Strategic Platform Search Design in the Two-Sided Platform Business Mode versus in the Merchant Business Mode *

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Abstract

Major digitalized intermediaries like Amazon, Taobao and Sam’s Club spend a significant amount of investment each year on designing the search environments on their websites. In this paper, we endogenize an intermediary’s search design by allowing it to determine its consumers’ search cost and check how product diversity affects the intermediary’s search design strategy in two extreme business modes: the two-sided platform business mode and the merchant business mode. We find that the effect of product diversity on the intermediary’s equilibrium search design is not monotonic and its influence on the market outcomes depends heavily on the intermediary’s business mode. In this context, our model implies that when the intermediary’s product diversity is extremely high, the two-sided platform business mode could reach a higher profit than the merchant mode. Otherwise, it is the merchant mode, on the contrary, that may bring more profit to the intermediary. Using unique data sets of Aliexpress’s website and Sam’s Club’s website, we get empirical results that are quite consistent with our theoretical search design predictions.

JEL Classification:  
Keywords: Consumer Search, Platform, E-Commerce, Digital Economics.

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1 Introduction

An increasing amount of consumers nowadays are using online shopping intermediaries to collect product information, compare prices and make purchases. Compared to conventional shopping methods, online platform intermediaries could largely lower the search cost that consumers incur in their shopping process, either by providing sophisticated search engines or numerous product filters. Indeed, online intermediaries like Alibaba, Amazon, Priceline, iTunes and Walmart spend enormous investment each year on information technologies and website constructions to meet the needs of their consumers. Depending on which website a consumer chooses, it could have absolutely different searching experience. For example, a consumer on Amazon who wants to buy a dress would have access to 870,865 products. At the same time, Amazon lists them according to the relevance and offers 9 refinement tools for the consumer to choose its desired attributes, including "whether it is Amazon prime", "whether it is new arrival", "brands", "whether it is add-on item", "its average customer review", "price range", "its current discount", "its seller" and "whether it is eligible for international shipping". While a consumer on Target who is also searching for a dress would have access to only 1,439 products. At the same time, Target also lists them according to the relevance and offers 12 refinement tools for the consumer to choose its desired attributes, including: "its category", "its type", "its color", "whether it is in deals", "its price", "whether it is eligible for shipping & pickup", "its brand", "its guest reviews", "garment length", "occasion", "its sleeve length" and "FPO/APO". Although Target has much less products available, it does offer more search tools for its consumers to choose from.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Business Mode</th>
<th>Default Ranking</th>
<th># Items</th>
<th># Search Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ebay</td>
<td>Two-sided Platform</td>
<td>Best Match</td>
<td>7,554,961</td>
<td>15</td>
</tr>
<tr>
<td>Kaola</td>
<td>Two-sided Platform &amp; Merchant</td>
<td>Comprehensive</td>
<td>2480</td>
<td>12</td>
</tr>
<tr>
<td>JD</td>
<td>Two-sided Platform</td>
<td>Comprehensive</td>
<td>over 910,000</td>
<td>14</td>
</tr>
<tr>
<td>Aliexpress</td>
<td>Two-sided Platform</td>
<td>Best Match</td>
<td>958,129</td>
<td>5</td>
</tr>
<tr>
<td>Amazon</td>
<td>Two-sided Platform &amp; Merchant</td>
<td>Relevance</td>
<td>870,765</td>
<td>9</td>
</tr>
<tr>
<td>Costco</td>
<td>Merchant</td>
<td>Best Match</td>
<td>34</td>
<td>10</td>
</tr>
<tr>
<td>Sam’s Club</td>
<td>Merchant</td>
<td>Most Relevance</td>
<td>28</td>
<td>6</td>
</tr>
<tr>
<td>Target</td>
<td>Merchant</td>
<td>Relevance</td>
<td>1562</td>
<td>13</td>
</tr>
<tr>
<td>Walmart</td>
<td>Merchant</td>
<td>Best Match</td>
<td>80,601</td>
<td>11</td>
</tr>
<tr>
<td>Sears</td>
<td>Merchant</td>
<td>Relevance</td>
<td>over 500</td>
<td>17</td>
</tr>
<tr>
<td>Saks Fifth Avenue</td>
<td>Merchant</td>
<td>Featured</td>
<td>6,668</td>
<td>19</td>
</tr>
</tbody>
</table>

In fact, search refinements which act as a main strategic element in current intermediaries’ search environment design varies a lot in different intermediaries. To illustrate this, we list the general website search information regarding several online shopping websites in table 1, including the default ranking, number of items and how many kinds of search refinements of more than ten online platform intermediaries in the category of "dresses". First of all, the

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1Those data comes from a search of "dress" in November 2017 on Amazon and Target
Table implies in each business mode the number of search tools is increasing in the number of products on the website. In addition, it seems that compared to the intermediaries in the two-sided platform business mode, the intermediaries in the merchant business mode have fewer items on the platform while they have more available search refinements for its consumers. Take JD’s (a two-sided platform intermediary) first webpage and Sears’s (a merchant intermediary) first webpage of the search of “dresses” for instance as showed in figure 1. Suppose there is a consumer who wants to buy a long sleeve dress. If it goes to JD, it can choose among more than 910,000 dresses on the website, however it cannot find sleeve lengths as a filter to find the ideal type. While Sears has only around 500 products online but it allows the consumer to screen this attribute. Although the consumer could learn about sleeve lengths on both Sears and JD, it may need to click each dress on JD and check the item descriptions one by one while on Sears one click on the filter divides all dresses according to the different sleeve lengths at once.

Figure 1: Search Aids on JD and Sears

We are interested in the factors that determine those different website constructions. Our study focuses especially on the influence of product variety on the intermediary’s strategic search environment design in different business modes, which is operationalized by the choice of its consumers’ search cost. Specifically, our paper checked two polar types of intermediaries: the merchant business mode and the two-sided platform business mode. The difference
between those two business modes could be simply summarized as follows: in the merchant business mode the intermediary buys sellers’ products and resells the goods to consumers for an individual price that it chooses. Whereas in the two-sided platform business mode the intermediary charges each seller a ”listing” fee for per unit sold through the platform, in exchange for which sellers can sell their goods directly to consumers affiliated with the intermediary for the price that they choose. According to [Hagiu 2007], an intermediary in the ”pure” merchant mode should reserve full control over the whole market, it should handle every aspect of the transaction with consumers including but not limited to marketing, consumer service, legal problem, shipping and selling. However, an intermediary in the ”pure” two-sided platform business mode leaves all control rights to sellers and the intermediary does not involve in any aspect of the transactions between consumers and sellers. In the two-sided platform business mode, it is the sellers who are responsible for every aspect of the transaction.

This paper for the first time compares the intermediary’s search design strategies in different business modes. We model consumers’ entry and sequential search in a horizontally differentiated market and allow the intermediary to design the search environment via its search cost. We find that both its business mode and product differentiation could be key factors that determine the intermediary’s strategy although the effect may not be obvious. Higher differentiation in its products, on one hand, could lower the equilibrium search cost because the intermediary needs to compensate for its lower demand. On the other hand, higher product diversity may increase the equilibrium search cost at the same time since it decreases the detrimental effect of low investment. However, figuring our the relationship between the product diversity and the equilibrium search design could be quite important because it indirectly determines the product price, the intermediary’s listing fee and its profit.

Based on our model, we find the effect of product diversity on the intermediary’s strategic search design is not monotonic. In addition, the intermediary may have the opposite reactions in different business modes. In our study, we follows [Anderson & Renault 1999]’s method to introduce product diversity to the search model. In the merchant business mode, we show that when products are not too differentiated, an increase in product diversity increases the intermediary’s equilibrium search cost. However, when products are sufficiently differentiated, an increase in product diversity decreases the intermediary’s equilibrium search cost. This is because in the merchant business mode, the intermediary makes revenue by selling products to consumers directly. An increase in product diversity lowers consumers’ expected utility of adopting the platform, thus decreasing the aggregate demand. Unlike the intermediary in the two-sided platform business mode, the intermediary in the merchant mode has full control of the product price. With a lower demand, the intermediary could increase its search cost to make products more differentiated and less competitive. Consequently, it could charge a higher price to make up for the loss in its demand. However, when products are too
differentiated and the product price is too high, the price elasticity of demand becomes quite large. Thus if the intermediary’s product diversity continues increasing the intermediary needs to lower its search cost to decrease its product price and to attract more consumers. In this context, the effect of product diversity on the intermediary’s product price in the merchant mode is also not monotonic. When products are not too differentiated, increasing product diversity generates higher search cost, both decreases competitions between products thus the intermediary could charge a higher product price. While when products are too differentiated and the product price is too high, the demand becomes quite elastic and it is optimal for the intermediary to lower its product price.

In the two-sided platform mode, we obtain a totally different result. We find that when products are not too differentiated, an increase in product diversity decreases the equilibrium search cost. However, when products are sufficiently differentiated, an increase in product diversity increases the intermediary’s equilibrium search cost. This is because in the two-sided platform business mode, the intermediary does not sell products directly to consumers. Instead, it makes money by charging each seller a listing fee. In other words, in the two-sided platform mode the intermediary loses control of product price and leaves it to individual seller. Consequently, its revenue depends heavily on the demand. An increase in product diversity hurts consumers’ expected utility of adopting the intermediary and consequently hurts the aggregate demand. Without the control over product price, the platform needs to lower its search cost to attract more consumers. However, when products are too differentiated and each seller maintains monopolistic power, further changes in search cost won’t influence consumers’ behaviors that much due to the huge difference between products. Consequently, it may be optimal for the intermediary to increase its search cost and save investment, both of which generate little effect on the demand when product diversity is high. Unlike the intermediary in the merchant business mode, the effect of product diversity on the product price is monotonic in the two-sided platform business mode. This is because in the two-sided platform mode, each seller only cares about its individual demand and does not have access to the aggregate one. Correspondingly, when their products become more differentiated, each of them faces less competition and charges a higher product price.

Since the intermediary has different strategies when running in different business modes, we are curious about the conditions under which the intermediary in one specific business mode is more profitable than the other. We find that in the two-sided platform business mode, when products are not too differentiated, an increase in product diversity decreases the intermediary’s profit. However when products are sufficiently differentiated, an increase in product diversity increases the intermediary’s profit. While in the merchant mode, it is the opposite: when products are not too differentiated, an increase in product diversity increases the intermediary’s profit. However when products are sufficiently differentiated, an increase in product diversity lowers the intermediary’s profit. This is caused by the intermediary’s
different strategies in different business modes. In the merchant business mode, the intermediary acts as a big retailer. Thus when product diversity is not too high, an increase in its variety makes products less competitive and helps the intermediary increase its profit. However, when products are too differentiated, the demand becomes very elastic. A further increase in its product diversity decreases its demand a lot thus decreases its profit. In the two-sided platform business mode, the intermediary acts like a pure intermediary instead of a big retailer. Thus when products are not too differentiated, an increase in product diversity enables sellers to have more monopolistic power and charge a higher price. However, it at the same time hurts the aggregate demand of the intermediary and the intermediary needs to pay more investment in search cost reduction to compensate for that. This moves the intermediary’s profit to a lower level. When products are sufficiently differentiated, the changes in its search cost won’t influence the demand that much thus the intermediary decreases its investment on search to achieve higher profit. We also compare the intermediary’s profits in two business modes. With the assumption that the search cost restriction is never binding we find that when products are extremely similar or extremely diverse, it is the two-sided platform mode that could bring more profits to the intermediary and between those two extremes it is the merchant mode that is more profitable. However empirically we need take binding search cost restriction into consideration, in this context our conclusion is quite similar to Hagiu’s theory that the intermediary is more profitable in the two-sided platform business mode when products are highly differentiated.

Using unique datasets from Aliexpress and Sam’s Club, we propose suggestive evidence to support our theory. Note that although we assume in our theoretical part that the search cost restriction is never binding, empirically we keep in mind that it is possible to have binding constraint and only the product diversity range during which the search cost restriction is non-binding could be observed. On Sam’s Club’s website we find that the unbinding product diversity interval has a lower bound that is between the thresholds of the equilibrium search cost design and the product price. Our results thus indicate that the search cost design is strictly decreasing in product diversity during the unbinding interval. In the two-sided platform business mode, we get quite different conclusions. On Aliexpress’s website, we find that when products are not too differentiated, an increase in product diversity decreases the intermediary’s equilibrium search cost in the two-sided platform mode, while when products are sufficiently differentiated it increases the equilibrium search cost. Both the result of Aliexpress, which runs mainly in the two-sided platform business mode, and the result of Sam’s Club, which runs in the merchant mode, show consistent conclusions with our theoretical predictions regarding the intermediary search investment strategies. For the product price, we also find supportive evidence in Sam’s club data. It implies that when products are not too differentiated, an increase in product diversity increases the intermediary’s product price in the merchant mode, while when products are sufficiently differentiated it decreases the
product price. Although Aliexpress is expected to embrace the two-sided platform business mode features, it turns out to be more similar to the merchant mode due to its different listing fee policy to our theoretical model. In our theoretical model we assume the intermediary in the two-sided platform business mode charges each seller a fixed amount of listing fee for per unit sold through the intermediary. However, empirically Aliexpress charges each seller a commission fee that is proportional to its total revenue instead of its total quantity sold. This implies that Aliexpress’s pricing strategies is to maximize the total revenue of sellers which makes it more similar to a big retailer in the merchant mode. Consequently, the product price strategy of Aliexpress is more similar to the merchant business mode than the two-sided platform business mode.

2 Literature Review

My paper mainly relates to two branches of literature: random search and intermediary business modes. It for the first time connects two previously separate strands of economics research together. On one hand, my paper contributes to the literature of random search by allowing the intermediary to optimize its search investment in an e-commerce search environment and by introducing business mode to the model as one important determinant. On the other hand, it contributes to the literature of intermediary business mode by investigating the effect of product diversity on the intermediary’s strategic search design and comparing the intermediary’s equilibrium in two polar types: the merchant business mode and the two-sided platform business mode.

A considerable amount of previous research has focused on analyzing the role that reducing search cost plays in the economics of platform. Imperfect information makes consumers incur search cost [Wolinsky 1986] and allows firms to keep monopolistic characteristics even when number of firms is quite large. However, current technology enables electronic marketplaces to dramatically lower the search cost by providing faceted navigation aids, search engines and tools selectively. Consequently, the intermediary nowadays affects competition, optimize productive resources allocation [Bakos 1997], and influence seller’s product design strategy imperatively [Kuksov 2004]. As pointed out by Dukes & Liu (2015), online intermediary faces two main conflicting goals: help consumers to find a desirable product and keep sellers’ competition in track. This required that the intermediary’s search environment embeds sufficiently high search cost to prevent consumers from evaluating too many products, but not too high to cause them to only partially evaluate the sellers. In addition, they also mentioned the point that compared to online retailer (merchant business mode), the most popular online shopping intermediary (two-sided platform business mode) seem to provide inferior navigation aids which is consistent with our finding in this paper. Besides business mode, product diversity could also influence the equilibrium dramatically and play a key role
in consumer’s search strategy (Chamberlin et al., 1948). In most previous studies researchers emphasize the distribution of random term in consumer’s utility to approach the product diversity. For example, Anderson & Renault (1999) measure the variance of the random term and Bar-Isaac et al. (2012), allows sellers’s design to influence the random term distribution. Similar definition could be also found in Zhong (2016)’s paper where he mentioned the concept of targeted search to filter items to a smaller product value range. However, he approached the intermediary’s optimization by controlling the lower bound of product value instead of deciding the search cost and did not take into account business modes as one important determinant. Our study use similar definition, specifically we assume the random term is uniformly distributed and use the length between maximum value to minimum value to represent product diversity.

It is widely believed that the intermediary could improve social welfare by minimizing inefficient searching between buyers and sellers in previous studies (Yavas, 1994; Biglaiser, 1993). However, unlike the traditionally defined intermediary who only acts more like a pure retailer, nowadays the intermediary could run in a wide variety of business types. Among them, we are most interested in two polar business modes: the merchant mode and the two-sided platform mode whose concepts were first clarified by Hagiu (2007). His study showed that the merchant mode could be more profitable when the degree of complementarity among sellers’ products is higher, while the two-sided platform could be more profitable when sellers’ investment incentives are important or when sellers’ information regarding product quality is asymmetric. His paper compares two polar intermediaries in the light of network and product substitutability/complementarity. My contribution to the literature departs from the popular network models (Hagiu & Wright, 2014; Armstrong, 2006; Caillaud & Jullien, 2003; Rochet & Tirole, 2003) and matching models (Rubinstein & Wolinsky, 1987; Wolinsky, 1987; Gehrig, 1993) by introducing sequential search to the model and enabling the intermediary to optimize its search investment in both business modes. This paper for the first time compares search design strategies of the intermediaries in two polar business modes. An important corollary in our research is that the intermediary may have the opposite search investment strategies depending on which business mode it runs in and the effect of product diversity on the strategic search design is not monotonic in both business modes. Allowing the intermediaries to optimize its search environment enables more flexibility in our model thus we get several interesting and innovative find outs. According to our model, the intermediary in the two-sided platform business mode is more profitable when product diversity is extremely high or extremely low while when product diversity is in the middle of these two extremes we get the opposite conclusion.

The remainder of the paper is organized as follows. We present our baseline model in section 3, then we proceed to use the variations of the baseline model to investigate the equilibriums in two polar modes: the two-sided platform business mode in section 4 and the
merchant business mode in section 5. Section 6 uses the results from previous models to compare the equilibriums in different business modes and discuss possible underlying mechanisms that generate the difference. In section 7, we talk in detail about the search cost constraints in two business modes. Lastly in section 8, we analyze unique data sets of Aliexpress and Sam’s Club to verify our theoretical predictions and conclude.

3 Baseline: random search on the platform

A platform intermediary could run business in either the merchant (one-sided mode) business mode or the two-sided platform business mode. In the merchant mode, the intermediary buys sellers’ products and resells the goods to consumers for an individual price that it chooses. In the two-sided platform mode, the intermediary charges each seller a "listing" fee for per unit sold through the platform, in exchange for which sellers can sell their goods directly to consumers affiliated with the intermediary for the price that they choose. There are \( n \) distinct brands (sellers) who offer products with the same cost, namely \( C(q) = cq + F \), where \( c \) is the marginal cost, \( q \) is the quantity and \( F \) is the fixed cost. In the two-sided platform business mode, each seller pays a listing fee to the platform and determines the product price simultaneously. In the merchant business mode, sellers sell their products to the intermediary and then leave the market. We follow the assumption of Hagiu (2007) that all bargaining power lies within the intermediary when it makes offers to sellers in the merchant mode. Thus the intermediary could always buy products from sellers at sellers’ marginal cost in the merchant business mode. Suppose that there are \( Q=q \) consumers adopt the intermediary, each with a unit demand. After adoption, each consumer in the intermediary knows the total number of available brands \( n \), but does not know the price and the value of each brand. Consumers engage in sequential search in order to obtain information about prices and values. We assume that consumers have perfect recall and that there is no replacement in the search. Once they decide to engage, in each round they need to pay a search cost \( s \) to learn the price and value of a particular brand and determine whether they want to continue searching or stop.

The timeline of moves in the model is as follows. First, for each business mode, the intermediary determines the design of consumers’ search cost on its platform firstly. Second, if it runs in the two-sided platform business mode the intermediary decides the listing fee and then each of the \( n \) sellers simultaneously chooses a price for its product; if it runs in the merchant business mode the intermediary buys the products from sellers at their marginal cost \( c \) and decides the product price then. Consumers are not informed of the value and the price of each product available through the intermediary, nor of the search cost. However, they know the price index and the certain fixed utility term \( x_0 \), hence they take the worst as guaranteed and decide whether they adopt the intermediary. After consumers adopt the
intermediary, they have access to search cost and product distribution. They optimize the search and choose the best product among the evaluated goods.

We start with the consumer’s choice. If consumer \( j \) adopts the platform intermediary, its utility from buying a unit of good \( i \) through the platform is of the form \( x_j(i) = x_0 - p_i + \mu v_{ij} \), where \( x_0 \) is the base level [Dukes & Liu, 2015] of consumer \( i \)'s utility that is constant and known to consumer, \( v_{ij} \) is the random terms that are i.i.d. and uniformly distributed in interval \( [0, \bar{v}] \) and \( \mu > 0 \) is a scale parameter that captures the heterogeneity of products on the platform [Anderson & Renault, 1999]. This implies that there is no correlation between consumers’ tastes and we only focus on horizontal differentiation. We use \( G(v) \) to denote the CDF of \( v \) and \( g(v) \) to denote the density distribution of \( v \) for simplicity. We focus as Wolinsky (1986) on a situation in which all brands charge the same price \( p^* \) and construct our model based on the study of Anderson & Renault (1999). Consumers expect firms which it has not yet visit all charge price \( p^* \) and correspondingly we defined the reservation value \( w^* \) to be the unique solution of\(^{3}\)

\[
\mu \int_{w^*}^{\bar{v}} (v - w^*)dG(v) = s
\]  

(1)

After consumers adopt the intermediary, it is optimal for them to follow the sequential search rule with the reservation value \( w^* \): each consumer stops searching and buys the first brand \( j \) whose value \( v_j \geq w^* \); if no such product exists, then the consumer exhaust all the products and choose the maximal among them. However, if the expected utility of adopting the intermediary is too low, consumers could choose not to adopt the intermediary and instead choose the outside choice \( O \) whose value is uniformly distributed between \( [o_1, o_2] \). Our study assumes that before adopting the intermediary consumers do not know the product values and the search cost. However, they know that if they adopt the intermediary they will get \( x_0 \) utility for certain and they are informed about the price index of the intermediary although they are not informed about prices\(^4\). Consequently consumers take the worst as guaranteed and their expected value of adopting the intermediary takes the form \( x_0 + \mu \hat{W} - \hat{p} \), where \( \hat{p} \) is the price

\(^{2}\)We assume that \( x_0 \) is large enough to guarantee the intermediary’s aggregate demand is positive throughout our study. In the absence of this requirement, there will always exist trivial equilibria so that all firms charge a such high price that the demand of adopting the intermediary is zero. The requirement of sufficient large \( x_0 \) rules out these equilibria and only admits nontrivial equilibria in which transactions happens on the intermediary. Actually this is true for most intermediaries, although price index might be different on different website but usually it won’t be too high such that on one uses the intermediary.

\(^{3}\)If there is no solution to the equation above, we let \( w^* = 0 \) and the scenario is discussed in detail in section 7

\(^{4}\)This assumption mimick the fact a majority of websites require membership or registration before using it thus consumers have little information regarding their search experience, product values and prices on the website before adopting the intermediary. However, using the available information like the intermediary’s review or reputation, consumers could form an expectation based on the certain utility they will get by adopting the intermediary and the price index of the intermediary.
index of the platform that could not be determined by individual seller (product) and \( W \) is
the lowest reservation value at the maximum search cost \( \frac{\bar{v}}{2} \) beyond which consumers stop
searching. Correspondingly, the aggregate demand of consumers who adopt the intermediary
equals the probability \( p^r(o \leq x_0 + \mu W - \bar{p}) = \frac{x_0 + \mu W - \bar{p} - o}{o_2 - o_1} \) which could be written as
\( Q = a - b\bar{p} \) where \( a = \frac{x_0 - o_1}{o_2 - o_1} \) and \( b = \frac{1}{o_2 - o_1} \).

With the reservation value \( w^* \), consumers keep searching until they find a product whose
value is no less than \( w^* \). After consumers exhaust all \( n \) brands, they buy the product with
the highest value among their sample of \( n \) brands. Thus the expected demand of a particular
firm is

\[
D(p, p^*, n) = Q\{\frac{[1 - G(w(p))] [1 - G(w^*)^n]}{n [1 - G(w^*)]} + \int_p^{w(p)} G\left(\frac{p^* - p}{\mu} + v\right)^{n-1} g(v) dv\} \tag{2}
\]

where \( p \) is the firm’s price level, \( p^* \) is the price level of all other firms and \( w(p) = w^* + \frac{p - p^*}{\mu} \).
The right-hand side of the equation is a product of the number of consumers who adopt the
platform and the probability that a randomly drawn consumer buys from the particular firm.
The first term in the curly brackets captures the probability that the consumer stops and
purchases from the seller with price \( p \) immediately after sampling it while the second term
captures the probability that the consumer at the end chooses the seller after exhausting all
available options. At the symmetric equilibrium, we have \( p = p^* \) and correspondingly we have

\[
D(p^*, p^*, n) = Q\frac{[1 - G(p^*)^n]}{n} \tag{3}
\]

In what follows, we shall assume that \( \bar{v} \) is quite large such that the restriction is non-binding
and \( w^* > 0 \), consequently at least some searches will take place at the symmetric equilibrium.

4 Two-sided platform business mode

4.1 Equilibrium analysis

We first analyze the case in which the intermediary runs in the two-sided platform business
mode. In the two-sided platform business mode, the intermediary charges each seller a fixed
amount of listing fee for per unit sold through its platform and allows firms (sellers) to sell
directly to consumers who adopted the intermediary. Firstly we study firm’s pricing behavior
in the two-sided platform mode and then establish the optimal platform listing fee and search
design chosen by the intermediary.

We focus on the symmetric equilibrium where all firms charge the same price \( p^* \). Per unit

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5 As previously noted, it is possible that the reservation value equals 0 when the search cost is too high such
that at equilibrium no consumer searches and each firm charges a sufficiently high price as showed in [Diamond
(1971)]. We will discuss this scenario specifically in section 7. However, those trial equilibria in which \( \frac{1}{p} > \frac{\mu}{\bar{v}} \)
are not our focus in this study.
sold through the platform, each firm incurs a marginal cost $c$ and a listing fee $p_s$. Thus the expected profit of the firm whose price is $p$ when all other firms charge $p^*$ is

$$\pi(p, p^*, n) = (p - c - p_s)D(p, p^*, n) - F$$  \hspace{1cm} (4)$$

If the equilibrium price $p^*$ is higher than the reservation value $w^*$, then no consumer searches or buys at the equilibrium thus demand is zero. In this paper, we are not interested in this case. We adopt the same method of Wolinsky (1986) who proved that there exists a pure strategic symmetric equilibrium price $p^*$ such that $p^* < w^*$ and get the equilibrium price that equals:

$$p^* = c + p_s - \frac{D(p^*, p^*, n)}{D_p(p^*, p^*, n)},$$  \hspace{1cm} (5)$$

where $D_p = \frac{\partial D}{\partial p}$.

Given the assumption that $v \sim U[0, \bar{v}]$ we have

$$p^* = c + p_s + \frac{\mu(\bar{v} - w^*)}{1 - (\frac{w^*}{\bar{v}})^n}$$  \hspace{1cm} (6)$$

where $w^* = \bar{v} - \sqrt{\frac{2\pi w^*}{\mu}}$. Using the backward induction, we then optimize the listing fee $p_s$ and the design of consumer search cost on the platform. To reduce the search cost incurred by the consumers on the platform, the intermediary could invest an effort $e$ in the following way.

$$s = \left(\bar{s} - \frac{e}{\rho}\right)^2$$

where $\bar{s}$ is the initial consumer search cost before the investment and $\rho$ is the parameter which measures the efficiency of its effort. When $\rho$ is large, the efficiency of search cost reduction is low. The investment of search reduction by the intermediary takes the form $C_e = fe^2$, where the marginal cost is positively related to parameter $f$ and the total cost is strictly increasing and convex in the effort level $e$. Thus the intermediary’s profit in the two-sided platform business mode could be written as:

$$\pi^i_p = p_s * Q(\hat{p}) - f \rho^2 (\bar{s} - \sqrt{s})^2$$  \hspace{1cm} (7)$$

where $\hat{p}$ is the price index and $Q(\hat{p}) = a - b\hat{p}$.

With the symmetric equilibrium assumption, the equilibrium prices of all firms are exactly the same. Correspondingly, without loss of generality we let $\hat{p} = p^*$. In this paper, we are only interested in the scenario when the fixed cost is approximately zero hence the number of brands $n$ is very large, like what could be found in most current online intermediary websites. According to Wolinsky (1986), even when the production fixed cost of firms is quite small
thus the number of firms is large, the model does not approach to perfectly competitive market and it still reserves monopolistic competition characteristics. This is caused by the fact that consumers’ imperfect information reduces the effective substitutability among brands. In this setting, the equilibrium price $p^*$ converges to $c + p_s + \sqrt{2s\mu\bar{\nu}}$ as $n \to \infty$. Correspondingly, the intermediary’s profit function can be rewritten as

$$\pi_p = p_s[a - b(c + p_s + \sqrt{2s\mu\bar{\nu}})] - f \rho^2(\bar{s} - \sqrt{s})^2$$

(8)

Note that although we assume the intermediary first chooses the optimal consumer search cost $s$ and then decides the listing fee $p_s$ it charges for each unit sold, the results do not change if we assumed a simultaneous choice of effort and fee. However, we have in mind a setup where the design of the platform is chosen upfront and remains fixed when the intermediary chooses the listing fee. The following proposition gives us the optimal choice of the intermediary regarding the listing fee and search cost.

**Proposition 1** Assume that the following conditions on the parameters hold: $\bar{s} > \frac{a-bc}{2\sqrt{b}b^2}$, $\mu \leq \frac{2f\rho^2}{\bar{s}}$. Then, the optimal listing fee $p_s^*$ chosen by the intermediary in the two-sided platform business mode is

$$p_s^* = \frac{a - bc - b\sqrt{2s\mu\bar{\nu}}}{2b}$$

(9)

and, the optimal search cost $s^*$ is

$$s^* = \left(\frac{4\bar{s}f\rho^2 - \sqrt{2\mu\bar{\nu}(a - bc)}}{4f\rho^2 - 2b\mu\bar{\nu}}\right)^2$$

(10)

**Proof.** We obtain $p_s^*$ and $s^*$ by the method of backward induction. We first obtain the first order condition of the optimal listing fee and set it equal to zero. Correspondingly we get

$$p_s^* = \frac{a - bc - b\sqrt{2s\mu\bar{\nu}}}{2b}$$

then we plug the optimal listing fee $p_s^*$ into the intermediary’s profit and derive

$$\pi_p = \frac{(a - bc - b\sqrt{2s\mu\bar{\nu}})^2}{4b} - f \rho^2(\bar{s} - \sqrt{s})^2$$

Thus we have

$$\frac{\partial \pi_p}{\partial \sqrt{s}} = -\frac{\sqrt{2\mu\bar{\nu}}}{2} \left(a - bc - b\sqrt{2s\mu\bar{\nu}}\right) + 2\bar{s}f\rho^2 - 2f \rho^2 \sqrt{s}$$

To guarantee the concavity of the profit function, we need to have $\mu \leq \frac{2f\rho^2}{\bar{s}}$. Hence the
optimal search cost in the two-sided platform business mode is

\[ s^* = \left( \frac{4s fp^2 - \sqrt{2\mu \bar{v}(a - bc)}}{4f p^2 - 2b\mu \bar{v}} \right)^2 \]

and we need to have \( \bar{s} > \frac{3(a - bc)\sqrt{2\bar{v}(\max\{\mu\})}}{8f p^2} \) to guarantee the optimal search to be positive.

4.2 Comparative statistics

For comparative statics, we have several important find outs. Among them, we are especially interested in how the intermediary’s strategic search design changes with respect to \( \mu \) that we interpreted as a measure of product diversity in the platform. This definition follows [Anderson & Renault, 1999] which introduces product differentiation via parameter before the random term. The following corollary shows one of our main results in the two-sided platform business mode: the equilibrium search cost is not monotonic in the product variety in the two-sided platform business mode.

**Corollary 1** Under the conditions of Proposition 1, the equilibrium search cost and the equilibrium listing fee in the two-sided platform business mode are unique. Furthermore,

1. there exists some value \( \hat{\mu} \in [0, \frac{2fp^2}{bs}] \) that the optimal search cost, \( s^* \) is decreasing for \( \mu \leq \hat{\mu} \), and increasing otherwise.

2. the equilibrium listing price \( p^* \) is decreasing in \( \mu \).

3. the equilibrium product price \( p^* \) is increasing in \( \mu \).

**Proof.** Please see the Appendix.

Corollary 1 shows that in the two-sided platform business mode the effect of product diversity on the search design is not monotonic. When products on the platform are not too differentiated the equilibrium search cost is decreasing in product diversity, however, when products on the platform are too differentiated the equilibrium search cost is strictly increasing in product diversity.

This is because when the product diversity is not too high, sellers become less competitive towards each other as their products become more differentiated. Responding to this, each seller tends to charge a higher price and hurts the aggregate demand consequently. However, in the two-sided platform business mode the intermediary’s revenue depends heavily on the number of consumers who adopt the intermediary. It charges sellers based on the product quantity sold through the platform, not the product price or total revenue. In order to weak this effect, the intermediary needs to lower its search cost on the platform, motivates consumers to search more and firms to charge a lower product price. When products
are sufficiently differentiated, changes in consumers’ search cost may no longer be able to influence consumers’ search activity that much due to the huge difference between products. Consequently it might be optimal for the intermediary to increase the search cost and correspondingly save investments on search design. The product price is influenced by a combination of two effects. On one hand, increasing product variety diminishes competitions among sellers and motives them to charge a high product price. On the other hand, decreasing listing fee and decreasing search cost may move the product price in the opposite direction. Although those two effects work oppositely, the product diversity effect is more dominate in the two-sided platform business mode. One main reason for the dominant product diversity effect is that each seller in the two-sided platform business mode cares about its individual demand only and has no interest in the aggregate demand of the intermediary. As a result, higher differentiated products enable each seller more monopolistic power thus moves them to higher product price.

We also check how the equilibrium changes with respect to the marginal cost and the efficiency of the search investment. Note that in our model, a large value of \( \rho \) implies a low efficiency in the search reduction while a high value of \( f \) indicates a high marginal cost for per unit of effort. Corollary 2 characterizes the equilibrium when the search reduction technology changes. This could be easily predicted: when the initial search cost is not too low, either an decrease in the efficiency or an increase in the marginal cost of search investment makes the search cost reduction more expensive thus decreases the intermediary’s optimal search cost.

**Corollary 2** Under the conditions of Proposition 1 the optimal search cost, \( s^* \) is decreasing in \( \rho \) and \( f \) as long as \( \bar{s} \geq \frac{\sqrt{2(a-bc)}}{2b \sqrt{f/a}} \).

**Proof.** Please see the Appendix.

5 Merchant mode

5.1 Equilibrium analysis

In this section we analyze the case that the intermediary runs in the merchant business mode. In the merchant mode, the intermediary buys sellers’ products and resells the goods to consumers for an individual price that it chooses. Firstly we study its product price and then establish the optimal platform design chosen by the intermediary in the merchant mode.

Firms in the merchant mode can either sell their products through the intermediary or exit the market. The intermediary offers a buyout bid \( \beta_i \) for product \( i \) and firm \( i \) is willing to sell its product to the platform as long as \( \beta_i \) is no less than its marginal cost, \( c \). Hence with the assumption of Hagiu (2007) that all bargaining power lies within the intermediary, the marginal cost of each product in the merchant mode exactly equals to the firms’ marginal
The intermediary’s game in the merchant business mode could be viewed as follow: the intermediary buys products from sellers at the marginal cost first, then it chooses its optimal search design. After that, it maximizes the total profit with respect to product price. The same as in the two-sided platform business mode, we are interested in the symmetric equilibrium where all products are sold at \( \tilde{p} \). Correspondingly, without loss of generality we have price index \( \hat{p} \) equals to price \( \tilde{p} \). Thus we could derive the profit of the intermediary in the merchant mode as:

\[
\pi^m_p = n(\tilde{p} - c)D(p, \tilde{p}, n) - f \rho^2(\overline{s} - \sqrt{s})^2
\]

where the first term is the sum of profits of \( n \) products and the second term is the search investment. With the set up, we derive the equilibrium product price and the optimal search cost listed in the following proposition.

**Proposition 2** The optimal price chosen by the intermediary in the merchant mode is

\[
\tilde{p} = c + \sqrt{2s\mu\overline{v}},
\]

and, the optimal search cost chosen by the intermediary is

\[
\tilde{s} = \left( \frac{2\overline{s}f\rho^2 + (a - bc)\sqrt{2s\mu\overline{v}}}{2(f\rho^2 + 2b\mu\overline{v})} \right)^2
\]

**Proof.**

Following the same method as in the two-side platform business mode, we get the equilibrium price that equals:

\[
\hat{p} = c - \frac{D(\hat{p}, \hat{p}, n)}{D_p(\hat{p}, \hat{p}, n)}
\]

where \( D_p = \frac{\partial D}{\partial p} \).

Given the assumption that \( v \sim U[0, \tilde{v}] \) we have

\[
\hat{p} = c + \frac{\mu(\overline{v} - w^*)}{1 - (w^*/\overline{v})^n}
\]

where \( w^* = \overline{v} - \sqrt{2s\mu\overline{v}} \).

When \( n \) is sufficiently large, the equilibrium price \( \hat{p} \) converges to \( c + \sqrt{2s\mu\overline{v}} \) as indicated in the baseline model. Correspondingly, we have the platform’s profit function

\[
\pi^m_p = \sqrt{2s\mu\overline{v}}[a - b(c + \sqrt{2s\mu\overline{v}})] - f \rho^2(\overline{s} - \sqrt{s})^2
\]

\(^6\)With the symmetric assumption, maximizing the sum of profits of \( n \) products with respect to product price is equivalent to maximizing profits of one individual product in the merchant mode.
We then optimize \( \tilde{s} \) using the first order condition and get

\[
\tilde{s} = \left( \frac{2\tilde{s}f\rho^2 + (a - bc)\sqrt{2\mu\bar{v}}}{2(f\rho^2 + 2b\mu\bar{v})} \right)^2
\]

\[\blacksquare\]

### 5.2 Comparative statistics

For comparative statics, we have several important find outs. Among them, we are especially interested in how the intermediary’s strategic search design changes with respect to product diversity. The following corollary shows one of our main results in the merchant business mode: both the equilibrium search design and the optimal product price are not monotonic in product diversity.

**Corollary 3** Under the conditions of Proposition 4, the equilibrium search cost and the equilibrium product price in the merchant business mode are unique. Furthermore,

1. there exists some value \( \hat{\mu} \in [0, \frac{2f\rho^2}{3\sqrt{b}\rho^2}] \) that the optimal search cost, \( \tilde{s} \) is increasing for \( \mu \leq \hat{\mu} \), and decreasing otherwise.

2. if the initial search cost \( \frac{(a - bc)}{2\sqrt{b}/\rho^2} < \tilde{s} < \frac{2(a - bc)}{3\sqrt{b}/\rho^2} \), then the optimal product price, \( \tilde{p} \) always is increasing.

3. if the initial search cost \( \tilde{s} \geq \frac{2(a - bc)}{3\sqrt{b}/\rho^2} \), then there exist some value \( \hat{\mu}_p \) that the optimal product price, \( \tilde{p} \) is increasing for \( \mu \leq \hat{\mu}_p \), and decreasing otherwise.

**Proof.** Please see the Appendix. \[\blacksquare\]

Corollary 3 shows that in the merchant business mode the effect of product diversity on the optimal search design is not monotonic. When products on the platform are not too differentiated, the optimal search cost is strictly increasing in product diversity. However it is the opposite when the products are sufficiently differentiated.

When products are not too differentiated, increasing product diversity decreases the aggregate demand. To compensate for the loss of demand the intermediary increases search cost, makes products less competitive and charges a higher price. The opposite reaction of the intermediary in the merchant mode is caused by the fundamental difference between two business modes: In the merchant business mode, the intermediary makes money by selling products directly to consumers instead of charging each seller a listing fee based on quantity sold. As a result, unlike in the two-sided platform business mode where the high price goes to the sellers, in the merchant business mode the high price goes to the intermediary in the form of revenue. Thus with a higher diversity it might be more profitable for the intermediary to
increase its search cost. In this way, it could save the search cost investment and at the same time give each product more monopolistic power thus charge a higher price. When products are sufficiently differentiated and the product price is very high. The price elasticity of the demand becomes quite large thus further increasing product price may hurt the intermediary’s profit. To avoid this situation, the intermediary needs to lower its search cost, move price downward and encourage more demand. The optimal price is influenced by a combination of two effects. On one hand, increasing product variety enables the intermediary to charge a higher product price. However, when product diversity is sufficiently high, a decrease in the optimal search cost might move the product price in the opposite direction. Based on our previous analysis, when products are not too differentiated and the demand is relatively inelastic, an increase in the product diversity moves the product price up. However, when the product diversity is sufficiently high, large elasticity will motive intermediary to lower the product price on the contrary.

We also check how the equilibrium changes with respect to the marginal cost and the efficiency of search investment. Note that in our model, a large value of $\rho$ implies a low efficiency in the search reduction while a high value of $f$ indicates a high marginal cost for per unit of effort. Corollary 4 characterizes the equilibrium when the search reduction technology changes. This could be easily predicted: when the initial search cost is sufficiently high, both decreasing efficiency and increasing marginal cost of the search investment make search cost reduction more expensive thus lower intermediary’s optimal search cost.

**Corollary 4** Under the conditions of Proposition 7, the optimal search cost $\tilde{s}$ is decreasing in $\rho$ and $f$ as long as $\tilde{s} \leq \frac{(a-bc)}{2\sqrt{df}\sqrt{mp}}$.

**Proof.** Please see the Appendix.

6 Comparison between two business modes

In the previous sections, we observe that the intermediary could derive different strategies depending on which business mode it runs in. This is because in different business modes, the intermediary makes revenue from different sources and has different control over the platform market. In the two-sided platform business mode, the intermediary makes money by charging a fixed amount of listing fee per unit and leaves all market control including advertising, marketing and pricing to sellers. However, in the merchant mode the intermediary remains full control over the market and sells products directly to consumers. In this section we compare the equilibrium outcomes of the two-sided platform business mode and that of the merchant business mode and analyze the underlying mechanisms that could generate the huge differences. The next proposition concerns the comparison of the equilibrium search cost between the two-sided platform business mode and the merchant business mode.
Figure 2: Search Cost in two-sided platform business mode and in merchant mode

![Optimal Search Cost](image)

**Proposition 3** Under the assumptions of Proposition 1, there exists some value $\mu_1 \in [0, \frac{2f^2}{\sigma^2}]$ such that the intermediary’s optimal consumer search cost in the merchant mode is larger than that in the two-sided platform business mode if $\mu \in [0, \mu_1]$, while the intermediary’s optimal consumer search cost in the two-sided platform mode is larger than that in the merchant business mode when $\mu \in [\mu_1, \frac{2f^2}{\sigma^2}]$.

**Proof.** See the Appendix. ■

From proposition 3 we have several interesting findings. Firstly, it seems that there is no motivation for the intermediary to invest on its search cost in both business modes when products are exactly the same. When products are identical, changes in its search cost do not affect consumers’ behavior. After consumers adopt the platform, they know that all products on the platform give them the same utility thus each consumer searches only once and buys the first product directly. Secondly, we find that when products are not identical while not too differentiated, it is optimal for the intermediary in the merchant business mode to have a higher consumer search cost than the intermediary in the two-sided platform business mode. This is because compared to the intermediary in the two-sided platform business mode, the intermediary in the merchant business mode runs like a big retailer and has full...
control over the market. However, the intermediary in the two-sided platform does not have any control over the product price while its aggregate demand depends heavily on that. Thus when products are not identical while not too differentiated the intermediary in the two-sided platform business mode needs to have a lower search cost to weak the adverse effect of product diversity on the demand. The revenue of the intermediary in the merchant business mode depends on both the product price and the demand. When products are not identical while not too differentiated and the demand is relatively inelastic, it is optimal for the intermediary in the merchant business mode to have a higher search cost which could lower competitions and increase product price. Thus when the product diversity is not too high, the intermediary in the merchant mode offers a higher search cost for its consumers than in the two-sided platform business mode. However, it is the opposite when products are sufficiently differentiated. When products are too differentiated, changes in the search cost can no longer affect demand much but high elasticity makes the demand quite sensitive to the changes in price. For the intermediary in the two-sided platform business mode who has no control over the product price, a high search cost does not decrease its demand much while can bring a large amount of saving in its investment. However, for the intermediary in the merchant mode although a high search cost may not hurt the demand much directly, its indirect effect on the demand through its product price price could be detrimental. Consequently, the intermediary in the merchant mode optimizes its search design by having a smaller search cost than in the two-sided platform business mode.

**Proposition 4** Under the assumptions of Proposition 7

1. there must exist some value $\bar{\mu}_t \in [0, \frac{2f^2}{b\phi}]$ such that the profit of the intermediary in the two-sided platform business mode is decreasing in product diversity when $\mu \in (0, \bar{\mu}_t)$ and the opposite otherwise.

2. there must exist some value $\bar{\mu}_m \in [0, \frac{2f^2}{b\phi}]$ such that the profit of intermediary in the merchant business mode is increasing in product diversity when $\mu \in (0, \bar{\mu}_m)$ and the opposite otherwise.

3. there must exist some interval $(\bar{\mu}_{pr}, \tilde{\mu}_{pr}) \subseteq [0, \frac{2f^2}{b\phi}]$ such that the intermediary in the merchant mode is more profitable when $\mu \in (\bar{\mu}_{pr}, \tilde{\mu}_{pr})$ and the opposite otherwise.

**Proof.** See the Appendix. ■
Besides the intermediary’s search design, we are especially interested in the question which business mode could bring more profit to the intermediary? To answer this question, we first investigate the intermediary’s profit functions in the two-sided platform business mode and in the merchant business mode one by one and then we compare them together. We find that in both business modes, the effect of product diversity on the profit is not monotonic. In addition, it could change the intermediary’s profit oppositely depending on which business mode that the intermediary runs in. In the two-sided platform business mode when products are not too differentiated, increasing product diversity decreases the platform’s profit. It is due to the fact that when products are not sufficiently differentiated, an increase in product diversity decreases the demand. Thus this forces the intermediary to spend more money on the search investment and hurts its profit at first. However, when products are too differentiated and each product has more monopolistic power, both the changes in product diversity and search cost can have quite minor effects on the demand. Consequently, the intermediary decreases its investment on the search cost reduction and this improves its profit. In the merchant mode, when products are not too differentiated an increase in the product diversity weaks competition and enables each product more monopolistic power. Consequently, more differentiation between products helps the intermediaries to achieve a higher profit level in the merchant mode where the intermediary has full control over price. However, when products are sufficiently differentiated, the demand become quite elastic in product price. Consequently, further increasing product diversity motivates the intermediary to invest heavily on the search cost reduction and hurts the intermediary’s profit.
From the previous analysis of the optimal search cost, we know that when products are exactly the same the intermediary in either the two-sided platform business mode or the merchant business mode has no motivation to invest on its search cost reduction. With the same initial search cost, the intermediary in the two-sided platform business mode is more profitable. This is because when products are identical, the market on the platforms is similar to a perfect competitive market. In a perfectly competitive market, product price is relative low, quantity is relative high and there is little profit for each product. This implies that the intermediary in the merchant mode who acts like a big retailer is less profitable when products are exactly identical. However, the intermediary in the two-sided platform mode charges a listing fee from each seller which is equivalent to an increase in each seller’s marginal cost. Although each seller may have little profit in a perfectly competitive market, the profit of the intermediary in the two-sided platform business mode can still maintain a higher profit level than in the merchant business mode. Those features of perfectly competitive market still exist but are diminishing when products become more differentiated. When the product diversity reaches some level, the advantage of the merchant mode begins to appear. In the merchant mode, the intermediary has full market controls. Unlike the two-sided platform mode where individual seller determines the product price, the intermediary in the merchant mode could internalize underlying substitute and complement relationships between products. Thus when product diversity is high while not too high, the merchant mode becomes more desired because of its control over the whole market. However when products are too differentiated, the ability of the merchant mode to solve price distortion disappears and on the contrary its control over the market makes it more costly to run. When products are too differentiated, the intermediary in the two-sided platform who has less control does not need to worry about the indirect effect of search design on the demand through its price. As a result, the intermediary in the two-sided platform business mode is able to decrease its investment on the search cost and reaches a higher profit than the merchant mode. In summary, when the product diversity is extremely high or extremely low, it is the two-sided platform business mode that could bring to the intermediary more profit. However, if the product diversity is the median of two extreme, it is the merchant business mode that could bring more profit to the intermediary.

7 Binding Constraint

In the previous theoretical models, we focus on the scenario where the maximum value of the random utility term $v_{ij}$ is so large such that the search cost in our model can always motivate consumers to do some search at the symmetric equilibrium. In this section, we discuss in detail about the binding consumer search cost constraint in the intermediary.

As noted previously, the reservation value plays a cardinal role in our model. It not only
Figure 3: search cost with binding restriction

affects the probability that any given consumer goes on searching after visiting a firm [Anderson & Renault, 1999] but also influences the intermediary’s search design and price strategies. The larger the reservation value is, the higher possibility that the consumer continues searching thus it is less likely for it to stop and choose the product. However, according to the definition of the reservation value, its value is largely determined by the search cost of the intermediary. Consumer’s reservation value is quite large when the search cost is small and vice versa. If the search cost of the intermediary is zero, the reservation value approaches to the maximum value of consumers’ utility random term thus each consumer would keep searching until it finds the produce with the highest value $\bar{v}$. However, if the search cost of the intermediary is too large, then it is possible that the reservation value defined in equation(1) becomes even smaller than the minimum random utility term. For example if the search cost $s$ is bigger than $\mu \bar{v}$ then consumers have no incentive to search beyond the first product since the largest possible utility increment can not cover the search cost. As stated in section 3, this implies that consumers have no motivation to search thus they always stop at the first product. Consequently, each firm could always charge a sufficiently high price and moves the aggregate demand decreases to zero. Clearly, those trivial equilibria with binding constraint are not our focus in this study and we don’t expect to see those scenarios in real market.
In reality, each intermediary will control its platform to make sure its aggregate demand to be positive thus we assume the intermediary only runs in the unbinding product diversity interval. In fact, we can find that there exist a unique maximum search cost $\hat{s}$ beyond which consumers stop searching. Specifically, we define the upper bound of the search cost $\hat{s}$ as the largest expected utility increment $\mu \int_0^{\hat{s}} (v - 0)g(v)dv$. It could be further simplified as $\frac{\hat{s}}{2}\mu$ which is a linear function with respect to $\mu$. If the intermediary’s search cost is higher than that, then even when the consumer get the worst product the expected increment in utility of one additional search won’t offset the search cost.

With the restriction, we should keep in mind that empirically we may not be able to see the relationships listed in section 4 and 5. This is because in reality, we are only able to observe the unbinding interval of $\mu$ in the intermediary. With the consideration of the search cost restriction, the possible scenarios in each business mode could be summarized as follows. In the two-sided platform business mode, we could have the following possible scenarios as showed in figure 3:

$(T_1)$ If the slope of the search restriction ($\frac{\hat{s}}{2}$) is so small such that there is no intersection or only one intersection between $\hat{s}$ and the equilibrium search cost $s^*$ in the two-sided platform business mode, then the intermediary has no motivation to invest on search cost reduction and firms always charge a infinite price.

$(T_2)$ If the slope of the search restriction $\frac{\hat{s}}{2}$ is so large such that there are two intersections $\mu_1^* < \mu_2^*$ between $\hat{s}$ and the equilibrium search cost $s^*$ in the two-sided platform business, then the equilibrium search cost $s^*$ is decreasing in $[\mu_1^*, \hat{s}]$ and is increasing in $[\hat{s}, \mu_2^*]$. Consequently, the product price is strictly increasing in the unbinding interval $[\mu_1^*, \mu_2^*]$.

Unlike the two-sided platform business mode, the product price is not monotonic in product diversity in the merchant mode. According to the previous analysis, the product price at the symmetric equilibrium equals $c + \sqrt{2\bar{v}\mu\hat{s}}$. Consequently, the threshold $\hat{\mu}$ at which the product price begins to decrease in product diversity is bigger than the threshold $\hat{\mu}$ at which the equilibrium search cost begins to decrease in product diversity. Thus in the merchant business mode, we could have the following possible scenarios as showed in figure 3:

$(M_1)$ If the intersection $\mu_m$ between the search restriction $\hat{s}$ and the equilibrium search cost is no less than $\hat{\mu}$, then the intermediary’s equilibrium search cost and product price both are strictly decreasing in product diversity in the unbinding interval $[\mu_m, 2\bar{v}\mu^2]$.

$(M_2)$ If the intersection $\mu_m$ between the search restriction $\hat{s}$ and the equilibrium search cost is in the interval $[\hat{\mu}, \hat{\mu}]$, then the intermediary’s equilibrium search cost is decreasing in product diversity in the unbinding interval $[\mu_m, 2\bar{v}\mu^2]$ while the product price is not monotonic. It is increasing in product diversity when $\mu \in [\mu_m, \hat{\mu}]$ and is decreasing when $\mu \in [\hat{\mu}, 2\bar{v}\mu^2]$. 

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(M₃) If the intersection $\mu_m$ between the search restriction $\hat{s}$ and the equilibrium search cost is less than $\hat{\mu}$, then both the intermediary’s equilibrium search cost and the product price are not monotonic in product diversity in the unbinding interval $[\mu_m, \frac{2}{b0} \nu^2]$. The search cost is increasing in product diversity when $\mu \in [\mu_m, \hat{\mu}]$ and is decreasing when $\mu \in [\hat{\mu}, \frac{2}{b0} \nu^2]$, the product price is increasing in product diversity when $\mu \in [\mu_m, \hat{\mu}_p]$ and is decreasing when $\mu \in [\hat{\mu}_p, \frac{2}{b0} \nu^2]$.

In summary, empirically we expect to see one of the above scenarios to happen. In the two-sided platform business mode, the product price should be always increasing in product diversity while the search cost decreases in product diversity when product diversity is small and increases in product diversity when product diversity is large. In the merchant business mode, there are three possible scenarios. One possibility is that both the product price and the search cost may always be decreasing in product diversity in the unbinding interval. Second possible scenario is that the search cost is decreasing in product variety and the product price is increasing in product diversity when $\mu$ is relatively small and is decreasing when $\mu$ is relative large. Lastly, it may be the case that both the search cost and the product price are increasing in product diversity when $\mu$ is relatively small and are decreasing when $\mu$ is relatively large.

8 Empirical Study

In this section we use unique Aliexpress dataset and Sam’s Club dataset to present some suggestive evidence of our theoretical predictions. As analyzed in section 7, we expect that empirically the intermediary only runs in the product diversity interval during which the search cost restriction is not binding. In this context, we find that the search cost in the two-sided platform business mode is decreasing in the product diversity when the product diversity is low and is increasing in the product diversity when the product diversity is high as predicted in property $T_2$, while the search cost in the merchant business mode is always decreasing in the product diversity as predicted in property $M_2$. In addition, we also observe a non-monotonic relationship between the product price and the product diversity in the merchant mode.

Our data is scraped from two professional online shopping platforms in US: Aliexpress and Sam’s Club from September, 2017 to December, 2017. Aliexpress is a B2C platform in US that was launched by Alibaba in 2010. Different to several online shopping platforms like Neiman Marcus, Walmart, Target and Amazon, it does not have any self-branded products thus looks like an intermediary in the two-sided platform business mode. Our Aliexpress data comes from ten major categories listed on Aliexpress’s homepage including cellphones& telecommunications, consumer electronics, home& garden, jewelry& accessories, luggage& bags, men’s clothing, mother& kids, computer & office, shoes and women’s clothing. In each
Table 2: Summary Statistics of Aliexpress Search Outcomes

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<td>0.70</td>
<td>3307.57</td>
<td>51.69</td>
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<td>average product rating</td>
<td>4366</td>
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<td>94.56</td>
<td>96.58</td>
<td>95.28</td>
<td>97.97</td>
<td>100.00</td>
<td>5.95</td>
</tr>
<tr>
<td>average number of review</td>
<td>4366</td>
<td>1.00</td>
<td>8.36</td>
<td>34.38</td>
<td>159.82</td>
<td>153.78</td>
<td>11698.00</td>
<td>384.89</td>
</tr>
<tr>
<td>ratio of top rated seller</td>
<td>4985</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.13</td>
<td>0.17</td>
<td>1.00</td>
<td>0.21</td>
</tr>
<tr>
<td>number of search outcomes</td>
<td>5003</td>
<td>0</td>
<td>11</td>
<td>122</td>
<td>18387</td>
<td>1996</td>
<td>18450422</td>
<td>307857.40</td>
</tr>
<tr>
<td>number of filter</td>
<td>5003</td>
<td>0.00</td>
<td>16.00</td>
<td>58.00</td>
<td>82.01</td>
<td>126.00</td>
<td>574.00</td>
<td>84.01</td>
</tr>
</tbody>
</table>

category, we search different combinations of keywords and for each search we collect information regarding the search outcomes that are ranked by Aliexpress’s default ranking on the first page. The keywords are found on Aliexpress’s webpage and are chosen randomly. Specifically, for each search we have the total number of search outcomes, number of keywords used in the search, keywords used for the search, category tag, average price of the products, average quantity sold, average shipping fee from China to US, average rating of the products, average number of people who rated the products and average ratio of top rated sellers. In total, we have over 50,000 observations in Aliexpress data set whose main statistics could be summarized as in table 2.

Table 3: Summary Statistics of Sam’s Club Search Outcomes

<table>
<thead>
<tr>
<th>Variable</th>
<th>obs</th>
<th>Min</th>
<th>P25</th>
<th>Median</th>
<th>Mean</th>
<th>P75</th>
<th>Max</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>keyword numbers</td>
<td>1592</td>
<td>0.00</td>
<td>3.00</td>
<td>4.00</td>
<td>3.70</td>
<td>4.00</td>
<td>7.00</td>
<td>1.03</td>
</tr>
<tr>
<td>average price</td>
<td>1271</td>
<td>1.00</td>
<td>25.02</td>
<td>62.45</td>
<td>217.73</td>
<td>207.26</td>
<td>4275.35</td>
<td>438.94</td>
</tr>
<tr>
<td>average number of review</td>
<td>1314</td>
<td>1.00</td>
<td>6.14</td>
<td>16.43</td>
<td>109.34</td>
<td>49.68</td>
<td>10265.76</td>
<td>512.92</td>
</tr>
<tr>
<td>number of search outcomes</td>
<td>1592</td>
<td>0.00</td>
<td>10.00</td>
<td>27.00</td>
<td>167.02</td>
<td>66.25</td>
<td>56795.00</td>
<td>1564.86</td>
</tr>
<tr>
<td>number of filter</td>
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<td>0.00</td>
<td>22.00</td>
<td>32.00</td>
<td>36.26</td>
<td>56.00</td>
<td>607.00</td>
<td>26.72</td>
</tr>
</tbody>
</table>

On the contrary to Aliexpress, Sam’s Club, which is the 8th largest retailer in US, does not allow individual seller to sell directly to consumers on the website. All the items listed on Sam’s Club are private labeled and are sold from Sam’s club to its consumers without involving producers. In addition, Sam’s Club is responsible for every aspect of the market including customer service, legal problem, advertising and so on. All these characteristics imply that Sam’s Club runs in the pure merchant business mode. Our Sam’s Club data comes from eleven major categories listed on Sam’s Club’s homepage including grocery, pharmacy& health & beauty, baby & kids, jewelry & flower & gifts, auto & tires, electronic, home and appliances, toys and games, furniture & mattresses, sports & fitness and outdoor
& patio & garden. In each category, we search different combinations of keywords and for each search we collect information regarding the search outcomes that are ranked by Sam’s Club’s default ranking on the first page. The keywords are found on Sam’s Club’s webpage and are chosen randomly. Because of its merchant business mode features, Sam’s Club has much less products on its website and for each product it offers less information compared to the two-sided platform websites like Aliexpress. For example, since Sam’s club only sells its self-labeled products on its website, it does not need to offer any rating or other detail information regarding the producers. In addition, its total number of available products is much less than the e-commerce platform due to its concentrated control rights. Based on the availability of data, for each search we collect the total number of search outcomes, the number of keywords used in the search, keywords used for the search, category tag, product average price, and average rating of the products. In total, we have over 10,000 observations in Sam’s Club dataset with main statistic summary as showed in table 3.

In the empirical work, we use the number of outcomes of each search to measure the product diversity and use the number of available website search filters on the website to measure the search cost. The intuitions behind these are that we expect a higher product diversity when there are more products available on the website and consumers concur a smaller search cost when there are more search aids available on the website. Our empirical model follows the same method as Zhong (2016) which “includes the category fixed effect to control for the differences in market sizes across different categories and takes the log to account for its skewness which greatly increases the model fit”.

We first check the correlation between the product variety and the search cost on the websites of Aliexpress and of Sam’s Club. Our theoretical model predicts that a higher product diversity should generate a lower (higher) equilibrium search cost when products are not too differentiated while it should generate a higher (lower) equilibrium search cost when products are sufficiently differentiated in the two-sided platform (merchant) business mode. With the consideration of search restriction, it is expected to find properties $T_2$ on Aliexpress’s website and find one among the three properties $M_1$, $M_2$ and $M_3$ on Sam’s Club’s website. Figure 4 shows the relationship between the equilibrium search cost and the product diversity on Aliexpress’s website and on Sam’s club’s website. As predicted by our model, on Aliexpress’s website when its product diversity is not too high, an increase in its product diversity

---

7Defined by the negative of the residual of log number of search filters regressing on review number and category fixed effect on Sam’s Club’s club, defined by the negative of the residual of log number of search filters regressing on average ratings, average number of review, ratio of top rated seller and category fixed effect on Aliexpress’s website

8Defined by the residual of log number of search outcomes number regressing on review number and category fixed effect on Sam’s Club’s website, defined by the residual of log number of search outcomes number regressing on average ratings, average number of review, ratio of top rated seller and category fixed effect on Aliexpress’s website

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diversity result in a lower search cost, while when the product diversity is sufficiently high the opposite is true; on Sam’s Club, an increase in its product diversity result in a lower search cost as predicted in $M_2$.

We also check the correlation between the product variety and the product price on Aliexpress’s website and on Sam’s Club’s website. According to our theory, the intermediary in the pure two-sided platform business mode should have a product price that is increasing in its product diversity, the intermediary in the pure merchant mode should have a product price that is increasing in its product diversity when products are not too differentiated while decreasing in its product diversity when products are sufficiently differentiated. It is clear indicated in figure 5 that the relationship between the product price\(^9\) and the product diversity\(^10\) on the website of Sam’s club shows strong merchant business mode features. As

\(^9\)Defined by the residual of log price regressing on review number and category fixed effect on Sam’s Club’s website, defined by the residual of log price regressing on average ratings, average number of review, ratio of top rated seller and category fixed effect on Aliexpress’s website

\(^10\)Defined by the residual of log number of search outcomes number regressing on review number and category fixed effect on Aliexpress’s website
predicted in our theoretical model, on Sam’s club’s website when its product diversity is not too high, an increase in its diversity results in a higher product price, while when product diversity is sufficiently high the opposite is true. On Aliexpress’s website, we find its price strategy is more similar to the merchant business mode than the two-sided platform business mode. This is because although Aliexpress runs in the two-sided platform business mode, its real listing fee policy is quite different to our theoretical model. Specifically, due to the following reasons its pricing policy is expected to have the characteristics of merchant business mode instead of the two-sided platform business mode. First of all, according to the definition of "pure" two-sided platform business mode by [Hagiu & Wright, 2014], all control rights of the intermediary should rest within independent suppliers in the two-sided platform business mode. Although Aliexpress does not claim legal "possession" of products on its website, it still maintains control over the market including but not limited to advertising, customer fixed effect on Sam’s Club’s website, defined by the residual of log number of search outcomes number regressing on average ratings, average number of review, ratio of top rated seller and category fixed effect on Aliexpress’s website.
service and responsibility for order fulfillment. Secondly, the listing fee policy on Aliexpress is a fixed percentage of individual seller’s total revenue rather than a fixed amount listing fee per unit sold through the website. However, in our model we assume that the intermediary in the two-sided platform business mode only charges a fixed amount of listing fee for each unit sold through the platform, which implies that the intermediary’s revenue depends heavily and directly by quantity only. This huge difference between two listing fee policies makes the pricing policy on website have less two-sided platform business mode features while more similar to a big retailer in the merchant business mode. Due to above facts, we expect that Aliexpress has the merchant mode features in its product pricing. In fact, our empirical results do show consistent evidence that the product price of Aliexpress indicates strong merchant business mode features as showed in figure 5.

In summary, our empirical analysis of the unique data set from Sam’s Club’s website and Aliexpress’s website shows strong supports to our theoretical models. We find the search cost on Aliexpress’s website is not monotonic: it is decreasing in product diversity when the product diversity on the website is low and is increasing in product diversity when the product diversity is high. We also observe a decreasing search cost on Sam’s Club’s website with respect to its product variety. In addition, consistent result regarding the product price could be found on Sam’s Club’s website: It is increasing in product diversity when products are not too differentiated while is decreasing in product diversity when products are too differentiated.
References


A Appendix

A.1 Proof of Corollary 1

To simplify our notation, we use \( \Delta_\mu \) to denote \( \sqrt{\mu} \). Notice that

\[
\frac{\partial s^*}{\partial \mu} = \frac{\sqrt{\bar{s}}}{\sqrt{\bar{s}}} \cdot \frac{\partial \sqrt{s^*}}{\partial \Delta_\mu}
\]

Thus \( \frac{\partial s^*}{\partial \mu} \) must have the same sign as \( \frac{\partial \sqrt{s^*}}{\partial \Delta_\mu} \). For simplicity, we analyze the derivative of \( \sqrt{s^*} \) instead of \( s^* \) with respect to \( \Delta_\mu \). Plugging \( \Delta_\mu \) into the optimal search cost in the two-sided platform mode we get

\[
\frac{4f\rho^2\bar{s} - (a - bc)\sqrt{2}\Delta_\mu}{4f\rho^2 - 2b\Delta_\mu^2}
\]

Hence we have

\[
\frac{\partial \sqrt{s^*}}{\partial \Delta_\mu} = -\frac{2\sqrt{2}(a - bc)\Delta_\mu^2 + 16f\rho^2bs\Delta_\mu - 4\sqrt{2}f\rho^2(a - bc)}{(4f\rho^2 - 2b\Delta_\mu^2)^2}
\]

Since the denominator is positive under the conditions of Proposition I, the sign of \( \frac{\partial \sqrt{s^*}}{\partial \Delta_\mu} \) must be the same as the concave function \( -2\sqrt{2}(a - bc)\Delta_\mu^2 + 16f\rho^2bs\Delta_\mu - 4\sqrt{2}f\rho^2(a - bc) \). The function has a negative value when \( \Delta_\mu = 0 \) and two positive solutions to \( \Delta_\mu^2 + 16f\rho^2bs\Delta_\mu - 4\sqrt{2}f\rho^2(a - bc) = 0 \). To see how does the value of the partial derivative change in \( \Delta_\mu \in [0, \sqrt{\frac{2f\rho^2}{b}}] \) that monotonically corresponds to \( \mu \in [0, \frac{2f\rho^2}{b}] \), we need to check its value when \( \Delta_\mu \) reaches the maximum value. Under the condition of Proposition I its value at \( \sqrt{\frac{2f\rho^2}{b}} \) is positive. Thus there must exist some \( \hat{\mu} \in [0, \frac{2f\rho^2}{b}] \) such that \( \frac{\partial \sqrt{s^*}}{\partial \Delta_\mu} \) is negative when \( \mu \in [0, \hat{\mu}] \) and positive when \( \mu \in [\hat{\mu}, \frac{2f\rho^2}{b}] \).

At the symmetric equilibrium \( p^*_s = \frac{a - bc - 2\sqrt{2}\bar{s}\mu}{2b} \), thus \( \frac{\partial p^*_s}{\partial \Delta_\mu} \) could be written as \( -\frac{\sqrt{2}}{2} \frac{\partial \Delta_\mu \sqrt{s^*}}{\partial \Delta_\mu} \). Hence the derivative of \( p^*_s \) with respect to \( \Delta_\mu \) equals

\[
\frac{\partial p^*_s}{\partial \Delta_\mu} = -\frac{4\sqrt{2}f\rho^2}{(4f\rho^2 - 2b\Delta_\mu^2)^2} \left( b\bar{s}\Delta_\mu^2 - \sqrt{2}(a - bc)\Delta_\mu + 2f\rho^2\bar{s} \right)
\]

Since the first term \( -\frac{4\sqrt{2}f\rho^2}{(4f\rho^2 - 2b\Delta_\mu^2)^2} \) on the right side is negative for certain, the sign of the first order derivative thus is determined by the convex function \( b\bar{s}\Delta_\mu^2 - \sqrt{2}(a - bc)\Delta_\mu + 2f\rho^2\bar{s} \) with respect to \( \Delta_\mu \). To derive the sign of the convex function, we need calculate the minimum value of the equation. If we plug in \( \Delta_\mu = \frac{\sqrt{2}(a - bc)}{2b\bar{s}} \) into the function, then due to the restriction of \( \bar{s} \) we get

\[
2f\rho^2\bar{s} - \frac{(a - bc)^2}{2b\bar{s}} > 0
\]

Thus \( p^*_s \) is strictly decreasing in \( \Delta_\mu \). Since \( \Delta_\mu \) is strictly increasing in \( \mu \), then \( p^*_s \) is strictly decreasing in \( \mu \). With the notation of \( \Delta_\mu \) the equilibrium product price \( p^* \) could be written
as
\[ p^* = c + \frac{a - bc}{2b} + \frac{\sqrt{2}}{2} \Delta_\mu \sqrt{s^*} \]

This implies that \( \frac{\partial p^*}{\partial \Delta_\mu} \) must have the opposite sign as \( \frac{\partial \sqrt{s^*}}{\partial \Delta_\mu} \). Thus \( p^* \) must be strictly increasing in \( \Delta_\mu \) thus is \( \mu \).

A.2 Proof of corollary 2

To see how does the optimal search cost \( s^* \) change if \( f \) or \( \rho^2 \), we only need to check \( \frac{\partial \sqrt{s^*}}{\partial \rho^2} \) since the position of \( f \) and \( \rho^2 \) is symmetric in \( s^* \). Since we could write \( \sqrt{s^*} \) as

\[ \sqrt{s^*} = \bar{s} + \frac{2b\mu \bar{s} - (a - bc)\sqrt{2\bar{\mu}}} {4f\rho^2 - 2b\bar{\mu}} \]

Thus if \( 2b\mu \bar{s} - (a - bc)\sqrt{2\bar{\mu}} > 0 \), then the optimal search cost \( s^* \) is strictly decreasing in \( f \) and \( \rho \) and increasing otherwise.

A.3 Proof of corollary 3

Notice that \( \frac{\partial \sqrt{s}}{\partial \rho} \) must have the same sign as \( \frac{\partial \sqrt{s}}{\partial \rho} \). For simplicity, we analyze the derivative of \( \sqrt{s} \) with respect to \( \sqrt{\mu} \). To simplify the notations, we let \( \Delta_\mu = \sqrt{\mu\bar{\mu}} \) and plug it into the \( \sqrt{s} \) in the merchant mode and get

\[ \frac{2\bar{s}f\rho^2 + (a - bc)\sqrt{2}\Delta_\mu} {2(f\rho^2 + 2b\Delta_\mu^2)} \]

Hence we have

\[ \frac{\partial \sqrt{s}}{\partial \Delta_\mu} = -4\sqrt{2}b(a - bc)\Delta_\mu^2 - 16f\rho^2b\bar{s}\Delta_\mu + 2\sqrt{2}f\rho^2(a - bc) \]

\[ 4(f\rho^2 + 2b\Delta_\mu^2)^2 \]

Since the denominator is positive under the condition of Proposition 1, the sign of \( \frac{\partial \sqrt{s}}{\partial \Delta_\mu} \) must be the same as the concave function \( -4\sqrt{2}b(a - bc)\Delta_\mu^2 - 16f\rho^2b\bar{s}\Delta_\mu + 2\sqrt{2}f\rho^2(a - bc) \). Its value is positive and strictly decreasing when \( \Delta_\mu \) equals 0 and it has only one positive solution when the concave function equals 0. To see how does the value of partial derivative change when \( \mu \in [0, \frac{2f\rho^2}{b}] \), we need to check its value when \( \mu \) reaches the maximum value. Under the condition of Proposition 1, its value at \( \frac{2f\rho^2}{b} \) is negative. Thus there must exist come \( \hat{\mu} \in [0, \frac{2f\rho^2}{b}] \) such that \( \frac{\partial \sqrt{s}}{\partial \mu} \) is positive when \( \mu \in [0, \hat{\mu}] \) and positive when \( \mu \in [\hat{\mu}, \frac{2f\rho^2}{b}] \).

With the optimal search cost \( \bar{s} \), the derivative of \( \bar{p} \) with respect to \( \Delta_\mu \) equals

\[ \frac{-8f\rho^2b\bar{s}\Delta_\mu^2 + 4\sqrt{2}f\rho^2(a - bc)\Delta_\mu + 4f^2\rho^4\bar{s}}{(2f\rho^2 + 4b\Delta_\mu^2)^2} \]

The denominator is positive for certain, the sign of the first order derivative thus is determined by concave function \( -8f\rho^2b\bar{s}\Delta_\mu^2 + 4\sqrt{2}f\rho^2(a - bc)\Delta_\mu + 4f^2\rho^4\bar{s} \). It is positive and strictly
increasing when $\mu$ equals 0. To see how does the value of partial derivative change when $\mu \in [0, \frac{2f\rho^2}{b\theta}]$, we need to check its value when $\mu$ reaches the maximum value. When $\mu = \frac{2f\rho^2}{b\theta}$ we have $\Delta_{\mu} = \sqrt{\frac{2f\rho^2}{b\theta}}$, the value of the concave function becomes $(\frac{8f^2\rho^3(a-bc)}{\sqrt{b}} - 12\bar{s}f^2\rho^4)$. If $(\frac{8f^2\rho^3(a-bc)}{\sqrt{b}} - 12\bar{s}f^2\rho^4) > 0$, then under the condition of Proposition 1 $\tilde{p}$ is always increasing. If $(\frac{8f^2\rho^3(a-bc)}{\sqrt{b}} - 12\bar{s}f^2\rho^4) \leq 0$, then there must exist some $\hat{\mu}_p \in [0, \frac{2f\rho^2}{b\theta}]$ such that $\frac{\partial \tilde{p}}{\partial \mu}$ is positive when $\mu \in (0, \hat{\mu}_p)$ and is negative otherwise.

A.4 Proof of Corollary 4

To see how does the optimal search cost $\tilde{s}$ change if $f$ or $\rho^2$, we only need to check $\frac{\partial \sqrt{s}}{\partial f}$ since the position of $f$ and $\rho^2$ is symmetric in $s^*$. Since we could write $\sqrt{s}$ as

$$\sqrt{s} = s + \frac{(a-bc)\sqrt{2\bar{v}\mu} - 4b\bar{v}\mu s}{2f\rho^2 + 4b\bar{v}\mu}$$

Thus if $(a-bc)\sqrt{2\bar{v}\mu} - 4b\bar{v}\mu s > 0$, then the optimal search cost $\tilde{s}$ is strictly decreasing in $f$ and $\rho$, otherwise it is increasing in $f$ and $\rho$.

A.5 Proof of Proposition 3

We use $D$ to denote $\sqrt{s_i} - \sqrt{s_m}$. Plugging in the optimal functions of $\sqrt{s_i}$ and $\sqrt{s_m}$, we derive

$$D = \frac{-2\sqrt{2b}(a-bc)\Delta_{\mu}^3 + 20bf\rho^2\bar{s}\Delta_{\mu}^2 - 6\sqrt{2}f\rho^3(a-bc)\Delta_{\mu}}{4(f\rho^2 + 2b\Delta_{\mu}^2)(2f\rho^2 - b\Delta_{\mu}^2)}$$

From the equation above, we find that when $\mu$ equals zero the optimal search costs in the two-sided platform business mode and the merchant business mode are the same. If we go back and check the equilibrium search cost equations one by one. The optimal search cost in the two-sided platform business mode is at first decreasing when $\mu$ is small and then it is increasing when $\mu$ becomes large. However, the optimal search cost in the merchant mode changes oppositely. Since they both equals approximate to $\tilde{s}$ when $\mu \to 0$, there must exist a small value $\mu_h \in [0, \frac{2f\rho^2}{b\theta}]$ such that $D < 0$ at $\mu_h$. However, if we check the value of two equilibrium search costs in two business mode when $\mu$ approximate to $\frac{2f\rho^2}{b\theta}$ we find the $D > 0$ when $\mu$ is approximate to the upper bound of $\mu$. Due to the continuity of the numerator in $D$, there must exist some value $\mu_l \in [0, \frac{2f\rho^2}{b\theta}]$ such that the optimal consumer search cost is larger in the merchant mode if $\mu \in [\mu_l, 2\frac{f\rho^2}{b\theta}]$ while the optimal consumer search cost is larger in the two-sided platform mode when $\mu \in [\mu_l, 2\frac{f\rho^2}{b\theta}]$.
A.6 Proof of Proposition 4

To compare the optimal profits in the two-sided platform mode and the merchant mode, we need to at first plug the optimal platform price and the optimal search cost in two modes into two profit functions respectively. Thus we could get the optimal profit function in the two-sided platform mode:

$$\pi_p^i = \frac{(a - bc)^2}{4b} - f\rho^2\bar{s} + \frac{(2\sqrt{2}f\rho^2\bar{s} - (a - bc)\Delta_\mu)^2}{4(2f\rho^2 - b\Delta_\mu^2)}$$

and the optimal profit function in the merchant mode:

$$\pi_p^m = -f\rho^2\bar{s} + \frac{(\sqrt{2}f\rho^2 + (a - bc)\Delta_\mu)^2}{2(2f\rho^2 - b\Delta_\mu^2)}$$

Correspondingly, we have

$$\frac{\partial \pi_p^i}{\partial \Delta_\mu} = \frac{(4f\rho^2\bar{s} - \sqrt{2}(a - bc)\Delta_\mu)(4f\rho^2b\bar{s}\Delta_\mu - 2\sqrt{2}f\rho^2(a - bc))}{4(2f\rho^2 - b\Delta_\mu^2)^2}$$

which implies that the sign of $\frac{\partial \pi_p^i}{\partial \Delta_\mu}$ is the same as $4f\rho^2b\bar{s}\Delta_\mu - 2\sqrt{2}f\rho^2(a - bc)$. Since $4f\rho^2b\bar{s}\Delta_\mu - 2\sqrt{2}f\rho^2(a - bc)$ is a linear function which is strictly increasing in $\mu$. To see how does the value of partial derivative change when $\mu \in [0, \frac{2f\rho^2}{b}]$, we need to check its value when $\mu$ reaches the maximum value. If $\Delta_\mu$ reaches the maximal boundary $\frac{2f\rho^2}{b}$, the value of the linear function becomes positive under the assumption of Proposition 4. Thus the profit function must be a function of $\Delta_\mu$ which is decreasing in $[0, \frac{(a - bc)}{\sqrt{2b\bar{s}}}]$ and increasing in $[\frac{(a - bc)}{\sqrt{2b\bar{s}}}, \frac{2f\rho^2}{b}]$.

The derivative of the optimal profit in the merchant mode with respect to $\mu$ takes the form of

$$\frac{\partial \pi_p^m}{\Delta_\mu} = \frac{(2f\rho^2\bar{s} + \sqrt{2}(a - bc)\Delta_\mu)(\sqrt{2}(a - bc)f\rho^2 - 4bf\rho^2\bar{s}\Delta_\mu)}{2(2f\rho^2 + b\Delta_\mu^2)^2}$$

which is positive at 0 and always strictly decreasing in $\Delta_\mu$. To see whether it may reach negative value when $\mu$ equals the maximum, we plug $\mu = \frac{2f\rho^2}{b}$ into $\Delta_\mu = \sqrt{\bar{s}}\bar{v}$. With the conditions in Proposition 4, its value is negative at the $\sqrt{\frac{2f\rho^2}{b}}$. Thus the optimal profit function in the merchant mode is a function of $\Delta_\mu$ which is increasing in $[0, \frac{(a - bc)}{2\sqrt{2b\bar{s}}}]$ and decreasing in $[\frac{(a - bc)}{2\sqrt{2b\bar{s}}}, \frac{2f\rho^2}{b}]$.

To compare the optimal profits in two modes, we need to check whether those two curves cross with each other. From the the analysis of shapes of profit functions in the two-sided platform business mode and the merchant business mode, we know that when the product diversity is sufficiently high or sufficiently low, it is more profitable for intermediary to stay in the two-sided platform business mode. To see whether the value of $\Delta_\pi = \pi_p^i - \pi_p^m$ may change to negative value during some interval, we could check its value when $\pi_p^i$ reaches its
minimum value. If it is negative, then there must exist some interval in which intermediary
in the merchant is more profitable than the two-sided platform.

According to the derivative of $\pi_i'$ with respect to $\Delta_{\mu}$, the profit function in the merchant
mode is a function with respect to $\Delta_{\mu}$ which is increasing in $[0, \frac{(a-bc)}{\sqrt{2bs}}]$ and decreasing in
$[\frac{(a-bc)}{\sqrt{2bs}}, \sqrt{\frac{2f\mu^2}{b}}]$. Hence intermediary in the two-sided platform business mode reaches its
maximum value at $\frac{(a-bc)}{\sqrt{2bs}}$. If we plug in the value of $\frac{(a-bc)}{\sqrt{2bs}}$ into $\Delta_{\pi} = \pi_i' - \pi_m'$, we derive

$$\Delta_{\pi}^{i} = \frac{(a - bc)^4}{2b^2s^2(2f\rho^2 - b\Delta_{\mu}^2)(f\rho^2 + 2b\Delta_{\mu}^2)} \left( \frac{(a - bc)^2}{4bs^2} - f\rho^2 \right)$$

With the restriction that $s \geq \frac{a-bc}{2\sqrt{hf\rho^2}}$, we have the value of $\Delta_{\pi} < 0$ must be negative. Thus
there must exist some interval $(\mu_{pr}, \bar{\mu}_{pr})$ that the intermediary in the merchant mode is more
profitable when $\mu \in (\mu_{pr}, \bar{\mu}_{pr})$ and the opposite otherwise.