How Can Downstream Firms Benefit from Upstream Collusion?

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Abstract

Motivated by the recent antitrust cases in which Japanese auto parts suppliers colluded to raise supply prices against their long-term collaborators and partial owners, the Japanese carmakers, we study whether and how downstream firms may benefit from an upstream collusion. Oligopoly competition in successive industries is shown to give rise to a vertical externality and a horizontal externality. If a collusion leads to an outcome in which the two externalities are better balanced, the joint profits of all firms in the two industries will increase. Downstream firms can then benefit from such a collusion if they are entitled to a sufficient share of upstream profits through, say, passive ownership. We link the incidence of such collusions to industry characteristics in terms of demand, cost and competition intensity, and demonstrate that a higher collusive input price, although raising the joint profits, makes collusion more difficult due to a business-stealing effect.

Keywords: upstream collusion, successive oligopoly, vertical externality, horizontal externality, passive ownership

JEL Code: L1, L2, L4, D43

1 Introduction

In 2008 the United States Department of Justice (DOJ) and Federal Bureau of Investigation (FBI) started investigating automobile parts suppliers for anticompetitive conducts in the US. The perpetrators were mostly Japanese firms including Furukawa, Yazaki, Denso, Mitsubishi, Sumitomo, Aisan, etc. They were accused of price fixing and bid rigging for more than a decade, from year 2000 to 2011, in the sales of automotive wire harnesses, bearing, air conditioning systems, windshield washer and wiper systems, compressors and condensers,
radiators, seatbelts, etc.¹ By March 2017, 48 companies and 65 individuals had been charged; all had pleaded guilty and agreed to pay a combined fine of $2.9 billion (DOJ, 2017). Alerted by the US investigation, antitrust authorities in Japan, the European Union, Canada, South Korea, Mexico, Australia, and China subsequently carried out their own investigations of Japanese suppliers operating in their countries. All companies pleaded guilty to the respective jurisdictions and paid various amounts of fines (DOJ, 2013; DG Competition, 2013; NDRC, 2014).

What’s puzzling about this antitrust case is that the victims were mostly Japanese carmakers including Nissan, Toyota, Honda, Mazda, Mitsubishi, and Fuji Heavy Industries. Japanese companies, especially those in the automobile industry, are well known for maintaining long-term relationship and close collaboration with their suppliers. Many observers believe that such tight cooperation contributed greatly to the success of Japanese automobile industry, as close collaboration brings many benefits in the form of reduced time of new model development, shared cost of innovation, quick response to fluctuations in market demand, and so on (Ahmadjian and Lincoln, 2001). Why would the suppliers conspire against their long-term customers? Why did not the Japanese carmakers report or complain about the price fixing and bid rigging? Given that the victims were global heavyweights such as Toyota, Honda and Nissan, it is hard to imagine that the carmakers were ignorant of the conspiracy or lacked the power to strike back. Then why had the practice persisted for so long?

We suggest that in the vertically related industries, it is possible for downstream firms to benefit from upstream collusion if they receive proper compensations from the colluding firms. While there can be many ways for the two industries to share the benefits of collusion, a particularly simple mechanism is to transfer some profits from the upstream to the downstream through ownership structure. This is clearly the case with the Japanese automobile industry, as most Japanese auto parts suppliers are partially owned by one or more major Japanese carmakers. For example, Toyota holds 41% of Denso and 33% of Aisan. In turn, Denso and Aisan sell 40% of their products to Toyota. In China, Japanese car manufacturing operates mostly through joint ventures, with Toyota holding 5.3% of Nachi Bearing and 22% of JTEKT, while Mitsubishi holds 6.3% of NTN Bearing.

Given such a premise, this paper investigates the conditions under which upstream firms are able to collude to raise the input price without hurting downstream firms. We demonstrate that a necessary condition for such upstream collusion to take place is that it raises the joint profits of all firms in the upstream and downstream industries. Once the necessary condition

¹Executives of the companies held regular meetings and telephone communications in the United States and Japan to rig bids, set prices, and allocate supplies. They used code names and met in remote locations to keep secrecy, and had measures to monitor and enforce the collusive agreements (DOJ, 2013). Most of the involved parts are standard products and were used by carmakers in a modular way. Although the costs of these parts accounted for only about 5-8% in the final prices of the automobiles, they were needed frequently in large quantities, which facilitated collusion and caused substantial damages to the carmakers.
is satisfied, a sufficient condition is that downstream firms hold a sufficiently high share of the upstream profits, but ignore the partial ownership when making their own competitive choices (i.e., a passive ownership).

What, then, determines whether a higher input price can raise the joint profit? Based on a standard, general two-stage model of successive oligopoly, we show that firms’ equilibrium choices give rise to two externalities: a vertical externality by which a downstream firm ignores the benefits that its output quantity brings to upstream firms, and a horizontal externality by which a downstream firm ignores the damages that its output brings to other downstream firms. The vertical externality makes firms produce too little as compared with the monopoly level, whereas the horizontal externality makes firms produce too much. If the horizontal externality dominates, firms over-produce in the oligopoly equilibrium. By raising the input price and therefore reducing the total output, then, an upstream collusion moves the outcome closer to the monopoly level and thereby raises the joint profit. Of course this would reduce both the consumer surplus and the social welfare, which justifies the antitrust case in the first place.

We then analyze passive ownership as a compensation scheme. For upstream collusion to take place, downstream firms must hold a sufficiently high share of the upstream profits through share-holding. If the demand concavity is constant, we show that a higher collusive input price hurts the downstream increasingly more than it benefits the two industries’ combined profits, which makes the compensation harder even though the upstream and joint profits are both higher. In other words, there exists a tension between collusive profit maximization and the feasibility of the collusion due to this business-stealing effect. We also demonstrate that for passive ownership to work as a compensation scheme in upstream collusion, downstream firms need to ignore such ownership structure when making their competitive choices. Otherwise, not only the downstream firms are worse off, but also the upstream collusion itself will never take place.

The decomposition of the two externalities and the analysis of passive ownership help us identify a number of variables that facilitate upstream collusion. First, a greater number of firms in either industry or a more balanced structure between the upstream and downstream facilitates upstream collusion. Second, a smaller product differentiation or demand concavity for the final products are conducive to upstream collusion. The degree of demand concavity matters because it determines how much the downstream firms can pass an increase in input price to the final consumers. Third, greater cost convexity or dispersion in the downstream hinders upstream collusion. In the upstream, by contrast, the cost convexity has no impact, while the cost dispersion may facilitate or hinder collusion depending on which upstream firm’s output is reduced by the collusion.

This research adds to the literature in three ways. First, it provides a unifying framework for understanding upstream collusion and, more broadly, vertical conducts in general. While most papers on upstream collusion study the stability condition, i.e., an upstream firm’s incentive to join the collusion (Choe and Matsushima 2013, Jullien and Rey 2007, Piccolo
and Reisinger 2011, and Nocke and White 2007, 2010), we focus on whether and how an upstream collusion can benefit downstream firms. To the extent that “victims” of upstream collusions are powerful downstream corporations which have every means to strike back, we believe the question studied here deserves much attention (see also Schinkel et al., 2008). The decomposition of a competitive outcome into a vertical externality and a horizontal externality proves very useful. In fact we find that the two externalities are so general that almost all conducts in a vertical setting can be understood within such framework: retail price maintenance (Rey and Tirole, 1986), quantity constraint (Schinkel et al., 2008), exclusive dealing (Lin, 1990; O’Brien and Shaffer, 1993), exclusive territories (Rey and Stiglitz, 1988), and vertical disintegration (Bonanno and Vickers, 1988; Lin, 1988).

In a series of studies, Winter and his coauthors (Mathewson and Winter, 1984; Winter, 1993; Krishnan and Winter, 2007) have used vertical and horizontal externalities to analyze vertical controls. Their models typically assume a monopoly supplier who can extract all of the surplus, so the major purpose of these studies is to explain whether a particular vertical conduct can achieve the monopoly profit. By contrast, our upstream industry is oligopoly, and we focus on the conditions under which upstream collusion will not hurt downstream firms.

Second, we highlight a new channel through which passive ownership may affect competition. The few existing papers in the literature on passive ownership have focused on how a vertical ownership may change vertical and horizontal conducts, and have found either no effect (Flath, 1989; Greenlee and Raskovich, 2006) or anticompetitive effects (Hunold and Stahl, 2016). In our model, ownership structure does not directly coordinate firms’ competition choices. Rather, it facilitates upstream collusion by providing a mechanism for downstream firms to share the benefit of the collusion. As such, welfare is determined by the nature of the collusion (whether it raises or reduces the input price) rather than the ownership per se. Furthermore, different from the existing literature, our ownership is passive in a stronger sense in that it affects the choices of neither the downstream firms nor the upstream firms. Otherwise, passive ownership cannot be the compensation mechanism to support upstream collusion.

Third, our study can inform antitrust policies. We demonstrate that firms in different stages of a supply chain can collude to benefit at the expense of consumers. In such a situation, the real victims (i.e., consumers) and the conspirators (i.e., the upstream firms) do not interact directly, as they are separated by legitimate and innocuous businesses (i.e., the downstream firms). Since the antitrust authority cannot rely on the “victims” to report any wrongdoing, this kind of collusion is hard to uncover (see also Schinkel et al., 2008). As such, our findings about the conditions that facilitate upstream collusion can help the antitrust authority better detect anticompetitive conducts. For example, a substantial share of an upstream firm being held by its major customer should sound a warning alarm, especially when capacity constraint in the downstream is relaxed (corresponding to less convex production cost), downstream technologies are similar (i.e., cost dispersion is smaller), or the demand for the final products
is less elastic (corresponding to a smaller demand concavity).

Schinkel et al. (2008) have also suggested that upstream collusions may benefit downstream firms. They argue that the US Supreme Court’s ruling that allowed only direct purchasers to claim antitrust damages would facilitate upstream collusion, as an upstream cartel can share the collusive profits with its direct purchasers to avoid litigation. Their particular mechanism is different from ours: Quantity rationing to raise the total profit, and collusive lower input price to transfer profits. More importantly, Schinkel et al. (2008) directly assume that collusion can always achieve higher joint profits, while we demonstrate that in the absence any coordination across the two industries, an upstream collusion does not always raise the joint profit. In fact, we devote a substantial part of our analysis to the conditions under which this will happen.

The rest of the paper is organized as follows. After setting up the model in section 2, we analyze the equilibrium choices in Section 3 and identify the condition under which an upstream collusion may raise the joint profits. Section 4 highlights passive ownership as a profit-sharing mechanism and characterizes its conditions. Section 5 extends the model to different settings to show that the major finding and its driving force are general and robust. Section 6 concludes.

2 Model setting

Consider two vertically related industries with \( m \) (\( m \geq 2 \)) identical upstream firms and \( n \) identical downstream firm(s). A homogeneous input is produced by upstream firms at constant marginal cost, \( f \), and sold at a uniform price to downstream firms, which then transform the input into a homogeneous final product at constant marginal cost, \( c \), on a one-for-one basis. Let \( p = p(Q) \) be the inverse demand for the final product where \( p \) is the price and \( Q \) is the total quantity of sales.

Firms in both industries compete à la Cournot, and the interaction between the two industries is through an endogenous demand function for the input, which is determined in the following way. For any given input price, \( t \), downstream Cournot competition results in an equilibrium output quantity for each downstream firm and hence a total quantity, \( Q \). The relationship between \( t \) and \( Q \) thus derived, \( t(Q) \), is regarded by upstream firms as the demand function for the input. More specifically, the successive oligopoly game is played as follows (Lewis et al., 1986; Salinger, 1988). All upstream firms simultaneously choose their quantities with the understanding that the resulting input price is determined

\footnote{Which mechanism is more relevant is an empirical question. The Japanese auto parts case clearly fits into our setting. In addition, quantity rationing with lower input price involve some form of (tacit) collusion across the two industries. By contrast, downstream firms in our model are truly devoid of any collusive behavior with upstream firms. This makes the upstream collusion easier to sustain and harder to uncover. As a result, colluding firms may optimally choose such a mechanism, to which the antitrust authority needs to pay special attention.}
from $t = t(\sum_{j=1}^{m} q^{u}_j)$, where $q^{u}_j$ is the quantity produced by upstream firm $j$. The profit of firm $j$ is therefore $\pi^{u}_j = q^{u}_j \left[ t(\sum_{j=1}^{m} q^{u}_j) - f \right]$. Given any input price $t$, downstream firms simultaneously choose their quantities, $q^{d}_i$ for downstream firm $i$, and the profit of firm $i$ is given by $\pi^{d}_i = q^{d}_i \left[ p(\sum_{j=1}^{m} q^{d}_j) - c - t \right]$. In equilibrium, $\sum_{j=1}^{m} q^{d}_j = \sum_{i=1}^{n} q^{d}_i = Q$.

The demand function $p(Q)$ is assumed to be such that $p(Q) > 0$ and $p'(Q) < 0$ for any $Q > 0$ and $\lim_{Q \to \infty} p(Q) = 0$. In addition, to ensure that the successive oligopoly equilibrium is well behaved, we assume that $p(Q)$ satisfies the following two properties for any $Q > 0$:

**A1.** $p''(Q)Q + p'(Q) < 0$;

**A2.** $p'''(Q)Q^2 + (n + 3)p''(Q)Q + (n + 1)p'(Q) < 0$.

A1 implies that output quantities of downstream firms are strategic substitutes, and each firm’s profit is strictly concave in its quantity choice, whereas A2 implies similar properties for the upstream industry given that the demand for input is derived from downstream competition. A1 is common in Cournot competition (Novshek 1985; Nocke and Whinston 2009), while A2 is a counterpart of A1 and is implicitly assumed in the literature of successive oligopoly (Greenhut and Ohta 1979; Salinger 1988). Under these two assumptions, there exists a unique Nash equilibrium in quantities in each industry, and the equilibrium is stable.

Suppose that the successive oligopoly was originally at equilibrium with total output $Q^*$ and input price $t^* = t(Q^*)$, and consider the following collusion game. All the upstream firms decide whether or not to collude. A collusion is a collective, binding decision by the upstream firms to charge a different input price $\tilde{t} \neq t^*$. If they do so, the collusive input price $\tilde{t}$ will result in a corresponding $\tilde{Q}$ that still satisfies the input demand function, i.e., $\tilde{t} = t(\tilde{Q})$. Alternatively and equivalently we may think that collusion is to produce total quantity of input at $\tilde{Q}$ rather than the equilibrium $Q^*$ (so each upstream firm produces $\tilde{Q}/m$ rather than the equilibrium $Q^*/m$). An upstream firm’s profit in the collusive outcome is then $\pi^{u}_i = (\tilde{t} - f)\tilde{Q}/m$, whereas a downstream firm $i$’s profit is $\pi^{d}_i = q^{d}_i \left[ p(\sum_{k=1}^{n} q^{d}_k) - c - \tilde{t} \right]$ when it produces $q^{d}_i$.

Due to concerns about antitrust lawsuits, a successful collusion is carried out only if the colluding firms are sure that no firms affected, including downstream firms, would sue the upstream firms for damage or report the collusion to the antitrust authority. In other words, collusion must make every firm, both upstream and downstream, better off. Motivated by the auto parts antitrust case, we assume that $l \leq m$ upstream firms are partially owned by downstream firms, while the remaining $(m - l)$ firms are not. Without loss of generality, we assume that $\beta \in [0,1]$ percent of each of the $l$ upstream firms is owned equally by all downstream firms, i.e., each downstream firm is entitled to $\beta/n$ share of the profit of each of the $l$ upstream firms.\(^3\) Ownership here only results in profit transfer; downstream firms do

\(^3\)It is easy to show that the collective choice of collusion can be implemented in a non-cooperative, repeated game such that no firm has any incentive to deviate as long as the discount factor is sufficiently large.

\(^4\)If the share is unequal, the binding condition falls on the firm with the smallest share. This would apparently make the collusion more difficult.
not intervene in upstream firms’ operation and decision-making, nor do they consider how their own competitive choices affects upstream profits.

3 Joint profits and the two externalities

3.1 Demand for input

For any given input price $t$, a downstream firm $i$ chooses its output quantity $q_i^d$ to maximize its profit $\pi_i^d$. In equilibrium, all downstream firms choose the same quantity $q_i^d = Q/n$, so the first-order condition $\frac{\partial \pi_i^d}{\partial q_i^d} = 0$ leads to:

$$t(Q) = p'(Q) \frac{Q}{n} + p(Q) - c,$$

which defines the endogenous demand function for input.\(^5\) Note that $t'(Q) = p''(Q)Q/n + p'(Q)\frac{n+1}{n}$, then A1 implies that $t'(Q) < 0$, or equivalently $Q'(t) < 0$, meaning that there is a one-to-one mapping between $t$ and $Q$ such that a larger input price always results in a smaller total production quantity.

The input demand function, $t(Q)$, is valid in both oligopoly competition and collusion. In what follows, it will be convenient to treat $Q$ as the instrument of collusion rather than $t$. It is also useful to define the downstream markup as $\delta(Q) \equiv p(Q) - t(Q) - c$. Given (1), $\delta(Q) = -p'(Q)\frac{Q}{n} > 0$. Then $\frac{\partial \delta}{\partial Q} = -\frac{1}{n}[p''(Q)Q + p'(Q)] > 0$. Given the inverse relationship between $t$ and $Q$, $\frac{\partial \delta}{\partial t} > 0$ is equivalent to $\frac{\partial t}{\partial Q} < 0$. That is, when the input price is larger, the downstream markup becomes smaller. This is because A1 and A2 of the demand function give rise to cost absorption, i.e., any increase in the input price will be absorbed, but only partially, by downstream firms.

**Lemma 1** When the input price increases, the industrial total output ($Q$) and the downstream markup ($\delta$) both decrease.

3.2 Successive oligopoly

The successive oligopoly equilibrium is solved by backward induction. The downstream competition has been derived above and captured by equation (1). Facing such demand function for input, upstream firm $j$ chooses quantity $q_j^u$ to maximize its profit $\pi_j^u$. The first-order condition, $\frac{\partial \pi_j^u}{\partial q_j^u} = 0$, then leads to:

$$\left[ p'(Q) \frac{Q}{n} + p(Q) - c - f \right] + \left[ \frac{n+1}{n} p'(Q) + \frac{Q}{n} p''(Q) \right] q_j^u = 0$$

\(^5\)The second derivative is $\frac{\partial^2 \pi_i^d}{\partial q_i^d \partial t} = p''(Q)q_i^d + 2p'(Q)$. If $p''(Q) < 0$, then $p''(Q)q_i^d + 2p'(Q) < 0$; if $p''(Q) \geq 0$, then $p''(Q)q_i^d + 2p'(Q) \leq p''(Q)Q + 2p'(Q) < p''(Q)Q + p'(Q) < 0$, where the last inequality is due to A1. In subsequent analysis, the second order conditions are always satisfied given A1 and A2. Thus, they are omitted to save space.
for $j = 1, 2, \ldots, m$. In equilibrium, each upstream firm produces the same amount: $q^u_j = \frac{Q}{m}$, so we have

$$
\left[(m + n + 1 - nm)p'(Q^*) + p''(Q^*)Q^*\right] \frac{Q^*}{nm} + \left[p'(Q^*)Q^* + p(Q^*)\right] = c + f. \quad (3)
$$

The first term on the left-hand side of (3) is the net externality ($NE$) which we will analyze in details shortly, and the second term is the marginal revenue from all firms combined; the right-hand side of (3) is the (joint) marginal cost. By A1 and A2, the equilibrium quantity, $Q^*$, is determined uniquely by (3). Given $Q^*$, the equilibrium values of all the other variables can then be uniquely determined.

### 3.3 Joint-profit maximization

Since a collusion requires the consent from all firms in the two industries, a necessary (but not sufficient) condition is that the collusion raises the firms’ joint total profits. For this purpose, consider the total profits as a function of the total quantity, $Q$:

$$
\Pi(Q) = [p(Q) - c - f] Q,
$$

which is maximized at $\frac{\partial \Pi(Q)}{\partial Q} = 0$, or

$$
p'(Q^{**})Q^{**} + p(Q^{**}) = c + f. \quad (4)
$$

(4) equates the marginal revenue from all firms combined with the marginal cost, and thus defines a unique quantity, $Q^{**}$, which maximizes the joint profit. We refer to $Q^{**}$ as the monopoly quantity.

### 3.4 Vertical and horizontal externalities

Now compare (3) with (4). If

$$
NE \equiv \left[(m + n + 1 - nm)p'(Q^*) + p''(Q^*)Q^*\right] \frac{Q^*}{nm} > 0, \quad (5)
$$

we will have $p'(Q^*)Q^* + p(Q^*) < c + f = p'(Q^{**})Q^{**} + p(Q^{**})$, which in turns implies $Q^* > Q^{**}$ given that $\Pi(Q)$ is concave in $Q$ by A1. That is a situation in which firms over-produce in successive oligopoly as compared to the monopoly quantity. If (5) is violated, firms under-produce, i.e., $Q^* \leq Q^{**}$.

**Lemma 2** The equilibrium total output in successive oligopoly ($Q^*$) is greater than the monopoly output ($Q^{**}$) if and only if (5) holds.

To understand the economics behind (5), we show below that $NE$ (net externality) reflects the discrepancy between the individual and the collective incentives, and can be further
decomposed into two externalities:\(^6\)

\[
NE = \frac{\partial \pi^d}{\partial q^d} - \frac{\partial \Pi(Q)}{\partial q^d} = -(t^* - f) + \left[ -\frac{n - 1}{n} p'(Q^*)Q^* \right].
\]

Vertical externality \((-\)

Horizontal externality \((+)

When choosing its output quantity, an individual downstream firm ignores two effects on other firms’ profits, as captured by the two terms on the right hand side of equation (6). The first term, \(-(t^* - f)\), comes from \(-\frac{\partial (\sum_{j=1}^{m} \pi_j^u)}{\partial q^d} = -\frac{\partial (t - f)Q}{\partial q^d} = -(t^* - f)\). It measures how a downstream firm’s output affects the total upstream profits and is therefore referred to as a vertical externality. Since upstream firms enjoy a positive markup (i.e., \(t^* > f\)) in equilibrium, an additional unit of downstream sales would have benefited upstream firms, but downstream firms ignore such benefits, so they produce too little from the viewpoint of all firms’ collective interest. The second term in (6), \(-\frac{n - 1}{n} p'(Q^*)Q^*\), comes from \(-\frac{\partial (\sum_{k \neq i} \pi_i^d)}{\partial q^d} = -\frac{n - 1}{n} Q^* \frac{\partial[p'(Q) - t - c]}{Q} \frac{\partial Q}{\partial q^d} = -\frac{n - 1}{n} p'(Q^*)Q^*\). It measures how an individual downstream firm’s output affects the profits of all other downstream firms and is thereafter referred to as a horizontal externality. Because a downstream firm’s output hurts its competitors but it ignores such damage, it tends to produce too much from the viewpoint of collective interest.

In sum, competition in successive oligopoly results in a vertical externality and a horizontal externality. From the perspective of joint profits, vertical externality tends to reduce the total output while horizontal externality tends to increase it. The firms’ joint profit is maximized if the two externalities exactly cancel out. If the horizontal externality dominates, i.e., if (5) holds, firms produce too much to the detriment of their collective interest.

### 3.5 The upstream, downstream, and joint total profits

We now look at how a change in \(Q\) affects the upstream and downstream firms. Figure 1 draws the total profits of the upstream (\(\Pi^u\)), the downstream (\(\Pi^d\)) and all firms combined (\(\Pi\)) as functions of \(Q\). Since \(t(Q)\) decreases in \(Q\) (Lemma 1), there exists a \(Q^{max}\) such that \(t(Q^{max}) = f\), at which point the upstream profit is zero. Earlier we have demonstrated that the total profit of all firms, \(\Pi\), is concave in \(Q\) with the maximum at \(Q^{**}\).

\(^6\)In successive oligopoly, a downstream firm’s individual incentive is captured by:

\[
\frac{\partial \pi^u}{\partial q^u} = \frac{\partial \pi^u(Q)}{\partial Q} \frac{\partial Q}{\partial q^u} = p'(Q)\frac{Q}{n} + p(Q) - c - t
\]

\[
= [p'(Q) + p(Q) - c - f] + (f - t) - \frac{n - 1}{n} p'(Q)Q
\]

\[
= \frac{\partial \Pi(Q)}{\partial q^u} + (f - t) + \left[ -\frac{n - 1}{n} p'(Q)Q \right],
\]

where \(\frac{\partial \Pi(Q)}{\partial q^u} \equiv \frac{\partial \Pi(Q)}{\partial q^u} \frac{\partial Q}{\partial q^u} = p'(Q)Q + p(Q) - c - f\) represents the collective incentive, i.e., how this downstream firm’s output affects the upstream and downstream firms’ joint profit.
The downstream total profit is $\Pi^d(Q) = \delta(Q)Q$, where $\delta(Q)$ is the downstream markup. Then $\frac{\partial \Pi^d(Q)}{\partial Q} = \delta'(Q)Q + \delta(Q)$. By Lemma 1, $\delta'(Q) > 0$, so $\frac{\partial \Pi^d(Q)}{\partial Q} > 0$, meaning that the downstream total profit increases with industry total output. This is because a larger total output indicates a smaller input price, which allows a greater downstream markup (Lemma 1). Greater total output and greater markup reinforce each other to lead to a greater total downstream profit. Therefore, $\Pi^d$ is a monotonically increasing function of $Q$, reaching the largest level at $Q^{\text{max}}$.

The upstream firms’ total profit, $\Pi^u(Q) = (t - f)Q$, is maximized when

$$Q \frac{Q}{n} [p''(Q)Q + 2p'(Q)] + [p'(Q)Q + p(Q)] = c + f.$$  \hspace{1cm} (7)

Given A2, $\Pi^u(Q)$ is concave, so (7) has a unique solution, denoted as $Q^u$ in Figure 1. When upstream firms decrease $Q$, they will ignore the damage imposed on downstream firms, which is captured by the first term of the left-hand side of (7). Later we are going to discuss in more details this business-stealing effect. For now, it suffices to note that, because $p''(Q)Q + 2p'(Q) < 0$, a comparison between (7) and (4) reveals:

$$Q^u < Q^{**}.$$  \hspace{1cm} (8)

That is, the upstream total profit is maximized at an output level smaller than the monopoly quantity. Comparing (7) with (3), we know

$$Q^u < Q^*.$$  \hspace{1cm} (9)
That is, the upstream firms would collectively prefer an output level that is smaller than the equilibrium level.

Therefore, \( Q_u < Q^{**} \) and \( Q_u < Q^* \) both hold unconditionally, whereas the comparison between \( Q^{**} \) and \( Q^* \) depends on whether (5) holds or not. Figure 1 shows the situation of \( Q^{**} < Q^* \) when (5) holds, so we have \( Q_u < Q^{**} < Q^* \). The comparison between \( Q^{**} \) and \( Q^* \) is crucial because, starting from the equilibrium \( Q^* \), a smaller \( Q \) (resulting from a higher collusive input price) raises the joint profit only when \( Q^* > Q^{**} \).

The results are summarized in the following proposition:

**Proposition 1** Consider the equilibrium total output \( Q^* \) in successive oligopoly.

(i) When (5) holds, \( Q_u < Q^{**} < Q^* \). Then a small reduction of total output from \( Q^* \) raises the upstream and joint profits but reduces the downstream profits.

(ii) When (5) fails, \( Q_u < Q^* < Q^{**} \). Then a small increase of total output from \( Q^* \) raises the downstream and joint profits but reduces the upstream profits.

### 4 Passive ownership and profit sharing

We now investigate passive ownership as a compensation mechanism for downstream firms to benefit from upstream collusion. We will first look at collusions that marginally raises the input price from the equilibrium level, and then move on to collusions that raises the input price discretely, and finally discuss collusions that reduce the input price.

#### 4.1 Marginal analysis

An upstream firm’s final profit is \((1 - \beta)\pi_u(Q^*)\) if it is partially owned by downstream firms, whereas a downstream firms’ final profit is \(\pi_d(Q^*) + \frac{l}{n} \beta \pi_u(Q^*)\). In Figure 1, starting from \(Q^*\), when \(Q\) is slightly smaller, upstream firms always benefit \(\frac{\partial \pi_u(Q^*)}{\partial Q} < 0\) given that \(Q_u < Q^*\).

For downstream firms to also benefit, a necessary (but not sufficient) condition is that the joint profit decreases in \(Q\) \(\left(\frac{\partial \Pi(Q^*)}{\partial Q} < 0\right)\), which is the case if (5) holds and consequently \(Q^{**} < Q^*\).

Given condition (5) is satisfied, the sufficient condition for downstream firms to benefit from a small decrease in \(Q\) is \(\frac{\partial \pi_d(Q^*)}{\partial Q} + \beta \frac{\partial \pi_u(Q^*)}{\partial Q} \leq 0\), or

\[
\beta \geq \frac{m^2}{l(m-1)(n+1) + \rho(Q^*)} \equiv \beta(Q^*),
\]

where

\[
\rho(Q) \equiv \frac{p''(Q)Q}{p'(Q)}
\]

Here and later when we say (5) fails, it means \(NE < 0\). We skip the borderline case of equality, which implies the equilibrium outcome happens to maximize the joint profit, i.e., \(Q^* = Q^{**}\).
is the degree of demand concavity (Fauli-Oller, 1997; Ziss, 2001). Given A1, we have $\rho(Q) > -1$ and hence $\beta(Q^*) > 0$.

**Proposition 2** Starting from the successive oligopoly equilibrium (i.e., $t^* = t(Q^*)$), upstream firms carry out a collusion that marginally raises the input price, which also benefits the downstream firms, if and only if (10) holds. When this happens, social welfare and consumer surplus both drop.

For upstream collusion to take place, downstream firms must hold a sufficiently high share of the upstream profits (i.e., $\beta \geq \beta(Q^*)$). Given that $\tilde{t} > t^*$ in collusion, we have the corresponding collusive quantity $\tilde{Q} < Q^*$ and thus the final price $p(\tilde{Q}) > p(Q^*)$. Because the final price is higher, consumers are worse off. Because the final price moves further away from the social marginal cost (i.e., $c + f$), social welfare also drops. In other words, the loss in consumer surplus exceeds the gain in producer profits. Such unambiguous negative effects of upstream collusion justifies the antitrust lawsuits mentioned in Introduction.

### 4.2 Facilitators of upstream collusion

The previous subsection establishes (10) as the sufficient condition for upstream collusion. Note that for the collusion to be feasible, we must have $\beta(Q^*) \leq 1$, which is equivalent to

$$\rho(Q^*) \leq \frac{l(m-1)(n+1) - 2m^2}{m^2 - l(m-1)}.$$  

Condition (11) is more stringent than condition (5), as (5) only requires the total profits of all firms to rise, whereas (11) requires the total profits of a subset of firms (excluding independent upstream firms, which are unambiguously better off) to rise. If there is no independent upstream firms (i.e., $l = m$), the two conditions coincide.

Given $\beta$, anything that makes it easier to satisfy $\beta \geq \beta(Q^*)$ (condition (10)) must imply a lower $\beta(Q^*)$, which must simultaneously make it easier to satisfy the requirement of $\beta(Q^*) \leq 1$ (condition (11)). In terms of what facilitates upstream collusion, therefore, studying (10) is equivalent to studying (11). From now on, we will focus on (11) to discuss how a parameter affects the likelihood of upstream collusion.

Condition (11) involves the demand concavity $\rho(Q^*)$, which in general is endogenous. In many cases, however, $\rho(Q^*)$ is a constant, i.e., $\rho(Q^*) \equiv \rho$. We will focus on this case for simplicity.

Fixing the number of upstream firms that are (partially) owned by downstream firms, $l$, an increase in $m$ indicates there are more independent upstream firm, which takes away some of the increased profits without compensating downstream firms, so collusion becomes less

---

8For example, for $a > 0$ and $b > 0$, we have $\rho(Q) = 0$ for linear demand $p = a - bQ$; $\rho(Q) = x - 1$ for $p = a - bQ^*$ (for $x > 0$).
likely. Fixing \( m \), an increase in \( l \) leads to the opposite effect. From now on, we will focus on the case of \( l = m \). Then condition (11) becomes

\[
(m - 1)(n - 1) \geq \rho + 2. \tag{12}
\]

Apparently, (12) is easier to satisfy when \( m \) or \( n \) is larger or \( \rho \) is smaller. The following proposition summarizes the factors that affect the likelihood of upstream collusion, in the sense of a more relaxed requirement on the lower bound of downstream firms’ holding of the upstream firms (i.e., a lower \( \beta(Q^*) \)):

**Proposition 3** Suppose that demand concavity is constant and no upstream firm is independent. Then upstream collusion is facilitated by:

1. a less concave demand (i.e., a smaller \( \rho \));
2. a greater number of firms in the downstream (i.e., a larger \( n \)); or
3. a greater number of firms in the upstream (i.e., a larger \( m \)); or
4. a more balanced vertical structure (i.e., a smaller \( |m - n| \) for fixed \( m + n \)).

To understand the intuition for (i), note that \( \rho \) determines the allocation of profits between the two industries when \( Q \) changes. A smaller \( \rho \) indicates a larger pass-through rate \( \left( \frac{\nu'(Q)}{\nu(Q)} = \frac{n}{\rho + (n+1)} \right) \). That is, the smaller is \( \rho \), the larger proportion of an increase in input price can be passed to final consumers. As a result, the increase of input price does less damage to downstream firms, making it easier to compensate them.

The intuition for (ii) is as follows. When there are more downstream firms (i.e., \( n \) increases), the horizontal competition in the downstream is intensified, strengthening horizontal externality and therefore making it more likely for the horizontal externality to dominate the vertical externality. At the same time, the profit of downstream firms is smaller and therefore compensation becomes easier. Both makes upstream collusion more likely.

For (iii), when there are more upstream firms, the upstream markup drops so the vertical externality decreases. At the same time, a greater number of upstream firms increases the total output, so each downstream firms exerts a greater damage to its downstream rivals when it expands its output, i.e., the horizontal externality increases. Both make it more likely for the horizontal externality to dominate the vertical externality.

Finally, (iv) can be understood from two extreme cases. If \( n \) is very large but \( m \) is small, the vertical externality will be large. If, on the other hand, \( n \) is small while \( m \) is large, the horizontal externality will be small. In both cases, it is harder for the horizontal externality to dominate the vertical externality.

The proposition leads to some further implications. Since upstream collusion requires downstream firms to own a sufficient share of upstream profits, partial vertical integration may facilitate upstream collusion. Since a greater number of firms in either industry is
conducive to collusion, entry of new competitors into the upstream or downstream industry may precipitate and facilitate upstream collusion, provided firms are sufficiently patient. Exit has the opposite effect: collusion becomes less likely. Finally, horizontal merger in either industry reduces the number of competitors and hence the intensity of horizontal competition, making collusion more difficult. In response, firms may increase cross-sharing between the two industries in order to sustain collusion.

4.3 Intra-marginal analysis

So far we have been discussing an upstream collusion that raises the input price marginally from the equilibrium level $t^*$. Such an approach has the convenience of deriving the sufficient condition based on the equilibrium first-order conditions. But what happens if the increase in input price is intra-marginal?

Suppose that (5) continues to hold, i.e., $Q^u < Q^{**} < Q^*$ (see Figure 1). Then the collusive $Q$ must satisfy the downstream firm’s participation constraint:

$$\pi^d(Q) + \frac{1}{n} \pi^u(Q) \geq \pi^d(Q^*) + \frac{1}{n} \pi^u(Q^*).$$

(13)

When $Q$ decreases, on the one hand, $\pi^u(Q)$ is larger, which means a smaller $\beta$ can satisfy (13). On the other hand, however, $\pi^d(Q)$ is smaller, so $\beta$ has to be larger to satisfy (13). In general, therefore, the net effect of $Q$ on (13) is ambiguous. Nevertheless, for $Q \in [Q^u, Q^*]$, (13) is equivalent to

$$\beta \geq \hat{\beta}(Q) \equiv \frac{n}{l} \pi^d(Q^*) - \frac{1}{n} \pi^u(Q) = \int_Q^{Q^*} \beta(Q) \left[ -\frac{\partial \pi^u(Q)}{\partial Q} \right] dQ,$$

where $\beta(Q) \equiv \frac{n}{l} \left[ -\frac{\partial \pi^u(Q)}{\partial Q} \right]$ is the minimal share holding to keep downstream firms unaffected by a marginal change of output at arbitrary $Q$. Notice that since $\left[ -\frac{\partial \pi^u(Q)}{\partial Q} \right] \geq 0$, $\hat{\beta}(Q)$ is a weighted average of $\beta(Q)$ along the path between $Q$ and $Q^*$ with the weight at any arbitrary $Q$ being $\int_Q^{Q^*} \left[ -\frac{\partial \pi^u(Q)}{\partial Q} \right] dQ$. Consequently, if $\hat{\beta}(Q)$ is monotonic on $[\bar{Q}, Q^*]$, $\hat{\beta}(Q)$ will also be monotonic.9

We now turn to the properties of $\hat{\beta}(Q)$. By substituting $\frac{\partial \pi^d(Q)}{\partial Q}$ and $\frac{\partial \pi^u(Q)}{\partial Q}$, $\hat{\beta}(Q)$ can be rewritten as

$$\hat{\beta}(Q) \equiv \frac{m}{l} \left[ 1 + \frac{1}{\frac{\rho'(Q) Q + \rho(Q) - c - f}{Q \rho''(Q) Q + 2 \rho'(Q)}} \right].$$

Note that $\hat{\beta}(Q)$ and $\hat{\beta}(Q^*)$ are the same function and differ only in the argument ($Q^*$ is a particular value of the general $Q$). We first show that $\hat{\beta}'(Q) < 0$ for every $Q$ if $\rho(Q)$ is

9Similar approach has been used by Farrell and Shapiro (1990) in their merger studies.
constant. In the expression of $\beta(Q)$, the term $p'(Q)Q + p(Q) - c - f \equiv J(Q)$ represents how a change in $Q$ affects the joint profits of all firms in the two industries and is referred to as the joint effect, whereas the term $\frac{Q}{n} [p''(Q)Q + 2p'(Q)] \equiv B(Q)$ represents how a change in $Q$ shifts profits between the two industries and is referred to as the business-stealing effect.

To see this decomposition, note that when $Q$ changes slightly, the total profit of all upstream firms changes by

$$\frac{\partial \Pi_u(Q)}{\partial Q} = \frac{Q}{n} [p''(Q)Q + 2p'(Q)] + [p'(Q)Q + p(Q) - c - f].$$

Earlier we have seen that $\frac{\partial \Pi(Q)}{\partial Q} \equiv p'(Q)Q + p(Q) - c - f$, so the second term represents how a change in $Q$ increases the joint profit, i.e., the joint effect. The first term then captures how a change in $Q$ transfers profits from the downstream to the upstream, i.e., the business-stealing effect.

Now, $\beta(Q) = \frac{m}{n} \frac{1}{1+\frac{J(Q)}{P(Q)}}$ with $B(Q) < 0$, and $J(Q) \geq 0$ when $Q \in [Q^u, Q^*]$ and $J(Q) < 0$ when $Q \in (Q^*, Q^u]$. Apparently, $J'(Q) = (\rho + 2)p'(Q) < 0$ by A1, and $B'(Q) = \frac{\rho+1}{n} (\rho + 2)p'(Q) < 0$ by A1 given that the demand concavity is constant. For $Q \in [Q^u, Q^*]$, it is obvious that when $Q$ is smaller, the joint effect becomes weaker while the business stealing effect becomes stronger, which leads to a larger $\beta(Q)$. For $Q \in (Q^*, Q^*)$, a smaller $Q$ leads to a stronger joint effect as well as a stronger business-stealing effect. If $\rho$ is constant, we are able to show that as $Q$ decreases, the business-stealing effect becomes stronger relative to the joint effect (see the appendix for details). This leads to $\beta'(Q) < 0$ when $Q \in (Q^*, Q^*)$.

In summary, $\beta'(Q) < 0$ for any $Q \in [Q^u, Q^*]$, meaning that as $Q$ decreases, a downstream firm needs a greater $\beta$ in order to break even. Given that $\beta'(Q) < 0$, we can prove that $\beta'(\tilde{Q}) < 0$ (again, formal proof is in the appendix). Then we have the following Proposition.

**Proposition 4** Suppose the demand concavity ($\rho$) is constant. Then $\beta(Q)$ decreases with $Q \in [Q^u, Q^*]$.

The proposition indicates that if upstream firms further increases the input price, although the joint profit and upstream profit both increase, upstream collusion becomes more difficult as it places a more stringent requirement on the shares of upstream firms that must be owned by the downstream. This is because a higher input price (corresponding to a smaller collusive output) allows the upstream to steal more profit from the downstream. Given the constant demand concavity, the business-stealing effect is always the dominant force, which means the downstream firms must receive a larger share of the upstream profit in order not to be worse off.

**Example 1** Consider linear final demand, i.e., $p = a - bQ$. Then $\rho = 0$. For simplicity, let’s assume $a = 1$ and $c = f = 0$. 
In successive oligopoly equilibrium, the input price is $t^* = \frac{1}{m+1}$, the final price is $p^* = 1 - \frac{mn}{(m+1)(n+1)}$, and the firms’ profit are

$$\pi_d^* = \frac{b}{n^2} \left[ \frac{mn}{b(m+1)(n+1)} \right]^2;$$

$$\pi_u^* = \frac{b(n+1)}{m^2n} \left[ \frac{mn}{b(m+1)(n+1)} \right]^2.$$

If the collusive input price is $\bar{t}$, the final price will be $\bar{p} = \frac{1+n\bar{t}}{m+1}$. The firms’ profit become

$$\pi_d^c = \frac{b}{n^2} \left[ \frac{n(1-\bar{t})}{b(n+1)} \right]^2;$$

$$\pi_u^c = \frac{b(n+1)\bar{t}}{mn(1-\bar{t})} \left[ \frac{n(1-\bar{t})}{b(n+1)} \right]^2.$$

Obviously, the threshold share-holding of intra-marginal collusion is $\hat{\beta} = \frac{n}{l} \frac{\pi_d^c - \pi_d^u}{\pi_u^c - \pi_u^u}$. To ensure that upstream firms benefit from collusion, the denominator should be positive, which requires $\bar{t} \in \left( \frac{1}{m+1}, \frac{m}{m+1} \right)$. Notice that the industry joint profit is maximized when $t^{**} = \frac{n-1}{2m}$, but upstream total profit is maximized when $t^u = \frac{1}{2}$. Given the over-produce condition (5), it is easy to verify $\frac{n-1}{2m} \in \left( \frac{1}{m+1}, \frac{m}{m+1} \right)$. Besides, our analysis in the section focuses on $\bar{t} \in \left( \frac{1}{m+1}, \frac{1}{2} \right)$.

By simplifying the expression, we have $\hat{\beta} = \frac{n}{l(n+1)} \left[ \frac{m}{m-(m+1)t} + 1 \right]$. Obviously, $\hat{\beta}$ increases with $\bar{t}$. In other words, as the collusive output decreases, the required minimal share-holding rises.

Proposition 4 and Example 1 reveal two properties of upstream collusion with passive ownership. First, as long as the share-holding is large enough, the colluding firms prefer an output ($Q^u$) that is smaller than the one that maximizes the joint profit of the two industries ($Q^{**}$). Second, if the demand concavity is constant, a smaller collusive output makes upstream collusion harder. In other words, there is a tension between upstream profit maximization and the feasibility of the upstream collusion. Both properties differentiate passive ownership from other compensation schemes. For example, if upstream firms use two-part tariffs, the optimal collusive output will be $Q^{**}$, which maximizes the joint profit. Collusion is also easier, as upstream firms have more surpluses to compensate the downstream. As a collusive compensation mechanism, therefore, passive ownership may not be as powerful as two-part tariffs, but it is much harder to be detected by antitrust authorities. That is why it must be thoroughly understood.

4.4 Collude to reduce the input price

We have established that the necessary condition for upstream collusion is over-production in successive oligopoly (so that a slightly higher $t$ would raise the total profit), which is captured
by condition (5). If (5) fails, the equilibrium total output $Q^*$ falls below the monopoly level (i.e., $Q^* \in (Q^u, Q^*)$) and thus $\frac{\partial \Pi(Q^*)}{\partial Q} > 0$ (Proposition 1). If upstream collusion raises the input price, all upstream firms gain, but the joint profit decreases. Then there is no way for upstream firms to compensate the downstream firms to make every firm better off. However, if upstream firms collude to reduce the input price, the total profit would increase. A lower input price benefits all downstream firms but hurts upstream firms (Proposition 1). In that case, if there is a mechanism for downstream firms to properly compensate upstream firms, for example, if there is a lump-sum transfer (e.g., franchise fee) from downstream firms to upstream firms, then all firms can be better off.

Since consumer surplus and total welfare increases if and only if the final price decreases or, equivalently, total output increases, such collusion improves consumer surplus and total welfare.

**Proposition 5** If (5) fails, then price-reducing upstream collusion takes place if there exist mechanisms for downstream firms to compensate upstream firms properly. When that happens, firm profits, consumer surplus and social welfare all increase.

If the demand for final product is linear, $\rho = 0$, and (5) boils down to $(m - 1)(n - 1) \geq 2$, which fails when $m = n = 2$. In that case, upstream firms may collude to reduce the input price, which raises the profit of all four firms as well as the consumer surplus and social welfare. It is a win-win-win.

When upstream collusion reduces the input price, the compensation can also be implemented if upstream firms are passive owners of downstream firms. In that case passive ownership improves welfare. Therefore, passive ownership by itself does not always hurt welfare. It only facilitates upstream collusion, the nature of which (i.e., whether it raises or reduces the input price) determines how social welfare changes.

## 5 Extensions

The analysis so far is carried out in a setting where the marginal production cost is constant, and firms carry out Cournot competition. In what follows we will explore alternative settings with increasing marginal cost, differential cost, or Bertrand competition, and show that the major insights continue to be valid. These extensions also reveal additional facilitating conditions for upstream collusion. Before discussing alternative settings of the business environment, we first elaborate on the meaning of passive ownership, which is different from the typical definition in the literature (e.g. Hunold and Stahl, 2016), and demonstrate why it is crucial for upstream collusion.

### 5.1 Passive ownership

In the main model we have assumed that when choosing its competitive output, a downstream firm ignores the effect of its choice on upstream profits, in which it holds some shares. It
is well known in both reality and economic theory (Fershtman and Judd, 1987; Creane and Davidson 2004; Kalnins and Lafontaine 2004) that sometimes a company may be instructed to maximize something that is different from its true profit, so the arrangement is not surprising. Nevertheless, let us consider what happens if downstream firms do make such considerations. Then a downstream firm’s objective function is 

$$
\pi_i^d = [p(Q) - t - c]q_i + \frac{\beta}{n} t \frac{1}{m} (t - f)Q.
$$

Taking \( t \) as given, the FOCs lead to the demand for input as:

$$
t(Q) = \frac{mn}{mn - l\beta} \left[ \frac{p'(Q) Q}{n} + p(Q) - c - \frac{1}{mn} \beta \right]. \quad (14)
$$

Compared to the formulation of demand for input in the main model, which corresponds to \( \beta = 0 \), here the slope is steeper (i.e., \( \frac{mn}{mn - l\beta} > 1 \)), while the intercept may be higher or lower.

Given this \( t(Q) \), an upstream firm solves \( \max_{q_j} \pi_j^u = (t(Q) - f)q_j \). Re-arranging the first-order condition, we have

$$
\left[ (m + n + 1 - nm)p'(Q) + p''(Q)Q \right] \frac{Q}{nm} + \frac{p'(Q)Q}{n} + p(Q) = c + f,
$$

which is exactly the same as the equilibrium condition (3) in the main model, i.e., it does not contain any \( \beta \). Therefore, the equilibrium output is the same. The equilibrium input price then is:

$$
t(Q^*) = \frac{mn}{mn - l\beta} \left[ \frac{p'(Q^*) Q^*}{n} + p(Q^*) - c - \frac{1}{mn} \beta \right]$

$$
= \left[ \frac{p'(Q^*) Q^*}{n} + p(Q^*) - c \right] + \frac{l \beta}{mn - l \beta} \left[ \frac{p'(Q^*) Q^*}{n} + p(Q^*) - c - f \right]$

$$
> \frac{p'(Q^*) Q^*}{n} + p(Q^*) - c
$$

The last inequality is due to the fact that \( p'(Q^*) \frac{Q^*}{n} + p(Q^*) - c - f \) is the upstream margin in the main model and is therefore positive in equilibrium. This means the equilibrium input price is higher when share holding affects downstream’s competitive choices. The reason is the steeper demand in (14) means that \( Q \) is less responsive to \( t \). That is, when the upstream raises the input price, the downstream wouldn’t reduce the output by much, which in turn gives the upstream firms an incentive to further raise the input price. In equilibrium, the two forces (i.e., a higher input price and a less responsive downstream output) exactly cancel out when it comes to the total output, but the equilibrium input price will be higher than when downstream firms do not consider their share holdings.

In other words, when downstream firms partly internalize the positive externality of a larger output on the upstream profits, they will be reluctant to reduce the output even when the input price is higher. This makes them vulnerable because upstream firms will take advantage of this concern by charging an even higher input price. In equilibrium, the total output remains unchanged, but the input price is higher, which hurts downstream firms.

The fact that downstream firms will be worse off justifies our assumption that down-
stream firms ignore their ownership structure when making their competitive choices. Furthermore, if downstream firms insist considering the share-holding when making their competitive choices, a downstream firm’s objective function is

\[ d_i = \left[ p(Q) - c - \frac{\beta}{m} Q(t-f) Q \right] q_i + n l(t) Q, \]

Then by the Envelope Theorem, we have

\[ \frac{\partial d_i}{\partial t} = \frac{\beta l Q}{n} - q_i = \frac{Q}{n} (\beta \frac{l}{m} - 1), \]

where \( q_i = \frac{Q}{n} \) in equilibrium. Since \( \beta \leq 1 \leq \frac{m}{T} \), \( \frac{\partial d_i}{\partial t} \leq 0 \). It takes equality if and only if \( l = m \) and \( \beta = 1 \). If either condition fails, the downstream firm must be hurt by a higher \( t \). Therefore, there is no way for a collusion to benefit the downstream firms even though they receive a share of the increased upstream profits. This provides yet another justification of our consideration of passive ownership in the main model.

**Proposition 6** If downstream firms consider their partial holding of the upstream shares when making their competitive choices, then

(i) downstream firms are worse off while upstream firms are better off as compared to the case in which downstream firms ignore such share-holding;

(ii) upstream collusion will never take place.

### 5.2 Increasing marginal cost

In the main model, we have assumed that the marginal costs in both industries are constant. It is evident that the driving force does not depend on this particular cost structure. In this extension we demonstrate that our results continue to hold when marginal costs are increasing. More specifically, assume that an upstream firm \( j \)'s cost is \( F(q_j), j \in \{1, ..., m\} \), and a downstream firm \( i \)'s cost is \( C(q_i), i \in \{1, ..., n\} \), with \( F'(q_j) \geq 0, F''(q_j) \geq 0, C'(q_i) \geq 0, C''(q_i) \geq 0 \). The other settings remain the same.

Following the same procedure as in the main model (see the appendix for more details), we can show that the sufficient condition for upstream collusion is:

\[ \beta \geq \frac{m^2}{l(m-1)} \rho(Q^*) + 1 + \frac{\rho(Q^*) + 1}{\rho'(Q^*) - C''(Q^*)} \equiv \beta^I(Q^*), \]

where \( \gamma(Q) \equiv \rho''(Q)Q + \rho'(Q) \rho'(Q) - C''(Q^*) \) is a variable that reflects the curvatures of the demand and marginal cost functions. Following previous exercise, we assume \( \rho \) is constant and \( l = m \), and look at the condition \( \beta^I(Q^*) \leq 1 \), which becomes:

\[ (m-1)(n-1) \geq \frac{1 + \gamma}{\gamma} (\rho + 1). \]

Noting that \( \gamma(Q) \in (0, \rho(Q) + 1] \) (with \( \gamma = \rho + 1 \) when \( C''(.) = 0 \)), we have:

\[ \frac{\rho''(Q)Q + \rho'(Q)}{\rho'(Q) - C''(Q^*)} \]

---

10 This is where our passive ownership differs from its standard definition. If firm \( A \) partially owns firm \( B \), the term “passive ownership” usually means \( A \) does not intervene in \( B \)'s operation. But in our paper, we additionally assume that \( A \) does not internalize how its own choice affects \( B \)'s profit, part of which will be \( A \)'s own profit.

11 Farrell and Shapiro (1990) and Gaudet and Salant (1991) use a similar expression \( \frac{\rho''(Q)Q + \rho'(Q)}{\rho'(Q) - C''(Q^*)} \).
Proposition 7 Consider upstream collusion when marginal costs increase with output.

(i) Suppose $p$ is constant and $l = m$. Compared with the case of constant marginal cost, an increasing marginal cost in the downstream hinders upstream collusion, whereas increasing marginal costs in the upstream has no impact.

(ii) Fixing the equilibrium output $Q^*$, upstream collusion is facilitated by a less convex downstream cost, i.e., $\beta^I$ increases with $C''(Q^*_n)$; and $\beta^I = \beta$ when $C''(Q^*_n) = 0$.

The Proposition indicates that an increasing marginal cost in the downstream makes it more difficult for upstream firms to collude, more so when the downstream cost is more convex. When the downstream cost is convex, the equilibrium output $Q$ is less responsive to the input price $t$. To see this, note that if $t$ is smaller, each downstream firm would like to expand, but the expansion would be limited as production is increasingly costly due to the convex cost structure. Because $Q$ is less responsive to $t$, when upstream firms raises $t$, it does not change the final price by much, meaning that it has a small impact on the joint effect but a large impact on the business-stealing effect. For this reason, it becomes increasingly difficult to compensate downstream firms, and therefore upstream collusion is more difficult.

Earlier in Proposition 3 we have demonstrated that demand concavity is detrimental to upstream collusion. Here we establish that cost convexity in the downstream is also detrimental. A more convex cost function results in a steeper supply curve, whereas a more concave demand indicates a steeper demand curve. In both cases, the equilibrium total quantity becomes less sensitive to changes in input price. This means that at equilibrium, the downstream has already been squeezed very much; further raising the input price will mostly damage the downstream firms without raising the joint profit by much. In other words, a more concave final demand and a more convex downstream cost both increase the business-stealing effect and reduce the joint effect. This makes it more difficult to compensate the downstream firms and hence upstream collusion less likely.

5.3 Differential costs

We now consider how upstream collusion is affected if the cost differs with either industry. As will be shown, the dispersion of upstream cost makes upstream collusion easier or more difficult depending on whether the collusion reduces the output of an inefficient or efficient upstream firm, whereas the dispersion of downstream cost always makes upstream collusion more difficult.

5.3.1 Differential marginal cost in the upstream

Suppose the costs of upstream firms are different, but the average cost remains the same. That is, $\frac{1}{m} \sum_{j=1}^{m} f_j = \bar{f} = f$. The other assumptions remain the same. By similar method, we have \[(m+n+1-nm)p'(Q) + p''(Q)Q\frac{Q}{nm} + p'(Q)Q + p(Q) = c + \bar{f}\]. That means the
dispersion of upstream marginal cost does not change the total output at successive oligopoly. The total profit of upstream firms is \( \Pi^u = \sum_{j=1}^{m} [t(Q) - f_j]q_j \), where \( t(Q) = p'(Q)\frac{Q}{n} + p(Q) - c \).

Suppose upstream firms collude to marginally reduce the output of firm \( k \), whose marginal cost is \( f_k \). The change of upstream firms' total profit is then

\[
\frac{\partial \Pi^u}{\partial Q} = \frac{m-1}{nm}Q[p''(Q)Q + (n+1)p'(Q)] + \Delta f_k.
\]

where \( \Delta f_k = \bar{f} - f_k \) represents the relative efficiency of the firm whose output is reduced.

Meanwhile, the downstream firms' total profit is \( \Pi^d = \sum_{i=1}^{n}[p(Q) - \frac{t(Q)}{n} - c]Q = -p'(Q)\frac{Q}{n} \), and collusion changes the profit by \( \frac{\partial \Pi^d}{\partial Q} = -\frac{Q}{n}[p''(Q)Q + 2p'(Q)] \).

Assume that upstream colluding firms share the collusion gain equally. The condition \( \frac{\partial \Pi^u(Q^*)}{\partial Q} + \beta \frac{\partial \Pi^d(Q^*)}{\partial Q} \leq 0 \) leads to

\[
\beta \geq \frac{m^2[2 + \rho(Q^*)]}{l(m-1)[(n+1)+\rho(Q^*)] + mnl\frac{\Delta f_k}{p'(Q)Q}} \equiv \beta^u.
\]

>From the above expression we can easily conclude that \( \beta^u > \beta \) when \( \Delta f_k > 0 \); \( \beta^u < \beta \) when \( \Delta f_k < 0 \); and \( \beta^u = \beta \) when \( \Delta f_k = 0 \). In other words, the condition for upstream collusion becomes more relaxed when it reduces the output of an inefficient firm, but is more stringent if it reduces the output of an efficient firm.

### 5.3.2 Differential marginal cost in the downstream

Suppose the cost of downstream firms are different, but the average cost remains the same. That is \( \frac{1}{n} \sum_{i=1}^{n} c_i = \bar{c} = c \). The other assumptions remain the same.

In this case, the gain of upstream firms from the collusion is the same as before (i.e., \( \frac{\partial \Pi^u}{\partial Q} = \frac{m-1}{nm}Q[p''(Q)Q + (n+1)p'(Q)] \)), but the loss of downstream firms has changed (i.e., \( \frac{\partial \Pi^d}{\partial Q} = -\frac{1}{n}[p''(Q)Q + p'(Q)]q_i + [-p'(Q)\frac{Q}{n} + c - c_i]\frac{\partial q_i}{\partial Q} \)). Note that a higher input price hurts an efficient downstream firm more than it hurts an inefficient downstream firm. This is because an efficient downstream firm has a larger output (i.e., a larger \( q_i \)), so a given reduction in the downstream markup results in a larger profit loss. At the same time, a higher input price reduces the output of an efficient downstream firm by more (i.e., a larger \( \frac{\partial q_i}{\partial Q} \)). Both make an efficient downstream firm lose more from the collusion than does an inefficient downstream firm. Therefore, we conclude that the dispersion of downstream cost makes upstream collusion more difficult because compensating the efficient downstream firm becomes more demanding.

The above two results are summarized in the following proposition.

**Proposition 8** Assume that marginal costs differ across firms within either industry but the average cost remains unchanged.

(i) Dispersion of downstream cost always hinders upstream collusion.

(ii) Dispersion of upstream cost facilitates upstream collusion if output reduction happens to upstream firms that are less efficient than average, and hinders upstream collusion otherwise.
5.4 Bertrand competition

Now suppose that downstream firms carry out Bertrand competition (the nature of upstream competition will be specified later). If the final products continue to be perfect substitutes, then each downstream firm’s profit is zero, and the equilibrium total production quantity is such that \( Q^* > Q^u = Q^{**} \). In that case, upstream collusion always increases upstream profit without hurting downstream firms.

Now suppose that the final products are differentiated, with downstream firm \( i \) facing demand \( q_i(p_i, \mathbf{p}_{-i}) \) with \( \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} < 0 \) and \( \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_{-i}} > 0 \) for \( i \in \{1, \ldots, n\} \), where \( \mathbf{p}_{-i} \) is the price vector of all other downstream firms except \( i \).\(^{12}\) The equilibrium price \( p^*_i \) must satisfy the individual FOC:

\[
q_i(p_i, \mathbf{p}_{-i}) + (p_i - t - c) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0. \quad (16)
\]

On the other hand, the joint profit of all firms, \( \Pi = \sum_{i=1}^{n} (p_i - c - f) q_i(p_i, \mathbf{p}_{-i}) \), is maximized by firm \( i \)'s price, \( p^*_i \), if the following FOC holds:

\[
q_i(p_i, \mathbf{p}_{-i}) + (p_i - c - f) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} + \sum_{j \neq i} (p_i - c - f) \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0. \quad (17)
\]

Rearranging (16), we have

\[
\frac{\partial \pi^d_i}{\partial p_i} - \frac{\partial \Pi}{\partial p_i} = -\frac{\partial \Pi^u}{\partial p_i} + \left( \frac{\partial \pi^d_i}{\partial p_i} - \frac{\partial \Pi^d}{\partial p_i} \right)
\]

\[
= \left[ (f - t) \sum_j \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} \right] + \left[ -\sum_{j \neq i} (p_j - c - t) \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} \right].
\]

Vertical externality 

\[
(+) \quad \text{Horizontal externality} 
\]

\[
(-) \quad \text{Vertical externality}
\]

To analyze the magnitude of vertical externality and horizontal externality, we need to explicitly specify the vertical relation. If upstream firms sell the input through a “market interface” (Inderst, 2010; Ghosh et al., 2014), then the equilibrium input price mainly depends on upstream competition (given the input demand arising from downstream competition).

For example, if upstream firms carry out Bertrand competition, then we have \( t = f \) and the vertical externality vanishes completely. Horizontal externality always dominates. Consequently a slight increase of input price always raises the joint profit. If upstream firms carry out Cournot competition, \( t \) is endogenous determined by the system of \( \frac{\partial \pi^u_i}{\partial p_i} = 0, \ i = 1, \ldots, m \), where \( \pi^u_i = (t - f) q_i(p(t)) \), and \( q_i(p(t)) \) is characterized by \( q_i(p_i, \mathbf{p}_{-i}) \) and downstream firms’ FOCs, i.e., \( q_i(p_i, \mathbf{p}_{-i}) + (p_i - t - c) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0, \ i = 1, \ldots, n \).

\(^{12}\)In addition, we assume that an increase of any individual downstream firm’s final price reduces the total output, i.e., \( \sum_{j} \frac{\partial q_j(p_j, \mathbf{p}_{-j})}{\partial p_j} < 0 \). If upstream firms sell the input at price \( t (t \geq f) \), then downstream firm \( i \) chooses price, \( p_i \), to maximize \( \pi^d_i = (p_i - t - c) q_i(p_i, \mathbf{p}_{-i}) \), treating \( t \) as given. We assume \( \frac{\partial^2 \pi^d_i}{\partial (p_i)^2} < 0 \) to guarantee that the FOCs are sufficient to characterize the equilibrium, \( \frac{\partial^2 \pi^d_i}{\partial p_i \partial p_{-i}} > 0 \) to ensure that downstream firms’ prices are strategic complements, and \( \frac{\partial^2 \pi^d_i}{\partial p_i \partial p_{-i}} + \sum_{j \neq i} \frac{\partial^2 \pi^d_i}{\partial p_i \partial p_j} < 0 \) to ensure stability of the equilibrium.
Example 2 Consider linear demand functions \( q_i = 1 - p_i + \lambda \sum_{j=1}^{n} (p_j - p_i) \) for \( i = 1, \ldots, n \) and with \( \lambda > 0 \). In successive oligopoly, \( t^* = \frac{1}{m+1} (1 - c - f) + f \) and \( p_i^* = 1 - \frac{(1+(n-1)\lambda)m}{2+(n-1)\lambda|m|} (1 - c - f) \). Consequently the vertical externality is \( \frac{1}{m+1} (1 - c - f) \) and the horizontal externality is \( -m \frac{(n-1)\lambda}{2+(n-1)\lambda} (1 - c - f) \). Horizontal externality dominates if and only if

\[
(m - 1)(n - 1) \geq \frac{2}{\lambda}.
\]

A larger number of upstream or downstream firms makes upstream collusion easier as overproduction is more likely, just like in the case of Cournot competition. Since we do not require \( \lambda \leq 1 \), \( m \) and \( n \) can be quite small such as \( m = n = 2 \). A smaller degree of product differentiation (i.e., a greater \( \lambda \)) facilitates upstream collusion, as greater similarity among the products intensifies downstream competition, which leads to overproduction.

If every upstream firm supplies exclusively to one downstream firm under the constraint that the number of upstream and downstream firms are equal (Lin, 1988, 1990; O’Brien and Shaffer, 1993), then we only need to replace \( t \) with \( t_i \) in the above expressions, where \( t_i \) maximize \( \pi_i = (t_i - f)q_i(p(t)) \), and \( q_i(p(t)) \) is characterized by \( q_i(p_i, p_{-i}) \) and downstream firms’ FOCs, i.e., \( q_i(p_i, p_{-i}) + (p_i - t_i - c) \frac{\partial q_i(p_i, p_{-i})}{\partial p_i} = 0 \), \( i = 1, \ldots, n \). Again, a larger number of firms in either industries or a smaller degree of product differentiation intensifies downstream competition, and therefore makes it more likely for the horizontal externality to dominate the vertical externality.

Example 3 Let \( m = n = 2 \) and the demand for firm \( i \) \( i = 1, 2 \) is \( q_i = 1 - p_i + \lambda (p_{-i} - p_i) \) with \( \lambda > 0 \). Then in equilibrium we have \( t_i^* = \frac{3\lambda + 2}{\lambda^2 + 7\lambda + 4} (1 - c - f) + f \) and \( p_i^* = \frac{2(3\lambda + 2)(\lambda + 1)}{(\lambda + 2)(\lambda^2 + 7\lambda + 4)} (1 - c - f) + c + f \). Hence, the vertical externality is \( \frac{3\lambda + 2}{\lambda^2 + 7\lambda + 4} (1 - c - f) \) and the horizontal externality is \( -\frac{\lambda(\lambda^2 + 4\lambda + 2)}{(\lambda + 2)(\lambda^2 + 7\lambda + 4)} (1 - c - f) \). Horizontal externality dominates vertical externality if and only if \( \lambda > 2.32 \). That is, when the final products are sufficiently homogeneous, total profit is raised by an upstream collusion that raises the input price.

The above results are summarized in the following proposition:

**Proposition 9** Suppose that downstream firms carry out Bertrand competition with differentiated products. Then upstream collusion is facilitated by a greater number of firms in either industry, or a smaller degree of production differentiation.

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\(^{13}\) Note that equilibrium stability requires self-price effect to be stronger than cross-price effect, which always holds: \( \left| \frac{\partial q_i}{\partial p_i} \right| > \sum_{j \neq i} \left| \frac{\partial q_i}{\partial p_j} \right| \) given that \( \frac{\partial q_i}{\partial p_i} = -1 - (n - 1)\lambda \) and \( \frac{\partial q_i}{\partial p_j} = \lambda \). So there is no need to restrict \( \lambda \) to be smaller than one. Since \( \frac{1}{\left| \frac{\partial q_i}{\partial p_i} \right|} = \frac{(n-1)\lambda}{1+(n-1)\lambda} \) increases with \( \lambda \), a greater \( \lambda \) indicates more homogeneous products.
6 Conclusion

In this paper we have demonstrated that downstream firms may benefit from an upstream collusion with passive ownership. This requires two conditions: first, the collusion must raise the combined total profits of upstream and downstream firms, and second, there must be a mechanism for the upstream firms to share the benefits of the collusion with the downstream firms. More broadly, in vertically related industries characterized by oligopoly competition, there always exist a vertical externality and a horizontal externality. From the viewpoint of the firms’ joint profit, the horizontal competition can be too fierce if horizontal externality is stronger than vertical externality, or too gentle if otherwise is true. Linear wholesale price often fails to completely balance the two externalities in equilibrium, resulting in distortions in total output and hence a sub-optimal level of total profits for all the firms concerned. If the firms can find a way to move the total output to a different level, they will be able to mitigate the imbalance between the two externalities and thereby raise the total profit. If, in addition, they can find a mechanism to compensate impaired firms, no firm will oppose the move.

More specifically, the implementation of the collusion consists of two elements: the concrete way to swing the total output, and the mechanism for the two industries to share the benefits. In our particular case, the total output is changed by an upstream collusion that raises the input price, and the sharing is through downstream firms’ partial and passive ownership of some upstream firms. In real life, both elements can take a richer format. Most of the commonly used vertical restraints can be understood as a way to increase the total profit, as mentioned in the introduction. There are various ways to divide the profit between upstream and downstream firms, such as two-part tariff. However, as a compensation and profit-sharing mechanism, passive ownership has its own peculiar characteristics. First, it is naturally embedded among industries and thus difficult to be detected by the antitrust authority. Second, it draws a tension between the intensity and the feasibility of the collusion. As the collusive input price increases, the threshold share-holding required to make the downstream break-even also increases, which puts a constraint on how far the collusion can go. This means the collusion may not be so aggressive and, hence, harder to detect.

In addition to the analysis presented here, we have also studied other extensions including the cases of downstream collusion and the vertical disintegration. Our main results and the underlying driving forces remain valid.14 Our setting and assumptions are fairly general discussing the collusive profits and the two externalities, but we have made an additional assumption of constant demand concavity when analyzing the passive ownership. It remains a future research agenda to see what will happen if we relax it.

14These extensions are omitted to save space, but available upon request.
7 Appendix

7.1 Proof of Proposition 4

Note that $eta(Q) \equiv \frac{\mathcal{J}(Q)}{\mathcal{B}(Q)} = \frac{\pi_i(Q) - \pi_i(Q)}{\pi(Q) - \pi(Q)}$. Given constant $\rho$, we first show $\beta'(Q) < 0$, and then prove $\beta'(Q) < 0$.

By taking derivative of $\beta(Q)$ with respect to $Q$, we have

$$\left[ \frac{\mathcal{J}(Q)}{\mathcal{B}(Q)} \right]' = nQ[p''(Q)Q+2p'(Q)]^2 - [p'(Q)Q + p(Q) - c - f] [p''(Q)Q^2 + 4p''(Q)Q + 2p'(Q)] \frac{1}{\mathcal{B}(Q)}.$$ 

Notice that for any $Q \in \{Q^u, Q^s\}$, we have $p'(Q)Q + p(Q) - c - f > 0$; for any $Q \in \{Q^s, Q^s\}$, we have $[p'(Q)Q + p(Q) - c - f] \in \left[ -\frac{Q}{nm} [p''(Q)Q + (m+n+1-nm)p'(Q)], 0 \right]$.

Given that the demand concavity $\rho$ is constant, we have $p''(Q)Q^2 + 4p''(Q)Q + 2p'(Q) < 0$. Therefore, for any $Q \in \{Q^u, Q^s\}$, we have $\left[ \frac{\mathcal{J}(Q)}{\mathcal{B}(Q)} \right]' > 0$; for any $Q \in \{Q^s, Q^s\}$, $\left[ \frac{\mathcal{J}(Q)}{\mathcal{B}(Q)} \right]' \in \left[ nQ \frac{(p+m+1)(p+n+1)}{nm(p^2+1)}, \frac{n}{Q} \right]$. That is, $\left[ \frac{\mathcal{J}(Q)}{\mathcal{B}(Q)} \right]' > 0$ for $Q \in \{Q^u, Q^s\}$.

Therefore, for any $Q \in \{Q^u, Q^s\}$, $\frac{\mathcal{J}(Q)}{\mathcal{B}(Q)}$ increases on $Q$, and thus $\beta(Q)$ always decreases with $Q$.

Since $\beta'(Q) < 0$, for any $Q_1 < Q_2 < Q^s$, we have $\beta(Q_1) > \beta(Q_2)$ and thus

$$\hat{\beta}(Q_2) = \frac{\int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] \beta(x) dx}{\int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx} < \frac{\beta(Q_2) \int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx}{\int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx} = \beta(Q_2).$$

By decomposing $\hat{\beta}(Q_1)$, we have

$$\hat{\beta}(Q_1) = \frac{\int_{Q_1}^{Q_2} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] \beta(x) dx + \int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] \beta(x) dx}{\int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx} > \frac{\beta(Q_2) \int_{Q_1}^{Q_2} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx + \hat{\beta}(Q_2) \int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx}{\int_{Q_2}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx} > \frac{\hat{\beta}(Q_2) \int_{Q_1}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx}{\int_{Q_1}^{Q^s} \left[ -\frac{\partial \pi_i(Q)}{\partial x} \right] dx} = \hat{\beta}(Q_2).$$

Therefore, when $\beta'(Q) < 0$, we have $\hat{\beta}(Q)$ is also decreasing in $Q$.

7.2 Increasing marginal cost

In successive oligopoly, given input price $t$, each downstream firm maximizes its profit: $\max_{q_i \geq 0} \pi_i \equiv q_i^d [p(\sum_{i=1}^{n} q_i^d) - t] - C(q_i^d)$, which gives rise to the inverse demand for input:

$$t(Q) = \frac{p'(Q)}{n} + p(Q) - C'(\frac{Q}{n}).$$
Substitute this input demand function into an upstream firm’s profit function to obtain
\[ \pi_j^u = t(\sum_{i=1}^{m} q_j^u) q_j^u - F(q_j^u), \]
the FOC of which determines the unique equilibrium in successive oligopoly, which is characterized by the total output \( Q^* \):
\[ \frac{Q^*}{nm} \left[ p''(Q^*)Q^* + (m + n + 1 - nm)p'(Q^*) - C''(\frac{Q^*}{n}) \right] + p'(Q^*)Q^* + p(Q^*) = C'(\frac{Q^*}{n}) + F'(\frac{Q^*}{m}). \]

The joint profit is \( \Pi(Q) = p(Q)Q - nC'(\frac{Q}{n}) - mF'(\frac{Q}{m}) \). It is maximized at \( Q^{**} \), which is characterized by:
\[ p'(Q^{**})Q^{**} + p(Q^{**}) = C'(\frac{Q^{**}}{n}) + F'(\frac{Q^{**}}{m}). \]

Similar to condition (5), the firms over produce (i.e., \( Q^* > Q^{**} \)) if and only if
\[ p''(Q^*)Q^* + (m + n + 1 - nm)p'(Q^*) - C''(\frac{Q^*}{n}) > 0. \]

A downstream firm’s individual incentive can be decomposed as:
\[ \frac{\partial \pi^d}{\partial q^d} = \frac{\partial \Pi(Q)}{\partial q^d} + \left[-\frac{\partial \Pi^u(Q)}{\partial q^d} \right] + \left[\frac{\partial \pi^d}{\partial q^d} - \frac{\partial \Pi^d(Q)}{\partial q^d} \right] \]
\[ = \left[\frac{\partial \Pi(Q)}{\partial q^d} \right] + \left[\frac{F'(Q/m) - t}{n} \right] + \left[\frac{-n-1}{n} p'(Q) \right]. \]

Vertical externality

Horizontal externality

(−)

(+)

On the other hand, upstream firms’ joint profit is \( \Pi^u(Q) = t(Q)Q - mF'(\frac{Q}{m}) \), and its optimization leads to a solution of \( Q^u \), which satisfies:
\[ \frac{Q^u}{n} \left[ p''(Q^u)Q^u + 2p'(Q^u) - C''(\frac{Q^u}{n}) \right] + \left[p'(Q^u)Q^u + p(Q^u) - C'(\frac{Q^u}{n}) - F'(\frac{Q^u}{m}) \right] = 0. \]

It is easy to show that this optimal output is smaller than the monopoly level (i.e., \( Q^u < Q^{**} \)).

Now suppose that upstream firms collude to increase input price at successive oligopoly equilibrium. Notice that
\[ \frac{\partial \Pi^u(Q^*)}{\partial Q} = \frac{Q^*}{n} \left[ p''(Q^*)Q^* + 2p'(Q^*) - C''(\frac{Q^*}{n}) \right] \]
\[ - \frac{Q^*}{nm} \left[ p''(Q^*)Q^* + (m + n + 1 - nm)p'(Q^*) - C''(\frac{Q^*}{n}) \right] \]
\[ = \frac{m-1}{nm} Q^* \left[ p''(Q^*)Q^* + (n + 1)p'(Q^*) - C''(\frac{Q^*}{n}) \right] < 0, \]
which means upstream firms always benefit from such collusion given our A1. At the same time, downstream firms benefit from the collusion if and only if
\[ \frac{\partial \pi^d(Q^*)}{\partial Q} + \beta \frac{\partial \pi^d(Q^*)}{\partial Q} \leq 0, \]
equivalently
\[ \beta \geq \beta^I. \]

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References


