provide an alternative explanation. In a costless quality setting, firms may, in equilibrium, differentiate along any attribute dimension, while in a costly quality setting, firms should, in equilibrium, differentiate along the least-costly attribute dimension.

We now address the policy implications. We focus on one main implication. Our results suggest that consumers may optimally select information strategies that do not fully eliminate their uncertainty, i.e., they *may choose to be rationally inattentive* when making a purchase decision. This creates market power for firms and relaxes price competition. Regulators can countervail this market power by providing conditions for the existence of a sufficient large group of "informed" consumers. This group of consumers intensifies price competition and serves as a "market competition guardian".

6 Conclusion

We contribute to the literature of product positioning under consumer information frictions (Kuksov, 2004; Bar-Isaac, Caruana and Cuñat, 2012; Larson, 2013; Fishman and Levy, 2015) in two aspects. First, we examine product positioning on a *multi-attribute* space. Second, we model the process of gathering and processing information according to the *rational inattention framework*. We show that, under this setting, the competition is inherently different from the standard search model. Firms do have an incentive to respond to lower information costs by increasing differentiation but solely *if the proportion of "informed" consumers in the market is small* and *along the least-costly attribute dimension*. This implies that equilibrium prices may, as the unit cost of gathering and processing information decreases, increase in some markets and decrease in others. Further, it implies also that when the cost of quality improvement in a market changes, there can be radical shifts in equilibrium product attributes.

The paper considers a set of assumptions whose relaxation seem a very interesting area of future research. We highlight the following: (i) considering a higher number of firms in the market, (ii) assuming that consumers' income is not large enough for all consumers to find a product that generates a positive utility in equilibrium, (iii) relaxing the additive separability between income and prices in the conditional indirect utility function, and (iv) including reputation issues that arise in multiple period settings.

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Online Appendix A - Literature Review

In this Appendix, we provide a more detailed description of the literature on product positioning under perfect information and under consumer Information Frictions.

Product Positioning under Perfect Information

The theory of product positioning under perfect information begins with Hotelling (1929), who introduces the idea that products compete on more than just price. As discussed in the main text, price is an important aspect, but it is definitely not the sole one. Hotelling (1929) argues that a firm does not "*lose all his trade instantly when he raises his price only a trifle. Many customers will still prefer to trade with him because they live nearer to his store than to the others, or because they have less freight to pay from his warehouse to their own, or because his mode of doing business is more to their liking, or because he sells other articles which they desire (...) or for a combination of reasons.*" He illustrates this idea by developing a strategic duopoly model that, in addition to price, incorporates a "location" attribute, which can be interpreted literally as a product's geographic location in real space or figuratively as a product's "location" in some specification spectrum.

The strategic problem of each firm is to select its price and "location" so to maximize its own profit, recognizing that the other firm is doing exactly the same. The two firms do so in two stages. In the first stage, each firm chooses the "location" of its product. In the second stage, firms set prices. The intuition behind the two-stage structure assumption lies on the fact that prices are more flexible than "locations" in the short run. Thus, as discussed above, the second stage can be interpreted as the short-run where only prices are flexible, while the first stage can be viewed as the long-term when strategic decisions to determine the "location" position are taken.

Horizontal Differentiated Attributes

Hotelling (1929) assumes a continuum of consumers that, after observing the available "location" attributes and prices, make indivisible and mutually exclusive purchase decisions involving the two products. That is to say, consumers are not given the option of making no purchase (i.e., implicitly assuming that the two products serve the whole market). Further, he assumes that there is no *a priori* ranking consensus among consumers for those "location" attributes. In this sense, "location" reflects an horizontal differentiation attribute (like the mode of doing business, assortment, color, style, etc.). We will address this attribute as *design*. At equal prices, *some consumers prefer and purchase design A*, while *others prefer and purchase design B*. In order to capture this feature, consumers are considered to be heterogeneous in terms of their ideal design and to bear an utility loss whenever purchasing a product with a design that differs from theirs. This implies that, all else constant, consumers prefer (and purchase) the product that has the design closest to their preference. As a consequence, the solution to the strategic problem of firms involves trading-off two opposing forces. On the one hand, firms have a *demand* incentive to choose a design similar to each other for any given prices, so to increase market share, by capturing consumers "located" within the two product designs. On the other hand, they also have a *strategic* incentive to choose designs as different as possible in order to relax price competition. Hotelling (1929) examines this trade-off under the assumption that consumers ideal designs are uniformly distributed along a compact linear real space and that the utility loss is linear in the distance between the consumer and the product designs. He concludes that the above trade-off is dominated by the demand incentive. As a consequence, firms should choose designs close to each other, near the "center" of the market, which establishes a *principle of no differentiation*, i.e., of *minimum differentiation*.

However, subsequent research by d'Aspremont, Gabszewicz and Thisse (1979) asserts this result to be invalid. Due to a flawed key calculation in Hotelling (1929), who neglects to consider strategies through which a firm undercuts the price of the rival to capture the whole market. They show that when these undercutting strategies are considered, no equilibrium price solution in pure strategies will in fact exist if the product designs are close to each other. In order to circumvent this outcome, d'Aspremont, Gabszewicz and Thisse (1979) suggests a slight modification to Hotelling (1929)'s example. Instead of considering the utility loss to be linear in the distance between the *actual* design of the product and the *ideal* design of the consumer, they assume it to be *quadratic*. This seems more appropriate since it allows the marginal loss to be increasing in that distance. Under this new more realistic assumption, they conclude that firms should choose designs as different as possible from each other, which implies that the strategic incentive in fact dominates the demand incentive and establishes a *principle of differentiation*. In this particular case, yielding *maximum differentiation*.[18]

[18] The result rests, though, on the assumption that consumers' ideal "location" is uniformly distributed. Neven (1986) relaxes this assumption by considering non-uniform distributions. He argues that when firms solve the strategic problem under this new assumption, they must trade-off three (and not only two) oppositing forces. The additional force is related

Vertical Differentiated Attributes

Hotelling (1929)'s formulation cannot be used, however, to capture a product's "location" in a *quality* specification spectrum, a vertical differentiation attribute for which there is *a priori* ranking consensus among consumers. In order to introduce this feature, Spence (1975) and Mussa and Rosen (1978) model consumers to be homogeneous in terms of their ideal quality (which is infinite quality), but heterogeneous in terms of their valuation (i.e., in terms of how much they are willing to pay) for it.[19] This implies that, at equal prices, *every* consumer *prefers a high-quality product* to *a low-quality product.* However, *some consumers purchase the former* (i.e., are willing to pay for it), while *others purchase the latter* (i.e., are not willing to pay for it).

Although Spence (1975) and Mussa and Rosen (1978) augment Hotelling (1929)'s formulation to cope with a vertical differentiation attribute, they focus only on the strategic problem of a monopolist. Shaked and Sutton (1982) are the first to examine the above trade-off for a duopoly setting. The solution to the firms' strategic problem in this setting involves trading-off two opposing forces. On the one hand, firms have a *demand* incentive to supply high-quality products, since consumers, for any given prices, prefer high-quality products to low-quality products. On the other hand, they also have a *strategic* incentive to differentiate the two products in order to relax price competition. In order to examine this trade-off, Shaked and Sutton (1982) assume a continuum of consumers that, after observing the available qualities and prices, make indivisible and mutually exclusive purchase decisions involving the two products, but are given the option of making no purchase (i.e., implicitly assuming that the two products may not serve the whole market). Further, they assume consumers' valuation for quality to be uniformly distributed on some positive support. Under these assumptions, they concluded that one of the firms should choose the highest feasible quality and that the other firm should choose a lower quality. This establishes that the *differentiation principle* also holds for vertical differentiated attributes. Tirole (1988) examines the exact same product positioning problem, but without giving consumers the option of not purchasing any of the two products. He concludes that, under this new assumption, one of the firms should choose the highest feasible quality and that the other firm should choose the lowest feasible quality, reconfirming Shaked and Sutton (1982)'s differentiation principle - in this particular case, yielding *maximum differentiation.*

Subsequent research by Moorthy (1988) points out that the above product differentiation equilibria would not exist if there were no upper bound on quality. The reason being that Shaked and Sutton (1982) assume (in line with all previous research) that quality is costless. This implies that the high-quality product would always have an incentive, given the low-quality product, to increase its quality further.

to the fact that, if consumers are non-uniformly distributed, firms also have an incentive to position close to the dense areas of the distribution. Neven (1986) examines this trade-off and concludes the product design equilibrium will depend on how concentrated the distribution of consumers' ideal designs really is, establishing that the differentiation principle still holds conditionally. When the distribution is not too concentrated, firms should choose designs as different as possible from each other, as in the uniform case. However, as the distribution becomes more concentrated, firms may eventually choose designs less far apart in order to position themselves close to the peak of the distribution.

[19] Consumer's heterogeneity in the valuation for a product's quality can be motivated, for example, by differences in income levels (Gabszewicz and Thisse, 1979).

This would increase revenues (and profits) for both firms since consumers are willing to pay more for better quality and the extra-differentiation (towards the low-quality product) relaxes price competition. In order to circumvent this outcome, Moorthy (1988) suggests a slight modification to Shaked and Sutton (1982)'s formulation. He assumes that each firm has a *quadratic marginal cost* (but no fixed cost) of supplying quality. This implies that a higher quality product costs more to produce than a lower quality product. And so, increasing quality drives revenues up, but costs more. Under this new more realistic assumption, he concludes that Shaked and Sutton (1982)'s *differentiation principle* still holds, with one of the firms supplying high quality (but now not the highest feasible one, which is too costly) and the other supplying a lower quality.

Product Positioning under Consumer Information Frictions

Consumer Search

The theory of product positioning (price wise) under consumer search dates back to Diamond (1971)' seminal paper, that examines the effect of consumer imperfect information (and consequently, of the costs consumers have to incur to search for information) on equilibrium prices. To do so, he considers a *homogeneous* product setting under sale at a variety of different firms in a multitude of time periods. The product is assumed durable and therefore consumers purchase it only once. However, consumers are uncertain about the current and future prices at which the product is (and will be) available at the different firms. In each period, in order to reduce this uncertainty, consumers can, at a cost, visit one firm and learn perfectly the corresponding current price. Consumers either purchase the product at this particular firm or choose to postpone the decision to the following period. They do so by comparing the cost of searching further with the expected gain from finding a lower future price at that particular firm or at a competing firm. Given this imperfect information setting and consumer behaviour, the strategic problem of each firm is to select its price so to maximize its own profit, recognizing that the other firms are doing exactly the same. Diamond (1971) concludes that the solution to this problem yields equilibrium prices that, in the presence of any search costs whatsoever, coincide with the joint profit maximizing price. This implies that (i) consumers' imperfect information relaxes price competition and creates market power (otherwise inexistent in this homogeneous product setting) for firms, and (ii) lower search costs do not decrease equilibrium prices.

Wolinsky (1986) and Anderson and Renault (1999) augment Diamond (1971)'s formulation to cope with an exogenous differentiated product setting, in which consumers are assumed to be heterogeneous in terms of their valuation for the products available in the market. They consider that consumers are uncertain not only about the price of the different products, but also about the values they attach to them. Under this new setting, the solution to the firms' strategic problem confirms that (i) consumers imperfect information relaxes price competition and provides an additional source of market power to firms, but does not present the quantitative conclusion (joint profit maximizing price) which was proposed by Diamond (1971). Instead, the solution yields that (*ii*) lower search costs decrease equilibrium prices.

Rational Inattention

Sims (1998) argues that they do so *rationally*, i.e., they choose to gather and process the information that maximizes their objective in the decision situation, subject to the aforementioned capacity-constraint. To do so, decision-makers have to quantify information flows. Sims (1998) follows the information theory literature and suggests quantifying this flow as the reduction that the information flow renders in the decision-maker's uncertainty, where uncertainty is measured using Shannon (1948)'s entropy function (this reduction in uncertainty is denoted mutual information in the language of information theory). Sims (2003) provides the first application of the above rational inattention idea, by examining its implications to the dynamic programming problem typically featured in many macroeconomic models.

Sims (1998, 2003) original rational inattention specification assumes that decision-makers do not incur in any cost to gather and process information, but cannot attend all the information that is freely available because of a *fixed* capacity-constraint to gather and process information flows. This implies, as discussed above, that decision-makers have to choose only what information to attend to and what information to ignore. An alternative specification of rational inattention considers that decision-makers *incur costs* in gathering and processing information (for a discussion, see, e.g., Caplin and Dean, 2013; de Oliveira, 2014; Matějka and McKay, 2015), typically assumed to be proportional to the flow of information gathered and processed. This implies that decision-makers, in addition to choosing what information to gather and process, must choose also *how much* information to gather and process. The total quantity of information gathered and processed is the amount that is optimal given the aforementioned cost. Under this new more realistic specification, decision-makers can (as pointed out by Matějka and McKay, 2015) gather and process more information when the stakes are high. Although the two specifications are not, in general, equivalent, for local statements they are (de Oliveira, 2014).[20]

The rational inattention literature has applied both specifications to a variety of different problems. In addition to those more related to our setting, these applications include problems of (*i*) consumption-saving decisions by individuals that are uncertain - and as a consequence must gather and process information - about wealth (see, e.g., Sims, 2003, 2006; Luo, 2008; Tutino, 2013), (*ii*) price setting decisions by firms that are uncertain - and as a consequence must gather and process information - about economic conditions (see, e.g., Mackowiak and Wiederholt, 2009; Woodford, 2009; Paciello and Wiederholt, 2014; Matějka, 2016), and (*iii*) portfolio decisions by investors that are uncertain - and as

[20] Under Sims (1998, 2003)' specification, the Lagrange function, that solves the decision-makers optimal choice of what information to gather and process, incorporates the constraint as a linear representation, where the Lagrange multiplier (the shadow price associated with the limited flow of information constraint) measures the cost of gathering and processing a unit of information.

a consequence must gather and process information - about asset payoffs (see, e.g., Van Nieuwerburgh and Veldkamp, 2009, 2010; Mondria, 2010; Cabrales, Gossner and Serrano, 2013; Yang, 2015).

Online Appendix B - Proofs

In this Appendix, we provide the proofs to the various propositions stated in the main body of the paper.

Proof to Proposition 1. The probability that each consumer i purchases, in the second stage, product j, conditional on the realization $(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$ and the information strategy, $F(\mathbf{sg}_i, \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, chosen in the first stage, is given by $\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) = \int_{\mathbf{sg}_i \in \Gamma_j} F(d\mathbf{sg}_i | \mathbf{x}, \boldsymbol{\delta}, \mathbf{p})$, where Γ_j denotes the set of signals that lead to the choice of product j.

Matějka and McKay (2015) show (see Corollary 1 therein) that the collection of the conditional probabilities above for consumer i, $\mathcal{P} = \{\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\}_{j \in \{1,2\}}$, is induced by a solution to her decision problem if and only if it solves the following optimization problem.

$$\max_{\mathcal{P}=\{\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\}_{j\in\{1,2\}}} \sum\nolimits_{j\in\{1,2\}} \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} u_{ij} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\, G(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}) - c(\mathcal{P}, G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}); \gamma_i), \tag{18}$$

subject to:

$$\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) \geq 0, \quad \forall j \in \{1,2\} \text{ and } \forall (\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) \in \mathbb{R}^6 \tag{19}$$

$$\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) = 1, \quad \forall (\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}) \in \mathbb{R}^6 \tag{20}$$

where the cost of information (given in equation (3)), can be calculated from $\mathcal{P}$, as follows:

$$\begin{aligned} c(\mathcal{P}, G(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}); \gamma_i) &= \gamma_i \Bigg(-\sum\nolimits_{j\in\{1,2\}} \Pr^0_{ij} \log\left(\Pr^0_{ij}\right) \\ &\quad + \int_{(\boldsymbol{\delta},\mathbf{x},\mathbf{p})} \left(\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i) \log\left(\Pr_{ij}(\mathbf{x}, \boldsymbol{\delta}, \mathbf{p}; \gamma_i, \theta_i, v_i)\right)\right) G(d\mathbf{x}, d\boldsymbol{\delta}, d\mathbf{p}) \Bigg). \end{aligned} \tag{21}$$

The Lagrangian of the problem above is:

$$\begin{aligned}\mathcal{L}(\mathcal{P}) \quad = \quad & \sum\nolimits_{j\in\{1,2\}} \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} u_{ij} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}) - \gamma_i \Bigg(-\sum\nolimits_{j=1}^{2} \Pr^0_{ij}\log\left(\Pr^0_{ij}\right) \\ & + \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \left(\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\log\left(\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right)\right) G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p})\Bigg) \\ & + \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}) \\ & - \int\limits_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \rho_i(\boldsymbol{\delta},\mathbf{x},\mathbf{p}) \left(\sum\nolimits_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) - 1\right) G(d\mathbf{x},d\boldsymbol{\delta},d\mathbf{p}),\end{aligned} \tag{22}$$

where $\lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \geq 0$ denotes the Lagrange multipliers associated to restriction (19) and $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ denotes the Lagrange multipliers associated to restriction (20).

If $\Pr^0_{ij} > 0$, then the first order conditions with respect to the conditional probabilities associated to the two products, $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$ and $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$, are given by:

$$u_{i1} + \lambda_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) - \rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) + \gamma_i\left(\log\left(\Pr^0_{i1}\right) + 1 - \log\left(\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) - 1\right) = 0 \tag{23}$$

$$u_{i2} + \lambda_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) - \rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) + \gamma_i\left(\log\left(\Pr^0_{i2}\right) + 1 - \log\left(\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) - 1\right) = 0 \tag{24}$$

Given that we follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough for all consumers to find a product that generates a positive utility in equilibrium, we have that $u_{ij} > 0$. As a consequence, the above set of first order conditions implies that if $\Pr^0_{ij} > 0$ for all $j \in \{1,2\}$, then $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$ and $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$ almost surely.[21]

In order to see why whenever $\Pr^0_{ij} > 0$, we must have $\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$, suppose (without loss of generality) that $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, which implies $\log\left(\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\right) = -\infty$, on a set of positive measure with respect to $G(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$. Since we assume that $\Pr^0_{i1} > 0$, we have $\log\left(\Pr^0_{i1}\right) > -\infty$. This implies, since $\lambda_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) \geq 0$, that $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$ on a set of positive measure to make the first order condition (23) hold. However, if $\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$, then, in order for the first order condition (24) to hold for all realizations $(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$, we must have $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$ or $\lambda_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = \infty$. But $\lambda_{i2}(\boldsymbol{\delta},\mathbf{x},\mathbf{p}) > 0$ will only be satisfied if $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, when restriction (19) is binding. This implies (without loss of generality) that if $\Pr_{i1}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)$, then $\Pr_{i2}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$. However, this is not possible, since then: $\sum_{j\in\{1,2\}} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = 0$, which contradicts restriction (20).

As a consequence, whenever $\Pr^0_{ij} > 0$, we must have $\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$. This implies that restriction (19) does not bind, and so we must have $\lambda_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p}) = 0$. Therefore, the first order condition

[21] This result does not hold point-wise because the consumer's decision problem is unaffected by deviations in her choices on a measure-zero state of realizations.

for any product $j \in \{1,2\}$ can be rearranged to:

$$\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = \Pr_{ij}^0 e^{(u_{ij}-\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p}))/\gamma_i} = \frac{\Pr_{ij}^0 e^{u_{ij}/\gamma_i}}{e^{\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})/\gamma_i}}. \tag{25}$$

If we substitute this result into restriction (20), we have that $e^{\rho_i(\mathbf{x},\boldsymbol{\delta},\mathbf{p})/\gamma_i} = \sum_{k\in\{1,2\}} \Pr_{ik}^0 e^{u_{ik}/\gamma_i}$, which yields:

$$\Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i) = \frac{\Pr_{ij}^0 e^{u_{ij}/\gamma_i}}{\sum_{k\in\{1,2\}} \Pr_{ik}^0 e^{u_{ik}/\gamma_i}}. \tag{26}$$

We assumed until this point that $\Pr_{ij}^0 > 0$ for all $j \in \{1,2\}$. However, note that the proposition holds even for $\Pr_{ij}^0 = 0$, as otherwise $\Pr_{ij}^0 = \int_{(\mathbf{x},\boldsymbol{\delta},\mathbf{p})} \Pr_{ij}(\mathbf{x},\boldsymbol{\delta},\mathbf{p};\gamma_i,\theta_i,v_i)\, G(\mathbf{x},\boldsymbol{\delta},\mathbf{p})$ could not hold. ■

Proof to Lemma 1. The heart of the proof lies in establishing that, in this setting of single-products firms in which firm j and firm $-j$ set prices to maximize profits, the aggregate demand function and the cost function faced by each firm satisfy Mizuno (2003)'s five conditions for the existence of a unique (pure strategies) price equilibrium. These five conditions are:

(i) $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p})$ is strictly positive and strictly decreasing in p_j on $\Re^2$

(ii) $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}) = D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1})$ for all τ, where $\mathbf{1} = (1,1)'$

(iii) $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j^H,p_{-j}^H)\, D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j^L,p_{-j}^L) \geq D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j^H,p_{-j}^L)\, D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j^L,p_{-j}^H)$ for $p_j^H > p_j^L$, and $p_{-j}^H > p_{-j}^L$

(iv) $C_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\varphi)$ is convex in $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p})$

(v) $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p})$ is increasing in p_{-j} on $\Re^2$

Condition (i) consists of two parts. The first part of condition (i) requires aggregate demand to be strictly positive for every price vector on $\Re^2$. From equation (10) it is straightforward to show that this condition is satisfied in our model. For every price vector on $\Re^2$, $\Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,\theta_i,v_i) > 0$ almost surely for every consumer i and product j, and consequently $D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}) > 0$ almost surely for every product j. The second part of condition (i) establishes the standard law of demand. In our model, note that from (10), we have almost surely:

$$\begin{aligned}
\frac{\partial D_j(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p})}{\partial p_j} &= \int_{\gamma_i}\int_0^1\int_0^1 \frac{\partial \Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,,\theta_i)}{\partial p_j} P_\gamma(d\gamma_i)\, P_v(dv_i)\, P_\theta(d\theta_i) \qquad (27)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i)\left(1-\Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i)\right) P_v(dv_i)\, P_\theta(d\theta_i)\right) P_\gamma(d\gamma_i)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i)\, P_v(dv_i)\, P_\theta(d\theta_i)\right.\\
&\quad \left. -\int_0^1\int_0^1 \Pr_{ij}(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i)^2 P_v(dv_i)\, P_\theta(d\theta_i)\right) P_\gamma(d\gamma_i).
\end{aligned}$$

Since $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)>0$ almost surely for all i and j, we have $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)<1$ almost surely for all i and j, because $\Pr_{i1}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)+\Pr_{i2}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)=1$. This result implies that the integrand $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)>\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2$ almost surely, and therefore, using the inequality rule for definite integrals, we must have $\int_0^1\int_0^1\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)>\int_0^1\int_0^1\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)$ almost surely. Since $\gamma_i>0$, this establishes that the second part of the condition is also satisfied, since we have that $\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)/\partial p_j<0$ almost surely for every product j.

Condition (ii) requires aggregate demand for a product to depend only on price differences, which is also satisfied by our aggregate demand function almost surely for every product j:

$$
\begin{aligned}
D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1}\right) &= \int_{\gamma_i}\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}+\tau\mathbf{1};\gamma_i,v_i,\theta_i\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{\left(-p_j-\tau-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{\left(-p_k-\tau-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{(-\tau/\gamma_i)}e^{\left(-p_j-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{(-\tau/\gamma_i)}e^{\left(-p_k-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \\
&= \int_{\gamma_i}\int_0^1\int_0^1 \left(\frac{e^{\left(-p_j-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{\left(-p_k-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \\
&= D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right).
\end{aligned} \tag{28}
$$

Condition (iii) requires the aggregate demand function $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ of every product j to be totally positive of order 2 in prices. In order to show that this condition is, in fact, satisfied, it suffices to show that the population distribution function $P\left(\gamma_i,v_i,\theta_i\right)=P_\gamma\left(\gamma_i\right)P_v\left(v_i\right)P_\theta\left(\theta_i\right)$ is log concave. As Mizuno (2003) shows, if $P\left(\gamma_i,v_i,\theta_i\right)$ is log concave, $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is log concave by the Prekópa–Borel theorem for every product j. Furthermore, since, under condition (ii), aggregate demand for a product depends only on price differences, we can always rewrite the aggregate demand function as $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)=g_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j-p_k\right)$ for every products j and $k\neq j$, which by the duality between log concave functions and totally positive of order 2 functions, establishes that $g_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j-p_k\right)$ and hence $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is totally positive of order 2 in p_j and p_k. It remains to be shown that $P\left(\gamma_i,v_i,\theta_i\right)=P_\gamma\left(\gamma_i\right)P_v\left(v_i\right)P_\theta\left(\theta_i\right)$ is, in fact, log concave. The proposition establishes that $P_\gamma\left(\gamma_i\right)$ is a log concave function. Further, in our model, $P_v\left(v_i\right)$ and $P_\theta\left(\theta_i\right)$ are assumed to denote a uniform distribution. Since uniform distributions are log concave, and the product of log concave functions, is log concave, condition (iii) is, in fact, satisfied.

Condition (iv) requires the cost function to be convex in demand, which is satisfied in our model since we have that $\partial^2 C_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\varphi\right)/\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)^2=0$ almost surely.

Finally, condition (v) requires that any two product are gross substitutes, which again is satisfied in our model since $\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)/\partial p_{-j}>0$ almost surely. In order to see why, note that from (10), we have almost surely:

$$\frac{\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)}{\partial p_{-j}} = \int_{\gamma_i}\int_0^1\int_0^1 \frac{\partial \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)}{\partial p_{-j}} P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \tag{29}$$

$$= \int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)\Pr_{i-j}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\right)P_\gamma\left(d\gamma_i\right)$$

$$= \int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)\left(1-\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\right)P_\gamma\left(d\gamma_i\right),$$

where the last equality is just a consequence of the fact that $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)+\Pr_{i-j}\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p};\gamma_i,v_i,\theta_i\right)=1$, for every consumer i. This result establishes that our model implies $\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)/\partial p_{-j}=-\partial D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)/\partial p_j$ almost surely, which, using condition (i), ensures that condition (v) is, in fact, satisfied. ■

Proof to Lemma 2. Note that $\Pi_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j,p_{-j};\varphi\right)<\Pi_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},mc_j,p_{-j};\varphi\right)$ for any $p_j<mc_j=mc\left(\delta_j;\varphi\right)$, so that $p_j^*\geq mc_j,\forall j=1,2$. Furthermore, note also that $\partial\Pi_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},mc_j,p_{-j}^*;\varphi\right)/\partial p_j=D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},mc_j,p_{-j}^*\right)>0$ since $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is strictly positive for any price vector on $\Re^2$.[22] Finally, note that since $D_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},\mathbf{p}\right)$ is strictly decreasing in p_j, there must be some $p_j\in\left(mc_j,\infty\right)$ for which $\partial\Pi_j\left(\bar{\mathbf{x}},\bar{\boldsymbol{\delta}},p_j,p_{-j}^*;\varphi\right)/\partial p_j<0$, so that $p_j^*<\infty,\forall j=1,2$. ■

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