Confusion, Indecisiveness and Polarization

Andreas Hefti, Shuo Liu and Armin Schmutzler*

December 14, 2018

Abstract

We analyze endogenous consumer confusion in a market with differentiated goods. Contrary to the well-studied homogeneous goods case, obfuscation is not necessarily an equilibrium phenomenon. If the taste distribution features a concentration of indecisive consumers, confusion is beneficial for firms and obfuscation an equilibrium strategy. By contrast, confusion is not an equilibrium if the taste distribution is polarized. If confusion does arise, however, its adverse welfare consequences are more severe than with homogeneous goods, as consumers may not only pay higher prices, but also end up with the wrong product. Our model can also be adapted to offer new insights on the incentives for political candidates to induce polarized opinions by confusing voters.

Keywords: obfuscation, consumer confusion, differentiated products, price competition, polarized/indecisive preferences.

JEL Classification: D43, L13, M30.

---

*Hefti: Department of Economics, University of Zurich and School of Management and Law, Zurich University of Applied Science, heft@zhaw.ch. Liu: Department of Economics, University of Zurich, shuo.liu@econ.uzh.ch. Schmutzler: Department of Economics, University of Zurich and CEPR, armin.schmutzler@econ.uzh.ch. We wish to thank Paul Heidhues, Justin Johnson, Rani Spiegler, Alexander Rasch, Kfir Eliaz, Kai Konrad, Markus Reisinger and Mike Riordan as well as participants at seminars in Tel Aviv (TAU), Beijing (UIBE), Heraklion (CRESSE), Shanghai (SUFE), Dusseldorf (DICE), Athens (EARIE), Frankfurt, Glasgow, Munich (MPI) and Zürich for many great discussions. Shuo Liu would like to acknowledge the financial support by the Forschungskredit of the University of Zurich (grant no. FK-17-018).

1 Introduction

Some of the most important consumption decisions are inherently complex. For instance, when faced with the choice between two different cars, smartphones or insurance contracts, consumers often have a hard time figuring out which alternative they prefer. Assessing the difference in the monetary value of two goods is an even more challenging task. For complex consumption decisions, it therefore appears reasonable to assume that consumers' judgments are noisy at best.

The extent of such noise in consumer decisions is typically not an entirely exogenous characteristic of the goods under consideration. On the one hand, firms can engage in measures to educate consumers. They can describe the products' properties in a transparent fashion and discuss the exact needs of consumers with them. On the other hand, firms can also deliberately confuse consumers. For instance, insurance companies may write contracts in such a way that comparison becomes difficult. Smartphone manufacturers may add features with unclear value to their products. More generally, when advertising differentiated products, firms may emphasize irrelevant product details rather than those characteristics that really matter for consumer valuations.

This paper asks whether firms want to *educate* consumers or whether instead they want to engage in *obfuscation* activities to confuse consumers. A quick glance at the special case of homogeneous goods might suggest that firms' incentives are clear-cut. Oligopolistic producers of such goods suffer from the temptation to undercut each others' prices, resulting in a zero-profit equilibrium under well-known conditions.[1] To alleviate this problem, firms may seek to reduce the undercutting temptation. With homogeneous goods, obfuscation may allow competitors to escape the "Bertrand trap" by reducing market transparency. The literature has made this point in several variants, but the bottom line is that producers of homogeneous goods can obtain positive profits by confusing consumers, even when this would otherwise be impossible.[2]

While the case of homogeneous goods is an important theoretical benchmark, in many industries firms offer differentiated products to cater to the needs of heterogeneous consumers. In such environments, the role of obfuscation is more subtle. On the one hand,

[1]Such an equilibrium arises, e.g., if the following conditions hold simultaneously: static interaction, identical and constant marginal costs, no capacity constraints; see, e.g., Tirole 1989.

[2]For instance, firms can benefit by using hidden fees (Gabaix and Laibson (2006); Heidhues et al. (2016)), spurious differentiation resulting from the credulity of consumers (Spiegler, 2006), complex price formats (Carlin, 2009; Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013), intransparent webpages (Ellison and Ellison, 2009), or more generally from increasing consumer search costs (Ellison and Wolitzky, 2012).

the scope for confusion is larger than with homogeneous goods. For example, there can be many ways to present the differences between products, and the dimensions that firms emphasize are likely to influence the perceived valuations. On the other hand, the incentives to confuse consumers are less obvious. Firms usually obtain positive profits in differentiated markets even without obfuscation. It therefore is possible in principle that, by blurring the perception of consumers, obfuscation reduces rather than increases profits. It could thus potentially be in the interest of firms to educate consumers.

We seek to identify conditions under which consumer confusion arises in markets with differentiated products. To this end, we study a duopoly framework, where the population of consumers is characterized by a distribution of valuations for the two goods with an arbitrary correlation structure, encompassing standard discrete choice models, such as Hotelling or Salop. The two firms first decide on their marketing activities. Thereafter they compete in prices. Finally consumers choose which product to buy. Firms can choose their marketing activities from an exogenously given set of options. The activities jointly determine the noise in consumer perceptions, thereby resulting in a distribution of perceived valuation differences in the consumer population that may differ from the true valuation distribution.[3] We abstract from any cost heterogeneity between different activities.[4] As we are asking whether and how firms want to influence the noise in consumer decisions, we assume that the stochastic perturbations do not bias valuations systematically. More precisely, we assume that marketing does not affect the expected valuation *differences*.[5] In this sense, firms cannot systematically fool consumers.

Our main result establishes that both confusion and education can be equilibrium phenomena, with the outcome depending on the true valuation distribution. *Consumer confusion* arises if the distribution of perceived valuation differences and the distribution of true valuation differences do not coincide. Any marketing profile that induces confusion is called *obfuscating*, and any profile inducing (or restoring) the true distribution is called *educating*. Perhaps surprisingly, the degree of taste differentiation itself plays no vital role for whether obfuscation occurs in equilibrium. Our first main result identifies simple properties of the true preference distribution determining whether firms will engage in obfuscation activities. We distinguish between *indecisive* preferences, for which,

[3]As we will discuss in Section 2.4.1, our analysis is thus related to Johnson and Myatt (2006) who consider a monopolist's incentive to influence valuation distributions

[4]This decision reflects an uncertainty about the adequate relation between the costs of marketing activities and their effects on consumer perception; see Section 2.4.3.

[5]From a theoretical perspective, unbiasedness can be seen as playing a similar disciplining role in our (non-Bayesian) analysis as the assumption of "conformity with the prior" in models of persuasion (Kamenica and Gentzkow, 2011) or costly information acquisition (Caplin and Dean, 2015)

roughly speaking, indifferent consumers are relatively common and *polarized* preferences, for which indifferent consumers are relatively rare. For instance, in the standard textbook Hotelling model, the former (latter) case arises when the density of the consumer distribution has a maximum (minimum) in the middle of the interval.[6] We find that consumer confusion arises in equilibrium if preferences are indecisive, whereas it does not arise if preferences are polarized.

To illustrate the relevant properties of the taste distribution, consider for instance the hospitality industry. It is hard to imagine that a guest will be indifferent when faced with the choice between a “family” hotel and a “business” hotel – instead, most consumers will clearly prefer one alternative over the other, resulting in a polarized distribution. However, if the comparison is between business hotel A and business hotel B, the situation may be better described by a substantial amount of indecisive consumers.

Our second result requires more structure on the set of feasible marketing activities. If the marketing profiles can be compared by the amount of confusion generated (e.g., by a mean-preserving spread of the noise distribution), then with indecisive tastes, the unique subgame-perfect equilibrium features maximal consumer confusion, while full education arises as the only subgame-perfect equilibrium for polarized tastes.

A crucial condition for the results achieved so far is that obfuscation only leads to small perturbations of the true valuations. Suppose instead that the marketing tools are so powerful that even the most loyal consumer of a firm can be confused enough to perceive the other firm’s good as better, which seems plausible if there is only very little true taste dispersion. When such ”massive confusion” is possible, a U-shaped relation between confusion and firm payoffs results when tastes are polarized. Our third result shows that confusion can then arise in equilibrium outcome despite polarized tastes. In particular, in the limit case of homogeneous goods, true taste differentiation is negligible, so that obfuscation is an equilibrium phenomenon independently of the shape of the taste distribution.

The welfare analysis of obfuscation differs from the case with homogeneous goods, for which, in the absence of binding outside options, the main effect is redistribution from consumers to firms. With differentiated goods, whenever firms choose to obfuscate, this not only increases prices, but it also leads to a mismatch between consumers and products. The size of the mismatch is increasing in the extent of obfuscation.

We also show that our results are robust to modifications of the setting. First, we find that similar results arise with a strictly binding outside option, except that obfuscation

[6] For definiteness, think of a situation with firms located at the ends of a compact interval that is symmetric around the mid-point; moreover, take transportation costs to be linear in distance.

is typically less attractive for firms. Second, we use a location model to address the idea that consumers are confused about their own needs rather than about objective characteristics of the product.

Our results shed lights on the advertising literature which has discussed firms' incentives to engage in informative advertising. Interpreting a reduction in the noise of relative valuations as informative advertising, we see that the preference distribution determines whether such advertising arises as an equilibrium phenomenon.

The general logic of our model applies beyond the oligopoly setting, for instance, to competition between candidates who compete for voters. Candidates can choose how much information to provide to voters about their platforms. In addition, they can engage in other measures to convince voters, such as promises and campaigning efforts. Promises are costly only if the candidate wins the election; efforts are costly even when she does not. In the former case, our oligopoly model applies directly. The latter case requires modifications of the setting, as unconditional efforts lead to a contest structure. In both situations, we find that candidates will want to engage in obfuscation activities only if the voter preference distribution displays indecisiveness. Intuitively, with indecisive preferences obfuscation distorts the preference distribution so that it becomes more polarized. This reduces the necessary efforts of the candidates when competing for voters.

The paper is organized as follows. Section 2 introduces, analyzes and discusses the general framework. We present extensions in Section 3. Section 4 contains the application to political economy. The related literature is discussed in Section 5, and Section 6 concludes. All proofs are relegated to Appendices A and B.

2 General Framework and Analysis

We now introduce our general framework. We present the model set-up and assumptions in Section 2.1 and the main results in Section 2.2.

2.1 The model

Consider a duopoly where each firm $i = 1, 2$ produces a good x_i (or simply i) at zero marginal cost. There is a unit measure of consumers indexed by k. Consumer k has a (true) valuation $v_i^k \in \mathbb{R}$ for good i. The joint valuation distribution for the consumer population is $F_0\left(v_1^k, v_2^k\right)$, with support $S_0 \subseteq \mathbb{R}^2$. The firms play a two-stage complete information game. In the first stage, they simultaneously choose marketing activities $a_i \in A_i$. The marketing profile $\mathbf{a} = (a_1, a_2) \in \mathcal{A} \equiv A_1 \times A_2$ determines the distribution

$F_{\mathbf{a}}\left(\tilde{v}_1^k, \tilde{v}_2^k\right)$ of *perceived* valuations $(\tilde{v}_1^k, \tilde{v}_2^k)$. In the second stage, firms observe $\mathbf{a}$ and infer the resulting distribution $F_{\mathbf{a}}$. Then they simultaneously choose prices $p_i \in \mathbb{R}_+$ so as to maximize expected profits. Finally, each consumer k makes her consumption choice given her perceived valuations $(\tilde{v}_1^k, \tilde{v}_2^k)$. Throughout the main analysis, we assume there is no outside option (or it is not binding if there is one).[7] Accordingly, each consumer k acquires the alternative with the highest perceived net utility $\tilde{u}_i^k = \tilde{v}_i^k - p_i$, $i \in \{1, 2\}$. Thus, the population distribution of perceived valuation *differences* $\tilde{v}_\Delta^k \equiv \tilde{v}_2^k - \tilde{v}_1^k$ suffices for studying the model.

To make the economic forces in the two stage games most salient, we focus on marketing activities $\mathbf{a} \in \mathcal{A}$ that affect perceived valuation *differences* according to

$$\tilde{v}_\Delta^k = v_\Delta^k + \varepsilon_{\mathbf{a}}, \tag{1}$$

where the random variable $\varepsilon_{\mathbf{a}}$, $\mathbf{a} \in \mathcal{A}$, is independent of true valuation differences $v_\Delta^k = v_2^k - v_1^k$. With (1), the distribution function of $\tilde{v}_\Delta^k$, $G_{\mathbf{a}}$, is given by the convolution

$$G_{\mathbf{a}}(x) = \int_{-\infty}^{+\infty} G_0(x - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon), \ x \in \mathbb{R},$$

for every $\mathbf{a} \in \mathcal{A}$. For given prices $p_1, p_2 \in \mathbb{R}_+$ and marketing activities $\mathbf{a} \in \mathcal{A}$, the expected demand of firm 1 is $D_1(p_1, p_2; \mathbf{a}) \equiv \Pr(\tilde{v}_\Delta^k \leq p_2 - p_1) = G_{\mathbf{a}}(p_2 - p_1)$. Accordingly, $D_2(p_1, p_2; \mathbf{a}) \equiv 1 - G_{\mathbf{a}}(p_2 - p_1)$ is the expected demand of firm 2. Further, expected profits are $\Pi_i(p_1, p_2; \mathbf{a}) \equiv p_i D_i(p_1, p_2; \mathbf{a})$, $i = 1, 2$.

Main assumptions We introduce some notation and impose several assumptions on G_0 and $\varepsilon_{\mathbf{a}}$. We say that a random variable X is *symmetric at zero* if its distribution function is symmetric at zero.[8] Further, we say that X is *degenerate at zero* if $X = 0$ holds with probability one, and we denote such a random variable by O.

Assumption 1 (Distributional assumptions) *The following assumptions on the distributions of v_Δ^k and $\varepsilon_{\mathbf{a}}$ are imposed:*

(A1.1) *v_Δ^k is symmetric at zero and has a density g_0.*

(A1.2) *G_0 is log-concave on supp(g_0).*

(A1.3) *g_0 is continuous at zero and $g_0(0) > 0$.*

(A1.4) *$\forall \mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ is symmetric at zero.*

[7] This is relaxed in Section 3, the main effect of the outside option being to make obfuscation less appealing for the firms.

[8] The distribution function G is *symmetric at zero* if $G(x) = 1 - G(-x)$ $\forall x \in \mathbb{R}$, or (ii) $G(x) = 1$ if $x \geq 0$ and $G(x) = 0$ else.

(A1.5) *If $\varepsilon_{\mathbf{a}} \neq O$, $\varepsilon_{\mathbf{a}}$ has a density $\gamma_{\mathbf{a}}$ that is log-concave on supp$(\gamma_{\mathbf{a}})$.*

The log-concavity assumptions (A1.2) and (A1.5) will assure that the first-order conditions for payoff maximization in the pricing stage also are sufficient. Jointly with (A1.3), this is useful for establishing equilibrium existence. Conditions (A1.1) and (A1.4) are of economic importance. They state that no firm has a pre-existing systematic advantage over the other, nor can it gain such an advantage by the various marketing activities. We discuss the economic rationale for (A1.4) in Section 2.4. Further, the assumption that v_{Δ}^{k} has a density g_0 implies that true consumer valuations are heterogeneous.[9]

Assumption 1 guarantees that a density function $g_{\mathbf{a}}$ of $\tilde{v}_{\Delta}^{k}$ exists in the pricing stage and is given by

$$g_{\mathbf{a}}(x) = \int_{-\infty}^{\infty} g_0(x-\varepsilon)d\Gamma_{\mathbf{a}}(\varepsilon), \ \forall x \in \mathbb{R}. \tag{2}$$

In particular, $g_{\mathbf{a}}(x) = g_0(x)$ $\forall x \in \mathbb{R}$ if $\varepsilon_{\mathbf{a}} = O$.

Finally, we define $\kappa \equiv supp(g_0)$. κ parameterizes the range of valuation differences in the consumer population. It is thus a measure of *taste differentiation*.

2.2 Main Results

Section 2.2.1 contains the main results requiring only little structure on $\mathcal{A}$ and the marketing technology. In Section 2.2.2, we derive stronger results under additional assumptions.

2.2.1 Endogenous confusion or education

Our first result characterizes the second-stage pricing equilibrium.

Lemma 1 *Under Assumption 1 there is a unique symmetric pure-strategy equilibrium in every pricing subgame. Each firm chooses the price $p_{\mathbf{a}}^{*} = \frac{1}{2g_a(0)}$, $\mathbf{a} \in \mathcal{A}$.*

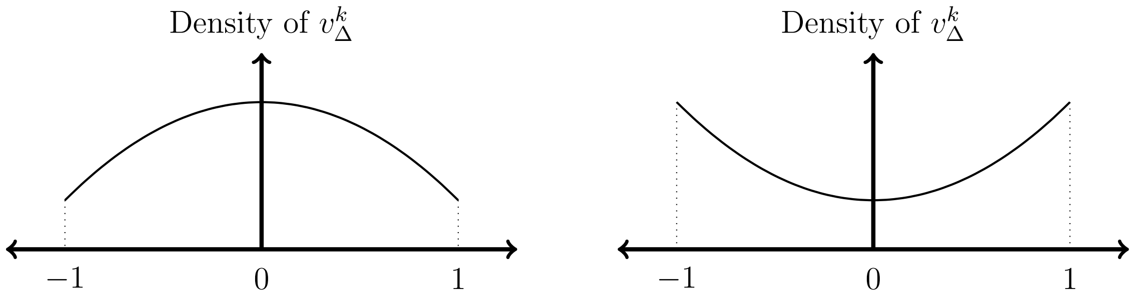
As perceived valuation differences are dispersed with a zero-symmetric log-concave density (2) for any $\mathbf{a} \in \mathcal{A}$, equilibrium existence is guaranteed. Crucially, Lemma 1 shows

[9] It is straightforward to derive G_0 from the more primitive joint distribution F_0. For example, if F_0 has a density function f_0, G_0 can be expressed as

$$G_0(v_{\Delta}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{v_{\Delta}} f_0(v, v+x)dxdv, \ \forall v_{\Delta} \in \mathbb{R}.$$

It is then easy to derive sufficient conditions on F_0 under which the symmetry condition (A1.1) holds. For example, it holds if F_0 itself is symmetric, i.e., $F_0(x,y) = F_0(y,x)$ $\forall x, y \in \mathbb{R}$. Likewise, log-concavity (A1.2) can be checked by standard calculus tools if F_0 has a differentiable density function; a sufficient condition is that v_1^k and v_2^k are independent and each is drawn from a log-concave distribution function. Alternatively, (A1.2) holds if F_0 is an (arbitrary) multivariate normal distribution.

that the equilibrium price $p^*_{\mathbf{a}}$ is pinned down by $g_{\mathbf{a}}(0)$, the measure of perceptually indifferent consumers. Such consumers react most sensitively to price changes. If $g_{\mathbf{a}}(0)$ is low, there are many inframarginal customers whom the firms want to exploit. As a result, high equilibrium prices become sustainable. Absent a binding outside option, higher prices always correspond to higher revenues, so that firms seek to play marketing profiles that reduce the measure of perceptually indifferent consumers.



(a) Indecisive distribution

(b) Polarized distribution

Figure 1: Examples of preference distributions

Our first main result shows that the distribution of true valuation differences v^k_Δ determines whether consumer confusion arises in equilibrium, or whether consumers are fully educated instead. The conclusion will depend on differences in the shape of the valuation distribution as depicted in the two parts of Figure 1. The following definition makes the relevant concepts more precise.

Definition 1 *Let* $\delta > 0$ *be such that* $[-\delta, \delta] \subset supp(g_{\mathbf{0}})$*.*

(i) True preferences are a) ***weakly*** δ***-indecisive*** *if* $g_0(0) > g_0(x)$ $\forall x \in [-\delta, 0) \cup (0, \delta]$*, b)* δ***-indecisive*** *if* $g_{\mathbf{0}}$ *is strictly increasing on* $[-\delta, 0]$ *and strictly decreasing on* $[0, \delta]$*, and c)* ***strongly*** δ***-indecisive*** *if* $g_{\mathbf{0}}$ *is strictly concave on* $[-\delta, \delta]$*.*

(ii) True preferences are a) ***weakly*** δ***-polarized*** *if* $g_0(0) < g_0(x)$ $\forall x \in [-\delta, 0) \cup (0, \delta]$*, b)* δ***-polarized*** *if* $g_{\mathbf{0}}$ *is strictly decreasing on* $[-\delta, 0]$ *and strictly increasing on* $[0, \delta]$*, and c)* ***strongly*** δ***-polarized*** *if* $g_{\mathbf{0}}$ *is strictly convex on* $[-\delta, \delta]$*.*

The tastes represented in Figure 1 feature strongly δ-indecisive and strongly δ-polarized preferences, respectively, where $[-\delta, \delta] = [-1, 1] = supp(g_{\mathbf{0}})$. Given the zero-symmetry of v^k_Δ, strong δ-indecisiveness implies δ-indecisiveness. δ-indecisiveness implies weak δ-indecisiveness; the difference between the two concepts is the monotonicity requirement in the definition of δ-indecisiveness (similarly for polarization). For δ-indecisive preferences, less pronounced valuation differences occur more frequently than more pronounced valuation differences, while weakly δ-indecisive preferences only require that indifference ($v_\Delta^k = 0$) occurs more often than all other alternatives on $[-\delta, \delta]$. The most general result of this section, Theorem 1, only requires *weak* indecisiveness (*weak* polarization); the more restrictive concepts are useful to obtain stronger results.

It will be convenient to indicate marketing activities by real numbers, so that $A_i \subset \mathbb{R}$ and $\mathcal{A} \subset \mathbb{R}^2$. We use the convention that $\varepsilon_{\mathbf{0}} = O$. Thus, O corresponds to a transparent marketing strategy that leads to full education if chosen by both firms. We impose the following minimal structure on the set of marketing profiles $\mathcal{A}$.

Assumption 2 *The set $\mathcal{A}$ satisfies the following two conditions:*

(i) $\mathbf{0} \in \mathcal{A}$.

(ii) $\forall i = 1, 2,\ j \neq i$ *and* $\forall a_j \in A_j$, $\exists a_j \in A_j$, *such that* $\varepsilon_{(a_i, a_j)} \neq O$.

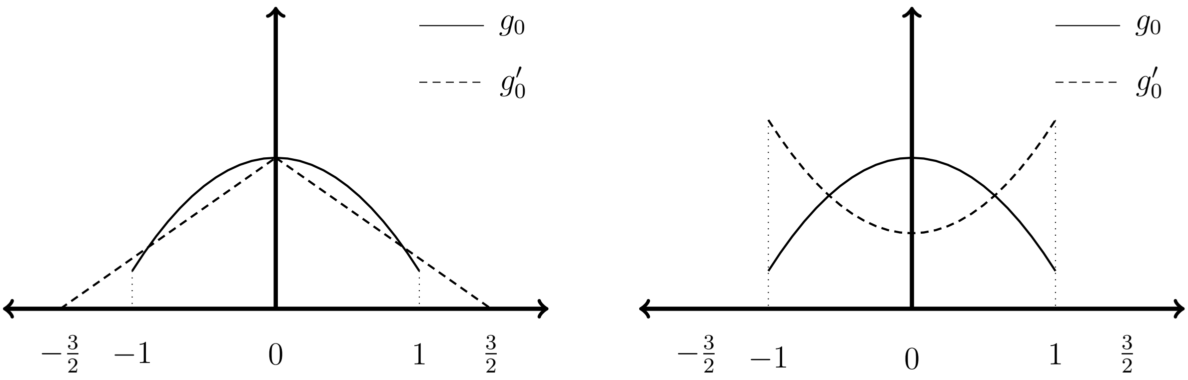
Assumption 2(i) assures that full consumer education is among the feasible options. Assumption 2(ii) means that each firm can induce some consumer confusion unilaterally. Together, (i) and (ii) imply that, while a transparent market is a possible outcome, it cannot be enforced unilaterally. Moreover, Assumption 2 rules out the trivial case where both firms are trapped in an education equilibrium even though they would jointly benefit from confusion. We now state our main result.

Theorem 1 *Suppose that Assumptions 1 and 2 hold.*

(i) If there exists a $\delta > 0$ *with supp*$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ *such that preferences are weakly δ-polarized, then an SPE without consumer confusion (an education equilibrium) exists.*

(ii) If there exists a $\delta > 0$ *with supp*$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ *such that preferences are δ-indecisive, then no SPE without consumer confusion exists.*

For the statements in Theorem 1, only the shape of the valuation difference distribution g_0 matters, while the extent of taste differentiation κ plays no role. Even with a small degree of taste differentiation, there will be no consumer confusion with polarized preferences. Conversely, equilibrium confusion arises with indecisive preferences even with an arbitrarily large degree of taste differentiation. As an illustration, the distributions g_0 and g_0' in Figure 2 a) differ in the degree of taste differentiation, but both yield the same



(a) Unequal taste differentiation (b) Same taste differentiation

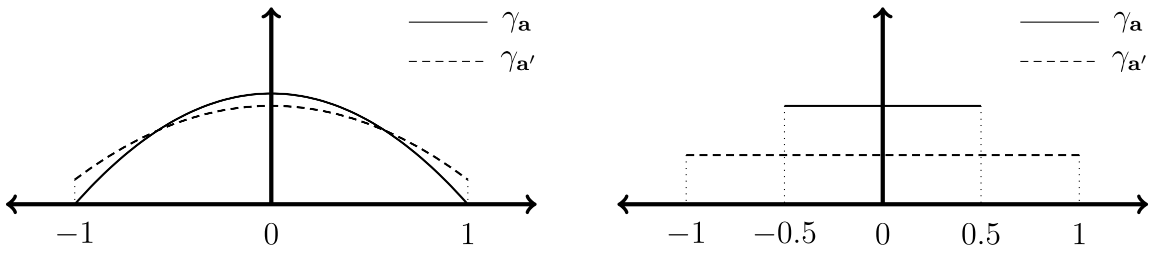
Figure 2: Taste differentiation and taste distribution

SPE as $g(0)$ is the same. By contrast, g_0 and g_0' in Figure 2 b) have the same degree of taste differentiation but yield different SPE, as g_0 is indecisive while g_0' is polarized.

The rationale for Theorem 1 is as follows. If firms choose an obfuscating marketing profile, some truly indifferent consumers perceive one good as strictly superior, while some consumers who strictly prefer one good over the other become indifferent. Firms benefit from obfuscation if the former effect dominates the latter. With polarized tastes, confusion breaks pre-existing allegiance with a firm, as more consumers are pushed towards indifference than vice versa. Therefore, the firms seek to avoid the intensified price competition caused by an obfuscated market. Because full education is feasible by Assumption 2, it must be part of an SPE. By contrast, confusion is beneficial for firms if true tastes are indecisive. In such a situation, confusion tends to reduce the measure of indifferent consumers, as more indifferent consumers end up perceiving one of the products as superior than vice versa. As any individual firm can always force some confusion on the market by Assumption 2, full education cannot be supported as an equilibrium.

The essential role of Assumption 2 is to provide meaningful restrictions on the possible outcomes. If, contradicting Assumption 2 (ii), consumer education could be enforced unilaterally, then part (i) of Theorem 1 could even be strengthened in that any SPE involves full education.[10] Further, there could be SPE without consumer confusion despite indecisive Specifically, if for some $a \in A$ any marketing profile (a, a') leads to full education, then full education (with both firms choosing a) always constitutes a Nash equilibrium in the first stage because neither firm can unilaterally affect the distribution of perceived valuation differences. Such an SPE, however, is strictly dominated by any

[10]Heidhues et al. (2016) study a model where each single firm can choose to perfectly educate all consumers in the market. In our framework, this corresponds to the case where there is some $a \in A$ that enforces full education independent of the other firm's marketing activity.



(a) $\gamma_{\mathbf{a}'} \succ_S \gamma_{\mathbf{a}}$, same support.

(b) $\gamma_{\mathbf{a}'} \succ_S \gamma_{\mathbf{a}}$, variable support.

Figure 3: Examples of SSC orderings

possible SPE with consumer confusion.

2.2.2 Minimal and Maximal confusion

On intuitive grounds, one should expect that firms seek to confuse consumers as much as possible if preferences are indecisive. We now study this idea more formally.

Throughout this section we let $A_1 = A_2 = A$, so that $\mathcal{A} = A^2$. We will assume that the noise distributions can be ordered in one of two ways. The first result appeals to the familiar notion of a mean-preserving spread.[11] The second result relies on an alternative ordering. Let $\gamma_{\mathbf{a}}, \gamma_{\mathbf{a}'}$ be two zero-symmetric density functions with supports $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $[-\omega'_{\mathbf{a}}, \omega'_{\mathbf{a}}]$, respectively, where $\omega'_{\mathbf{a}} \geq \omega_{\mathbf{a}}$. These two density functions satisfy the sidewise single-crossing (SSC) property, denoted by $\gamma_{\mathbf{a}'} \succ_S \gamma_{\mathbf{a}}$, if $\forall e, e' \in [0, \omega'_{\mathbf{a}}]$ with $e' > e$

$$\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) \geq 0 \Longrightarrow \gamma_{\mathbf{a}'}(e') - \gamma_{\mathbf{a}}(e') > 0. \tag{3}$$

SSC means that the two densities intersect only once in $[-\omega'_{\mathbf{a}}, 0]$ and $[0, \omega'_{\mathbf{a}}]$, respectively;[12] see Figure 3 for illustrations.

We use the two orderings in the following alternative assumptions.

Assumption 3 *$A \subset \mathbb{R}^+$ is compact with $\mathbf{0} \in \mathcal{A}$,* $\max A = \bar{a} > 0$*, $\varepsilon_{\mathbf{a}} = O$ if and only if $\mathbf{a} = \mathbf{0}$. Moreover, one of the following conditions holds:*
(i) $\forall \mathbf{a}, \mathbf{a}' \in \mathbf{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'}$ is MPS of $\Gamma_{\mathbf{a}}$.
(ii) $\forall \mathbf{a}, \mathbf{a}' \in \mathbf{A}$ with $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$, $\omega'_{\mathbf{a}} \geq \omega_{\mathbf{a}}$ and $\gamma_{\mathbf{a}'} \succ_S \gamma_{\mathbf{a}}$.

[11] A random variable Y is a mean-preserving spread (MPS) of X, if Y has the same distribution as $X + \eta$, where $\eta \neq O$ and $E[\eta|X] = 0$. Intuitively, Y is a noisy version of X. **?** showed that if the involved distribution functions have a uniformly bounded support, then the (partial) MPS ordering between distributions is equivalent to the (partial) order induced by second-order stochastic dominance. **?** shows how to extend the result to the case of an unbounded support.

[12] Hence they intersect at most twice on the entire domain.

Assumptions 3(i) and (ii) each imply Assumption 2.[13] Moreover, $\mathcal{A}$, $(\bar{a}, \bar{a})$, uniquely induces the maximum feasible amount of consumer confusion in the MPS sense.

Theorem 2 *Suppose Assumption 1 holds.*

(i) Suppose there exists $\delta > 0$ such that supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and (a) or (b) holds:

(a) True preferences are strongly δ-indecisive and Assumption 3(i) is satisfied.

(b) True preferences are δ-indecisive and Assumption 3(ii) is satisfied.

There exists a unique SPE. Consumer confusion is maximal ($a_1^* = a_2^* = \bar{a} \equiv \max A$).

(ii) Suppose there exists $\delta > 0$ such that supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and (a) or (b) holds:

(a) True preferences are strongly δ-polarized and Assumption 3(i) is satisfied.

(b) True preferences are δ-polarized and Assumption 3(ii) is satisfied.

There exists a unique SPE; in this SPE, there is full education ($\mathbf{a}^* = \mathbf{0}$).

The conditions (a) and (b) in Theorem 3(i) and (ii) cannot be ranked according to their generality. Assumption 1 implies that, if the distributions $\{\varepsilon_{\mathbf{a}}\}_{\mathbf{a} \in \mathcal{A}}$ are (partially) ordered by the SSC criterion, they are also ordered by the MPS criterion. The converse statement does not hold. However, the requirements on true preferences in Theorem 3(i)(a) are stronger than those in Theorem 3(i)(b); similarly for Part (ii): We only need δ-indecisive or δ-polarized tastes in the SSC case, but their strong counterparts in the MPS case.

The proof of Theorem 2 shows that the MPS requirement in Assumption 3(i) can be further weakened. We merely require that $\varepsilon_{a,\bar{a}}$ is an MPS of $\varepsilon_{a,a'}$ for any $a, a' \in A$ with $a' < \bar{a}$; similarly for $\varepsilon_{\bar{a},a}$. To see this, note that the proof argues that, if $\varepsilon_{\mathbf{a}'}$ is MPS of $\varepsilon_{\mathbf{a}'}$ and $g_0(\cdot)$ is strictly concave, then the noisier version $\varepsilon_{\mathbf{a}'}$ more effectively reduces the measure of perceptually indifferent consumers $g_{\mathbf{a}}(0)$. Therefore, $a_i = \bar{a}$ is a strictly dominant strategy for each player, as $\varepsilon_{a,\bar{a}}$ is an MPS of any $\varepsilon_{a,a'}$ with $a' < \bar{a}$. Hence, in any SPE, the marketing profile is $(\bar{a}, \bar{a})$.[14]

To illustrate the assumptions, suppose that for every $\mathbf{a} \neq \mathbf{0}$, $\Gamma_{\mathbf{a}}$ follows the uniform distribution with support $supp(\gamma_{\mathbf{a}}) = [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\omega_{\mathbf{a}} > 0$, so that an increase of $\omega_{\mathbf{a}}$ means more consumer confusion in the sense of a larger range of possible opinions. Interesting special cases are given by (i) $\omega_{(a_1,a_2)} = a_1 + a_2$, (ii) $\omega_{(a_1,a_2)} = \max\{a_1, a_2\}$, and (iii)

[13] This follows from the assumptions that i) $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = 0$ and ii) $\bar{a} > 0$, which imply that $\varepsilon_{\mathbf{a}} \neq O$ can always be forced unilaterally.

[14] Finally, if the set of distributions $\{\varepsilon_{\mathbf{a}}\}_{\mathbf{a} \in \mathcal{A}}$ verifies the MPS ordering (Assumption 3), then the maximal element $(\bar{a}, \bar{a})$ of $\mathcal{A}$ always is the one with the largest variance.[15] If additionally consumer tastes are strongly δ-indecisive, Theorem 3 thus implies that the equilibrium marketing profile yields the highest feasible dispersion as measured by the variance of the possible opinion changes.

$\omega_{(a_1,a_2)} = \min\{a_1, a_2\}$. In case (i), individual marketing activities have an independent incremental effect on the overall level of confusion, while with (ii) the level of confusion depends only on the firm that engages most in obfuscation. If $0 \in A_1, A_2$ then (i) is consistent with Assumptions 1-3. (ii) is consistent with Assumption 2, but violates Assumption 3. (iii) means that the firm with the lower level of obfuscation determines the prevailing level of confusion. Thus, Assumption 2(ii) does not hold (and Assumption 3 does not hold either).

2.2.3 Massive confusion

Theorems 1 and 2 assume that, $\forall \mathbf{a} \in \mathcal{A}$, $supp(\gamma_{\mathbf{a}}) \subset [-\delta, \delta] \subset supp(g_0)$. Thus the scope for consumer confusion is constrained by the degree of the existing taste differentiation. Obfuscation can never convert consumers with the *most extreme* true valuations in favor of one firm to the other firm. We now show that if such reversals are possible, so that the potential confusion is "massive", then firms may even choose obfuscation when preferences are indecisive.

Theorem 3 (Massive confusion) *Suppose that (A1.1) - (A1.3) are satisfied, and, for every* $\mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ *is uniformly distributed on* $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, *where* $\omega_{\mathbf{a}} > \kappa \geq 0$. *Then, an SPE with maximal confusion exists under either of the following conditions: (i) Preferences are indecisive on* $supp(g_0)$. *(ii) Preferences are polarized on* $supp(g_0)$ *and* $\bar{\omega} \equiv \max_{\mathbf{a} \in \mathcal{A}} \omega_{\mathbf{a}}$ *is sufficiently large.*

Thus, under the simplifying assumption of uniform noise distributions, our results extend to the case of massive confusion: For indecisive consumers, the result that firms want to obfuscate as much as possible generalizes, independently of $\omega_{\mathbf{a}}$, the maximal degree of possible confusion. Even with polarized consumers, maximal obfuscation arises when it is possible to confuse consumers sufficiently. As we show in the proof, this result holds whenever the support of true preferences is bounded. The intuition is simple: When firms can induce differences in perceived valuations that are arbitrarily large compared to the true valuation differences, the mass of indifferent consumers will eventually become negligible, regardless of the true valuation distribution. As a result, firms always benefit from confusing consumers if the scope for confusion is sufficiently large. This discussion suggests a reinterpretation of the idea that obfuscation always arises in equilibrium with homogeneous good: In this case, *any* obfuscation is massive, so that firms benefit from introducing it, no matter whether true preferences are polarized or not.

2.2.4 Welfare

In the homogeneous goods case, obfuscation increases prices and therefore benefits firms at the expense of consumers. In our setting, obfuscation could, in principle, reduce prices. However, whenever this would be the case (when preferences are polarized), firms avoid obfuscation according to Theorem 1.

The effects on total welfare are more complex than the effects on prices. When firms engage in obfuscation, there are potential welfare losses because some consumers buy the wrong good. Taking the equilibrium action profile of the firms in the first stage, $\mathbf{a}^*$, as given, we can compute the total expected welfare loss from mismatch as follows:

$$
\begin{aligned}
L &= \int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx + \int_{-\infty}^{0} (-x)(1-\Gamma_{\mathbf{a}^*}(x))g_0(-x)dx \\
&= 2\int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx, \qquad (3)
\end{aligned}
$$

where the second equality uses the symmetry of $\Gamma_{\mathbf{a}}$ and g_0. To understand the measure L, note that if consumer k buys from the wrong firm, the welfare loss is her true valuation difference $|v_2^k - v_1^k|$. Without loss of generality, suppose that $x = v_2^k - v_1^k > 0$. Then, $g_0(x)$ captures the likelihood of type x and $\Gamma_{\mathbf{a}^*}(-x)$ is the probability that type x buys from the wrong firm. Clearly, the welfare loss is zero (i.e., $L = 0$) if and only if $\varepsilon_{\mathbf{a}^*}$ is degenerate at zero. In what follows, we shall focus on the case where $\varepsilon_{\mathbf{a}^*}$ is non-degenerate. The next result shows that, for suitable noise distributions, more confusion always leads to larger welfare loss, regardless of the shape of the true preference distribution.

Proposition 1 *Suppose that $\varepsilon_{\mathbf{a}^*}$ is uniformly distributed on $[-\omega_{\mathbf{a}^*}, \omega_{\mathbf{a}^*}]$, where $\omega_{\mathbf{a}^*} > 0$. The expected welfare loss from mismatch is strictly increasing in $\omega_{\mathbf{a}^*}$.*

Intuitively, increasing obfuscation increases the chances that consumers buy from the wrong firm. A more subtle question is how the size of the welfare loss depends on the distribution of valuation differences and, in particular, on whether preferences are indecisive or polarized. Equation (3) reflects two competing intuitions. On the one hand, when confusion arises, chances are high that almost indifferent consumers will buy the wrong product, and there are many such consumers when the preference distribution is weakly indecisive (contrary to when it is weakly polarized). On the other hand, however, when almost indifferent consumers buy from the wrong firm, the welfare loss is smaller than when consumers with strong preferences do so. Thus, the net effect of preference polarization/indecisiveness on welfare loss is not obvious in general. In the next subsection, we resolve this issue for a concrete example.

2.3 Application: Competition on the Line

Equipped with the formal tools developed above, we now make the informal discussion of the Hotelling example in the introduction precise. In addition, the analysis will deliver additional insights on how the welfare loss from obfuscation depends on the true preference distribution.

We suppose that each consumer is characterized by a parameter θ, which is drawn from a commonly known distribution H_0 with support $\Theta \subset \mathbb{R}$. The true valuation of a type θ consumer for product $i \in \{1, 2\}$ is $v_i^\theta = \mu - (x_i - \theta)^2$, where $\mu > 0$ and x_i is the location of firm i. We assume that $\Theta = [-\lambda, \lambda]$ and $x_2 = \lambda = -x_1$, where $\lambda > 0$, and H_0 has the symmetric density $h(\theta) = \alpha\theta^2 + \beta$ on Θ, where $\alpha \in \left[-\frac{3}{4\lambda^3}, \frac{3}{2\lambda^3}\right]$ and $\beta = \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}$.[16] H translates into a distribution G of valuation differences on $[-4\lambda^2, 4\lambda^2]$. If $\alpha > 0$, the density of G has a minimum at 0 and the true preferences are $4\lambda^2$-polarized. Conversely, if $\alpha < 0$, the true preferences are $4\lambda^2$-indecisive. As in the general framework, obfuscation introduces noise to the perceived valuation difference for each consumer, resulting in a density $g_{\mathbf{a}}$. We maintain Assumptions (A1.4) and (A1.5). Applying Theorem 1, we obtain the following result.

Lemma 2 *In the model with competition on the line, suppose that* $\alpha \leq \hat{\alpha} \equiv (6-3\sqrt{3})/4\lambda^3$. *There exists a unique pure-strategy equilibrium in every pricing subgame, where each firm chooses the same price* $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$ $\forall \mathbf{a} \in \mathcal{A}$.

Compared with the heuristic discussion in the introduction, the result sharpens the conditions for a unique symmetric pricing equilibrium: For low values of the polarization measure α and the differentiation parameter λ, the conditions of Theorem 1 apply.

The following proposition is a direct application of Theorems 1 and 2.

Proposition 2 *In the model with competition on the line, suppose that* $\alpha \leq \hat{\alpha}$, *and* $supp(\gamma_{\mathbf{a}}) \subset [-\lambda, \lambda]$ $\forall a \in A$.

(i) There exists (does not exist) an SPE without consumer confusion if $\alpha > 0$ $(\alpha < 0)$.

(ii) If Assumption 3(i) also holds, then $\forall \alpha \neq 0$ *there exists a unique SPE; this SPE features minimal (maximal) consumer confusion if* $\alpha > 0$ $(\alpha < 0)$.

Proposition 2 formalizes the general intuition that polarization prevents obfuscation for the Hotelling example.[17] It also shows that obfuscation does not require homogeneous

[16] As in the general case, we use λ to parameterize product differentiation.

[17] The main ingredient in the proof is to show that the location distribution is log-concave for $\alpha \leq \hat{\alpha}$.

goods. As long as preferences are weakly indecisive, firms engage in obfuscation under quite general conditions.

The proposition below describes how the expected welfare loss depends on α, the measure of polarization.

Proposition 3 *Consider the model with competition on the line under the assumptions of Proposition 3. Suppose that $\varepsilon_{\mathbf{a}^*}$ is uniformly distributed on $[-\omega_{\mathbf{a}^*}, \omega_{\mathbf{a}^*}]$, where $\omega_{\mathbf{a}^*} > 0$. If $\omega_{\mathbf{a}^*} < \hat{\omega} \equiv 64\lambda^2/15$, then the expected welfare loss is strictly decreasing in α. If $\omega_{\mathbf{a}^*} > \hat{\omega}$, then the expected welfare loss is strictly increasing in α.*

Thus, if the maximal confusion is small (large) relative to product differentiation, the expected welfare loss always decreases (increases) as preferences become more polarized. A complete welfare analysis must also identify the circumstances under which obfuscation arises in SPE. Under the conditions of Lemma 2, obfuscation only arises for indecisive consumers ($\alpha < 0$). There is no welfare loss with polarized consumers, as firms do not obfuscate. When the maximal degree of confusion is sufficiently large, however, obfuscation even arises with polarization (see Theorem 3), so that Proposition 4 applies to this case as well.

2.4 Discussion

In the following, we discuss several aspects of our framework. In Section 2.4.1, we link our approach to the treatment of demand rotations by Johnson and Myatt (2006). Section 2.4.2 discusses the firms' marketing activities. Section 2.4.3 deals with default activities and marketing costs. Finally, Section 2.4.4 shows how our model can incorporate different degrees of consumer sophistication.

2.4.1 Demand Rotations

At an abstract level, our paper asks whether competing firms benefit from changes in demand that increase the (perceived) valuations of some consumers and decrease those of others. Johnson and Myatt (2006) explore a related question for a monopolist. They suppose the firm can engage in measures related to advertising, marketing or product design that result in a rotation of the demand functions (or, equivalently, the valuation distribution). A (clockwise) rotation increases the distribution function below a threshold value, but reduces it above the threshold. In particular, a rotation around the mid-point takes away mass from this mid-point. In our setting, a rotation of the distribution G of valuation differences around zero would therefore always be desirable for firms. In fact, a local rotation around zero would be sufficient. However, obfuscation, that is, adding noise to the distribution as in (1), does not necessarily lead to a local rotation. When the preferences satisfy weak polarization, the mass at zero increases under obfuscation, which is not consistent with a (clockwise) rotation of the distribution. Therefore, while firms would want to induce a (local) rotation, they do not want to engage in obfuscation in this case.

In Johnson and Myatt (2006), the monopolist does not even necessarily want to induce a rotation of the valuation distribution. Doing so is only desirable if the monopolist follows a *niche strategy* according to which he only serves consumers with valuations above the rotation point. When consumers are sufficiently homogeneous, the monopolist will instead follow a *mass market* strategy, that is, he also serves consumers with valuations below the rotation point. In this case, a rotation is undesirable because it necessitates a price reduction. The difference in the role of rotations in Johnson and Myatt (2006) and our paper reflects the fact that we consider distributions of valuation differences rather than valuations themselves.

Contrary to Johnson and Myatt (2006), our focus is on demand changes resulting from valuation distortions rather than changes in true valuations. As far as the positive analysis (Theorems 1-3) is concerned, this difference is immaterial: All our results can, for instance, be applied to changes in product design that some consumers truly appreciate, while others do not, as long as Condition (1) is satisfied. The welfare analysis of Section 2.2.4, however, refers only to the case that there is a conflict between true and perceived valuations, so that confused consumers take wrong decisions.

2.4.2 Marketing Activities

Our analysis requires symmetry of the distribution Γ capturing the noise induced by marketing activities. This assumption is consistent with activities that increase the relative valuations for some consumers, but decrease them for others. These activities fall into two broad categories. The first category contains several activities affecting only the valuation for a firm's own good; the second one contains activities that also affect the valuation for the competitor's product.

Confusion about a firm's own good A firm's marketing activity may relate to the description of product features. For instance, a firm can describe the properties of a product in a transparent way, so that consumers easily understand how valuable it is. Instead, it can present unnecessary details, thereby confusing consumers to over- or undervalue the product. On a closely related note, a firm may instruct its sales persons to provide useful product information in response to consumer queries about the product or, instead, to remain vague and unclear.

Firms can also influence the precision of consumer information with product design. For instance, high-tech products such as smartphones have numerous features that most consumers are unlikely to ever use. The exact number and nature of these features may therefore be of limited relevance for the true product valuation of most consumers. Their existence may, however, distract consumers from assessing the value of the product correctly by focusing on the properties that will really matter. It therefore seems plausible that an increase in the number of features increases the noise in the perception of a product's value and therefore acts as an obfuscation device.

Even when a firm uses advertising to create a better image of its product (for instance, by exaggerating quality), consumers may react to these activities in a heterogeneous way. While some consumers may take the firm's advertising at face value, others may be put off by overselling attempts and may therefore be less willing to buy the good, reflecting the well-known nuisance effect of advertising. On a closely related note, the practice of "shockvertising" (the use of provocative advertising motives by firms such as Benetton) can trigger heterogeneous reactions, ranging from enthusiasm to disgust. It is thus seems plausible that such advertising adds noise to consumer decisions..[18]

Confusion about the market The second broad category of marketing instruments consists of activities that not only affect a firm's own perceived valuation, but also the perceived valuation for the competitor's good. To fix ideas, suppose each good has two characteristics A and B. Firm 1 (2) is known to be strong with respect to characteristic A (B). Consumers differ with respect to their needs for the different characteristics: Some consumers require characteristic A more than characteristic B and vice versa. To capture this, suppose firms are located at the ends of a Hotelling line; consumers are spread over the line, where consumers who have relatively strong needs for characteristic A (B) are located near Firm 1(2). Now suppose that, to understand their needs (location) exactly, consumers need advice by the firms – without advice, the consumers perceive their needs with noise. Then if the firms choose not to educate the consumers precisely about their needs, some consumers will incorrectly perceive their valuations for Firm 1 as higher than their true valuations (and those for Firm 2 as lower), whereas others perceive their

[18]One may argue that these effects are at least partly due to genuine changes of valuations: A consumer who sees through such claims may be annoyed and thus arguably experiences a true reduction in the valuation for the product by the advertising. Similarly, the brand image effects of provocative advertising arguably correspond to true preference effects.

valuation for Firm 2 (relative to Firm 1) as higher than the true valuation.[19]

2.4.3 Default Strategies and Consumer Costs

So far, we have implicitly assumed that marketing strategies differ only with respect to the amount of noise they induce. We now address two further, closely related sources of heterogeneity. First, there could be a default marketing strategy which corresponds to "inactivity". Second, there could be heterogeneous marketing costs. It is useful to assume the firm can achieve the default marketing activity with zero costs.[20] In principle, the default could be $a_i = 0$, so that if a firm does nothing, perceived and true valuations coincide; it would then be natural that the cost function is increasing in obfuscation for an order satisfying (A3)(i) or (ii). Alternatively, there could just as well be pre-existing confusion, which remains unresolved unless the firms engage in costly education activities; in this case, the default satisfies $a_i \neq 0$. Assuming that larger deviations from the default are more costly would be consistent with a cost function that is decreasing in obfuscation (if the default corresponds to maximal confusion) or non-monotone (if the default corresponds to an interior level of confusion).

With default activities involving pre-existing confusion, Theorem 2 applies directly if one abstracts from marketing costs: With (strongly) indecisive tastes, firms still choose marketing activities that induce maximal confusion; with polarized tastes, they strive to eliminate any confusion. The previous analysis is useful even with heterogeneous marketing costs, because it identifies how the willingness-to-pay for consumer confusion or education depends on the preference distribution. With marketing costs, the analysis would change in two related ways. First, extreme marketing profiles as predicted by Theorem 2 would no longer arise if they are too costly for the firms. Second, there would be a potential conflict of interest between the firms who would have to coordinate on who engages in more costly marketing activities. As a result, even if obfuscation (or education) would increase joint profits, it would not necessarily arise in equilibrium.

2.4.4 Obfuscation and Sophistication

Many papers in the literature have worked with the assumption that consumers differ with respect to how naive they are, distinguishing between a group of perfectly rational ("sophisticated") consumers and a group of "naive" consumers who have various types of cognitive limitations; the relative size of these groups is usually a central parameter

[19] Similarly, in a setting as just described, consumers may be clear about the needs, but they may not be sure about the exact attribute qualities of the two firms.

[20] Essentially, this is just a normalization.

in this literature.[21] In our setting, one can think of the noise distribution as capturing the degree of sophistication in the population in a continuous rather than in a discrete fashion: For completely sophisticated consumers, the perceived valuation differences are equal to zero; the more naive a consumer is, the greater the difference between perceived and actual valuations. Our assumptions on the noise distribution can thus be interpreted as assumptions on the distribution of consumer sophistication.

3 Extensions

In this section, we extend the model in two directions. First, we introduce outside options. Second, we illustrate the channels through which consumer confusion may arise in a slightly modified setting.

3.1 Outside options

We now illustrate how the presence of binding outside options affects the analysis. For simplicity, assume that the perceived valuations are given by

$$\tilde{v}_1^k = \frac{\mu}{2} + \frac{\upsilon}{2} + \frac{\varepsilon}{2}, \quad \tilde{v}_2^k = \frac{\mu}{2} - \frac{\upsilon}{2} - \frac{\varepsilon}{2},$$

where $\mu > 0$ is some constant, and $\upsilon \in [-1, 1]$ is drawn from some distribution G_0.[22] We now suppose that the consumers have a reservation value $u_0 = 0$, so that a consumer k will only buy from firm i if $\tilde{v}_i^k - p_i \geq \max\{\tilde{v}_j^k - p_j, 0\}$. We use $p_{\mathbf{a}}^m$ to denote the solution to the monopoly problem

$$\max_{p \geq 0} \ \Pi^m(p) \equiv p\left(1 - G_{\mathbf{a}}(2p - 1)\right).$$

Note that a unique solution of this maximization problem exists if $G_{\mathbf{a}}$ is log-concave on *supp*($g_{\mathbf{a}}$). The following extension of Theorem 1 characterizes the equilibrium.

Proposition 4 *Suppose that G_0 and $\Gamma_{\mathbf{a}}$ satisfy (A1). In the game with outside options, there exists a unique symmetric pure-strategy equilibrium in every pricing subgame, and each firm chooses the price*

$$p_{\mathbf{a}}^* = \begin{cases} \frac{1}{2g_{\mathbf{a}}(0)} & \text{if } g_{\mathbf{a}}(0) > \frac{1}{\mu}, \\ \frac{\mu}{2} & \text{if } g_{\mathbf{a}}(0) \in \left[\frac{1}{2\mu}, \frac{1}{\mu}\right], \\ p_{\mathbf{a}}^m & \text{otherwise.} \end{cases}$$

[21] For examples, see Gabaix and Laibson (2006) and Heidhues et al. (2016).

[22] As in Section 3.3, we therefore assume that valuations are negatively correlated; however, we now assume that the support is a straight line in $\mathbb{R}^2$.

Thus, when the concentration of consumers around indifference is sufficiently high, everything is as in the equilibrium without outside options: Competition for the indifferent consumers keeps prices so low that everybody is served. Below a certain threshold concentration of indifferent consumers, firms charge lower prices than without outside options ($p = \mu/2$), but the market remains completely covered. Finally, for a very low concentration of indifferent consumers, firms give up on these consumers and charge higher (monopoly) prices.

Thus, with a binding outside option and monopolistic pricing there is another inefficiency associated with confusion: Not only do some consumers purchase the wrong product, but there are also some consumers who do not purchase at all though a transaction would be socially desirable.

In this sense, the result from the homogeneous good case extrapolates: Firms can never earn less than zero there (Spiegler-Piccione); in our case: Firms can never earn less than the full information benchmark. The main difference is that sometimes firms do not wish to obfuscate. If obfuscation occurs in equilibrium, it must be profitable.

3.2 Salop Model

So far, we have been agnostic about the exact source of confusion. Consumers may not correctly understand the (objective) properties of the good or their own (subjective) needs. In the following, we specify the framework in such a way that confusion concerns consumer needs rather than objective properties. The details are in the appendix. Here, we summarize the main ideas briefly.

Specifically, we suppose two firms are located at antipodal locations of a Salop circle parameterized by $\theta \in [0, 1]$ (Firm 1 is located at 0, Firm 2 at 1/2). Consumers are distributed on the circle with a continuous and positive density that is symmetric at both axes, so that it takes the same values for any two locations with identical distance from 0 and 1, but also for two locations with identical distance from 1/4 (and thus also from 3/4). Transportation costs are linear. We capture obfuscation as inducing a uniform location shock $e \in [-\varepsilon(\mathbf{a}), \varepsilon(\mathbf{a})]$, corresponding to a confusion in consumer needs.

Contrary to our main model, in this setting obfuscation never generates any perceived valuations that cannot arise without obfuscation.[23] Moreover, it is straightforward to see that, for suitable values of e, the distribution of possible perceived valuation distributions

[23] This is true because the perceived valuation of a confused consumer corresponds to the true valuation of a consumer with another location.

has a smaller support for consumers who are strongly attached to one of the competitors (located near 0 or 1/2) than for consumers who are close to indifference (located near 1/4 or 3/4). Thus, this model violates Condition (1). Moreover, there is an upper bound to the extent of obfuscation: If $e \geq 1/2$, then, independently of a consumer's true location, the perceived location will be uniformly distributed on the circle.

In the appendix, we show that the first-order conditions suggest interesting effects of obfuscation. With indecisive preferences, profits and prices are inverse-U shaped in ε. With polarization, we instead obtain an interior minimum. In both cases, the extreme value is between 1/4 and 1/2. To see the intuition, consider the case of indecisive preferences. As in our baseline model, a small amount of obfuscation benefits firms by moving consumers away from indifference. As a result, as obfuscation becomes sufficiently strong, the perceived valuation distribution no longer displays indecisiveness, but instead polarization. Then further obfuscation changes the valuation difference towards uniformity, which is no longer beneficial for the firms: If consumers are maximally confused about their needs, the two goods become identical from an ex-ante perspective.

4 Competition for voters

The ideas of our model can be applied to other types of competitive interactions. We illustrate this for the case of competition for voters. This involves extending our formal apparatus to deal with contests.

We consider a two-stage model where candidates $i \in \{1, 2\}$ compete for voter k by first choosing the clarity of their platform and then engaging in costly activities to persuade voters. Paralleling the set-up of Section 2, we assume there is a population of voters that can be described by a joint valuation distribution $F_0\left(v_1^k, v_2^k\right)$ of idiosyncratic "baseline valuations" for the politicians. However, we assume that politicians can influence the perceived valuations of voters in two ways. In the first stage, they choose to present their political platforms more or less clearly: A candidate can use mixed messages that can be interpreted positively or negatively by any recipient, leading to a noisy picture of the candidate, whereby some voters get a too positive impression of the candidate's value for them and others get a too negative impression. We model this by assuming that candidate i can choose from a set of platform descriptions A_i which are characterized by additive noise distributions $\Gamma_{\mathbf{a}}$ distorting the underlying baseline valuations and generating perceived baseline valuations $\tilde{v}_k^i$. We maintain the same assumptions on G_0 and $\Gamma_{\mathbf{a}}$ as imposed in Assumption 1 for the oligopoly model.

However, we now also assume that, once the perceived baseline valuations are determined, the candidates can engage in costly effort s_i to convince voters to vote for them, with an effort cost function $c(s_i)$ satisfying $c(0) = 0$, $c'(s_i) > 0$, $c'(0) = 0$ and $c''(s_i) > 0$ for $s_i > 0$. As a result of exerting effort, the perceived utility of voter k from candidate i becomes

$$U_i^k = \tilde{v}_i^k + s_i. \tag{4}$$

The effort cost s_i allows for two different interpretations. First, it is well known that, e.g., a strong media presence of a candidate can persuade voters to favor the candidate. In this interpretation, a persuasion effort of s_i is associated with advertising expenditure $c(s_i)$ which, like in an all-pay auction, the politician needs to pay, no matter whether she is successful or not. Second, s_i could correspond to a commitment to a policy which the candidate needs to fulfil should she be elected. This commitment comes at a cost $c(s_i)$ which only needs to be paid in case of success: We think of this cost as reflecting the reduction in credibility coming from an inability to live up to the expectations generated by the commitments. The politician could be concerned about this loss of credibility because of intrinsic feelings of shame from being perceived as dishonest or because the inability to deliver will impede the election chances in future elections.

In both cases, the probability that candidate i wins if $\tilde{v}_\Delta^k \equiv \tilde{v}_j^k - \tilde{v}_i^k$ and $s_\Delta = s_i - s_j$ is

$$Pr(\tilde{v}_\Delta^k \leq s_\Delta + \varepsilon_{\mathbf{a}}) = \int G_0(s_\Delta + e)\gamma_{\mathbf{a}}(e)de \tag{5}$$

We normalize the payoff of the candidate from winning to 1, which is wlog for our purpose. Candidate i's payoff in the advertising case is

$$\Pi_i(s_1, s_2; \mathbf{a}) = Pr(\tilde{v}_\Delta^k \leq s_\Delta + \varepsilon_{\mathbf{a}}) - c(s_i)$$

and in the commitment case

$$\Pi_i(s_1, s_2; \mathbf{a}) = Pr(\tilde{v}_\Delta^k \leq s_\Delta + \varepsilon_{\mathbf{a}}))(1 - c(s_i)).$$

The commitment case is essentially isomorphic to the model of Section 2. In particular, a higher effort in the second stage has an analogous effect as a lower price in the previous model: It increases the chances of winning, but it is costly only in case of success.[24] By contrast, the second stage of the advertising example is a contest: The candidate incurs the advertising cost no matter whether she wins the election or not.

In spite of these differences, the intuition from Theorem 1 applies in both cases. As stated in Proposition 1A in the Appendix, voter preferences fully determine whether or

[24]Similarly a price reduction in the original model increases demand, but results in a cost only in those cases where a consumer buys the good.

not candidates engage in (small) obfuscation: Candidates will only use obfuscation when preferences are indecisive rather than polarized. In particular, ex ante indecisive preferences provide a breeding ground for voter confusion. With ex-ante indecisive preferences, both candidates are forced to choose high levels of commitments in order to win the campaign if there is no confusion, because small differences in commitments have a strong effect on the voter shares. With obfuscated campaigns, the perception of commitments (of their differences) becomes noisy, which reduces the measure of perceptually indecisive consumers and thereby allows candidates to reduce their commitments.

It is useful to think more about the nature of the "baseline preferences" and the sources of indecisiveness. In reality, voters form judgments about policy platforms under potentially large uncertainty about their consequences. Even if candidates describe their position in the clearest possible way, this uncertainty will not be resolved. For instance, the effects of a politician's approach to climate policy are subject to scientific uncertainty as well as uncertainty about the strategies of other countries. We should therefore think of the baseline valuations as reflecting the valuation that a voter puts on a candidate when being clearly informed about the platform – but this valuation leaves large scope for uncertainty about the true effects of different policies if implemented. One reason for indecisiveness may then be that this uncertainty is so large that, absent obfuscation strategies, many voters will be unsure whom to vote for. This does not reflect indifference about policy platforms, but uncertainty about the relation between candidate platforms and outcomes. In such a situation, our model then predicts polarizing obfuscation strategies as a means to overcome voter indifference.[25]

5 Relation to the Literature

Our paper belongs to the literature on competition with boundedly rational consumers.[26] The most distinctive feature of our contribution is that we treat the case of heterogeneous

[25] In principle, this idea is applicable to the oligopoly context as well. For instance, even when producers of health food describe the contents in a perfectly transparent fashion, there can be irreducible uncertainty about the long-term health effects, reflecting a lack of knowledge at the firm level. Then consumers may be indecisive because even with the best possible information they simply cannot tell which good serves them better.

[26] A peripherally related literature going back to Grossman and Hart (1980), Grossman (1981), Milgrom (1981) and Milgrom and Roberts (1986) asks under which conditions firms want to disclose product information to consumers who otherwise only have stochastic information about quality, assuming (unlike we do) that the consumers are sufficiently sophisticated to use Bayesian updating to interpret the disclosure decisions of firms.

preferences, where the incentives for confusing consumers are less immediate than for homogeneous goods.

Building on the intuition of Scitovsky (1950), several authors have argued that producers of homogeneous goods can escape the Bertrand trap if they manage to mislead, deceive or confuse consumers. For example, firms may present prices in such a way that comparison becomes difficult (Carlin, 2009; Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013). Hence, consumers may sometimes choose from high-price firms even if better offers are available. Firms exploit this by playing mixed-strategy equilibria in which expected profits are positive. Several authors have shown how firms can benefit if consumers perceive homogeneous goods as better than they really are. For instance, in Gabaix and Laibson (2006) firms hide add-on costs of products, so that naïve consumers ignore these costs in their purchasing decisions. The authors show how firms and sophisticated consumers can benefit from the presence of naïve consumers. Heidhues et al. (2016) establish that fooling naïve consumers with hidden fees becomes particularly profitable for socially wasteful products. On a related note, Spiegler (2006) shows how firms producing valueless services can benefit if consumers apply simple sampling procedures to evaluate the services. In our framework, obfuscation does not necessarily bias consumer valuation upwards. Thus, firms cannot be systematically fooled (Spiegler, 2017).[27] Nevertheless, firms may still benefit from confusion. In fact, with indecisive preferences, firms may benefit from obfuscation even if it causes a decrease of expected (absolute) consumer valuations.[28]

More broadly, our paper sheds light on the older advertising literature. First, this literature deals with the role of advertising that is informative "about the existence, prices and characteristics of goods" (Belleflamme and Peitz, 2010). We can think of our model as capturing the choice between relatively informative advertising (little or no obfuscation) and uninformative advertising (more obfuscation); thus informative advertising reduces the noise in perception of the relative product valuations. Our analysis thus shows that informative advertising may or may not increase prices, contrary to the effects of information about product existence, which typically reduces prices by increasing competition (Bagwell, 2007).

The paper can also be put into the perspective of the persuasive advertising literature.

[27]Spiegler (2017) asks when decision-makers can be fooled if they hold a misspecified causal model. He defines "fooling" in terms of the expected outcome or prediction from the mispecified model relative to the correct model.

[28]While we assume that obfuscation leaves expected valuation *differences* unaffected, there is no such statement for expected valuations.

This literature considers advertising activities that firms use to increase the willingness to pay for their product or, similarly, shift the valuation distributions in their favor. The literature argues that games of persuasive advertising have the structure of a prisoners' dilemma: Firms engage in costly advertising races, which, in equilibrium, do not effect prices and gross profits. Contrary to the bulk of this literature, the marketing activities in our model can be interpreted as activities that persuade some consumers at the cost of alienating others.[29] At first glance, it would appear that such activities are less attractive to firms than persuasive advertising activities that shift demand systematically in the direction of a firm. Our equilibrium analysis shows, however, that this is not the case: Firms either refrain from such advertising measures (with polarized preferences) or they use the measures to soften competition – under no circumstances do they end up in the more standard prisoners' dilemma.

6 Conclusion

The paper extends the analysis of firm-driven consumer confusion to the prevalent case where consumers differ in their opinion about the values of the existing alternatives. We find that the overall dispersion of consumer tastes has decisive implications for the propensity of firms to confuse or educate consumers and for the subsequent price competition. If undecided consumers are relatively common, firms benefit from slight confusion, because it results in more consumers conceiving one product as better than the other, allowing the firm to increase its price without losing too much demand. Consumers can be harmed in up to three ways by such obfuscation: they might purchase the "wrong" product, they pay higher prices and some consumers may forego purchases that would be socially efficient. With a polarized dispersion of tastes, we obtain the opposite result that consumer confusion is harmful for firms. In this case, firms seek to increase market transparency by educating confused consumers, e.g., by providing qualified information and guidance that helps consumers understand the offered alternatives in the market.

Finally, no matter whether preferences are polarized or not, obfuscation tends to be beneficial for firms and harmful for consumers if confusion becomes so strong that it dominates true preference differences, no matter what the shape of the true preference distribution is.

In sum, our results suggest that research on market transparency and confusion cannot be agnostic about the nature of the existing dispersion of opinions, as this interacts non-trivially with the firms' incentives to influence the effective perception of consumers.

[29] As an example, consider the cold-calls of tele-marketing agents (see Schumacher and Thysen, 2017).

References

BAGWELL, K. (2007): "The economic analysis of advertising," *Handbook of industrial organization*, 3, 1701–1844.

CAPLIN, A. AND M. DEAN (2015): "Revealed preference, rational inattention, and costly information acquisition," *American Economic Review*, 105, 2183–2203.

CARLIN, B. I. (2009): "Strategic price complexity in retail financial markets," *Journal of Financial Economics*, 91, 278–287.

CHIOVEANU, I. AND J. ZHOU (2013): "Price competition with consumer confusion," *Management Science*, 59, 2450–2469.

ELLISON, G. AND S. F. ELLISON (2009): "Search, obfuscation, and price elasticities on the internet," *Econometrica*, 77, 427–452.

ELLISON, G. AND A. WOLITZKY (2012): "A search cost model of obfuscation," *The RAND Journal of Economics*, 43, 417–441.

GABAIX, X. AND D. LAIBSON (2006): "Shrouded attributes, consumer myopia, and information suppression in competitive markets," *The Quarterly Journal of Economics*, 121, 505–540.

GROSSMAN, S. J. (1981): "The informational role of warranties and private disclosure about product quality," *The Journal of Law and Economics*, 24, 461–483.

GROSSMAN, S. J. AND O. D. HART (1980): "Disclosure laws and takeover bids," *The Journal of Finance*, 35, 323–334.

HEIDHUES, P., B. KŐSZEGI, AND T. MUROOKA (2016): "Inferior products and profitable deception," *The Review of Economic Studies*, 84, 323–356.

JOHNSON, J. P. AND D. P. MYATT (2006): "On the simple economics of advertising, marketing, and product design," *American Economic Review*, 96, 756–784.

KAMENICA, E. AND M. GENTZKOW (2011): "Bayesian persuasion," *American Economic Review*, 101, 2590–2615.

MILGROM, P. (1981): "Good news and bad news: Representation theorems and applications," *The Bell Journal of Economics*, 380–391.

MILGROM, P. AND J. ROBERTS (1986): "Relying on the information of interested parties," *The RAND Journal of Economics*, 18–32.

PICCIONE, M. AND R. SPIEGLER (2012): "Price competition under limited comparability," *The Quarterly Journal of Economics*, 127, 97–135.

SCHUMACHER, H. AND H. THYSEN (2017): "Equilibrium Contracts and Boundedly Rational Expectations," .

SCITOVSKY, T. (1950): "Ignorance as a source of oligopoly power," *The American Economic Review*, 40, 48–53.

SPIEGLER, R. (2006): "Competition over agents with boundedly rational expectations," *Theoretical Economics*, 1, 207–231.

——— (2017): "Can agents with causal misperceptions be systematically fooled?" *Working Paper*.

Appendix A: Proofs of Main Results

Proof of Lemma 1 First, we claim that $g_{\mathbf{a}}(0) > 0 \; \forall \mathbf{a} \in \mathcal{A}$, and hence $p^*_{\mathbf{a}}$ is well-defined. If $\varepsilon_{\mathbf{a}}$ is degenerate, this follows directly form (A3), so suppose that $\varepsilon_{\mathbf{a}}$ is non-degenarte. By (A5), $\Gamma_{\mathbf{a}}$ then has a density $\gamma_{\mathbf{a}}$ which is log-concave on its support. By definition, it follows that $supp(\gamma_{\mathbf{a}})$ must be a convex set, i.e., an interval on $\mathbb{R}$. It then follows from (A4) that $0 \in supp(\gamma_{\mathbf{a}})$, for if $0 \notin supp(\gamma_{\mathbf{a}})$ then $supp(\gamma_{\mathbf{a}})$ must reside entirely either in $(-\infty, 0)$ or in $(0, \infty)$, contradicting the symmetry of $\Gamma_{\mathbf{a}}$ at zero. Symmetry of $\Gamma_{\mathbf{a}}$ further assures that 0 is an interior point of $supp(\gamma_{\mathbf{a}})$. By (A3), $g_0(0)$ is continuous at the point $x = 0$ and $g_0(0) > 0$. This implies that there must exist $\delta > 0$ such that $g_0(x) > 0$ $\forall x \in [-\delta, \delta]$. Because 0 is an interior point of $supp(\gamma_{\mathbf{a}})$ and $supp(\gamma_{\mathbf{a}})$ is an interval, we can choose this $\delta > 0$ so small to assure that $[-\delta, \delta] \subset supp(\gamma_{\mathbf{a}})$. Accordingly, we conclude that

$$g_{\mathbf{a}}(0) = \int_{-\infty}^{+\infty} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon \geq \int_{-\delta}^{\delta} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon > 0.$$

Now consider the profit maximization problems of the firms. As G_0 is non-degenerate, choosing $p_i = 0$ would never be optimal for any firm. For every $p_j > 0$, let $\underline{p}(p_j) \equiv \sup\{p_i \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_j - p_i) = 1\}$ and $\bar{p}(p_j) \equiv \inf\{p_i \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_j - p_i) = 0\}$. Taking firm j's price p_j as given, it is clear that, from firm i's perspective, any $p_i < \underline{p}(p_j)$ is strictly dominated by $\underline{p}(p_j)$, and any $p_i > \bar{p}(p_j)$ is strictly dominated by $\bar{p}(p_j)$. Hence, when solving the firms' optimization problems, we can assume without loss of generality that, by taking $p_j > 0$ as given, each firm i will only choose a price p_i from the interval $\big[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\big]$. This leads to the first-order conditions of the firms:

$$\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1} = G_{\mathbf{a}}(p_2 - p_1) - p_1 g_{\mathbf{a}}(p_2 - p_1) = 0,$$
$$\frac{\partial \Pi_2(p_1, p_2)}{\partial p_2} = 1 - G_{\mathbf{a}}(p_2 - p_1) - p_2 g_{\mathbf{a}}(p_2 - p_1) = 0.$$

By (A1) and (A4), it is easy to verify that $G_{\mathbf{a}}$ is symmetric for all $\mathbf{a} \in \mathcal{A}$. Hence, by plugging $p_1 = p_2$ into the first-order conditions, we obtain the unique candidate for a symmetric equilibrium in the pricing subgame:

$$p_1 = p_2 = p^*_a \equiv \frac{G_{\mathbf{a}}(0)}{g_{\mathbf{a}}(0)} = \frac{1}{2g_a(0)},$$

where the last equality follows again the symmetry of $G_{\mathbf{a}}$.

Finally, we argue that the log-concavity assumptions (A2) and (A5) guarantee that the price profile $(p_1, p_2) = (p^*_a, p^*_a)$ indeed constitutes an equilibrium. To see this, first note that $G_{\mathbf{a}}(x)$ is log-concave on $supp(g_{\mathbf{a}})$ for all $\mathbf{a} \in \mathcal{A}$. This is trivial if $G_{\mathbf{a}}$=G_0, i.e., if $\Gamma_{\mathbf{a}}$ is degenerate. If $\Gamma_{\mathbf{a}}$ is non-degenerate, the claim holds since both G_0 and $\gamma_{\mathbf{a}}$ are log-concave

((A2) and (A5)), and the convolution of log-concave functions is also log-concave. Since the function $f(p) = p$ is strictly log-concave on $[0, +\infty)$, it is then straightforward to verify that the profit function $\Pi_i(p_i, p_j)$ is *strictly* log-concave (and hence strictly quasi-concave) in p_i on $\left[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\right]$ for all $p_j > 0$. Since $\Pi_i(\underline{p}(p_j), p_j) = \Pi_i(\bar{p}(p_j), p_j) = 0$, strict quasi-concavity implies that $p_1 = p_a^*$ must be a global maximum of the function $\Pi_1(p_1, p_{\mathbf{a}}^*)$. Hence, when firm j plays $p_j = p_{\mathbf{a}}^*$, it is a best response for firm i to choose the same price $p_i = p_{\mathbf{a}}^*$. We can now conclude that $(p_{\mathbf{a}}^*, p_{\mathbf{a}}^*)$ constitutes a unique symmetry equilibrium for the pricing subgame where firms choose $\mathbf{a} \in \mathcal{A}$ in the first-stage. □

Proof of Theorem 1 (i) If $\varepsilon_{\mathbf{a}}$ is degenerate for any $\mathbf{a} \in \mathcal{A}$ the claim is trivial, so suppose otherwise. Then, for every $\mathbf{a} \in \mathcal{A}$, such that $\Gamma_{\mathbf{a}}$ is non-degenerate, we have

$$g_{\mathbf{a}}(0) = \int_{supp(\gamma_{\mathbf{a}})} g_0(-\varepsilon)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon > \int_{supp(\gamma_{\mathbf{a}})} g_0(0)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon = g_0(0),$$

where the inequality follows from $supp(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ and polarized preferences. Hence, by Theorem 1 we can conclude that $p_{\mathbf{a}}^* < p_0 \equiv \frac{1}{2g_0(0)}$ for all such $\mathbf{a} \in \mathcal{A}$. It immediately follows that any choice of $\mathbf{a} \in \mathcal{A}$ which leaves $\Gamma_{\mathbf{a}}$ degenerate must be a Nash equilibrium of the marketing stage, followed by $p_1^* = p_2^* = p_0$ in the pricing stage. In this SPE, marketing strategies are thus chosen such that no consumer confusion results. (ii) can be proven analogously. □

Proof of Theorem 2 (i) First suppose (a) holds. Take any $\mathbf{a}, \mathbf{a}' \in \mathcal{A}$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. Then $\varepsilon_{\mathbf{a}'}$ has the same distribution as $\varepsilon_{\mathbf{a}} + \eta$, $\eta \neq O$, by the MPS property. Thus

$$\begin{aligned} g_{\mathbf{a}'}(0) = \textstyle\int_{-\delta}^{\delta} g_0(-e)d\Gamma_{\mathbf{a}'}(e) &= E\left[g_0(\varepsilon_{\mathbf{a}'})\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}'}\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}} + \eta\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &< E\left[g_0\left(E[\varepsilon_{\mathbf{a}} + \eta|\varepsilon_{\mathbf{a}}]\right)\right] \\ &= E\left[g_0\left(\varepsilon_{\mathbf{a}}\right)\right] = \textstyle\int_{-\delta}^{\delta} g_0(-e)d\Gamma_{\mathbf{a}}(e) = g_{\mathbf{a}}(0). \end{aligned} \tag{6}$$

The second line follows from the law of iterated expectations, the third because $\varepsilon_{\mathbf{a}'}$ and $\varepsilon_{\mathbf{a}}+\eta$ are equal in distribution, and the forth line follows from Jensen's inequality because g_0 is strictly concave. The order of the marketing strategies in Assumption 3 implies that $g_{\mathbf{a}}(0)$ achieves its unique min at $\mathbf{a} = (\bar{a}, \bar{a})$ which by Lemma 1 maximizes the equilibrium price and profit on $\mathcal{A}$. For this reason, the marketing profile $(\bar{a}, \bar{a})$ is part of an SPE. Moreover, $(\bar{a}, \bar{a})$ is the only possible part of an SPE, because for any alternative marketing profile $(a_1, a_2) \in \mathcal{A}$, the firm with $a_i < \bar{a}$ would always deviate to $\bar{a}$.

Now suppose (b) holds. Take any $\mathbf{a}, \mathbf{a}'$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{0} \neq \mathbf{a} \leq \mathbf{a}'$. Assumption 3 (ii) implies that there exists a unique $\hat{e} \in (0, \omega(\mathbf{a}'))$, such that $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) < 0$ $\forall e \in [0, \hat{e})$, and $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) > 0$ $\forall e \in (\hat{e}, \omega(\mathbf{a}')]$.

Since g_0 is strictly decreasing on $[0, \omega(\mathbf{a}')] \subset [0, \delta]$, we further have

$$\begin{aligned}\int_{\hat{e}}^{\omega(\mathbf{a}')} g_0(e)\left(\gamma_{a'}(e) - \gamma_a(e)\right) de &< \int_{\hat{e}}^{\omega(\mathbf{a}')} g_0(\hat{e})\left(\gamma_{a'}(e) - \gamma_a(e)\right) de \\ &= \int_0^{\hat{e}} g_0(\hat{e})(\gamma_a(e) - \gamma_{a'}(e)) de \\ &< \int_0^{\hat{e}} g_0(e)(\gamma_a(e) - \gamma_{a'}(e)) de, \end{aligned} \tag{A1}$$

where the equality makes use of that, by symmetry and $\omega(\mathbf{a}) \leq \omega(\mathbf{a}')$, we have

$$\frac{1}{2} = \int_0^{\omega(\mathbf{a}')} \gamma_{\mathbf{a}'}(e) de = \int_0^{\omega(\mathbf{a})} \gamma_{\mathbf{a}}(e) de = \int_0^{\omega(\mathbf{a}')} \gamma_{\mathbf{a}}(e) de.$$

Exploiting again the symmetry of g_0, $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{a}'}$, we further have

$$\begin{aligned}&\int_{-\omega(\mathbf{a}')}^{\omega(\mathbf{a}')} g_0(-e)\gamma_{\mathbf{a}'}(e) de - \int_{-\omega(\mathbf{a})}^{\omega(\mathbf{a})} g_0(-e)\gamma_{\mathbf{a}}(e) de \\ =& \int_{-\omega(\mathbf{a}')}^{\omega(\mathbf{a}')} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\ =& 2\int_0^{\omega(\mathbf{a}')} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\ =& 2\left[\int_0^{\hat{e}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega(\mathbf{a}')} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] \\ =& 2\left[\int_0^{\hat{e}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega(\mathbf{a}')} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] < 0,\end{aligned}$$

where the last inequality follows from (A1). We have now shown that $g_{\mathbf{a}'}(0) < g_{\mathbf{a}}(0)$ for any feasible $\mathbf{a} \neq \mathbf{a}'$ with $\mathbf{0} \neq \mathbf{a} \leq \mathbf{a}'$. Hence, if preferences are δ-indecisive, $g_{\mathbf{a}}(0)$ must be uniquely minimized at $\mathbf{a}^* = (\bar{a}, \bar{a})$, which in turns implies that the equilibrium price is uniquely maximized at $\mathbf{a}^*$.

(ii) As the argument for (b) is similar, we only provide the argument for the case that (a) holds, so that g_0 is strictly convex. Then, the inequality in (6) is reversed. By Lemma 1, any $\varepsilon_{\mathbf{a}'}$ which is MPS of $\varepsilon_{\mathbf{a}}$ therefore is payoff dominated by $\varepsilon_{\mathbf{a}}$. Further, any $\varepsilon_{\mathbf{a}} \neq O$ trivially is a MPS of O. Because $\varepsilon_{\mathbf{a}} = O \Leftrightarrow \mathbf{a} = 0$, it follows that $\mathbf{0}$ is the only part of an SPE. Indeed, setting $a_i = 0$ is a dominant action for each firm i, because for any given $a_{-i} \in A$, $(0, a_{-i})$ is a strict best reply of firm i, because any alternative (a_i, a_{-i}) is a MPS of $(0, a_{-i})$. □

Proof of Theorem 3 First, under the assumptions of the theorem, the distribution function $\Gamma_{\mathbf{a}}$ is given by

$$\Gamma_{\mathbf{a}}(x) = \begin{cases} 1 & \text{if } x > \omega_{\mathbf{a}} \geq 0, \\ \frac{x+\omega_{\mathbf{a}}}{2\omega_{\mathbf{a}}} & \text{if } x \in [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}], \\ 0 & \text{otherwise.} \end{cases}$$

Moreover, for every $\omega \geq 0$, let

$$g_{\omega}(0) \equiv \int_{-\omega}^{\omega} g_0(\varepsilon) d\Gamma_{\omega},$$

where Γ_{ω} is the degenerate distribution that put all mass at zero if $\omega = 0$, and it is the uniform distribution on $[-\omega, \omega]$ if $\omega > 0$.

(i) We now show that the local density $g_{\omega}(0)$ is strictly decreasing in ω on $[0, +\infty)$. Since preferences are strongly indecisive on the support of g_0 and, by definition, $g_0(x) = 0$ $\forall x \notin supp(g_0)$, we immediately have

$$g_{\omega}(0) > \int_{-\omega}^{\omega} g_0(0) d\Gamma_{\omega} = g_0(0) \ \forall \omega > 0.$$

Furthermore, since partial derivative of $g_{\omega}(0)$ with respect to ω exists for all $\omega > 0$, we have

$$\begin{aligned} \frac{\partial g_{\omega}(0)}{\partial \omega} &= \frac{g_0(-\omega)}{2\omega} + \frac{g_0(\omega)}{2\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2} g_0(-\varepsilon) d\varepsilon \\ &= \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2} g_0(\varepsilon) d\varepsilon \\ &< \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2} g_0(\omega) d\varepsilon \\ &= \frac{g_0(\omega)}{\omega} - \frac{g_0(\omega)}{\omega} \\ &= 0, \end{aligned}$$

where the inequality follows that g_0 is strongly indecisive on $supp(g_0)$ and $g_0(x) = 0$ $\forall x \notin supp(g_0)$.

By Theorem 1, in every pricing subgame there exists a unique symmetric equilibrium in which $p_1 = p_2 = \frac{1}{2g_{\mathbf{a}}(0)} = \frac{1}{2g_{\omega_{\mathbf{a}}}(0)}$. Since $g_{\omega}(0)$ is decreasing in ω, it immediately follows that $g_{\omega_{\mathbf{a}}}$ is minimized at $\omega_{\mathbf{a}} = \bar{\omega}$. Therefore, the supgame equilibrium price is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$, which implies that there must exist an SPE with maximal obfuscation.[30]

[30]This SPE will be unique if we further assume that $\omega_{\mathbf{a}} = \varphi(a_1, a_2)$ $\forall \mathbf{a} \in \mathcal{A}$, where φ is strictly increasing in both a_1 and a_2.

(ii) As in Theorem 4, let $g_\omega \equiv g_\omega(0) \equiv \int_{-\omega}^{\omega} g_0(\varepsilon)d\Gamma_\omega$, where Γ_ω is the degenerate distribution that put all mass at zero if $\omega = 0$, and it is the uniform distribution on $[-\omega, \omega]$ if $\omega > 0$. We first prove the following two lemmas.

Lemma A1. *If $supp(g_0)$ is bounded, then* $\lim_{\omega\to+\infty} \ g_\omega(0) = 0$.

PROOF. Since $supp(g_0)$ is bounded, we must have $supp(g_0) \subset [-\omega, \omega]$ for sufficiently large ω. As a result,

$$\lim_{\omega\to+\infty} \ g_\omega(0) = \lim_{\omega\to+\infty} \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = \lim_{\omega\to+\infty} \int_{supp(g_0)} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = 0.$$

Lemma A2. *If $supp(g_0)$ is bounded, then $g_\omega(0)$ is strictly decreasing in ω on* $(\sup supp(g_0), +\infty)$.

PROOF. For every $\omega > 0$, we have

$$\frac{\partial g_\omega(0)}{\partial\omega} = \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{g_0(\omega)}{2\omega^2} d\varepsilon.$$

Since $supp(g_0)$ is bounded, we must have $\sup supp(g_0) < +\infty$, and $g_0(\omega) = 0$ for all $\omega > \sup supp(g_0)$. It then follows that

$$\frac{\partial g_\omega(0)}{\partial\omega} = -\int_{-\omega}^{\omega} \frac{g_0(\omega)}{2\omega^2} d\varepsilon < 0 \quad \forall\omega > \sup supp(g_0).$$

Hence, $g_\omega(0)$ is strictly decreasing in ω whenever ω is larger than the supremum of the support of g_0.

Now consider any preference distribution that is strongly polarized on $supp(g_0)$. By definition, g_0 is strictly decreasing on $(\inf supp(g_0), 0]$ and is strictly increasing on $[0, \sup supp(g_0))$. This implies that the support of the support of g_0 must be bounded, because otherwise we would have

$$\int_{supp(g_0)} g_0(x)dx = \int_{-\infty}^{+\infty} g_0(x)dx \geq \int_{-\infty}^{+\infty} g_0(0)dx = +\infty,$$

contradicting to that g_0 being a density function. Applying Lemmas 1 and 2, we can conclude that $\lim_{\omega\to+\infty} g_\omega(0) = 0$ and that $g_\omega(0)$ is strictly decreasing on $(\sup supp(g_0), +\infty)$. Hence, there must exist $\hat{\omega} > 0$, such that if $\omega \geq \hat{\omega}$, then $g_\omega(0) \leq g_0(0)$. Therefore, whenever $\bar{\omega} \geq \hat{\omega}$, the subgame equilibrium price is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$, which implies that there must exist an SPE with maximal obfuscation. □

Proof of Proposition 1 Let $\eta \equiv \sup supp\,(g_0)$. We can write the expected welfare loss from mismatch as a function of the degree of confusion:

$$L(\omega) = 2\int_0^{\min\{\omega,\eta\}} \left[x \cdot \frac{-x+\omega}{2\omega} \cdot g_0(x)\right] dx = \int_0^{\min\{\omega,\eta\}} \left[x\left(1-\frac{x}{\omega}\right) g_0(x)\right] dx.$$

Taking the first derivative, we obtain

$$L'(\omega) = \int_0^{\eta} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega \geq \eta$, and

$$L'(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x) + \omega\left(1-\frac{\omega}{\omega}\right) g_0(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega < \eta$. Since by assumption G_0 is a non-degenerate distribution, we plainly have $L'(\omega) > 0\ \forall \omega > 0$. Hence, the expected welfare loss is strictly increasing in ω. □

Appendix B: Proofs of Additional Results

B.1 Hotelling

Proof of Proposition 2 Recall that we use $G_{\mathbf{a}}$ and $g_{\mathbf{a}}$ to denote the distribution and density functions of the perceived valuation differences. To prove the proposition, it suffices to show that the assumptions of Theorem 1 are satisfied if $\alpha \leq \hat{\alpha}$. Among these assumptions, (A1) , (A3), (A4) and (A5) hold trivially. It remains to verify that (A2) also holds, that is, $G_{\mathbf{0}}$ is log-concave on its support. To show this, we will make use of the following lemma, which states that H_0 is log-concave if $\alpha \leq \hat{\alpha}$.

Lemma A3. *If $\alpha \leq \hat{\alpha}$, then H_0 is log-concave on $[-\lambda, \lambda]$.*

Proof. If $\alpha \leq 0$, the statement of the lemma immediately follows because in these cases the density function h_0 is log-concave, which is sufficient (but not necessary) for the distribution function H_0 to be log-concave on $[-\lambda, \lambda]$.

Suppose now that $\alpha \in (0, \hat{\alpha}]$. We will show that H_0 remains to be log-concave despite that the density function h_0 is actually log-convex. By continuity, it suffices to show that H_0 is log-concave on the open interval $(-\lambda, \lambda)$. Since h_0 is differentiable on $(-\lambda, \lambda)$, H_0

is log-concave on this interval if and only if for all $\theta \in (-\lambda, \lambda)$,

$$\begin{aligned}
& h_0'(\theta)H_0(\theta) - (h_0(\theta))^2 \leq 0 \\
\iff & 2\alpha\theta \cdot \left(\frac{1}{3}\alpha\theta^3 + \beta\theta + \frac{1}{2}\right) \leq \left(\alpha\theta^2 + \beta\right)^2 \\
\iff & \frac{2}{3}\alpha^2\theta^4 + 2\alpha\beta\theta^2 + \alpha\theta \leq \alpha^2\theta^4 + \beta^2 + 2\alpha\beta\theta^2 \\
\iff & -\frac{1}{3}\alpha^2\theta^4 + \alpha\theta \leq \left(\frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)^2. \qquad (3.1)
\end{aligned}$$

Given that $\alpha > 0$, the inequality obviously holds when $\theta \leq 0$. But given that $\theta > 0$, the LHS of (3.1) is increasing in θ on $[0, \lambda]$, since

$$\left(\frac{1}{3}\alpha^2\theta^4 + \alpha\theta\right)' = -\frac{4}{3}\alpha^2\theta^3 + \alpha \geq -\frac{4}{3}\alpha^2\lambda^3 + \alpha > 0,$$

where the last inequality holds as $\hat{\alpha} < 3/(4\lambda^3)$. Hence, inequality (3.1) holds for all $\theta \in (-\lambda, \lambda)$ if and only if

$$\begin{aligned}
& -\frac{1}{3}\alpha^2\lambda^4 + \alpha\lambda \leq \frac{1}{4\lambda^2} + \frac{\alpha^2\lambda^4}{9} - \frac{\alpha\lambda}{3} \\
\iff & -\frac{4}{9}\alpha^2\lambda^4 + \frac{4}{3}\alpha\lambda \leq \frac{1}{4\lambda^2} \\
\iff & -\alpha^2\lambda^4 + 3\alpha\lambda \leq \frac{9}{16\lambda^2} \\
\iff & \left(\alpha\lambda^2 - \frac{3}{2\lambda}\right)^2 \geq \frac{27}{16\lambda^2}. \qquad (3.2)
\end{aligned}$$

Since $\lambda > 0$ and $\hat{\alpha} \leq 3/(2\lambda^3)$, (3.2) is further equivalent to $\alpha \leq \frac{6-3\sqrt{3}}{4\lambda^3} = \hat{\alpha}$. □

To finish the proof of Proposition 2, note that

$$G_{\mathbf{0}}(x) = \Pr(4\lambda\theta \leq x) = \Pr\left(\theta \leq \frac{x}{4\lambda}\right) = H_0\left(\frac{x}{4\lambda}\right).$$

Since the function $t(x) = x/4\lambda$ is increasing and concave in x, $G_{\mathbf{0}}$ is log-concave on $[4\lambda a, 4\lambda b]$ if H_0 is log-concave on $[a, b] \subset \mathbb{R}$. Hence, by Lemma 1, $G_{\mathbf{0}}$ is log-concave on $[-4\lambda^2, 4\lambda^2]$ provided that $\alpha \leq \hat{\alpha}$. □

Proof of Proposition 3 From Proposition 2, we know that assumptions (A1) - (A5) are all satisfied given that $\alpha \leq \hat{\alpha}$. In addition, the preferences are polarized (indecisive, uniform) at $[-\lambda, \lambda]$ if $\alpha > 0$ ($\alpha < 0$, $\alpha = 0$). Since $[-\bar{\varepsilon}_a, \bar{\varepsilon}_a] \subset [-\lambda, \lambda]$ $\forall a \in A$, the conditions of Theorems 2, 3 and 4 are all satisfied, thus the statement of the proposition immediately follows. □

Proof of Proposition 4 We start with some preliminary calculations. Recall that in the Hotelling example, we have

$$G_0(x) = H_0\left(\frac{x}{4\lambda}\right)$$

and

$$g_0(x) = \frac{1}{4\lambda}h_0\left(\frac{x}{4\lambda}\right) = \begin{cases} \frac{1}{4\lambda}\cdot\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right) & \text{if } x \in [-4\lambda^2, 4\lambda^2] \\ 0 & \text{otherwise.} \end{cases}$$

As argued, the parameter α can be viewed as a measure of polarization. In particular, as α increases, preferences are more polarized. The welfare loss can now be written as a function of α:

$$\begin{aligned} L(\alpha) &= \int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right)\cdot\frac{1}{4\lambda}\cdot h_0\left(\frac{x}{4\lambda}\right)\right]dx \\ &= \frac{1}{4\lambda}\int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right)\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)\right]dx. \end{aligned}$$

Taking derivative with respect to α, we have

$$L'(\alpha) = \frac{1}{4\lambda}\int_0^{\min\{\omega,4\lambda^2\}} \left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right]dx.$$

Note that $L'(\alpha)$ does not depend on α, which implies that the expected welfare loss always changes monotonically with respect to the degree of polarization. The sign of the change is non-obvious, and it depends on the exact degree of confusion as well as the degree of product differetiation. Nevertheless, as the next result shows, the expected welfare loss always decreases (increases) as preferences become more polarized if confusion small (large) relative to product differentiation. We are now in a position to prove the statements of Proposition 4.

First, suppose that $\omega \le 4\lambda^2$. In this case, we obtain

$$\begin{aligned} L'(\alpha) &= \frac{1}{4\lambda}\int_0^{\omega}\left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right]dx \\ &= \frac{1}{4\lambda}\left[\int_0^{\omega}\left(\frac{x^3}{16\lambda^2} - \frac{\lambda^2 x}{3}\right)dx - \int_0^{\omega}\left(\frac{x^4}{16\lambda^2\omega} - \frac{\lambda^2x^2}{3\omega}\right)dx\right] \\ &= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2} - \frac{\lambda^2x^2}{6}\right)\Bigg|_0^{\omega} - \left(\frac{x^5}{80\lambda^2\omega} - \frac{\lambda^2x^3}{9\omega}\right)\Bigg|_0^{\omega}\right] \\ &= \frac{1}{4\lambda}\left[\frac{\omega^4}{64\lambda^2} - \frac{\omega^4}{80\lambda^2} - \frac{\lambda^2\omega^2}{6} + \frac{\lambda^2\omega^2}{9}\right]. \end{aligned}$$

Hence, provided that $\omega \in (0, 4\lambda^2]$, we further have

$$L'(\alpha) < 0 \iff \left(\frac{1}{64\lambda^2} - \frac{1}{80\lambda^2}\right)\omega^2 < \frac{\lambda^2}{6} - \frac{\lambda^2}{9} \iff \omega < \frac{4\sqrt{10}}{3}\lambda^2.$$

Since $(4\sqrt{10}/3) \approx 4.22 > 4$, it follows that $L'(\alpha) < 0$ whenever $\omega \leq 4\lambda^2$.

Next, consider the case where $\omega > 4\lambda^2$. Expanding the equation $L'(\alpha)$ again, we have

$$\begin{aligned} L'(\alpha) &= \frac{1}{4\lambda}\int_0^{4\lambda^2}\left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2-\frac{\lambda^2}{3}\right)\right]dx \\ &= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2}-\frac{\lambda^2x^2}{6}\right)\Bigg|_0^{4\lambda^2}-\left(\frac{x^5}{80\lambda^2\omega}-\frac{\lambda^2x^3}{9\omega}\right)\Bigg|_0^{4\lambda^2}\right] \\ &= \frac{1}{4\lambda}\left[\left(\frac{4^4\lambda^8}{64\lambda^2}-\frac{4^2\lambda^6}{6}\right)-\frac{1}{\omega}\left(\frac{4^5\lambda^{10}}{80\lambda^2}-\frac{4^3\lambda^8}{9}\right)\right] \\ &= 4\lambda^5\left[\left(\frac{16}{64}-\frac{1}{6}\right)-\frac{\lambda^2}{\omega}\left(\frac{64}{80}-\frac{4}{9}\right)\right]. \end{aligned}$$

Hence, provided that $\omega > 4\lambda^2$, we further have

$$L'(\alpha) > 0 \iff \frac{\lambda^2}{\omega}\left(\frac{4}{5}-\frac{4}{9}\right) < \frac{1}{4}-\frac{1}{6} \iff \omega > \frac{64}{15}\lambda^2.$$

Note that $64/15 \approx 4.27 > 4$. We can now conclude that $L'(\alpha) < 0$ whenever $\omega < \hat{\omega} \equiv 64\lambda^2/15$, and $L'(\alpha) > 0$ whenever $\omega > \hat{\omega}$.

□

B.2 Outside Options

Proof of Proposition 5 The demand function of firm 1 is given by

$$\begin{aligned}D(p_1,p_2) =& \int \Pr\left(\tilde{v}_1^k - p_1 \geq \max\{\tilde{v}_2^k - p_2, 0\}\right) d\Gamma_{\mathbf{a}} \\ =& \int \Pr\left(\upsilon \geq p_1 - p_2 - \varepsilon,\ \upsilon \geq 2p_1 - m - \varepsilon\right) d\Gamma_{\mathbf{a}} \\ =& \int \min\left\{\Pr\left(\upsilon \geq p_1 - p_2 - \varepsilon\right), \Pr\left(\upsilon \geq 2p_1 - m - \varepsilon\right)\right\} d\Gamma_{\mathbf{a}} \\ =& 1 - \int \max\left\{G_0\left(p_1 - p_2 - \varepsilon\right), G_0\left(2p_1 - m - \varepsilon\right)\right\} d\Gamma_{\mathbf{a}}.\end{aligned}$$

Recall that, for all $x \in \mathbb{R}$, we write

$$G_{\mathbf{a}}(x) = \int G_0(x-\varepsilon) d\Gamma_{\mathbf{a}}, \text{ and } g_{\mathbf{a}}(x) = \int g_0(x-\varepsilon) d\Gamma_{\mathbf{a}}$$

Note that

$$D(p,p) = \begin{cases} \frac{1}{2} & \text{if } p \leq \frac{m}{2}, \\ 1 - G_{\mathbf{a}}(2p-1) & \text{if } p > \frac{m}{2}. \end{cases}$$

Let $\Pi_1(p_1,p_2) = p_1 D(p_1,p_2)$. For every $p_2 > 0$, function Π_1 is differentiable in p_1 almost everywhere. In particular, if $p_1 < m - p_2$, we have

$$\frac{\partial \Pi_1(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(p_1 - p_2) - p_1 g_{\mathbf{a}}(p_1 - p_2),$$

which is also the left derivative of $\Pi_1(p_1,p_2)$ at $p_1 = m - p_2$. Similarly, if $p_1 > m - p_2$, we have

$$\frac{\partial \Pi_1(p_1,p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(2p_1 - m) - 2p_1 g_{\mathbf{a}}(2p_1 - m),$$

which is also the right derivative of $\Pi_1(p_1,p_2)$ at $p_1 = m - p_2$.

Finally, let $p_{\mathbf{a}}^m$ be the solution to the monopoly problem

$$\max_{p \geq 0} \ \Pi^m(p) \equiv p\left(1 - G_{\mathbf{a}}(2p - m)\right).$$

Note that the log-concavity of G_0 and $\Gamma_{\mathbf{a}}$ implies that the above objective function is strictly quasi-concave. Therefore, a unique $p_{\mathbf{a}}^m$ exists for every $\mathbf{a} \in \mathbf{A}$.

Since log-concavity is preserved under convolution, the function $G_{\mathbf{a}}$ is log-concave on its support $supp(g_{\mathbf{a}})$. In addition, since $G_{\mathbf{a}}$ is a distribution function, its log-concavity also hold on $[0, +\infty)$. Hence, for all $p_2 > 0$ and $\mathbf{a} \in \mathcal{A}$, the demand function D must be log-concave in p_1 on both $[0, m - p_2]$ and $[m - p_2, +\infty)$. Note that we are not claiming that D is log-concave in p_1 on the entire interval $[0, +\infty)$. In what follows, we will show that although the global log-concavity of the demand function is not guaranteed, assumptions (A1)-(A5) are still sufficient to guarantee the existence of a unique symmetric equilibrium in every pricing subgame.

First, suppose that $g_{\mathbf{a}}(0) > \frac{1}{m}$. Suppose also that firm 2 is choosing $p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}$. In this case, the whole market is guaranteed to be covered (i.e., every consumer will buy from one of the firms) if $p_1 \leq m/2$. In addition, since

$$\left.\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{1}{2g_{\mathbf{a}}(0)}<\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{1}{2g_{\mathbf{a}}(0)} \cdot g_{\mathbf{a}}(0) = 0,$$

and the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is strictly quasi-concave in p_1 on $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$, $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ is a maximum of the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ over the range $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$. We now argue that

$$\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right) \leq \Pi_1\left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right) \ \forall p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}.$$

To see this, note that $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ implies that the market will no longer be fully covered and, in particular, there will be consumers who choose to stick to their outside options even though they prefer firm 1 over firm 2. Therefore, a deviation to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ must be less profitable than it would have been in the case without outside option. But then, as we have shown in Theorem 1, in the absence of the outside option, choosing $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ actually maximizes firm 1's expected profits over $[0, +\infty)$ given that its competitor plays $p_2 = \frac{1}{2g_{\mathbf{a}}(0)}$. This implies that deviating to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be profitable either in the presence of the outside option. Therefore, $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ must be a global maximum of the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$, and $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ indeed constitutes an equilibrium in the pricing subagme. It is easy to see that this is the only symmetric equilibrium with a price strictly less than $\frac{m}{2}$. In addition, since

$$\left.\frac{\partial \Pi^m(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) < \frac{1}{2} - 1 < 0$$

and $\Pi^m(p)$ is strictly quasi-concave, even a monopoly will not choose a price $p \geq \frac{m}{2}$. Hence, when $g_{\mathbf{a}}(0) > 1/m$, there cannot be any symmetric equilibrium in which both firms choose a price larger than $\frac{m}{2}$. As a result, $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) > 1/m$.

Next, consider the case $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$. Taking $p_2 = \frac{m}{2}$ as given, we will show that $p_1 = \frac{m}{2}$ is a best response for firm 1. As mentioned, the profit function $\Pi_1(p_1, p_2)$ is semi-differentiable at the point $p_1 = m - p_2$. In particular, we have

$$\left.\frac{\partial^- \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{1}{2} \cdot g_{\mathbf{a}}(0) = \frac{m}{2} - \frac{m}{2} \cdot g_{\mathbf{a}}(0) \geq 0,$$

and

$$\left.\frac{\partial^+ \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - g_{\mathbf{a}}(0) = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq 0.$$

Since $\Pi_1\left(p_1, \frac{m}{2}\right)$ is strictly quasi-concave on both $\left[0, \frac{m}{2}\right]$ and $[\frac{m}{2}, +\infty)$, the above inequalities show that $p_1 = \frac{m}{2}$ is a maximum of the $\Pi_1\left(p_1, \frac{1}{2}\right)$ on each of these two intervals. This immediately implies that $p_1 = \frac{m}{2}$ is a global maximum of $\Pi_1\left(p_1, \frac{m}{2}\right)$ on $[0, +\infty)$. Hence, if $g_{\mathbf{a}}(0) = [\frac{1}{2m}, \frac{1}{m}]$, the pricing subgame admits a symmetric equilibrium with $p_1 = p_2 = \frac{m}{2}$. Since $\frac{1}{2g_{\mathbf{a}}(0)} \geq \frac{m}{2}$, a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ cannot exist. In addition, because

$$\left.\frac{\partial \Pi^m(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq \frac{1}{2} - \frac{1}{2} = 0$$

and $\Pi^m(p)$ is strictly quasi-concave, even a monopoly will not choose a price strictly higher than $\frac{m}{2}$. Hence, no symmetric equilibrium with $p_1 = p_2 > \frac{m}{2}$ can exist either. As a result, $(p_1, p_2) = \left(\frac{m}{2}, \frac{m}{2}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right]$.

Finally, suppose that $g_{\mathbf{a}}(0) < \frac{m}{2}$. Observe that in this case, we must have $p^m_{\mathbf{a}} > \frac{m}{2}$, since

$$\left.\frac{\partial \Pi^m(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0$$

and $\Pi^m(p)$ is strictly quasi-concave on $[0, +\infty)$. Now suppose that firm 2 plays $p_2 = p^m_{\mathbf{a}}$, and consider firm 1's profit function $\Pi_1(p_1, p^m_{\mathbf{a}})$. Given the formula of the demand function and $p^m_{\mathbf{a}} > m - p^m_{\mathbf{a}}$, we have

$$\Pi_1(p^m_{\mathbf{a}}, p^m_{\mathbf{a}}) = \Pi^m(p^m_{\mathbf{a}}) \geq \Pi^m(p_1) \geq \Pi_1(p_1, p^m_{\mathbf{a}}) \ \forall p_1 \in [0, +\infty),$$

which further implies that $p_1 = p^m_{\mathbf{a}}$ is a best response for firm 1. Hence, $(p_1, p_2) = (p^m_{\mathbf{a}}, p^m_{\mathbf{a}})$ indeed constitutes an equilibrium in the pricing sugame where $g_{\mathbf{a}}(0) < \frac{m}{2}$. Moreover, given $\frac{1}{2g_{\mathbf{a}}(0)} > \frac{m}{2}$, there cannot exist a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$. Since

$$\left.\frac{\partial^- \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} > \left.\frac{\partial^+ \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0,$$

$p_1 = p_2 = \frac{m}{2}$ does not constitute an equilibrium either. In conclusion, $(p_1, p_2) = (p^m_{\mathbf{a}}, p^m_{\mathbf{a}})$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) < \frac{m}{2}$. □

To be more concrete, we assume further that υ is uniformly distributed on $[-1,1]$, and $\varepsilon_{\mathbf{a}}$ is uniformly distributed on $[-\omega,\omega]$, where $\omega > 0$. We can distinguish the following two cases (see Killmann and von Collani, 2001):

Case 1. $\omega \leq 1$.

$$g_{\mathbf{a}}(x) = \begin{cases} \frac{x+1+\omega}{4\omega} & \text{if } -1-\omega \leq x < -1+\omega, \\ \frac{1}{2} & \text{if } -1+\omega \leq x < 1-\omega, \\ \frac{-x+1+\omega}{4\omega} & \text{if } 1-\omega < x \leq 1+\omega, \\ 0 & \text{otherwise.} \end{cases}$$

Case 2. $\omega > 1$.

$$g_{\mathbf{a}}(x) = \begin{cases} \frac{x+1+\omega}{4\omega} & \text{if } -1-\omega \leq x < 1-\omega, \\ \frac{1}{2\omega} & \text{if } 1-\omega \leq x < -1+\omega, \\ \frac{-x+1+\omega}{4\omega} & \text{if } -1+\omega < x \leq 1+\omega, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\bar{\omega}$ be the maximal ω that can induced by the obfuscation activities. The following result is a direct application of Proposition 1 (and Theorem 1).

Corollary 1. *Suppose that $m > 2$ and υ is uniformly distributed on $[-1,1]$. There exists an SPE with the equilibrium price $p_1 = p_2 = p^*(\bar{\omega})$, where*

$$p^*(\bar{\omega}) = \begin{cases} 1 & \text{if } \bar{\omega} < 1, \\ \bar{\omega} & \text{if } 1 \leq \bar{\omega} < \frac{m}{2}, \\ \frac{m}{2} & \text{if } \frac{m}{2} \leq \bar{\omega} < m \\ p^m(\bar{\omega}) & \text{if } \bar{\omega} > m, \end{cases}$$

where $p^m(\bar{\omega})$ is the unique solution to the problem

$$\max_{p \in [\frac{m}{2}, \bar{\omega}+1]} \; p\left(1 - G_{\bar{\omega}}(2p-m)\right)$$

and

$$G_{\bar{\omega}}(x) = 1 - \frac{1}{2\bar{\omega}} + \frac{1}{4\bar{\omega}}\left[-\frac{x^2}{2} + (1+\bar{\omega})x + \frac{3}{2} - \bar{\omega} - \frac{\bar{\omega}^2}{2}\right].$$

B.3 Salop Model

We parameterize the circle by $i \in [0,1]$. Two firms are located at antipodal locations (Firm 1 is located at 0, Firm 2 at 1/2). Consider a point i on the unit circle. Let $\hat{i} \in [0,1/2]$ denote the smallest arc distance between this location and the location of firm 1. Consumption utilities of a consumer with location i then are

$$\begin{aligned} U_i(1) &= V - p_1 - t\hat{i} \\ U_i(2) &= V - p_2 - t(1/2 - \hat{i}), \end{aligned} \tag{7}$$

where $t > 0$. Fix prices $p_1, p_2 \geq 0$ and define $\Delta \equiv \frac{p_2 - p_1}{2t}$. Given that $|\Delta| \leq 1/4$, the market segment S_1 of firm 1 is $S_1 = \{i \in \mathcal{I} : \hat{i} \leq \Delta + 1/4\}$. The market segment of firm 2 is $S_2 = S_1^C$.[31] Further $S_1 = \emptyset$ if $\Delta < -1/4$, and $S_1 = [0,1]$ if $\Delta > 1/4$.

For symmetry reasons, it suffices to consider only the half-circle on the right-hand side. This is like a Hotelling line of length 1/2, with firms sitting at the vertices. Consumers are dispersed over the half-line according to the bounded function $h : \mathbb{R} \to \mathbb{R}_+$ with the following properties

$$\begin{aligned} &(1) \quad h(i) > 0 \Leftrightarrow i \in [0,1/2] \\ &(2) \quad h(\cdot) \text{continuous on } (0,1/2) \\ &(3) \quad \textstyle\int_0^{1/2} h(i)di = 1/2 \\ &(4) \quad h(\cdot) \text{ symmetric at } \tfrac{1}{4} \end{aligned} \tag{8}$$

Obfuscation We suppose consumer confusion means that each consumer's perceived location is distorted by a 0-symmetric, uniformly distributed locational shock $e \in [-\varepsilon, \varepsilon]$, where $\varepsilon \in [0,1/2]$ is the obfuscation parameter. A consumer consumes at firm 1 if her perceived location belongs to S_1, i.e., if the perceived distance to firm 1 is less than $\Delta + 1/4$ (assuming that $|\Delta| < 1/4$). For each $i \in [0,1/2]$, the perceived distance to 1 is[32]

$$\hat{i} = \begin{cases} |i+e|, & i+e \leq 1/2 \\ 1-(i+e), & i+e > 1/2. \end{cases}$$

The market demand of firm 1 from consumers on the right half-circle, $d_1 \in [0,1/2]$, corresponds to the mass of consumers that have their original location on the right half-circle ($i \in [0,1/2]$) and their perceived valuations belong to S_1. Hence

$$d_1 = \{i \in [0,1/2] : |i+e| \leq 1/4 + \Delta, e \in [-\varepsilon, \varepsilon]\} \cup \{i \in [0,1/2] : i \geq 3/4 - \Delta - e, e \in [-\varepsilon, \varepsilon]\}.$$

Overall demand of firm 1 (taking consumers on both sides of the circle) then is $D_1 = 2d_1$. In the appendix, we derive an explicit expression for d_1, assuming that $\Delta \in (-1/4, 1/4)$.

[31] Because there is a continuum of consumers we can ignore the zero-mass of indifferent consumers.
[32] Note that as $\varepsilon \in [0,1/2]$, $e \in [-\varepsilon, \varepsilon]$ and $i \in [0,1/2]$, we must have that $i + e \in [-1/2, 1]$.

We now distinguish the case of small to intermediate obfuscation $0 < \varepsilon < 1/4$, and intermediate to maximal obfuscation $1/4 < \varepsilon \leq 1/2$.

Proposition 1 ***A*** *The following cases can be distinguished:*

(i) *If $h(i) = 1$ on $[0, 1/2]$ (uniform dispersion), then confusion has no effects on equilibrium profits and prices at all.*

(ii) *If $h'(i) > 0$ on $(0, 1, 4)$, and hence $h'(i) < 0$ on $(1/4, 1/2)$ (indecisive preferences), then there is a unique $\varepsilon_0 \in (1/4, 1/2)$ such that profits and prices increase in confusion up to ε_0, and decrease thereafter.*

(iii) *If $h'(i) < 0$ on $(0, 1, 4)$, and hence $h'(i) > 0$ on $(1/4, 1/2)$ (polarization), then there is a unique $\varepsilon_0 \in (1/4, 1/2)$ such that profits and prices decrease in confusion up to ε_0, and increase thereafter.*

As an example, suppose that $h(i)$ is given by the following function (see Figure 2), where

$$h(i) = \begin{cases} 2 - m + 8i(m-1), i \in [0, 1, 4] \\ 3m - 2 - 8i(m-1), i \in (1, 4, 1/2], \end{cases} \tag{9}$$

and $m \in (0, 2)$ is a parameter.

$h(i)$ induces a valuation distribution featuring polarization for $m < 1$ and indecisiveness for $m > 1$.

The equilibrium condition from Theorem 1 thus becomes

$$p(0) = \frac{t}{2h(1/4)} = \frac{t}{2m}.$$

Therefore, $\Pi(0) = \frac{p(0)}{2} = \frac{t}{4m}$.

Figure 3 illustrates how equilibrium profits depend on obfuscation. In the benchmark case of a uniform taste distribution, there is no effect of obfuscation and profits. The case $m = 1/4$ represents a case of polarization. In line with Proposition 1, profits are initially decreasing in obfuscation. As obfuscation becomes sufficiently large, however, profits start increasing until they reach a second local maximum at $\varepsilon = 0.5$. Intuitively, this obfuscation level corresponds to maximal obfuscation. In this case, no matter what a consumer's true location is, his perceived location will be uniformly distributed over the unit circle – true preferences are therefore irrelevant. Clearly, profits in this case are still smaller than without obfuscation – firms obtain higher profits when consumers have polarized tastes than when they are randomly distributed over the circle. The case $m = 7/4$ represents a case of indecisive consumers. In line with Proposition 1, profits are initially increasing in obfuscation. As obfuscation becomes sufficiently large, however, profits start decreasing again until they reach a second local minimum at $\varepsilon =$

0.5. Intuitively, small obfuscation moves the consumers from being indecisive towards polarization. As obfuscation becomes large, however, polarization is reduced again, until the distribution ultimately becomes uniform again. Therefore, even with free obfuscation firms facing indecisive consumers would not engage in maximal obfuscation.

Let I denote the random variable with support $[0, 1/2]$ and density (8) capturing consumer dispersion over the half circle, where $F_I(i)$ is the (half)-distribution function associated with $g(i)$. Hence

$$F_I(i) = \begin{cases} 0 & i < 0 \\ \int_0^i g(\iota)d\iota & i \in [0, 1/2] \\ 1/2 & i > 1/2 \end{cases}$$

and $F_I(1/4) = 1/4$. Let E denote the random variable pertaining to obfuscation, i.e., E is uniform on $[-\varepsilon, \varepsilon]$. Then

$$d_1 = \Pr\left[-1/4 - \Delta \leq I + E \leq 1/4 + \Delta\right] + \Pr\left[I + E \geq 3/4 - \Delta\right]$$

For $\varepsilon = 0$, this expression reduces to

$$d_1 = \Pr\left[I \leq 1/4 + \Delta\right] + \Pr\left[I \geq 3/4 - \Delta\right] = F_I\left(1/4 + \Delta\right)$$

For $\varepsilon \in (0, 1/4]$ we obtain

$$\begin{aligned} d_1(\Delta, \varepsilon) &= \int_{-\varepsilon}^{\varepsilon} \tfrac{1}{2\varepsilon} \Pr\left[-1/4 - \Delta \leq I + e \leq 1/4 + \Delta\right] de + \int_{-\varepsilon}^{\varepsilon} \tfrac{1}{2\varepsilon} \Pr\left[I \geq 3/4 - \Delta - e\right] de \\ &= \int_{-\varepsilon}^{\varepsilon} \tfrac{1}{2\varepsilon} \left(F_I(1/4 + \Delta - e) - F_I(-1/4 - \Delta - e)\right) de + \int_{-\varepsilon}^{\varepsilon} \tfrac{1}{2\varepsilon} \left(1/2 - F_I(3/4 - \Delta - e)\right) de \\ &= \tfrac{1}{2\varepsilon} \left(\int_{1/4+\Delta-\varepsilon}^{1/4+\Delta+\varepsilon} F_I(i)di - \int_{-1/4-\Delta-\varepsilon}^{-1/4-\Delta+\varepsilon} F_I(i)di - \int_{3/4-\Delta-\varepsilon}^{3/4-\Delta+\varepsilon} F_I(i)di \right) + 1/2 \end{aligned} \tag{10}$$

It is easy to see that (use de l'Hospital rule)

$$\lim_{\varepsilon \to 0} d_1(\Delta, \varepsilon) = F_I(1/4 + \Delta) = d_A(\Delta, 0).$$

The payoff function of firm 1 is

$$\Pi_1(\Delta, \varepsilon) = 2p_1 d_1(\Delta, \varepsilon) = 2p_1 d_1\left(\frac{p_2 - p_1}{2t}, \varepsilon\right) \tag{11}$$

Evaluating the first-order condition for $p_2 = p_1 = p$ yields

$$p(\varepsilon) = \frac{t}{2\frac{\partial d_1(0,\varepsilon)}{\partial_\Delta}} \tag{12}$$

in a symmetric equilibrium.

This implies that

$$sign(\Pi_1'(\varepsilon)) = -sign\ h(\varepsilon), \tag{13}$$

where $h(\varepsilon) \equiv \frac{\partial^2 d_1(0,\varepsilon)}{\partial_\Delta \partial_\varepsilon}$. Expression (10) evaluates to

$$\begin{aligned} \frac{\partial d_1}{\partial \Delta} &= \frac{F_I(1/4+\Delta+\varepsilon)-F_I(1/4+\Delta-\varepsilon)}{2\varepsilon} \\ &+ \frac{F_I(-1/4-\Delta+\varepsilon)-F_I(-1/4-\Delta-\varepsilon)}{2\varepsilon} \\ &+ \frac{F_I(3/4-\Delta+\varepsilon)-F_I(3/4-\Delta-\varepsilon)}{2\varepsilon} \end{aligned} \tag{14}$$

Thus (noting that $F_I(i)$, while continuous, may not be differentiable at the kink points $i = 0$ and $i = 1/2$)

$$\begin{aligned} h(\varepsilon) &= \tfrac{1}{2\varepsilon^2} \left(\varepsilon \left[g(1/4+\varepsilon) + g(1/4-\varepsilon) + g(-1/4+\varepsilon) + g(-1/4-\varepsilon) + g(3/4+\varepsilon) + g(3/4-\varepsilon) \right] \right) \\ &- \tfrac{1}{2\varepsilon^2} \left(F(1/4+\varepsilon) - F(1/4-\varepsilon) + F(-1/4+\varepsilon) - F(-1/4-\varepsilon) + F(3/4+\varepsilon) - F(3/4-\varepsilon) \right) \end{aligned} \tag{15}$$

Proof of Proposition 1 Let $\varepsilon \in (0, 1/4)$. Then (15) simplifies to

$$h(\varepsilon) = \frac{(2\varepsilon g(1/4+\varepsilon) - (F(1/4+\varepsilon) - F(1/4-\varepsilon)))}{2\varepsilon^2}.$$

Exploiting the fact that, by symmetry of g at $i = 1/4$,

$$F_I\left(\frac{1}{4}+\varepsilon\right) - F_I\left(\frac{1}{4}-\varepsilon\right) = 2F_I\left(\frac{1}{4}+\varepsilon\right) - \frac{1}{2}$$

we find that

$$h(\varepsilon) \geq 0 \quad \Leftrightarrow \quad g\left(\frac{1}{4}+\varepsilon\right)\varepsilon \geq F_I\left(\frac{1}{4}+\varepsilon\right) - \frac{1}{4} = \int_{1/4}^{1/4+\varepsilon} g(i)di. \tag{16}$$

Therefore we can establish the claims in (i), (ii) and (iii) for the case where $\varepsilon \in (0, 1/4)$.

(i) $\int_{1/4}^{1/4+\varepsilon} g(i)di = 1/4 + \varepsilon - 1/4 = \varepsilon$, and $g\left(\frac{1}{4}+\varepsilon\right)\varepsilon = \varepsilon$, thus $h(\varepsilon) = 0$ by (16). Hence obfuscation has no effects on profits by (13) for $\varepsilon \in (0, 1/4)$.

(ii) $\int_{1/4}^{1/4+\varepsilon} g(i)di > \int_{1/4}^{1/4+\varepsilon} g(1/4+\varepsilon)di = g(1/4+\varepsilon)\varepsilon$, thus $h(\varepsilon) < 0$ by (16). Hence profits increase in obfuscation by (13) for $\varepsilon \in (0, 1/4)$.

(iii) $\int_{1/4}^{1/4+\varepsilon} g(i)di < \int_{1/4}^{1/4+\varepsilon} g(1/4+\varepsilon)di = g(1/4+\varepsilon)\varepsilon$, thus $h(\varepsilon) > 0$ by (16). Hence profits decrease in obfuscation by (13) for $\varepsilon \in (0, 1/4)$.

Now, suppose that $\varepsilon \in (1/4, 1/2]$ (the case $\varepsilon = 1/4$ is not problematic, because $\frac{\partial d_A(0,\varepsilon)}{\partial \Delta}$ is continuous at $\varepsilon = 1/4$). Then (15) simplifies to

$$h(\varepsilon) = \frac{\varepsilon\left(g(\varepsilon - 1/4) + g(3/4 - \varepsilon)\right) - \left(1 + F(\varepsilon - 1/4) - F(3/4 - \varepsilon)\right)}{2\varepsilon^2}, \tag{17}$$

where the sign of $h(\varepsilon)$ depends solely on the nominator. Using $g(\varepsilon - 1/4) = g(3/4 - \varepsilon)$ and $F(\varepsilon - 1/4) = 1/2 - F(3/4 - \varepsilon)$ (both follow from symmetry of g at 1/4), the nominator of $h(\varepsilon)$ becomes

$$z(\varepsilon) = 2\varepsilon g(3/4 - \varepsilon) - 3/2 + 2F(3/4 - \varepsilon). \tag{18}$$

If $g(i) = 1$ on $(1/4, 1/2]$, then $z(\varepsilon) = 0$ on $(1/4, 1/2]$, completing the proof for claim (i). Suppose now that $g(i)$ features anti-polarization as in (ii). Hence $g(1/2) < 1$ and $g(1/4) > 1$, and

$$\begin{aligned} \lim_{\varepsilon \to 1/4} z(\varepsilon) &= 1/2 g(1/2) - 1/2 < 0 \\ \lim_{\varepsilon \to 1/2} z(\varepsilon) &= g(1/4) - 1/2 > 0. \end{aligned}$$

Further, for $\varepsilon \in (1/4, 1/2)$ we have $z'(\varepsilon) = -2\varepsilon g'(3/4 - \varepsilon) > 0$. These arguments, together with the continuity of $z(\varepsilon)$, assure the existence of a unique $\varepsilon_0 \in (1/4, 1/2)$, such that for $\varepsilon \in [1/4, 1/2]$

$$z(\varepsilon) \begin{cases} < 0 & \varepsilon < \varepsilon_0 \\ = 0 & \varepsilon = \varepsilon_0 \\ > 0 & \varepsilon > \varepsilon_0. \end{cases}$$

It follows from (13) that $\Pi_A(\varepsilon)$ must have a unique maximum on $[1/4, 1/2]$ at ε_0, which completes the proof for (ii). Case (iii) is proved in the same way. ■

B.4 Competition for Voters

Given $\mathbf{a} \in \mathbf{A}$ an interior symmetric second stage equilibrium necessarily solves

$$\frac{\partial \Pi_j(s, s; \mathbf{a})}{\partial s_j} = 0$$

with solution $s(\mathbf{a})$. Finally, we suppose that for any $\mathbf{a} \in \mathbf{A}$ the payoff function $\Pi_j(s_1, s_2; \mathbf{a})$ is strictly quasiconcave in s_j, such that the above condition is also sufficient, and assures that $(s_1, s_2) = (s(\mathbf{a}), s(\mathbf{a}))$ is a symmetric equilibrium in any second stage subgame induced by $\mathbf{a} \in \mathbf{A}$.[33]

Theorem 4 *Consider the political economy application just described.*

(i)If there exists a $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subseteq [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ such that preferences are weakly δ-polarized, then there exists an SPE without voter confusion.

[33] In the commitment example, strict quasiconcavity follows from the convexity assumptions imposed on $c(\cdot)$. In the advertising example, this may additionally require that c is sufficiently convex.

(ii) If there exists a $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subseteq [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ such that preferences are δ-indecisive, then no SPE without consumer confusion exists.[34]

Proof. (i) As the commitment case is isomorphic to the oligopoly model, the result follows by direct application of Theorem 1. (ii) Now consider the case

$$\Pi_j(s_1, s_2; \mathbf{a}) = Pr(\tilde{v}^k_\Delta \leq s_\Delta + \varepsilon_{\mathbf{a}}) - c(s_i).$$

Given that $\Pi_i(s_1, s_2; \mathbf{a})$ is strictly quasiconcave in s_1 for each $\mathbf{a} \in \mathbf{A}$, the second stage symmetric equilibrium is described by

$$\int g_0(e)\gamma_{\mathbf{a}}(e)de = c'(s_i), \tag{19}$$

with equilibrium profits

$$\Pi^*_i = 1/2 - c(s_i).$$

Hence each firm benefits from an equilibrium which involves a lower s. By the proof of Theorem 1,

$$\int g_0(e)\gamma_{\mathbf{a}}(e)de > g_0(0)$$

with weakly δ-polarized preferences whenever $\varepsilon_{\mathbf{a}}$ is non-degenerate. Hence the choice of $\mathbf{a}$ such that $\varepsilon_{\mathbf{a}}$ is degenerate constitutes a SPE. The remaining claim is proved similarly.■

[34] In the knife-edge case where g_0 is constant on $[-\delta, \delta]$, obfuscation has no effect and there are SPE both with and without voter confusion.