Endogenous quality choice and environmental impact in a model of continual technological change

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Abstract

This model of a vertically-differentiated product market combines features of optimization and bounded rationality in firms’ and consumers’ decisions. Their interactions impact the variety and level of technology available to consumers, as well as the effect of ongoing technological change on the natural environment, where waste enters total surplus as a negative externality. Consumers, through their upgrade choices and preferences for innovation, are intermediaries for the effects that the rate of technological change has on the natural environment. From the policy-maker’s perspective, this model illustrates the trade-offs that exist in a dynamic equilibrium of producer improvement and costly waste creation.

Exploration of the model’s characteristics investigates the usual first-order questions of the relationships between pairs of variables—connections between consumers’ preferences and the rate of technological change, between costs of production and environmental impact, and the effect of various incentive mechanisms on producer and consumer behaviors. The region of interest in the model’s parameter space can also be calibrated to mimic observed and desirable stylized facts. This means that multi-dimensional relationships can also be investigated within the bounds of expected stylized facts.

1 Introduction

This agent-based model explores the interplay between a product’s rate of technological change and its impacts on the natural environmental, through the waste created by product replacement. Technology proceeds endogenously through firms’ market research on consumers’ preferences given the current level of embodied technology and proposed product introductions found through a stochastic R&D process. Investigating the question in this fashion allows an environmental considerations to take into account the behaviors in which both firms and consumers interact dynamically to create producer profits and consumer...
surplus. New phones are brought to market to replace the handsets currently in use; some succeed and some fail.

Minimizing the devices’ toxic components’ effects on land and water is important to policy makers. According to EPA (2011), only 8% of the 141 million mobile devices “retired” in 2009 were recycled. That is a statistic widely reported in the years since, but EPA (2016) notes that estimating percentages like these is difficult even with the thorough methodology provided in that report, after stating (in Appendix C) that only 1% of the phones at end-of-life in 2013 can be accounted for in known recycling programs. A variety of interventions can be considered (OECD 2001), including end-of-life taxes and advance disposal fees to incentivize consumers’ actions, or related methods to incentivize producers decision-making. This model incorporates the analysis of policy interventions within the context of innovation decisions made by (a) producers, as they innovate, and (b) consumers, as they replace existing devices for newer, higher quality models.

The model considers the endogenous determination of product characteristics in a one-dimensional quality space, modeling the firm’s decision to introduce a new products and illustrating the steady-state condition of continuous product innovation. To begin the exploration of the parameters’ effects on market characteristics, we explore the relationship between the average quality of phones and introduction cost. We also illustrate the extent to which a firm chooses to refrain from introducing a new model that cannibalizes its existing cell phone models.

As an initial evaluation of the environmental impacts of the deposit-refund program, consider the effect of a deposit-fee and variation in consumers’ nuisance costs incurred when recycling used phones. Finally, the quantity of new phones purchased and old phones landfilled are compared as entry costs are varied to illustrate the relationships between landfill rates and the levels of deposit fee and entry costs.

2 Overview, design concepts, and details

The following description roughly follows the ODD format described in Grimm, Berger, DeAngelis, Polhill, Giske, and Railsback (2010).

2.1 Entities, State Variables, and Scales

The three types of agents are consumers, brands, and models—the last being types of cell phones sold by a producer of a given brand.

A consumer’s primary state variable is the consumer surplus it receives from the current phone. In any period that this consumer considers replacing that model of phone, the state variables also include the set of models considered for replacement and their consumer surplus. When the consumer decides to replace the current phone, a final state variable is the utility received from recycling versus disposing of the phone in the landfill.
Each brand’s (firm’s) primary state variables are a set of models that it sells, its current profit, and its estimated future profit. The estimated profit is determined by the subset of consumers that the firm surveys as it considers whether to introduce a new model.

The phone models each belong to a brand. A given model’s primary state variable is its (vertically differentiated, one-dimensional) quality. The phone’s quality never changes and this model of phone is never sold by another firm. (It is possible for a firm to consider selling a model with the identical quality of one of another firm’s existing models, but this is highly unlikely to occur in a probabilistic sense even if it were to be considered by the firm to be a profitable introduction.) The final phone model state variables is its price. Firms consider the effects of nearby models prices on their phone’s profits, so these may be considered part of a phone’s state variables as well.

2.2 Process Overview and Scheduling

The main procedure in the implementation of this model contains the following sub-procedures, executing in the following order.

1. Each model updates its knowledge of the nearby models, the nearest above and below this model’s quality.

2. The firm chooses prices for all of its phones by best-responding to the prices known for models in the previous period.

3. Each consumer determines the consumer surplus created by their current phone and compares it to a randomly-chosen set of available phones, switching to the best alternative if it provides great consumer surplus than their current phone.

4. All firms become aware of the lowest-quality phone that could be sold to at least one consumer. This quality will be lower than any currently-held phone’s quality because the consumer’s outside good is their current phone.

5. Firms innovate by considering the profitability of a new phone.
   
   (a) This phone has quality at least as great as the lowest-acceptable quality level determined in the previous step.

   (b) The randomly-chosen quality level is also less than a highest-possible quality level (that is increasing at an exogenously-determined rate).

   (c) The firm chooses the phone’s price as a best-response to the existing models and polls a randomly-chosen group of consumers to see if they will buy it.

   (d) The phone is introduced if the firm’s profits are expected to increase as a result.

6. The firm checks to see if it can increase its profits by considering each of the existing models at a time to see if its withdrawal would result in a rise in firm profits.
2.3 Design Concepts

- This basic principles behind this ACE model are to consider vertically-differentiated products’ innovation and the resulting steady-state stream of consumers’ discards.

- All agents in this model adapt to changes in their environment. Consumers adjust their phone ownership to increase their surplus compared to the outside good (their current phone). Firms (brands) adjust their current models’ prices (thus inducing adaption by the phone models) and by both introducing new models and withdrawing models that are no longer profitable.

- Firms engage in some remedial learning through surveys of consumers when they consider new model introduction. Their price adjustments can also be seen as the group of firms “learning” the multi-product Nash equilibrium, although the parameter settings may not allow them to actually reach those equilibrium prices.

- Consumers sense the price and quality of a random subset of models at any given time. Firms sense all products’ quality and price.

- In addition to the stochastic processes described above for firms’ consumer “surveys” and each consumer’s knowledge of a subset of the available phones, firms decision to innovate at any time is a stochastic realization.

- The collectives in the model include the models that are grouped into a given brand and the replication of consumers with identical tastes and choices. Groups of consumer agents differ in their preferences for the phones and their nuisance costs of recycling disposal.

- The model’s outputs are the measured variables that include the rate of technological change (introduction of new models and their average improvement over existing quality) and the rate at which phones are sent to the landfill versus recycled.

2.4 Initialization

The system is initialized with consumers that have no phones and firms with one phone model each.

3 Agents’ behaviors

Consumers receive consumer surplus, firms receive profits, and each agent acts to improve the level of their goal variable. Their ability to improve its level is constrained by the
timing at which they make a decision, by the information they have when a decision are made, and by other agents’ behaviors and decisions.

### 3.1 Consumers’ surplus

Consumer surplus is determined by a standard linear marginal utility of quality for income, the parameter $\theta_i$ in the formula below for consumer $i$’s utility from product $j$ of quality $s_j$ and price $p_j$. We can also refer to $\theta_i$ as consumer $i$’s willingness-to-pay (WTP) for (a unit increase in) quality.

$$U_{ij} = \theta_i s_j - p_j$$

This may, of course, be a special case that has undesirable properties. An alternative with decreasing marginal utility would be $U_{ij} = \theta_i \sqrt{s_j} - p_j$.

For some example maps of constant consumer surplus, see Fig. 1, two pairs of constant-CS curves for two different consumers. A given consumer $i$’s set of indifference curves all have the same slope. As illustrated in the figure, the set of more steeply-sloped curves reflect that consumer’s greater marginal utility of quality for income.

![Figure 1: Constant-CS maps for two consumers with differing willingness-to-pay for quality.](image)

Consumers are uniformly distributed over a range of WTP for quality from $\theta_{\min}$ to $\theta_{\min}$: $\theta \epsilon [\theta_{\min}, \theta_{\min}]$. Note that either a non-linear relationship between product quality $s_j$ and consumer $j$’s surplus or a non-uniform distribution of consumers would complicate the
derivations below, adding integration (rather than simply interval length) in some of the calculations.

3.1.1 Initial purchase of any phone

When deciding among some set of products, consumer $i$ picks the one that provides the largest consumer surplus. There is always an outside good providing $u = u_0$.

3.1.2 Replacing a phone

The situation is more complicated if the consumer already holds a product. In that case the ‘outside good’ is the model that the consumer owns. Firms have a limited understanding that each consumer potentially has a different the outside good. When best-responding to other firms’ previous prices, firms instead

- treat all consumers as having the same value of an outside good and
- (appropriately) don’t attribute a price to the outside good.

The second point would be true even if firms recognized individual consumers’ current phone as their outside good. This distinction is also only important when firms are setting price (as a best-response to other firms’ prices), not when they are calculating profit using “market research” because in the latter case the firm surveys (a random subset of) consumers, who report whether they would buy they new phone or not. Since consumers know which phone they own and its value to them (based on each consumer $i$’s willingness to pay for increased quality, $\theta_i$), the information that firms get through this channel does recognize that varying ’outside good’ that is the consumer’s current phone.

Note, too, that a zero price associated with the outside good means consumer surplus immediately increases after buying a phone. This is true for both the consumer (who knows what phone they hold) and the the firm (that treats all consumers’ outside good option homogeneously). There may be behavioral reasons to argue against treating demand this way, but there seem to be others (including the loss of liquidity that comes with purchasing a new phone) arguing in its favor.

3.1.3 Depreciation

Physical reasons for depreciation are not part of this model. This is realistic in the sense that the rate at which phones physically wear out is slow compared to the rate of technological change. Thus physical reasons for depreciation of a phone are irrelevant.
3.2 Implications for efficiency dynamics and firms’ decisions

With the above characteristics of demand, consumer surplus must be non-decreasing over time. Holding a product provides constant consumer surplus each period. With constantly-decreasing marginal cost of production (at a given level of quality), this means that the total surplus created by a new phone sale is constantly decreasing even without innovation. Innovation then adds to this upward trend in consumers’ surplus.

These characteristics of demand and production place constraints on the viability of both new product introductions and the firm’s existing product line. In particular, firms are aware of the products that are available to consumers, and they are aware that there is some minimum level of quality below which only even a price of zero would be insufficient to generate sales.

3.2.1 New-product introduction

Firms do not consider introducing new products that would have no demand given the consumer surplus received by both the phones that consumers currently hold and the currently-available phones on the market. What does this mean?

The advantage of producing a low-quality phone is that it may also have a lower price than higher-quality phones. The lowest price possible is zero.

3.2.2 Withdrawing phones

Phones are withdrawn from the market, based on two considerations:

- cannibalizing the firm’s demand for another phone and
- falling below the minimum quality that customers would be willing to buy at zero price,

where the latter case is contingent on the products currently available. That lower limit naturally trends up as marginal cost decreases—see the description above.

3.3 Firms’ profits each period

Each firm collects profits from the products it sells. Each product \( j \)'s profit calculated from its price, marginal cost, quantity sold, and fixed cost:

\[
\pi_j = (p_j - m_j)q_j - F_j
\]

See Fig. 2 for an example of products’ mapping from quality \( s \) into the space of consumers’ willingness to pay for quality, \( \theta \). This mapping changes as any firm changes a product’s (or products’) price(s) relative to other products. In that figure, \( \theta_y \) is the upper bound of demand for product 2 with quality \( s_2 \), and \( \theta_x \) is the lower bound.
To generalize this labeling, the subscript to $\theta$ indicates the product that has a lower bound corresponding to that value. Applied to the product with quality $s_2$ in Fig. 2, $\theta_2 = \theta_x$, $\theta_1 = \theta_y$, and $\theta_5 = \theta_z$. As can be seen for the case of $s_3$ in that figure, there may not be a boundary $\theta_j$ for every product $j$.

![Figure 2: Mapping products’ quality to consumers’ willingness-to-pay. Note that some consumers purchase the outside good, and one product has been priced relative to its neighbors so that it receives zero demand.](image)

With the above definitions of consumer surplus and the bounds of demand in the $\theta$ space of consumers’ WTPs, we can calculate demand from the width of the interval over which consumers choose product $j$ based on total demand. (As mentioned above, this process is simplified by assuming a uniform distribution of consumers.) Quantity demanded for product $j$ is the fraction of consumers choosing that product multiplied by total market size:

$$q_j = \frac{\theta_{j+1} - \theta_j}{\theta_{\text{max}} - \theta_{\text{min}}} D_T,$$

where it only remains to define the boundary for the highest-quality product, $J$. Because these are vertically-differentiated goods, as long as any consumer wants to buy some good other than the outside good, there will be no segment of consumers at the highest-WTP levels (the far right of Fig. 2) that buys the outside good: the highest-quality $s_J$ product has upper bound $\theta_{\text{max}}$.

4 Firms’ optimal pricing

Although agents don’t optimize in this model, firms’ behaviors do have best-response features. They thus may find—or at least approximate—optimal prices and quantities. In
addition, understanding the conditions under which we expect various outcomes can help verify the model’s behavioral coding, avoiding behavioral assumptions, initial conditions, and programming mistakes that take the simulation out of the realm of the target social system.

After fixing product qualities we can find conditions for optimal pricing under various scenarios. We will also assume that there is no product entry. Firms are thus backwards-looking, employing adaptive expectations. The ideas follow the structure of models in Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and the literature that followed them.

4.1 Local monopoly: demand and optimal price

Conditions of a local monopoly occur when a product j’s segment of the market does not border on another product’s market segment: the indifferent consumers at the border of this good’s coverage (other than that consumer at \( \theta_{max} \) with the highest willingness-to-pay for quality) are indifferent between j and the outside good. With vertically-differentiated products, this case only applies to the single-product monopolist. This contrasts with the case of horizontal differentiation.\(^1\) In the case of vertically-differentiated products, if there is even one product worth buying in comparison to the outside good for some consumer with \( \theta_j \), then there are no consumers with \( \theta > \theta_j \) who will choose to buy the outside good. This will become more interesting when considering buyers who may already have a good, but for the analysis at hand (where all buyers have the same outside good rather than holding a previously-purchased product), there can be no local monopoly other than a single-product monopolist.

4.1.1 No consumers choose to buy the outside good

In addition to this case only being applicable to the single-product monopolist, it is also a special case of the conditions derived in the next section. Quantity is simply \( D_T \), the entire market. What matters is the condition under which the price necessary to secure this demand is optimal. Given that the firm is selling to everyone, it wants to set the highest price possible. Price is thus just high enough to make the consumer with \( \theta = \theta_{min} \) exactly indifferent between the this product j and the outside good: \( \theta_{min}s_j - p_j \geq u_0 \), or

\[
p_j \leq \theta_{min}s_j - u_0 ,
\]

resulting in \( \pi = (s_j\theta_{min} - u_0 - m)D_T - F \).

\(^1\)If products were horizontally differentiated, then when any firm’s product j is priced high enough relative to the outside good, with nearest neighbors that are also priced high enough, there will be segments of consumers between j and its neighbor(s) buying the outside good. Thus although the first conditions we derive will be for the single-product monopolist, we will then generalize them to be general conditions for products that, when priced optimally, have no direct competition (other than the outside good).
4.1.2 Some consumers buy the outside good

As defined above, \( \theta_j \) is the lower bound of product \( j \)'s demand. To have a local monopoly, the outside good must be purchased by an adjacent, non-empty segment of consumers with \( \theta < \theta_j \) (to the left) of those buying the product of quality \( s_j \) (as illustrated in Fig. 2 by consumers with \( \theta < \theta_x \)). In the case of a single-product monopolist, this is the only good, so there is no \( \theta_{j+1} \) and all buyers with \( \theta > \theta_j \) buy the good with quality \( s_j \). This case defines \( \theta_1 \). The segment of consumers buying the outside good has \( \theta s_j - p_j > u_0 \), so they are those with

\[
\theta < \theta_1 = \frac{p_j + u_0}{s_j},
\]

where this defines the cutoff desired above. This boundary depends on the product chosen by buyers to its right (good \( j = 1 \)) and the product chosen by buyers to its left (the outside good, which produces consumer surplus \( u_0 \)).

To be sure that some segment of consumers will purchase the product, we require that \( \theta_{max}s_j - p_j > u_0 \), so we need a low enough

\[
p_j < s_j \theta_{max} - u_0. \tag{2}
\]

The producer will thus produce and sell the quantity

\[
q_j = \frac{D_T}{\theta_{max} - \theta_{min}}(\theta_{max} - \theta_1) = \frac{D_T}{\theta_{max} - \theta_{min}}(\theta_{max} - \frac{p_j + u_0}{s_j})
\]

and will maximize the profit \((p - m)q - F\), or

\[
\frac{D_T}{(\theta_{max} - \theta_{min})s_j}(p + u_0 - s_j \theta_{max})(p - m) - F
\]

The resulting optimal price

\[
p_j^* = \frac{1}{2}(m + s_j \theta_{max} - u_0) \tag{3}
\]

increases with marginal cost (a supply-side effect) and the largest WTP (a demand-driven effect). Because marginal cost's functional form does not depend on quality (even though it does in the model), the increase in optimal price with product quality is purely demand-driven here (and it would still be a positive derivative if marginal cost increased with quality). Importantly, optimal price decreases as the value of the outside good increases: competition disciplines the firm with market power.

The quantity of the outside good chosen instead of the product with quality \( s_j \) is

\[
\theta_1 - \theta_{min} = \frac{1}{2}(m + u_0) + \frac{\theta_{max}}{2} - \theta_{min}
\]

\[2\]As noted above, all consumers with \( \theta > \theta_1 \) will purchase the good due to the nature of vertically differentiated products.
which is positive as long as
\[ \theta_{\text{min}} \leq \frac{\theta_{\text{max}}}{2} \text{ or both } \theta_{\text{min}} > \frac{\theta_{\text{max}}}{2} \text{ and } s_j < \frac{m + u_0}{2\theta_{\text{min}} - \theta_{\text{max}}}. \]

If neither the first condition nor the second pair of conditions is met, then no consumers buy the outside good and (as derived above) the monopolist sets price so that the customer with WTP for quality of \( \theta_{\text{min}} \) is exactly indifferent between the outside good and the good with quality \( s_j \).

Once we assume that the firm prices optimally, we can describe a bound on the underlying parameters to ensure that there is a set of customers who want to buy the product at that price. Substituting the optimal price (3) into the expression for the lower edge of consumers buying the good \( s_j \),

\[ \theta_1|_{p^*_j} = \frac{p^*_j + u_0}{s_j} = \frac{\theta_{\text{max}}}{2} + \frac{m + u_0}{2s_j}. \]

That should be less than \( \theta_{\text{max}} \): \( u_0 \) and \( m \) can’t be too large and \( s_j \) can’t be too small. The relationship that this implies among the four parameters is

\[ s_j > \frac{m + u_0}{\theta_{\text{max}}}, \tag{4} \]

ensuring that the optimal price will generate demand for the product \( s_j \).

With these conditions on the optimal price satisfied, the quantity that the monopolist sells is

\[ q^*_j = \frac{(s_j\theta_{\text{max}} - m - u_0)D_T}{2(\theta_{\text{max}} - \theta_{\text{min}})s_j}. \tag{5} \]

Some of the effects are signed by inspection of this expression: Optimal quantity decreases with increasing marginal cost (a supply-side effect interacting with the demand side) and the value of the outside good (a competitive effect, motivated by demand-side decisions that have a greater effect than the output effect of the fall in the optimal price). Taking a derivative signs the effect of increased product quality: \( \frac{\partial q^*_j}{\partial s_j} = \frac{(m - u_0)D_T}{2(\theta_{\text{max}} - \theta_{\text{min}})s_j^2} > 0. \)

Unsurprisingly (but subject to the other conditions outlined above) increasing product quality will increase the quantity sold.

The effects of the bounds on WTP \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \), are not immediately intuitive. It is easy to see that the effect of increasing \( \theta_{\text{min}} \) in (5) is to decrease the denominator and thus increase the optimal quantity. Why does decreasing the width of the WTP range increase the optimal quantity? Because we are holding the size of the market fixed at \( D_T \), and this decrease in the range of consumer heterogeneity increases the density of consumers in any given segment of the interval. A similar effect occurs when an increase in \( \theta_{\text{max}} \) increases the range of consumer heterogeneity, thus decreasing consumer density everywhere in that range). In preparation, note that the lower limit on the optimal price in (3) is given by
the inequality (1), and at the boundary of that inequality, $s_j \theta_{\min} = p_j + u_0$. Now taking the derivative, $\partial q_j^*/\partial \theta_{\max} = \frac{(m + u_0 - s_j \theta_{\min})D_T}{2(\theta_{max} - \theta_{min})^2 s_j}$, which at the boundary just mentioned is $-\frac{(p_j - m)D_T}{2(\theta_{max} - \theta_{min})^2 s_j}$. Since we derived this with a lower bound on the optimal price $p_j^*$, this derivative must be negative. The decrease in the optimal quantity as $\theta_{max}$ increases is a result of the lower consumer density in any interval of the $[\theta_{min}, \theta_{max}]$ support.

4.2 The multi-product monopolist: demand

We’ve seen in the development above that there is no general ‘local monopoly’; only a single-product monopolist, and other forms with multiple products. This section still deals with monopoly, and two following sections consider the (single and multi-product) oligopoly cases.

Put differently: whether there is one firm or many, all products compete with their nearest two neighbors, except the product of highest quality and the product of lowest quality. Each of these exceptions has one neighbor, and the latter may compete directly with the outside good. There is one more exception: a product may have been priced out of the market, relative to the goods that would otherwise be its neighbors.

Refer back to Fig. 2 in each of the cases below. At times there will be direct mention of its examples, at times references are left to the reader.

4.2.1 Demand for each product with two neighbors

Product $j$ with price $p_j$ has closest neighbors $j - 1$ and $j + 1$, with qualities $s_{j-1} > s_j > s_{j+1}$; the other products’ prices are $p_{j-1}$ and $p_{j+1}$. Label the lower bound of each product’s range of demand with the same subscript as that product. The demand for the product with quality $s_j$ is thus the interval $[\theta_j, \theta_{j+1}]$.

Writing it this way ignores the possibility of a mass of consumers at either end of the interval (in other words, there must be measure-zero consumers at each end of the interval). A more-general definition, adding the assumption that a consumers indifferent between two or more goods buys the higher-quality good, would have the interval of demand for $j$ specified instead as open on the right: $[\theta_j, \theta_{j+1})$.

To find the lower-bound of $j$’s demand, we consider the consumer $i$ indifferent between $s_j$ and $s_{j-1}$, so that $\theta_i s_j - p_j = \theta_i s_{j-1} - p_{j-1}$. Solving for this consumer’s WTP for quality gives the lower bound,

$$\theta_j = \frac{p_j - p_{j-1}}{s_j - s_{j-1}}.$$
Thus the range of consumers buying \( s_j \) is \( \theta_{j+1} - \theta_j \) and

\[
q_j = \left( \frac{p_{j+1} - p_j}{s_{j+1} - s_j} - \frac{p_j - p_{j-1}}{s_j - s_{j-1}} \right) \frac{D_T}{\theta_{\text{max}} - \theta_{\text{min}}}.
\]

When can we be sure that this quantity is positive? To consider only one product at a time, we can presume that demand for goods with qualities \( s_{j-1} \) and \( s_{j+1} \) are both positive. Then the good with quality \( s_j \) must have a low enough price relative to its neighbors that to ensure that it has positive demand: \( \theta_{j+1} > \theta_j \). After substitution and simplification, this becomes

\[
p_j < \frac{p_{j+1}(s_j - s_{j-1}) + p_{j-1}(s_{j+1} - s_j)}{s_{j+1} - s_{j-1}}.
\]

### 4.2.2 Demand for the highest-quality product

The highest-quality good \( s_j \) appeals to customers with WTP for quality in the range \([\theta_j, \theta_{\text{max}}]\), with

\[
q_j = \left( \theta_{\text{max}} - \frac{p_j - p_{j-1}}{s_j - s_{j-1}} \right) \frac{D_T}{\theta_{\text{max}} - \theta_{\text{min}}}.
\]

This demand is positive as long as

\[
p_j < p_{j-1} + (s_j - s_{j-1})\theta_{\text{max}}
\]

### 4.2.3 Demand for the lowest-quality product

Begin by assuming that some consumers purchase the outside good, so the condition opposite of (1) holds, \( p_1 > s_1\theta_{\text{min}} - u_0 \), so that \( \theta_1 > \theta_{\text{min}} \). The range of consumers who purchase the lowest-quality good \( s_1 \) is \( \theta_1 - \theta_{\mid j=2} \), resulting in quantity demanded

\[
q_1 = \left( \frac{p_2 - p_1}{s_2 - s_1} - \frac{p_1 - u_0}{s_1} \right) \frac{D_T}{\theta_{\text{max}} - \theta_{\text{min}}}.
\]

If its price is low enough, as given by (1), all customers in the left-most segment of the \( \theta \) distribution will buy quality \( s_1 \). This is the case where \( \theta_1 = \theta_{\text{min}} \), and we will need to consider the boundary condition when making calculations of optimal price.

### 4.2.4 Multiple products priced out of the market

When all boundaries in the \( \theta \) interval are non-overlapping, there need be no considerations in addition to those described above. If there is only one overlapping pair of boundaries, then the situation is also very simple: pull out the excluded good. If none of the goods priced out of the market are neighbors, the situation remains the same and simple: each of the goods are excluded and the boundaries recalculated without those goods.
Things become more involved when there are two adjacent products that will be excluded. In this case, there may be three possibilities (the first, the second, or both received zero demand). With three adjacent products; there may be $3 + 2 + 1 = 6$ possibilities to consider. Etc.

**References**


