A Model of Church Exit and the Effect of Denominational Ties

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February 2018

Abstract

This paper examines the exit behavior of churches - specifically the occurrence of a congregation disbanding (closing). Churches exhibit much lower exit rates than other entities (such as for-profit firms). Also, churches affiliated with established denominations have lower exit rates than non-affiliated churches. This paper develops a model of the exit decision of religious entrepreneurs that reflects a non-profit-maximizing objective and differences in denominational affiliation. I then test the implications of the model using a hazard analysis to study the determinants of church exit in historical data.

Preliminary Draft - Please do not cite without permission of author

JEL classification: L2,L3,Z12.

Keywords: Exit, religious organizations

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1 Introduction

Economists study many varied industries. Industry differences include whether a good is manufactured, a service is performed or retail transaction is provided. The objectives of the firms (or organizations) in different industries can vary from dominantly profit-maximizing, non-profit-maximizing or a combination of both. Despite industry differences, the same behaviors and characteristics are studied in each; for example, entry and exit, productivity and firm-size distributions. This paper focuses on the market for religious organizations, specifically the exit behavior of these organizations.

There is a large literature studying firms’ exit decisions both theoretically and empirically. Much of the literature deals with for-profit firms, though Lakdawalla and Philipson (2006) and Harrison and Laincz (2008) both present models of how non-profits behave differently than for-profits and how this behavior affects exit of non-profits. One existing study, Anderson et al. (2008), specifically calculates the exit rate for churches in the United States. Generally, non-profits have lower exit rates than for-profits, and churches have an even lower exit rate than non-profits generally. Anderson et al. (2008) also note that churches that are

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1 These are of course very broad differences and there are much finer distinctions that can be made in type of good or service produced; for one overview see Shepherd and Shepherd (2003).

2 Potential objectives of non-profit organizations are discussed in section 2.

3 The theoretical literature looks at industry dynamics, the processes of both entry and exit. Just a few notable papers include Hopenhayn (1992), Jovanovic (1982), Ericson and Pakes (1995) and Pakes and Ericson (1998).


5 Unfortunately there are no direct comparisons available, but looking across different papers Jarmin et al. (2003) reports that the annual exit rate for the US retail sector between 1992-1997 is 9.7%, Harrison and Laincz (2008) reports that between 1989-2000 the highest exit rate of any non-profit sector is 3.5% and Anderson et al. (2008) reports that the annual exit rate for churches in the US between 1998-2005 is 1.0%.
affiliated with a denomination have lower exit rates than non-affiliated churches.⁶ This paper develops a model in which the alternative objective of a church (compared to a for-profit firm) and also the difference between being affiliated and non-affiliated affect a church’s exit decision.⁷ This paper adds to the empirical exit literature with a hazard analysis of historic church data.

One reason to particularly study exit of churches is to add to the existing literature treating religion as an industry. Much work deals with the “demand” side, looking at individuals’ decisions regarding religious attendance and the relationship between religious participation and various outcomes.⁸ Generally, positive outcomes are associated with religious participation, so it is understandable that U.S. subsidizes religious activity and organizations through various policies.⁹ These subsidies directly affect the “supply” side – the decisions of religious organizations. Given the (implicit and explicit) government support for religious organizations, it is important to understand their behavior. Some work focusing on the decisions of religious organizations include Rennhoff and Owens (2012) which studies a church’s decision of whether or not to offer weekday daycare, Walrath (2015) estimates the effect of centralized decision making on entry behavior between different religious denominations and Ferguson and Mochrie (2014) analyze the relationship between number of different denominations and

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⁶According to Pew (2008) “denomination’ refers to a set of congregations [churches] that belong to a single administrative structure characterized by particular doctrines and practices.” Just a few examples of specific denominations include Roman Catholic, Evangelical Lutheran Church of America and United Methodist.

⁷Although being affiliated or non-affiliated is binary, the effects of affiliation are more nuanced. There are differences in strength of denominational ties, which religious researchers refer to as polity; this is discussed in more detail in section 3.

⁸For example, religious participation is negatively associated with criminal behavior (Lipford et al. (1993), Hull and Bold (1995)) and drug and alcohol use (Mellor and Freeborn (2011), Gruber and Hungerman (2008)) and positively related to electoral participation (Gerber et al. (2016)) and educational attainment and income (Gruber (2005)).

⁹A significant policy benefiting churches is their non-profit status, which, like other non-profits, typically exempts them from federal income taxes and property taxes (Harrison and Seim (2013)). There is also the itemized tax deduction for charitable giving. More generally, the variety of religious organizations in the U.S. have been supported by the First Amendment through permitting all religious practice and not allowing explicit government support for any one religion (no state sponsored religion) (Finke and Stark (1992), 59).
religious participation in Scottish towns.

The market for religion can serve as an interesting benchmark in the study of mixed markets, in which both for-profit firms and non-profit organization compete (e.g. education, health care). The market for religion is exclusively made up of non-profits. (Though one may question the objectives of various religious organizations, religious organizations on the whole in the U.S. receive the same tax treatment from the I.R.S. and thus must meet certain standards regarding its purpose and “net earnings may not inure to the benefit of any private individual or shareholder” (see IRS (b), 2).) By providing a market with a uniform type of organizations (non-profit), the study of religious organizations may prove a valuable comparison to help better understand mixed markets.

The market for religion also offers a situation in which there are multi-church denominations competing with non-denominational single churches. Also, not all multi-church denominations act the same; across denominations there are differences regarding the degree of closeness between churches within a denomination. This level of connectedness within denominations is referred to as polity (more detail on polity in section 3). Differences in polity means there are different levels of intra-denominational support within churches of a single denomination. Varying degrees of cross-subsidization within different denominations is similar to different retail franchising relationships. The religious market thus has potential similarities with retail markets in which non-denominational churches (or single-unit firms – “mom-and-pop shops”) compete with denominationally affiliated churches (multi-unit firms – retail chains). And the fact that for different denominations, the strength of the relationship between the churches within a denomination reflects various strengths of relationships

10 Cordes and Weisbrod (1998), Hansmann (1987), Harrison and Seim (2013) and Lakdawalla and Philipson (2006) provide just a few examples of the effects of for-profits and non-profits competing in the same market; these papers address both differences in an organization’s objective and also the effects of the organization’s tax treatment.

11 An explanation of franchising from Lafontaine and Blair (2008) is that under a franchise agreement, the franchisee pays the franchisor for the right to sell the franchisor’s product or use the franchisor’s business format. See Lafontaine and Shaw (1998) for more detail.
between units within a multi-unit firm (retail chain). Various units (individual chain locations) could be exclusively owned by one firm (entirely corporate owned), or different units could be owned by individual franchisees (this could be a mix of some corporate-owned franchises, franchisees who own one unit, or franchisees who own multiple units).\(^{12}\)

This paper proceeds as follows. Section 2 presents a model of church exit which allows for different possible objectives and single or multi-unit status and the model’s implications for exit rates. Section 3 describes the data. Section 4 presents the hazard analysis and section 5 concludes.

### 2 Model and Implications

#### 2.1 Model Description

This section presents a simple model of the exit decision of a church. The primary focus of the model is to study the interaction between objective (for-profit or non-profit) and number of units (whether an organization is single or multi-unit).

An assumption must be made regarding the objective of the church. The current literature posits a number of different objectives for a church.\(^{13}\) Hanson and Xiang (2013) use an objective function which weights both profit and quantity produced by a church. This objective is consistent with the objective Harrison and Laincz (2008) use to study the entry and exit behavior of non-profits generally. This paper considers both the case of a church acting to maximize profits and the case in which a church maximizes output. The model in this pa-


\(^{13}\)Ekelund (1996) assumes the Medieval Catholic church acted as a profit-maximizer. Walrath (2015) assumes that religious entrepreneurs act as profit-maximizers when making a decision of whether or not to enter a market. Rennhoff and Owens (2012) use a reduced form value (or profit) function to be flexible (or, to use their term, “agnostic”) regarding a church's objective of adding a daycare center. Another assumption is that churches maximize consumer welfare (maximizes the aggregate utility of consumers) (see Iannaccone (1992) and Zaleski and Zech (1992)).
per will just look at the two ends of the spectrum, either for-profit or output-maximization, and compare those extremes.

In addition to a difference of objective, another key difference between a for-profit and a non-profit relates to potential scrap value earned from shutting down. The Internal Revenue Service states that a non-profit cannot distribute profits to those in control.\(^{14}\) Whereas for-profits distribute profits to owners (shareholders).\(^{15}\) If a for-profit firm has a single plant and decides to close it, the scrap value is transferred to the shareholders. If a certain religious organization is not affiliated with a denomination and chooses to sell its church, no one in control of the church can receive the proceeds. However, if a single Catholic church (for example) closes and is sold (for a scrap value) the proceeds can be used to support a different Catholic church.\(^{16}\) Thus, exit decisions of a religious organization take into account whether the organization is single or multi-unit. This model is focusing on the extremes of affiliation; either non-affiliated (single-unit) or extremely strong affiliation ties (a single decision-maker making decisions for both units jointly).\(^{17}\)

Entry and exit are naturally dynamic decisions; however, since the primary focus of this model is to examine how the non-distribution of assets interacts with a church’s single or multi-unit status, this model abstracts away from most inter-temporal issues. A religious entrepreneur (or firm) wakes up with either one or two churches (units). The religious

\(^{14}\)To qualify for tax-empt status as a non-profit, an organization must file a 1023 form in which the organization shows: “No part of the organizations net earnings will inure to the benefit of private shareholders or individuals. You must establish that your organization will not be organized or operated for the benefit of private interests, such as the creator or the creators family, shareholders of the organization, other designated individuals, or persons controlled directly or indirectly by such private interests.” See IRS (a), page 21.

\(^{15}\)Harrison and Laincz (2008) refer to this condition that a non-profit cannot distribute profits as the non-redistribution constraint (NRC).

\(^{16}\)The Internal Revenue Service states that if a non-profit has any assets remaining when it dissolves the assets must go to other tax-exempt organizations (see IRS (a) page 23).

\(^{17}\)Later, when looking at the data, I would expect that denominations with weaker denominational ties would behave more like a non-affiliated organization, and denominations with stronger denominational ties would act more like an organization with theoretically extremely strong ties. The various strength of denominational ties are described more in section 3.
entrepreneur (firm) incurs a fixed cost for each church (unit) it chooses to operate. The religious entrepreneur (firm) can choose to shut down a church (unit) and receive a scrap value.\textsuperscript{18} The only input of a church (firm) is labor, \( l \). Output, \( q \), is determined by:

\[
q = \varphi l^\alpha
\]  

(1)

where \( \alpha \in (0, 1) \) and \( \varphi \geq 0 \) is a productivity shock with strictly increasing cumulative distribution function \( F(\varphi) \).\textsuperscript{19} Assume both churches and firms are producing the same good. Demand for the good is perfectly elastic at price \( p \). The supply of labor is perfectly elastic at wage \( w \). If an entrepreneur or firm chooses to operate a unit, it incurs fixed cost \( \theta \). If an entrepreneur or firm chooses to shut down a unit, it receives scrap value \( \psi \).

\subsection*{2.1.1 Profit-maximizing with single unit}

If a church (firm) is maximizing profit, it will choose an amount of labor, \( l^* \), that maximizes the following profit equation:

\[
\Pi(l^*) = p\varphi l^\alpha - wl - \theta
\]  

(2)

If a for-profit firm chooses to shut down a unit it can distribute the scrap value \( \psi \) to its shareholders. A firm’s decision of whether or not to operate a unit needs to take the opportunity cost of staying open into account. The firm will shut down a unit if profit is less than the scrap value, \( \Pi(l^*) \leq \psi \), and keep the unit open if profit is greater than the scrap value, \( \Pi(l^*) \geq \psi \). Since the church is unable to distribute the scrap value, it will stay open as long as \( \Pi(l^*) \geq 0 \). In this framework a religious entrepreneur is more likely to keep a single church open than a firm is, because the firm’s non-zero scrap value.

\textsuperscript{18}Due to limits on non-profit distribution of assets, the value of the scrap value depends on the specific situation, which will be discussed in further detail soon.

\textsuperscript{19}The multiplicative productivity shock is similar to Jovanovic (1982) and consistent with Hopenhayn (1992).
2.1.2 Profit-maximizing with multiple units

If a religious entrepreneur (firm) has two churches (units), the churches (units) differ in their productivity shocks, $\varphi_1$ and $\varphi_2$. The church (firm) chooses $l_1^*$ and $l_2^*$ that maximize equation 2. The religious entrepreneur (firm) compares the following:

$$
\begin{cases}
\Pi(l_1^*) + \Pi(l_2^*) & \text{(operate both units)} \\
\Pi(l_1^*) + \psi & \text{(operate more productive unit, close down less productive)}
\end{cases}
$$

or, the church (firm) can close both units in which the payoffs would be

$$
\begin{cases}
0 & \text{(church closes both units)} \\
2\psi & \text{(firm closes both units)}
\end{cases}
$$

The religious entrepreneur (firm) chooses the option that results in highest total profit (from both units).

2.1.3 Output-maximizing with single unit

Due to alternate objectives for religious organizations, this section considers a religious entrepreneur with one unit which chooses employment level $\hat{l}$ that maximizes output subject to a non-negative profit constraint:

$$
\max_l \quad \varphi^{l^\alpha} \\
\text{s.to} \quad p\varphi^{l^\alpha} - wl - \theta \geq 0
$$

For a religious entrepreneur with only 1 unit, the scrap value $\psi$ has no affect on its decision since there are neither share-holders nor another unit that could benefit from $\psi$. 
2.1.4 Output-maximizing with two units

A religious entrepreneur with two units has more possibilities. One possibility is that the entrepreneur operates both units independently, choosing an amount of labor for each unit that maximizes that unit’s output subject to non-negative profits. Another alternative is that one unit (with higher $\varphi$) subsidizes a unit (with lower $\varphi$) such that the low-value $\varphi$ unit produces more than it otherwise could have due to the non-negative profit constraint. Or the firm can close down the unit with lower $\varphi$ and use scrap value $\psi$ to subsidize the remaining unit (with higher $\varphi$).

The entrepreneur must consider each possibility. In order to operate each church independently, the entrepreneur must determine $\{\hat{l}_1, \hat{l}_2\}$ that satisfies:

$$\max_{l_i} \varphi_i l_i^\alpha \quad \text{s.to} \quad p\varphi_i l_i^\alpha - w l_i - \theta \geq 0 \quad \text{for} \quad i = 1, 2$$

(6)

In order to operate the two churches jointly (and benefiting from cross-subsidization) the entrepreneur must determine $\{\tilde{l}_1, \tilde{l}_2\}$ that satisfies:

$$\max_{l_1, l_2} \sum_{i=1}^{2} \varphi_i l_i^\alpha \quad \text{s.to} \quad \sum_{i=1}^{2} (p\varphi_i l_i^\alpha - w l_i - \theta) \geq 0$$

(7)

Or, if shutting down the less productive unit and using the proceeds to subsidize the more productive unit, the entrepreneur must determine $\bar{l}$ that satisfies

$$\max_{l_i} \varphi_i l_i^\alpha \quad \text{s.to} \quad p\varphi_i l_i^\alpha - w l_i - \theta + \psi \geq 0$$

(8)

where $\varphi_i = \max\{\varphi_1, \varphi_2\}$
The religious entrepreneur must then choose which labor allocation maximizes combined output across the two units.

2.2 Model Implications

For each situation above I am interested the probabilities that a religious entrepreneur and a firm would shut down a unit (or two). Specifically, I am interested in cutoffs for the productivity shock, \( \hat{\phi} \), such that for \( \varphi \geq \hat{\phi} \) the entrepreneur (firm) will operate the unit and for \( \varphi < \hat{\phi} \) the entrepreneur (firm) will shut down the unit and the exit probabilities that result from the cutoffs.

First, consider the exit decisions of for-profit firms with either one or two units. Suppose the productivity cutoff of a for profit-firm with one unit is represented by \( \hat{\phi}_{f,1} \) and the productivity cutoff of a for-profit firm with two units is represented by \( \hat{\phi}_{f,2} \). A for-profit firm with two units makes the same decision as a for-profit single unit firm, two times.\(^{20}\) Thus \( \hat{\phi}_{f,1} = \hat{\phi}_{f,2} \) and these cutoffs can be represented by \( \hat{\phi}_f \).

Now, consider the productivity cut-offs and probability of exit of for-profit firms compared to cut-offs for a religious entrepreneur with 1 and 2 units. A religious entrepreneur with two units has two different productivity draws.

**Proposition 2.1.** For \( \alpha \in (0, 1) \) and \( \psi > 0 \) if the religious entrepreneur maximizes profits then the probability of exit is highest for for-profit firms (single or multi-unit), next highest for single-unit religious entrepreneur and lowest for a two-unit religious entrepreneur.

**Proof.** See appendix. \( \square \)

\(^{20}\)A for-profit firm with two units can be thought of as either solving:

\[
\max_{l_i} p\varphi_1 l_i^{\alpha} - wl_i - \theta \quad \text{for } i = 1,2
\]

or

\[
\max_{l_1, l_2} p\varphi_1 l_1^{\alpha} + p\varphi_2 l_2^{\alpha} - wl_1 - wl_2 - 2\theta
\]

Looking at the first order conditions it can be seen that solving either problem results in the same \( l_1^*, l_2^* \).
If both the for-profit firm and the religious entrepreneur are choosing labor to maximize profit, the only difference between their decisions is what can be done with scrap value $\psi$. For the for-profit firm to operate a unit, $\hat{\varphi}_f$ must ensure profits are larger than $\psi$. On the other hand, for a religious entrepreneur to operate a single unit productivity cut-off, $\hat{\varphi}_{r,1}$, must ensure profits are just greater than zero. Since profits are increasing in $\varphi$, $\hat{\varphi}_f > \hat{\varphi}_{r,1}$ when $\psi > 0$, leading to a lower probability of exit in single-unit religious entrepreneur versus for-profit firms (with either one or two units). A religious entrepreneur with two units can scrap the unit with lower $\varphi$ and use $\psi$ to subsidize the other unit. A two unit religious entrepreneur will operate at least one unit as long as combined profits from both units are greater than zero. Given this potential cross-subsidy from scrap value $\psi$ the probability of exit for a multi-unit religious entrepreneur is less than the probability of exit for a single-unit entrepreneur (more detail in Appendix).

Also consider the case in which a religious entrepreneur maximizes output subject to non-negative profits. First consider a very restricted analytic case.

**Proposition 2.2.** For $\alpha = 0.5$, $\psi > 0$ and $\psi$ not too much larger than $\theta$, if religious entrepreneurs are maximizing output then the probability of exit is highest for for-profit firms (single or multi-unit), next highest for single-unit religious entrepreneur and lowest for a two-unit religious entrepreneur.

*Proof.* See appendix.

Intuitively, the key difference between the three cases again involves the scrap value, $\psi$. The for-profit firm has the highest productivity cutoff because in order for the firm to operate a unit its profits must be high enough to cover scrap value $\psi$, leading to the highest probability of exit. Whereas, a religious entrepreneur with just one unit only needs a $\varphi$ high enough to ensure zero profits. A religious entrepreneur with two units can close one unit to
subsidize the other unit; with the subsidy the remaining unit can maintain zero profits with a productivity shock lower than the two previous cases.\textsuperscript{21}

As mentioned above, the key to the differences in \( \hat{\varphi} \) is driven by \( \psi \). If \( \psi = 0 \) then \( \hat{\varphi}_f = \hat{\varphi}_{r,1} = \hat{\varphi}_{r,2} \) (for both the case when the religious entrepreneur maximizes profit and when the religious entrepreneur maximizes output). With zero scrap value there will be no benefit to a religious entrepreneur from scrapping a second unit, thus there would be no difference between a one-unit and two-unit non-profit. With no scrap value a for-profit firm would start operating as soon as \( \varphi \) was high enough to ensure zero profits – the same as a religious entrepreneur with one unit.

3 Description of Data

To test the above implications of the model a panel dataset of churches is constructed. The dataset starts with city directories from 1901, 1906, 1916, 1926 and 1936. From these directories the name, address and denomination of every church in seven cities in Iowa is collected.\textsuperscript{22,23} By comparing name, address and denomination across directories a panel set is created with a church’s first appearance and how long it remained open (or if it remained open at the last observation in 1936). Table 1 provides church counts and average market population for the above years.\textsuperscript{24}

\textsuperscript{21}Theorem 2.2 restricts \( \alpha \) to 0.5, but the above intuition holds for all \( \alpha \in (0, 1) \). Also the above result was verified numerically for \( \alpha \in (0, 1) \) for a few specific values of \( w, p, \psi, \theta \).

\textsuperscript{22}This time period is used for a couple reasons. Directories in this time period listed churches in specific denominational categories (whereas more current phone books may or may not include denomination). Also, over this time period the United States Census Bureau conducted a Census of Religious Bodies, which allows for denominational information to be used.

\textsuperscript{23}The cities in Iowa are Burlington, Cedar Rapids, Council Bluffs, Davenport, Des Moines, Sioux City and Waterloo.

\textsuperscript{24}Population for each of these cities is obtained from the U.S. Census for year 1900, 1910, 1920, 1930 and 1940. Since the church data is collected for 1901, 1906, 1916, 1926 and 1936, city population for these years is projected using a second order polynomial.
Table 1. Church Counts and Market Population

<table>
<thead>
<tr>
<th>City</th>
<th>Year</th>
<th>1901</th>
<th>1906</th>
<th>1916</th>
<th>1926</th>
<th>1936</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burlington</td>
<td>Church Count</td>
<td>33</td>
<td>33</td>
<td>34</td>
<td>29</td>
<td>30</td>
</tr>
</tbody>
</table>
<pre><code>                    | Population     | 23,245 | 23,755 | 24,667 | 25,431 | 26,049 |
                    | Pop per church | 704   | 720   | 725   | 877   | 868   |
</code></pre>
<p>| Cedar Rapids  | Church Count | 31   | 34   | 42   | 50   | 51   |
| Population     | 25,653 | 30,993 | 41,192 | 50,749 | 59,666 |
| Pop per church | 828   | 915   | 981   | 1015  | 1170  |
| Council Bluffs| Church Count | 26   | 26   | 34   | 37   | 44   |
| Population     | 25,529 | 28,806 | 34,387 | 38,670 | 41,656 |
| Pop per church | 982   | 1108  | 1011  | 1045  | 947   |
| Davenport     | Church Count | 35   | 35   | 41   | 43   | 39   |
| Population     | 35,610 | 41,301 | 51,113 | 58,833 | 64,461 |
| Pop per church | 1017  | 1180  | 1247  | 1368  | 1653  |
| Des Moines    | Church Count | 93   | 92   | 101  | 137  | 149  |
| Population     | 63,312 | 80,361 | 110,395 | 135,008 | 154,200 |
| Pop per church | 681   | 873   | 1093  | 985   | 1035  |
| Sioux City    | Church Count | 41   | 42   | 51   | 65   | 69   |
| Population     | 33,639 | 44,672 | 62,609 | 75,045 | 81,978 |
| Pop per church | 820   | 1064  | 1228  | 1155  | 1188  |
| Waterloo      | Church Count | 20   | 26   | 39   | 42   | 49   |
| Population     | 14,180 | 21,039 | 32,968 | 42,512 | 49,686 |
| Pop per church | 709   | 809   | 845   | 1012  | 1014  |
| <strong>Total</strong>     | Church Count | 279  | 288  | 342  | 403  | 431  |</p>

*Note:* Population for each city is obtained from the U.S. Census for 1900, 1910, 1920, 1930 and 1940. The populations reported are predicted based on a second-order polynomial fit of the population data.
From the directory data it can be determined if a given church is “independent” (not affiliated with a denomination) or affiliated with a larger denomination. Information regarding these affiliated denominations can be gathered from the U.S. Census of Religious Bodies (U.S. Dept. of Commerce (1941)). Particularly, this paper focuses on the polity structure of the denomination. Broadly, a denomination’s polity describes its governance structure. There are variations in how much control a denomination has over individual congregations (churches). An “independent” church has no polity. The loosest form of polity is a congregational control structure (for example, two denomination with a congregational polity are the Southern Baptist Convention and the United Church of Christ); within the congregational polity, individual congregations (churches) retain the most authority. The most hierarchical organizational structure is referred to as episcopal (examples of denominations with an episcopal polity are the United Methodist Church and the Episcopal Church in the USA (Takayama and Cannon (1979))). Another organizational structure, somewhere between congregational and episcopal, is presbyterian (the Presbyterian Church USA has this organizational structure). One attribute of the Religious Census is that for each denomination in the United States it describes its polity, generally placing the denomination in one of four categories (none, congregational, presbyterian, episcopal).

For some denominations the Religious Census describes polity as some combination of the above – due to these mixed polities, I create two different polity classification systems; the main difference is how denominations described by the Religious Census as “synodical” are classified (though the Census usually sticks to one of the four structures mentioned above, it occasionally used synodical to describe a polity somewhere between presbyterian and episcopal). One of my classifications is “more stringent;” in this classification, synodical denominations are considered presbyterian (the idea being that I stricter with regard to what denomination is considered episcopal – thus synodical denominations are lumped into presbyterian). In the other, “less stringent” classification, synodical is grouped with episcopal
Table 2. Church Counts Based on Polity

<table>
<thead>
<tr>
<th>Type of Classification</th>
<th>Polity Type</th>
<th>None</th>
<th>Congregational</th>
<th>Presbyterian</th>
<th>Episcopal</th>
<th>Catholic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>“More Stringent” &amp; Catholics as Episcopal</td>
<td>90</td>
<td>166</td>
<td>117</td>
<td>166</td>
<td>0</td>
<td>539</td>
<td></td>
</tr>
<tr>
<td>“More Stringent” &amp; Catholics as Own Group</td>
<td>90</td>
<td>166</td>
<td>117</td>
<td>120</td>
<td>46</td>
<td>539</td>
<td></td>
</tr>
<tr>
<td>“Less Stringent” &amp; Catholics as Episcopal</td>
<td>90</td>
<td>161</td>
<td>91</td>
<td>197</td>
<td>0</td>
<td>539</td>
<td></td>
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<td>91</td>
<td>151</td>
<td>46</td>
<td>539</td>
<td></td>
</tr>
</tbody>
</table>

(I am being less strict with regard to what constitutes episcopal). One other consideration needs to be made; it has been argued (for example by Takayama (1975)) that the Roman Catholic church exhibits a strict level of hierarchy beyond presbyterian and episcopal structures. For this reason, I run tests lumping Catholics with other episcopal denominations and also run tests in which Catholics are their own category. Table 2 presents the counts for number of churches for each type of polity, depending on type of classification being used (these counts reflect each church that ever appears in the data).

Table 3 shows exit rates based various polities (this specific table is using the “Less Stringent” classification, but results are similar when using “More Stringent.”) Exit rates for each period are calculated by taking the number of churches that exited a market over that time range divided by the number of churches at the beginning of the time range. (For example, the exit rates for 1916-1926 begins with all churches that existed in 1916, then subtract number of churches still open in 1926, and divide that quantity by the number of
Table 3. Exit Rates for Churches in Different Polity Groupings, and Grocery Stores, All Ages

<table>
<thead>
<tr>
<th>Years</th>
<th>1901-1906</th>
<th>1906-1916</th>
<th>1916-1926</th>
<th>1926-1936</th>
</tr>
</thead>
</table>

**Broad Affiliation**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Affiliated Churches</td>
<td>0.08</td>
<td>0.14</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Non-Affiliated Churches</td>
<td>0.24</td>
<td>0.65</td>
<td>0.56</td>
<td>0.50</td>
</tr>
<tr>
<td>All Churches</td>
<td>0.09</td>
<td>0.19</td>
<td>0.17</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**Polity Types**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Congregational</td>
<td>0.09</td>
<td>0.18</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Presbyterian</td>
<td>0.12</td>
<td>0.09</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>Episcopal</td>
<td>0.03</td>
<td>0.18</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>Catholic</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>Non-Affiliated Churches</td>
<td>0.24</td>
<td>0.65</td>
<td>0.56</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**Grocery Stores**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.47</td>
<td>0.64</td>
<td>0.70</td>
<td>0.65</td>
</tr>
</tbody>
</table>

*Note:* This particular table uses affiliation classification “Less Stringent” and Catholic as episcopal. Results similar using the alternative classification of “More Stringent.”

Churches in 1916.) Table 3 presents exit rates for all churches, not dependent on age, it is presented to give an overall sense of exit behavior depending on denomination.

When looking at exit, age of the firm is important. The data is left-censored since for churches that exist in 1901 there is no way to know when the church first entered. For analysis in section 4, this is corrected for by excluding churches that existed in 1901 and only using new churches that entered after 1901 (this is not perfect, since the first observation after 1901 is 1906, so those churches could be between zero and five years of age, but this decreases the range of potential ages). Table 4 presents exit rates for this cohort of churches.

In order to compare exit behavior of churches with that of for-profits, I use data on entry and exit of grocery stores for five of the same cities as I have church data for.\(^{25}\)^\(^{26}\) Although not an ideal comparison group, this grocery store data allows for some comparison between

\(^{25}\)Due to data issues, I do not have grocery store data for Davenport of Sioux City.

\(^{26}\)This grocery data is collected the same way the church data is collected, using directories for various Iowa cities and matching across time using name and address.
Table 4. Exit Rates for Churches in Different Polity Groupings, Churches Entering After 1901

<table>
<thead>
<tr>
<th>Broad Affiliation</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1906-1916</td>
</tr>
<tr>
<td>Affiliated Churches</td>
<td>0.04</td>
</tr>
<tr>
<td>Non-Affiliated Churches</td>
<td>0.70</td>
</tr>
<tr>
<td>All Churches</td>
<td>0.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Polity Types</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1906-1916</td>
</tr>
<tr>
<td>Congregational</td>
<td>0.00</td>
</tr>
<tr>
<td>Presbyterian</td>
<td>0.12</td>
</tr>
<tr>
<td>Episcopal</td>
<td>0.00</td>
</tr>
<tr>
<td>Catholic</td>
<td>0.00</td>
</tr>
<tr>
<td>Non-Affiliated Churches</td>
<td>0.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grocery Stores</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1906-1916</td>
</tr>
<tr>
<td></td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: This particular table uses affiliation classification “Less Stringent” and Catholic as episcopal. Results similar using the alternative classification of “More Stringent.”

the behavior of churches to a specific type of for-profit firm. Table 5 provides counts for grocery stores by year and city.

Tables 3 and 4 provide exit rates for grocery stores. At casual glance, the exit rates for the grocery stores are higher than the exit rates for churches. This very superficial look at the data seems to support the implication of the model that exit rates increase when comparing exit rates of churches to for-profit firms.
Table 5. Counts for Grocery Stores by City

<table>
<thead>
<tr>
<th>Year</th>
<th>1901</th>
<th>1906</th>
<th>1916</th>
<th>1926</th>
<th>1936</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burlington</td>
<td>61</td>
<td>66</td>
<td>72</td>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>Cedar Rapids</td>
<td>89</td>
<td>92</td>
<td>172</td>
<td>218</td>
<td>209</td>
</tr>
<tr>
<td>Council Bluffs</td>
<td>74</td>
<td>80</td>
<td>108</td>
<td>162</td>
<td>145</td>
</tr>
<tr>
<td>Des Moines</td>
<td>217</td>
<td>238</td>
<td>399</td>
<td>564</td>
<td>456</td>
</tr>
<tr>
<td>Waterloo</td>
<td>23</td>
<td>36</td>
<td>88</td>
<td>115</td>
<td>145</td>
</tr>
</tbody>
</table>

4 Empirical analysis

4.1 Hazard analysis of effect of polity

To test differences in church exit depending on affiliation and polity, a Cox Proportional Hazard model is used. The standard form of the proportional hazard is:

\[ h(t) = h_0(t) e^{(x\beta)} \] (9)

where \( h_0(t) \) is the baseline hazard (the probability a church exits at time \( t \); it is estimated non-parametrically), \( x \) is various co-variates and \( \beta \) is the effect of the co-variate. One benefit of the Cox Proportional-Hazard model is that it works with right-censored observations; specifically, churches that are still open in 1936. This model cannot handle left-censored observations, so the data is limited to churches who did not appear until 1906 (all churches observed in 1901 are dropped); this way the birth year of all churches is observed.\(^{27}\)

First, I test whether affiliation, in a general sense, affects exit. A church is considered

\(^{27}\)Since the Cox model is for continuous data, ties would not occur. However, given the structure of this data there are a number of ties (exit for multiple observations happening at the same \( t \)). The nature of the ties is due to imprecise measuring of the event, so the best method to deal with ties is the “exact method” in which it is assumed there is some true ordering of these seemingly tied exits and that is accounted for in the analysis.
affiliated if its denomination has either congregational, presbyterian or episcopal (or Catholic) polity structures. This is a very minimal analysis in which the only coefficient of interest is the effect of being affiliated. Consider this to be $\beta_{aff}$; the value is -1.592 with a standard error of 0.220. This coefficient is significant at the 1 percent level. As expected, the affiliation of a church with a denomination (no matter the polity) decreases the probability of church exit.

Next, I look at the effect on exit depending on specific polity type. Each specific polity has a statistically significant negative effect on church exit; Catholic churches have a particularly lower hazard. Table 6 presents the results. The results are robust to various classifications of polity. The lower exit of episcopal denominations is not simply being determined by the very low exit rate of churches (when Catholics are split off from episcopal, the episcopal coefficient is significantly negative).

### 4.2 Important Caveat for now / this draft

A key variable in exit analysis is size of the firm. At this time it could be that low hazard rates for affiliated churches is being driven by affiliated denominations being larger, and denominations of stricter polity being larger than denominations of less strict policy. Though determining exact size of individual churches is not possible, there are ways to control for size at the denominational level, and this will be done in future drafts. Potential variables

---

28 Location of the church is controlled for using dummies for each city (other than Burlington). The location estimates are suppressed. A key assumption of proportional hazard is that a change in covariates shifts the baseline hazard, $h_0(t)$, proportionately, not in a time-dependent way. One way to test this assumption is to include regressors interacted with $\log(t)$; if the time-interacted covariates are statistically significant, that variable does not meet the proportionality assumption. If a variable fails the proportionality assumption, one fix is to include the time-interacted covariate to control for its dependence on time. Affiliation meets the proportional hazard assumption, however the city dummies do not. To correct for the non-proportionality of the city dummies, variables interacting the dummies with time are included; the estimates for these variables are also suppressed.

29 Since the only distinction is affiliated or not affiliated, the different classifications (more or less stringent) are moot.

30 Again, dummies and time-interacted dummies are estimated, but the results not presented.
Table 6. Hazard Regression, Effect of Polity-type on Exit

<table>
<thead>
<tr>
<th>Type of Classification</th>
<th>None</th>
<th>Congregational</th>
<th>Presbyterian</th>
<th>Episcopal</th>
<th>Catholic</th>
</tr>
</thead>
<tbody>
<tr>
<td>More Stringent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catholics as</td>
<td>-1.272***</td>
<td>-1.960 ***</td>
<td>-1.856 ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Episcopal</td>
<td>(0.268)</td>
<td>(0.385)</td>
<td>(0.307)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>More Stringent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catholics as</td>
<td>-1.314***</td>
<td>-2.039 ***</td>
<td>-1.626 ***</td>
<td>-3.551 ***</td>
<td></td>
</tr>
<tr>
<td>Own Group</td>
<td>(0.274)</td>
<td>(0.389)</td>
<td>(0.338)</td>
<td>(1.017)</td>
<td></td>
</tr>
<tr>
<td>Less Stringent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catholics as</td>
<td>-1.266***</td>
<td>-1.540 ***</td>
<td>-2.042 ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Episcopal</td>
<td>(0.272)</td>
<td>(0.372)</td>
<td>(0.307)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less Stringent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catholics as</td>
<td>-1.264***</td>
<td>-1.548 ***</td>
<td>-1.775 ***</td>
<td>-3.506 ***</td>
<td></td>
</tr>
<tr>
<td>Own Group</td>
<td>(0.272)</td>
<td>(0.372)</td>
<td>(0.316)</td>
<td>(1.017)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. *** denotes significance at the 1 percent level. Dummy variables for each city (other than Burlington) and a variable interacting location dummy with log(t) are calculated, but suppressed.
would be number of members, number of employees number of total congregations at the denominational level; these variables should be available in the Religious Census at geographic / denominational level. (In some cases, denominational families may need to be used; for example it may not possible to determine whether a church belongs to the denomination United Norwegian Lutheran Church in America or the denomination Norwegian Synod of the American Evangelical Lutheran Church.)

4.3 Comparison exit rates of churches and grocers

The hazard model can also be used to test differences in exit between churches and grocery stores (as our imperfect for-profit comparison market). We again use equation 9. Now, the dataset is all observed “units,” both churches and grocery stores. For a simple first regression, the variable of interest would be a dummy variable indicating if a unit is a church or not (if it is a church, the dummy if 1); call the coefficient on this dummy, $\beta_{np}$, for non-profit. In this simple regression, $\beta_{np}$ is negative and statistically significant. See Table 7.31

I then look at the effect of affiliated churches; specifically if beyond just being a church (non-profit), being affiliated makes exit even less likely. In this case $\beta_{np}$ is no longer statistically significant, but $\beta_{aff}$ is both negative and statistically significant. This implies that non-affiliated churches are not statistically significantly more likely to exit than grocery stores. To support this, I also used a sample with only grocery stores and non-affiliated churches (not including affiliated churches), and in this case $\beta_{np}$ is again not statistically significant, supporting the idea that non-affiliated churches do not have exit at lower rates than for-profits (grocery stores). This is just a cursory analysis for now.

31Similar to the above regressions, city dummies and city dummies interacted with time are included; the estimates for these variables are suppressed.
Table 7. Hazard Regression, Effect of Non-Profit Status

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{np}$</th>
<th>$\beta_{aff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Units</td>
<td>-1.281***</td>
<td>-0.197 -1.361***</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.144) (0.172)</td>
</tr>
<tr>
<td>Grocery Stores</td>
<td>-0.102</td>
<td>(0.786)</td>
</tr>
<tr>
<td>and Non-Affil Churches</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. *** denotes significance at the 1 percent level. Dummy variables for each city (other than Burlington) and a variable interacting location dummy with log($t$) are calculated, but suppressed.

5 Conclusion

This paper presents a model of firms’ exit decisions that depend on whether or not the firm is for-profit or a religious entrepreneur and single-unit or multi-unit. This model predicts that the exit rate of for-profits is higher than the exit rate of single-unit churches, which is higher than the exit rate of multi-unit churches.

The interaction of an organization’s objective (and in practicality, tax status as for-profit or non-profit) and single or multi-unit status is very important. For churches, it is not only whether a church belongs to a denomination or not, but the strength of denominational ties. The stronger denominational ties (polity) are, the more important denominational status would be to probability of exit. This is seen in the analysis, in which denominations of stronger polity typically have lower probability of exit. The difference between exit of for-profit or non-profit is also supported by a (currently) cursory analysis of church exit compared to exit of grocery stores.

In the theoretical model the number of units has no effect on the exit rate of for-profit firms. Potential future research includes making the number of units endogenous. Whether
or not a firm is multi-unit is not random. We would expect a firm with multiple units to be the result of a higher initial productivity draw. In that case I would expect the productivity shocks of multi-unit firms would be correlated and both be high, decreasing the exit rates of multi-unit for-profit firms, which does not occur in this model.

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R. E. Caves. Industrial organization and new findings on the turnover and mobility of firms.  


IRS. Tax-Exempt Status for Your Organization, a.


M. Walrath. Entry models applied to churches: Could protestants use a catholic bishop to solve excess entry? 2015.

A Proofs

A.1 Proof of Proposition A.1

Proposition A.1. For $\alpha \in (0, 1)$ and $\psi > 0$ if the religious entrepreneur maximizes profits then the probability of exit is highest for for-profit firms (single or multi-unit), next highest for single-unit religious entrepreneur and lowest for a two-unit religious entrepreneur.

Proof. First look at the cutoff of a for-profit firm, $\hat{\varphi}_f$. This firm chooses $l^*$ to satisfy the maximization problem in equation 2. $l^*$ must satisfy the first order condition:

$$\alpha p \varphi l^{*\alpha - 1} - w = 0 \quad (10)$$

The above can be re-written, to obtain $l^* = \left(\frac{\alpha p \varphi}{w}\right)^{\frac{1}{1-\alpha}}$. This equation for $l^*$ can be substituted into the equation for profit to obtain an expression for profit in terms of $\varphi$:

$$\Pi(\varphi) = p \varphi^{\frac{1}{1-\alpha}} \left(\frac{\alpha p}{w}\right)^{\frac{\alpha}{1-\alpha}} - w \varphi^{\frac{1}{1-\alpha}} \left(\frac{\alpha p}{w}\right)^{\frac{1}{1-\alpha}} - \theta \quad (11)$$

The $\hat{\varphi}_f$ must satisfy:

$$\Pi(\hat{\varphi}_f) = \psi \quad (12)$$

Differentiating equation 11 with respect to $\varphi$ shows that profits are strictly increasing in $\varphi$.

Now consider the decision of of a religious entrepreneur with one unit. Assuming the religious entrepreneur is also maximizing profits, $\hat{\varphi}_{r,1}$ must satisfy:

$$\Pi(\hat{\varphi}_{r,1}) = 0 \quad (13)$$

Since $\Pi(\varphi)$ is increasing in $\varphi$, $\hat{\varphi}_f > \hat{\varphi}_{r,1}$ as long as $\psi > 0$.

A religious entrepreneur with two units chooses labor that results in highest profit of three alternatives in equation (4). The probability of a two-unit entrepreneur keeping at
least one church open is higher than the probability of keeping a single church open due to the possible cross-subsidization with $\psi$ from shutting down the less productive church. As long as one of the two churches has a $\varphi$ draw high enough to earn more than $-\psi$ at least one church will stay open (since the other church will be closed to subsidize the other church; a combined profit greater than zero is better than shutting down both churches and receiving zero). Thus, the productivity cut-off to keep one church open is $\varphi_{r,2}$, where it satisfies:

$$\Pi(\varphi_{r,2}) = -\psi$$

(14)

Since, $\Pi(\varphi)$ is increasing in $\varphi$, $\varphi_{f,2} < \varphi_f, 1$.

To compare the exit rates between a for-profit firm and a single and multi-unit religious entrepreneur, let’s look at the exit probabilities for each situation. Compare the exit probabilities for two single-unit firms, two single-unit religious entrepreneurs and a two-unit entrepreneur. The probability of observing zero, one or two churches (units) is given by:

<table>
<thead>
<tr>
<th>Number churches (units)</th>
<th>Two Single-unit Religious Entrepreneurs</th>
<th>Two Single-unit Religious Entrepreneurs</th>
<th>Two-unit Religious Entrepreneur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remaining open</td>
<td>$F(\varphi_f)^2$</td>
<td>$F(\varphi_{r,1})^2$</td>
<td>$F(\varphi_{r,2})^2$</td>
</tr>
<tr>
<td>0</td>
<td>$2F(\varphi_f)(1 - F(\varphi_f))$</td>
<td>$2F(\varphi_{r,1})(1 - F(\varphi_{r,1}))$</td>
<td>$1 - F(\varphi_{r,2})^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$-(1 - F(\varphi_{r,1}))^2$</td>
</tr>
<tr>
<td>1</td>
<td>$(1 - F(\varphi_f))^2$</td>
<td>$(1 - F(\varphi_{r,1}))^2$</td>
<td>$(1 - F(\varphi_{r,1}))^2$</td>
</tr>
</tbody>
</table>

Either two religious entrepreneur each with a single church or one entrepreneur with two churches result in the same probability of having two churches which remain open. This is because the two-church entrepreneur will only keep both churches open as long as both churches earn a profit above $\psi$ – otherwise it is beneficial to shut-down the less productive church to subsidize the other church. However, the two-church entrepreneur has a lower probability of having zero or only one church open, so the exit rate amongst two-unit churches will be lower than the exit rate of single-unit churches.
A.2 Proof of Proposition 2.2

Proposition A.2. For $\alpha = 0.5$, $\psi > 0$ and $\psi$ not too much larger than $\theta$, if religious entrepreneurs are maximizing output then the probability of exit is highest for for-profit firms (single or multi-unit), next highest for single-unit religious entrepreneur and lowest for a two-unit religious entrepreneur.

Proof. First look at the cutoff of a for-profit firm, $\hat{\varphi}_f$. This firm chooses $l^*$ to satisfy the maximization problem in equation 2. With $\alpha = \frac{1}{2}$, $l^*$ must satisfy the first order condition:

$$\frac{1}{2}p_\varphi l^*-\frac{1}{2} - w = 0 \quad (15)$$

The above can be re-written, to obtain $l^* = \left(\frac{p_\varphi}{2w}\right)^2$. This equation for $l^*$ can be substituted into the equation for profit to obtain an expression for profit in terms of $\varphi$:

$$\Pi(\varphi) = p_\varphi \left(\frac{p_\varphi}{2w}\right) - w \left(\frac{p_\varphi}{2w}\right)^2 - \theta \quad (16)$$

The $\hat{\varphi}_f$ must satisfy:

$$\Pi(\hat{\varphi}_f) = \psi \quad (17)$$

Differentiating equation 16 with respect to $\varphi$ shows that profits are strictly increasing in $\varphi$.

Now consider the decision of of a religious entrepreneur with one unit. The religious entrepreneur chooses $\hat{l}$ that satisfies equation 5. Since $p_\varphi l^\alpha$ is increasing in $l$, $\hat{l}$ will be the largest $l$ that satisfies the non-negative profit constraint:

$$p_\varphi \hat{l}^\frac{1}{2} - w\hat{l} - \theta = 0 \quad (18)$$

The above can be re-written as:

$$w^2\hat{l}^2 + (2w\theta - p_\varphi^2\hat{l}^2)\hat{l} + \theta^2 = 0 \quad (19)$$
Applying the quadratic formula:

\[
\hat{l} = \frac{p^2 \varphi^2 - 2\theta w + p\varphi \sqrt{p^2 \varphi^2 - 4w\theta}}{2w^2}
\] (20)

Given the above we can see that \( \hat{l} > 0 \) only if \( p^2 \varphi^2 > 4\theta w \). This condition can be re-written in terms of \( \varphi \). For \( \varphi > \frac{2}{p} \sqrt{\theta w} \) the firm operates the unit. For \( \varphi \leq \frac{2}{p} \sqrt{\theta w} \) the firm does not operate the unit (\( \hat{l} = 0 \)). Thus \( \hat{\varphi}_{r,1} = \frac{2}{p} \sqrt{\theta w} \).

To compare \( \hat{\varphi}_f \) to \( \hat{\varphi}_{r,1} \), plug the above value of \( \hat{\varphi}_{r,1} \) into the profit equation for a for-profit firm (equation 16); given \( \hat{\varphi}_{r,1} \) the for-profit firm’s profit is zero. Since \( \psi > 0 \), a for-profit firm would not operate a unit given \( \hat{\varphi}_{r,1} \) since profits would be less than the scrap value (\( \hat{\varphi}_f \) satisfies \( \Pi(\hat{\varphi}_f) = \psi \)). Since profits are increasing in \( \varphi \), one can see \( \hat{\varphi}_f > \hat{\varphi}_{r,1} \).

A religious entrepreneur with two units chooses labor that satisfy one of the three equations 6, 7 or 8. Define \( \hat{\varphi}_{r,2} \) as the minimum \( \varphi_1 \) necessary to keep unit one operating if the other plant has \( \varphi_2 = 0 \). A single unit will stay open if:

\[
p\varphi \hat{l}^2 - w\hat{l} - \theta + \psi = 0
\] (21)

The above equation is just like equation (18), except in this case a single unit remains open as long as it earns more than zero after receiving \( \psi \) from the less productive unit. Thus, the solution to equation (21) will be the same as the solution to equation (18), except \( \theta \) will be replaced by (\( \theta - \psi \)). Thus, \( \hat{\varphi}_{r,2} = \frac{2}{p} \sqrt{(\theta - \psi)w} \). First assume \( 0 < \psi < \theta \), we can see that \( \hat{\varphi}_{r,2} < \hat{\varphi}_{r,1} \). If \( \psi > \theta \), then \( \hat{\varphi}_{r,2} = \frac{2}{p} \sqrt{w} \).

Again, compare the exit probabilities for two single-unit firms, two single-unit religious entrepreneurs and a two-unit entrepreneur. The probability of observing zero, one or two churches (units) is given by: Whether or not a two-unit religious entrepreneur keeps both units open depends on \( \psi \). The larger \( \psi \) is the more likely the less productive unit will be shut-down to subsidize the other unit. To show that the exit rate of a two church entrepreneur
<table>
<thead>
<tr>
<th>Number churches (units)</th>
<th>Two Single-unit for-profits</th>
<th>Two Single-unit Religious Entrepreneurs</th>
<th>Two-unit Religious Entrepreneur</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$F(\varphi_f)^2$</td>
<td>$F(\varphi_{r,1})^2$</td>
<td>$F(\varphi_{r,2})^2$</td>
</tr>
<tr>
<td>1</td>
<td>$2F(\varphi_f)(1 - F(\varphi_f))$</td>
<td>$2F(\varphi_{r,1})(1 - F(\varphi_{r,1}))$</td>
<td>$1 - F(\varphi_{r,2})^2$</td>
</tr>
<tr>
<td></td>
<td>$(1 - F(\varphi_f))^2$</td>
<td>$(1 - F(\varphi_{r,1}))^2$</td>
<td>$(1 - F(\varphi_{r,1}))^2$</td>
</tr>
</tbody>
</table>

is less than the exit rate of two single church entrepreneurs, focus on a situation in which both productivity shocks for the two-unit entrepreneur are $\varphi_{r,1} = \frac{2}{\rho} \sqrt{\theta w}$. The entrepreneur keep both units open as long as the output of the less productive unit is greater than the additional output the more productive unit can produce hiring $\psi/w$ additional workers (that would be the added output resulting from shutting down the less productive unit. Again, assuming both units have productivity $\varphi_{r,1} = \frac{2}{\rho} \sqrt{\theta w}$, each unit produces:

$$\varphi_{r,1} \hat{l}^{0.5}$$ \hspace{1cm} (22)

where $\hat{l}$ is defined by equation (20). Then the resulting output for a single unit is $2\theta/p$ and the combined output for both units is $4\theta/p$. Compare this output to the output of a single “more productive” unit using additional labor $\psi/w$:

$$\varphi_{r,1}(\hat{l} + \psi/w)^{0.5}$$ \hspace{1cm} (23)

The resulting output is $\frac{2\sqrt{\theta} \sqrt{\theta + \psi}}{\rho}$. Then, the entrepreneur keeps both units open as long as $\psi < 3\theta$. \hfill $\Box$