Firm productivity and the Variety of Inputs and Outputs: Evidence from Chinese Trade Data

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Abstract
This paper studies how the trade liberalization in China changes the firm productivity. We develop a framework to estimate revenue productivity (TFPR) and real productivity (TFPQ) with multi-product firms. We find that the aggregate TFPR increases 30% from 2002-2007 and TFPQ increases 22%, suggesting that the observed TFPR increase is mainly driven by real productivity change rather than the markup change. We further decompose the change of productivity into three channels: (1) access to foreign inputs; (2) technology upgrade; (3) resource re-allocation within the firm. We find the most significant channel is the last one, which explains half of the aggregate productivity increase. We also find that the SOEs and private firms significantly improve the TFPR. However, private firms TFPR increase mainly come from the increase of TFPQ, while only 65% of the TFPR increase of SOEs can be attributed to change of TFPQ.

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1 Introduction

Since China joined the WTO, the export and import of China grow significantly. Now China is the largest export country and the second largest import country in the world. At the same time, the average firm level productivity of China increases as well. Figure 1 plots the average firm level productivity from year 2000-2007. As we can see, the firm average productivity increases by 12%.

Our paper is to understand the link between international trade and the change of firm level productivity. Three possible channels are considered: (1) Access to the international inputs. Foreign inputs may affect firm productivity through two channels: as in quality ladder models, foreign inputs may have a higher price-adjusted quality, and as in product-variety models they imperfectly substitute domestic inputs. Halpern et al. (2015) find that a quarter of Hungarian productivity growth is attributed to imported inputs. (2) Resource allocations within the firm. International competition may help firms to focus on products they have the largest comparative advantage and improve the resource allocations within a firm. Redding et al. (2006) documents that the unproductive products will be dropped when access to the international market improves. (3) Firms improve their productivity by upgrading their technology or management. Bustos (2011) finds the Argentina firms increase the R&D after the trade liberalization. We quantify the increase of firm productivity in China into these three channels.

Our starting point is a Chinese custom data, which tracks the firms’ import and export information at the product level. We combine this data set with the Chinese manufacturing firm survey data. For each firm, we know its export products and imported products as well as other resources the firm uses (capital and labor). We then build a structural dynamic industry equilibrium model that allow firms to optimally choose their export products and imported inputs. The model is quite flexible to permit rich heterogeneity across products and firms.

We estimate this model in the micro data. In doing so, we face two key empirical challenges. First, the imports are chosen endogenously by the firm. We deal with this identification problem using a structural approach following Halpern et al. (2015). Our model implies that the effect of imports on the firm production function is only through the number of imported varieties and a time-shifter capturing the relative quality-adjusted price change between foreign goods and domestic goods. We then can identify the productivity gain from the imported

1 The firm level TFP is computed using the Olley-Pakes (1994). We normalize the average TFP in year 2000 to be 1.

2 Old trade theory, as Melitz (2003) focuses on the resource allocations across firms when trade cost declines but fixing the firm productivity as given.
inputs channel. Second, we do not observe the resources allocation within the firm. However, our structural model implies that the resources allocations within the firm is related to the revenue shares of different products in a firm. Through the revenue shares distribution change within a firm, we can identify the productivity gain through the second channel. Our model also introduces an unobserved firm productivity, similar as Olley and Pakes (1996). This term is firm-wide and same across all products within a firm. We interpret it as the firm upgrading channel. We follow the identification strategy in Olley and Pakes (1996) and uses the firm investment to identify it.

A great benefit of our data is that we can observe the import and export prices at very disaggregated product levels. It helps us to separate the mark-up and real productivity changes. We are not the first one to do this job. De Loecker (2011) also estimate the markup and real productivity in a multi-product firm model using an Indian data. They assume that for a fixed product, the technology of single product firm and multiple product firm are the same. There are two drawbacks of this approach. First, they are unable to perform counter-factual analysis since they do not model the multiple products firms’ pricing and resource allocation decisions. Second, they can not analyze those products which are only produced by multi-product firms. Our approach can overcome both drawbacks.

Our result shows that the aggregate revenue productivity (TFPR) improves by 30% from 2001-2007, which doubles the productivity change estimated from Olley-Pakes(1994). Within this change, 17.4% can be attributed to the firm productivity increase and the rest can be attributed to the resource re-allocation across firms. We further decompose the firm level productivity into the three channels mentioned before. The most significant contribution at the firm level comes from resources allocation within the firm, 11.9%. The upgrade of the technology is also important. It increases aggregate productivity by 5.5%. While the contribution of access to the foreign imported goods is very small.

Comparing the real productivity change and markup change, we find that most TFPR improvement in China comes from the real productivity (TFPQ) change. However, when we group firms by ownerships, we find that both SOEs and private firms significantly improve the TFPR. However, private firms TFPR increase mainly come from the increase of TFPQ, while only 65% of the TFPR increase of SOEs can be attributed to change of TFPQ.

Besides the papers mentioned above, our paper relates to several other literature. First, our paper contributes to the empirical literature of exploring the firm productivity gain of the international trade. In a multi-product firms model setup, De Loecker et al. (2016) and Dhyne et al. (2016) uses a different approach and gets a similar condition as ours.
Dhyne et al. (2016) find that trade liberalization primarily affected markets by causing firms to find ways to reduce marginal costs as opposed to causing output prices to fall. Our paper suggests how the firms can cut their marginal cost and highlights the channel of domestic inputs improvement and allocation of resources across products.

Second, our paper is related to the literature that focus on productivity in developing countries. The low productivity of the developing countries usually is attributed to resource mis-allocation across firms (Hsieh and Klenow (2009)) or lack of competition across firms (Bloom et al. (2007)). When the frictions are reduced (such as trade cost), people usually start to think the reallocation across firms (Restuccia and Rogerson (2008)). Our paper contributes to the literature by focusing on how the allocation within the firm will change in response of the decline of the trade cost.

The remainder of the paper is organized as follows. In the next section, we provide a brief overview about the data used in the analysis and document some motivation facts. In section 3, we lay out the structural model. Section 4 discusses our estimation strategy and section 5 shows the main results. Section 6 concludes.

2 Data and Motivation Facts

2.1 Data

In this paper, we match Chinese manufacture firm survey data and Chinese customs database. The first dataset covers operating information of all Chinese manufacture firms whose annual sales are above 500 million RMB (71 million USD) or SOEs. The Chinese customs data records the export and import price and quantity information of each firm at the product level (HS6). We match the two datasets by matching the names of legal representative, head-quarter’s address and telephone number. The efficiency of the match process turns out to be good. Take year 2007 as an example, the number of firms in Chinese manufacture firm survey data is 298992, among which 75930 firms are exporters. In the same year, there are 193567 exporters in the Chinese customs database. We can match 46604 firms in the two datasets.

At the end, we can observe the capital, number of employees, total expenditure of materials at the firm level. We also observe other firm characteristics, such as the set-up year and the ownership. At the product level, we know the prices and quantities of all imported inputs and exports.4

We choose three industries in our analysis: chemistry, home appliance and clothes. They

4We define the domestic output as one product (j=1) and the price of the domestic output is normalized to 1. So for those firms who only sell domestically, we consider these firms sell a single product with unit price.
are very large industries, and actively involved in export and import. We define a product as a HS5 code. Table 1 reports the summary statistics in our data. On average a firm exports 5 products and imports 20 products.

2.2 Motivation Facts

2.2.1 Fact 1: Export Varieties increases, but the revenue shares are more dispersed

Figure 2 plots the average counts of export varieties per firm. We define one export variety as an HS5 product. From 2000 to 2007, the average export varieties of a firm increases from 5 to 6.5. The increase of export varieties may be an indicator of firm productivity increase, as pointed out by Goldberg et al. (2009). In this paper, we will try to see whether the firm productivity increases in China and how does it contribute to the aggregate productivity change.

Then we try to compare the distribution of export revenues from each variety: we compute the Herfindahl index of revenue shares of each export variety within a firm-year observation. The Herfindahl index is computed as follows: fixing a firm-year observation, we compute the export revenue share for each HS5 product. The Herfindahl index is defined as

\[ H = \frac{\sum_{i=1}^{N} s_i^2 - \frac{1}{N}}{1 - \frac{1}{N}}, \]

where \( s_i \) is the revenue share of product \( i \) and \( N \) is the number of export varieties. Figure 3 plots the average Herfindahl index of revenue shares. As we can see, the Herfindahl index increases from 0.42 to 0.44 in our data sample. Why the allocations of revenue shares become more dispersed? What is the productivity implication of the change of revenue share dispersion? We will try to explore these questions through the lens of our model.

2.2.2 Fact 2: Import Varieties decreases

Figure 4 plots the average count of import varieties per firm. Similarly, we define one import variety as an HS5 product. Our data shows that the firms’ import varieties decline from 25 in year 2000 to 18 in year 2006 and then jumps to 19 in year 2007. Overall, a firm imports fewer varieties. Goldberg et al. (2009) documents that the increase of the import varieties increases the export varieties in India and along with the increase in output variety, the export share has also increased. The case of China is different. How will it change the aggregate productivity?

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5We restrict the number of export varieties to be greater than 5 to exclude the measurement errors of a few varieties within a firm.

6The total number of import varieties increases, although the number of import varieties per firm decreases
3 The Model

Motivated by the above facts, in this section we build a model of industry equilibrium in which firms use both domestic and imported intermediate goods to produce multiple products.

3.1 Model Setup

3.1.1 Production Technology

Firms are indexed by $i$, time is indexed by $t$. Firms produce multiple products and each product is indexed by $j = 1, 2, ..., J_{it}$, where $J_{it}$ is the number of products to produce in $t$. The firm uses capital $K_{ijt}$, labor $L_{ijt}$ and intermediate inputs to produce it. The intermediate inputs are indexed by $n = 1, 2, ..., \bar{N}$, where $\bar{N}$ is the total counts of intermediate input varieties.

For each product $j$, firm $i$ uses

$$Q_{ijt} = \Omega_{ijt} \left(K_{ijt}\right)^{\alpha_j} \left(L_{ijt}\right)^{\beta_j} \prod_{n=1}^{\bar{N}} \left(X_{nijt}\right)^{\gamma_{jn}}$$

where $\Omega_{ijt}$ denotes the firm-product specific productivity, $K_{ijt}$, $L_{ijt}$ and $X_{nijt}$ denotes the capital, labor and intermediate inputs $n$ that firm $i$ allocates to good $j$. We assume that the Cobb-Douglas weight $\alpha_j$, $\beta_j$ and $\gamma_{jn}$ depends on different product $j$. We denote $\gamma_j = \sum_{n=1}^{\bar{N}} \gamma_{jn}$ as the elasticity of the intermediate inputs when producing $j$. The capital and labor the firm uses is $K_{it} = \sum_{j=1}^{J_{it}} K_{ijt}$ and $L_{it} = \sum_{j=1}^{J_{it}} L_{ijt}$.

Each intermediate good $X_{nijt}$ is assembled from a combination of a foreign and a domestic variety

$$X_{nijt} = \left[(B^n_{nijt} X_{nijt}^{n,F} \right)^{\frac{\theta-1}{\theta}} + \left(X_{nijt}^{n,H}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

where $X_{nijt}^{n,F}$ and $X_{nijt}^{n,H}$ are foreign and domestic variety of intermediate good $n$. $B_{nijt}$ is the relative quality of foreign variety $n$. $\theta$ is the elasticity of substitution between foreign variety and domestic variety. We assume different products share the same $\theta$.

Assume that the prices of foreign and domestic varieties are $P_t^{n,F}$ and $P_t^{n,H}$ and denote $A^n_t = \frac{B^n_t P_t^{n,H}}{P_t^{n,F}}$ is the relative price of the domestic input adjusted by the quality of goods. If the intermediate inputs are solely come from domestic variety (such as non-tradable goods), we let $A^n_t = 0$. Following Halpern et al. (2015), we assume that $A^n_t$ of all traded intermediate inputs is the same across all traded inputs $n$, $A^n_t = A_t$. 

6
Firms pay costs to access foreign intermediate inputs. Let $N_{it}$ denote the number of imported foreign varieties of firm $i$ in year $t$. We assume the cost of getting access to foreign inputs is $F(N_{it})$ is increasing and convex in $N_{it}$: firms need to pay more costs to get access to one more foreign variety.

We denote the log firm-product productivity $\Omega_{ijt}$ as $\omega_{ijt}$ and we assume $\omega_{ijt}$ can be decomposed into two parts

$$\omega_{ijt} = \omega_{it} + \varepsilon_{ijt}$$

(3)

where $\omega_{it}$ is the firm productivity which is same across all products within the same firm, $\varepsilon_{ijt}$ is the firm-product productivity component.

### 3.1.2 Demand Curve

We assume that for product $j$, the demand curve facing by firm $i$ is

$$\ln p_{ijt} = -\sigma_{jt} \ln Q_{ijt} + D_{jt} + u_{ijt}$$

(4)

where $D_{jt}$ is the demand shifter of product $j$, $p_{ijt}$ is the price of product $j$ firm $i$. $\sigma_{jt}$ is the inverse demand elasticity. We would like to highlight that we allow a time varying demand elasticity. The demand elasticity may change due to more foreign competitors after the import tariff reduction in China or it may change because the Chinese firms can sell more products to foreign consumers.

### 3.1.3 Firm Investment Decision

The firm can invest on innovation to expand their product range. For simplicity, we assume that the innovation process does not have any uncertainty: by investing $\chi(I_{it}^J)$, the firm can expand the goods from $J_{it}$ to $J_{it} + I_{it}^N$ next period.\footnote{Klette and Kortum (2004) assume that the innovation arrives at a Poisson process and firms choose the arrival rate of the Poisson process. But by law of large number, when firms have lots of product lines, the number of new products is certain.} The product $j$ may die and out of the firm’s production set next period. We assume the realization of the product death shock is $\delta^J$. The firm can also invest $I_{it}^K$ to expand the the physical capital. The depreciation rate of the physical capital is denoted as $\delta^K$.

The timing of the model is as follows: we assume firm level capital $K_{it}$ and the number of products $J_{it}$ are predetermined in $t$. At the beginning of $t$, the firm level productivity $\omega_{it}$ and each product $j$’s demand $u_{ijt}$ are realized. Then the firm chooses number of intermediate inputs $N_{it}$ by paying the cost $F(N_{it})$. Then $\varepsilon_{ijt}$ are realized and the firm chooses the prices.
and resources used on each product \( j \). Firms then choose the investment on physical investment \( I_{it}^K \) and innovation investment \( I_{it}^I \).

In the following analysis, we neglect the footnote \( i \) and \( t \) if it does not cause any confusion. The firm problem is as follows

\[
V(K, J, \omega, \{u\}_{j=1}^J) = \max_{I^K, I^J, N, \{\varepsilon\}_{j=1}^J} \left\{ E_\varepsilon[\pi(K, J, N, \omega, \{u\}_{j=1}^J, \{\varepsilon\}_{j=1}^J)] - I^K - \chi(I^J) - F(N) + \beta E_{\omega', u'} \left[ V(K', J', t) \right] \right\}
\]

s.t. \( J' = (1 - \delta^J) J + I^J \)

\( K' = (1 - \delta^K) K + I^K \)  

where \( \pi(K, J, \omega, \{u\}_{j=1}^J, \{\varepsilon\}_{j=1}^J) \) is the per period profit of the firm, which will be defined later. The firm chooses the physical investment \( I^K \), production innovation \( I^J \) and the number of foreign intermediate inputs \( N \). Next period, \( \delta^J \) fraction of products will get out of the production set of the firm. The number of products next period is determined by equation (6). Capital will depreciate at rate \( \delta^K \) and next period the capital will follow equation (7).

The static profit after the realization of product productivity \( \{\varepsilon\}_{j=1}^J \) is defined as follows:

\[
\pi(K, J, N, \omega, \{u\}_{j=1}^J, \{\varepsilon\}_{j=1}^J) = \max_{L_j, X_j^N, K_j, p_j} \left\{ \sum_{j=1}^J p_j Q_j - w \sum_{j=1}^J L_j - \sum_{n=1}^N \sum_{j=1}^J P^n X^n_j \right\}
\]

s.t. \( \sum_{j=1}^J K_j = K \)

Equations (1), (2) and (4)

where \( P^n \) is the price of intermediate input \( n \). We arrange the order of \( n \) so that parts of the intermediate goods \( n = 1, 2, ..., N \) will be imported from foreign countries and intermediate goods \( n = N + 1, ..., \hat{N} \) will only be purchased from domestic market. The price of the intermediate \( n \) is

\[
P^n = \left\{ \begin{array}{ll}
P^{n, H} (1 - \theta)^{1 - \theta} + (P^{n, F}/B^n)^{1 - \theta} & \text{if } n \leq N \\
P_n^H [1 + A^n]^{-\theta} & \text{if } N + 1 \leq n \leq \hat{N} \end{array} \right.
\]

In the profit equation (8), the first term is the sum of revenues from all products; the second and the third term are the labor cost and the intermediate inputs cost. The firm chooses the labor \( L_j \) and the input \( X_j^n \). The restrictions include the production function (1) and the demand equation (4).
3.2 Model Solution

3.2.1 Resource Allocation within the Firm

Let $R_{ijt}$ denote the revenue of product $j$ of firm $i$ at period $t$. And define $\rho^K_{ijt}$, $\rho^L_{ijt}$, $\rho^n_{ijt}$ as the shares of capital, labor and intermediate input $n$ that are allocated to produce product $j$ respectively. The optimality conditions of equation (8) yield

$$\rho^K_{ijt} = \frac{K_{ijt}}{K_{it}} = \frac{\alpha_j (1 - \sigma_{jt}) R_{ijt}}{\sum_{j=1}^{J} \alpha_{j'} (1 - \sigma_{j't}) R_{j't}} \tag{10}$$

$$\rho^L_{ijt} = \frac{L_{ijt}}{L_{it}} = \frac{\beta_j (1 - \sigma_{jt}) R_{ijt}}{\sum_{j=1}^{J} \beta_{j'} (1 - \sigma_{j't}) R_{j't}} \tag{11}$$

$$\rho^n_{ijt} = \frac{P_n X^n_{ijt}}{P_n X^n_{it}} = \frac{\gamma_j^n (1 - \sigma_{jt}) R_{ijt}}{\sum_{j=1}^{J} \gamma_{j'}^n (1 - \sigma_{j't}) R_{j't}} \tag{12}$$

Define the intermediate expenditures of product $j$ as $M_{ijt} = \sum_{n=1}^{N} P^n_t X^n_{ijt}$ and the total intermediate input expenditure is $M_{it} = \sum_{j=1}^{J} \sum_{n=1}^{N} P^n_t X^n_{ijt}$. We can get the share of intermediate expenditures of product $j$ from the FOC of equation (8) as

$$\rho_M^{ijt} = \frac{M_{ijt}}{M_{it}} = \frac{\gamma_j^n (1 - \sigma_{jt}) R_{ijt}}{\sum_{j=1}^{J} \gamma_{j'}^n (1 - \sigma_{j't}) R_{j't}} \tag{13}$$

3.2.2 Import Inputs Choice

Then we solves how the firm chooses the inputs to use. The expenditure share on the foreign good in the the spending for variety $n$ in product $j$ is

$$S^n_{ijt} = \frac{P_t^{n,F} X^n_{ijt}^{n,F}}{P^n_t X^n_{ijt}} = \frac{A_t^{\theta - 1}}{1 + A_t^{\theta - 1}}$$

where the second equality follows the CES aggregator (2). Notice that $S^n_{ijt}$ is same across all goods $n$, products $j$ and firm $i$. We denote $S^n_{ijt} = S_t$.

Consider a firm $i$ the expenditure share of imported inputs $n$ is

$$\frac{M^n_{it,F}}{M_{it}} = \sum_{j=1}^{J} \frac{M^n_{ijt,F}}{M_{it}} = \sum_{j=1}^{J} S_t \frac{M_{ijt}}{M_{it}} \frac{\gamma^n_j}{\gamma_j} = \sum_{j=1}^{J} S_t \rho^n_{ijt} \frac{\gamma^n_j}{\gamma_j} \tag{14}$$

The second equality uses the facts that expenditure share of foreign good $n$ in the $M^n_{ijt}$ is
$S_t$ and $\frac{M_{it}^n}{M_{ijt}} = \frac{\gamma^n_{i}}{\gamma_j}$ which follows the Cobb-Douglas production function (1).

Thus, we can define the share of imported inputs as

$$M_{it}^F = \sum_{n=1}^{N_{it}} \frac{M_{it}^n}{M_{it}} = S_t \sum_{j=1}^{J_{it}} \rho_{ijt}^M G_j (N_{it})$$

where $G_j (N_{it}) = \sum_{j=1}^{N_{it}} \frac{\gamma^n_{i}}{\gamma_j}$. Following Halpern et al. (2016), we use a smooth function to approximate $G_j (N_{it})$ when we estimate the model.

### 3.2.3 Production Function

Let $\Delta_{jt} = \sum_{n=1}^{\bar{N}} \frac{\gamma^n_{i}}{\gamma_j} \log P_{n,t}^H - \sum_{n=1}^{\bar{N}} \frac{\gamma^n_{i}}{\gamma_j} \log \frac{\gamma^n_{i}}{\gamma_j}$. It captures the price index of material of product $j$. We show in the appendix that the production function (1) can be rewritten as

$$q_{ijt} = \alpha_j k_{it} + \beta_j l_{it} + \gamma_j (m_{it} - \Delta_{jt}) + \gamma_j a_t G_j (N_{it}) +$$

$$\alpha_j \log \rho_{ijt}^K + \beta_j \log \rho_{ijt}^L + \gamma_j \log \rho_{ijt}^M + \omega_{it} + \varepsilon_{ijt}$$

where $q$, $k$ and $l$ denote the logs of corresponding variables. $m$ is the log values of the intermediate expenditures. $a_t = \frac{1}{\theta - 1} \log (1 + A_{t}^{\theta-1})$ is a time-shifter measuring the relative technology change of foreign inputs. The first line of the equation (16) contains variables at the firm level: firm capital, firm labor, firm level number of imported goods. Fixing the expenditure of intermediate inputs, when we increase the number of imported varieties, the output quantity will increase. And from equation (16), we can see this effect is captured by $G_j (N_{it})$. This part is very similar as the Halpern et al. (2016), with two important differences. First, all elasticities and function $G$ depend on product $j$. This difference allows us to quantify the firm productivity change at the product level. Second, we allow the a time varying relative quality between foreign inputs and domestic inputs. If the domestic inputs quality improves, $a_t$ will decline. Then the gain of getting access to foreign inputs will shrink.

The second line of equation (16) captures the unobserved variables: allocations of the resources within the firm, and productivity. Comparing with De Locker et al. (2016), our model builds the link between multiple inputs and multiple output products, while they neglect this channel.
3.3 Firm’s Other Decisions

The firm choose the number of imported varities by equating the marginal revenue and marginal cost

\[
\frac{E_v[\pi(N_{it})]}{\partial N_{it}} = F'(N_{it})
\]

(17)

We are going to use this condition to recover the cost function \( F(N) \) later.

The capital investment and innovation investment are similar:

\[
\beta \frac{\partial E_{\omega, u'}}{\partial K'} [V(K')] = 1
\]

\[
\beta \frac{\partial E_{\omega, u'} [V(J')]}{\partial J'} = \chi'(I^J)
\]

4 Estimation

In this section, we introduces our estimation strategies. First, we estimate the demand elasticity \( \sigma_{jt} \) from equation (4). Second, we estimate the \( G_j(N_{it}) \) from equation (15); Finally, we estimate the production function (??).

4.1 Estimating the demand elasticity

We first estimate the demand curve (4). For simplicity, we assume that \( \sigma_{jt} \) follows

\[
\sigma_{jt} = \begin{cases} 
\sigma_j & \text{if } t \leq 2003 \\
\sigma'_j & \text{if } t > 2003
\end{cases}
\]

where the year 2003 is the time when China joined the WTO. To estimate the demand curve, the classical endogeneity problem will arise since the price change may reflect marginal cost difference as well as preference change. Following Wei et al. (2017), we use the average relative price of the firm in other markets, input material’s price deflator, log capital, log labor, log material, import variety counts and export variety counts as the instrument variables. The idea is those variables are related to the marginal cost change rather than the consumers’ preference change. Table 2 shows the estimation of the inverse of demand elasticity \( \sigma_{jt} \) of all product-year pairs. The two columns in the table 2 report the summary statistics of the point estimation and standard error estimation of \( \sigma_{jt} \). The average demand elasticity inverse is around 0.50, which suggests the demand elasticity is about 2. Most products demand elasticity
4.2 Estimating the marginal benefit of increases in input variety

In the second step, we estimate equation (15). We assume a parametric functional form

$$G_j(N) = \begin{cases} 
\bar{G}_j \left(1 - \left(\frac{N}{\bar{N}_i}\right)^{\lambda_j} \right)^{\frac{1}{\lambda_j}} & \text{if } N \leq \bar{N}_I \\
\bar{G}_j & \text{if } N > \bar{N}_I 
\end{cases}$$

Here $\lambda_j \in (0, 1)$ and $\bar{G}_j \in (0, 1)$. This functional form implies that when number of varieties increases, the marginal benefit will decline. And there is a cutoff value, if the import varieties exceed $\bar{N}_I$, the marginal benefit of expanding varieites declines to 0. $\bar{N}_I$ is the total number of traded varieties in the market. If a firm’s number of imported varieties equals to $\bar{N}_I$, $\bar{G}_j$ equals to the share of total imported share in the intermediate inputs.

There are four groups of unknowns to estimate $S_t$, $\bar{G}_j$, $\lambda_j$ and $\gamma_j$. However, $\gamma_j$ and $\bar{G}_j$ cannot be separately identified because they enter equation (15) in the same way. We normalize $\bar{G}_j$ to be 0.8 to match the aggregate total imported share in the intermediate inputs from China’s input-output table. We then estimate the nonlinear equation to get $\lambda_j$. The estimation results $\gamma_j$ will be ignored in this step. We will estimate $\gamma_j$ in the next step.

Table 3 reports the estimation of $S_t$ and $\lambda_j$. The first two columns reports the estimation of $S_t$ for each year. There is a declining trend: $S_t$ drops from 0.267 in year 2000 to 0.213 in year 2007. Since the import share is declining on average, it must suggest that there is a growing technology improvement of domestic material goods, which drives down the relative price of domestic goods adjusted by the quality. The last two columns in table 3 reports the estimation of $\lambda_j$ for each product. The average $\lambda_j$ is 0.624 and the standard deviation is small, close to 0.028.

4.3 Estimating the production function

In the third step, we estimate the equation (16) following the methodology of Olley and Pakes (1994). Define $\xi_{it} = \omega_{it} - E[\omega_{ijt}|\omega_{i,t-1}]$. The estimation equation becomes

$$q_{ijt} = E[\omega_{ijt}|\omega_{i,t-1}] + \xi_{it} + \epsilon_{ijt} + \alpha_j k_{it} + \beta_j l_{it} + \gamma_j (m_{it} - \Delta_{jt}) + \gamma_j a_t G_j(N_{it}) + h(r_{ijt}, r_{i-jt})$$

8Since we have a great number of $\sigma_{jt}$ to estimate, it is possible that the estimated $\sigma_{jt}$ is negative or above 1. In this case, we truncate the estimation to be at the 1 percentile level.
where $h(r_{ijt}, r_{i-jt})$ captures is a function of the polynomial approximations of $\rho^K_{ijt}$, $\rho^L_{ijt}$ and $\rho^M_{ijt}$.

The estimation condition is

$$E[\xi_{it} + \epsilon_{ijt}] = 0.$$  

The key difference from the single product case is, as in 7, that we need instruments for $r_{ijt}/r_{it}$. Following 7, we also use lagged values of $r_{i-j,t}$ and inputs lagged even further back. The set of conditioning variables are

$$x_{ijt} = (r_{i-j,t-1}, k_{it}, k_{it-1}, m_{i,t-1}, m_{i,t-2}, l_{i,t-1}).$$

The conditional moment restriction we utilize for estimation are given by

$$g(x_{ijt}; \theta) = E[\xi_{it} + \epsilon_{ijt} | x_{ijt}; \theta]$$

$$= E[h(i_{k,t-1}, k_{it-1}, J_{i,t-1}) + \alpha_j k_{it} + \beta_j l_{it} + \gamma_j (m_{it} - \rho_{jt}) + \gamma_j a_t G_j (N_{it}) + h(r_{ijt}, r_{i-jt}) | x_{ijt}; \theta]$$

$$= 0$$

Table 4 reports the estimation of $\alpha_j, \beta_j$ and $\gamma_j$. The first two columns report the point estimation of $\alpha_j$. On average the capital elasticity is not large, around 0.083, and $\alpha'_j$'s 99% percentile is only around 0.148. The next two columns report the point estimation of $\beta_j$. On average the labor elasticity is 0.186. The last two columns report the estimation of $\gamma_j$. The mean of the material’s elasticity is 0.278. Overall, the production function demands show strong decreasing returns to scale (0.083+0.148+0.278=0.51).

### 4.4 Recovering other parameters

From the estimation of equations (15), we get $S_t = A_t^{\theta-1}/(1+A_t^{\theta-1})$. In the equation (16), we replace $a_t$ by $a_t = \frac{1}{\theta-1} \log(1 + A_t^{\theta-1}) = \frac{1}{\theta-1} \log(1 + S_t/1-S_t$). Then from the parameters in the equation (16), we back up the substitution parameters $\theta$.

The $A_t$ is identified from the following equation

$$\log A_t = \frac{1}{\theta-1} \log \left( \frac{S_t}{1-S_t} \right)$$

(18)

Our estimation of $\theta$ is around 1.6 and figure 7 plots the relative price of the domestic input from 2000-2007. $\log A_t$ shows a significant declining trend with (declines 3% on average), which suggests that the domestic input becomes cheaper or the quality improves.
The cost of getting more foreign varities \( F(N) \) is obtained by using the FOC (17) and we impose the condition that \( F(0) = 0 \). Figure xxx draws \( F(N) \).

5 Results

5.1 Aggregate Productivity Decomposition

The revenue productivity of each firm is defined as

\[
TFPR_{it} = \sum_{j=1}^{J_{it}} p_{ijt} \exp(\gamma_j a_t G_j (N_{it})) \exp(\alpha_j \log \rho_i^K + \beta_j \log \rho_i^L + \gamma_j \log \rho_i^M) \Omega_{ijt}
\] (19)

There are three parts in the revenue productivity: (1) the access to the foreign inputs \( \exp(\gamma_j a_t G_j (N_{it})) \); (2) the resource allocation within the firm \( \exp(\alpha_j \log \rho_i^K + \beta_j \log \rho_i^L + \gamma_j \log \rho_i^M) \); and the firm-product real productivity \( \Omega_{ijt} \). Figure XXX plots the estimation of the log revenue productivity. Table XXX reports the summary statistics of the estimated log TFPR. The average of the log TFPR is 0.321 and the standard deviation is 1.173.

We define the aggregate revenue productivity as the weighted average of each firm’s TFPR

\[
\overline{TFPR}_t = \sum_i \frac{R_i}{R_t} TFPR_{it}
\]

where \( \frac{R_i}{R_t} \) is the revenue share of firm \( i \).

We are interested in the change of \( \overline{TFPR} \). The first row of table XXX reports the percentage change of aggregate \( \overline{TFPR} \) between year 2002 and 2007. The aggregate TFP increases by 30.3%. We then decompose \( \overline{TFPR} \) as follows:

(1) We fix the firms who survive through the year 2002 to 2007 and call those firms as set \( I \). The aggregate productivity change of firms in set \( I \) is defined as

\[
\ln TFPR_{i,2007} - \ln TFPR_{i,2002} = \ln \sum_{i \in I} \frac{R_{i,2007}}{R_{2007}} TFPR_{i,2007} - \ln \sum_{i \in I} \frac{R_{i,2002}}{R_{2002}} TFPR_{i,2002}
\] (20)

(2) For those firms in set \( I \), we first fix the resource allocation within the firm \( \exp(\alpha_j \log \rho_i^K + \beta_j \log \rho_i^L + \gamma_j \log \rho_i^M) \) and the firm-product real productivity \( \Omega_{ijt} \), using the value in year 2002. At the same time, we also fix the product set \( J_{it} \) as well. We only allow the channel of accessing to foreign inputs. The counterfactual productivity of each firm by allowing only the
import channel is defined as

\[ TFPR_{i,2007}^{IMP} = \sum_{j=1}^{J_{i,2002}} p_{ij,2007} \exp(\gamma_j a_{2007} G_j (N_{i,2007})) \exp(\alpha_j \log \rho_{ij,2002}^K + \beta_j \log \rho_{ij,2002}^L + \gamma_j \log \rho_{ij,2002}^M) \Omega_{ij,2002} \]

We can get the aggregate productivity change from equation (20), replacing \( TFPR_{i,2007} \) with \( TFPR_{i,2007}^{IMP} \). The aggregate productivity change from this step is contributed to access to foreign inputs.

(3) We then allow the \( \Omega_{ij} \) to change and define the productivity as

\[ TFPR_{i,2007}^{\Omega} = \sum_{j=1}^{J_{i,2002}} p_{ij,2007} \exp(\gamma_j a_{2007} G_j (N_{i,2007})) \exp(\alpha_j \log \rho_{ij,2002}^K + \beta_j \log \rho_{ij,2002}^L + \gamma_j \log \rho_{ij,2002}^M) \Omega_{ij,2007} \]

The aggregate productivity change from this step is contributed to the technology upgrade.

(4) Next, we allow the resource allocation within the firm to change, but we still fix the product set \( J_i \). We contribute the productivity change to the intensive margin change of allocating resources to existing products.

\[ TFPR_{i,2007}^{Intensive} = \sum_{j=1}^{J_{i,2002}} p_{ij,2007} \exp(\gamma_j a_{2007} G_j (N_{i,2007})) \exp(\alpha_j \log \rho_{ij,2007}^K + \beta_j \log \rho_{ij,2007}^L + \gamma_j \log \rho_{ij,2007}^M) \Omega_{ij,2007} \]

(5) We now allow the entry and exit of products. The productivity change is called the extensive margin change of allocating resources to existing products.

\[ TFPR_{i,2007}^{Extensive} = \sum_{j=1}^{J_{i,2007}} p_{ij,2007} \exp(\gamma_j a_{2007} G_j (N_{i,2007})) \exp(\alpha_j \log \rho_{ij,2007}^K + \beta_j \log \rho_{ij,2007}^L + \gamma_j \log \rho_{ij,2007}^M) \Omega_{ij,2007} \]

(6) Finally, we allow the entry and exit of firms. That is we do not restrict firms in the set \( I \).

Table XXX reports the decomposition results. The first column reports the decomposition results of the aggregate TFPR. The aggregate TFPR increases by 30.3% from year 2002 to 2007. The second to the fifth row reports the decomposition of step 1 to step 5. We can see that the access to foreign inputs contributes very little to the aggregate productivity change. The most significant contribution at the firm level comes from resources allocation within the firm, 11.9%. The entry and exit of products and reallocation of resources among existing products contribute 4.3% and 7.6% respectively. The upgrade of the technology is also importance. It
increases aggregate productivity by 5.5%. In total, the productivity increase at the firm level can change the aggregate TFPR by 17.4%. The last row in the table is the productivity change if we allow firms to enter and exit. It significantly increases the aggregate TFPR by 12.8%.

**DISCUSSION**

5.1.1 Firm Characteristics and Productivity Change

Which type of firm can increase its productivity the most, state owned firm or private firm? What is the source of its productivity growth, foreign inputs, reallocation within the firm or technology upgrade? To answer these questions, we regress the counterfactual firm level productivity from step 1 to step 5 to the firm’s characteristics. Table XXX reports the results. There are 8 columns in the table. The first 4 columns regress change of $TFPR_i$, $TFPR_i^{IMP}$, $TFPR_i^\Omega$ and $TFPR_i^{Intensive} + TFPR_i^{Extensive}$ on ownership dummies as well as location and industry dummies. The first column suggests that xxx firms can improve its TFPR more than xxx firms. The access to the foreign market can improve xxx productivity more than ....As we can see, the private firms and foreign state owned firms both increase their revenue productivity a lot. While the state owned firms increase 4.3% more than private firms.

The last 4 columns regress the change of TFPQ and the four decomposition parts on ownership dummies as well. As we can see, the state owned firms do not change too much on the real productivity.

5.2 The Role of Domestic Input Relative Price

An interesting pattern in our estimation is that getting access to foreign inputs do not contribute too much to the overall aggregate productivity. The reason is that the domestic goods quality improves. In this section, we want to see if $A_t$ did not decline, what would happen to the aggregate productivity?

5.3 The Role of Elasticity Change

6 Conclusion

This paper studies how the trade liberalization in China changes the firm productivity. We develop a framework to estimate revenue productivity (TFPR) and real productivity (TFPQ) with multi-product firms. We find that the aggregate TFPR increases 30% from 2002-2007 and TFPQ increases 22%, suggesting that the observed TFPR increase is mainly driven by real productivity change rather than the markup change. We further decompose the change
of productivity into three channels: (1) access to foreign inputs; (2) technology upgrade; (3) resource re-allocation within the firm. We find the most significant channel is the last one, which explains half of the aggregate productivity increase. We also find that both SOEs and private firms significantly improve the TFPR. However, private firms TFPR increase mainly come from the increase of TFPR, while only 65% of the TFPR increase of SOEs can be attributed to change of TFPQ.

References


Figure 1: The Average of Firm TFP

NOTE: This figure plots the average firm level TFP. The firm productivity is estimated through the Olley-Pakes (1996). The TFP in year 2000 is normalized to 1.

Figure 2: Counts of Export Varieties per Firm

NOTE: This figure plots the average counts of export varieties per firm. One export variety is an HS5 product.
Figure 3: Average Herfindahl Index of Export Revenues Within the Firm

NOTE: This figure plots the average Herfindahl Index of export revenues within a firm. We restrict the number of export varieties greater than 5.

Figure 4: Counts of Import Varieties per Firm

NOTE: This figure plots the average counts of import varieties per firm. One import variety is an HS5 product.
Figure 5: Demand Elasticity Inverse Distribution

NOTE: This figure plots the distribution of demand elasticity inverse estimation.

Figure 6: log TFPR Distribution

NOTE: This figure plots the distribution of log TFPR.
Figure 7: log At

NOTE: This figure plots the trend of log At.

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>No. obs</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>373,356</td>
<td>173084.2</td>
<td>1168077.0</td>
</tr>
<tr>
<td>Revenue</td>
<td>373,364</td>
<td>143345.5</td>
<td>490274.6</td>
</tr>
<tr>
<td>Employee</td>
<td>373,374</td>
<td>486.3</td>
<td>1718.9</td>
</tr>
<tr>
<td>Intermediate Inputs</td>
<td>373,364</td>
<td>115742.6</td>
<td>434432.7</td>
</tr>
<tr>
<td>Export Counts</td>
<td>286,987</td>
<td>5.7</td>
<td>10.1</td>
</tr>
<tr>
<td>Import Counts</td>
<td>195,411</td>
<td>20.7</td>
<td>32.1</td>
</tr>
</tbody>
</table>

NOTE: This table reports the summary statistics of variables. The unit is 1000 RMB.

Table 2: Demand Elasticity Inverse

<table>
<thead>
<tr>
<th></th>
<th>Point est</th>
<th>Sd error est</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.498</td>
<td>0.183</td>
</tr>
<tr>
<td>std</td>
<td>0.253</td>
<td>0.141</td>
</tr>
<tr>
<td>1%</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>25%</td>
<td>0.317</td>
<td>0.091</td>
</tr>
<tr>
<td>50%</td>
<td>0.635</td>
<td>0.186</td>
</tr>
<tr>
<td>75%</td>
<td>0.656</td>
<td>0.192</td>
</tr>
<tr>
<td>99%</td>
<td>0.807</td>
<td>0.302</td>
</tr>
</tbody>
</table>

NOTE: This table reports point estimation and standard error estimation of demand elasticity inverse of each product-year pair.
### Table 3: The Marginal Benefit of Increasing Input Variety

<table>
<thead>
<tr>
<th>Year</th>
<th>Point est</th>
<th>Sd error est</th>
<th>$S_t$</th>
<th>Point est</th>
<th>Sd error est</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.267</td>
<td>0.019</td>
<td>mean</td>
<td>0.624</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>0.240</td>
<td>0.014</td>
<td>std</td>
<td>0.028</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>0.254</td>
<td>0.018</td>
<td>1%</td>
<td>0.534</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>0.250</td>
<td>0.020</td>
<td>25%</td>
<td>0.612</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>0.269</td>
<td>0.016</td>
<td>50%</td>
<td>0.623</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>0.241</td>
<td>0.014</td>
<td>75%</td>
<td>0.635</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>0.243</td>
<td>0.013</td>
<td>99%</td>
<td>0.707</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>0.213</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** This table reports point estimation and standard error estimation of $S_t$ and $\lambda$.

### Table 4: Production Function Estimation

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Point est</th>
<th>Sd error est</th>
<th></th>
<th>$\beta$</th>
<th>Point est</th>
<th>Sd error est</th>
<th></th>
<th>$\gamma$</th>
<th>Point est</th>
<th>Sd error est</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.100</td>
<td>0.117</td>
<td>0.124</td>
<td>0.115</td>
<td>0.577</td>
<td>0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>0.076</td>
<td>0.057</td>
<td>0.020</td>
<td>0.008</td>
<td>0.009</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>0.001</td>
<td>0.152</td>
<td>0.113</td>
<td>0.119</td>
<td>0.571</td>
<td>0.022</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.046</td>
<td>0.142</td>
<td>0.122</td>
<td>0.116</td>
<td>0.577</td>
<td>0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>0.088</td>
<td>0.129</td>
<td>0.132</td>
<td>0.112</td>
<td>0.581</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>0.135</td>
<td>0.109</td>
<td>0.179</td>
<td>0.094</td>
<td>0.599</td>
<td>0.034</td>
<td></td>
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</tr>
<tr>
<td>99%</td>
<td>0.354</td>
<td>0.211</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** This table reports point estimation and standard error estimation of capital, labor and material elasticity.

### Table 5: Estimated log(TFPR)

<table>
<thead>
<tr>
<th>Year</th>
<th>mean</th>
<th>std</th>
<th>1%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>0.940</td>
<td>0.913</td>
<td>-0.859</td>
<td>0.333</td>
<td>0.807</td>
<td>1.417</td>
<td>3.799</td>
</tr>
<tr>
<td>2007</td>
<td>1.047</td>
<td>0.923</td>
<td>-0.566</td>
<td>0.417</td>
<td>0.918</td>
<td>1.521</td>
<td>3.827</td>
</tr>
</tbody>
</table>

**NOTE:** This table reports the summary statistics of the estimated log(TFPR) in year 2002 and 2007.
Table 6: Decomposition Results

<table>
<thead>
<tr>
<th></th>
<th>TFPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate productivity change</td>
<td>0.359</td>
</tr>
<tr>
<td>Access to foreign input</td>
<td>0.000</td>
</tr>
<tr>
<td>Technology upgrade</td>
<td>0.142</td>
</tr>
<tr>
<td>Allocation within firm: intensive margin</td>
<td>0.053</td>
</tr>
<tr>
<td>Allocation within firm: extensive margin</td>
<td>0.073</td>
</tr>
<tr>
<td>Firm prod change</td>
<td>0.267</td>
</tr>
<tr>
<td>Firm entry-exit</td>
<td>0.092</td>
</tr>
</tbody>
</table>

NOTE: This table reports the decomposition of aggregate TFPR change. Firm prod change is the summation of the 2nd row to the 4th row.

Table 7: Ownership and Firm Productivity

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.236***</td>
<td>0.103***</td>
<td>0.076***</td>
<td>0.852***</td>
<td>0.375***</td>
<td>0.211***</td>
<td>0.180***</td>
<td>0.045***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>SOE</td>
<td>0.661***</td>
<td>0.654***</td>
<td>0.640***</td>
<td>-1.196***</td>
<td>-0.827***</td>
<td>-0.744***</td>
<td>0.786***</td>
<td>0.761***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.004)</td>
<td>(0.005)</td>
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<tr>
<td>Location</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Industry</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Obs</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
<td>373,356</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.016</td>
<td>0.092</td>
<td>0.165</td>
<td>0.138</td>
<td>0.259</td>
<td>0.416</td>
<td>0.020</td>
<td>0.104</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: The table reports the ownership and the firm productivity change.
Appendix

In this section, we prove equation (16). For simplicity, we ignore footnote $t$ in this section. Given the Cobb-Douglas structure, intermediate expenditure

$$M_{ij} = \Gamma_j \prod_{n=1}^{\bar{N}} (P_n)^{\gamma_j^n / \gamma_j} \prod_{n=1}^{\bar{N}} (X_{ij}^n)^{\gamma_j^n / \gamma_j}$$

where $\Gamma_j = \prod_{n=1}^{\bar{N}} \left( \frac{\gamma_j^n}{\gamma_j} \right)$. Substitute the equation (9), we can get

$$M_{ij} = \Gamma_j \prod_{n=1}^{\bar{N}} (P_n^{n,H})^{\gamma_j^n / \gamma_j} \prod_{n=1}^{\bar{N}} \exp \left( a\frac{\gamma_j^n}{\gamma_j} \right) \prod_{n=1}^{\bar{N}} (X_{ij}^n)^{\gamma_j^n / \gamma_j}$$

$$= \Gamma_j \exp \left[ -aG_j (N_i) \right] \prod_{n=1}^{\bar{N}} (P_n^{n,H})^{\gamma_j^n / \gamma_j} \prod_{n=1}^{\bar{N}} (X_{ij}^n)^{\gamma_j^n / \gamma_j}$$

where the second line uses the function $G_j$ to smooth $\gamma_j^n / \gamma_j$. So

$$\prod_{n=1}^{\bar{N}} (X_{ij}^n)^{\gamma_j^n} = M_{ij}^{\gamma_j} \exp[a\gamma_j G_j (N_i)] \exp (\Delta_j) = (\rho^{M}_{ij} M_i)^{\gamma_j} \exp[a\gamma_j G_j (N_i)] \exp (\Delta_j)$$

where $\exp(\Delta_j) = \Gamma_j^{\gamma_j} \prod_{n=1}^{\bar{N}} (P_n^{n,H})^{\gamma_j^n}$ is the price index of the material. Taking logs in the above equation, and combine the result with the production function (1), we can get equation (16).