Using Cost Functions to Estimate Productivity and Abatement Efficiency: An Application to Coal-Fired Power Plants

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Abstract

Assuming profit maximization is problematic for regulated industries where firms/plants minimize costs to produce output targets. A prime example is electricity generation where plants minimize costs subject to production and pollution control requirements. Assuming cost minimization while estimating a production function following OP/LP/ACF produces a non-viable control function. Derived input demand is a function of input prices, productivity, and output quantity. Upon inversion, replacing productivity in the production function with its proxy yields output as a function of itself. We avoid this by assuming cost minimization of production and pollution control subject to an output constraint and then deriving and estimating the dual cost function with productivity terms for both activities. Using 1995-2005 data on the 80 largest US coal-fired power plants, we compute reasonable implicit prices of sulfur content, Btu content, and emission permits, moderate scale economies, heterogeneity in the two productivity terms, and modest negative correlation between them. Counterfactual analyses consider changes in environmental restrictions and the price of coal. We also examine the impact of estimated productivity, vintage, and environmental restrictions on plant closings.

KEYWORDS: Electric power plants, production functions, cost functions, generation productivity, abatement efficiency, emission permit prices

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1 Introduction

The traditional approach of Olley and Pakes (1996) (OP), Levinsohn and Petrin (2003) (LP), and Ackerberg, Caves, and Frazer (2015) (ACF) to production function estimation requires the assumption of maximization of restricted profits. In theory, from the restricted profit function one would derive an input demand function, whose arguments are variable input prices, fixed input quantities, output prices, and unobserved productivity. By inverting this function one would obtain a control function which proxies for unobserved productivity in a composite error term. The assumption of OP is that the firm maximizes profits and that the derived input demand equation for investment is a function of vintage, capital, and productivity. Given a monotonic relationship between investment and productivity, they invert this function, obtaining a control function for productivity whose arguments are investment, capital, and vintage. Since output quantity is not an argument of this function, they substitute this control function for unobserved productivity in the production function. Its parameters are identified and estimated, using additional assumptions about a Markov process for productivity. The papers by LP and ACF follow essentially the same methodology, where LP replace investment with materials while ACF make labor endogenous.

However, profit maximization is a problematic assumption for regulated US and foreign electric utilities, plants in a variety of industries (including electricity generation) which minimize costs to produce exogenous output targets, and regulated railroads and airlines in many countries. Arguably, the assumption of output-constrained cost minimization is a more appropriate model. For electric utilities, plants owned by both regulated and deregulated firms are best treated as cost minimizers subject to exogenously determined production goals. Regulated utilities are cost minimizers subject to prices determined by regulatory commissions and output determined by consumers. Even deregulated utilities
will set production goals for their plants.


If the researcher wishes to assume minimization of variable costs and follow the OP/LP/ACF approach of estimating a production function which includes a productivity term, he would derive an input demand equation that is a function of variable input prices, output quantity, fixed inputs quantities, and productivity. Assuming invertibility of this function, a proxy function for productivity is a function of the variable input prices, the output quantity, and the quantities of fixed inputs. After substituting this proxy function into the production function, one would express output as a function of itself. We avoid this problem by assuming constrained cost minimization and deriving a cost function by minimizing the cost of production and pollution control subject to an output constraint. We then estimate this cost function and productivity terms for both production and abatement, which allows
identification of the parameters of the production function and the pollution control cost function. We utilize a function involving lagged productivity derived from the production function in place of a control function. Thus, our dual cost-function approach allows us to consistently model both productivity terms, the control of pollution, and the production of electricity.

To apply these techniques we study the efficiency of plants in the production of electricity and the abatement of sulfur dioxide (SO₂) using a balanced panel of the 80 largest US coal-fired power plants from 1995-2005. We compute reasonable implicit prices of sulfur content, Btu content, and emission permits, moderate scale economies, heterogeneity in the two productivity terms, and a modest negative correlation between them.

One important policy question is whether restructuring has improved plant efficiency and lowered the cost of electricity. A number of researchers conclude that restructuring the generation sector so that it is competitive appears to reduce costs. Rather than estimate a production function directly, Fowlie (2010) examines the effect of restructuring on plant input efficiency and emissions of nitrogen dioxide (NOₓ) using a pooled panel of 702 coal-fired US electric generating plants from 2000-04. She finds that deregulated plants in restructured electricity markets are less likely to install more capital intensive pollution control technologies compared to similar regulated and public plants. Hiebert (2002) estimates a variable cost frontier for US power plants and finds evidence of greater efficiency as coal plants are restructured. Using state-level data Joskow (2006) finds that restructuring significantly reduces the price of electricity in the residential and industrial markets by 5% to 10%. Rungsuriyawiboon and Stefanou (2007) estimates a dynamic cost function for US utilities and find that TE of inputs improves for utilities in restructured jurisdictions. However, none of these studies derive a cost function assuming minimization of both generation and abatement costs.
Another important policy question is whether the substantial number of recent closings of coal-fired generating units is due to increased environmental regulations or to plant age and reduced efficiencies of generation and abatement. Web pages of coal-fired plants that have recently shuttered power generation units typically blame increased environmental restrictions. However, the typical retired generation unit began operation in the mid 1950s and is operating well beyond its expected lifetime of 40-50 years. It may also exhibit low efficiencies of production and abatement, which may be far more important than increased environmental restrictions. In this case, efforts by the Administration to bring back thermal coal are likely to be ineffective. Finally, counterfactual analyses consider recent dramatic drops in permit prices and changes in the price of coal due to state subsidies for consumption of locally-produced coal.

2 Data

Our data is a balanced panel of the 80 largest coal-fired power plants in the U.S. from 1995 to 2005. The majority of our sample plants are located in the Southern, Mid-Atlantic, or Midwestern states, with a few in the Rocky Mountain and Far Western regions. The technology modeled in this study consists of one good output, one bad output, three good inputs, and one bad input. The good output is total electrical generation from coal measured in megawatt hours (mWh), while the bad output is the SO$_2$ in short tons. The good inputs are the capital stock (measured in 1973 year dollars), the number of employees, and the heat content in millions of Btu (MMBtu) of coal at each power plant. The single bad input is the sulfur content of the coal burned in short tons, $s_{jt}$. While the power plants in our sample consume coal and either oil or natural gas, on average 99% of the Btu generated by each plant comes from coal consumption. The quantity of capital $k_{jt}$ used in

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$^1$See the Appendix for the list of plants.
the production function is the capital for generation only. It is calculated using the plant’s total capital and subtracting the plant’s FGD capital obtained from EIA Form 860.

We obtained our data from a number of different sources. FERC Form 1 provides labor and capital data for private electric power plants, while the EIA-412 survey is the source of this data for public power plants. Although an increasing number of private utilities did not report capital and labor data after 2005, none of these are in our sample. While DOE halted the EIA-412 survey after 2003, the Tennessee Valley Authority voluntarily posted 2004-06 data for its electric power plants on-line. The U.S. Department of Energy (DOE) Form EIA- 767 survey (see U.S. Department of Energy, various years) is the source of information about fuel consumption and net mWh generation by plant. The SO\textsubscript{2} emissions data at the plant level are collected by the EPA as part of its Continuous Emissions Monitoring System. The sulfur content of the coal burned by plant comes from EIA Form 423\textsuperscript{2}.

Unfortunately, price data for inputs or outputs does not exist at the plant level, which is our level of analysis. However, an important source for generating latent plant-level prices is firm-level prices of energy, capital, and labor inputs, which are available. We compute the user cost of capital at the firm level using the corporate tax rate, the corporate property tax rate, the depreciation rate, the firm’s yield, and the Handy-Whitman Index as in Atkinson, Primont, and Tsionas (2015). The yield on the firm’s latest issue of long-term debt comes from Moody’s Public Utility Manual (before 2001) and from Mergent’s Public Utility Manual after that time. From FERC Form 1 we collect the wage paid by the firm as salaries plus wages for electric operating and maintenance workers divided by the number of full time workers plus one-half the number of part-time workers for the firm. From EIA Form 423 we compute the price of coal per MMBtu to the plant, using the price of the weighted-average monthly deliveries of coal to the plant. Although these deliveries

\textsuperscript{2}We wish to thank Carl Pasurka for supplying us with some of the data that we utilized, namely the data on input and output quantities.
are provided at the plant level, frequently 90-day stockpiles of coal are maintained. Thus, we do not have exact data on the price of the actual quantities of coal burned during a calendar year.

The papers by OP and LP assume that the Law of One Price holds, which would mean that input prices are not correlated with endogenous variables. If the Law of One Price holds, variation in prices occurs only because of unobserved quality differences, rendering prices correlated with the error term, which contains these quality measures. However, with electric utilities, omitted quality differences are minimal. For wages, unmeasured quality differences should be important only for higher-level management. Other tasks are highly mechanized in a very capital-intensive process. The price of energy is in terms of thermal content, so there is no omitted quality differential. The price of capital is typically a function of the price index for equipment and structures, the yield on utility bonds, tax rates, the ratio of equity to total capitalization, and depreciation. This calculation includes all important quality differentials. Finally, the price of output is measured as price per kilowatt hour in terms of a standardized voltage, which is the relevant quality measure.

We argue that numerous market imperfections should cause differential exogenous input price variation across time and firms. A differential degree of union power, which is greater in the Northeast, but which has diminished over time nationwide will cause both temporal and cross-sectional exogenous price variation in wages. The Northeast historically receives less natural gas than the South, but because of recent fracking natural gas prices relative to coal have fallen dramatically, increasing supply and consumption of natural gas in the Northeast. This will cause both types of exogenous coal price variation. Recent subsidies by some Eastern states for the purchase of high-sulfur coal within same-state boundaries will also cause both types of exogenous price variation. The same is true for the declining market power of Eastern coal mines as air quality regulations tighten over time requiring
more use of low-sulfur Western coal by Eastern plants.

We also collected a number of variables that measure coal quality and environmental costs. These include the $s_{jt}$, the SO$_2$ removal rate of scrubbers, the percent of total plant capacity that is scrubbed, the removal rate of the plant’s flue gas desulfurization (FGD) system, and finally the O&M and capital costs of FDG devices. These data come from EIA Forms 767 and 860.

Table 1 shows the summary statistics of the data. Among the 80 plants, only 18 plants employ FGD units in all years from 1995 to 2005. The other 62 plants have never installed FGD units. The left panel is for non-FGD plants and the right panel is for FGD plants. The FGD plants are larger than the non-FGD ones on average, with greater electrical generation, capital, labor, and coal-consumption. They also use coal with a significantly higher $s_{jt}$ and somewhat lower $b_{jt}$. The median $s_{jt}$ is 1.478% for FGD plants, but only 0.887% for non-FGD plants. The median $b_{jt}$ is 22.521 million/ton for FGD plants and 24.144 million/ton for non-FGD plants. The median coal price is lower for FGD plants because of they buy coal with a higher sulfur content and a lower $b_{jt}$. Median coal prices are $38.065/ton for non-FGD plants and $25.619/ton for FGD plants. The capital yield and the labor wage are similar for the two types of plants: median capital yields are 7.55% for non-FGD plants and 7.54% for FGD plants, while median wages are $43,560 for non-FGD plants and $43,339 for FGD plants.
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Table 1: Data Summary Statistics for Plants

- **Generation (10^6 mWh)**: The median generation for non-FGD plants is 4.417 x 10^6 mWh with a minimum of 0.116 x 10^6 mWh and a maximum of 22.329 x 10^6 mWh. For FGD plants, the median is 7.663 x 10^6 mWh with a minimum of 1.860 x 10^6 mWh and a maximum of 20.321 x 10^6 mWh.

- **Capital (10^8 $)**: The median capital for non-FGD plants is 7.582 x 10^8 $ with a minimum of 1.832 x 10^8 $ and a maximum of 4.418 x 10^8 $. For FGD plants, the median is 13.090 x 10^8 $ with a minimum of 3.675 x 10^8 $ and a maximum of 37.162 x 10^8 $.

- **Labor**: The median labor usage for non-FGD plants is 129 with a minimum of 23.744 and a maximum of 578. For FGD plants, the median is 212 with a minimum of 64.634 and a maximum of 538.

- **Coal (10^6 tons)**: The median coal usage for non-FGD plants is 1.967 x 10^6 tons with a minimum of 0.610 x 10^6 tons and a maximum of 12.308 x 10^6 tons. For FGD plants, the median is 3.999 x 10^6 tons with a minimum of 0.865 x 10^6 tons and a maximum of 9.135 x 10^6 tons.

- **Sulfur (%)**: The median sulfur content for non-FGD plants is 0.887% with a minimum of 0.125% and a maximum of 3.788%. For FGD plants, the median is 1.478% with a minimum of 0.326% and a maximum of 3.947%.

- **Btu (10^6/ton)**: The median Btu content for non-FGD plants is 24.144 x 10^6/Btu with a minimum of 16.212 x 10^6/Btu and a maximum of 26.348 x 10^6/Btu. For FGD plants, the median is 22.521 x 10^6/Btu with a minimum of 15.451 x 10^6/Btu and a maximum of 24.639 x 10^6/Btu.

- **Removal (%)**: The median removal rate for non-FGD plants is 85.041% with a minimum of 27.021% and a maximum of 97.700%. For FGD plants, the median is 85.041% with a minimum of 27.021% and a maximum of 97.700%.

- **Yield (%)**: The median yield for non-FGD plants is 7.550% with a minimum of 5.380% and a maximum of 8.950%. For FGD plants, the median is 7.54% with a minimum of 5.380% and a maximum of 97.700%.

- **Wage (10^4 $)**: The median wage for non-FGD plants is 4.356 x 10^4 $ with a minimum of 2.491 x 10^4 $ and a maximum of 9.468 x 10^4 $. For FGD plants, the median is 4.339 x 10^4 $ with a minimum of 2.675 x 10^4 $ and a maximum of 8.320 x 10^4 $.

- **Coal price ($/ton)**: The median coal price for non-FGD plants is 38.065 $/ton with a minimum of 11.382 $/ton and a maximum of 141.481 $/ton. For FGD plants, the median is 25.619 $/ton with a minimum of 9.473 $/ton and a maximum of 53.348 $/ton.

- **N**: The number of observations for non-FGD plants is 682, and for FGD plants is 198.

Generation capital has been slowly increasing for all plants. The average annual growth of plant capital is 3.66% for all plants. The level of labor used in generation has been decreasing for all plants, with an annual growth of −4.06%. Plant-level heat input from coal has increased slightly over time, with a mean growth rate of 0.96%. Plant-level electricity generation growth rate is close to the heat growth rate, with a mean of 0.96%. Over our sample period, total electricity generated has increased, while the SO\(_2\) emitted per mWh of electricity has fallen for both non-FGD and FGD plants. Average electricity generation in 10\(^6\) mWh increased from 5.711 in 1995 to 6.647 in 2005.

In Figure 1a, we represent the total generation for all plants with squares, for non-FGD plants with triangles, and for FGD plants with circles. Non-FGD plants generate about twice as much electricity as their FGD counterparts. In Figure 1b, we represent the average SO\(_2\) per mWh generation for all plants with squares, for FGD plants with circles, and for non-FGD plants with triangles. We see that SO\(_2\) production in thousands of short tons per mWh has fallen substantially for both types of plants. The decline has been almost 50% for FGD plants. The non-FGD plants emit from 40-80% more SO\(_2\) per mWh of electricity.
than the FGD plants.

(a) Total generation by plant type  
(b) $\text{SO}_2$ Emission per mWh Generation

Figure 1: Total Generation and $\text{SO}_2$ Emission per mWh Generation

In Figure 2a, we plot the plant-year $b_{jt}$ (in millions) against $s_{jt}$, indicating a substantial area of tradeoff between $b_{jt}$ and $s_{jt}$. We represent the non-FGD plants with triangles and the FGD plants with circles, where the size of each indicates the magnitude of coal purchases. The range of substitution possibilities is greater for FGD plants since they have the option of mixing fuel types and employing FGD. These plants accept a higher $s_{jt}$ only if $b_{jt}$ also increases. Most non-FGD plants, although more concentrated in two areas, exhibit a wide range of similar substitution possibilities. A cluster of non-FGD plants are willing to consume coal with a lower $s_{jt}$ and a lower $b_{jt}$. Another cluster chooses a substantially higher $b_{jt}$ with a slightly higher $s_{jt}$. However, many other non-FGD plants consume coal with a higher $s_{jt}$ so long as its $b_{jt}$ increases. No plants are willing to consume in the lower triangular portion of this table, where $s_{jt}$ would increase without any increase in $b_{jt}$, relative to existing points. In Figure 2b, we plot the plant-year generation of $\text{SO}_2$ against $s_{jt}$. As expected, for the non-FGD plants, there is a linear relationship between the two variables with a slope of approximately 2.0, as expected from the chemistry of converting
sulfur into SO$_2$ without controls. The FGD plants exhibit a wide range of differences in the percent of emissions per ton relative to $s_{jt}$, since plants differ substantially in the percent of emissions that are controlled as well as the control efficiencies of FGD devices as seen from Table 1.

In Figure 2a we graph total generation against total Btu for non-FGD and FGD plants. The linear relationship between the two variables indicates little difference in production efficiency. That is both types of plants appear to be nearly equally efficient in transforming Btu to mWh of electricity, although the non-FGD plants may be slightly more efficient at lower levels of Btu input. In Figure 2b we see that the non-FGD plants between .5 and 1.5 $10^8$ Btu exhibit a slightly greater mWh per Btu than FGD plants.

Figure 2: Sulfur Content, Btu Content, and SO$_2$ Emission
3 Cost Minimization Problem of Coal Plants

We model the production decisions for a set of coal-fired electricity generation plants which are attempting to minimize the costs of coal consumption and pollution control subject to constraints on total generation and environmental degradation. In each period, a plant chooses \( b_{jt} \) and \( s_{jt} \), its two principle characteristics.

3.1 Production Costs

In period \( t \), plant \( j \) first observes its generation capacity used to produce electricity, \( k_{jt} \), its labor stock, \( l_{jt} \), its generation productivity, \( \omega_{jt} \), and its exogenously determined target output, \( y_{jt} \), and then chooses the quality and the quantity of coal to produce \( y_{jt} \). The two key characteristics of coal are its Btu content per ton of coal, \( b_{jt} \), and its sulfur content per ton of coal, \( s_{jt} \). Both affect the total cost of coal and the cost of pollution control for a plant. First, both \( b_{jt} \) and \( s_{jt} \) affect the coal price, which increases with \( b_{jt} \) and decreases with \( s_{jt} \). Second, given the output level, the amount of coal a plant needs depends \( b_{jt} \).
The higher $b_{jt}$, the less coal the plant needs to consume. Lastly, $b_{jt}$ and $s_{jt}$ both affect the abatement cost to control SO$_2$. While $b_{jt}$ determines the total amount of coal, $s_{jt}$ determines the amount of sulfur per ton of coal.

Let the total Btu consumed (total heat input) be $h_{jt}$ and assume that the plant’s non-stochastic production function for electricity has a Cobb-Douglas form,

$$y_{jt} = e^{(\beta_0 + \omega_{jt})} k_{jt}^{\beta_k} l_{jt}^{\beta_l} h_{jt}^{\beta_h}, \tag{1}$$

where $(\beta_0, \beta_l, \beta_k, \beta_h)$ are parameters. Given the production function in (1), the $h_{jt}$ needed to produce $y_{jt}$ for a given $(k_{jt}, l_{jt}, \omega_{jt})$ is

$$h_{jt}(y_{jt}, l_{jt}, k_{jt}, \omega_{jt}) = \left( y_{jt} e^{-(\beta_0 + \omega_{jt})} l_{jt}^{\beta_l} k_{jt}^{\beta_k} \right)^{\frac{1}{\beta_h}}.$$  

Total $h_{jt}$ decreases as the productivity $\omega_{jt}$ increases or as the output $y_{jt}$ decreases. If we apply the traditional approach of OP/LP/ACF, inverting this function would yield $\omega_{jt}$ as a function of $(k_{jt}, l_{jt}, h_{jt}, y_{jt})$. Substituting this into (1) would yield a non-viable estimation equation, since output would be a function of output. Instead, we formulate our cost function approach.

For any $b_{jt}$, the tons of coal required to produce $y_{jt}$ by plant $j$ is

$$n(b_{jt}; y_{jt}, l_{jt}, k_{jt}, \omega_{jt}) = \frac{h_{jt}(y_{jt}, l_{jt}, k_{jt}, \omega_{jt})}{b_{jt}} = \left( y_{jt} e^{-(\beta_0 + \omega_{jt})} l_{jt}^{\beta_l} k_{jt}^{\beta_k} \right)^{\frac{1}{\beta_h}} b_{jt}^{-1}. \tag{2}$$

The higher the $b_{jt}$, the less amount of coal the plant must consume to produce the given electricity output.

The price of coal per ton as delivered to the utility plant, $w^c_{jt}$, is a function of $b_{jt}$, $s_{jt}$,
and freight or shipping costs per ton, $f_{jt}$, from the mine to the plant. Thus,

$$w^c_{jt} = w^c_{jt}(b_{jt}, s_{jt}, f_{jt}).$$

However, coal shipping rates are confidential. We work around this since the marginal rate of substitution by the power plant between $b_{jt}$ and $s_{jt}$ is independent of the distance between the plant and the mine, which determines $f_{jt}$. That is, the generating plant would make the same tradeoff between $b_{jt}$ and $s_{jt}$ at a given plant regardless of the distance between the mine and the plant. Therefore, we assume separability of $b_{jt}$ and $s_{jt}$ from $f_{jt}$ and write the delivered price of coal as

$$w^c_{jt}(b_{jt}, s_{jt}, f_{jt}) = \bar{w}^c_{jt}(b_{jt}, s_{jt}) + w_{jt}(f_{jt}).$$

Thus, only $b_{jt}$ and $s_{jt}$ are choice variables for the firm in terms of minimizing total variable costs in the separable cost function. The total cost of coal is

$$w^c_{jt}(b_{jt}, s_{jt}, f_{jt})n(b_{jt}; y_{jt}, l_{jt}, k_{jt}, \omega^y_{jt}).$$

### 3.2 Pollution Control Costs

The total pollution control cost function for FGD plants can be very different from that for non-FGD plants. For an FGD plant, total pollution control cost includes the expenditures on FGD and on the purchase of pollution permits (and the opportunity cost of any allowances held by the plant) for uncontrolled emissions. A non-FGD plant solely relies on emission permits to comply with environment regulation. Its pollution control cost includes only the expenditures to purchase permits and the opportunity cost of any allowances held by the plant.
3.2.1 Pollution Control Costs of Plants with Scrubbers

The plant \( j \)'s total \( \text{SO}_2 \) abatement and emission cost depends on FGD expenditures and costs to purchase permits, and the opportunity cost of any allowances held by the plant. The total operating costs for an FGD unit depends on the amount of coal, \( s_{jt} \), and its abating efficiency, \( \omega_{jt} \). A plant knows its abating efficiency when choosing the \( s_{jt} \). If a plant lacks a scrubber or does not scrub all of the \( \text{SO}_2 \) that is generates, it must cover uncontrolled emissions with a combination of allowances (which can be traded) and permits, which are allowances purchased from other plants of the EPA.

Let the amount of coal used in generation units that have scrubbers be \( n_{jt}^a (\leq n_{jt}) \). Assume that the total abatement cost function is \( C^a(n_{jt}^a, s_{jt}, \omega_{jt}) \). The abatement cost is expected to increase with the amount of sulfur scrubbed, \( n_{jt}^a s_{jt} \). It will be nonlinear if the marginal pollution control cost changes with the arguments. We assume that the abatement cost is a power function of the total amount of sulfur in the consumed coal:

\[
C^a(n_{jt}^a, s_{jt}, \omega_{jt}) = e^{-\omega_{jt} F(n_{jt}^a s_{jt})} = e^{-\omega_{jt} (n_{jt}^a s_{jt})^\lambda}. \tag{3}
\]

The amount of coal used in a plant’s generating units without scrubbers is \( n_{jt} - n_{jt}^a \). This plant needs to either use its free permit allowances or buy permits to emit \( \text{SO}_2 \) without abating. Let the permit price faced by plant \( j \) in year \( t \) be \( p_{jt} \). Since a plant can trade its endowed free allowances, there is also an opportunity cost to holding allowances.

Given the \( \text{SO}_2 \) removal rate \( r_{jt} \), the amount of sulfur abated from the \( n_{jt}^a \) tons of coal is \( n_{jt}^a r_{jt} \). Thus, the remaining sulfur in the coal is \( s_{jt}(n_{jt} - n_{jt}^a r_{jt}) \) (tons), which will be turned into \( S^c_{jt} = 2s_{jt}(n_{jt} - n_{jt}^a r_{jt}) \) tons of \( \text{SO}_2 \) (based on molecular weight) assuming that all sulfur is transformed into \( \text{SO}_2 \). Then the cost of buying permits for the emission for
Plant $j$ in year $t$ is
\[ p_{jt} S_{jt} = 2p_{jt}s_{jt}(n_{jt} - n_{jt}^a r_{jt}). \] (4)

Therefore, the total pollution control costs has two parts, the costs of using the scrubbers and the cost of using purchased permits/allowances.

### 3.2.2 Tradeoff between Abating and Emitting Sulfur Dioxide

For a given total coal, the plant needs to choose how much SO$_2$ to abate and how much to emit in order to minimize its total pollution control costs. The minimization problem is:

\[
\min_{n_{jt}^a} \left\{ e^{-\omega_{jt}^a} (n_{jt}^a s_{jt})^{\lambda} + 2p_{jt}s_{jt}(n_{jt} - n_{jt}^a r_{jt}). \right\}
\]

Then the optimal amount of $n_{jt}^a$ must satisfy the first-order condition (FOC):

\[ e^{(-\omega_{jt}^a)} \lambda (n_{jt}^a)^{\lambda-1} s_{jt}^{\lambda} - 2p_{jt}s_{jt}r_{jt} = 0, \]

Denote the optimal amount of coal that should be scrubbed by $n_{jt}^{a*}$. With the convex abating cost function assumption, we know that the marginal cost of abatement increases with the scrubbed coal quantity. Then there is a unique optimal abatement level.

Thus, the optimal amount of coal to abate for plant $j$ in year $t$ is a function of $(s_{jt}, r_{jt}, \omega_{jt}^a)$.

\[ n_{jt}^{a*} = \left( \frac{2p_{jt}r_{jt}}{\lambda e^{\omega_{jt}^a}} \right)^{\frac{1}{\lambda-1}} \frac{1}{s_{jt}}. \] (5)

Plugging $n_{jt}^{a*}$ into the pollution control cost function, we obtain the minimum pollution control cost as a function of the quantity of coal and $s_{jt}$. Because the coal quantity depends on the output, $b_{jt}$ and other plant-year specific variables as in equation (2), we can write the pollution control cost as a function of the Btu content and the sulfur content. Let
$X_{jt} = (r_{jt}, p_{jt}, y_{jt}, l_{jt}, k_{jt})$ be the plant-year specific variables. The pollution control cost function is

$$C^s(s_{jt}, b_{jt}; X_{jt}, \omega^y_{jt}, \omega^a_{jt}) = (1 - \lambda) e^{\frac{s_{jt}}{\lambda}} \left( \frac{2p_{jt} r_{jt}}{\lambda} \right) + 2p_{jt} s_{jt} \frac{h_{jt}(y_{jt}, l_{jt}, k_{jt}, \omega^y_{jt})}{b_{jt}}. \quad (6)$$

When $\lambda > 1$, the first term in the cost is negative, and it is the savings in cost for an FGD plant compared to non-FGD plants. The second term is the cost of permits if a plant completely relies on permits to comply to the pollution regulations.

### 3.2.3 Pollution Control Cost of Plants without Scrubbers

For a plant without the scrubbers, the amount of SO$_2$ generated and emitted is $S^e_{jt} = 2s_{jt}n_{jt}$ tons. The cost of permits to emit $S^e_{jt}$ is the same as that for plants with scrubbers.

$$C^s(n_{jt}, s_{jt}, p_{jt}) = p_{jt}S^e_{jt} = 2p_{jt} s_{jt} n_{jt} = 2p_{jt} s_{jt} \frac{h_{jt}(y_{jt}, l_{jt}, k_{jt}, \omega^y_{jt})}{b_{jt}}, \quad (7)$$

### 3.3 Minimization of Total Variable Cost

#### 3.3.1 Plants with FGD

A plant minimizes the sum of its total production costs and abatement cost by choosing $b_{jt}$ and $s_{jt}$. For a plant with FGD, the optimization problem is

$$\min_{b_{jt}, s_{jt}} w^c_{jt}(b_{jt}, s_{jt}) n(b_{jt}; X_{jt}, \omega^y_{jt}) + C^s(s_{jt}, b_{jt}; X_{jt}, \omega^y_{jt}, \omega^a_{jt}),$$

subject to the coal needed to produce the electricity as in equation (2).

The FOC with respect to $b_{jt}$ is

$$\frac{\partial w^c_{jt}(b_{jt}, s_{jt})}{\partial b_{jt}} n_{jt} + w^c_{jt}(b_{jt}, s_{jt}) \frac{\partial n_{jt}}{\partial b_{jt}} + \frac{\partial C^s(s_{jt}, b_{jt}; X_{jt}, \omega^y_{jt}, \omega^a_{jt})}{\partial b_{jt}} = 0. \quad (8)$$
The first term measures the marginal cost of choosing higher Btu-content coal. The second term is the marginal savings in the price of coal when choosing higher Btu-content coal. The last term measures the marginal savings in sulfur abatement costs when less coal is used due to a higher Btu-content. The first-order condition with respect to the sulfur content of coal is

\[
\frac{\partial w_c^e(b_{jt}, s_{jt})}{\partial s_{jt}} n_{jt} + \frac{\partial C_s^e(s_{jt}, b_{jt}; X_{jt}, \omega^y_{jt}, \omega^a_{jt})}{\partial s_{jt}} = 0.
\]  

(9)

The first term is the marginal reduction in the price of coal from choosing a higher sulfur content, while the second term is the marginal abatement cost of using higher sulfur content coal.

Using the derived pollution control cost function, we can derive the explicit functional forms of the two FOCs. We have

\[
\frac{\partial w_c^e}{\partial b_{jt}} \frac{w_c^e}{b_{jt}} - \frac{2p_{jt} s_{jt}}{b_{jt}} = 0.
\]  

(10)

\[
\frac{\partial w_c^e}{\partial s_{jt}} + 2p_{jt} = 0.
\]  

(11)

From the FOCs, we know the plants face permit prices that are equal to their marginal savings on coal when the sulfur content increases. Only when they are equal, are the plants minimizing the total variable costs. From the two FOCs, we can see that the \( b_{jt} \) and the sulfur content choices do not depend on the unobserved productivity and abating efficiency. The optimal \( b_{jt} \) does not change with productivity because that the productivity affects the coal cost and the pollution control cost only through the total amount of heat and that the costs are linear in the amount of heat. The optimal sulfur content does not change with the unobserved productivity for the same reason. It also does not depend on the abating efficiency which only affects the lump-sum savings by using FGD equipment.
Denote the optimal \( b_{jt} \) and sulfur content by \( (b^*_{jt}, s^*_{jt}) \). They are functions of the variables in \( X_{jt} \) and the unobserved efficiencies \( (\omega^y_{jt}, \omega^a_{jt}) \). We can write down the total variable cost function.

\[
C(X_{jt}, \omega^y_{jt}, \omega^a_{jt}) = \left( w^c_{jt}(b^*_{jt}, s^*_{jt}) + 2p_{jt}s_{jt} \right) \frac{h_{jt}(X_{jt}, \omega^y_{jt})}{b^*_{jt}} + (1 - \lambda) e^{\frac{\omega^a_{jt}}{\lambda}} \left( \frac{2p_{jt}r_{jt}}{\lambda} \right)^{\frac{\lambda}{1-\lambda}}. \tag{12}
\]

### 3.3.2 Plants without FGD

For non-FGD plants, the optimization problem is

\[
\min_{b_{jt}, s_{jt}} w^c_{jt}(b_{jt}, s_{jt}) n(b_{jt}, \omega^y_{jt}, X_{jt}) + C^s(n_{jt}, s_{jt}, p_{jt}),
\]

subject to the coal needed to produce the electricity as in equation (2). Using the permit cost in equation (7), the FOCs for \( b_{jt} \) and the sulfur content are

\[
\frac{\partial w^c_{jt}}{\partial b_{jt}} - \frac{w^c_{jt}}{b_{jt}} - \frac{2p_{jt}s_{jt}}{b_{jt}} = 0.
\]

\[
\frac{\partial w^c_{jt}}{\partial s_{jt}} + 2p_{jt} = 0.
\]

The FOCs for non-FGD plants are the same as the FOCs for FGD plants. Denote the optimal \( b_{jt} \) and sulfur content by \( (b^*_{jt}, s^*_{jt}) \), the total variable cost for the plant is

\[
C(X_{jt}, \omega^y_{jt}) = \left( w^c_{jt}(b^*_{jt}, s^*_{jt}) + 2p_{jt}s^*_{jt} \right) \frac{h_{jt}(X_{jt}, \omega^y_{jt})}{b^*_{jt}}. \tag{13}
\]
4 Econometric Model

4.1 Coal Price Function

The delivered price of a ton of coal depends on the $b_{jt}$, the sulfur content, the location of the mine, the location of the plant, the year effect, and the contract type of the purchase. To control for the freight charges on transporting the coal, we use the state dummies of the mine and the plant. There are three types of coal purchase contracts depending on the length of the contract and whether the contract is new, C, NC, and S. The price is expected to increase with $b_{jt}$ and decrease with the sulfur content.

Therefore, we assume that plant $j$ faces an average coal price function, $\bar{w}_{jt}(b, s, f) = \bar{w}^c(b, s; n) + w(f)$, and the non-stochastic, hedonic, coal price function is

$$
\bar{w}_{jt}^c(b, s, f) = \alpha_0 + \alpha_1 b_{jt} + \alpha_2 s_{jt} + \alpha_3 b_{jt}^2 + \alpha_4 s_{jt}^2 + \alpha_5 b_{jt} * s_{jt} + \alpha_6 \log(n_{jt}) + \alpha_7 s_{jt} * A_t + \sum_{q=1}^{Q} \alpha_{8q} d_q + \sum_{t=1}^{T} \alpha_{9t} d_t + \sum_{m=1}^{M} \alpha_{10,m} d_m + \sum_{j=1}^{J} \alpha_{11,j} d_j, \tag{14}
$$

where $d_q$ is the dummy for the contract type, $d_t$ is the year dummy for year $t, t = 1, \ldots, T$, $d_m$ is the state dummy for mine $m, m = 1, \ldots, M$, that sells coal to plant $j$, $d_j$ is the state dummy for plant $j$ that buys coal from mine $m$, and $A_t$ is the total amount of SO$_2$

\(^3\)Type C contracts are for purchases received under a purchase order or contracts that has a duration of one year or longer. Type NC contracts are new contracts or renegotiated contract purchases under which deliveries were first made during the reporting month. Type S are for the spot-market purchases of coal received under a purchase order or contract that has a duration of less than one year.
allowances in the United States in year \( t \). The realized price of coal is

\[
w^c_{jt}(b, s) = \overline{w}^c_{jt}(b, s) + w_{jt}(f) + \epsilon^c_{jt},
\]

where \( \epsilon^c_{jt} \) is a coal price shock.

### 4.2 Transition of Productivity and Abating Efficiency

In each period, the plant’s current productivity depends on its productivity in the last period and the current vintage. That is,

\[
\omega^y_{jt+1} = h^y(\omega^y_{jt}, \tau_{jt+1}) + \xi^y_{jt+1} = \rho^y_0 + \rho^y_1 \omega^y_{jt} + \rho^y_2 (\omega^y_{jt})^2 + \rho^y_3 (\omega^y_{jt})^3 + \rho^y_4 \tau_{jt+1} + \xi^y_{jt+1},
\]

where \( \xi^y_{jt+1} \) is the shock to the productivity. Plant \( j \) observes \( \xi^y_{jt+1} \) before it chooses the coal characteristics, so it is correlated with \( s_{jt+1}, b_{jt+1}, n_{jt+1}, \) and \( n^a_{jt+1} \), but it is uncorrelated with the input (labor, capital, and coal) prices in periods \( t + 1 \) and \( t \). Similarly, the plant’s current abating efficiency and vintage affect its abating efficiency in the next period.

\[
\omega^a_{jt+1} = h^a(\omega^a_{jt}, \tau_{jt+1}) + \xi^a_{jt+1} = \rho^a_0 + \rho^a_1 \omega^a_{jt} + \rho^a_2 (\omega^a_{jt})^2 + \rho^a_3 (\omega^a_{jt})^3 + \rho^a_4 \tau_{jt} + \xi^a_{jt+1},
\]

where \( \xi^a_{jt+1} \) is the shock to the abating efficiency. Plant \( j \) observes \( \xi^a_{jt+1} \) before it chooses the coal characteristics, so it is correlated with \( s_{jt+1}, b_{jt+1}, n_{jt+1}, \) and \( n^a_{jt+1} \). We assume that \( \xi^a_{jt+1} \) and \( \xi^y_{jt+1} \) are independent of each other. Thus, \( \xi^a_{jt+1} \) is uncorrelated with \( s_{jt}, b_{jt}, n_{jt}, \) and \( n^a_{jt} \).

---

4 The total amount includes both the new allowances issued in that year and the allowances banked from previous years.
5 Estimation

The parameters to be estimated include those in the production function, the coal price function, and the marginal pollution control cost functions. The coal price function can be estimated using plant transaction level data. The plants’ generation function parameters are estimated using two-stage GMM in STATA. With the estimates of the coal price function and the generation function, we use the FOCs with respect to $b_{jt}$ and $s_{jt}$ to calculate each plant’s marginal pollution control costs. Then we use OLS regression to get the estimates of the parameters in the the marginal cost functions.

5.1 OLS Estimation of the Coal Price Function

The stochastic coal price equation (15) can be estimated using coal transaction data at the plant level directly via OLS regression. We aggregate the transaction level data to get the plant-year level $b_{jt}$, $s_{jt}$, and the cost shock, all weighted by the coal quantity. Using plant level data, we evaluate the plant-year level marginal costs as

$$\frac{\partial w^c_{jt}(b_{jt},s_{jt})}{\partial b_{jt}} = \alpha_1 + 2\alpha_3 b_{jt} + \alpha_5 s_{jt},$$

(18)

$$\frac{\partial w^c_{jt}(b_{jt},s_{jt})}{\partial s_{jt}} = \alpha_2 + 2\alpha_4 s_{jt} + \alpha_5 b_{jt} + \alpha_7 A_t.$$  

(19)

The marginal costs of coal will be used in the FOCs of the cost minimization problem. With the estimates of $\frac{\partial w^c_{jt}(b_{jt},s_{jt})}{\partial s_{jt}}$, we can compute the plant-year specific permit prices as the FOCs for the optimal $s_{jt}$ for both types of plants.

5.2 Estimation of the Abating Cost Function

The parameters to be estimated in this step are $\lambda$ in the abating cost function, equation (3), and the $\rho^a = (\rho^a_0, \rho^a_1, \rho^a_2, \rho^a_3, \rho^a_4)$ in the abating efficiency transition function, equation
We use the data on the observed abating cost, $s_{jt}$, and the coal quantity, $n_{jt}$, to estimate this equation. The logarithm of abating cost is

$$\log C^a(s_{jt}, n_{jt}^a, \omega_{jt}) = -\omega_{jt} + \lambda \log s_{jt} + \lambda \log n_{jt} + \epsilon_{jt}^a,$$

where $\epsilon_{jt}^a$ is an idiosyncratic error term.

Because the abating efficiency $\omega_{jt}^a$ is unobserved, we use its transition function and the optimal quantity of abated coal to get its value. From the optimal quantity of abated coal in (5), we know that

$$\omega_{jt-1}^a = (\lambda - 1) \log(s_{jt-1}n_{jt-1}^a) + \log \left( \frac{\lambda}{2p_{jt-1}r_{jt-1}} \right). \quad (20)$$

Plugging this lagged abating efficiency into its transition function, $\omega_{jt}^a = h^a(\omega_{jt-1}^a, \tau_{jt}) + \xi_{jt}^a$, and replacing the abating efficiency in the log abating cost function, we get

$$\log C^a(s_{jt}, n_{jt}^a, \omega_{jt}) = -h^a(\omega_{jt-1}^a, \tau_{jt}) + \lambda \log s_{jt} + \lambda \log n_{jt} + \xi_{jt}^a + \epsilon_{jt}^a, \quad (21)$$

where the new error term is $(\xi_{jt}^a + \epsilon_{jt}^a)$. We can estimate $\lambda$ and $\rho^a$ in the transition function using two-stage GMM. The moment conditions assume orthogonality between $(\xi_{jt}^a + \epsilon_{jt}^a)$ and the input (coal, labor, and capital) prices and their lagged values. Denote the prices and their lagged values by $Z_{jt}^a$.

$$E[Z_{jt}^a(\xi_{jt}^a + \epsilon_{jt}^a)] = 0.$$ 

With the estimates $\hat{\lambda}$, we can compute the unobserved abating efficiency using equation (20). 22
5.3 Estimation of the Production Function

The parameters to be estimated in this step includes the Cobb-Douglas coefficients in the generation function, \( \beta = (\beta_0, \beta_k, \beta_l, \beta_h) \), and the linear parameters in the generation productivity transition function, \( \rho^y = (\rho^y_0, \rho^y_1, \rho^y_2, \rho^y_3, \rho^y_4) \). We use three equations in the estimation, the Cobb-Douglas generation function in equation (1), the total variable cost function in equation (12) and (13), and the AR(1) transition process of productivity in equation (16). Rearranging the total cost functions in equation (12) and (13) and taking logarithms, we get

\[
\log \tilde{C}_{jt} = \log \left( \frac{w^*_jt(b^*_jt, s^*_jt) + 2pjt s^*_jt}{b^*_jt} \right) + \frac{1}{\beta_h} (\log y_{jt} - \beta_0 - \beta_k \log k_{jt} - \beta_l \log l_{jt} - \omega^h_{jt}) + \epsilon_{jt}^c,
\]

where we use \( h_{jt} = \frac{1}{\beta_h} (\log y_{jt} - \beta_0 - \beta_k \log k_{jt} - \beta_l \log l_{jt} - \omega^h_{jt}) \), and \( \epsilon_{jt}^c \) is a plant-year specific variable cost shock. On the left-hand side, the new cost term is \( \tilde{C}_{jt} = C(X_{jt}, \omega^y_{jt}, \omega^a_{jt}) - (1 - \lambda) e^{\omega^y_{jt}} \frac{2pjt r_{jt}}{\lambda} \right) \) for an FGD plant and \( \tilde{C}_{jt} = C(X_{jt}, \omega^y_{jt}) \) for a non-FGD plant.

Taking the logarithm of the production function lagged one year, we get

\[
\omega^y_{jt-1} = \log y_{jt-1} - \beta_0 - \beta_k \log k_{jt-1} - \beta_l \log l_{jt-1} - \beta_h \log h_{jt-1}.
\]

Plug this expression into the the transition function, \( \omega^y_{jt} = h^y(\omega^y_{jt-1}, \tau_{jt}) + \xi^y_{jt} \) and replace the \( \omega^y_{jt} \) in equation (22) with the transition function. The log total variable cost function becomes

\[
\log \tilde{C}_{jt} = \log \left( \frac{w^c_{jt}(b^*_jt, s^*_jt) + 2pjt s^*_jt}{b^*_jt} \right) + \frac{1}{\beta_h} (\log y_{jt} - \beta_0 - \beta_k \log k_{jt} - \beta_l \log l_{jt} - h^y(\omega^y_{jt-1}, \tau_{jt}) + \frac{1}{\beta_h} \xi^y_{jt} + \epsilon_{jt}^c.
\]

We can estimate the parameters using two-stage GMM, in which the moments condi-
tions are based on the orthogonality between the composite error term and instrumental variables.

\[ E \left[ \left( \frac{1}{\beta_h} \epsilon^y_{jt} + \epsilon^c_{jt} \right) * Z^c_{jt} \right] = 0, \]  

(23)

where \( Z^c_{jt} \) includes the logarithm of the current plant-year specific prices of labor, coal, capital, and their lagged values.

6 Estimation Results

Table 2 shows the regression results of the quadratic coal price function, controlling for mine-state fixed effects, plant-state fixed effects, year fixed effects, month fixed effects, and contract-type fixed effects. From Table 2, we also find that as both \( s_{jt} \) and total annual allowances increase, the marginal price of sulfur becomes less negative, implying that relaxing regulation leads to smaller impact of sulfur on coal price. Coal price also decreases slightly with the delivered quantity, which reflects the spreading of the fixed cost associated with larger shipments.
Table 2: Coal Price Function

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{jt}$</td>
<td>283.596***</td>
</tr>
<tr>
<td></td>
<td>(80.780)</td>
</tr>
<tr>
<td>$b_{jt}$</td>
<td>2.580***</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
</tr>
<tr>
<td>$s^2_{jt}$</td>
<td>3071.451***</td>
</tr>
<tr>
<td></td>
<td>(384.000)</td>
</tr>
<tr>
<td>$b^2_{jt}$</td>
<td>-0.005***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>$s_{jt} \ast b_{jt}$</td>
<td>-27.565***</td>
</tr>
<tr>
<td></td>
<td>(2.980)</td>
</tr>
<tr>
<td>$s_{jt} \ast allowances$</td>
<td>2.739***</td>
</tr>
<tr>
<td></td>
<td>(0.810)</td>
</tr>
<tr>
<td>$\log(n_{jt})$</td>
<td>-0.098***</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

Parentheses contain estimated asymptotic standard errors.

The symbol *** indicates significance at the .01 level using a two-tailed t-test.

We compute the marginal prices in equations (18) and (19). The results show that the marginal price of $b_{jt}$ is positive for all plants in all years, and the marginal price of sulfur content is negative for all plants in all years. Table 3 shows that the price per short ton of coal goes up by $2.032 on average if $b_{jt}$ increases by one million Btu. The standard deviation of the marginal price of $b_{jt}$ is $0.239 among all plants. The coal price per short ton drops by $2.254 if the sulfur content increases by 1%. The standard deviation of the marginal price of sulfur is $0.775.
Table 3: Marginal Coal Prices ($) for Sulfur and Btu Content

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\partial w_{jt}}{\partial s_{jt}}$</th>
<th>$\frac{\partial w_{jt}}{\partial b_{jt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>2.032***</td>
<td>-2.254***</td>
</tr>
<tr>
<td></td>
<td>(0.239)</td>
<td>(0.775)</td>
</tr>
</tbody>
</table>

Parentheses contain estimated asymptotic standard errors.

The symbol *** indicates significance at the .01 level using a two-tailed t-test.

Using the estimates of the marginal prices of sulfur content and the FOCs with respect to the sulfur content, we compute the plant-year specific permit prices using the FOC with respect to the sulfur content. Table 4 shows the average estimated permit price, its standard deviation, and the actual average permit price by year. These estimates reflect the marginal cost/saving of sulfur content. Our estimates are very close to actual permit prices until 2001, when we begin to substantially underestimate actual prices. This occurs because from 2001 to 2005 a number of elements not captured in our model pushed permit price to very high levels. In 2001 the Clear Skies initiative was first announced. This would cut SO$_2$ emissions by 73 percent from 2002 emissions of 11 million tons to a cap of 4.5 million tons in 2010, and 3 million tons in 2018. This rule never became law. In 2004 the Clean Air Interstate Rule (CAIR) was finalized. This rule in part required 28 eastern states to make reductions in SO$_2$ of over 70% from 2003 levels. In 2005 the unforeseen events of train derailments of Powder River Basin coal shipments and disruptions of natural gas transmission caused by hurricanes Katrina and Rita pushed the average permit price to about $690. The CAIR initiative was vacated in 2008 after permit prices began to plunge from their peak in 2005 to current prices of less than one dollar.
Table 4: Plant-Year \( \text{SO}_2 \) Permit Prices ($/ton)

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimates</th>
<th>Actual Avg. Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>124.30</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>(39.52)</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>118.89</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>(38.11)</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>115.28</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>(38.82)</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>113.19</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>(38.82)</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>113.14</td>
<td>201</td>
</tr>
<tr>
<td></td>
<td>(40.60)</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>107.66</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>(40.28)</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>107.34</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>(39.02)</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>109.97</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>(38.36)</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>110.37</td>
<td>172</td>
</tr>
<tr>
<td></td>
<td>(37.52)</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>109.51</td>
<td>260</td>
</tr>
<tr>
<td></td>
<td>(37.19)</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>110.19</td>
<td>690</td>
</tr>
<tr>
<td></td>
<td>(36.56)</td>
<td></td>
</tr>
</tbody>
</table>

Parentheses contain estimated standard deviations.

Table 5 shows the results from estimating the abating cost function (21). The estimate of \( \lambda \) is greater than one, which implies that the marginal abating cost of sulfur is increasing. It also means that FGD equipment can reduce pollution control costs compared with using permits and emitting all the \( \text{SO}_2 \). This occurs because the marginal cost is lower than the permit prices up to a certain amount of sulfur. The estimates of the parameters in \( \rho^a \) show that the lagged abating efficiency and vintage do not significantly affect the current abating efficiency.
Table 5: Abatement Cost Function

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>1.672</td>
</tr>
<tr>
<td></td>
<td>(0.768)**</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>5.701</td>
</tr>
<tr>
<td></td>
<td>(7.829)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.263</td>
</tr>
<tr>
<td></td>
<td>(2.757)</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>0.482</td>
</tr>
<tr>
<td></td>
<td>(1.423)</td>
</tr>
<tr>
<td>$N$</td>
<td>180</td>
</tr>
</tbody>
</table>

Parentheses contain estimated asymptotic standard errors.
The symbol ** indicates significance at the .05 level using a two-tailed t-test.

Table 6 shows the estimation results of the Cobb-Douglas production function and the parameters in the productivity transition equation. The first two columns show the results using just the data of FGD plants and non-FGD plants. The third column shows the results using all the 80 plants. The last two columns show the results using the ACF model with all inputs and the value-added version with only the inputs of capital and labor.

Using the full sample, the estimates of $(\hat{\beta}_k, \hat{\beta}_h)$ are positive and significant, with $\hat{\beta}_k = 0.452$, $\hat{\beta}_h = 0.943$. The impacts of labor on the total generation is small and insignificant. This is because the generation process of electricity in coal plants is directly affected by capital and heat. Plants exhibit increasing returns in electricity generation on average, since $\hat{\beta}_l + \hat{\beta}_k + \hat{\beta}_h = 1.502$. This is consistent with findings in the literature as summarized by Atkinson (2018). Smaller plants tend to exhibit substantial scale economies, while the largest plants exhaust scale economies. The lagged productivity significantly influences the current productivity. More productive plants are consistently more productive over time. Firm vintage has a little impact on its generation productivity.
Table 6: Production Function Estimation

<table>
<thead>
<tr>
<th></th>
<th>(1) AL-FGD</th>
<th>(2) AL-No FGD</th>
<th>(3) AL-All</th>
<th>(4) ACF</th>
<th>(5) ACF-VA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>-4.103</td>
<td>11.443</td>
<td>-2.190</td>
<td>1.365</td>
<td>3.833</td>
</tr>
<tr>
<td></td>
<td>(4.022)</td>
<td>(1.379)****</td>
<td>(6.767)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( \beta_k )</td>
<td>-0.221</td>
<td>0.386</td>
<td>0.452</td>
<td>0.020</td>
<td>0.444</td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td>(0.265)</td>
<td>(0.208)**</td>
<td>(0.090)</td>
<td>(0.400)</td>
</tr>
<tr>
<td>( \beta_l )</td>
<td>-0.219</td>
<td>-0.168</td>
<td>0.107</td>
<td>-0.006</td>
<td>0.436</td>
</tr>
<tr>
<td></td>
<td>(0.258)</td>
<td>(0.294)</td>
<td>(0.405)</td>
<td>(0.140)</td>
<td>(0.519)</td>
</tr>
<tr>
<td>( \beta_h )</td>
<td>1.311</td>
<td>0.997</td>
<td>0.943</td>
<td>0.966</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.052)****</td>
<td>(0.126)****</td>
<td>(0.341)****</td>
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<td>( \rho_y^0 )</td>
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<td>1.820</td>
<td>9.487</td>
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<td>(3.499)</td>
<td>(0.392)****</td>
<td>(3.479)****</td>
<td>(49.921)</td>
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<td>( \rho_y^1 )</td>
<td>3.853</td>
<td>1.366</td>
<td>8.573</td>
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<td>(0.598)****</td>
<td>(0.014)****</td>
<td>(0.363)****</td>
<td>(30.622)</td>
<td>(5.407)</td>
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<td>( \rho_y^2 )</td>
<td>1.013</td>
<td>0.016</td>
<td>1.480</td>
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<td>(1.706)</td>
<td>(0.001)****</td>
<td>(0.465)****</td>
<td>(4.709)</td>
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<td>( \rho_y^3 )</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.000</td>
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<td></td>
<td>(0.006)</td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.004)*</td>
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<td>( N )</td>
<td>180</td>
<td>620</td>
<td>800</td>
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* \( p < 0.1; ** p < 0.05; *** p < 0.01 \)

Figure 4 shows the estimated \( \omega_{jt}^y \) and \( \omega_{jt}^a \) for the 18 plants with FGD from 1995 to 2005. The circle sizes reflect the plants’ generation in a year. The generation productivity and the abating efficiency are negatively correlated. The partial correlation is \(-0.225\) after controlling for the logarithm of total generation. This implies that, to a moderate degree, more productive plants are less efficient in abating \( \text{SO}_2 \). We also find that the larger plants have lower generation efficiencies than smaller plants, with a partial correlation of \(-0.878\). Both partial correlation coefficients are significant at less than the 0.01 level.
7 Conclusion

The traditional approach of OP/LP/ACF to production function estimation requires the assumption of unconstrained profit maximization which may be inappropriate for many firms and plants. Using this approach they derive an input demand equation from a profit function and invert this function to obtain a control function that proxies for unobserved productivity. Since the input demand function depends on input and output prices (or their quantities if fixed), but not output, the inclusion of the control function allows a clear causal interpretation. However, profit maximization is a problematic assumption for regulated US and foreign electric utilities, plants in a variety of industries (including electricity generation) which minimize costs to produce exogenous output targets, and regulated railroads and airlines in many countries. Arguably, the assumption of output-constrained cost minimization is more realistic. In fact, approximately 96% of empirical studies of the
electric utility, railroad, and airline industries since 1965 assume output-constrained cost
minimization by plants and firms rather than profit maximization. Using the standard
OP/LP/ACF framework for estimating a production function, if one assumes cost mini-
mization, from the cost function one would derive an input demand equation and hence a
control function that includes output as an argument. Such a function would imply causal
inconsistencies when it replaces productivity in the production function. Output would be
a function of itself. Alternatively, we assume minimization of the cost of production and
pollution control subject to an output constraint, which allows derivation and estimation of
a cost function. Rather than specifying a control function, we express lagged productivity
as a lagged function of the other arguments of the production function. This allows empir-
ical identification of the parameters of the production function and the pollution control
cost function.

We apply these techniques to a balanced panel of the 80 largest US coal-fired power
plants from 1995-2005, where the bad output is SO$_2$. Our most important results are the
following. First, all plants face coal prices that increase with Btu content and decrease
with sulfur content. The implicit price of sulfur is $ -2.25 and for Btu content is $ 2.03.
Second, while SO$_2$ permit prices vary across plants in a year, which affect their optimal
sulfur content choices, our estimated permit prices, assuming cost minimization, are very
close to actual permit prices until 2001. The discrepancies after 2001 are due to speculative
pressures. Third, consistent with engineering studies, plants’ marginal abatement costs of
sulfur are increasing with the amount of sulfur abated. Forth, generation productivity and
abating productivity are moderately negatively correlated for FGD plants, with a partial
correlation coefficient of $-0.225$. Lastly, scale economies exists for the average plant, while
capital and heat are the main inputs that determine electricity generation. Counterfactual
analyses consider the implications of restructuring, low permit prices, state subsidies for
consumption of locally-produced coal, and factor causing the increasing number of coal-

fired plant retirements.

References


## Appendix

### Table A.1. Plants and Firms

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