Abstract

It is often claimed that horizontal mergers cannot cause price increases if one of the merging firms is capacity constrained. The argument is that the constraint prohibits the post-merger recapture of lost sales, and thereby disables the mechanism by which merger-induced price increases occur. In this paper we show that this claim is incorrect. We show that mergers between substitutes in which one of the merging firms faces a binding pre-merger capacity constraint unambiguously increase prices in a price-setting Stackelberg game. The intuition is simple. Even if there is no recapture of lost sales, a merger causes the acquiring entity to take into account the positive effect of a change in the price of its good on the price of the acquired good. If the constraint continues to bind in the post-merger equilibrium, this reduces to the fact that an increase in the price of the unconstrained good increases the price at which the constrained good sells out its capacity. We also show a similar result for a Cournot quantity-setting game. These results points to the danger of focusing only on lost sales, and ignoring the internalization of higher prices.

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1 Introduction

In recent years it has become common to explain the price effects of horizontal mergers by reference to the "recapture of lost sales" intuition. Before a merger, each firm chooses the price that equates the negative effect of an infinitesimal price increase (lost marginal sales) against the positive effect (higher price on all infra-marginal sales). Those lost marginal sales will be divided among competing firms, except for those that drop out of the market. After a merger with one of the competing firms, the fraction of total sales lost that go to that firm (equal to the diversion ratio), and the associated profits, will be recaptured and will no longer be lost by the merged entity. The negative effect of a price increase gets smaller after the merger, while the positive effect stays the same, which means that the pre-merger price can no longer be the profit-maximizing price; absent efficiencies, the new profit-maximizing price must be higher.¹

This familiar recapture intuition has been used to justify the claim that if one merging party faces a capacity constraint that binds in the pre-merger equilibrium, and continues to bind in the post-merger equilibrium, then the merger cannot increase prices. This claim was made explicitly in public testimony by an economic expert in a recent litigated merger case.

Q. So that was a lot about Hershey. What about Pinnacle?

A. Pinnacle is in a different situation because while there is some demand side substitutability as there is between Hershey and Pinnacle, but between Pinnacle and Hershey, as well, there’s some overlap, it’s just in the context of one being community and the other being academic, it is not all that substantial, but there’s some, to be sure, without taking the capacity constraint into account.

But once capacity is taken into account, there can’t be substantial diversion of patients from anywhere, but, in particular, from Pinnacle to Hershey, say as a result of some

¹This intuition has entered antitrust practice in the form of Upward Pricing Pressure (UPP) analysis. See the DOJ/FTC Horizontal Merger Guidelines (2010) and Farrell and Shapiro (2010). In its simplest form, UPP analysis involves comparing the recaptured profits from an infinitesimal price increase (equal to the diversion ratio times the profit margin on recaptured sales) against the marginal cost efficiencies in order to determine whether or not a merger will increase price. There are now a number of simulation methods that extend this idea to generate quantitative predictions on the magnitudes of the price increases, as well as research evaluating the accuracy of those methods. See Weyl and Jaffe (2013) and Miller et al. (2016, 2017). This idea is referred to as the "value of diverted sales" in Section 6.1 of the Horizontal Merger Guidelines.
imagined price increase or some competitive event that is being assessed, because Hershey just doesn’t have the capacity to take on a major influx of patients from Pinnacle as a possible result of whatever, a natural disaster, we hope not, or a price rise. No, it just – it won’t happen.

So that the practical diversion between Pinnacle and Hershey is insignificant due to Hershey’s capacity constraint, as well as due to the differentiation of their services. But that means that the diversion ratio, in practical terms, because of the capacity constraint, is de minimis. And the conclusion is at the bottom of the slide. That means that upward pricing pressure in that direction, the upward pricing pressure from the merger on Pinnacle is negligible.  

This argument can be summarized as follows. The pre-merger price of the constrained good is set so that the quantity demanded at that price equals the capacity constraint, given the prices of rival firms. After the merger, there is no incentive to raise the price of the unconstrained good because none of the customers lost as a result of that higher price can purchase the constrained good instead. Since the price of the unconstrained good will not change, there is no benefit to increasing the price of the constrained good, as that is the price at which profits are already maximized. Thus, the merger will have no effect.

The purpose of this paper is to explain why the above argument is incorrect. Our primary insight is that a merger causes the price setter of the unconstrained good to become the residual claimant of the constrained good. Before the merger, the unconstrained firm does not internalize the positive effect of an increase in its price on the price of the constrained firm, which if the constraint continues to bind in the post-merger equilibrium will simply be the the price at which the constrained firm continues to sell out its capacity. After the merger, the merged entity does internalize this effect. We show that as long as the products are substitutes, this creates an incentive to increase both prices. This is true in both the quantity-setting and the price-setting games that we analyze.

2 Expert testimony of Professor Robert Willig in Federal Trade Commission and Commonwealth of Pennsylvania vs. Penn State Hershey Medical Center and Pinnacle Health System, April 15, 2016. A similar incorrect claim is made by Neurohr (2016) with regard to the case where one of the merging firms faces a capacity constraint that binds in both the pre-and post-merger equilibria. (Neurohr correctly points out that prices will increase when the constraint does not bind in the post-merger equilibrium.) In addition, the authors of the present paper have commonly heard this claim made in non-public correspondence in merger cases.
Using a Cournot model with homogeneous goods, we provide a general proof that mergers increase the market price when one of the merging firms is capacity constrained before the merger. The key intuition, again, is that the owner of the unconstrained plant becomes the residual claimant on the sales of the constrained plant. This makes demand for the merged entity more inelastic; the merged entity will reduce quantity relative to the pre-merger quantity level in response to this change in elasticity. The remaining firms will increase quantity, but by less than the quantity reduction by the merged entity, so total market quantity will decrease, which raises the market price of the good. This is true whether or not the constraint continues to bind in the post-merger equilibrium.

This intuition is very similar to the standard intuition for mergers in Cournot models: mergers increase prices because the merged entity internalizes the effect of a price increase on its merger partner. There is no recapture of lost sales in these models, so the confusion that has led to the incorrect argument discussed above is avoided. It is only in price-setting models that the issue arises.

The natural model to use for a price-setting game would be differentiated Bertrand. However, this approach faces the well-known problem that pure strategy equilibria may not exist in price-setting games with upward-sloping marginal cost curves. A pure strategy equilibrium is certain not to exist when the marginal cost curve is vertical at the capacity constraint, as we assume. One option would be to model a mixed strategy Bertrand game. We do not do this, as it does not seem reasonable to think that firms choose their prices stochastically in a game with no uncertainty. Instead, we use a Stackelberg price setting game in which the constrained firm chooses its price in the second stage.

In the Stackelberg game, mergers unambiguously raise both prices when one of the merging firms faces a capacity constraint that binds in the pre-merger equilibrium. As in the case of Cournot competition, the merger causes the price setter of the unconstrained good to become the residual claimant of the constrained good. Specifically, before the merger, the unconstrained firm does not internalize the positive effect of its price on the price of the constrained good. After the merger, the merged entity internalizes this effect, which holds even if there are no recaptured sales.

The pre-merger price of the constrained good in the second stage of the Stackelberg game is the price at which its quantity demanded just equals the capacity constraint, given the prices that its rivals choose in the first stage. If the two goods are substitutes, then an increase in the price of the unconstrained good diverts some demand to (i.e., shifts out demand for) the constrained good, which creates excess demand for that good. This means that the price of the constrained good can
be increased by at least a small amount while continuing to sell out at the constraint. This higher capacity-clearing price increases the profits of the constrained good, as the same quantity is sold at a higher price. And since the unconstrained firm’s pre-merger profit-maximizing price has the property that the firm is indifferent between increasing it by an infinitesimal amount and not, internalizing this effect is sufficient for the merged entity to increase both prices post-merger. Therefore, the result is just like the standard Stackelberg (or Bertrand) result in the unconstrained case: absent efficiencies, mergers between firms with positive diversion ratios unambiguously increase the price of both goods by at least a small amount. Note that the actual post-merger price increases may be large enough that the capacity constraint ceases to bind.

The remainder of the paper proceeds as follows. Section 2 contains a review of relevant literature. Section 3 contains the Cournot results, including numerical examples. Section 4 contains the Stackelberg Results. Section 5 concludes.

2 Related Literature

A substantial theoretical literature has explored the effect of capacity constraints on competition in oligopolistic markets (e.g. Levitan and Shubik (1972), Bresnahan and Suslow (1989), and Compte, Jenny, and Rey (2002)). Only a few papers in this literature deal with the specific question that we address, namely the effect of capacity constraints on unilateral price effects of mergers.

Perhaps the best known of these papers is Froeb et al. (2003), who use a Bertrand-based computational oligopoly model to simulate the effects of hypothetical mergers of parking garages. They show that the presence of a capacity constraint that is binding on one of the pre-merger firms reduces the price effect of a merger by about half.\(^3\) Unlike our paper, Froeb et al. (2003) do not show that merger effects are necessarily positive even in the presence of a binding capacity constraint on one of the merging firms, nor do they articulate a reason why this must be so. However, their finding that a binding pre-merger capacity constraint on one of the merging firms mitigates the magnitude of the

\(^3\)Froeb et al. (2003) do not distinguish between the case where the constrained good continues to sell out post-merger vs. the case where post-merger quantity of the constrained good is reduced. Therefore, it is not clear to which of these cases their results apply. Moreover, despite their finding of a positive merger effect when one of the merging firms is capacity constrained pre-merger, the paper contains language that suggests the opposite: “In the case where the merged firm is capacity-constrained, there is no merger price effect.” This comment may have contributed to the confusion surrounding this subject.
price effect is consistent with the results of our own parametric examples, and with the findings of prior literature including Hosken et al. (2002) and Higgins et al. (2004).

Another related paper is Neurohr (2016), which adjusts the UPP framework used by competition authorities in merger investigations to account for pre-merger capacity constraints. That paper finds that a capacity constraint that binds one of the firms pre-merger mitigates merger effects. However, it deals exclusively with the case where the merged entity chooses not to sell out the constrained good post-merger. The paper does contain a brief mention of the case in which the constraint continues to bind post-merger. Neurohr says that, “If the constraint still holds post merger, the merger has no effect.” As we show in the present paper, this claim is incorrect.

Greenfield and Sandford (2017) independently discovered some of the same results as the present paper, though their description of the underlying intuition is somewhat different from ours. Their paper also develops simulation methods for predicting merger effects quantitatively, allowing for differences in capacity, cost, and demand across firms.

As discussed in Section 4 below, introducing a capacity constraint into a Bertrand model means that the unconstrained firm faces a convex kink in its demand curve, which eliminates the possibility that a pure strategy pre-merger equilibrium exists. This well-known problem is discussed by Shapley and Shubik (1969) and by Levitan and Shubik (1972), who characterized a mixed strategy equilibrium for Bertrand games with capacity constraints.

The papers already discussed in this review either implicitly ignore this problem or simply assume that the problem is quantitatively small enough that it can be ignored in their quantitative analysis. However, Chen and Li (2018) confront this problem directly in a Bertrand game (they do not model a quantity-setting game). They analyze mixed strategy equilibria in a symmetric firm, homogenous good Bertrand model with symmetric capacity constraints in order to determine merger price effects. In their model, a merger between two identically capacity constrained firms (where the rest of the firms in the market are also constrained) can produce a merger price effect. Unlike our paper, their model does not allow for differentiated goods or for one firm to be unconstrained pre-merger. Moreover, they do not develop a general intuition for why a merger in which one firm is capacity constrained pre-merger must increase prices.
In this section we analyze merger effects in a Cournot model where one of the merging firms faces a capacity constraint that binds in the pre-merger equilibrium. We show that for general demand, a merger between the constrained firm and one of any number of symmetric unconstrained firms with identical cost functions results in a higher post-merger price. This is true whether or not the constrained good remains constrained after the merger. We also discuss how this result can be extended to a model of firms with non-identical cost functions.

The intuition behind this result is straightforward. Consider a standard symmetric Cournot model with no capacity constraints and marginal cost functions that have the necessary properties for a merger between two of the firms to be profitable. In the pre-merger equilibrium, each firm’s quantity is the one at which the loss from a small quantity reduction (fewer sales) is equal to the benefit (higher market price, which the firm will enjoy for all infra-marginal sales, as well as movement down the marginal cost curve). After the merger, each firm takes into account the fact that the price increase resulting from a quantity reduction will also benefit the merger partner, so these pre-merger equilibrium quantities cannot be the post-merger equilibrium quantities, which must be lower. This reduction in quantity by the merging firms results in an expansion by the non-merging firms, because

Relaxing the symmetry assumption would cause different firms to have different cost functions, which would cause asymmetric equilibrium quantities among non-constrained firms. The proof method that we use below can be extended to accommodate this. But doing so would require defining a sequential mapping firm by firm, which is an algebra intensive process that adds no additional intuition, so we omit it for brevity. However, later in this section we present an intuitive discussion of the results of relaxing the symmetry assumption.

In a well-known paper, Salant, Switzer, and Reynolds (1983) show that in a symmetric Cournot model with constant marginal costs and no capacity constraints, a merger of two firms is never jointly profitable for the merging firms if there are \( n > 2 \) firms in the pre-merger market. The intuition in their model is that the merging firms want to reduce quantity to raise price, but the other competitors increase quantity so much that the profits from selling \( 1/(n-1) \) of the post-merger quantity at the higher price is less profitable than selling \( 2/n \) of the pre-merger quantity at the lower price. Perry and Porter (1985) show that this result is not robust to the case where Cournot competitors face upward-sloping marginal cost. The upward-sloping marginal cost curves of the non-merging firms make expanding quantity more costly relative to the case with constant marginal cost curves. Informally, the Perry and Porter insight is that a merger between two small firms creates a big firm, not a single firm that is no different than either of the pre-merger firms. For steep enough marginal cost, the effect of non-merging firm expansion is not great enough to completely offset the gains to the merged entity from reducing quantity, and so the merger can be profitable. The numerical examples that we present in Section 3.3 satisfy this condition.
the price is higher and sales are more profitable, but not by enough to fully restore the pre-merger equilibrium quantity.

The analysis when one firm has a capacity constraint $k$ that binds in the pre-merger equilibrium is fundamentally similar. Pre-merger, the constrained firm produces quantity $k$ by assumption. The unconstrained firm chooses the quantity that balances the marginal loss and the marginal benefit of reducing quantity. But the merged entity internalizes the fact that a quantity reduction for the unconstrained good benefits the constrained good, which will still sell $k$ units but at a higher price. So it reduces its quantity by at least a small amount. If this reduction is not too large, then the constrained good will remain constrained in the post-merger equilibrium. If it is large enough, then the constraint will no longer bind. In either case, the merger reduces total quantity. Note that because this is a Cournot model of quantity competition with undifferentiated products, it is clear that our result does not depend on Bertrand (or Stackelberg)-style diversion between the merging parties.

3.1 Setup

There are $n$ firms indexed by $i \in \{1, 2, \ldots, n\}$, each with a single plant with identical cost curves. Each plant $j$ has a total cost curve, $c(q_j)$ with $c' > 0$ and $c'' \geq 0$. Without loss of generality, the last $n - 1$ firms have no capacity constraint, while Firm 1 has a capacity constraint at $q_1 = k$. Firm 1’s cost curve is given by $c(q)$ for $q \leq k$ and is infinite for any units in excess of $k$. The market demand curve is denoted by $p = d(\sum_i q_i)$ with $d' < 0$. Each firm chooses a quantity and firm $j$ earns a profit of $\pi_j = d(\sum_i q_i) - c(q_j)$. The demand and cost functions $d(\cdot)$ and $c(\cdot)$ are such that for any $\sum_{i \neq j} q_i$, $\pi_j$ is concave in $q_j$. In addition, the marginal revenue of firm $j$ is decreasing in $\sum_{i \neq j} q_i$. The equilibrium concept is the standard Cournot equilibrium in quantities.

3.2 Equilibrium

We first describe the premerger equilibrium in which Firm 1 produces at the constraint $k$. This will occur whenever $d(\cdot)$ and $c(\cdot)$ are such that the $n$-firm unconstrained (symmetric) equilibrium firm quantity $q_j^*$ is greater than $k$. We let Firm 1 merge with (WLOG) Firm 2, and call the merged entity Firm 1,2. We consider only those $d(\cdot)$, $c(\cdot)$ and $k$ combinations for which the new Firm 1,2 would continue to be capacity constrained at Plant 1 (the plant of the pre-merger Firm 1) post-merger. This would happen if and only if Firm 1,2’s optimal post-merger quantity exceeds $2k$. We now state and prove the proposition that any such merger would result in a price increase.
Proposition 1 If Firms 1 and 2 merge, and if the merged entity’s profits are maximized by operating the constrained plant at its capacity post-merger, then equilibrium market quantity will be lower, and price will be higher, in the post-merger equilibrium than in the pre-merger equilibrium.

Proof. The proof proceeds through the following series of lemmas.

Lemma 1 The optimal quantity of any firm $j$ depends only on the sum of the quantity choices of the remaining firms and is independent of the distribution of quantity across firms.

Proof. This is a well-known property of Cournot equilibria and is stated without proof. □

Lemma 2 Post merger, the merged Firm 1,2 would produce a total quantity less than $q_j^* + k$ if each of the other $n-2$ firms produced $q_j^*$. That is, the merged entity would choose to reduce its quantity if all non-merging rivals continued to produce their pre-merger quantities.

Proof. The pre-merger equilibrium quantity of Firms 2 through $n$, $q_j^*$, satisfies each unconstrained firm’s (identical) first-order condition $\frac{\partial \pi_j}{\partial q_j} = d'((n-1)q_j^* + k)q_j^* + d((n-1)q_j^* + k) - c'(q_j^*) = 0$. Firm 1,2’s post-merger profit function is $\pi_{1,2} = d(\sum_i q_i)(k + q_2^{post}) - c(k) - c(q_2^{post})$, where $q_2^{post}$ denotes the quantity produced in the plant owned by Firm 2 pre-merger. Since the post-merger quantity of Plant 1 is fixed at $k$ by assumption, the FOC for Firm 1,2 is $\frac{\partial \pi_{1,2}}{\partial q_2} = d'((n-2)q_j^* + k + q_2^{post}) + d((n-2)q_j^* + k + q_2^{post}) - c'(q_2^{post}) = 0$. Comparing to the pre-merger FOC, it is clear that the derivative $\frac{\partial \pi_{1,2}}{\partial q_2}$, evaluated at $q_2^{post} = q_j^*$, is negative, because $d' < 0$ and because reducing quantity lowers marginal cost. Firm 1,2 has a larger number of sales affected by a price change, so it will reduce its quantity; its best-response quantity for Plant 2 given the optimal pre-merger production of the other $n-2$ firms is less than $q_j^*$. We denote this best response quantity $q_2^\#$. □

Lemma 3 Each of the $n-2$ non-merging rivals’ best responses to $q_2^\#$ is: (i) greater than $q_j^*$, and (ii) less than $\frac{(n-1)q_j^*-q_2^\#}{n-2}$. That is, each of the other $n-2$ firms would increase its quantity in response to Firm 1,2 reducing quantity at Plant 2 from $q_j^*$ to $q_2^\#$, but not by enough that total quantity of all $n-1$ unconstrained firms would be as high as the total premerger quantity.

Proof. Suppose that each non-merging firm were to produce $q_j^*$ in response to $q_2^{post} = q_2^\#$. The total quantity of firms $i \neq j$ would then be lower than under the pre-merger equilibrium. Since the marginal revenue of $j$ is decreasing in $\sum_{i \neq j} q_j$, $j$’s first-order condition is positive and it would want to produce more than $q_j^*$. That is, the fact that quantities are strategic substitutes is sufficient to prove (i).
To prove (ii), let $\Delta$ denote the difference between $q^*_j$ and $q^\#_j$ and assume that each unconstrained firm increases its quantity to $q^*_j + \frac{\Delta}{n-2}$ in response to $q^\text{post}_2 = q^\#_2$, so that total post-merger quantity equals total pre-merger quantity. In that case, the first-order condition for each non-merging firm would be

$$\frac{\partial \pi_j}{\partial q_j} = d'((n-1)q^*_j + k)(q^*_j + \frac{\Delta}{n-2}) + d((n-1)q^*_j + k) - c'(q^*_j + \frac{\Delta}{n-2}) < 0.$$ 

Comparing the two FOCs, and recalling that total quantity is, by assumption, equal to the pre-merger quantity, the market price must be unchanged (i.e., the second term in the two FOCs are identical). Comparing the first term in the two FOCs, we see that the effect on the market price of an additional unit of quantity is the same, but each non-merging firm now has a larger quantity that is affected by that price change. Finally, comparing the third term in the two FOCs, we see that each non-merging firm now has a higher marginal cost compared to the pre-merger equilibrium. For these reasons, each non-merging firm would want to produce a quantity lower than the level that would fully replace the post-merger reduction in quantity at Plant 2 from $q^*_j$ to $q^\#_j$. By continuity and symmetry, this implies there is a $q^\#_j \in (q^*_j, q^*_j + \frac{\Delta}{n-2})$ at which $\frac{\partial \pi_j}{\partial q_j} = 0$. □

**Lemma 4** Given quantities of $q^\#_j$ by each non-merging firm, Firm 1,2’s best response quantity for Plant 2 is less than $q^\#_2$.

**Proof.** Since $q^\#_j > q^*_j$, and marginal revenue for Firm 1,2 is decreasing in the total quantity of the non-merging firms, Firm 1,2’s first-order condition evaluated at $q_j = q^\#_j$ for each $j \neq 1, 2$ and at $q^\text{post}_2 = q^\#_2$ is negative. Thus, Firm 1,2’s best response quantity for Plant 2 is less than $q^\#_2$. □

**Lemma 5** There exists a post-merger Nash equilibrium in which total quantity is less than the pre-merger quantity level.

**Proof.** Define an iterative mapping process beginning with Lemmas 2 through 4. Denote the round of the process with superscript $r$. In round $r$, Firm 1,2’s best-response quantity given the $n-2$ rivals’ previous best-response quantities is $q^\#_{2}^{(r)}(q^\#_{j}^{(r-1)})$. In round $r + 1$, each of the $n-2$ rivals’ best-response quantity given Firm 1,2’s previous best-response quantity is $q^\#_{j}^{(r+1)}(q^\#_{2}^{(r)})$. This mapping is a contraction mapping. That is, for each $q^\#_{2}^{r}$, $q^\#_{2}^{(r+1)}$ is mapped into the interior of the interval $[k, q^\#_{2}^{r}]$. Similarly, for each $q^\#_{j}^{r}$, $q^\#_{j}^{(r+1)}$ is mapped into the interior of the interval $[q^\#_{j}^{r}, (n-1)q^\#_{n-2}^{r}]$. Because $q^\#_{2}^{r}$ is bounded by $k$ and $q^\#_{j}$, the intervals are compact. It is well known that such a mapping has a fixed point and that fixed point is the Nash equilibrium of the post-merger game. □

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Lemma 2 contains the key result. When Firm 2 purchases Firm 1, it becomes larger without changing the slope of its residual demand curve. Thus, its demand becomes more inelastic at the pre-merger quantity levels, so it wants to reduce quantity to raise price. This is the main effect of the merger. The rest of the proof simply says that, in response to a reduction in quantity by the merged firm, the remaining firms will increase their quantity, but by less than the merged firm’s reduction. While the formal proof only shows the existence of equilibrium, under relatively weak conditions, the equilibrium will be unique (Gaudet and Salant, 1991).

The above proof considered the case where the constraint continues to bind post-merger. But the intuition from Proposition 1 is also easily applied to the case where it does not. To see this, start by observing that if the pre-merger constrained firm is not constrained post-merger, its quantity must be lower post-merger than it was pre-merger. Because cost curves are symmetric, the quantity of the unconstrained plant of the merged entity must also be lower (since symmetry and cost-minimization imply identical quantity in the two unconstrained plants post-merger). One need only then apply the result that the other firms will not increase quantity by enough to completely offset the merging firms quantity reduction to obtain the result.

None of this analysis involves the recapture of lost sales. Recapture as a mechanism by which mergers increase prices is not present in the Cournot model. Therefore, the argument that a merger cannot increase prices in the presence of a capacity constraint because a constrained firm cannot recapture lost sales does not even arise; in the Cournot model mergers with capacity constraints raise prices for the same reason as they raise prices without capacity constraints, with only slight additional complication.

3.3 Numerical Example

In the previous sub-section, we showed that a merger among Cournot-competing firms where one firm faces a capacity constraint that binds both before and after the merger must reduce quantity and increase price. It remains to be shown that such a merger is both possible and profitable. We do this by means of a numerical example. It is easy to construct such an example; we simply use the Perry and Porter (1985) model, introduce a capacity constraint, assume symmetric costs and linear demand, and choose parameter values such that the merger is profitable.

Let there be a linear market demand curve with intercept $a$ and slope $b$. There are $n$ firms with identical linear marginal cost curves that intersect the origin and have slope $d$, except that Firm 1
has a capacity constraint at $q_1 = k$. Firms compete as Cournot competitors pre- and post-merger. For $a = 25$, $b = -1$, $n = 4$, $d = 6$, and $k = 2$, pre-merger equilibrium quantity for each of the three (unconstrained) firms is 2.3 units. Firm 1 produces 2 units (its capacity constraint $k$), so the total pre-merger quantity is 8.9. The pre-merger price is $16.10. The pre-merger profit of Firm 1 is $20.20, and the pre-merger profits of the unconstrained firms are $21.16 each. Following the merger, quantity in the constrained plant of the merged entity remains at $k = 2$, quantity in the unconstrained plant of the merged entity falls to 2.04, and quantity at each of the rivals increases to 2.33, for a total quantity of 8.7, which is 2.2% below the pre-merger quantity. The price increases by 1.2% to $16.30.

It is easy to obtain larger price effects if we let the marginal cost curve be strictly convex instead of linear. This assumption allows us make the non-merging firms larger while making the marginal cost curve steeper at the equilibrium quantity levels, both of which reduce the non-merging firms’ incentives to increase quantity in response to the merged firm’s quantity reduction. This convexity is most easily accomplished by assuming a piecewise linear cost curve where marginal cost is constant up to some point $z$ and then has a positive constant slope for units beyond $z$. Alter the example above by letting $k = 3.8$, and letting marginal cost be 0 up to $z = 4$, beyond which point marginal cost has slope $d = 6$. With these parameters, the merger keeps the quantity of the constrained plant constant at 3.8, reduces the quantity of the unconstrained plant from 4.5 to just over 4, and increases the quantity at the rivals from 4.5 to 4.58. The quantity reduction is 2.2%, and the price increase is 5%, from $7.62 to $8.01.

One could also induce asymmetries among the firms to obtain larger price effects. For example, one could allow the acquiring and the acquired firms to both have flatter marginal cost curves than the non-merging firms. This would again reduce the incentives of the non-merging firms to increase quantity in response to a quantity reduction by the merged firm. This result should not be surprising when one notes that the merging firm with the flatter marginal cost curve would have a larger premerger market share, and mergers among firms with larger shares typically result in larger price effects.

It is also easy to construct a numerical example to show that there are conditions under which the constraint ceases to bind post-merger, but the merger is still profitable and prices still increase. In fact if we take the example from above with $a = 25$, $b = -1$, $n = 4$, $d = 6$, and increase $k$ from 2

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\(^6\)If the MC curve began at the origin, then making it steeper would increase merger effects, but it would also reduce the size of each firm, making it more difficult for the capacity constraint to bind in equilibrium. The piecewise MC function eliminates this problem, because it combines a steeper MC curve with larger firms.
to 2.2, we find that the post-merger quantity in each of the two merged plants fall to 2.035 (which is less than the constraint of 2.2) and the price effect of the merger is about 2%.

4 Stackelberg Model

We now consider how capacity constraints influence merger effects when competition is in prices rather than in quantities. As above, Firm 1 faces a capacity constraint $K$ that binds in the pre-merger equilibrium, Firm 2 is the merger partner, and the merged entity is Firm 1,2. The key intuition is very similar to that in the Cournot case above: the merged entity (which now owns the plants of each pre-merger firm) takes into account the effect of the actions of the unconstrained plant on the prices of the constrained plant. In Cournot, the merged entity takes into account the fact that a lower quantity by the unconstrained plant increases the market price, which increases the profits at the constrained plant. In Stackelberg, the merged entity takes into account the fact that a higher price for the unconstrained good increases the price of the constrained good. When the constraint continues to bind in the post-merger equilibrium, this is simply an increase in the price at which the constrained plant sells out its capacity. So while there is no net recapture of lost sales following a merger (which is often cited as the source of merger effects in models of price competition), there is an internalization of higher prices, which is sufficient for a merger effect. The result is similar for the case where the merger causes the constraint not to bind in the post-merger equilibrium.

A natural model to use for price competition would be differentiated products Bertrand, in which all firms choose prices simultaneously. However, introducing a capacity constraint into a Bertrand model creates a technical problem, namely that it causes the unconstrained firm to face a convex kink in the demand curve, which in turn causes there not to exist any pure strategy pre-merger equilibrium. The reason is as follows. Consider a price pair such that Firm 1 has quantity demanded exactly equal to its capacity constraint. At this price pair, Firm 2 faces an asymmetry in the effect of slightly increasing its price vs. slightly decreasing it. Slightly increasing $p_2$ would cause some customers to want to switch from Good 2 to Good 1. But since Firm 1 is constrained when $p_1$ is held constant (per the Bertrand assumption) this would cause excess demand at Firm 1; not every customer who wishes to purchase Good 1 at $p_1$ can be accommodated. Some of these excess Firm 1 customers will return to Firm 2, which makes Firm 2’s demand curve steeper relative to the case where Firm 1 is not constrained. However, decreasing $p_2$ does not have a symmetric effect, because at a lower $p_2$ Firm
1 will not cause excess demand. The slope of the demand curve for price reductions by Firm 2 will therefore have a flatter slope resulting from Firm 1 not being constrained.

To see this, consider the price pair \((p_{1}^{pre}, p_{2}^{pre})\), which is the equilibrium price pair under the assumption that Firm 2 ignores this effect of recapture of unserved Firm 1 customers. This pair cannot be an equilibrium when Firm 2 does take the effect into account, because at that price pair Firm 2 would want to increase \(p_{2}\). Now consider a different price pair \((\tilde{p}_{1}^{pre}, \tilde{p}_{2}^{pre})\) that would prevail if Firm 2 did take this effect into account. This also cannot be an equilibrium because Firm 2 would then want to reduce its price. This means that there is no equilibrium in pure strategies.\(^7\)

One way to address this problem and still use Bertrand would be to model the mixed strategy equilibrium and examine how that equilibrium is affected by the merger. We do not take this approach. Aside from the technical difficulties, it seems unreasonable to imagine that firms choose their prices stochastically in a game with no uncertainty. Another way to address the problem is to assume that Firm 2 does not recapture any of the customers that are not accommodated by Firm 1 when it faces excess demand at its posted price. This is the approach taken by Froeb et al. (2003) and by Kalniņš et al. (2018), but we do not take it here.\(^8\) Instead, we use a Stackelberg model for the price-setting game. Specifically, we consider the case in which one or more unconstrained firms simultaneously choose their prices in Stage 1, and Firm 1 (the capacity constrained firm) chooses its price in Stage 2. Analysis of the case in which Firm 1 chooses its price in Stage 1 and and one or more rivals simultaneously choose their prices in Stage 2 is in process, but mergers increase prices in that case as well.

\(^7\)An alternative way to model Firm 1’s capacity constraint would be to assume that its MC curve is arbitrarily close to, but not quite, vertical. In that case, \((p_{1}^{pre}, p_{2}^{pre})\) would be an equilibrium price pair. For an arbitrarily steep MC curve, there will be an increase in \(p_{2}\) small enough that Firm 1 can fully accommodate the resulting increase in quantity demanded. However, this is only a local equilibrium, not necessarily a global one. Though a sufficiently small increase in \(p_{2}\) will not lead to excess demand at Firm 1, a larger increase in \(p_{2}\) might do so. The recapture by Firm 2 of some of these unserved customers would reintroduce the kink in demand, which might make a larger price increase profitable. The flatter the MC curve, the less likely this is to occur, but for an MC curve steep enough to be considered a capacity constraint, it remains a concern.

\(^8\)While we do not rely on the result, it is worth noting that if we do take this approach we find that a merger increase both prices.
4.1 Setup

There are $n$ single-product, price-setting firms indexed by $i \in \{1, 2, \ldots, n\}$. All goods have continuously differentiable demand functions with respect to own prices and rival prices, which are strictly decreasing in own prices and strictly increasing in rival prices, and prices are strategic complements. The firms produce differentiated goods at the same constant marginal cost $c$, which for ease of notation we set to zero. Firm 1 has a maximum capacity equal to $K$, and Firms $2 - n$ are not capacity constrained. Each firm sets a price $p_i$ and faces demand $q_i = q_i(p_1, p_2, \ldots, p_n, K)$. Let $p_{-1}(\cdot)$ denote a vector of prices $\{p_2, \ldots, p_n\}$.

We begin with a Stackelberg game in which each of the $n$ firms is independently owned. This game is structured as follows. In Stage 1, Firms $2 - n$ all set prices simultaneously. In Stage 2, Firm 1 observes these prices and then sets its price $p_1$. We then consider a merger of Firms 1 and 2 (WLOG). The structure of the game is unchanged; prices $p_{-1}(\cdot) \equiv \{p_2, \ldots, p_n\}$ are still set simultaneously in Stage 1 and then $p_1$ is set in Stage 2. But after the merger the merged entity sets a price for one of its products (Good 2) in Stage 1 and the other of its products (Good 1) in Stage 2.

We assume that $K$ is small enough that Firm 1’s quantity demanded is strictly greater than $K$ when all firms choose the unconstrained Stackelberg pre-merger equilibrium prices. This ensures that Firm 1’s capacity constraint binds in the pre-merger equilibrium and also binds following a small post-merger price increase.

4.2 Equilibrium

We first describe the pre-merger equilibrium in which Firm 1 produces at the capacity constraint $K$. We then let Firm 1 merge with Firm 2 (WLOG), and call the merged entity Firm 1,2. We now state and prove the proposition that any such merger would cause both prices to increase.

**Proposition 2** If all goods are strategic complements (each firm’s best-response function is non-decreasing in every other price) and if the capacity constraint of Firm 1 is strictly binding in the pre-merger equilibrium, then following a merger between Firm 1 and Firm 2, a small increase in $p_2$, accompanied by an increase in $p_1$ such that $q_1$ remains equal to $K$, is profitable. Further, no non-merging firm will lower its price in response.

Proof. The structure of the proof is to show that, evaluated at the pre-merger prices, the merged entity has an incentive to raise the prices of both of the goods that it owns by at least a small amount,
and the non-merging firms have an incentive to (weakly) increase their prices in response. The proof proceeds through the following series of lemmas, beginning with the pre-merger equilibrium.

**Lemma 6** In Stage 2, Firm 1 sets \( p_1 \) so that \( q_1 = K \).

*Proof.* Suppose that, given the price vector \( p_{-1}(\cdot) \), \( p_1 \) were such that \( q_1 > K \). Then Firm 1 could increase \( p_1 \) and continue to sell \( K \) units, increasing its profits. If \( p_1 \) were such that \( q_1 < K \), this would violate the assumption that the constraint is binding in the pre-merger equilibrium. Define \( f(p_{-1}) \) as a function that maps \( p_{-1} \) into the \( p_1 \) that results in \( q_1 = K \). □

**Lemma 7** The first-order condition for the profit-maximizing price of each Firm \( i \neq 1 \) is \( p_1 \left[ \frac{\partial q_i}{\partial f(p_{-1})} \frac{\partial f(p_{-1})}{\partial p_i} + \frac{\partial q_i}{\partial p_i} \right] + q_i(f(p_{-1}), p_{-1}) = 0 \).

*Proof.* Each firm’s profit is \( \Pi_i = p_i q_i(f(p_{-1}), p_{-1}) \). Taking derivates with respect to each price gives the FOCs stated in the lemma. The solution to these FOCs, along with Firm 1’s choice to employ \( f(p_{-1}) \), constitute the equilibrium price vector, denoted \( \{ f(p_{-1}^*), p_{-1}^* \} \), for the pre-merger game. □

**Lemma 8** In response to a small increase in \( p_2 \), the merged entity will set \( p_1 \) so that \( q_1 = K \).

*Proof.* If the merged entity set a lower \( p_1 \), then it would earn lower profit on Good 1 (since it is constrained to sell only \( K \) units) and since Goods 1 and 2 are substitutes, a lower \( p_1 \) would reduce the demand for Good 2 which would further reduce profits. The assumption that the constraint is strictly binding in the pre-merger equilibrium ensures that the merged entity will not increase \( p_1 \) in response to a small increase in \( p_2 \) by so much that the constrint no longer binds. □

**Lemma 9** Post-merger, Firm 2’s FOC is \( p_2 \left[ \frac{\partial q_2}{\partial f(p_{-1})} \frac{\partial f(p_{-1})}{\partial p_2} + \frac{\partial q_2}{\partial p_2} \right] + q_2(f(p_{-1}), p_{-1}) + \frac{\partial f(p_{-1})}{\partial p_2} K = 0 \).

*Proof.* Firm 2’s post-merger profit function is \( \Pi_{1,2} = p_2 q_2(f(p_{-1}), p_{-1}) + f(p_{-1}) K \). Taking the derivative with respect to \( p_2 \) gives the FOC stated in the lemma. □

**Lemma 10** Post-merger, a small increase in \( p_1 \) and \( p_2 \) above \( f(p_{-1}^*) \) and \( p_2^* \) is profitable.

*Proof.* Post-merger, the FOCs of Firms 3 – \( n \) are \( p_i \left[ \frac{\partial q_i}{\partial f(p_{-1})} \frac{\partial f(p_{-1})}{\partial p_i} + \frac{\partial q_i}{\partial p_i} \right] + q_i(f(p_{-1}), p_{-1}) = 0 \), the same as in the pre-merger game. We know from Lemma 9 that the FOC of Firm 2 becomes \( p_2 \left[ \frac{\partial q_2}{\partial f(p_{-1})} \frac{\partial f(p_{-1})}{\partial p_2} + \frac{\partial q_2}{\partial p_2} \right] + q_2(f(p_{-1}), p_{-1}) + \frac{\partial f(p_{-1})}{\partial p_2} K = 0 \). Evaluating these FOCs at the pre-merger equilibrium price vector \( \{ f(p_{-1}^*), p_{-1}^* \} \) yields that the FOCs of Firms 3 – \( n \) are equal to 0 (as at the
pre-merger equilibrium), while the FOC of the merged entity with respect to \( p_2 \) is positive because the first two terms sum to zero (because they are the FOC of Firm 2 in the pre-merger game), while the last term \( \frac{\partial f(p-1)}{\partial p_2} K \) is positive because Goods 1 and 2 are substitutes. These conditions (at a given point the FOCs for one of the firms in \( i \in \{2, \ldots, n\} \) is positive and the rest are equal to zero), combined with the fact that prices are strategic complements, imply that there exists a vector of prices such that every price is greater than or equal to the corresponding price in \( p_{-1}^* \). Because the merged entity’s FOC with respect to \( p_2 \) is strictly positive at \( p_{-1}^* \), the post-merger equilibrium \( p_2 \) is strictly greater than \( p_{2}^* \). From Lemma 8 we see that \( f(p-1) \) is increasing in \( p_2 \) (and non-decreasing in all other prices), which means that the post-merger \( p_1 \) is greater than \( f(p_{-1}^*) \). Specifically, it is the (higher) price at which \( q_1 = K \) when the prices of other firms increase. \( \square \)

QED

The intuition behind this proof is simple. Both before and after the merger, Firms 2 – n choose their prices knowing that Firm 1 will charge the price that will cause it to exactly sell out its constraint. At the pre-merger equilibrium, the effect of a small increase in own price for each Firm \( i \in \{2, \ldots, n\} \) is zero. The only thing that changes after the merger is that the merged entity takes into account the effect of a change in \( p_2 \) on \( p_1 \). Specifically, a small increase in \( p_2 \) increases the \( p_1 \) at which Good 1 just sells out its constraint. This means that \( p_2 \) will increase by at least a small amount, as can be seen in the additional term \( \frac{\partial f(p-1)}{\partial p_2} K \) in the merged entity’s FOC with respect to \( p_2 \), and \( p_1 \) will increase in response. And since all prices are strategic complements the merged entity’s increases in prices creates an incentive for competing firms to (weakly) raise their prices as well. This is the very simple intuition behind this paper. While it is true that a constrained firm cannot recapture lost sales when its merger partner increases its price, it can still benefit from that higher price because it causes an increase in its own price.

Proposition 2 shows that the merged entity can increase profits by raising both prices by a small amount, choosing \( p_1 \) so that the quantity demanded for Good 1 remains at \( K \). Because prices are strategic complements, the post-merger equilibrium must be characterized by higher prices for all firms (DeGraba, 1995). But in the post-merger equilibrium, the price increase may be large enough that the profit-maximizing quantity for Good 1 will be less than \( K \). That is, the constraint could cease

\[9\text{See Characterizing solutions of supermodular games; intuitive comparative statics, and unique equilibria, Economic Theory February 1995, Volume 5, Issue 1, pp 181188.}\]
to bind in the post-merger equilibrium. In that case, the post-merger equilibrium will simply be the equilibrium that would exist if there were no capacity constraint.\textsuperscript{10}

5 Discussion and Concluding Remarks

Standard merger theory says that mergers between competitors increase prices (absent efficiencies). The purpose of this paper is to answer a very simple question: does that standard result continue to hold when one of the merging firms faces a capacity constraint that binds both before and after the merger? We find that the answer is yes for every game that we formally study.\textsuperscript{11} While some previous work has found results along these lines, to our knowledge the literature has not provided a general statement or articulated the mechanism by which this is likely to be true in general. (but see Greenfield and Sandford, 2017, for an independent development of related ideas). Moreover, it has been mistakenly suggested that the answer is in fact the opposite, namely that a capacity constraint that binds both pre- and post-merger makes merger effects impossible.

This confusion is a result of the fact that it has become common to think of merger effects in price-setting games in terms of recapture of lost sales. That is, the price effect of a merger between substitutes is often described by reference to the merged entity internalizing the fact that some of the lost sales following a price increase will be recaptured by the merger partner. For example, this framing is at the heart of Upward Pricing Pressure analysis, which has become common in merger evaluation. But if the merger partner faces a binding capacity constraint, then no sales can be recaptured, which might appear to mean that no merger effect is possible.

This way of describing merger effects, while correct, is incomplete even in the standard case, and is misleading in the presence of capacity constraints. It misses the fact that recapture of lost sales is not the only benefit of a post-merger price increase. Another benefit, which accrues even if the capacity constraint prevents recapture, is that by acquiring a competing good, a firm benefits from any price increase that accrues to the acquired good. This also creates an incentive to increase the

\textsuperscript{10}Whether the constraint continues to bind in the post-merger equilibrium will depend on the difference between the constrained and the unconstrained equilibrium quantities. All else equal, the smaller this difference, the more likely the constraint will cease to bind.

\textsuperscript{11}This paper addresses the most basic version of the question. It ignores interesting complications such as whether capacity truly binds pre-merger (in our model production beyond the constraint is impossible), or whether and how quickly capacity could be adjusted (in our model capacity is exogenous and constant).
price of the original good, because that price increase will allow the merged entity to increase the price of the acquired good. Once it is understood that recapture is not the only mechanism by which mergers cause price increases, it is easy to see that the claim that capacity constraints prevent merger effects is incorrect. Note that no such confusion exists a Cournot model.

As discussed in Section 4, differentiated product Bertrand games in the presence of upward-sloping marginal cost curves present problems involving a lack of pure strategy equilibria. For this reason, we provide formal general results in a homogeneous product Cournot game for the analysis where firms choose quantities and we use a Stackelberg game for the analysis where firms choose prices. We expect our results to hold more generally for price-setting games, but we leave this for future work.

Another topic for future work is to examine the extent to which capacity constraints mitigate the magnitude of merger effects. As discussed in Section 2 above, there is some literature suggesting that constraints do mitigate merger effects. For example, Froeb et al. (2003) find that merger effects are mitigated when one firm is capacity constrained, and are eliminated when both firms are constrained. Greenfield and Sandford (2017) also show that if both goods remain constrained post merger, then there are no price effects. Moreover, it is obvious that constraints must mitigate merger effects in settings where the capacity constraint does not bind in the post-merger equilibrium; the post-merger equilibrium is the same as in the unconstrained case, but the constraint causes higher pre-merger prices, so the effect must be smaller. This is true for both the Cournot and Stackelberg models. We have not proven a general result for whether the constraint mitigates merger effects when the constraint does continue to bind post-merger. We do know that it mitigates merger effects in the parametric Cournot example presented in Section 3 above, and more generally in any Cournot model with linear demand and piecewise linear marginal cost as described in Section 3. This result may be more general, but proving this and also identifying the factors that increase or decrease the mitigation are questions for future research.

References


