1 Introduction

With more than two-decades development of online retail industry, online marketplaces have been playing an important role in boosting the continuous prosperity of e-commerce. Dramatic grows in product sales have be seen on many international online marketplaces, like Amazon Marketplace in the US, Taobao Mall in China, FlipKart in India, and Rakuten in Japan.

Marketplaces are intermediaries to link sellers and buyers. One of the important design of marketplace is what products to show and what products to display on prominent locations (i.e. to spotlight the product) when consumers search for potential purchase. The design of product spotlight is common and faced by other Internet giants such as search engines (e.g. Google, Baidu in China) for search queries, news aggregators (e.g. Google news) for news reports, and travel agent websites (e.g. Hotwire, priceline) for hotels or airline tickets.

\[\text{According to the data in 2014, Amazon Marketplace has 244 million users globally, and its retail sales increased by 27.2\% while that of the online retail industry in the US increased by 17\%. (Source: } \text{http://fortune.com/2014/07/01/10-largest-retailers-amazon/). Taobao Mall in China served 350 million users and its retail sales increased by 60\%, compared to 35\% increasing rate of the online retail industry in China (Source: } \text{http://expandedramblings.com/index.php/alibaba-statistics/ and http://www.businesswire.com/news/home/20150129005475/en/Alibaba-Group-Announces-December-Quarter-2014-Results}).\]
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etc. Hundred thousands sellers gather on the marketplace and provide various products with different features, qualities or popularity to consumers. Consumers seek and evaluate product information to find their desirable ones from all the alternatives. The marketplace needs to decide which products to spotlight as to facilitate consumers search and boost transactions. The decision of product spotlight affects consumer’s search process. It is well known that information presentation format affects consumers information acquisition patterns (Bettman and Kakkar 1977). Specifically, experimental evidence shows that in an online environment consumers’ attention and product evaluation process follow top-down and left-right manners. This renders significant prominence to the products that are displayed on the top of a search result compared to the ones that are listed downwards.

The conventional wisdom on shopping intermediaries presumes that platforms design a search environment to help consumers to find the desirable product with reduced search costs (Spulber 2006). More recent studies show that the marketplace may have its own concerns. It may divert search (Hagiu and Jullien 2011), or even obfuscate their offering to soften competition (Ellison and Ellison 2009). Even if marketplace acts to the best interest of consumer, it also has to decide whether the display order should emphasize product fitness or prices (Dinerstein et al. 2017). All studies indicate the product spotlight decision is one essential element in marketplace’s design, and will affect prices and volume of trade through consumers’ search and purchase behavior.

The revenue structure of marketplace adds extra complexity to the spotlight design. First, marketplaces charge sellers commission fees in the form of a fixed fraction of revenues. In its spotlight decision, the marketplace needs to facilitate consumer’s search and purchase while keeping the seller’s competition in check. Second, bidding revenue from sponsored ads is also an important source. Relating to the design of product spotlight, marketplaces also allow sellers to bid for spotlight position. As the number of products available on an online marketplace is often overwhelming to consumers, sellers also care the spotlight location and
would like to make their products more prominent in the shopping environment to attract consumers attention. To resolve the tension, a marketplace often allows sellers to compete for spotlight positions in the shopping environment and collects advertising fees. Thus, an online marketplace acts not only as a retail platform but also as an advertising publisher for search advertising. Marketplaces make profit from both commission fees and from bidding revenue on search advertising. They differ from the conventional advertising publishers: search engines (e.g., Google) and social media (e.g., Facebook), which collect revenue merely from search advertising market. Such difference may also affect their consideration of product spotlight decisions.

In this paper, we assess marketplace’s spotlight rule. We consider two different rules: spotlight decision chosen by marketplace (no bidding rule) and by seller’s bidding in a second price auction (bidding rule). Given the spotlight rule, the marketplace also sets commission fee rate. Sellers then compete on the marketplace. Under no bidding rule, sellers compete via prices. Under bidding rule, sellers compete by bidding in the auction first, and then compete via prices after knowing the spotlight decision. Then consumers search and make purchase decision given the product spotlight. Thus we are able to assess marketplace’s spotlight rule by considering both commission fee revenue and bidding revenue (with bidding rule) with an economic foundation of consumer’s search decision and sellers’ bidding and pricing decisions.

To be specific, in our model, we consider one marketplace with two sellers. Seller $H$ provides a product (product $H$) with a wider breadth of appeal to consumers than the one provided by seller $L$ (product $L$). In other words, product $H$ provides a better chance to fit consumers tastes than product $L$. For simplicity, we assume that the marketplace has one

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2The winning sellers can spotlight their products to consumers through sponsored ads, and they pay only when consumers click their ads. Figures B1 and B2 in Appendix B illustrate the sponsored ads on Amazon Marketplace. The sponsored ads have been very successful in booming sellers business [Source:https://advertising.amazon.com/amazon-display-advertisingandhttps://advertising.amazon.com/sponsored-products].

3For example, Taobao’s revenue from search advertising market was $6.3 billion in 2014, making it the second largest publisher in China (next to Baidu). Amazon earned over $1.03 billion in advertising revenue in 2015.
spotlight position in its shopping environment. We refer to the situation in which product \( H/L \) is spotlighted as high/low spotlight (denoted by \( HL/LH \) for short). In addition, we refer to the product in the spotlight location as spotlighted product and to that not in the spotlight location as underplayed product. A consumer is attracted to the spotlighted product and is assumed to learn its fit for free. She then decides whether to costly learn the fit of the underplayed product. We assume that she knows the products prices at no cost, because the price information are often vivid and easily observable in a marketplace's shopping environment. We conduct a subgame perfect equilibrium analysis in the marketplace's product spotlight decision by integrating sellers price competition and their bidding behaviors. The value of the spotlight position is captured by the consumers navigation and click behavior, which is formally modelled through a sequential search process.

The first result from our analysis relates to the marketplace’s spotlight decision. We find that marketplace chooses to spotlight product with low fitness under some conditions of search cost, a phenomenon referred to as “position paradox” in literature. If product \( L \) is underplayed, it can use low prices to induce the consumer to search its product, especially when search cost is low. This will intensify price competition and lower the marketplace's profit from the commission fee. To avoid this, spotlighting product \( L \) is optimal to the marketplace. But position paradox occurs only when search cost is small. When search cost is high, it will be difficult for seller \( L \) to attract consumer to search his product by lowering prices. Instead, it is better off for seller \( L \) to maintain relatively high prices to exploit the consumers whose tastes can be satisfied by product \( L \). Thus, under this condition, spotlighting product \( H \) is optimal to the marketplace.

The second result is about non-linearity of optimal commission fee rate and marketplace’s equilibrium profit with bidding rule. When search cost is small, product \( L \) is spotlighted. If search cost increases, his profit may increase or decrease, from both pricing competition and bidding competition. Though higher search cost softens pricing competi-
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tion, it may lead to more fierce bidding competition. Thus, the marketplace has to increase or decrease commission fee rate to guarantee product $L$ a certain level. So the commission fee rate is non-linear in search cost, and so it its profit. Such non-linear effect is only possible when we discuss both pricing competition and bidding competition. When we only consider pricing competition, when product $L$ is spotlighted, the commission fee rate is always increasing in search cost as higher search cost softens pricing competition.

In addition, we find that bidding revenue becomes more important when search becomes more difficult. While search cost affects both commission fee and bidding revenue, the latter is usually larger for majority of market conditions. The effect comes from shift of commission fee rate, as well as relative change of seller’s profit. The result implies that the marketplace should focus more on advertising auction market when search is difficult.

Our work relates to the literature on search and product display on Internet retailers or search engines. Earlier studies show that online retailers and search engines favor consumers, such as Spulber (2006) and Chen and He (2011). More recent research shows it’s not always the case. Hagiu and Jullien (2011) shows that search engines may divert search as to earn more click fees from ads. Ellison and Ellison (2009) shows that small retailers have incentive to obfuscate consumers to increase differentiation and avoid price competition on price comparison websites. Dinerstein et al. (2017) exploits eBay’s reform of product display to study how price-based and product fitness-based prominence affects consumer’ choice set and utility. Our work joins this strand of research from different angles, by considering both commission fee and bidding revenue of marketplace.

Our work also contributes to the growing literature on design of online marketplaces (Chen et al., 2002, Iyer and Pazgal, 2003, Iyer and Padmanabhan, 2006, Hagiu and Jullien, 2011, Jiang et al., 2011, Dukes and Liu, 2015, Liu et al. 2016). These studies have largely focused on the marketplaces role in affecting consumers product information based on the
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outcome in the product market without considering the role of search advertising market.

This paper connects to the literature of position auction (Edelman et al. 2007, Varian 2007, Chen et al. 2009, Ghose and Yang 2009, Wilbur and Zhu 2009, Katona and Sarvary 2010, Liu et al. 2010, Chen and He 2011, Jerath et al. 2011, Xu et al. 2011, Yao and Mela 2011, Zhu and Wilbur 2011, Desai et al. 2014, Liu and Viswanathan 2014, Sayedi et al. 2014, Yang et al. 2014, Amaldoss et al. 2015, Lu et al. 2015, Shin 2015). Most studies have focused on position auction in conventional online publishers such as search engines (e.g., Google and Bing) and social media (e.g., Facebook and LinkedIn), which collect revenue from search advertising market only. In contrast, we study a new type of online publisher: online marketplace (e.g., Amazon Marketplace and Taobao Mall), which collects revenue from product market in addition to search advertising market. This requires the marketplace to keep sellers’ competition in check. To the best of our knowledge, our study is the first to explore the interactions between the marketplace and sellers in both markets with consumer navigation behaviors.

In addition, our paper also relates to the “Position Paradox” in position auction literature (Katona and Sarvary 2009, Jerath et al. 2011, Amaldoss et al. 2015). Katona and Sarvary (2009) and Amaldoss et al. (2015) do not model consumers’ click-through rate or sellers’ pricing decision. Jerath et al. (2011) model clicks but do not consider prices, and they show that “Position Paradox” can occur for any search cost. In our model, we study the marketplace’s spotlight decisions, and thus we ought to study seller’s competitive pricing decisions. This will make “Position Paradox” occur in the marketplace’s environment only when search cost is sufficiently low. Thus, the condition for “Position Paradox” in our study is stricter compared to that in Jerath et al. (2011).
2. Model Setup

Assume that there are two sellers (H and L) on a marketplace, each providing a product (H and L). If product \( i \in \{H, L\} \) fits, it provides a positive utility \( v = 1 \). Otherwise, it provides zero utility. \( \alpha_i \) denote the fit probability of product \( i \), where \( 0 < \alpha_L < \alpha_H < 1 \). That is,

\[
v_i = \begin{cases} 
1 & \text{with probability } \alpha_i \\
0 & \text{with probability } 1 - \alpha_i 
\end{cases}
\]

The fit probability \( \alpha_i \) reflects the range of tastes product \( i \) can satisfy. Specifically, higher \( \alpha_i \) reflects that product \( i \) can cater to a wider range of tastes.\(^4\) The fit probability \( \alpha_i \) is known to the sellers and the marketplace. The potential fit value \( v \) is the same for both products if they both fit and is normalized to \( v = 1 \).\(^5\)

The two products are displayed to consumers orderly: the spotlighted product first. There is one unit mass of consumers, and each consumer has a unit demand. Because the spotlighted product draws the consumer’s attention, we assume that she always first clicks this product and learns its fit for free.\(^6\) In addition, as the price information of products are vivid in a marketplaces shopping environment,\(^7\) we assume that the consumer can observe the prices of both products for free. If she does not like the spotlighted product, she can click the underplayed product but has to incur search cost (denoted by \( \tau \geq 0 \)) to learn whether it fits. After evaluation, the consumer knows the fit and price for both products. Her ex post utility of product \( i \) is given by \( u_i = v_i - p_i \), where \( v_i \) is 1 if it fits and is 0 if otherwise, and \( p_i \) is the price of product \( i \). There is an outside option of not buying any products in the market and its value is normalized to zero. When only one product fits and its price is less than

\(^4\)For example, a mass market brands may meet a wider range of consumers’ tastes than a designer brand.
\(^5\)Athey & Ellison (2011) and Chen & He (2011) use a similar setting.
\(^6\)One can add a search cost for the consumer to learn the spotlighted product. As long as the search market is active (e.g., the search cost is sufficiently low), we can show that our main results still hold.
\(^7\)For example, prices are highlighted in red by Amazon Marketplace (see Figure B1 and B2 in Appendix B) and in bold on Taobao Mall.
one, the consumer buys this product. When both products provide positive fitted values, she selects the cheaper one. In the following text, we refer the former type of consumers as non-switchers and refer the latter type as switchers. She chooses the outside option if she does not like any products.

We consider two rules of spotlight determination. One is by marketplace’s choice. The marketplace unilaterally determines which product to spotlight based on its own interest. We call it “no bidding rule”. The other is by seller’s bidding. Sellers can compete for the spotlight position in the search advertising market. Consistent with business practice in reality, we consider the conventional second-price auction. Each seller submits a bid to compete for the spotlight position. The seller with the higher bid wins the auction and pays the other seller’s bid. We call it “bidding rule”. In addition, we denote $\pi_0(\geq 0)$ the seller’s guaranteed outside profit. We understand it as the fixed cost of operating the business on the marketplace and assume that $\pi_0$ is common for both sellers. Sellers provide products on the marketplace only when their profits are higher than $\pi_0$.

The marketplace’s profit potentially has two sources. Specifically, in the product market, the marketplace charges sellers commission fee (denoted by $\rho \in (0, 1)$), a percentage of the final prices.\(^8\) If the spotlight decision is by bidding rule, in the search advertising market, the marketplace collects bid submitted by the losing seller from the winning one. If there is no bidding, marketplace gets all the profit from the commission fee.

The timing of the game is as follows. Under no bidding rule: first, the marketplace chooses optimal commission fee rate and announces which seller is spotlighted. Second, sellers set prices simultaneously with the knowledge of spotlight decisions. At last, consumers make search and purchase decisions. Under bidding rule: first, the marketplace chooses optimal commission fee rate. Second, the two sellers submit their bids for the spotlight

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\(^8\)In this paper, our analysis is set on the ground that most online shopping intermediaries charge commission fee, and we do not consider the marketplaces optimal choice between different contractual policies (details on various contract designs can be found in Chen et al. 2002).
position. Third, product spotlight (low or high spotlight) is determined. The seller who bids higher is spotlighted. Fourth, sellers set prices simultaneously with the knowledge of whose product is spotlighted. Finally, consumers make search and purchase decisions. The last two steps are the same under two spotlight rules. Under bidding rule, sellers need to compete on both bidding and pricing stages. We provide an illustration of the game order below.

Figure 1: Order of the Game

The paper is organized as follows. Solving the game backward, we first focus on price competition and consumer search. Specifically, in section 3 we first analyze the benchmark situation with zero search cost, followed by the situations with positive search cost. In section 4 we explore sellers bidding decisions, the spotlight decision consequently, and the marketplace’s optimal choice of commission fee rate. We also discuss the implication of spotlight rules on marketplace’s two sources of profit. Section 5 concludes.
3 Consumer Search and Seller’s Price Competition

In this section, as a benchmark, we first study the situation in which the consumer incurs zero search cost ($\tau = 0$) to learn the underplayed product’s fit, and then explore the sub-games with positive search cost. We respectively discuss seller’s pricing strategy in the two sub-games where product $H$ (and $L$) is spotlighted.

3.1 Zero Search Cost

With zero search cost, the consumer learns the fit of both products for free. There are two possible purchase situations: (1) if both products provide position fit she will select the cheaper one; (2) if only one product provides positive fit she will select it. Obviously, if no product provides a positive fit, she will not buy anything and leave the market. Thus, the potential demands for the two products are given by

$$d_H = \begin{cases} (1 - \alpha_L)\alpha_H & p_H \geq p_L \\
\alpha_H & p_H < p_L \end{cases} \quad \text{and} \quad d_L = \begin{cases} \alpha_L & p_H \geq p_L \\
(1 - \alpha_H)\alpha_L & p_H < p_L \end{cases}$$

(1)

where $d_H$ and $d_L$ respectively denotes the demand for product $H$ and $L$. Expression (1) illustrates that, given that the product’s utility exceeds its price, when product $H$ has a higher price ($p_H \geq p_L$), only its non-switchers (size $(1 - \alpha_L)\alpha_H$) choose it while the switchers and product $L$’s non-switchers (size $\alpha_L$) choose product $L$; when product $H$ has a lower price ($p_H < p_L$) the switchers and its non-switchers (size $\alpha_H$) choose it while only product $L$’s non-switchers (size $(1 - \alpha_H)\alpha_L$) choose product $L$. Thus, the revenue for product $i$ is given by $\pi_i = d_i p_i$. The sellers make pricing decision to maximize the profits.

The following lemma summarizes the optimal pricing and equilibrium revenue when search cost is zero.
Lemma 1. (Zero Search Cost) If the search cost is zero, in equilibrium, sellers adopt a mixed pricing strategy on the range of \([1 - \alpha_L, 1]\). Seller H’s expected price and revenue are higher, or \(E(p_H) > E(p_L)\) and \(\pi_H > \pi_L\).

This lemma illustrates an intuitive result: on average seller H charges higher prices and has a higher revenue. With zero search cost, the consumer has perfect information about both products. Thus, sellers would like to lower prices to attract switchers. However, no sellers want to set prices too low to avoid losing revenue from their non-switchers. Consequently, both sellers randomize prices to balance revenues from switchers and non-switchers (as in Narasimhan, 1988). Because seller H has a larger fraction of non-switchers, it is less willing to cut prices, implying a higher expected price and revenue. With zero search cost, the spotlight position create no value to sellers, implying that they have no incentive to bid.

It is noteworthy that with zero search cost the two sellers have the same lowest prices because it is never beneficial for any seller to set prices lower than its rivals lowest price. However, this is not true with positive search cost because the underplayed seller can lower its price to induce consumer search, as we will see in the next subsection.

3.2 Positive Search Cost

With non-zero search cost, the fitness of the underplayed product is no longer transparent and the consumer has to decide whether to incur search cost to learn it. We focus on sellers’ price competition by first studying the sub-game in which product H takes the spotlight position and then the sub-game in which product L takes this position.

Spotlighting Product H (HL order)

When product H takes the spotlight position, the consumer knows the fit of product H and the prices of both products. She has to incur a search cost to evaluate the fit of product L.
3. CONSUMER SEARCH AND SELLER’S PRICE COMPETITION

When product $H$ provides zero fit, the consumer will search product $L$ if the expected gain exceeds the outside option, or $\alpha_L(1 - p_L) - \tau \geq 0$. And if product $L$ fits, she will buy the product $L$. When product $H$ provides a positive fit, the consumer may still search product $L$ if its price is attractive. When buying product $H$ yields a positive utility, the condition that the consumer chooses to evaluate product $L$ is given by

$$\alpha_L(1 - p_L) + (1 - \alpha_L)(1 - p_H) - \tau \geq 1 - p_H \quad (2)$$

which is reduced to $\alpha_L(p_H - p_L) \geq \tau$.

The terms on the right hand side of expression (2) represent the expected benefit of evaluating product $L$: the first two terms respectively represent the expected utility of obtaining a positive and a zero fit from product $L$, and the last term is the search cost. Define function $f_{HL}(p_L) = p_L + \tau/\alpha_L$. If $p_H \leq f_{HL}(p_L)$, the consumer will not evaluate product $L$ when she finds a sufficiently good fit from product $H$. Under this situation, the demands for products $H$ and $L$ are respectively $\alpha_H$ and $(1 - \alpha_H)\alpha_L$ and their revenues are $\alpha_H p_H$ and $(1 - \alpha_H)\alpha_L p_L$. Alternatively, if $p_H > f_{HL}(p_L)$, the consumer will evaluate product $L$ even if she finds a sufficiently good fit from product $H$ because the price of product $L$ is attractive. After evaluation, if product $L$ offers a good fit, she will buy it because its price is lower. Otherwise, she will return to buy product $H$. Consequently, the demands for product $H$ and $L$ are respectively $(1 - \alpha_L)\alpha_H$ and $\alpha_L$ and their revenues are $(1 - \alpha_L)\alpha_H p_H$ and $\alpha_L p_L$ respectively. Compared to the zero search case, we can see $\tau/\alpha_L$ measures the advantage of the spotlight position for product $H$.

Each seller chooses its price to maximize its revenue. The following lemma characterizes the equilibrium outcomes.

**Lemma 2. (High Spotlight-HL)** In equilibrium, sellers choose mixed pricing strategy. In
3. **CONSUMER SEARCH AND SELLER’S PRICE COMPETITION**

\[
\frac{d\pi^{HL}_i}{d\tau} \begin{cases} 
> 0 & \tau_1 < \tau < \alpha_L \\
= 0 & \tau \leq \tau_1 \text{ or } \tau \geq \alpha_L \end{cases} \quad \text{and} \quad \frac{d\pi^{HL}_L}{d\tau} \begin{cases} 
< 0 & \tau < \alpha_L \\
= 0 & \tau \geq \alpha_L \end{cases}
\]

where \(\pi^{HL}_i\) are expected revenue of seller \(i\) in \(HL\) order\(^9\).

This lemma illustrates that the expected revenue of product \(H\) is non-decreasing in search cost while that of product \(L\) is non-increasing. Specifically, when search cost is small \((\tau < \tau_1)\), it is not so costly for the consumer to evaluate product \(L\). Seller \(L\) can set attractive price to induce consumer search and compete for switchers. As the search cost increases, it becomes more difficult to search product \(L\). Thus, seller \(L\) has to further lower its price to ensure consumer traffic and this lowers its revenue. When search cost is intermediate \((\tau_1 < \tau < \alpha_L)\), further lowering price to induce consumer search is no longer optimal for product \(L\) because the search cost is so high that its price has to be very low to be attractive to switchers. Thus, seller \(L\) wants to keep high prices to better exploit its non-switchers. Its expected revenue is decreasing from losing demand because of higher search cost. In addition, seller \(H\)’s revenue increases in search cost because as search cost increases the consumer is more reluctant to evaluate product \(L\) especially when she finds a good fit from product \(H\). When search cost is too high \((\tau \geq \alpha_L)\), the consumer does not evaluate product \(L\) at all. Thus, seller \(H\) sets high prices as if he is monopolist seller and his revenue remains constant. Seller \(L\) also sets monopolist price, but he gets no demand so his revenue is zero.

In addition, this lemma illustrates that in equilibrium sellers mix their prices and sometimes charge lower prices to induce consumer search. The mixed prices seem to be consistent with the observation that sellers often change prices on online marketplaces and may update prices even in an hour-by-hour manner.

\(^9\)Details can be found in Appendix A.
Spotlighting Product $L$ ($LH$ order)

We similarly analyze the situation when product $L$ takes the spotlight position. If the consumer finds that product $L$ provides zero fit, she will search product $H$ under the condition that $\alpha_H(1 - p_H) - \tau \geq 0$. She buys product $H$ if it provides a positive fit. If she finds a positive fit from product $L$, she may still search product $H$ if it has a low enough price. Thus, the consumer evaluates product $H$ when

$$v - p_L \leq \alpha_H(v - p_H) + (1 - \alpha_H)(v - p_L) - \tau$$

which can be reduced to $\tau \leq \alpha_H(p_L - p_H)$. That is, the consumer evaluates product $H$ only when the net benefit of doing so exceeds the net cost.

Define function $f^{LH}(p_L) = p_L - \tau/\alpha_H$. If $p_H \geq f^{LH}(p_L)$, the consumer will not evaluate product $H$ when she finds a good fit from product $L$. Thus, the demands for products $H$ and $L$ are respectively $\alpha_H(1 - \alpha_L)$ and $\alpha_L$ and their revenues are respectively $\alpha_H(1 - \alpha_L)p_H$ and $\alpha_L p_L$. Alternatively, if $p_H < f^{LH}(p_L)$, the consumer will evaluate product $H$ even when she finds a good fit from product $L$ because the price of product $H$ is attractive. After evaluation, if she finds a good fit from product $H$, she will buy it because it is cheaper. Otherwise, she will buy product $L$. Thus, the demands for products $H$ and $L$ are respectively $\alpha_H$ and $\alpha_L(1 - \alpha_H)$ and their revenues are respectively $\alpha_H p_H$ and $\alpha_L(1 - \alpha_H)p_L$. The sellers again use mixed strategy. The following lemma characterizes the equilibrium outcome in the low spotlight situation.

**Lemma 3. (Low Spotlight-LH)** In equilibrium, sellers choose mixed pricing strategy. In addition,

$$\frac{d\pi^{LH}_H}{d\tau} = \begin{cases} < 0 & \tau < \alpha_H \\ = 0 & \tau = \alpha_H \\ > 0 & \tau > \alpha_H \end{cases} \quad \text{and} \quad \frac{d\pi^{LH}_L}{d\tau} = \begin{cases} < 0 & \tau < \alpha_H \\ = 0 & \tau = \alpha_H \\ > 0 & \tau > \alpha_H \end{cases}$$

where $\pi^{LH}_i$ are expected revenue of seller $i$ in LH order.\(^{10}\)

\(^{10}\)Details can be found in Appendix A.
This lemma illustrates that in low spotlight situation product $L$’s revenue is non-decreasing in search cost. Specifically, when search cost is low ($\tau < \alpha_H$), the consumer may evaluate product $H$, but she is less likely to do so as search cost increases. Accordingly, product $L$ is able to get the demand of switchers more often and its revenue is increased. On the other hand, product $H$’s revenue is decreased in search cost because the demand shrinks as fewer consumers searches his product. When search cost is high ($\tau \geq \alpha_H$), no consumers will search product $H$, no matter product $L$ provides good fit or not. Thus, seller $L$ sets monopolist price and benefits from both non-switchers and switchers. His revenue is no further affected by search cost. Seller $H$ also sets monopolist price, but he gets no demand so his revenue is zero.

To summarize, the expected revenues of both sellers under the two spotlight orders are listed in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$\tau &lt; \frac{\alpha_L}{\alpha_H}(\alpha_H - \alpha_L)$</th>
<th>$\frac{\alpha_L}{\alpha_H}(\alpha_H - \alpha_L) \leq \tau &lt; \alpha_L$</th>
<th>$\tau \geq \alpha_L$</th>
<th>$\tau \geq \alpha_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HL</strong></td>
<td>(1 - $\alpha_L$)$\alpha_H$</td>
<td>$\alpha_H[(1 - \alpha_H)(1 - \tau/\alpha_L) + \tau/\alpha_L]$</td>
<td>$\alpha_H$</td>
<td></td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>(1 - $\alpha_L - \tau/\alpha_L$)$\alpha_L$</td>
<td>$\alpha_L(1 - \alpha_H)(1 - \tau/\alpha_L)$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>LH</strong></td>
<td>$\alpha_H(1 - \alpha_L)(1 - \tau/\alpha_H)$</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>$\alpha_L(1 - \alpha_L + \frac{\alpha_L}{\alpha_H}\tau)$</td>
<td></td>
<td>$\alpha_L$</td>
<td></td>
</tr>
</tbody>
</table>

As we have characterized the consumer’s search process and sellers’ pricing strategies in sub-games of both display orders, we are ready to discuss on the marketplace’s spotlight decisions.
4 Determination of Spotlight

We consider two rules of determining the product spotlight. One is marketplace rule: the marketplace unilaterally decides which product to be spotlighted by its own interest. The other is bidding rule: sellers bid for spotlight by a second price auction. The seller with higher bid wins the spotlight position and pays the other seller’s bid. The two spotlight rules are analyzed respectively in the following two sections.

4.1 No Bidding Rule: Marketplace’s Choice of Spotlight

In this subsection, we look at the marketplace rule of spotlight allocation: the marketplace unilaterally decides which product to spotlight based on its own interest. Marketplace gets commission fees from both sellers. After paying commission fee, sellers’ profits decrease proportionally \( (1 - \rho)\pi_i^k \) where \( k = HL/LH \) as the commission fee rate \( \rho \) is the same for both sellers. The commission fee revenue for the marketplace is \( \rho(\pi^k_H + \pi^k_L) \). Thus, the marketplace’s spotlight decision and the corresponding choice of the commission fee rate can be summarized as following\(^\text{[11]}\)

\[
\max_{k, \rho} \rho(\pi^k_H + \pi^k_L) \\
\text{s.t.} \quad (1 - \rho)\pi^k_H \geq \pi_0 \\
\quad (1 - \rho)\pi^k_L \geq \pi_0
\]

where \( k \in \{HL, LH\} \) is the spotlight decision. That is, the marketplace chooses product spotlight and commission fee rate to maximize profit under the constraint that the sellers participate operation on the marketplace.

\(^{11}\)Throughout the main model, we assume that both sellers are required to participate to maintain a competitive environment on the marketplace. It happens in the situation that no buyers will buy from the marketplace if they do not have choices of products.
From equation (3), the marketplace cares both the total revenues and the commission fee rate he is able to charge. We first discuss total revenues and commission fee rate under the two spotlight orders separately, and then the choice of spotlight.

First, we show how the total revenues shift with search cost. The effect of search cost on total revenues is summarized in the following Proposition.

**Proposition 1.** There is a unique search cost \( \tau^* = \frac{\alpha_H \alpha_L}{\alpha_H + \alpha_L} \) such that:

(i). when \( \tau < \tau^* \), the total revenue of two sellers is higher in Low Spotlight (LH);
(ii). when \( \tau > \tau^* \), the total revenue of two sellers is higher in High Spotlight (HL).

Increased search cost softens (in most cases at least) price competition between the two sellers, but also drives some consumers out of the marketplace as they do not search the underplayed product. The effect differs under different spotlight orders. This proposition states that the relative magnitude of the total revenue is governed by a threshold of search cost \( \tau^* \).

Part (i) of the proposition states that when search cost is small \( (\tau < \tau^*) \), the total revenues are higher in Low Spotlight (LH order). With small search cost, if product \( L \) is underplayed, it can set low prices to attract consumer search and this will lower its revenue. Alternatively, if product \( L \) is spotlighted, seller \( H \) has lower incentive to induce consumer search through low prices because it has more non-switchers to exploit. Also, as search cost is not so large, the negative effect of crowding out consumers is not so large. Thus, the total revenue of the two sellers is higher under LH order.

In contrast, part (ii) of the proposition states that when search cost is large \( (\tau > \tau^*) \), the total revenue is higher in High Spotlight (HL order). With large search cost, it is difficult for the underplayed seller to use low prices to attract consumers. As seller \( H \) has a larger consumer base, spotlighting product \( H \) increases total revenues more than spotlighting product \( L \). Also, as search cost is large, the negative effect of crowding out consumers is
more severe. By spotlighting product $H$, fewer consumers are crowded out.

Next, we examine the commission fee rate. For a given product spotlight choice, the marketplace tries to set a high commission fee rate. The restriction is the participation of the seller with lower revenue, referred to as the binding seller. If $k = HL$ (spotlighting product $H$), as $\pi_{HL}^H > \pi_{HL}^L$ for any value of search cost, product $L$ is the binding seller. So the marketplace sets commission fee rate at $\rho^* = 1 - \frac{\pi_0}{\pi_H^H}$ and its profit is

$$\pi_{MP}^{HL} = \frac{\pi_{HL}^H - \pi_0}{\pi_L^H} (\pi_H^H + \pi_L^H) = \pi_H^H + \pi_L^H - \left(\frac{\pi_{HL}^H}{\pi_L^H}\pi_0 + \pi_0\right)$$

Similarly, if $k = LH$, or spotlighting product $L$, define $\pi_{min} = \min\{\pi_{HL}^L, \pi_{LH}^L\}$. The commission fee rate is $\rho^* = 1 - \frac{\pi_0}{\pi_{min}}$ and the marketplace’s profit is $\pi_{MP}^{LH} = \frac{\pi_{min} - \pi_0}{\pi_{min}} (\pi_H^H + \pi_L^H)$. As $\pi_L^{LH}$ is decreasing in search cost and $\pi_L^{LH}$ increasing, there is a threshold $\hat{\tau} = \frac{\alpha_H(\alpha_H - \alpha_L)(1 - \alpha_L)}{\alpha_H - \alpha_H \alpha_L + \alpha_L^2}$ such that $\pi_H^H > \pi_L^{LH}$ if $\tau < \hat{\tau}$ and $\pi_H^H \leq \pi_L^{LH}$ if $\tau \geq \hat{\tau}$. So

$$\pi_{MP}^{LH} = \begin{cases} 
\pi_H^H + \pi_L^{LH} - \left(\frac{\pi_{HL}^H}{\pi_L^H}\pi_0 + \pi_0\right), & \text{if } \tau < \hat{\tau} \\
\pi_H^H + \pi_L^{LH} - \left(\frac{\pi_{HL}^H}{\pi_H^H}\pi_0 + \pi_0\right), & \text{if } \tau \geq \hat{\tau}
\end{cases}$$

As both $\pi_H^{LH}$ and $\pi_L^{LH}$ are higher than $\pi_L^{HL}$, the commission fee rate is always higher when product $L$ is spotlighted.

The marketplace compares the profits (with optimal choice of commission fee rate) under the two display orders. Define the difference of its profit under the two orders as $\Delta \pi = \pi_{MP}^{HL} - \pi_{MP}^{LH}$. We have

$$\Delta \pi = \begin{cases} 
((\pi_H^H + \pi_L^H) - (\pi_H^H + \pi_L^{LH})) - \left(\frac{\pi_{HL}^H}{\pi_L^H}\pi_0 + \pi_0\right), & \text{if } \tau < \hat{\tau} \\
((\pi_H^H + \pi_L^{HL}) - (\pi_H^L + \pi_L^{LH})) - \left(\frac{\pi_{HL}^H}{\pi_H^L}\pi_0 + \pi_0\right), & \text{if } \tau \geq \hat{\tau}
\end{cases}$$

(4)
The first term in $\Delta \pi$ is the difference of total revenues of the two sellers, and the second term is the difference of profits earned by the seller with higher revenue. Intuitively, the marketplace has to share some revenue with the sellers. The binding seller only earns the outside profit. The non-binding seller earns more, which depends on the commission fee rate, or eventually the revenue ratio of the two seller’s revenues. When the seller’s revenues are less different, the marketplace can charge a higher commission fee rate, thus it only shares less revenue with the sellers.

The marketplace needs to balance total revenues and commission fee rate. As the commission fee rate is always higher when product $L$ is spotlighted, the marketplace chooses to spotlight product $H$ only when the increase of total revenues exceeds the increase of profit shared to the sellers.

We summarize the marketplace’s choice of spotlight decision and optimal commission fee rate in the following proposition.

**Proposition 2.** If the marketplace chooses spotlight decisions,

(i). When search cost is $\tau \leq \tau^*$, the marketplace selects to spotlight product $L$;

(ii). When search cost is $\tau > \tau^*$, there is a threshold value $\pi_0^*$ such that the marketplace selects to spotlight product $L$ (product $H$) when outside profit $\pi_0$ is larger (smaller) than $\pi_0^*$;

(iii). The optimal commission fee rate is $\rho^* = 1 - \frac{\pi_0}{\pi_L}$ when product $H$ is spotlighted; and is $\rho^* = 1 - \frac{\pi_0}{\min\{\pi_H, \pi_L\}}$ when product $L$ is spotlighted.

The result is very intuitive: the marketplace needs to balance total profits and commission fee rate when making spotlight decisions. As product $L$ is the bottleneck for commission fee rate, spotlighting product $L$ can give seller $L$ some advantage, and the marketplace can charge higher commission fee rate. If spotlighting product $L$ also generates higher total revenues ($\tau \leq \tau^*$), the two goals point to the same spotlight decision. The marketplace will spotlight product $L$. That is stated in part (i) of the proposition.
Otherwise, if spotlighting product $H$ generates higher total revenues, the marketplace needs to choose between a larger piece from a smaller pie or otherwise. The fraction of profit earned by the seller is proportional to outside profit $\pi_0$. When $\pi_0$ is small, the marketplace can charge a high commission fee rate, and the non-binding product earns a small portion of the revenue. Thus, the marketplace will spotlight product $H$. In the High Spotlight, the total revenues are higher and product $H$ does not earn a large fraction of the revenues. On the other hand, when $\pi_0$ is large, the commission fee rate is low and the non-binding product is able to get a large share of the revenue. So the marketplace will spotlight product $L$. Though the total revenues are lower, the marketplace can get a large fraction of the revenues. That is stated in part (ii) of the proposition. The corresponding choice of commission fee rate is summarized in part (iii).

We further examine the search cost threshold $\pi^*_0$. From equation (4),

$$\pi^*_0 = \begin{cases} 
(\frac{\pi^{HL}}{\pi^{LH}} - \frac{\pi^{LH}}{\pi^{HL}})^{-1}((\pi^{HL}_H + \pi^{HL}_L) - (\pi^{LH}_H + \pi^{LH}_L)) & \pi_{\text{min}} = \pi^{LH}_L \\
(\frac{\pi^{HL}}{\pi^{LH}} - \frac{\pi^{LH}}{\pi^{HL}})^{-1}((\pi^{HL}_H + \pi^{HL}_L) - (\pi^{LH}_H + \pi^{LH}_L)) & \pi_{\text{min}} = \pi^{LH}_H 
\end{cases}$$

The threshold depends relative magnitude of the difference of total revenues and the difference of commission fee rate. The difference of total revenues is usually smaller than that of commission fee rate, making $\pi^*_0$ a small number relative to the seller’s revenues, especially when $\alpha_L$ is relatively small. That indicates product $L$ is still very likely to be spotlighted when $\tau \geq \tau^*$. We provide two figures to illustrate the shape of $\pi^*_0$. 

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Figure 2: Shape of $\pi_0^*$

Shape of $\pi_0^*$: $\alpha_H = .9$; $\alpha_L = 0.75$

Figure 3: Shape of $\pi_0^*$

Shape of $\pi_0^*$: $\alpha_H = .9$; $\alpha_L = 0.6$
4. DETERMINATION OF SPOTLIGHT

Next, we examine how the market conditions affect the marketplace’s spotlight decisions. The next corollary summarizes how the total revenues of the two sellers are affected by products’ width of appeal ($\alpha_H$ and $\alpha_L$).

**Corollary 1.** $\tau^*$ increases in $\alpha_H$ and in $\alpha_L$.

Overall, total revenues of the two sellers are increased $\alpha_H$ and $\alpha_L$ under both display orders, as a result from expanded market. When $\alpha_H$ increases, Low Spotlight is more likely to arise. When product $H$ has a wider breath of appeal, though seller $H$’s revenue is increased in High Spotlight, the revenue is also increased in Low Spotlight. Besides, seller $L$ has to set even lower prices to induce consumer search if it is underplayed. Thus, the marketplace has a greater incentive to spotlight product $L$ to avoid this situation.

However, When $\alpha_L$ increases, Low Spotlight is also more likely to arise. Seller $L$’s revenue is increased in both display orders, but the increase in Low Spotlight is higher. On the other hand, seller $H$’s revenue is lowered in both display orders but by the same amount when search cost is small. Thus, the total revenues are higher under Low Spotlight.

We discuss how the optimal commission fee rate and the corresponding marketplace’s profit are affected by search cost. Proposition 3 summarizes the effect of search cost on commission fee rate.

**Proposition 3.** Under no bidding rule:

(i). if product $L$ is spotlighted, the optimal commission fee rate is non-monotonic in search cost;

(ii). if product $H$ is spotlighted, the optimal commission fee rate is decreasing in search cost;

(iii). if product spotlight decision changes from spotlighting product $L$ to $H$ ($H$ to $L$), optimal commission fee rate discontinuously jumps down (up).

Recall from part (iii) of proposition 2, the optimal commission fee rate is $\rho^* = 1 - \frac{\pi_0}{\pi_{HL}}$ when product $H$ is spotlighted; and is $\rho^* = 1 - \frac{\pi_0}{\min\{\pi_{LH}^{H}, \pi_{LH}^{L}\}}$ when product $L$ is spotlighted.
If product $L$ is spotlighted, as $\pi_{H}^{LH} > \pi_{L}^{LH}$ if $\tau < \hat{\tau}$ and $\pi_{H}^{LH} \leq \pi_{L}^{LH}$ if $\tau \geq \hat{\tau}$, the optimal commission fee rate is first increasing ($\tau < \hat{\tau}$) and then decreasing ($\tau > \hat{\tau}$). That is stated in part (i) of the proposition. On the other hand, if product $H$ is spotlighted, as $\pi_{L}^{HL}$ is decreasing in search cost, so is the optimal commission fee rate. That is part (ii) of the proposition.

When search cost increases, if the spotlight decision changes (which is only possible when $\tau > \tau^*$), as total revenues raise or drop discontinuously, so would the commission fee rate. That is stated in part (iii) of the proposition.

The effect on optimal commission fee rate is due to the changes in price competition and crowding out effect. When product $L$ is spotlighted, increased search cost intensifies product $L$’s advantage in the price competition (when $\tau < \hat{\tau}$), and the marketplace is able to charge higher commission fee rate. But as search cost further increases (when $\tau > \hat{\tau}$), product $H$’s revenue is decreased due to both price competition and consumers being crowded out, thus the marketplace has to decrease commission fee rate. When product $H$ is spotlighted, product $L$’s revenue is decreased due to both price competition and crowding out effect, so the commission fee rate is always decreasing.

Next, we are ready to show the effect of search cost on marketplace’s profit. If commission fee rate is fixed, the total revenues are increasing in search cost.\footnote{To be more specific, the total revenue in High Spotlight is V shaped (first decreasing when $\tau < \tau_1$, then increasing when $\tau > \tau_1$). As High Spotlight is never chosen when $\tau < \tau^*$, the decreasing part never happens in equilibrium. The total revenue in Low Spotlight is increasing as long as $\alpha_L$ is large enough to satisfy $(\alpha_H + \alpha_L) \alpha_L > \alpha_H$.} Increased search cost softens pricing competition between the two sellers but also crowds out more consumers.\footnote{The only exception is when search cost is small and product $H$ is spotlighted. But under this search cost, High Spotlight is never chosen by the marketplace.} The price competition effect increases total revenue of the two sellers, while the crowding out effect decreases total revenue. When $\alpha_L$ is not too small, the crowding out effect is not too large, and the total revenue increases in search cost.
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However, the commission fee rate is not fixed. We show in Proposition 3 that the optimal commission fee rate can be non-monotonic in search cost, and is generally decreasing when search cost is large. So the profit of the marketplace can be increasing or decreasing in search cost, depending how total revenue and commission fee rate are affected. 14

4.2 Bidding Rule: Seller’s Bidding for Spotlight

In this section, we consider the bidding rule of spotlight determination that the product spotlight is determined by seller’s bidding in a second price auction. The seller with higher bid wins the spotlight position and pays the other seller’s bid. Denote the bid submitted by seller by $b_i, i = H/L$. The winning seller’s expense on bidding and thus marketplace’s profit from search advertising is given by $b_i | b_i < b_{(-i)}, i = H/L$. If the biddings are tied, we assume that the marketplace would choose the spotlight by its own interest. 15

Now, we explore sellers’ incentive to win the spotlight position: such a position should yield higher profit so that a seller wants to compete for it in the auction. After paying the commission fee, seller $j$’s profit in the $k = HL/LH$ order is $(1 - \rho)\pi^k_j$. The following lemma illustrates that the spotlight position can always bring revenues advantages for the winning seller.

Lemma 4. For any commission fee rate, a seller’s revenue is (weakly) higher when it is spotlighted: $(1 - \rho)\pi^H_L \geq (1 - \rho)\pi^H_H$ and $(1 - \rho)\pi^L_H \geq (1 - \rho)\pi^L_L$.

The result of this lemma is intuitive: as long as the spotlighted product is attractive enough, the consumer will buy it even at a relatively high price in order to avoid search cost. Thus, the spotlight position yields profit advantages and sellers are inclined to compete for

---

14 We will go back to the marketplace’s profit in subsection 4.3. In that subsection, we provide figures from numerical calculation for illustration.

15 The assumption is inessential. When the biddings are tied, it’ll be indifferent as which product is spotlighted. To make discussion easy, we assume the spotlight decision is the same as in no-bidding rule when the biddings are tied.
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Due to the property of second-price auction, sellers bid at their true value for the spotlight position, which will be the profit advantage of being spotlighted. That is, \( b_H = (1 - \rho)(\pi_H^{HL} - \pi_H^{LH}) \) and \( b_L = (1 - \rho)(\pi_L^{LH} - \pi_L^{HL}) \). When \( b_H < b_L \), seller \( L \) will win the auction (Low Spotlight or LH) and pay \( b_H \); when \( b_H > b_L \), seller \( H \) will win the auction (High Spotlight or HL) and pay \( b_L \). The following proposition summarizes the spotlight decision.

**Proposition 4.** There is a unique search cost \( \tau^* \) such that:

(i). when \( \tau < \tau^* \), seller \( L \) wins the auction and the marketplace selects Low Spotlight (LH);

(ii). when \( \tau \geq \tau^* \), seller \( H \) wins the auction and the marketplace selects High Spotlight (HL).

This Proposition states the determination of product spotlight on the marketplace. Part (i) illustrates that, when search cost is low (\( \tau \leq \tau^* \)), seller \( L \) has a greater value for the spotlight position and will win the auction. Thus, seller \( L \) has a greater value for the spotlight position, and will submit a higher bid to win the auction. In contrast, part (ii) illustrates that, when search cost is high (\( \tau \geq \tau^* \)), seller \( H \) has a greater value for the spotlight position. Specifically, with large search cost, seller \( H \) knows that, if product \( H \) is spotlighted, it is difficult for seller \( L \) to use low prices to attract the consumers. Thus, seller \( H \) can maintain high profit. As seller \( H \) has a larger consumer base, he has a greater value for the spotlight position and will bid higher to win the auction.

From Proposition 1 and 4, the spotlight decision by bidding rule maximizes the total revenue of the two sellers. The winning seller outbids the other seller only when he can earn a higher profit compared to the situation when he is not spotlighted. Consequently, the total revenue of the two sellers when spotlighting the winning seller is higher than that when spotlighting the other seller.

Compare the spotlight decision under no bidding rule (proposition 2) and under bidding.
rule (proposition 4), we can see Low Spotlight is more likely to happen under no bidding rule. When search cost is small ($\tau < \tau^*$), product $L$ is spotlighted under both spotlight rules. When it’s large ($\tau \geq \tau^*$), product $H$ is always spotlighted under bidding rule. Under no bidding rule, he is spotlighted only when outside profit is small.

The difference comes from the marketplace’s ability to set commission fee rate. Under no bidding rule, it determines both commission fee rate and spotlight simultaneously. Potentially the commission fee rate can be different under the two display orders. When the marketplace can determine the spotlight, it is more intended to spotlight product $L$ so it is able to charge a higher commission fee rate. In contrast, under bidding rule, the commission fee rate is predetermined. The marketplace commits to the commission fee rate before the bidding. Although the product spotlight from bidding rule maximizes the total revenue of the two sellers, it does not achieve the largest possible commission fee, which is the target under no bidding rule.

In summary, Proposition 4 illustrates the marketplace’s product spotlight determination by seller’s bidding. We define seller’s net profit as the revenue from selling products minus commission fee and bidding expense, denoted as $\pi_{net}^H$ and $\pi_{net}^L$. The marketplace’s profits (denoted as $\pi^{mp}$) are from both commission fee and bidding revenue. After considering the spotlight decisions, we can summarize the profits of the marketplace as well as the net profits of the two sellers in Table 2.
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Table 2: Summary of Profits of Marketplace and Sellers

<table>
<thead>
<tr>
<th></th>
<th>LH order</th>
<th>HL order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau &lt; \tau^*$</td>
<td>$\tau \geq \tau^*$</td>
</tr>
<tr>
<td>Commission fee</td>
<td>$\rho(\pi_H^{LH} + \pi_L^{LH})$</td>
<td>$\rho(\pi_H^{HL} + \pi_L^{HL})$</td>
</tr>
<tr>
<td>Bidding</td>
<td>$(1 - \rho)(\pi_H^{HL} - \pi_H^{LH})$</td>
<td>$(1 - \rho)(\pi_L^{HL} - \pi_L^{LH})$</td>
</tr>
<tr>
<td>Net profit of seller $H$</td>
<td>$(1 - \rho)\pi_H^{LH}$</td>
<td>$(1 - \rho)(\pi_H^{HL} - \pi_L^{LH} + \pi_L^{HL})$</td>
</tr>
<tr>
<td>Net profit of seller $L$</td>
<td>$(1 - \rho)(\pi_L^{LH} - \pi_H^{HL} + \pi_H^{LH})$</td>
<td>$(1 - \rho)\pi_L^{HL}$</td>
</tr>
</tbody>
</table>

Before we go ahead and discuss the determination of commission fee rate, we summarize the effect of search cost on the net profits of the two sellers conditional on a fixed commission fee rate. To facilitate discussion, we define $V_H = \begin{cases} \pi_H^{LH} & \text{in LH order} \\ \pi_H^{HL} - \pi_L^{LH} + \pi_L^{HL} & \text{in HL order} \end{cases}$ and $V_L = \begin{cases} \pi_L^{LH} - \pi_H^{HL} + \pi_H^{LH} & \text{in LH order} \\ \pi_L^{HL} & \text{in HL order} \end{cases}$ so that $\pi_{i\text{net}} = (1 - \rho)V_i$, $i = H/L$. We call $V_i$ profit of seller $i$. $V_i$ is the net profit of seller $i$ as if the commission fee rate is zero. When the commission fee rate is not zero, the net profit of sellers is proportional to $V_i$. The property of seller’s profit $V_i$ is summarized in the following Proposition.

**Proposition 5.** Given a fixed commission fee rate, the profit of seller $L V_L$ is decreasing in search cost, and the profit of seller $H V_H$ is increasing in search cost when search cost is large ($\tau > \tau^*$) AND $\alpha_L$ is small, and is decreasing otherwise.

As the commission fee rate is fixed, we focus on the effect of search cost on $V_i$. The profits of seller $L V_L$ is decreasing in search cost. When search cost is small ($\tau < \tau^*$), seller $L$ values the spotlight position more and wins the auction. As seller $H$ can exploit his non-switching consumers without losing too much demand, he has less interest to bid high values for spotlight. The advantage of occupying spotlight position is increasing in search cost, so the revenue of seller $L$ increases. Meanwhile, the amount of bidding paid by seller $L$ (seller...
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H’s bid) is also increased. The increased bidding expense out-weights the increase of seller L’s revenue. Consequently, the profit of seller L is decreased due to the increasing bidding expense to win the spotlight. When search cost further increases \((\tau > \tau^*)\), seller H outbids seller L due to its larger width of appeal. The profit of seller L is decreased as he loses more demand when search cost increases.

Similar analysis can be applied to seller H’s profit. When search cost is small \((\tau < \tau^*)\), seller H is outbid, and his profit decreases in search cost due to increasing crowding out effect. When search cost is large \((\tau > \tau^*)\), whether his profit is increasing or decreasing depends on the relative magnitude of revenue and bidding expense (both of which are increasing in search cost). When \(\alpha_L\) is small, seller H can win the auction with a lower bidding expense and also enjoy a higher revenue due to less price competition, so increase in revenue out-weights the increase in bidding and his profit is increasing. Otherwise, when \(\alpha_L\) is large, his profit is decreasing.

It is worth to note that the winning sellers’ profit is not always increasing in search cost. Recall that the spotlight position has two effects on pricing strategy and revenues. Competition effect gives the spotlighted sellers an advantage: to let the spotlighted sellers be able to charge a higher price without losing demand. And crowding out effect gives the underplayed sellers a dis-advantage: some consumers are turned away because of the search cost. Seller bids both for the promotion of his product (competition effect), and also to foreclose the competitor’s product (crowding out effect). Both revenue and bidding expense are affected by the two effects, but differ in magnitude for different products under different display orders. That can explain why the revenue of the winning seller does not always have to increase more than the increase of bidding expense.

Optimal Commission Fee Rate

Now we are ready to determine the optimal commission fee rate. The marketplace will set the commission fee rate as high as possible under the condition that both sellers earn at
4. **DETERMINATION OF SPOTLIGHT**

least the outside profit $\pi_0$. The results are summarized in the following proposition.

**Proposition 6.** The optimal commission fee rate is determined as $\rho^* = 1 - \frac{\pi_0}{V_{\min}}$, where $V_{\min} = \min\{V_H, V_L\}$ is the lower value of the net profits of the two sellers. Specifically, if $\alpha_L$ is small, $V_{\min} = V_L$ for any search cost; if $\alpha_L$ is large,

$$
V_{\min} = \begin{cases} 
V_L & \tau < \tau^\dagger \text{ or } \tau > \tau^\dagger \\
V_H & \tau \in [\tau^\dagger, \tau^\ddagger]
\end{cases}
$$

where $V_H = V_L$ at $\tau^\dagger$ and $\tau^\ddagger$, and $\tau^\dagger < \tau_1 < \tau^\ddagger < \tau^*$. 

The marketplace’s profit comes from two sources: commission fee and bidding revenue. Apparently, the commission fee is increasing in $\rho$ and the bidding revenue is decreasing in $\rho$. As commission fee comes from both sellers while the bidding revenue comes from one seller and only by the amount of the other seller’s increased profit from the spotlight position, commission fee (when commission fee rate is 1) is always higher than bidding (when commission fee rate is 0). The commission fee out-weights bidding so marketplace will set the commission fee rate as high as possible, until the profit of the binding seller reaches the outside profit.

**Corollary 2.** $\rho^*$ is decreasing in $\tau$. And $\rho^*$ is decreasing in $\pi_0$.

From proposition 6, marketplace’s ability to charge high commission fee rate is restricted by the binding seller’s profitability, which depends on both revenue from pricing competition and bidding expense to win the spotlight location. As $\rho^*$ are positively correlated with $V_{\min}$, the shape of $\rho^*$ follows that of $V_{\min}$. The shape of $V_H$ and $V_L$ are discussed in Proposition 5. When search cost is small ($\tau < \tau^*$), $L$ wins the auction. Both $V_H$ and $V_L$ is decreasing in search cost, and so is $V_{\min}$. When search cost is large ($\tau > \tau^*$), $H$ wins the auction. Seller $L$ is the binding seller, and it’s profit $V_L$ is decreasing. Combining the two situations, from Proposition 5 and 6, $V_{\min}$ is decreasing in search cost, and so is $\rho^*$.

\[16\] A formal proof is provided in proof of prop 6 in appendix
Besides, \( \rho^* \) is decreasing in \( \pi_0 \). The result follows the determination of commission fee rate. The marketplace needs to secure the sellers at least the outside profit as to guarantee seller’s participation. Higher outside profit prevents the marketplace from setting the commission fee rate too high.

**Corollary 3.** The optimal commission fee rate under no bidding rule than is weakly higher than that under bidding rule for any search cost.

We compare the commission fee rate under no bidding rule (part (iii) of proposition 2) and under bidding rule (proposition 6). It’s not surprised to see that the rate under no bidding rule is higher. First, it comes from the product spotlight difference. When search cost is large (\( \tau > \tau^* \)), product \( H \) outbids product \( L \) under bidding rule, but product \( L \) is spotlighted under no bidding rule when outside profit is large. As product \( L \)’s revenue is higher when he is spotlighted, the marketplace can charge a higher commission fee rate. Second, even if the product spotlight is the same under the two rules, the marketplace can still charge a higher commission fee rate under certain conditions. Consider the case when product \( L \) is spotlighted but he has to pay non-zero amount of bidding for spotlight. If product \( L \) is the binding seller (\( V_{\min} = V_L \) under bidding rule and \( \pi_{\min} = \pi_H^L \) under no bidding rule), the commission fee rate is higher under no bidding rule. Under no bidding rule, product \( L \) is spotlighted without having to pay for the bidding, so his profit is higher. Thus, the marketplace can charge a higher commission fee rate.

The effect of search cost on platform’s profit is again undetermined. For fixed commission fee rate, both commission fee and bidding are increasing in search cost. But the commission fee rate itself is decreasing in search cost. So the overall effect on platform’s profit is not determined.

**Relative Importance of Advertising Market**

Under bidding rule, the marketplace has two channels of profits: commission fees and bidding revenue. Now we consider the relative importance of the two channels. We use profit
ratio to indicate the relative importance of the advertising market, defined as ratio of profits from bidding revenues to commission fees, or \( \Gamma = \frac{(1-\rho^*)\gamma}{\rho^*} \), where \( \gamma \) is defined as

\[
\gamma = \begin{cases} 
\frac{\pi_H^L - \pi_L^H}{\pi_H^L + \pi_L^H} & \text{if LH order} \\
\frac{\pi_L^H - \pi_H^L}{\pi_H^L + \pi_L^H} & \text{if HL order}
\end{cases}
\]

**Proposition 7.** \( \Gamma \) is increasing in \( \tau \) in most domain of \( \tau \), following the pattern of search cost (except a small region where \( \tau \) is modest and \( \alpha_L \) is small). And \( \Gamma \) is increasing in \( \pi_0 \).

It’s intuitive that \( \Gamma \) is increasing in \( \pi_0 \). As outside profit increases, the commission fee rate decreases, so \( \frac{(1-\rho^*)}{\rho^*} \) increases. The \( \gamma \) part is not affected, as commission fee rate does not directly affect seller’s pricing competition. So the relative importance of advertising market increases.

The result that \( \Gamma \) is increasing in \( \tau \) is more subtle. As both commission fee and bidding are increasing in \( \tau \), the results are driven by the relative magnitude of the change as well as the change of commission fee rate \( \rho^* \). While both sources of profits are increasing in search cost, the commission fee rate is non-linear. The overall effect is increasing, mostly driven by the increase in bidding revenue exceeding the increase in total profits of both sellers and the shift of commission fee rate. The following figure illuminates the results.
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Figure 4: Relative importance of advertising market

Figure 5: Relative importance of advertising market

Note: this figure shows the share of bidding revenue to marketplace's revenue, not the $\gamma$ we defined.
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4.3 Comparison of Marketplace’s Profit under Spotlight Rules

Next, we formally discuss under which spotlight rule the marketplace can earn a higher profit. One may naively think that the profit should always be higher under bidding rule, as the marketplace can get revenue from both commission fee and bidding revenues. Such intuition only holds when the commission fee rate is fixed. When the marketplace also sets commission fee rate, adopting the bidding rule is not at no cost. Recall from corollary 3 that the commission fee rate is always (weakly) higher under no bidding rule. The trade-off faced by the marketplace is clear: under no bidding rule, it can get a higher revenue from commission fee; under bidding rule, through the revenue from commission fee is lower, it can get extra revenue from bidding.

Intuitively, which spotlight rule generates higher profit for the marketplace depends on the commission fee rate. When the difference of seller’s revenue is similar under the two spotlight orders (High Spotlight and Low Spotlight) is small, the marketplace has less loss from committing to a fixed commission fee rate. Consider the extreme case: when $\alpha_L$ is very close to $\alpha_H$, High Spotlight or Low Spotlight makes less difference to the marketplace. It can charge the same commission fee rate. But the sellers do care whose product is spotlighted, and they are willing to bid for the spotlight position. In this case, the bidding rule can generate higher profit to the marketplace.

On the contrary, if the revenue differences of the two sellers under High Spotlight is very different to that under Low Spotlight, the marketplace will choose pretty different commission fee rate if it is able to. The marketplace would rather give up bidding revenue to keep the choice of choosing different commission fee rate under different spotlight orders. In this case, the no bidding rule leads to higher profit. Consider the other extreme case: when $\alpha_L$ is very small, his revenue can differ a lot if his product is spotlighted (Low Spotlight) or not (High Spotlight). If product $L$ is always the binding product, it means that the
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commission fee rate can be very different. Meanwhile, the price competition pressure to product $H$ is relatively small when $\alpha_L$ is small, so product $H$’s bidding will not be very large. In this case, the no bidding rule can generate higher profit to the marketplace.

Here is some examples for illustration. We fix $\alpha_H = .9$, change $\alpha_L = .6/.7/.8$ respectively, and show how does the marketplace’s profit under the two spotlight rules (y-axis) differ with respect to search cost (x-axis).

Figure 6: Marketplace’s profit under two spotlight rules: $\alpha_H = .9$ and $\alpha_L = .6$
Figure 7: Marketplace’s profit under two spotlight rules: $\alpha_H = .9$ and $\alpha_L = .7$

![Shape of Marketplace's Profit: $\alpha_H = .9; \alpha_L = .7$](image)

Figure 8: Marketplace’s profit under two spotlight rules: $\alpha_H = .9$ and $\alpha_L = .8$

![Shape of Marketplace's Profit: $\alpha_H = .9; \alpha_L = .8$](image)

We use numerical calculation of all possible values of $(\alpha_L, \tau)$ to show the above intuition, fixing $\alpha_H = .9$. By numerical calculate all possible situations, we find that given a search cost, the marketplace’s profit under no bidding rule is higher when $\alpha_L$ is small, and is lower when $\alpha_L$ is large. The results derives from the difference of commission fee rate. The difference of commission fee rate is increasing in $\alpha_L$, so the marketplace can earn more from
commission fee with increased $\alpha_L$. That effect dominates the increase in bidding revenues.

The following two figures illuminate the result. The x-axis is search cost ranging from 0 to possible maximal; and the y-axis is $\alpha_L$ ranging from .6 to .9 (the fixed value of $\alpha_H$). Every points on the figure is the marketplace’s profit difference under no bidding to under bidding rule on figure 9, its sign on figure 10, and commission fee rate difference on figure 11.

Figure 9: Difference of Marketplace’s profit under two spotlight rules: $\alpha_H = .9$
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Figure 10: Sign of Difference of Marketplace’s Profit under Two Spotlight Rules: $\alpha_H = .9$

Note: yellow = $\pi_{MP}$ is higher under bidding rule; blue = $\pi_{MP}$ is higher under no bidding rule

Figure 11: Difference of Marketplace’s Commission Fee Rate under Two Spotlight Rules: $\alpha_H = .9$
Next, we examine how the shift of product’s width of appealing affects the marketplace’s profit. To make the discussion easy, we fix $\alpha_H = 0.9$ and see how the profit is affected by $\alpha_L$. We also set outside profit $\pi_0$ at different levels, but it does not make much differences. The analysis has its own managerial implications: if the marketplace wants to screen sellers/products, it needs to consider how wide the range of products’ width of appealing it is willing to embrace. Should the marketplace sell only high width of appealing products, or should it allow a larger variety of products’ appliance. This exercise helps to answer managerial questions like this.

**Proposition 8.** The marketplace’s profit is non-linear $\alpha_L$. For some domain of search cost, the effect of $\alpha_L$ is decreasing, then increasing and then decreasing again, as a composed effect from pricing competition, shifts of optimal commission fee rate, and spotlight decisions.\(^17\) The effect is similar for optimal commission fee rate $\rho^*$, as well as ratio of bidding revenue $\Gamma$.

Figure 12: Comparative Statics of $\rho^*$

\(^{17}\)See appendix for math details.
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Figure 13: Comparative Statics of marketplace profit

Figure 14: Comparative Statics of $\Gamma$

The comparative statics of $\rho^*$, $\pi^{mp}$ and $\Gamma$ are illuminated in Figure 12, 13 and 14.
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respectively. The x-axis is domain of $\alpha_L$ from .2 to .9, and the y-axis is search cost $\tau$. On the figure, yellow stands for positive effect of $\alpha_L$, dark blue for negative effect and light blue for no effect. We can understand these comparative statics separately by different domain of search cost, as illuminated in figure 15. When $\tau$ is very small ($\tau < \tau_1$), product $L$ is displayed. When $\alpha_L$ is small, $V_{\min} = V_L$, $\rho^*$ is increasing in $\alpha_L$ as $V_L$ is increasing from a larger group of non-switchers. When $\alpha_L$ is large, $V_{\min} = V_H$ which is decreasing in $\alpha_L$, thus $\rho^*$ is also decreasing. The marketplace’s profit, however, is not always increasing in $\alpha_L$, even when the commission fee rate is increasing with a small $\alpha_L$. The effect is driven by more fierce pricing competition when the two products are less differentiated. The ratio of bidding revenue $\Gamma$ is zero and constant, as seller $H$ has no incentive to bid and the bidding revenue is always zero.

Figure 15: Threshold of search cost

When $\tau$ is modestly small ($\tau \in (\tau_1, \tau^*)$), product $L$ still wins the spotlight but need to pay for the location. In this case, the bidding expense also affects the value of $V_L$, but the shift of commission fee rate is similar to when $\tau$ is very small. The commission fee rate is
increasing when $\alpha_L$ is small and is decreasing when $\alpha_L$ is large. The marketplace’s profit is also increasing only with small $\alpha_L$ and decreasing otherwise, driven by both the commission fee rate change and more fierce competition. The change of $\Gamma$ is almost always opposite to that of $\rho^*$. $\Gamma$ is decreasing with small $\alpha_L$ and is increasing with large $\alpha_L$, driven mostly by the shift of $\rho^*$. When the commission fee rate decreases, bidding revenue becomes more important, and vice versa.

When $\tau$ is large ($\tau > \tau^*$), product $H$ wins the spotlight position. $V_{\min} = V_L$ always for this domain of search cost, and $V_L$ is increasing in $\alpha_L$, so commission fee rate is always increasing. In the same time, $\Gamma$ is decreasing, mostly driven by the change of commission fee rate. The marketplace’s profit is almost always increasing from an expanded demand, except a small region where $\alpha_L$ is large but $\tau$ is small where pricing competition is large.

The analysis has managerial implications for the operation of the marketplace. When the search cost is small, the marketplace shall not restrict niche products otherwise the pricing competition is too fierce. On the other hand, if the search cost is large, the marketplace should set a higher bar to exclude products with narrow demand. Meanwhile, the marketplace should focus more on the commission fees when search is easy, and more on the bidding market when search is difficult.

5 Conclusion

References

Yongmin Chen and Chuan He. Paid placement: Advertising and search on the internet*. 


A Proof

Proof of Lemma 1

We first show that the sellers’ prices are in the range of \([1 - \alpha_L, 1]\), and then use indifferent conditions to pin down the price distributions. The proof follows the logic of Varian (1980).

Since the consumer’s evaluation is at most 1, the sellers will not set the prices larger than 1. Seller \(H\) can at least get revenue of \(\alpha_H(1 - \alpha_L)\) when his price is 1 by fully exploiting his non-switchers, so he will not set the price lower than \(1 - \alpha_L\). Otherwise, his revenue is lower even if he is able to get demand from both switchers and non-switchers. Knowing that seller \(H\)’s lowest price is \(1 - \alpha_L\), seller \(L\) also has no incentive to set lower prices, as he is able to get the demand of switchers at that price. Hereby, we have shown that the price for both sellers are between \([1 - \alpha_L, 1]\).

Let the distribution of \(p_L\) be \(F_L(p)\), and that of \(p_H\) be \(F_H(p)\) with a probability mass \(\beta\) on \(\bar{p}_H = 1\)\(^{18}\). By using the iso-profit condition, we get the price distribution as following:

\[
F_L(p_L) = \frac{p_L - (1 - \alpha_L)}{\alpha_L p_L}
\]

where \(p_L \in [1 - \alpha_L, 1]\); and

\[
F_H(p_H) = \frac{p_H - (1 - \alpha_L)}{\alpha_H p_H}
\]

where \(p_H \in [1 - \alpha_L, 1]\); and probability mass \(\beta = 1 - \frac{\alpha_L}{\alpha_H}\) of \(p_H = 1\).

Given these distributions, one can verify that the expected profits for \(H\) product and \(L\) product are respectively given by \(\alpha_H(1 - \alpha_L)\) and \(\alpha_L(1 - \alpha_L)\), implying that \(H\) product is

\(^{18}\)If there is no probability mass on \(\bar{p}_H = 1\), the profit of seller \(L\) is always higher when his price is \(1 - \alpha_L\) than when his price is 1. That generates a contradiction.
more profitable than $L$ product. It is also straightforward to see that the expected price of product $H$ is higher than that of product $L$.

**Proof of Lemma 2**

When product $H$ is spotlighted, if product $H$ does not fit, consumer searches product $L$ when $\alpha_L(1 - p_L) \geq \tau$, or $p_L \leq 1 - \tau / \alpha_L$; if product $H$ fits, she searches product $L$ if it’s cheap enough:

$$\alpha_L(1 - p_L) + (1 - \alpha_L)(1 - p_H) - \tau \geq 1 - p_H \tag{5}$$

or $p_L \leq p_H - \tau / \alpha_L$.

From discussion in proof of Lemma 1, product $H$’s price is in the range of $[1 - \alpha_L, 1]$ at most. In reaction, product $L$ will not charge price higher than $1 - \tau / \alpha_L$ (otherwise, it has zero demand) or change price lower than $(1 - \alpha_H)(1 - \tau / \alpha_L)$ (otherwise, the profit is strictly lower than the price at $1 - \tau / \alpha_L$). Product $H$ can further adjust its prices depending on product $L$’s prices. Depending on the relative magnitude of $(1 - \alpha_H)(1 - \tau / \alpha_L) + \tau / \alpha_L$ and $1 - \alpha_L$, we discuss the two situations.$^{19}$

**Situation 1.**

When $(1 - \alpha_H)(1 - \tau / \alpha_L) + \tau / \alpha_L \geq 1 - \alpha_L$, or $\tau \geq \frac{\alpha_L}{\alpha_H}(\alpha_H - \alpha_L)$. When search cost is high, as prices of product $L$ are in the range of $[(1 - \alpha_H)(1 - \tau / \alpha_L), 1 - \tau / \alpha_L]$, product $H$ will not set the price lower than $(1 - \alpha_H)(1 - \tau / \alpha_L) + \tau / \alpha_L$. Consequently the price range of product $H$ is $[(1 - \alpha_H)(1 - \tau / \alpha_L) + \tau / \alpha_L, 1]$.

Let the distribution of $p_H$ be $F_H(p)$, and that of $p_L$ be $F_L(p)$ with a probability mass $\gamma$ on $\bar{p}_L = 1 - \tau / \alpha_L$.$^{20}$ By using the iso-profit condition, we get the price distribution as

$^{19}$We restrict the discussion to $\tau \leq \alpha_L$. When $\tau > \alpha_L$, product $L$ is never searched, and product $H$ will set monopolistic price at $p_H = 1$.

$^{20}$If there is no probability mass on $\bar{p}_L = 1 - \tau / \alpha_L$, the profit of seller $H$ is always higher when his price is $(1 - \alpha_H)(1 - \tau / \alpha_L) + \tau / \alpha_L$ than when his price is $1$. That generates a contradiction.
following:

\[ F_H(p_H) = \frac{(p_H - \tau/\alpha_L) - (1 - \tau/\alpha_L)(1 - \alpha_H)}{\alpha_H(p_H - \tau/\alpha_L)} \]

where \( p_H \in [(1 - \alpha_H)(1 - \tau/\alpha_L) + \tau/\alpha_L, 1] \); and

\[ F_L(p_L) = \frac{p_L - (1 - \tau/\alpha_L)(1 - \alpha_H)}{\alpha_L(p_L + \tau/\alpha_L)} \]

where \( p_L \in [(1 - \alpha_H)(1 - \tau/\alpha_L), 1 - \tau/\alpha_L] \) and probability mass of \( \gamma = \frac{\alpha_L \tau - (\alpha_H - \alpha_L)}{\alpha_L} \) at price \( p_L = 1 - \tau/\alpha_L \).

The expected revenues for product \( H \) and product \( L \) are \( \alpha_H((1 - \alpha_H)(1 - \tau/\alpha_L) + \tau/\alpha_L) \) and \( (1 - \alpha_H)\alpha_L(1 - \tau/\alpha_L) \) respectively. It is straightforward to see the former is increasing in \( \tau \) and the latter is decreasing.

Situation 2. when \( (1 - \alpha_H)(1 - \tau/\alpha_L) + \tau/\alpha_L > 1 - \alpha_L \), or \( \tau > \frac{\alpha_L \tau - (\alpha_H - \alpha_L)}{\alpha_L} \). When search cost is low, product \( L \) does not need to lower price to \( (1 - \alpha_H)(1 - \tau/\alpha_L) \) to win the switchers. Instead he only needs to lower the price to \( 1 - \alpha_L - \tau/\alpha_L \) as product \( H \) will not set the price to below \( 1 - \alpha_L \). Thus, the price range for product \( H \) is \([1 - \alpha_L, 1]\) and the price range for product \( L \) is \( [1 - \alpha_L - \tau/\alpha_L, 1 - \tau/\alpha_L] \).

Let the distribution of \( p_L \) be \( F_L(p) \), and that of \( p_H \) be \( F_H(p) \) with a probability mass \( \beta \) on \( \bar{p}_H = 1 \). By using the iso-profit condition, we get the price distribution as following:

\[ F_L(p_L) = \frac{p_L + \tau/\alpha_L - (1 - \alpha_L)}{\alpha_L(p_L + \tau/\alpha_L)} \]

where \( p_L \in [1 - \alpha_L - \tau/\alpha_L, 1 - \tau/\alpha_L] \); and

\[ F_H(p_H) = \frac{p_H - (1 - \alpha_L)}{\alpha_H(p_H - \tau/\alpha_L)} \]

where \( p_H \in [1 - \alpha_L, 1] \) and probability mass \( \beta = 1 - \frac{\alpha_L}{\alpha_H(1 - \tau/\alpha_L)} \).
The expected revenues for product $H$ and product $L$ are $(1 - \alpha_L)\alpha_H$ and $(1 - \alpha_L - \tau/\alpha_L)\alpha_L$ respectively. The former is irrelevant to $\tau$ and the latter is decreasing.

**Proof of Lemma 3**

Similar to the analysis in Lemma 2 if product $L$ does not fit, the consumer searches product $H$ when

$$\alpha_H(1 - p_H) \geq \tau$$

or $p_H \leq 1 - \tau/\alpha_H$; if product $L$ fits, she will search product $H$ when

$$\alpha_H(1 - p_H) + (1 - \alpha_H)(1 - p_L) - \tau \geq 1 - p_L$$

or $p_H \leq p_L - \tau/\alpha_H$.

Product $H$ will not set the price to be above $1 - \tau/\alpha_H$ (otherwise he has no demand), or set the price to be below $(1 - \alpha_L)(1 - \tau/\alpha_H)$ (otherwise the profit is less than the profit when price is $1 - \tau/\alpha_H$). As a reaction, the price of product $L$ is between $[(1 - \alpha_L)(1 - \tau/\alpha_H) + \tau/\alpha_H, 1]$. hereby, we have determined the price range of the two products.

Let the distribution of $p_L$ be $F_L(p)$, and that of $p_H$ be $F_H(p)$ with a probability mass $\beta$ on $\bar{p}_H = 1$. By using the iso-profit condition, we get the price distribution as following:

$$F_L(p_L) = \frac{p_L - \tau/\alpha_H - (1 - \alpha_L)(1 - \tau/\alpha_H)}{\alpha_L(p_L - \tau/\alpha_H)}$$

where $p_L \in [(1 - \alpha_L)(1 - \tau/\alpha_H) + \tau/\alpha_H, 1]$; and

$$F_H(p_H) = \frac{p_H - (1 - \alpha_L) + \frac{1-\alpha_L}{\alpha_H}\tau}{\alpha_H(p_H + \tau/\alpha_H)}$$

where $p_H \in [(1 - \alpha_L)(1 - \tau/\alpha_H), 1 - \tau/\alpha_H)$ and probability mass $\beta = 1 - \frac{\alpha_L}{\alpha_H} + \frac{\alpha_L}{\alpha_H}\tau$ at
The expected revenues for product $H$ and product $L$ are $\alpha_H(1 - \alpha_L)(1 - \tau/\alpha_H)$ and $(1 - \alpha_L)(1 - \tau/\alpha_H) + \tau/\alpha_H)\alpha_L$ respectively. The former is decreasing in $\tau$ and the latter is increasing.

**Proof of Proposition 1**

From Lemma 2 and 3 and as summarized in table 1 in High Spotlight,

$$p_H = 1 - \tau/\alpha_L.$$  

Total profit is

$$\pi_H^H + \pi_L^H = \begin{cases} 
(1 - \alpha_L)(\alpha_H + \alpha_L) - \tau & \text{if } \tau \leq \tau_1 \\
(1 - \alpha_H)(\alpha_H + \alpha_L) + \frac{\alpha^2_H}{\alpha_L} + \alpha_H\alpha_L - \alpha_L)\tau & \text{if } \tau > \tau_1
\end{cases}$$

It is straightforward to see $\pi_H^H + \pi_L^H$ is decreasing in $\tau$ when $\tau \leq \tau_1$ and increasing when $\tau > \tau_1$.

In Low Spotlight,

$$\pi_H^L = \alpha_H(1 - \alpha_L) - (1 - \alpha_L)\tau$$

$$\pi_L^L = \alpha_L(1 - \alpha_L) + \frac{\alpha^2_L}{\alpha_H}\tau$$

$$\pi_H^L + \pi_L^L = (\alpha_H + \alpha_L)(1 - \alpha_L) + \frac{1}{\alpha_H}(\alpha_H\alpha_L + \alpha^2_L - \alpha_L)\tau$$

We assume $\alpha_L$ is not too small so that $\alpha_H\alpha_L + \alpha^2_L - \alpha_H > 0$, or $\pi_H^L + \pi_L^L$ is increasing in
search cost.

By equating the total revenues under two display rules, we get \( \tau^* = \frac{\alpha_H \alpha_L}{\alpha_H + \alpha_L} \). The total revenue is higher under Low Spotlight (High Spotlight) when \( \tau \leq \tau^* \) \( (\tau > \tau^*) \).

**Proof of Proposition 2**

If \( k = HL \), as \( \pi_H^{HL} > \pi_L^{HL} \) for sure, \( \rho^* = 1 - \frac{\pi_0}{\pi_L^{HL}} \), and

\[
\pi^{MP} = \frac{\pi_L^{HL} - \pi_0}{\pi_L^{HL}} (\pi_H^{HL} + \pi_L^{HL}) \\
= \pi_H^{HL} + \pi_L^{HL} - \frac{\pi_H^{HL}}{\pi_L^{HL}} \pi_0 + \pi_0
\]

(6)

Similarly, if \( k = LH \), when \( \alpha_H(1 - \alpha_L) \geq \alpha_L \), \( \pi_H^{LH} \geq \pi_L^{LH} \) for any search cost, \( \rho^* = 1 - \frac{\pi_0}{\pi_L^{LH}} \), and

\[
\pi^{MP} = \frac{\pi_L^{LH} - \pi_0}{\pi_L^{LH}} (\pi_H^{LH} + \pi_L^{LH}) \\
= \pi_H^{LH} + \pi_L^{LH} - \frac{\pi_H^{LH}}{\pi_L^{LH}} \pi_0 + \pi_0
\]

(7)

The difference of the marketplace’s profit under the two display orders is

\[
\Delta \pi = ((\pi_H^{HL} + \pi_L^{HL}) - (\pi_H^{LH} + \pi_L^{LH})) - (\frac{\pi_H^{HL}}{\pi_L^{HL}} - \frac{\pi_H^{LH}}{\pi_L^{LH}}) \pi_0
\]

(8)

The first term is the difference of total profits, and the second term is the difference of profits gained by the sellers (product \( H \) in this case), and \( \frac{\pi_H^{HL}}{\pi_L^{HL}} - \frac{\pi_H^{LH}}{\pi_L^{LH}} > 0 \) always. If \( \Delta \pi \) is positive, the marketplace will spotlight product \( H \) and charge commission fee rate \( 1 - \frac{\pi_0}{\pi_L^{HL}} \); otherwise, the marketplace will spotlight product \( L \) and charge commission fee rate \( 1 - \frac{\pi_0}{\pi_L^{LH}} \). As \( \pi_L^{LH} > \pi_L^{HL} \), the commission fee rate is higher when product \( L \) is spotlighted.

Having characterized the marketplace’s decision on spotlight and the corresponding commission fee rate, we examine when \( \Delta \pi \) is positive (negative). Notice that \( \frac{\pi_H^{HL}}{\pi_L^{HL}} - \frac{\pi_H^{LH}}{\pi_L^{LH}} \) is always positive, so \( \Delta \pi \) would be positive when: 1. the difference of total profits \( ((\pi_H^{HL} + \pi_L^{HL}) - \pi_L^{LH} \)
\(\pi^H_L - (\pi^L_H + \pi^L_L)\) is positive; 2. \(\pi_0\) is sufficiently small. When \(\tau \leq \tau^*\), the difference of total profits is negative; and when \(\tau \geq \tau_3\), that term is zero, so in these two cases, \(\Delta \pi\) is negative. When \(\tau^* < \tau < \tau_3\), define the threshold of outside option as \(\pi^*_0 = \frac{(\pi^H_L + \pi^L_L) - (\pi^L_H + \pi^L_L)}{\pi^H_L - \pi^L_L}\).

When \(\pi^*_0 \leq \pi^*_L\), product \(H\) is spotlighted; otherwise, product \(L\) is spotlighted.

**Proof of Corollary 1**

From proof of proposition 1, \(\tau^* = \frac{\alpha_H \alpha_L}{\alpha_H + \alpha_L} = \frac{1}{\alpha_H} + \frac{1}{\alpha_L}\). The results follow.

**Proof of Proposition 3**

(i). If product \(L\) is spotlighted, the revenue of the two sellers are

\[
\pi^L_H = \alpha_H(1 - \alpha_L) - (1 - \alpha_L)\tau
\]

\[
\pi^L_L = \alpha_L(1 - \alpha_L) + \frac{\alpha_L^2}{\alpha_H} \tau
\]

As \(\pi^L_L\) is increasing in \(\tau\) and \(\pi^L_H\) is decreasing, there is a threshold \(\hat{\tau}\) such that \(\pi^*_{\min} = \pi^L\) when \(\tau < \hat{\tau}\) and \(\pi^*_{\min} = \pi^H\) when \(\tau > \hat{\tau}\). By equation the revenues of the two sellers, we get \(\hat{\tau} = \frac{\alpha_H(\alpha_H - \alpha_L)(1 - \alpha_L)}{\alpha_H - \alpha_H \alpha_L + \alpha_L^2}\). So the optimal commission fee rate is increasing when \(\tau < \hat{\tau}\) and is decreasing otherwise, from the monotonicity of \(\pi^*_{\min}\).

(ii). If product \(H\) is spotlighted, product \(L\) is always the binding product. As its revenue is decreasing in search cost, so is the optimal commission fee rate.

(iii). If product spotlight decision changes, it could only occur when \(\tau > \tau^*\) from Proposition 2. In this case, the total revenue in High Spotlight is higher. When spotlight decision shifts from product \(L\) to \(H\), total revenue jumps up. To keep the marketplace’s profit continuous at the shift point, it must be that the optimal commission fee rate jumps down.

**Proof of Lemma 4**
The profit of seller $i$ with display order $k$ is given in lemma 2 and lemma 3. It’s straightforward from the equations that the seller’s revenue is higher when his product is spotlighted.

**Proof of Proposition 4**

The sellers’ bidding are $(1 - \rho)(\pi_{HL}^H - \pi_{LH}^H)$ and $(1 - \rho)(\pi_{LH}^L - \pi_{HL}^L)$ respectively. To show that $(1 - \rho)(\pi_{HL}^H - \pi_{LH}^H) > (1 - \rho)(\pi_{H}^L - \pi_{HL}^H)$ when $\tau < \tau^*$, it’s sufficient to show $\pi_{HL}^H + \pi_{LH}^H > \pi_{L}^H + \pi_{HL}^H$ when $\tau < \tau^*$.

The other case when $\tau > \tau^*$ is the same. The rest follows the proof of proposition 2.

**Proof of Proposition 5**

The sellers’ bidding are $b_H = \pi_{HL}^H - \pi_{LH}^H$ and $b_L = \pi_{LH}^L - \pi_{HL}^L$, or

$$b_H = \begin{cases} 
\tau & \text{if } \tau < \tau_1 \\
-(\alpha_H - \alpha_L)\alpha_H + (\frac{\alpha^2}{\alpha_H} + 1 - \alpha_L)\tau & \text{if } \tau > \tau_1 \& \tau < \alpha_L 
\end{cases}$$

$$b_L = \begin{cases} 
(\frac{\alpha^2}{\alpha_L} + 1)\tau & \text{if } \tau < \tau_1 \\
(\alpha_H - \alpha_L)\alpha_L + (\frac{\alpha^2}{\alpha_H} + 1 - \alpha_H)\tau & \text{if } \tau > \tau_1 \& \tau < \alpha_H 
\end{cases}$$

When $\tau \leq \tau^*$, product $L$ wins the auction. We get the seller’s profit as

$$V_H = \alpha_H(1 - \alpha_L) - (1 - \alpha_L)\tau$$

$$V_L = \begin{cases} 
\alpha_L(1 - \alpha_L) - (1 - \alpha_L - \frac{\alpha^2}{\alpha_H})\tau & \text{if } \tau < \tau_1 \\
(1 - \alpha_L)\alpha_L - (\alpha_H - \alpha_L)\alpha_H - (1 - \alpha_L + \frac{\alpha^2}{\alpha_L} - \frac{\alpha^2}{\alpha_H})\tau & \text{if } \tau > \tau_1 \& \tau \leq \tau^* 
\end{cases}$$

Both $V_H$ and $V_L$ are decreasing in search cost.

When $\tau > \tau^*$, product $H$ wins the auction. We get the seller’s profit as

$$V_H = \alpha_H(1 - \alpha_H) - \alpha_L(\alpha_H - \alpha_L) - (1 - \alpha_H - (\frac{\alpha^2}{\alpha_H} - \frac{\alpha^2}{\alpha_L}))\tau$$
\[ V_L = (1 - \alpha_H)\alpha_L - (1 - \alpha_H)\tau \]

\( V_L \) is decreasing. \( V_H \) is increasing (decreasing) when \( 1 - \alpha_H - (\frac{\alpha_H^2}{\alpha_L} - \frac{\alpha_L^2}{\alpha_H}) \) is negative (positive), which happens when \( \alpha_L \) is large (small).

**Proof of Proposition 6**

To show that \( \rho^* = 1 - \frac{\pi_0}{V_{\text{min}}} \), it is sufficient to prove that the marketplace will set the commission fee rate as high as possible. To see this, notice the marketplace’s total profit is a weighted average of commission fees (when \( \rho = 1 \)) and bidding revenues (when \( \rho = 0 \)) and the commission fee rate \( \rho \) is the weight. From Table 2 the profit of the marketplace is \( \rho(\pi_H + \pi_L) + (1 - \rho)(\pi_H - \pi_L) \) in LH order. We can show that \( \pi_H + \pi_L > \pi_H - \pi_L \), so the marketplace will set the commission fee rate as high as possible. To see this, \( \pi_H + \pi_L > \pi_H - \pi_L \) and the first inequality sign follows proposition 4. The condition also holds when \( HL \) order is chosen. Under \( HL \) order, \( \pi_H + \pi_L > \pi_H + \pi_L > \pi_H - \pi_L \). So the marketplace tends to set a high commission fee rate but the seller’s profit is decreasing in the commission fee rate, so seller’s participation condition binds. The result follows.

We also need to show when \( V_{\text{min}} = V_L \). From Proposition 5

\[
V_H - V_L = \begin{cases} 
(\alpha_H - \alpha_L)(1 - \alpha_L) - \frac{\alpha_L^2}{\alpha_H} \tau & \text{if } \tau < \tau_1 \\
(\alpha_H - \alpha_L)(1 - \alpha_H - \alpha_L) + (\frac{\alpha_H^2}{\alpha_L} - \frac{\alpha_L^2}{\alpha_H}) \tau & \text{if } \tau \geq \tau_1 
\end{cases}
\]

So \( V_H - V_L \) is decreasing when \( \tau < \tau_1 \) and increasing otherwise. Also note that \( V_H - V_L > 0 \) at \( \tau = 0 \) and \( \tau = \tau^* \). If at \( \tau = \tau_1 \), \( V_H - V_L < 0 \), by continuity there are \( \tau^\dagger \) and \( \tau^\ddagger \) such that

\[
V_{\text{min}} = \begin{cases} 
V_L & \tau < \tau^\dagger \leq \tau \leq \tau^\ddagger \\
V_H & \tau \in [\tau^\ddagger, \tau^*] 
\end{cases}
\]

where \( V_H = V_L \) at \( \tau^\dagger \) and \( \tau^\ddagger \), and \( \tau^\dagger < \tau_1 < \tau^\ddagger < \tau^* \).
A. PROOF

If $V_H - V_L > 0$ at $\tau = \tau_1$, then $V_{\min} = V_L$ for any value of $\tau$.

at $\tau = \tau_1$, $V_H - V_L = (\alpha_H - \alpha_L)(\alpha_H^2 - \alpha_L(\alpha_H^2 + \alpha_L^2))$. So $V_H - V_L < 0$ when $\alpha_H^2 < \alpha_L(\alpha_H^2 + \alpha_L^2)$. The inequality holds when $\alpha_L$ is large.

Proof of Corollary 2

When $\tau \leq \tau^*$, both $V_H$ and $V_L$ are decreasing in search cost (see proof of Proposition 5), and so is $V_{\min}$. When $\tau > \tau^*$, $V_{\min} = V_L$ and is decreasing in search cost. So optimal commission fee rate is also decreasing.

The part that $\rho^*$ is decreasing in outside profit $\pi_0$ is straightforward. As $\rho^* = 1 - \frac{\pi_0}{V_{\min}}$ and $V_{\min}$ is not affected by $\pi_0$, the result follows.

Proof of Corollary 3

Recall that

$$V_{\min} = \begin{cases} V_L & \text{High Spotlight} \\ \min\{V_H, V_L\} & \text{Low Spotlight} \end{cases}$$

As $V_j \leq \pi_j$ for given display order, so $V_{\min} \leq \pi_{\min}$. The result follows.

Proof of Proposition ??

Recall that the profit of the marketplace is $\rho^*(\pi_{H}^{LH} + \pi_{L}^{LH}) + (1 - \rho^*)(\pi_{H}^{HL} - \pi_{H}^{LH})$ in LH order and $\rho^*(\pi_{H}^{HL} + \pi_{L}^{HL}) + (1 - \rho^*)(\pi_{L}^{LH} - \pi_{L}^{HL})$ in HL order. To analyze the comparative statics of $\tau$, we separately discuss the effect on commission fee rate $\rho^*$, total profit $\pi_{H}^{LH} + \pi_{L}^{LH}$ (or $\pi_{H}^{HL} + \pi_{L}^{HL}$ in HL order) and bidding revenue $\pi_{H}^{HL} - \pi_{H}^{LH}$ (or $\pi_{L}^{LH} - \pi_{L}^{HL}$ in HL order).

As search cost increases, the total profit increases. When search becomes more difficult, consumers are less intend to search underplayed product so the competition between sellers
is softened, the total profit increases consequently. Also, the bidding revenue increases in search cost. When search becomes more difficult, the spotlight position becomes more profitable to the sellers, so they are willing to bid more for the spotlight position. Finally, the effect on commission fee rate is non-monotonic, as analyzed in corollary 2. By combining these three effects, the marketplace’s profit is non-monotonic in search cost, and the result is driven by the shift of commission fee rate.

We discuss the comparative statics case by case by different domains of search cost. First, we assume $V_{\min} = V_L$ for all $\tau$, and discuss other cases later.

When $\tau < \tau_1$, bid $= \pi^H_L - \pi^L_H = 0$, the profit of the marketplace $\pi^p = (1 - \pi_0/\pi^L_H)(\pi^L_H + \pi^L_L)$. As both $1 - \pi_0/\pi^L_H$ and $\pi^L_H + \pi^L_L$ are increasing in $\tau$, the marketplace’s profit increases.

When $\tau \in (\tau_1, \tau^*)$, the marketplace’s profit is $(1 - \pi_0/(\pi^L_H - \pi^H_L + \pi^L_H))(\pi^L_H + \pi^L_L) + \pi_0/(\pi^L_H - \pi^H_L + \pi^L_H)(\pi^H_L - \pi^L_H)$. The commission fee rate is decreasing, while the total revenue of the two sellers and the bidding are increasing. The total effect is undetermined, as illuminated on figure ?? in the main text. Various numerical calculations show that the effect is positive (marked yellow in figures) when $\alpha_L$ is close to $\alpha_H$, but negative (marked dark blue) when $\alpha_L$ is sufficiently small (compared to $\alpha_H$). Figure ?? illuminates the effect at search cost $\tau^*$. 
Figure A.1: Range of parameter where $V_L > V_H$

When $\tau \in (\tau^*, \tau_2)$, the profit is $\pi^p = (1 - \pi_0 / \pi_L^H)(\pi_H^L + \pi_L^H) + (\pi_0 / \pi_L^H)(\pi_L^H - \pi_L^H)$. Now the commission fee rate $(1 - \pi_0 / \pi_L^H)$ is flat in $\tau$ as $\pi_L^H$ is constant to $\tau$. Both commission fee and bidding are increasing in search cost.

Similarly, when $\tau \in (\tau_2, \tau_3)$, $\pi^p = (1 - \pi_0 / \pi_L^H)(\pi_H^L + \pi_L^H) + \pi_0 / \pi_L^H(\pi_L^H - \pi_L^H)$. The commission fee rate and total revenue of the two sellers are constant, and the bidding is increasing. So the commission fee is constant, but profit from bidding is increasing in search cost.

For $\tau > \tau_3$, both commission fee and profit from bidding are constant, so is the commission fee rate.

If $V_{\min} = V_L$ does not hold for every value of search cost. We have $V_{\min} = V_H$ when $\tau \in (\tau^\dagger, \tau^\ddagger)$ where $\tau^\dagger < \tau_1 < \tau^\ddagger < \tau^*$. The discussion for $\tau < \tau^\dagger$ or $\tau > \tau^\ddagger$ holds as stated above. For $\tau \in (\tau^\dagger, \tau^\ddagger)$, as $V_H$ is constant in this domain of search cost, we have that both commission fee and profit from bidding are increasing in search cost. That completes the proof. Figure A.2 provides an illustration.
Proof of Proposition 7

The comparative statics of \( \Gamma \) to \( \tau \) is illuminated by figure 4. To show the comparative statics, we separately look at the effect on commission fee rate, bidding revenue and total revenues. Commission fee rate is non-monotonic in \( \tau \) as analyzed in corollary ??, while that bidding revenue and total revenues are both increasing in \( \tau \). Combining the three effects, which effect is dominant depends on domain of search cost, as we discuss below.

When \( \tau < \tau_1 \), seller \( L \) can win the spotlight with no cost, so the bidding revenue is zero. \( \Gamma \) is zero no matter how commission fee rate or total profit change with search cost.

When \( \tau \in [\tau_1, \tau^*), \rho^* \) is decreasing, so \( \frac{1-\rho^*}{\rho^*} \) is increasing. Even though the effect on \( \gamma \) can be not determined, the overall effect on \( \Gamma \) is positive, as shown from numerical calculation.
Lastly, when $\tau \geq \tau^*$, the commission fee rate is flat in search cost, and $\gamma$ can be increasing or decreasing. So the effect on $\Gamma$ is undetermined. However, numerical calculation shows that the effect is almost positive only except a small region where $\alpha_L$ is small and search cost is modest.

By combining the three domains of search cost, we show that $\Gamma$ is increasing in most domain of search cost $\tau$, only except a small region where $\alpha_L$ is small and search cost is modest. An illuminative figure is provided in figure A.3.

The second result that $\Gamma$ is increasing in $\pi_0$ is straightforward and provided in the main text.

Figure A.3: Range of parameter where $V_L > V_H$

Proof of Proposition 8

By numerical method, with a grid of .0014 for $\alpha_L$ on x-axis and .001 for $\tau$ on y-axis. The
attached figures show the comparative statics for the domain where \( \tau \in (\tau_1, \tau^*) \) for better illustration.

Figure A.4: Comparative Statics of \( \rho^* \)
Figure A.5: Comparative Statics of marketplace profit

Figure A.6: Comparative Statics of $\Gamma$
Figure B.1: Sponsored Ads on the First Page of Search Result at Amazon Marketplace
Figure B.2: Sponsored Ads below a First-Page Sponsored Product at Amazon Market