Optimal dynamic pricing of green goods under discounted network effects

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Abstract

This paper deals with optimal dynamic pricing for a dominant firm selling a green good, whose consumption generates network/conformity effects. We explicitly model the accumulation process for the network, with consumers discounting differently the choices of different cohorts of consumers, as a reflection of the fact that current consumers interact more with similarly aged cohorts. With linear per period demand, we get a unique closed-form solution for the infinite-horizon problem faced by the green good producer. The optimal pricing strategy and the evolution and steady-state network size for the green good crucially depend on the intensity of network effects. For weak peer effects, the steady-state network size is finite, whereas strong peer effects lead to "universal adoption". We compare the private optimal solution to the first-best outcome patronized by a benevolent social planner choosing to maximize overall welfare in order to account for both network externalities and environmental externalities. We find that the social planner would favor universal adoption more frequently (i.e. for a wider domain of network effects) than the dominant firm. We also find that even when both the dominant firm and the social planner endorse universal adoption, the network expansion is slower in the private solution than in the first-best outcome, opening the room for policy intervention.

JEL codes: Q50, L19, Q58.

Keywords: environmental goods, network effects, dynamic pricing, industry launch.

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1 Introduction

It is well established that environmental concerns influence consumers’ purchasing decisions (Björner et al. 2004; Teisl et al. 2002). For instance, the European Commission (2008) estimates that 75% of consumers are willing to pay a bit more for green products. Multiple reasons can be put forward to explain this "green premium", including "warm glow" effects, altruistic concerns – e.g., consumers wish to contribute to a healthier environment – and purely self-interest motives – e.g., avoiding health risks or having a feeling of compliance with ethical standards. A recent strand of literature has stressed that consumers’ behavior towards environmentally friendly goods may be affected by their perception of the behavior of their peers (Brécard, 2013, Hauck et al., 2014). Such a bandwagon effect may be partly due to a conspicuous consumption effect, or consumers wishing to keep up with, or to avoid criticism from, their neighbors, colleagues or their role models who favor the consumption of green goods.\footnote{Starting from Veblen (1887)’s pioneering book, several scholars have proposed different models to study various forms of conformity-based behavior in quite diverse economic settings (see e.g., Leibenstein, 1950, Bernheim, 1994, Karni and Levin, 1994, and Hong and Konrad, 1998). Other applications are covered below.} In addition, information transmission about the nature of the product and the availability of possible subsidy schemes may be gained through social networks. Such effects may also arise from a more rational view that the global effectiveness of a green good or technology on the environment depends on the number of users adopting it, and it might be very low or negligible when that number is too low. When this is the case, consumers are more inclined to join an environmental bandwagon the higher the chance of overall success they attribute to it, as attested by substantial survey and casual evidence.\footnote{Wiser (2007) finds that americans who are willing to pay a premium for renewable energy in the US believe other americans are also willing to do so. The same type of result is obtained by Welsch and Kuhling (2009) in Germany (for the case of solar thermal systems and green electricity). Indeed, the environmental effectiveness of domestic solar panels will be very low if only a limited number of households invest in this technology. As a result of this environmental externality, we expect consumers’ decision to acquire solar panels to depend substantially on the perceived number of households already using this technology.} A good example of a green good with network externalities is the case of electric vehicles (EV) being dependent on charging (hydrogen filling stations), leading to indirect network effects (see e.g., Greaker and Heggedahl (2010) or Greaker and Midttomme (2016)): As more consumers adopt the product, the easier it will be to find charging stations (e.g., Greaker and Heggedahl (2010) develop a one-shot model in which the density of filling stations is proportional to the number of EV adopters) and hence the higher is the utility of the consumers that opt for EV. On the environmental side, EV adoption allows for
the reduction of emissions. This, the global environmental effectiveness of the EV depends on the number of past adopters, in an effect that is akin to network externalities.\textsuperscript{3,4} When the consumers’ individual utility depends on the choices of other consumers in the market (as in the case of EV described above), the consumption of green goods is subject to network externalities of the sort investigated in the seminal studies by Rohlfs (1974), Katz and Shapiro (1985), Grilo et al. (2001) or Amir and Lazzati (2011).

A recent but growing literature, based on empirical, experimental and survey contributions, has established a significant role for network effects or social interaction on the dynamics of consumer adoption of green goods and technologies. As for experimental research, Pieters et al. (1998) show that the attitudes of Dutch consumers towards green products is affected by their perceptions of the behavior of other households. Frey and Stutzer (2008) find that 20% of American consumers consider word of mouth a key determinant of their choice to buy green. Finally, Jansen and Jager (2002) find that social processing and status-seeking are important determinants in the diffusion processes of green goods.

On the empirical side, the relevant contributions are even more recent. For example, in the context of energy systems, there is recent empirical evidence on how network effects have contributed to the increasing adoption of residential solar photovoltaic panels in various geographical areas. In a detailed and methodologically well-founded study of the diffusion of solar panels in California with data covering 11 years from 2001, Bollinger and Gillingham (2012) find "\textit{strong evidence for causal peer effects}" in inducing residents of particular areas to make the financially demanding decision to adopt residential solar panels. Similar results are reported by Rai and Robinson (2013). Other studies with similar findings are Graziano and Gillingham (2015), Noll et al. (2014) and Rode and Alexander (2016). In a study of group adoption of solar panels in Connecticut, Jacobsen et al. (2013) find that community-based financial incentives (in the form of subsidies for groups of buyers) are quite effective, but that the size of the subsidy itself appears less important. This is suggestive of a key role played by peer effects.

Despite the multi-faceted evidence pointing to the influence of network effects on the consump-

\textsuperscript{3}As network effects are considered as a positive externality on the demand side, it is worth adding that what might be viewed as the mirror image notion on the supply side, namely learning or knowledge spillovers, have also been recently investigated in the environmental economics literature (see e.g., Requate and Reichenbach, 2012, McDonald and Poyago-Theotoky, 2016, and Jaffe, Newell, and Stavins, 2005).

\textsuperscript{4}Throughout the text we shall use the terms network effects, banwagon effects, and peer effects interchangeably, when referring to the social dimension of the consumption of the green good.
tion of green goods, only a few studies so far have developed analytical models of consumption of green goods with network effects. Brécard (2013) and Hauck et al. (2014) propose static analytical models of vertical differentiation with network externalities in order to study optimal quality choices and pricing decisions of the green goods vis-à-vis brown goods. Hauck et al. (2014) extends Brécard (2013) in order to allow for partial market coverage. They find that as the intensity of network effects goes up: (i) the average quality of the environmental goods in the market may go down; (ii) total welfare may go down; (iii) there is room for regulatory intervention (through minimum environmental quality standard). Both models have a static nature and discuss the question of introduction and take-off of a green good with peer effects in consumption. Greaker and Midttømme (2016) study the diffusion of a clean substitute to a horizontally differentiated dirty durable good in a repeated multi-stage game wherein the current network includes past purchases with no discounting, and consumer utility for both durables increases in their respective market shares due to network effects.

In the present paper, we consider an environmentally friendly good or technology provided by a dominant firm, which is a substitute for a lower-quality brown (polluting) good supplied by a competitive fringe\(^5\). We investigate how pricing, adoption dynamics and social welfare are affected by the existence of environmental network effects in an infinite-horizon, discrete-time setting. To fix ideas, a good concrete example to keep in mind for this paper is that of EV, in which the density of filling stations depends on the accumulated number of EV adopters, so that the environmental value of the good depends on the total accumulated number of buyers so far, as in Gabszewicz and Garcia (2007, 2008), with the novel feature that each cohort’s additive contribution is discounted backwards. This serves twin purposes: It is a realistic way to account for the fact that current buyers interact more intensely with more recent cohorts, and a convenient way of ensuring a bounded network size over time. A key advantage of modeling the network formation process as an explicit dynamic recursion leads to the conclusion that the steady-state network size is the natural long run market size for the environmental industry at hand. In particular, a zero steady state will

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\(^5\)This market structure has also been considered in Greaker and Midttomme (2016), corresponding to the case of a monopolist supplier of the clean good and many suppliers of the dirty good. The authors also look at two other market configurations: monopoly provision of both goods (green and brown) and many suppliers of both goods. However, the case of the monopolist supplier of the green good and many suppliers of the brown good (which is the one we study here) is the most interesting one, as it captures the market asymmetry which often characterizes the provision of green and brown goods (which is quite evident, for example, in the case of EV, with the market for internal combustion engine vehicles being much more mature than EV).
mean a failed launch and an infinite steady state will reflect universal adoption. This allows us to establish an interesting link between the dynamic and the static formulations of network industries, by identifying the steady-state network with its static counterpart, namely what Katz and Shapiro (1985) called the fulfilled expectations network size. The present definition enjoys the advantage of avoiding any role for coordinated expectations on the part of both consumers and firms, a feature that makes Katz and Shapiro’s solution concept controversial due to its somewhat ad hoc nature.

In each period, there is a continuum of consumers differentiated by their concern for the environment, each of whom makes a one-time purchase of zero or one unit of the green good or the brown good, upon observing the current prices. Consumers cannot postpone their consumption choices. The dominant firm’s objective is to maximize its total discounted accumulated profits from selling the green good, while the fringe firms price the brown good at marginal cost.

Postulating a linear demand in both concurrent output and network size as a simplifying assumption, we address the following questions: What is the optimal pricing policy of a dominant firm selling a green good with dynamic network effects? How would the network evolve under the private optimal outcome versus a welfare-maximizing social planner solution? What intervention might be needed to set the industry in motion and launch a green good?

We derive a closed form solution for the linear-quadratic dynamic programming problem at hand and a characterization of the optimal pricing policy resulting from the green producers’ dynamic profit maximization problem. We investigate the properties of the resulting steady-state network size. We find that optimal prices are increasing in the current network size (i.e., the base of adopters of the green good). For weak network effects, price is strictly increasing in the network size. For sufficiently strong network effects, the green good producer would be interested in subsidizing initial cohorts of consumers in order to boost network effects (and increase future consumers’ willingness to pay for the green good). In this context, the green good producer sells its product at marginal cost for a finite number of periods, until the market gains momentum. Afterwards it starts making positive profits (with optimal prices being increasing in the network size). The steady-state properties are also crucially determined by the intensity of network effects. The dominant firm’s optimal pricing policy leads to universal adoption, when the network effects are strong enough. For weak network effects, there is a finite positive steady state.

The solution of the green producer’s dynamic problem (private solution) is then compared to the welfare-maximizing solution, which account’s not only for consumers’ individual utility but also for the environmental externality associated with the consumption of the green (less pollutant
Our results show that the social planner is only interested in patronizing the green good if its environmental quality is sufficiently superior to the environmental quality of the brown good. When that is the case, we find that the first-best pricing policy requires marginal cost pricing so that the network evolves as fast as possible. When network effects are sufficiently strong (with the proviso of a sufficiently large environmental quality gap), the social planner always fosters universal adoption of the green good. Only for weak network effects, the social planner would set a finite steady state. Our analysis reveals that a social planner would favor universal coverage more often than the dominant firm does. In particular, when the network effects are of intermediate strength, the social planner is interested in promoting universal coverage while the dominant firm is not. Moreover, when the networks are sufficiently strong (for both the social planner and the firm to envisage universal coverage), the network expansion in the optimal private solution is slower than in the first-best counterpart.

Our results enrich two lines of literature. First, we contribute to the recent theoretical literature studying how the adoption of green goods is affected by warm glow considerations related to peer effects. To the best of our knowledge, only Greaker and Midttømme (2016) have looked at this issue in a dynamic framework. They investigate the dynamics of diffusion of a green good with network effects that competes with an horizontally differentiated dirty good (also exhibiting network effects). Their analysis is focused on the domain of parameters for which a finite steady state is reached after a finite number of periods. They are particularly interested in comparing the private solution to the welfare maximizing solution. They find that there may be room for excess inertia and they show that the optimal tax level (yielding the socially optimal diffusion rates) depends on the network size (even if the marginal environmental damage does not). We depart from Greaker and Midttomme (2016) along several dimensions. First, we do not restrict our analysis to the cases in which steady-state network sizes are finite and we also investigate the economic and environmental properties of unbounded solutions, in which the universal adoption of the green good arises. Second, we propose a vertically differentiated set-up in which, at similar prices, all the consumers patronize the green good in detriment of the dirty good. Third, we allow for some asymmetry in the degree of the firms’ cost structure, reflecting a more realistic set-up in which the (higher-quality) green good is costlier to produce than the brown good. Finally, the nature of network effects in our model is quite different from Greaker and Midttomme (2016). While they look at network effects associated with future aftermarket benefits (in the tradition of Cabral (2011), Laussel and Resende (2014) or Long et al. (2015)) that may be generated both by the green good or the dirty good, we are more
interested in studying network effects that are specific to the green good itself, like for instance, phenomena of word of mouth related to green consciousness, warm glow, peer effects or the process of accumulating a critical mass of users in order to create the conditions for the development of the green good (e.g. the deployment of a charging infrastructure is an important condition to promote the take-off of the EV industry; similarly, it is important to have a critical mass of users in order to induce technology advancements in the production of solar panels and make them more attractive to consumers). Indeed, in a dynamic framework, the density of hydrogen charging stations at a given point in time, must depend on the accumulated number of EV adopters (with recent cohorts of consumers obviously having a great weight than the initial generations of consumers), corroborating our model with discounted backwards network effects.

The present paper also contributes to the industrial organization literature showing that important aspects of consumers’ and firms’ decisions in markets with network effects can only be captured within a fully dynamic set-up, either in the case of a monopoly industry (see e.g. Dhebar and Oren, 1985, 1986; Xie and Sirbu, 1995; Mason, 2000; Fudenberg and Tirole, 2000; Gabszewicz and Garcia, 2007, 2008; Laussel et al, 2015) or in an oligopoly framework (see, e.g. Laussel et al., 2004; Mitchell and Skrzypacz, 2006; Markovich and Moenius, 2009; Cabral, 2011; Laussel and Resende, 2014; Garcia and Resende, 2016). The present paper introduces the idea of decaying network effects over time to the literature on explicit dynamics of network effects by allowing for consumers to discount past sales in their assessment of the current importance of the network. As the firm discounts the future stream of profits as well, we propose a model with a double discounting structure. The closest papers to our work are Gabzewicz and Garcia (2007, 2008) who also study dynamic models of cumulative backwards network effects, without accounting for the environmental properties of the network good. Moreover, their modelling options are based on a finite time horizon setting without either type of discounting. We find that consumers and firms’ discounting factors play an important role in shaping the properties of the green good’s optimal dynamic pricing. In particular, we show that as firms become more impatient, steady-state prices may go up or down depending on the intensity of network effects. Differently, as consumers become more influenced by recent purchases (in the sense they value less the choice of early cohorts of consumers), their willingness to pay for the green good goes down. This necessarily leads to lower steady-state prices and translates into a narrower steady-state network for the environmentally friendly good.

This paper proceeds as follows: In Section 2 we introduce the model and the main results. Section 3 analyzes the welfare-maximizing outcomes, stressing some environmental policy implications.
Finally, Section 4 concludes and discusses possible further research.

2 The model and main results

This section lays out the basic dynamic pricing model for a dominant firm selling a green good characterized by network externalities over an infinite horizon. The solution is then solved for analytically (in closed-form), and a full characterization of its properties is provided.

2.1 The basic model

Consider an industry in which there are two variants of a product: a green good and a brown good, which are distinguished by their level of emissions. The two goods are substitutes, and we adopt the common assumption in the literature on network externalities that each consumer in this economy buys (at most) one unit of one of the variants.

At each period $t$, a new cohort of consumers enters the market. Consumers value each of the variants for its intrinsic utility (or stand-alone value), which depends on the environmental properties of each good. The green good in addition is valued for the network benefits that it entails; in other words, the green good is a mixed network good (see Amir and Lazzatti, 2011 for a discussion). Consumers are heterogeneous in their concerns for the environment so that a consumer of type $x$, with $x$ uniformly distributed in $[0, A]$ obtains utility $u^G_t(x)$ when he/she buys the green good, with:

$$u^G_t(x) = U + xq^G + \alpha s_t - P^G_t,$$

where $q^G$ stands for the environmental quality of the green good, the term $\alpha s_t$ represents the network effect in the consumption of the green good, the parameter $\alpha (\alpha > 0)$ measures the intensity of such network effect, and $U$ is the utility of the product that is independent of the environmental quality and of the network effect. The price of the green good good in period $t$ is denoted by $P^G_t$.

The relevant size of the network of the green good at time $t$ is $s_t$, which is the discounted accumulated demand of the green good until period $t - 1$. Alternatively, $s_t$ may be viewed as the discounted total actual past sales so that it naturally constitutes the relevant network and it is given

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6Note that the increment in the term $\alpha s_t$ can also be interpreted as a reduced-form utility component representing an increase in the quality of the product that stems from past usage.
by

\[ s_t = \sum_{i=0}^{t-1} \beta^{t-i} d_t^{G} + \beta^t s_0, \]  

(2)

where \( d_t^{G} \) represents the demand for the green good at time \( t \). Here, \( \beta \in (0, 1) \) is the discount factor used by consumers to evaluate the present backwards-discounted value of the network of all previous buyers. This backwards discounting represents a convenient way of keeping track of the relevant network up to the present time, and is meant to capture the idea that past consumption contributes less to the present utility of today’s consumers than more recent consumption. It is important to stress that this discounting is quite distinct from the usual discounting associated with standard time preference (the latter will be introduced later on when describing the green good producer’s profits). Instead, the \( \beta \) discounting is meant to capture several possible relevant features of such a model, including the attenuated interaction of current buyers with older cohorts (and the latter’s eventual death), and/or the decay and eventual expiration of the green product.

When a type \( -x \) consumer gets the brown good, it obtains a utility of \( u_t^B(x) \),

\[ u_t^B(x) = U + xq^B - P_t^B, \]  

(3)

where \( q^B < q^G \) stands for the environmental quality of the brown good and \( P_t^B \) represents the price of the brown good in period \( t \).

Given the utilities (1) and (3), in period \( t \), the consumer who is indifferent between buying the green good and getting the brown good is \( \hat{x}_t = \frac{P_t^B - P_t^G - \alpha_s}{q_t^G - q_t^B} \). The environmental qualities \( q_B \) and \( q_G \) depend negatively on the level of emissions released in the production of each good, \( e_B \) and \( e_G \) respectively. Namely we assume that

\[ q_i = \Omega - e_i, \]  

(4)

where \( \Omega \) is a constant and \( e_i \) represents the actual emissions due to the consumption of product \( i, i = B, G \). This modelling follows Brécard (2013). However, unlike Brécard, we assume that the environmental qualities are exogenously determined and the quality gap is normalized to 1, i.e. \( q_G - q_B = 1 \). Accordingly, given that consumers are uniformly distributed in \([0, A]\) according to their willingness to pay for environmental quality, it will follow that the consumers with higher environmental concerns (\( x > \hat{x}_t \)) buy the green good, whereas those consumers with lower environmental concerns (\( x < \hat{x}_t \)) buy the brown good.

\[ \text{In addition to its realistic reflection of current network effects with past purchases, } \beta \text{-discounting here also plays a critical mathematical role towards the well-foundedness of the model, since it ensures a bounded network size and thus a well defined discounted profit. This important implication will become clear below.}\]
As such, the demand for the green good is denoted by \( d^G (P^G_t, P^B_t, s_t) \) and is given by

\[
d^G (P^G_t, P^B_t, s_t) = A - P^G_t + P^B_t + \alpha s_t.
\]

The demand for the brown good is denoted by \( d^B (P^G_t, P^B_t, s_t) \) and is given by

\[
d^B (P^G_t, P^B_t, s_t) = P^G_t - P^B_t - \alpha s_t.
\]

As to the supply side for the two goods, we assume that the brown good sector is perfectly competitive, whereas the green good is provided by a single firm (which from now one is called the "green producer"). As such, the market structure consists of a dominant firm (the green producer) with a competitive fringe, providing a low-(environmental) quality variant of the good. The brown good must then be supplied at an equilibrium price equal to its marginal cost, \( (P^B_t)^* = 0 \) and the green good producer faces a residual demand that can be rewritten as (for the sake of simplicity, from now on we will drop the subscript \( G \) in the price of the green good, with \( P_t \equiv P^G_t \)).

\[
d^G (P_t, s_t) = A + \alpha s_t - P_t.
\]

The law of motion of the accumulated network is

\[
s_{t+1} = \beta (s_t + d_t),
\]

or equivalently

\[
s_{t+1} = \beta [A + (1 + \alpha) s_t - P_t].
\]

We assume that the dominant firm produces the green good at marginal cost \( c \). So, at each period \( t \), the green producer has the following (time-invariant) one-period profit function.

\[
\pi (P_t, s_t) = (P_t - c) (A + \alpha s_t - P_t)
\]

Thus, the green producer chooses optimally the sequence of markups \( p = \{p_t\}_{t=0}^\infty \), with \( p_t = P_t - c \) over an infinite horizon, taking in consideration, at each time, the future stream of profits. Therefore, for an initial network size \( s_0 \), the green producer’s payoff function in the infinite horizon problem is

\[
\Pi (p, s_0) = \sum_{t=0}^\infty \delta^t \pi (p_t, s_t) = \sum_{t=0}^\infty \delta^t p_t (a + \alpha s_t - p_t),
\]

Unlike Graeker and Midtomme (2016), we adopt a more realistic view that the green products are more costly to produce than the brown good. In fact, going back to the example of EV, for the same performance level, EV are still more costly than ICE (internal combustion engine) cars.
where \( a = A - c \), \( \mathbf{p} = \{p_t\}_{t=0}^{\infty} \) is the sequence of markups and \( \delta \in (0, 1) \) is the dominant firm’s discount factor for future profits.

The model reflects a double discounting structure. The firm discounts the future stream of profits in the standard way to form its present value. The consumers discount the past accumulated network to account for the fact that they enjoy environmental peer effects akin to network externalities with nearby cohorts of buyers more than they do with distant cohorts (back in time).

A key benefit of such a dynamic model of network formation is conceptual and relates to Katz and Shapiro’s (1985) notion of fulfilled expectations Cournot equilibrium as a solution concept for oligopolies with network effects. Though widely used, this concept remains controversial due to the fact that the fulfilled expectations component is quite ad hoc. As it requires superimposing a rational expectations requirement to a Nash equilibrium, some scholars find the concept too demanding in terms of coordination of expectations. By postulating a dynamic model where the natural state variable at time \( t \) is the accumulated network at time \( t \), one dispenses with the need for expectations as a critical ingredient of the model. Instead, the present approach offers a natural and transparent mechanism for network formation over time.

Finally, this structure reflects another tacit assumption as to the nature of network externalities. Consumers are backward looking with respect to the network effect, i.e., future buyers are not taken into account via some expectations-based manner in the utility of today’s consumers.\(^9\) In other words, consumers are not forward-looking in estimating their relevant network (see Garcia and Resende, 2011 for a survey on dynamic network externalities).

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\(^9\)As mentioned earlier, our objective here is to look at markets in which consumers’ utility depend on the choices of previous consumers’ cohorts. There are of however situations in which network benefits may result of interaction with future peers or future aftermarket benefits. Graker and Midttomme (2016) analyze how such forward-looking network effects may affect the competition between a competitive fringe of brown good producers and a monopolist supplier of a green good. This type of forward-looking expectations are also built into the Industrial Organization models of Cabral (2011), Laussel and Resende (2014) or Long, Laussel and Resende (2015) in a way that is reflected in their notion of dynamic equilibrium.
2.2 The optimal pricing solution

We now proceed to analyze the pricing decision of the dominant firm. Given any initial network size $s_0$ at time 0, denoted by $s \geq 0$, the dominant firm’s problem writes as

$$V(s) \triangleq \max_{p \in P} \Pi(p, s)$$

subject to

$$s_{t+1} = \max \{ \beta \left( a + (1 + \alpha) s_t - p_t \right), 0 \}.$$

This is clearly a Markov-stationary infinite-horizon dynamic programming problem. Denoting the next state by $s' = \beta \left( a + (1 + \alpha) s - p \right)$, i.e., dropping time subscripts to reflect time stationary, the associated Bellman equation is

$$V(s) = \max_{p \in [0, \alpha s + a]} \left\{ \pi(p, s) + \delta V(s') \right\}.$$ 

We proceed by determining first the Euler equation associated with the dynamic problem above. The first order condition is obtained by differentiating the Bellman equation with respect to $p$, as

$$\alpha s + a - 2p - \delta \beta V'[\beta (a + (1 + \alpha) s - p)] = 0 \tag{7}$$

where $V'$ denotes the derivative of the value function.$^{10}$

**Definition 1 Optimal (Markov-stationary) price policy**

Let $p(s)$ be a (time-invariant) rule defining the green-good producer’s optimal markup as a function of its accumulated consumer base $(s)$ in such a way that the optimal markup function yields a time path of sales that maximizes the discounted accumulated profits of the green firm, starting at any (date, state) pair $(t, s_t)$. Then, the optimal (Markov-stationary) price policy, $P(s)$, is simply $P(s) = p(s) + c.$

Taking into consideration definition 1, and denoting the optimal (Markov-stationary) markup candidate by $\rho(s)$, the Bellman equation writes as:

$$V(s) = \rho(s) (\alpha s + a - \rho(s)) + \delta V[\beta (a + (1 + \alpha) s - \rho(s))]$$

$^{10}$The second order condition is $-2 + \delta \beta^2 V'' < 0.$
Differentiating this identity with respect to the state \( s \) yields

\[
V'(s) = \rho'(s) (\alpha s + a - 2\rho(s)) + \alpha \rho(s) + \delta \beta \left((1 + \alpha) - \rho'(s)\right) V'[\beta(\alpha (1 + \alpha) s - \rho(s))]. \tag{8}
\]

Using (7), we can rewrite (8) as

\[
V'(s) = \rho'(s) [\alpha s + a - 2\rho(s)] + \alpha \rho(s) + \delta \beta \left[(1 + \alpha) - \rho'(s)\right] \frac{(\alpha s + a - 2\rho(s))}{\delta \beta}
\]

Simplifying yields the envelope result

\[
V'(s) = (\alpha + 1) (a + s\alpha) - (\alpha + 2) \rho(s). \tag{9}
\]

Combining (9) with (7), we obtain the Euler equation\textsuperscript{11}

\[
\alpha s + a - 2\rho(s) = \delta \beta \left((\alpha + 1) (a + s\rho') - (2 + \alpha) \rho(s')\right). \tag{10}
\]

To interpret this condition, observe that in light of network effects associated with the consumption of the green good, its producer faces a trade off between (i) set a higher markup in period \( t \), getting a higher revenue today (LHS of (10)); and (2) boost demand accumulation for tomorrow (in accordance with the transition equation), setting a lower price today.

Given the linear-quadratic structure of the problem, let us start by considering a linear optimal price policy candidate and using the standard guess-and-verify approach to determine the parameters \( \gamma \) and \( \phi \) of the policy function specified as follows:

\[
\rho(s) = \gamma s + \phi.
\]

For the problem to remain bounded and well-defined, we shall need some joint restrictions on the values of three key parameters, the two discount factors \( \delta \) and \( \beta \), as well as the parameter capturing the strength of network effects, \( \alpha \).

**Assumption 1** Assume that

\[
\beta \sqrt{\delta} (\alpha + 1) < 1.
\]

Throughout the paper, we shall maintain Assumption 1.

\textsuperscript{11}With time subscripts, The Euler equation can also be rewritten as

\[
\alpha s + a - 2p_t = \delta \beta \left((\alpha + 1) (a + s_{t+1}\alpha) - (2 + \alpha) p_{t+1}\right)
\]
Lemma 2 Let \( \rho(s) = \gamma s + \phi \). Then, the unique valid solution to the Euler Equation (10) is such that:

\[
\gamma = \gamma_1 = \frac{\Delta - \left(1 - \beta^2 \delta (\alpha + 1)^2\right)}{\beta^2 \delta (\alpha + 2)} > 0
\]

\[
\phi = \phi_1 = a \frac{\Delta - \beta \delta (1 - \beta) (\alpha + 1)}{\Delta - \beta \delta (1 - \beta) (\alpha + 1) + (1 - \beta)}
\]

with \( \Delta \triangleq \sqrt{(1 - \beta^2 \delta) \left(1 - \beta^2 \delta (\alpha + 1)^2\right)} \).

In the unique valid solution to the Euler equation, the green producer’s markup is increasing with the size of its accumulated consumer base (\( \gamma_1 > 0 \)). However, \( \phi_1 \) may be negative or positive. When \( \phi_1 < 0 \), the green producer’s markup would be negative, unless the firm already benefits from a sufficiently large installed base of adopters. Herein, we restrict our analysis to the domain of parameters in which the green good producer gets (at least) zero profit in every period, so that \( p(s) \geq 0 \). Taking this restriction into consideration, in what follows, we shall investigate the properties and the implications of the green producer’s optimal pricing policy. To this end, we first make precise the sense in which the parameter \( \alpha \) measures the intensity of the network effect, for use below.

Definition 3 The peer effects for the green good are called weak if \( (\alpha + 1)^2 \in (1, \hat{\alpha}) \).

The peer effects for the green good are called strong if \( (\alpha + 1)^2 \in \left(\hat{\alpha}, \frac{1}{\beta^2 \delta}\right) \), where

\[
\hat{\alpha} = \frac{(1 - \beta^2 \delta)}{\beta^2 \delta (\delta (1 - \beta) + (1 - \beta \delta))}. \tag{13}
\]

It can be shown via direct computation that

\[
\phi_1 > 0 \text{ if } (\alpha + 1)^2 \in (1, \hat{\alpha})
\]

and

\[\text{\footnotesize\\}
\text{\footnotesize\textsuperscript{12}}\text{If we allowed for negative markups, then we would have } p^*(s) = \gamma_1 s + \phi_1, \text{ regardless of the size of the network (with the firms getting negative profits until a critical mass of users is reached).}
\]

\[\text{\footnotesize\textsuperscript{13}}\text{Note that the threshold } \hat{\alpha} \text{ is decreasing in both the discount factors. As the consumers put a lower weight on the present utility, the lower the network effects of consumption must be for the peer effects to be qualified as strong.}
\]

\[\text{\footnotesize\textsuperscript{14}}\text{The denominator of } \phi_1 \text{ is always positive, and the numerator is positive (negative) if } \hat{\alpha} = \frac{(1 - \beta^2 \delta)}{\beta^2 \delta (1 - \beta) + (1 - \beta \delta)} > (\text{<})(\alpha + 1)^2.
\]
φ₁ < 0 if \((α + 1)^2 \in (\hat{α}, \frac{1}{β^2 δ})\).

Given that we must have \(p(s) ≥ 0\), it can be checked that the policy function is affine in the state when \(α\) is small, and piece-wise affine when \(α\) is large.\(^{15}\)

\[
p(s) = γ₁ s + φ₁ \text{ if } (α + 1)^2 \in (1, \hat{α})\]
\[
p(s) = \begin{cases} 0, & \text{if } s ≤ \hat{s} \\ φ₁ + γ₁ s, & \text{if } s > \hat{s} \end{cases} \text{ if } (α + 1)^2 \in (\hat{α}, \frac{1}{β^2 δ})
\]

where the threshold \(\hat{s}\) is given by \(\hat{s} = -\frac{φ₁}{γ₁}\).

\[
\hat{s} = -aβ²δ(α + 2) \frac{Δ - (α + 1) βδ (1 - β)}{(Δ + (α + 1)^2 β^2 δ - 1)(Δ + 1 - βδ - (α + 1) βδ (1 - β))}.
\]

Though it appears somewhat complex, the structure of the optimal pricing policy is in fact quite intuitive (see Figures 1 and 2 below). When network effects are weak, a simple affine price policy is optimal. More interesting is the case of strong network effects: For small current network sizes, it pays for the dominant firm to follow a zero markup policy in an effort to maximize immediate consumption and create momentum towards building a large consumer base. In fact, what the negative mark-up candidate policy in Lemma 1 suggests is that the firm would even subsidize the consumption of the green good by pricing at below marginal cost for small network sizes. When the consumer base gets past the threshold value of \(\hat{s}\), it becomes optimal again for the firm to follow an affine pricing policy (with strictly positive markup thereafter).

The optimal pricing policy of the green good with network effects recovers a familiar feature of optimal pricing for a network good. Early purchasers are rewarded by the dominant firm for offering critical participation in the firm’s effort to build an initial consumer base of the green good.

Next, we investigate the steady-state implications of the optimal policy.

2.3 The steady-state analysis

In this subsection, we investigate the asymptotic properties of the state and price trajectories that result from the unique Markov-stationary optimal markup policy \(p(\cdot)\).

\(^{15}\)One easily verifies that the candidate solution \(p(s) = γ₁ s + φ₁\) is negative over the initial interval \([0, \hat{s}]\), and that the optimal feasible solution then it to have a zero markup over this interval.
Along the optimal path, the state evolves according to the simple (Markovian) dynamic system

\[ s_{t+1} \overset{\Delta}{=} H(s_t) = \beta[(1 + \alpha)s_t + a - p(s_t)]. \]

Substituting in for \( p(\cdot) \) from (11) yields

\[ s_{t+1} = \beta((1 - \gamma_1)s_t + a - \phi_1) \text{ if } (\alpha + 1)^2 \in (1, \hat{\alpha}) \tag{12} \]
\[ s_{t+1} = \begin{cases} 
\beta((1 + \alpha)s_t + a), & s \leq \hat{s} \\
\beta((1 + \alpha - \gamma_1)s_t + a - \phi_1), & s > \hat{s} \end{cases} \text{ if } (\alpha + 1)^2 \in \left(\hat{\alpha}, \frac{1}{\beta^2 \delta}\right) \tag{13} \]

**Lemma 4** If \( \alpha + 1 < \bar{\alpha} \equiv (1 - \beta) + (1 - \beta \delta) \beta(1 - \beta \delta + 1 - \beta \delta) \), \( H(s) \) has all its slopes in \((0,1)\); otherwise, the slope of \( H(s) \) is greater than or equal to 1.

**Proof.** For \( s > \hat{s} \), \( H'(s) = \beta(1 - \alpha - \gamma_1) \), which is in \((0,1)\) if \( \alpha + 1 < \frac{(1 - \beta) + (1 - \beta \delta)}{\beta(1 - \beta \delta + 1 - \beta \delta)} \equiv \bar{\alpha} \). □

The threshold \( \bar{\alpha} \) introduces an upper bound on the strength of network effects for consumers for the steady state to be finite. Naturally, this upper bound is decreasing in both the discount factors. The more the firm becomes patient, the more it is important to keep low markups so as to create a large base of consumers. On the consumer side, a higher \( \beta \) increases consumers’ willingness to pay for the green good since consumers benefit more from the consumption of previous consumers’ cohorts (namely those consuming the period at initial stages). Therefore, the more consumers become patient the more it is important to create a large initial consumer base. The comparison between \( \bar{\alpha} \) and \( \hat{\alpha} \) is not straightforward. If \( \delta < 1/3 \) then \( \hat{\alpha} > \bar{\alpha}^2 \). If \( \delta > 1/3 \), then \( \bar{\alpha} \) is only higher than \( \bar{\alpha}^2 \) for \( \beta < \frac{\delta(3+\delta)-(1-\delta)\sqrt{3}\sqrt{(8+\delta)}}{4\delta} \). Hence, for sufficiently low \( \delta \), we have that whenever network effects are weak\((1 + \alpha)^2 < \bar{\alpha} \), in the sense of Definition 3, the steady-state network size will always be finite. When \( \delta > \frac{1}{3} \), then the consumers’ discounting factor needs to be sufficiently small for that to remain the case. In fact, it is worth noting that in our set-up, the effect of a reduction in the consumer’s discount factor \( \beta \) results in a reduction of consumers’ overall benefits from the network effects (due to the lower value attributed to older cohorts of consumers).\(^\text{16}\)

When finite, the steady-state network size \( s^* \) is given by

\[ s^* = \beta \left( \left( k - \frac{1}{\beta^2 \delta (k + 1)} (\Delta - (1 - \beta^2 \delta k^2)) \right) s^* + a - \frac{\Delta - \beta \delta (1 - \beta) k}{\Delta - \beta \delta (1 - \beta) k + (1 - \beta \delta)} \right) \]

where \( k = (1 + \alpha) \). This expression simplifies to

\(^{16}\)A reduction in \( \alpha \) was reduces the overall benefits from network effects. However, this reduction is independent of the consumers’ generation, whereas a reduction in \( \beta \) penalizes more the network benefits created by older generations of consumers.
\[ s^* = \frac{a \beta (1 - \beta \delta)}{(1 - \beta + 1 - \beta \delta) - \beta (1 - \beta \delta) + \beta \delta (1 - \beta)} (\alpha + 1). \] (14)

Figure 3 identifies the finite steady state (arising when \( \alpha + 1 < \tilde{\alpha} \)) for weak and strong network effects (in the sense of Definition 3). In each graph, the dashed line represents the 45 degree line, in which \( s_{t+1} = s_t \) and the solid line represents \( s_{t+1} \triangleq H(s_t) \), defined in (12) and (13), depending on the nature of network effects. The graph on the left represents the case of weak network effects (where \( (1 + \alpha)^2 < \hat{\alpha} \) implying \( p(s) = \rho(s), \forall s \)), whereas the graph on the right depicts the optimal evolution of the green good’s when the intensity of network effects is strong (in the sense that \( (1 + \alpha)^2 > \tilde{\alpha} \) implying a piecewise linear optimal price policy) but not too strong, in order to keep the steady state finite (in this case, we must have \( \hat{\alpha} < (1 + \alpha)^2 < \tilde{\alpha}^2 \).

For \( \alpha + 1 < \tilde{\alpha} \), the optimal steady-state price is then \( P(s^*) = c + p(s^*) = \gamma_1 s^* + \phi_1 \), which reduces to:

\[ P^* = c + \frac{a (1 - \beta) ((1 + \alpha) \beta \delta - 1)}{\beta (1 - \beta \delta) + \beta \delta (1 - \beta)} (\alpha + 1) - (1 - \beta + 1 - \beta \delta) > c. \]

Finally, the steady-state demand level reduces to

\[ d^* = \frac{a (1 - \beta) (1 - \beta \delta)}{2 (1 - \beta) (1 - \beta \delta) - \alpha \beta ((1 - \beta) \delta + 1 - \beta \delta)}. \]

The main properties of the optimal steady-state network size are described next.

**Proposition 5** If \( \alpha + 1 < \tilde{\alpha} \), the following hold.

(i) there exists a unique, non-zero, and finite steady-state equilibrium \( s^* > 0 \), with corresponding price \( P^* = c + \rho(s^*) > c \).

(ii) This steady-state network size is globally asymptotically stable (for all initial states \( s_0 \)).

(iii) For all \( s_0 \in [0, s^*) \), the optimal sequence of network sizes \( \{s_t\} \uparrow s^* \), and the optimal sequence of prices \( \{P(s_t)\} \nearrow P^* \).

(iv) For all \( s_0 > s^* \), the optimal sequence of network sizes \( \{s_t\} \searrow s^* \), and the optimal sequence of prices \( \{P(s_t)\} \searrow P^* \).

**Proof.** (i) The existence and uniqueness of \( s^* \) follows directly from the slope property of \( H \) given in Lemma 4. It is easy to check that \( P^* > c \). Indeed, when \( \alpha + 1 < \tilde{\alpha} \), the denominator of \( P^* \) (and \( s^* \)) is positive. Also, we observe that the numerator of \( P^* \) is positive, under Assumption 1.

(ii) It is easy to verify that the unique steady state must occur when \( s > \hat{s} \). In that region, the desired conclusion follows directly from Lemma 4.
The main implication of this Proposition is that, under the optimal pricing policy, when $\alpha + 1 < \tilde{\alpha}$, the dynamics of the network size follows a simple monotonic process with a unique steady-state equilibrium. For initial network sizes below the equilibrium, the firm prices low enough to build the effective network size up to its steady-state value, with actual prices monotonically increasing over time (since $\rho(\cdot)$ is an increasing function of $s$ and $p(s) = \rho(\cdot)$ for $s > \hat{s}$). For initial network sizes above the equilibrium, the firm prices so as to harvest its large consumer base and thereby bring down the effective network size up to its steady-state value, with actual prices monotonically decreasing over time. Naturally, this conclusion reflects the important feature of this model that consumers discount the participation of distant purchasers (back in time) in evaluating the relevant network size for their current purchase decision.

In the dynamic perspective offered by the present model, the unique steady-state network size emerges as the natural long run market size for the green good in question. As such, it appears to represent a more unambiguous determination of the right market size than Katz and Shapiro’s (1985) static analog, the so-called fulfilled-expectations Cournot equilibrium. For a thorough discussion of various aspects of this solution concept for network industries, see Amir and Lazzatti (2011).

There are several important practical consequences of this simple network size dynamics that are worth highlighting. The first is that this model is theoretically consistent with the possibility of observing a sequence of prices that are declining over time, if the initial network $s_0$ is sufficiently large. However, in practice, this is not likely to be very relevant, in particular for the case of new green goods. The reason is that, in real life, it is typical for goods with network effects to be launched with relatively small initial consumer bases. This is even more likely to be the case for environmentally friendly goods, as they need to displace an existing brown alternative.

Therefore, one would expect to observe prices that are monotonically increasing over time, until they stabilize at the steady-state level. In particular, one would expect to see introductory pricing (penetration pricing or marginal cost pricing) only in the early phases of the launch of the new green good. In other words, small network sizes cannot emerge again after the industry has been in operation for some time. This is an important finding in the case of environmentally friendly goods with network effects (as long as those green goods gain momentum by reaching a critical user base, there is no risk of reverting to the usually small initial base of consumers).\footnote{It is important to observe that, although these conclusions might appear to be at odds with actual pricing behavior observed in many real-world network industries, in which prices may go down as the market gains momentum, as it
Some simple comparative statics results of interest can be drawn for the finite steady-state values of the relevant endogenous variables.

**Proposition 6** The finite steady-state network size $s^*$ is

(i) increasing and convex in the strength of the network effect $\alpha$.

(ii) increasing in the firm’s discount factor $\delta$.

(iii) increasing in the consumers’ discount factor $\beta$.

In addition, the steady-state demand $d^* = d(P^*, s^*)$ is increasing and convex in $\alpha$.

**Proof.** This is straightforward upon computing $\frac{\partial s^*}{\partial \alpha}$, $\frac{\partial^2 s^*}{\partial \alpha^2}$, $\frac{\partial s^*}{\partial \delta}$, $\frac{\partial s^*}{\partial \beta}$, and $\frac{\partial d^*}{\partial \alpha}$. The verification details are left to the reader. ■

**Proposition 7** The finite steady-state price $P^*$ is

(i) increasing and convex in the strength of the network effect $\alpha$.

(ii) increasing in the firm’s discount factor $\delta$, as long as the network effect is strong enough, namely if $\alpha + 1 > 1/\beta$, and decreasing otherwise.

(iii) increasing in the consumers’ discount factor $\beta$.

**Proof.** This is straightforward upon computing $\frac{\partial P^*}{\partial \alpha}$, $\frac{\partial^2 P^*}{\partial \alpha^2}$; $\frac{\partial P^*}{\partial \delta}$; and $\frac{\partial P^*}{\partial \beta}$. The verification details are left to the reader. ■

These simple conclusions suggest an interesting interpretation of a central notion that has emerged in the economics literature on industries with network effects, but that so far has eluded any formal definition or systematic treatment (to the best of our knowledge). Indeed, such industries are often said to enjoy increasing returns to scale on the demand side. The facts that the steady-state network size $s^*$ and the price $P^*$ are all increasing at an increasing rate in the strength of the network effect $\alpha$ are all revealing reflections of such demand-side scale economies, and may constitute appropriate definitions for this useful notion (at least in a dynamic framework such as ours).

These comparative statics conclusions turn out to be quite intuitive, but only upon understanding that they pertain to the firm’s optimal behavior at the steady-state equilibrium. It is clearly expected to be the case with EV, distributed energy generation devices (e.g., PV solar panels) or energy demand management systems (e.g., smart meters), the declining prices in such industries are typically due to rapid technological progress and process R&D. These important features of network industries are outside the scope of the present paper (and would actually be interesting issues for further study).
intuitive for the steady-state network size to increase with the strength of the network effect $\alpha$. Since $s^*$ represents the long run network size, it is also to be expected that it would be higher for a more patient firm (higher $\delta$). Finally if consumers place more weight on past generations in assessing the relevant network size (i.e., they have a higher $\beta$), the long run network size will also be larger.

As to the effects on pricing, the fact that the long run price increases at an increasing rate with the strength of the network effect $\alpha$ does not contradict the fact that the firm’s best short run practise might be to give away the product at (or even below) marginal cost in the early phase of industry start up (which occurs precisely when network effects are strong). Our analysis unveils that the equilibrium steady-state price is always increasing with $\beta$. For higher $\beta-values$, consumers have a higher willingness to pay for the green good and the green producer is able to extract part of this extra consumer surplus by charging a higher steady-state price. The effect of $\delta$ on equilibrium steady-state prices is non-monotonic. For sufficiently strong network effects (in the sense of point (ii) in the Proposition above), steady-state prices increase with the firm’s discount factor. In this scenario, the green producer is likely to adopt a piecewise continuous (kinked) pricing policy aimed at boosting the initial size of the green good network. As a result of such strategy, consumers’ willingness to pay for the green good goes up (recall we are considering the case of strong network effects) and the firm is able to set higher steady-state prices. In other words, $P^*$ is increasing with $\delta$ for sufficiently strong network effects. For weak network effects, i.e. $\alpha + 1 < 1/\beta$, this is no longer the case: On the one hand, for weak network effects, the price tends to be strictly increasing with the state $s$ (which slows the initial network expansion in comparison to the initial flat price policy). On the other hand, for weak network effects, consumers’ willingness to pay for the green good is less responsive to changes in $s$.

Up to now, we have examined steady-state outcomes when $\alpha + 1 < \tilde{\alpha}$, so that the steady-state value of the accumulated demand is finite. In that case, in the steady state some consumers may actually (optimally) choose not to buy the green good. We now elaborate further on the properties of the steady state the intensity of network effects is so strong that it leads to an unbounded solution (in which the steady state is not finite, meaning that universal adoption of the green good is expected to occur).

**Remark 8** If $\alpha + 1 < \tilde{\alpha}$ is violated (with Assumption 1 still in place), the optimal path of network sizes diverges to $+\infty$, for any initial state $s_0$. This follows directly from the fact that, although $H(s)$ is piece-wise linear, the slopes of $H(s)$ are all $> 1$. This is intuitive, in that with sufficiently
strong network effects, it pays for the dominant firm to price in a way that allows for the size of the network to be built up indefinitely. Naturally, this implication of optimal pricing eventually turns unrealistic since the size of the population or the potential market size is finite (though perhaps very large).

The universal adoption outcome discussed in the above Remark is illustrated in Figure 4, below, where $s_{t+1}$ is growing at an increasing rate, implying that the steady-state network of green good consumers is unbounded. In figure 4, the dashed line represents the 45 degree line, in which $s_{t+1} = s_t$ and the solid line represents $s_{t+1} \triangleq H(s_t)$, for $(1 + \alpha)^2 > \max \{ \tilde{\alpha}^2, \hat{\alpha} \}$.

The figure depicts a steady-state outcome in which the strength of network effects is so high that the green good producer sets a piecewise affine optimal pricing policy and the steady-state network size is unbounded. From a theoretical point of view, we may observe that the steady state is unbounded despite the fact that the green good producer optimally follows an affine pricing policy. This result can only occur when $\delta$ is large enough and it arises for high $\beta$ and low $\alpha$. The intuition behind the result is as follows: if $\alpha$ is sufficiently small, the firm has no interest in adopting a penetration pricing strategy, choosing an affine optimal price function instead. Despite this fact, the accumulated network size, $s$, may grow fast if $\beta$ is sufficiently large (so that the first generation of consumers remain highly weighted).

The above Remark shows that for sufficiently strong network effects, the dominant firm’s optimal pricing strategy is such that there is ”universal adoption” in the steady state, in the sense that all consumers entering the market end up buying the network good. Differently, under weak network effects, i.e. $\alpha + 1 < \frac{(1-\beta) + (1-\beta\delta)}{\beta(1-\beta\delta + 1-\beta\bar{\delta})}$, the steady-state value of the accumulated demand is finite. In that case, in the steady state some consumers may actually (optimally) choose not to buy the green good.

[Comment to Rabah: The reviewer 2 says he is not satisfied with the previous treatment of the unbounded solution. We have extended this and we even included one of his remarks in a footnote]. Please see if there is something else worth adding.

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18 The unbounded steady-state outcome results from the fact that we let $\alpha$ in the demand function to stay constant over time so that the demand can increase in every period. If instead we have considered $\alpha$ to be a concave function of $s$, the unbounded solution could not arise. We thank an anonymous reviewer for bringing our attention to this point.

19 The domain of parameters for which this result may arise tends to be rather small. For example, the steady state will have the properties described above if $\delta = 0.5, \beta = 0.5$ and $\alpha = 1.6$. 

21
3 Welfare analysis

In this section, we determine the price policy of a first-best social planner whose objective is to maximize discounted social welfare over an infinite horizon. Naturally, we view this solution as a useful benchmark for comparison with the private solution, rather than a realistic planning solution.

The social planner maximizes total welfare that includes the consumer surplus of all consumers and profits discounted of the total environmental damage. We assume that the planner is unable to influence consumers’ behavior, so he also takes their $\beta$-discounting in forming the relevant network size as given.

The environmental damage function is given by $D(E) = \tau E$, where $\tau > 0$ measures the emissions’ degree of environmental damage (i.e. the social environmental marginal cost of emissions) and $E = e^G d^G + e^B d^B$ stands for total amount of emissions. As such, the one-period social welfare function is given by $w(p, s) - D(E)$, where

$$w(p, s) = \frac{1}{2}(a + \alpha s - p)^2 + \frac{1}{2}(p + c - \alpha s)^2 + p(\alpha s + a - p) - D(E)$$

$$= \frac{(a + \alpha s)^2 - p^2}{2} + \frac{1}{2}(p + c - \alpha s)^2 - D(E)$$

$$= s\alpha(a + sa) + (c - sa)(p + c) + \frac{a^2 - c^2}{2} - D(E).$$

Regarding the emissions defining the goods’ environmental quality in equation (4), we have that $E = e^G d^G + e^B d^B = e^B - d^G = e^B A - (a + \alpha s - p)$.

In order to see this, note that $d^G = A - d^B$ and $e^B = e^G + 1$. The problem of the social planner is to obtain the first-best (FB) markup rule $p^{FB}(s)$ that maximizes the intertemporal discounted total social welfare, given the law of motion for the accumulated network. Namely, the social planner’s problem is

$$\max_p \Omega(p, s_0) = \sum_{t=0}^{\infty} \mu^t(w(p_t, s_t) - D(E))$$

s.t.

$$s_{t+1} = \beta[a + (1 + \alpha)s_t - p_t],$$

where $\mu$ stands for the social planner’s discount factor. It is worth noting that the one-period payoff in the objective function of the dynamic problem (15) is linear in $p$, with slope given by

$^{21}$We could assume that social planner’s discount factor coincides with the dominant firm’s discount. This assumption would not impact the optimal solution of the social planner.
\(\alpha s - c + \tau\). When \(\tau \geq c - \alpha s\), i.e. when the environmental damage of consumption is higher than private marginal production cost discounted of the the network effect of the green good the one-period social welfare function is monotonically decreasing in the markup \(p\). On the contrary, when \(\tau < c - \alpha s\) or in other words, the environmental damage of consumption is lower than the private marginal production cost discounted of the the network effect of the green good, the one-period social welfare function is monotonically increasing in the markup \(p\).

Hence, in the socially optimal solution, the following properties must hold.

**Proposition 9**

i. The socially optimal markup policy is given by:

\[
p_{FB}(s) = \begin{cases} 
0 & \text{if } \alpha s \geq c - \tau \\
 a + \alpha s & \text{if } \alpha s < c - \tau 
\end{cases}
\]

ii. The socially optimal state transition is given by:

\[
s_{t+1} \triangleq G(s_t) = \begin{cases} 
\beta[a + (\alpha + 1)s_t] & \text{if } \alpha s_t \geq c - \tau \\
\beta s_t & \text{if } \alpha s_t < c - \tau 
\end{cases}
\]

**Proof.** The one period social welfare function is liner in \(p\) with slope given by \(\alpha s - c + \tau\). When \(\tau \geq c - \alpha s\), the one-period social welfare function discounted of the environmental damage is monotonically decreasing in the markup \(p\). Hence, it follows that the socially optimal markup is \(p_{FB}(s) = 0\). Therefore the socially optimal state transition is given equal to:

\[
s_{t+1} = \beta[a + (\alpha + 1)s_t].
\]  

(16)

When \(\tau < c - \alpha s\), the one-period social welfare function discounted of the environmental damage is increasing in \(p\) and it follows that the social planner sets the markup to be \(p = a + \alpha s\). In that case, the socially optimal state transition is given by the line through the origin

\[
s_{t+1} = \beta s_t.
\]  

(17)

**Lemma 10** If \(\alpha + 1 < \frac{1}{\beta}\), \(G(s)\) has all its slopes in \((0, 1)\), otherwise, the slope of \(G(s)\) is lower than 1 for \(s < \frac{c - \tau}{\alpha}\) greater than or equal to 1 and greater than 1 for \(s > \frac{c - \tau}{\alpha}\).
**Proof.** The slope of $G(s)$ is given by

$$G'(s) = \begin{cases} 
\beta (1 + \alpha) & \text{if } \alpha s \geq c - \tau \\
\beta & \text{if } \alpha s < c - \tau 
\end{cases}$$

which is positive, and $< 1$ if $\alpha + 1 < \frac{1}{\beta}$. 

Lemma (10) determines the conditions under which there is a finite or unbounded network size in the steady state. For $\alpha + 1 \geq \frac{1}{\beta}$, we either have a unique finite stable steady state in which the green good does not take off, $s^{**} = 0$ (for $\tau < c - \alpha s$) or an unbounded steady state, in which the accumulated network size would tend to infinite in the long-run (universal adoption solution), when $\tau > c - \alpha s$. Differently, when $\alpha + 1 < \frac{1}{\beta}$, we have two finite stable steady states which depend on the initial value of the accumulated network, $s_0$. The steady states are is given by

$$s^{**} = \begin{cases} 
0 & \text{if } s_0 < \frac{c - \tau}{\alpha} \\
\frac{a\beta}{1 - \beta(\alpha + 1)} & \text{if } s_0 > \frac{c - \tau}{\alpha}
\end{cases}$$  \hspace{1cm} (18)

The previous results show that, when the extra environmental damage of the brown good is not too excessive (in the sense $\tau < c - \alpha s$), the social planner never patronizes the green good consumption, disencouraging it (in favor of the cheaper brown good) by setting $p^{FB}(s) = a + \alpha s$. Differently, when $\tau > c - \alpha s$, in the first best solution, at least some consumers are expected to adopt the brown good. If network effects are strong, in the that $\alpha + 1 \geq \frac{1}{\beta}$, the first best solution results in universal adoption when $\tau > c - \alpha s$. When $\alpha + 1 < \frac{1}{\beta}$, the intensity of network effects is more limited and therefore the steady state accumulated network of the green good is strictly positive but finite.

Let us now turn our attention to the comparison between the private and the socially optimal price. The following proposition presents the general results.

**Proposition 11**  \hspace{1cm} (i) Under weak network effects, $p^{FB}(s) \geq p(s)$, for $s < \frac{c - \tau}{\alpha}$ and $p^{FB}(s) \leq p(s)$, otherwise.

(ii) Under strong network effects, $p^{FB}(s) > p(s)$ for $s < \max\left\{ \hat{s}, \frac{c - \tau}{\alpha} \right\}$ and $p^{FB}(s) \leq p(s)$ otherwise.

**Proof.** See the Appendix. 

Summing up, when the degree of environmental damage is higher than the private cost of production ($\tau > c$), the planner would always choose to incentivize the adoption of the green good
by pricing it at zero for all levels of the accumulated demand. Instead, the firm would choose a positive price policy that is increasing in the accumulated demand, except when the network effects are strong, in which case, it would also like to incentivize adoption by charging zero price.

When the degree of environmental damage is lower than the private cost of production \((\tau < c)\), we are in a case in which the consumption of the brown good is not very damaging for the environment. In this case, if the accumulated network is low, the social planner chooses to desincentivize adoption through a positive price which is higher than the green good’s producer price.

The comparison between the asymptotic implications of the private optimal price and the socially optimal price is then as follows.

**Proposition 12**

1. (i) If \(\alpha + 1 < \frac{1}{\beta}\), the social planner steady state is lower than the private optimum if \(s < \frac{c-\tau}{\alpha}\) and higher otherwise.

2. (ii) If \(\frac{1}{\beta} < \alpha + 1 < \frac{(1-\beta)+(1-\beta \delta)}{\beta((1-\beta)+1-\beta \delta)}\), the social planner steady-state network \((s^{**})\) is lower than the private optimum if \(s < \frac{c-\tau}{\alpha}\). However, when \(s < \frac{c-\tau}{\alpha}\), the social planner fosters universal adoption \((s^{**} = +\infty)\), whereas the green producer patronizes a finite (strictly positive) steady-state network, \(0 < s^* < s^{**} = +\infty\), with \(s^*\) given by (14).

3. (iii) If \(\alpha + 1 > \frac{(1-\beta)+(1-\beta \delta)}{\beta((1-\beta)\delta+1-\beta \delta)} > \frac{1}{\beta}\), the green producer always fosters universal adoption, with \(s^* = +\infty\). The social planner will only do it if \(s > \frac{c-\tau}{\alpha}\).

**Proof.** From Lemma 4 and 10 we know how the slopes of the state equation compare with 1. The threshold values for \((\alpha + 1)\) are \(\frac{1}{\beta}\) and \(\frac{(1-\beta)+(1-\beta \delta)}{\beta((1-\beta)+1-\beta \delta)}\), respectively for the planner’s and the dominant firm problems. We obtain that \(\frac{1}{\beta} < \frac{(1-\beta)+(1-\beta \delta)}{\beta((1-\beta)\delta+1-\beta \delta)}\) for \(\delta\) and \(\beta\) \(\in (0, 1]\). □

Proposition (12) compares the private and the socially optimal steady-state networks for the green good. For low values of the network effect, both the social planner and the dominant firm solutions yield finite steady states, where adoption of the green good is not universal. Indeed, in the first best solution, it can even be null for sufficiently low initial values of the accumulated network. For intermediate values of the network effect, namely for \(\frac{1}{\beta} < \alpha + 1 < \frac{(1-\beta)+(1-\beta \delta)}{\beta((1-\beta)+1-\beta \delta)}\), provided that the environmental effectiveness of the green good (vis-à-vis the brown good) is large enough, the planner’s problem yields an infinite steady state (which is interpreted as universal adoption), whereas the dominant firm would result in a finite steady state, with non-universal adoption. In other words, for this intermediate range of network effects, the steady-state adoption in the firm’s optimal solution is not socially optimal from a welfare perspective (some consumers are inefficiently out of the market for the green good). Finally for high values of the network effect,
i.e., \( \alpha + 1 > \frac{(1-\beta)+(1-\beta\delta)}{\beta(1-\beta)\delta+1-\beta\delta} \), while the social planner and the dominant firm choose distinct price policies, provided that the environmental effectiveness of the green good (vis-à-vis the brown good) is large enough, both will lead to eventual universal adoption if the initial value of the accumulated network is high enough, but with different convergence rates. This case deserves a special remark as it highlights the inertia problem associated to the network effect of the green good. Figure 4 illustrates this comparison for different values of \( \alpha \).

The interesting parametrization, from the point of view of policy is when the environmental effectiveness of the green good is sufficiently large for the social planner to be interested in patronizing this good. The most interesting case arises when the values of network effects are intermediate and the initial accumulated network is small as for new green goods. In that case, a suitable regulation that magnifies the intensity of the network effects would lead the green producer to choose a markup policy that fosters universal adoption in the steady state, as well. One possible policy could be advertising campaigns in which well recognized public figures or celebrities are shown to adopt the green product. This would increase the peer effect of the product and move the steady state from a finite to an infinite solution. Another possible beneficial intervention would be a subsidization policy that pays for adopters’ price, thus generating behavior compatible with marginal cost pricing. In the following discussion, let us look at cases in which \( \tau > c \). From an environmental perspective, this is the most interesting scenario in which the environmental quality of the green good is significantly superior to the environmental quality of the brown good. When we compare the network optimal evolution in the private solution:

\[
\begin{align*}
  s_{t+1} &= \beta ((1 + \alpha - \gamma_1) s_t + a - \phi_1) \text{ if } (\alpha + 1)^2 \in (1, \hat{\alpha}) \\
  s_{t+1} &= \begin{cases} 
    \beta ((1 + \alpha) s_t + a), & s \leq \hat{s} \\
    \beta ((1 + \alpha - \gamma_1) s_t + a - \phi_1), & s > \hat{s} \end{cases} \text{ if } (\alpha + 1)^2 \in \left(\hat{\alpha}, \frac{1}{\beta^2 \delta}\right)
\end{align*}
\]

and the first-best network path

\[
  s_{t+1} = \beta [(\alpha + 1) s_t + a]
\]

it is straightforward to conclude that the private solution only mimics the first-best solution when network effects are strong \((\alpha + 1)^2 > \hat{\alpha}\) and the size of the green good’s network \(s\) is below the critical threshold \(\hat{s}\). After the green good’s network reaches this critical value (or for any network size, in the case of weak network effects), the private solution no longer coincides with the first best. Actually, upon observation, it is easy to conclude that, in the last case, the linear function defining the network evolution in the first best is more slopped than the linear function defining the green good’s optimal solution (since \((\alpha + 1) > (1 + \alpha - \gamma_1)\), given \(\gamma_1 \geq 0\)). Therefore, after the critical
network size $\tilde{s}$ is reached (or for any network size in the case of weak network effects), we get that the network evolution in the private solution is too slow in comparison with the private solution. This result holds even when the steady-state network is unbounded both for the social planner and for the private green good producer. This means that although universal adoption eventually arises as an optimal outcome in both cases, the path to this outcome in the private solution is inefficiently slower (than the first-best one).

4 Conclusion

This paper investigates the optimal behavior of a dominant firm that produces a network green good in an industry with a competitive fringe that produces a brown good. We study the pricing behavior as well as the long run dynamics of the resulting network size. In addition, we compare the dominant firm’s solution to the socially optimal solution, in which both consumption network externalities and environmental externalities are taken into account. We model the problem as an infinite-horizon, discrete time decision problem of a firm selling a good whose utility depends on the backwards-discounted accumulated volume of past sales. The model reflects three key concepts. The first is to model environmental goods (such as solar panels or electric vehicles) as goods subject to network effects. The idea is that a person is more likely to buy such a product if he believes that its market will be sufficiently successful to give rise to a significant beneficial impact on the environment. The second is to treat accumulated past sales as a state variable, dispensing with the controversial role of expectations in network economies. The third is the concept of a backwards-discounted network size, which keeps the overall network bounded and makes the infinite horizon problem well-defined. In short, an objective of the paper is to harness some of the key insights derived for network industries in industrial organization to shed new light on markets for environmental goods and technologies. A key advantage of modeling the network formation process as an explicit dynamic recursion leads to the identification of the steady-state network size with its static counterpart, namely what Katz and Shapiro (1985) called the fulfilled expectations network size.

The optimal pricing of green goods depends in important ways on the strength of network effects. In general, early purchasers are rewarded by the dominant firm for offering critical participation in the firm’s effort to build an initial consumer base. For sufficiently strong network effects, the dominant firm would even be willing to subsidize earlier consumers by setting a price below to marginal cost. In that case, universal adoption of the green good may be reached in the profit-maximizing
solution. The dominant firm charges a markup equal to zero for low values of the network size and then charges a markup that is increasing in the network size. In fact, for strong network effects, the dominant firm would also be interested in subsidizing initial cohorts of consumers so that the market builds momentum, increasing future consumers’ willingness to pay for the good.

Comparing the private solution to the first-best outcome when the environmental benefits of the green good are sufficiently large for the social planner to be interested in patronizing this good, we find that the socially optimal steady state coincides with the private one for high peer effects, as both cases lead to universal adoption. For intermediate peer effects, a benevolent planner would foster universal adoption, whereas the dominant firm prefers to leave some consumers out of the market. Here the dominant firm is not willing to lose revenue on initial cohorts of consumers, and always sets gets a positive markup in the steady state. As a result, the network size for late consumers is lower in the private solution than in the social optimum.

In light of this, our analysis reveals that there is room for policy intervention in the context of green goods with network effects and substantial environmental benefits (vis-à-vis their more pollutant variants). We find that the design of the environmental policy crucially depends on the intensity of network effects. In particular, in the case of strong network effects are strong (such that both the green producer and the social planner patronize universal adoption), we find that the network expansion is too slow in the private solution. In the case of intermediate network effects, the social planner would patronize universal adoption of the green good, whereas the green producer doesn’t. In such a case, the attribution of a time-variant subsidy that allows market prices to mimic marginal cost pricing could be necessary to avoid such a drastic market failure.

As for possible future research, one could extend the current model to incorporate endogenous technological process innovation, which could possibly change the pricing dynamics. Other extensions of the model include the study of other forms of network effects, such as forward-looking consumers and the analysis of more complex market structures, such as oligopolist competition.
Appendix

**Proof of Lemma 2.** Start with the Euler equation (10) and consider \( p(s) = \gamma s + \phi \). Under this specification, the Euler equation assumes the form

\[
\alpha s + a - 2(\gamma s + \phi) = \delta \beta [[(\alpha + 1)[a + \alpha(\alpha + 1)s - (\gamma s + \phi)]]] - \left(2 + \alpha\right)[\gamma(\alpha + 1)s - (\gamma s + \phi)] + \phi]
\]

Identifying constants, we obtain two equations in \( \gamma \) and \( \phi \), namely

\begin{align*}
\alpha - 2\gamma &= \beta \delta (\alpha \beta (\alpha + 1)(\alpha - \gamma + 1) - \beta \gamma (\alpha + 2)(\alpha - \gamma + 1)) \\
\alpha - 2\phi &= -\beta \delta ((\phi + \beta \gamma (a - \phi))(\alpha + 2) - (\alpha + 1)(a + \alpha \beta (a - \phi)))
\end{align*}

(21)

Solving (21), there are two candidate solution pairs for the variables \((\gamma, \phi)\). The first pair, which we argue below is the unique valid solution, is given by

\begin{align*}
\gamma_1 &= \frac{\Delta - \left(1 - \beta^2 \delta (\alpha + 1)^2\right)}{\beta^2 \delta (\alpha + 2)} \\
\phi_1 &= a \frac{\Delta - \beta \delta (1 - \beta)(\alpha + 1)}{\Delta - \beta \delta (1 - \beta)(\alpha + 1) + (1 - \beta \delta)}
\end{align*}

where \( \Delta \triangleq \sqrt{(1 - \beta^2 \delta)(1 - \beta^2 \delta (\alpha + 1)^2)} \), with \( (1 - \beta^2 \delta)(1 - \beta^2 \delta (\alpha + 1)^2) > 0 \) iif Assumption 1 holds.

The second candidate solution pair is

\begin{align*}
\gamma_2 &= -\frac{1}{\beta^2 \delta (\alpha + 2)} \left(\sqrt{(1 - \beta^2 \delta)(1 - \beta^2 \delta (\alpha + 1)^2} + 1 - \beta^2 \delta (\alpha + 1)^2\right) \\
\phi_2 &= a \frac{\Delta + \beta \delta (1 - \beta)(\alpha + 1)}{\Delta + \beta \delta (1 - \beta)(\alpha + 1) - (1 - \beta \delta)}
\end{align*}

The candidate solution \( p(s) = \gamma_2 s + \phi_2 \) can be shown not to satisfy the transversality condition, and can therefore be ruled out.\(^{22}\) Alternatively, one could calculate the optimal Markovian (time-varying) policy for every finite-horizon, and then take the limit as the horizon tends to infinity.

\(^{22}\)The dynamic program at hand is part of the well known class of linear-quadratic problems (for a general treatment, see e.g., Bertsekas, 1976). Such problems do not have a bounded one-period reward and as such do not quite fit the usual setting in economics (Stokey, Lucas and Prescott, 1989). The transversality condition of the problem is

\[
\lim_{t \to \infty} \delta^t s, V'(s_t) = 0 \quad (22)
\]

This condition can be interpreted as saying that the present discounted value of the accumulated network at infinity should be zero, or that the accumulated network, \( s_T \) should not grow too fast as compared to its marginal value \( (\delta^T v'(s_T)) \). In other words, the transversality condition rules out the overaccumulation of network and a dominant
It is easy to verify that there is a unique Markovian optimal policy for every horizon, and that it converges (uniformly) to the solution

\[ p(s) = \max\{\gamma_1 s + \phi_1, 0\} \]

Either way, the result is that the latter price policy, which is affine in the network size, is the unique optimal solution for the infinite-horizon problem. Under Assumption 1, direct computations show that \( \gamma_1 > 0 \) and \( \gamma_2 < 0 \).

firm who prices so that the network over accumulates is not behaving optimally. (22) can be rewritten as:

\[
\lim_{t \to \infty} \delta^t ((\alpha + 1) (\alpha + s_t \alpha) - (\alpha + 2) p_t) s_t = 0.
\]
References


