Public Goods Through Markets: Do Charity Auctions Work?*

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Abstract

In competitive markets, prices arise at the intersection of the margins of revenues and costs. Firms that contribute part of their proceeds to a public good must also balance the consumer’s altruistic motives with free riding. As a result, prices do not necessarily increase. The paper discusses these mechanisms theoretically in the context of charity auctions. Counterfactuals from a structural auction model with externalities on unique data from Charitystars.com show that switching to non-charity auctions would enhance profitability. This result highlights how markets can fail to provide the right incentives for prosocial behavior.

JEL classifications: H41, C57, D44, L31
Keywords: public goods, externalities, charitable giving, auctions with externalities, charity auctions, structural estimation, nonparametric identification

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1 Introduction

Wealthy entrepreneurs and their foundations have long been the major players for charitable donations. Yet, younger, less-affluent individuals are increasingly emerging as tomorrow’s philanthropists (Blackbaud, 2013). They are less attracted by entrusting their names to buildings; rather they focus on the demonstrable impacts of the charities to which they donate. Recent reports support this view by, for example, documenting how the expansion of social media usage by charities has helped them achieve greater fundraising.¹

This trend is also encouraged by firms, which have developed new tools of corporate social responsibility (CSR), embracing accountability and transparency. For instance, the brand Product RED cooperates with several large corporations, ranging from Apple to Nike, to raise funds for combating HIV in Africa. Similarly, for every pair of TOMS shoes purchased, the company donates a pair to children in developing countries. All these markets share a common feature: a part of the price paid is donated to a charity. These products are commonly labelled impure public goods because their purchase forces consumers to simultaneously buy a private and a public good (i.e., the fundraising). There are many examples of these arrangements between profit and not-for profit firms. Yet, there is no agreement in the economic literature on their effectiveness.

Early commentators warned against CSR as it can endanger a firm’s profits and therefore its sustainability (Friedman, 1970, Baumol and Blackman, 1991). According to this theory, engaging in such activities falls outside of the logic of profit maximization and threatens the future of the firm. Due to market competition, this exposes its shareholders and employees to a far worse outcome than what CSR generates.

Recent work has shifted the focus on the additional consumer surplus that different types of CSR produce. For example, Kotchen (2006) studies the purchase decision when a consumer’s choice set includes private goods, direct contributions to fund public goods and impure public goods. This is the case of green markets where people can purchase either a regular tool or its environmentally friendly counterpart, but can also donate to a charity for the safeguard of the environment. The author demonstrates that the addition of the impure alternative can make consumers worse-off. This result is echoed in other studies that, for example, show how CSR efforts do not always result in a greater willingness to pay by consumers (Sen and Bhattacharya, 2001, David et al., 2005). Yet, the opposite result holds in other markets. For instance, a natural experiment in a major amusement park indicates that pay-what-you-want policies can raise consumers’ willingness to pay if connected to donation (Gneezy et al., 2010). In addition, other papers relate prosocial behavior to the presence of social norms (Bartling et al., 2014) and to market structures (e.g., Besley and Ghatak, 2007, Pecorino, 2016, Lai et al., 2017).

While all these papers target either consumers or firms, we learned little from their inter-
actions. In these markets, firms must not only balance the additional cost of producing the public good with unitary price changes, but also with the incentives it generates on consumers: while some consumers will be persuaded to purchase the impure good, others will not do so and free-ride. The way prices adjust when these margins move is ambiguous. Understanding how they respond to different sizes of the public good is a fundamental question that needs to be addressed in order to understand the welfare impact of such policies. This paper does so by bridging theory with empirics through an analysis of price formation under the lens of the optimal behavior of consumers. With this approach, mechanisms leading to socially responsible behavior can be told apart from the socially ill ones, while also describing the optimal actions of suppliers.

To this end, this paper studies a simple market structure in a context where parts of the proceeds are donated to a charity. This is the case of charity auctions, where bidders compete to win an item (the private good) knowing that the proceeds will be donated (the public good). The objective is to understand how altruistic motives shape decisions, and how these incentives affect market outcomes. These results are important as they also shed light on the functioning of more complex markets with elastic supply.

In charity auctions bidders are compelled by two forces. On one hand, they have a taste for winning the auction and donating, or warm glow (Becker, 1974, Andreoni, 1989). As a result, winning the auction carries additional utility, incentivizing bidders to raise their bids. On the other hand, they have a positive surplus even by losing the auction because a fraction of the proceeds of the auction will be invested in the production of a public good (Engelbrecht-Wiggans, 1994, Goeree et al., 2005, Engers and McManus, 2007). The last effect is an externality from the winner to the other bidders, who extract surplus from the donor who pays for the object. In equilibrium, the bidders who are most likely to lose increase their bids in an attempt to affect the final price. In addition, there are cases where high-value bidders decrease their bids. This happens as they try (i) to avoid overpaying, and (ii) because they also enjoy somebody else’s donation (shill bidding). This phenomenon is directly related to free-riding in the public good literature (e.g., Bergstrom et al., 1986, Andreoni, 1988) and highlights the complexity of balancing the charitable margin with the classic value-price dichotomy.

The theoretical model illustrates how prices, and thus the size of the public good generated, depend on the way these two altruistic forces, the warm-glow and the externality, interact. In certain cases prices may even be lower than equilibrium prices in similar non-charity auctions. This friction help explain the conflicting results in the related empirical literature. In fact, across different mechanisms and market environments there is both evidence for losses or negligible gains (e.g., Carpenter et al., 2008, Schram and Onderstal, 2009, Isaac et al., 2010), as well as for considerable profits from charity auctions (Elfenbein and McManus, 2010a, Leszczyc and Rothkopf, 2010, Elfenbein et al., 2012, Canals-Cerda, 2014). While these results seem surprising at first, as we would naively expect donations to always increase bids, they can be interpreted in terms of
the comparative statics of the agents’ optimal strategy with respect to these two forces.

This paper is also connected to a series of recent empirical studies dedicated to the estimation of charitable motives in different settings. For example, DellaVigna et al. (2012) focuses on the reason for people to donate in a door-to-door fund-raising experiment. While their results are consistent with warm-glow, the authors highlight the importance of the social pressure to donate resulting from the campaign. In addition, they associate social pressure to sizeable welfare losses for the solicitees (see also Huck et al., 2015). In a second study (DellaVigna et al., 2013) the authors decompose their previous findings by gender, highlighting the existence of a gender gap in prosocial preferences. Finally, drawing from a parallel between donation and taxation, Dwenger et al. (2016) take advantage of administrative tax records on local German churches and a field experiment to show that intrinsic motivation is a key driver of tax compliance.

The empirical analysis on charity auctions is based on a unique dataset from Charitystars.com, a European start-up headquartered in Los Angeles, London and Milan. The company offers an internet auction platform where users bid on VIPs’ belongings and where the proceeds are donated to charities. Since its foundation in 2013, the company generated over $10 million dollars in funds for charities and nonprofits such as Special Olympics, Save the Children, Make-A-Wish and WWF. The online charity auction market is large and expanding. For example, eBay for Charity, the charity auction branch of eBay, raised $84 million in 2017 worldwide. Having raised $810 million since 2003, eBay for Charity is on track to meet its goal of raising $1 billion by 2020.

While anyone can list an item on eBay for Charity, bidders on Charitystars always face the same auctioneer: Charitystars itself. In particular, auctioneers on eBay vary under many aspects, some of which may be payoff relevant for bidders. For example, other studies evidenced how auctioneers with poor reputation promote the donation of a portion of the proceeds in order to improve their standings (Elfenbein et al., 2012). Moreover, not all charity claims are verifiable on eBay for Charity, which often result in slightly lower sale probability (Elfenbein et al., 2017). Charitystars instead guarantees the donations by producing certificates. Charitystars auctions also have other interesting features. For instance, the auction countdown is automatically extended by 4 minutes anytime a bid is placed in the last 4 minutes of the auction. This feature impedes sniping (i.e. the practice of bidding in the very last seconds of an auction), a common feature of eBay auctions causing lower transaction prices (Elfenbein and McManus, 2010b, Backus et al., 2017). These feature of Charitystars auctions do not only remove additional confounding factors, but also permit building a novel structural model by easing the comparison of bids across auctions.

The need for structurally estimating the primitives of the model arises not only to address counterfactuals and simulations, but also directly from the nonlinear relation between expected

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2For example, Harbaugh et al. (2007) demonstrate the existence of considerable similarities in the neural activities arising when paying a tax and when donating.

prices and charitable motives. In fact, there exists no adequate instrument to capture the two altruistic forces in a reduced form fashion. Moreover, when they affect prices in opposite direction, they can cancel each other out, suggesting the erroneous conclusion that bidders do not have charitable motives.

The paper contributes to the empirical auction literature by providing nonparametric identification and estimation techniques to structurally approach the problem posed by auctions with externalities across bidders. While there is a growing theoretical literature in both microeconomic theory and computer science investigating how cross-bidders externalities affect auction outcomes (e.g., Jehiel et al., 1999, Varma, 2002, Lu, 2012, Gatti et al., 2015), the body of econometric research on this subject is rather slim (e.g., Kuehn, 2016). Typically, the existence of an invertible mapping from bids to private values would suffice to ensure identification of the primitives (e.g., Guerre et al., 2000, Athey and Haile, 2002). However, the fundraising from charity auctions naturally creates (identity-independent) externalities across bidders as losing bidders “win” the public good contribution of the winner. In this setting, the altruistic motives can be combined in multiple ways to yield the same distribution of private values, undermining their identification from observed data.

To fix this challenge, we leverage on the key feature of each Charitystar’s listing: the percentage of the final price that is given to a charity. This variable affects the bidders’ first order conditions through the altruistic parameters and varies across listings in a quasi-experimental fashion. The intuition for the nonparametric identification follows directly from the equivalence of the distributions of bids and values. The model’s primitives are identified by comparing bids across different auctions (varying with respect to the percentage donated) while keeping the quantile of the distribution of bids fixed. In fact, this corresponds to conditioning on the distribution of values, which is unknown. This approach is based on the restriction that private values are independent of the percentage donated, which is tested and confirmed in the data. Methodologically, the closest papers study the identification and estimation of risk-aversion in first-price auctions using either auxiliary data, variation in covariates or parametric restrictions (e.g., Lu and Perrigne, 2008, Guerre et al., 2009, Campo et al., 2011).

A second threat to identification in online auctions comes from jump bidding and from inattentive bidders who can forget or miss a chance to update their bids. These issues often result in the accidental recording of some bids as willingness to pay, while instead these bidders had different valuations. The empirical model deals with this concern by translating Charitystars online English auctions in “parallel” sealed bid second-price auctions. This operation produces a set of moment conditions under the rather mild assumption that the second-highest bidder exits the auction at a price equal to her willingness to pay, while all the other bidders are allowed to place non-optimal bids. This approach allows the estimation of both the private values and the altruistic parameters. Taken altogether, these estimates yield the value of the impure public good. Ultimately, computing these values for different bidders gives a demand function, which
is key to assessing the opportunity for Charitystars to engage in the production of the public good.

Estimating the model we learn that bidders are warm-glow givers (Andreoni, 1990). This means that they attach more utility on their own donations than on those of the other bidders. Thus, free-riding is less of a concern in this market as bidders have more incentives to win. Such a result is also reflected in Charitystars’ revenues. In fact, a set of counterfactual experiments demonstrate that the expected prices with the current format are greater than in an alternative non-charity auctions (by ca. €50). Interestingly, donations would not be greater if the auctioneer were to adopt the all-pay format. This format was highlighted in the literature as the optimal way to maximize charity revenues especially with a large number of bidders (Goeree et al., 2005, Engers and McManus, 2007). At the estimated parameters, and given the number of bidders that usually show up in the auction, English auctions outperform all-pay auctions. However, as the number of bidders grows, this gap is muted, consistent with the theory.

Despite greater revenues, the price expansion does not fully cover the donation. The company has a large profitable deviation and it could increase its net revenues by resorting to standard English auctions by as much as €260 per listing. It is intuitive to think that auction formats do not affect marginal costs. In this case, switching to non-charity auctions would grant higher profits to Charitystars by the same amount. Yet, VIPs could view donations as an economical form of advertising, which could curtail the company’s cost of collecting the objects. Nevertheless, the foregone profits are so large that undertaking such a deviation, or at least lowering the amount donated, seems profitable.

This finding is a related to a growing literature showing that socially responsible investors demand lower returns on their investments (Riedl and Smeets, 2017). Yet, this highlights the limits of the provision of public goods through markets, as consumers often only shoulder a small portion of the actual donation which is mostly borne by firms. In the charity auction case this is exacerbated by the interaction between the strategic environment and the free-riding typical of public good contributions, which can result in high value bidders decreasing their bids in equilibrium. In these circumstances, offering charity-linked products may not be incentive compatible. This analysis casts doubts on the opportunity for firms to supply charity linked products and encourage careful examination of the incentives that such donations instil in the market.

Alternatively, similar circumstances could also boost the number of objects that Charitystars can gather while keeping costs constant. In this case offering charity auctions could still be profitable. See also Ghosh and Shankar (2013) for an analysis of advertising and warm glow giving.
2  Auctions on Charitystars

Charitystars is an internet platform where charities do fundraising. On www.charitystars.com famous celebrities place their objects and memorabilia on auction. The higher bidder wins the object, pays his bid, which is then partially donated to a charity. The identity of the charity receiving the donation, as well as the fraction of the transaction price being donated are common knowledge before each auction.

In order to bid for an item, potential users sign up on the online platform. Once the account is online, users can bid in any live auction. All auctions employ an open, ascending-bid format analogous to eBay. Auctions involve a single item. Bidders can submit a cutoff price (this tool is called proxy bidding on eBay) instead of a bid. Once a cutoff is set, Charitystars.com will issue a bid equal to the smallest of the highest current price (or the reserve price if there is no bid yet) and the highest submitted cutoff price, plus the minimum increment. Thus, the winner pays the second highest valuation, plus a minimum increment.\(^5\)

A particularity of Charitystars’ auctions is the secrecy of the reserve price: at any point in time bidders only know whether the reserve price is met or not. Although the reserve price is never disclosed, not even after the end of an auction, it can be found in the source (HTML) code behind each listing. Secret reserve prices are not uncommon in online auctions. The empirical auction literature have treated these occurrences by simply adding an additional bidder who keeps the object in case no one bids above the reserve price (e.g., Bajari and Hortacșu, 2003).\(^6\) In addition, on Charitystars after each unsold auction there is an additional step where the highest bidder is given the option to purchase the object by paying the reserve price. To avoid possible concerns the following analysis is restricted to listings counting at least two bidders which concluded in a direct sale.\(^7\)

The company auctions a very broad spectrum of items, from VIP tickets to the Monaco Gran Prix to famous photographs and arts collectibles. However, soccer is one of the most popular item categories with over 4,000 auctions held in 3 years. Given the number of observations and the relative homogeneity of the soccer jerseys auctioned by the company, this marketplace seem to be a good place to investigate altruism among bidders and its outcomes.

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\(^5\)The following example explains the mechanism. Assume there are 3 bidders: Alice values the object 100, Bob 150 and Claudia 200, the reserve price is 80 and the minimum increment is 10. If Alice bids first an amount equal to her valuation, then the standing price of the object is 90 (80 + 10). Assume Claudia’s bid come next for an amount of 120. Then the standing price will increase to 100 (90+10, which is equal to Alice’s cutoff). To win Bob will bid his valuation, and the standing price will grow to 110 (100 + 10). Eventually, Claudia will bid her valuation (200), and pay 160 (bidder B’s valuation plus the minimum increment).

\(^6\)Knowledge of the reserve price does not affect the analysis.

\(^7\)The second (indirect) step is very rare, happening only in 7% of the surveyed auctions.
2.1 Description of the data

This paper uses publicly available data from Charitystars.com that were collected directly from the website. The dataset contains auctions of authentic soccer jerseys sold between July 1st, 2015 and June 12th, 2017.\(^8\) These dates were chosen as they mark the beginning and the end (after the Champions League final) of two consecutive football seasons. The dataset includes 1,583 auctions. Charitystars keeps a percentage of the proceeds from each auctions for itself, which is in general 15%, while the rest is either fully donated to a charity or divided between the provider of the object, Charitystars and a charity. There is large variation in the percentage that is finally donated though for most auctions the fundraising corresponds to either 10%, 78% or 85% of the final price. Figure 1 shows this variation for two subsets of the data. All auctions to the right of the black solid lines involve smaller than standard proceeds to Charitystars, and are therefore removed from the analysis. Many more auctions are at the 85% in both samples, while the amounts of auctions at 10% and 78% are almost equal. Throughout the paper we will refer to the percentage donated by \(q\).

All items are posted online on the platform website and advertised on the social media of the firm in a similar fashion. The listing webpage shows pictures of the item on the left of the screen, while bidders find a short description of the item on the bottom of the website, altogether with the information on the recipient charity. A picture of a typical webpage (Figure 8) is reported in Appendix A.

For each auction, all bids, the date and time of the bid, the bidder nationality and the charity receiving the money are observed. Unfortunately, auctions start dates are not available online, but the average length of Charitystars’ auctions is usually between 1 and 2 weeks. Length is therefore proxied using the distance in days between the first bid posted and the closing day. A larger list of auction variables which will be used in all regressions is available in Table 8 in Appendix B, while Figure 9a in Appendix B plots the pdf of the transaction price for the three most frequent auction formats (\(q \in \{10\%, 78\%, 85\%\}\)). Table 1 gives an overview of the main characteristics of the auctions for listings with transaction prices larger than €100 and smaller than €1,000, with at least 2 different bidders and whose minimum increment is within €25. The final price is greater than the reserve price in more than 95% of the listings. This database will be used throughout and consists of 1,108 auctions.

To better tackle unobservables, we will also refer to a subset of this database including only jerseys that were sold below €400. This upper limit (€400) was chosen because in the Summer of 2017 Charitystars decided to set a €50 minimum raise anytime the standing price reaches €400 (i.e. the minimum increment changes during the auction depending on the standing price and this is common knowledge across bidders). This is a high value which may suggest that the company believes that items reaching such a high price may differ under some characteristics.

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\(^8\)All other soccer related auctions not involving jerseys (such as shin guards, footballs and shorts) are excluded from the database.
Figure 1: Number of auctions by percentage donated

(a) Transaction price ∈ (100, 1000)

(b) Transaction price ∈ (100, 400)

Note: Number of auctions available in the dataset by percentage donated. The plots include only auctions that ended in a transaction and for which the reserve price was greater than 0, the number of bidders was at least 2 and the minimum increment is not greater than €25. Panel (a) shows the number of auctions available for each percentage donated when the dataset is restricted to auctions whose price is in €(100,1,000). There are 1,188 auctions in total. Panel (b) restricts the dataset to more homogeneous auctions (762 auctions). Charitystars always withholds a 15% share of the final price and therefore all auctions whose percentage donated is above 85% are excluded from the analysis as these are special one-off charitable events (these are all the auctions to the right of the solid vertical line).

Therefore, most of the analyses are also replicated with this smaller dataset to ensure that the results are not driven by unobservables.9

2.2 Descriptive statistics and correlation analysis

This section reports results from correlation analyses aimed at highlighting certain features of bidding in Charitystars that should be respected by the model developed in Section 3. The value of these analyses is twofold because upon estimation of the primitives of the structural model (Section 5), some of these results can be tested by simulation.

We maintain the hypothesis that bids and donations are positively related. That is we expect higher bids to be associated with greater portions of the final price donated (q). To test this hypothesis we run a series of regressions focusing only on the winning bid (i.e. the transaction

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9Note also that such a high minimum increment undermines the identification of the structural model in Section 4 (e.g., Haile and Tamer, 2003, Chesher and Rosen, 2017). For this reasons auctions with large minimum increments are not included. Table 1 shows that the mean of the Minimum increment variable is close to €1, which is equal to the value at the 90th quantile.
### Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Q(25%)</th>
<th>Q(50%)</th>
<th>Q(75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auction characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage donated (q)</td>
<td>0.70</td>
<td>0.27</td>
<td>0.78</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Transaction price in €</td>
<td>364.25</td>
<td>187.50</td>
<td>222.00</td>
<td>315.00</td>
<td>452.50</td>
</tr>
<tr>
<td>Reserve price in €</td>
<td>179.03</td>
<td>132.02</td>
<td>100.00</td>
<td>145.00</td>
<td>210.00</td>
</tr>
<tr>
<td>Minimum increment in €</td>
<td>1.71</td>
<td>3.15</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Number of bidders (in # days)</td>
<td>7.83</td>
<td>3.27</td>
<td>5.00</td>
<td>7.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Sold at reserve price (d)</td>
<td>0.04</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Length</td>
<td>8.08</td>
<td>3.07</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Extended time (d)</td>
<td>0.43</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Charity’s activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helping disables (d)</td>
<td>0.35</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Infrastructures in developing countries (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Healthcare (d)</td>
<td>0.23</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Humanitarian scopes in developing countries (d)</td>
<td>0.14</td>
<td>0.34</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Children’s wellbeing (d)</td>
<td>0.84</td>
<td>0.36</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Neurodegenerative disorders (d)</td>
<td>0.06</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Charity belongs to the soccer team (d)</td>
<td>0.10</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Improving access to sport (d)</td>
<td>0.63</td>
<td>0.48</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Overview of the main covariates used in all specifications in the reduced form analysis and in the structural model. (d) stands for dummies. Only auctions with price between €100 and €1000. Prices are in euros. If the listing was in GBP the final price was converted in euros using the exchange rate of the last day of auction. There are 1,108 auctions in total.

In fact, all the other bids do not provide useful information in an environment where bidders can update their bids multiple times. Table 2 performs the following OLS regression

$$\log(price) = \gamma_0 + x\gamma + q\gamma_q$$

(2.1)

where the matrix of covariates, $x$, includes all variables other than the percentage donated, $q$. The columns of the table vary based on the definition of $x$, which is described in the bottom panel of the table. The variables are defined in detail in Appendix B. From the estimates we learn that $\gamma_q$ is above 20% and significant (at 1% level) in most columns, confirming the hypothesis of positive correlation between $q$ and prices.

Two more observations are interesting. First, the transaction price is increasing in the number of bidders. This provides a simple test in favor of the Independent Private Value model (IPV), within which bidders draw private and independent values for the auctioned item. If instead valuations were affiliated, bidders would optimally shade their bids in order to escape from the winner’s curse (Milgrom and Weber, 1982). Because the size of the curse grows with the number of bidders, lower bids are expected when an additional bidder joins an auction (Bajari and Hortacsu, 2003). Since the number of bidders coefficient is positive, the IPV framework seems to
Table 2: Relation between log(Price) and percentage donated

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Bidders)</td>
<td>0.291</td>
<td>0.298</td>
<td>0.285</td>
<td>0.280</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>q</td>
<td>0.037</td>
<td>0.234</td>
<td>0.230</td>
<td>0.258</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.048)</td>
<td>(0.052)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td></td>
<td>0.353</td>
<td>0.341</td>
<td>0.342</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.117</td>
<td>2.348</td>
<td>2.533</td>
<td>2.642</td>
</tr>
<tr>
<td></td>
<td>(0.235)</td>
<td>(0.228)</td>
<td>(0.230)</td>
<td>(0.297)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Add. Charity Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>League/Match Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>32.31%</td>
<td>45.90%</td>
<td>47.20%</td>
<td>49.22%</td>
</tr>
<tr>
<td>BIC</td>
<td>1,404</td>
<td>1,162</td>
<td>1,269</td>
<td>1,333</td>
</tr>
<tr>
<td>N</td>
<td>1,108</td>
<td>1,108</td>
<td>1,108</td>
<td>1,108</td>
</tr>
</tbody>
</table>

Note: OLS regression of log of the transaction price on covariates. Only auctions with price between €100 and €1000. Control variables are defined in Appendix B.

* – $p < 0.1; ** – $p < 0.05; *** – $p < 0.01.

A second observation is related to unobserved heterogeneity across auctions. The sole inclusion of the reserve price in Table 2 is responsible for a jump in the Adjusted R-squared from 32% to 46%. According to Roberts (2013), this happens when the reserve price carries unobservable information to the researcher but observable to the bidders and auctioneer. For example, bidders’ willingness to pay may be higher if a player receives an important prize while one of his jerseys is up for auction with a high $q$. If no variables in the data reflect the prize, the regression may overestimate the impact of $q$ on prices. Given that the auctioneer anticipates that bidders are affected by the size of $q$ (after all the word “charity” is part of the name of the firm), we would expect the correlation between the reserve and the percentage donated to be high if the reserve price were not affiliated with unobservables. However this correlation is close to zero. This evidence supports affiliation between reserve prices and unobserved heterogeneity, and it will exploited in the structural model.

In addition, the reserve price is the lowest price at which the auctioneer is willing to sell the object and therefore embodies information on the lowest price at which she is happy to sell the item. The absence of correlation between these two variables also suggests a high degree of homogeneity of the goods across different $q$-auctions.

10 An additional justification for the last model is that bidders may support different football teams, players and causes, and therefore their valuation may be private and independent from that of the other bidders.

11 Spearman’s rank correlation test -0.0611 in the large sample and -0.0139 in the small sample.
Overall, this suggests that bidders react to giving incentives. Other papers found qualitatively comparable results. For example, Elfenbein and McManus (2010a) estimated that prices in eBay charitable listings are on average 6% larger than comparable non-charity ones, while Leszczyc and Rothkopf (2010) using a controlled experiment determined that a 40% donation leads to a 40% price increase.\footnote{To provide more robust results we can run (2.1) again restricting the sample to auctions with prices less than €400. Table 9 in Appendix C shows that the transaction price is still increasing in \( q \), and the coefficient is ca. 13\%.}

The charity auctions literature has also investigated the shape of the relation between \( q \) and prices (e.g., Elfenbein and McManus, 2010a). To this end, Table 10 in Appendix C adds \( q^2 \) to the covariates used in the OLS regression (2.1). The sign of the coefficients imply a concavity in \( q \). However, \( q \) is only marginally significant in column (III) where a large set of covariates is added, while \( q^2 \) is never significant. In contrast, the linear combination of \( q \) and \( q^2 \) is positive and significant across all columns (shown in the third panel), failing to reject the linearity of the relation between \( q \) and \( \log(price) \).

Table 3: Linearity of the relation between \( \log(Price) \) and percentage donated

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(\text{Reserve Price}) )</td>
<td>0.353***</td>
<td>0.521***</td>
<td>0.370***</td>
<td>0.298***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.037)</td>
<td>(0.030)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>( \log(\text{Bidders}) )</td>
<td>0.298***</td>
<td>0.254***</td>
<td>0.302***</td>
<td>0.354***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.034)</td>
<td>(0.037)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>( q )</td>
<td>0.234***</td>
<td>0.255***</td>
<td>0.274***</td>
<td>0.308***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.058)</td>
<td>(0.070)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>( \text{Constant} )</td>
<td>2.348***</td>
<td>1.236***</td>
<td>1.236***</td>
<td>2.726***</td>
</tr>
<tr>
<td></td>
<td>(0.228)</td>
<td>(0.310)</td>
<td>(0.336)</td>
<td>(0.378)</td>
</tr>
</tbody>
</table>

Main Variables | Y | Y | Y | Y

| Adjusted R-squared | 45.90% | 31.91% | 30.31% | 28.80% |
| N | 1,108 | 1,108 | 1,108 | 1,108 |

Note: OLS Regression and quantile regressions of the logarithm of the transaction price on covariates to test the linearity of donation. Only auctions with price between €400 and €1000. Boostrapped standard errors with 400 repetitions. The null hypothesis that \( q \) is the same in column (II), (III) and (IV) is not rejected beyond 70% level. Control variables are defined in Appendix B.

* – \( p < 0.1; ** – p < 0.05; *** – p < 0.01. \)

Quantile regressions are another commonly used approach to test linearity. Table 3 reports three quantile regressions in the same spirit of regression (2.1).\footnote{Column (I) reports the estimates from the OLS regression in the second column of Table 2.} All the coefficients of \( q \) are similar across the four columns, and in particular, we cannot reject the null hypothesis that the coefficients computed at the 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} quartiles are equal (F test p-value 0.82). The same result can be observed graphically in Figure 10, which plots these coefficients, and can also be
replicated in the smaller sample (Table 11). This evidence suggests a linear relation between the fundraising and the logarithm of the price and will be used in Section 3 to characterize the optimal strategy of a bidder who takes into account both her pecuniary payoff as well as the amount of fund raised.

Table 4: Relation between number of bidders and percentage donated

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 &lt; p &lt; 1000</td>
<td>100 &lt; p &lt; 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>0.032</td>
<td>-0.002</td>
<td>0.067</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.058)</td>
<td>(0.069)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.223</td>
<td>0.105</td>
<td>0.096</td>
<td>0.394</td>
</tr>
<tr>
<td></td>
<td>(0.235)</td>
<td>(0.241)</td>
<td>(0.334)</td>
<td>(0.360)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Charity Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>League/Match Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo R-squared</td>
<td>12.49%</td>
<td>13.92%</td>
<td>13.05%</td>
<td>14.32%</td>
</tr>
<tr>
<td>N</td>
<td>1,108</td>
<td>1,108</td>
<td>713</td>
<td>713</td>
</tr>
</tbody>
</table>

Note: Poisson regression of the number of bidders on covariates. The length of the auction is used as exposure variable and is not included among the covariates. Control variables are defined in Appendix B.

We now move from the intensive to the extensive margin of giving. The data show that a total of 2,247 bidders compete on average in 5.34 different auctions. This mean increases to 8.51 different auctions when excluding bidders who (naively) bid only in one auction. Most bidders take part in different auction formats (varying over the amount donated): excluding bidders who placed less than 2 bids more than 80% of the bidders bid at least on two auctions with different amount donated. This is a very large number given that \( q = 85\% \) for half of the auctions in the data (559 out of 1108). Also, note that the participation decision of Charitystars users is not correlated with the percentage donated. Therefore bidders seem to participate in all auctions and do not select on the basis of the amount donated. This is confirmed in Table 4 using two sets of Poisson regressions of the form

\[
\log(\mathbb{E}[\text{bidders}|x,q]) = \gamma_0 + x \gamma + q \gamma_q + \log(\text{length})
\]

where the variable Length is the length of the auction (in days) and is used as the exposure variable (the results are to be interpreted in terms of daily bidders). The first two columns refer to the larger dataset, while the second two are for the smaller dataset. The table suggests that the weak relation between \( q \) and the number of bidders holds also when conditioning on other

\[14\]The Spearman rank-order correlation test reveals that the correlation between \( q \) and the number of bidders is only 0.0840 in the large sample and 0.0628 in the small sample.
covariates.

As a result, contributions to the public good depend disproportionately on the intensive margin rather than on the extensive margin. Therefore, how bidders place their bids is the fundamental question that a revenue optimizing auctioneer should address, disregarding how bidders self-select in different auctions.

A final consideration can be made concerning symmetric bidding. Given the data availability, there are two main sources of asymmetry that can be tested in a reduced form fashion. First, since most jerseys belong to Italian teams (63% of the auctions) and most charities are Italian (90% of the auctions), we may ask whether bidders from different nationalities employ different strategies. Table 12 in Appendix C.1 reports the coefficients from three regressions similar to (2.1) where the dummies for the winner’s nationality are added to the covariate matrix, $x$. Overall, this source of asymmetry is rejected by this analysis.

Asymmetry may also come from recurrent winners as these bidders may be more interested in collecting soccer jerseys than in contributing to a public good. Recurrent winners are not unusual: in fact, the median number of auctions won by each winner in the sample is 3.\textsuperscript{15} Table 14 in Appendix C.1 investigates whether recurrent winners are willing to pay more on average. This is captured by the dummy variable Recurrent Winner that takes value 1 if the winner won more than 3 auctions. There seems to be no evidence for this type of asymmetry as the correlation in Column (I) between Recurrent Winner and the transaction price seems to be driven by unobservables. In fact, it vanishes when adding another covariate (i.e., the total number of bids placed), which accounts for the level of competition in the auction. Moreover, it is also not significant in the smaller dataset, where observations are more homogeneous.\textsuperscript{16}

In conclusion, while asymmetries are possible, there is not enough empirical evidence to clearly distinguish bidders across different groups.\textsuperscript{17} The next section develops and discusses an auction model building on the results just mentioned.

3 Charitable auctions

Bidders in charity auctions respond to two stimuli. First and foremost, they are moved by the willingness to purchase the auctioned item. In this case, as in the textbook auction, they will bid an amount that raises their chances of winning, while keeping a positive margin between own private value, $v$, and the final price. In an ascending auction, like those on Charitystars, the

\textsuperscript{15}The first, second and third quartiles of the number of auctions won are the same across the small and large samples and correspond to 1, 3 and 7 items won respectively.

\textsuperscript{16}Importantly, notice also that $q$ hardly varies across specifications.

\textsuperscript{17}In related papers with asymmetric bidders the distinction is immediate. In procurement auctions such a divide comes often from different marginal costs. For example, in their analysis of timber auction data Athey et al. (2011) distinguish between mills and loggers while Krasnokutskaya (2011) compares bids across small and large firms competing for highway contracts in California.
latter is equivalent to the second highest bid, \( b^{II} \), and in equilibrium a bidder will bid her private value.

Charitable motives add a layer of complexity, as a bidder’s utility increases in the size of the public good generated through their contribution. For the auctions described in the previous section, this contribution is proportional (\( q \)) to the transaction price. Therefore, in case of a victory a bidder will gain an additional satisfaction based on how much is paid, \( b^{II} \). We model this additional utility by the term \( \beta \cdot q \cdot b^{II} \), where \( \beta \in (0, 1) \) transforms the pecuniary contribution in utils and indicates the satisfaction from winning and being the donor.

In addition, the creation of a public good affects the whole community, and not just the winner of the auction, as it could be the purchase of a new ambulance or the funding of innovative cancer research. Therefore, the utility of the losing bidders is not zero as is for the textbook auctions. Instead, it must be increasing in the amount donated: the term \( \alpha \cdot q \cdot b^{II} \) captures the felicity to these bidders from the public good.

A bidder’s utility can be summarized by

\[
    u(v; \alpha, \beta, q) = \begin{cases} 
    v - b^{II} + q \cdot \beta \cdot b^{II} & \text{if } i \text{ wins} \\
    q \cdot \alpha \cdot b^{II} & \text{otherwise}
    \end{cases}
\]

This model extends the theoretical literature on charity auction (e.g., Engelbrecht-Wiggans, 1994, Goeree et al., 2005, Engers and McManus, 2007) to fit the main characteristic in the marketplace on Charitystars.com (i.e., the \( q \) part of the final price is donated). As charitable motives cannot explain the full amount of one’s bid, the literature assumes that \( 0 < \alpha, \beta < 1 \). Thus, \( \alpha \) and \( \beta \) transform the euro denominated funds raised by the auctioneer in terms of utility. Importantly, as \( q \rightarrow 0 \) a bidders’ utility reverts to that of fully selfish bidders (e.g., \( \alpha = \beta = 0 \)), notwithstanding altruistic motives.

We now turn to describing the optimal strategy in such a market. Before proceeding to the optimal bid some regularity assumptions are required:

**Assumption 1. Optimality:**

1. The values are private and independent.

2. All \( n > 1 \) bidders draw their values from a continuous distribution \( F(\cdot) \) with probability density \( f(\cdot) \) on a compact support \([v, \overline{v}]\).

3. The hazard rate of \( F(\cdot) \) is increasing.

The first condition defines the auction environment as an IPV framework, which implies that bidders hold independent and private valuations for the soccer jerseys. Note that their
total utilities, obtained by summing that from the private and public goods, depend on how much the other bidders value both the jerseys and to do charity ($\alpha$ and $\beta$). In the previous section we argued that this model seems appropriate based on some reduced form results (see the discussion of Table 2). Points two and three of Assumption 1 are simple regularity conditions common to most auction models within the IPV framework. In particular, condition three is key in order to establish that the equilibrium bidding function is a global optimum. This condition will also play a central role in proving identification of the primitives in Section 4. Therefore, the remainder of the paper studies a symmetric auction where the parameters $\alpha$ and $\beta$ and the distribution $F(\cdot)$ are common knowledge and constant for all the bidders.

To simplify the theoretical treatment of the English auction we resort to the equivalence with sealed-bid second-price auction, which was proved by Engers and McManus (2007) for the case $q = 1$ and $\beta \geq \alpha$. The expected utility to bidder $i$ with valuation $v$ is

$$
\mathbb{E}_{v,i}[u(v; \alpha, \beta, q)] = \mathbb{E}_{v,i}[v - (1 - q \cdot \beta) b^{II}, i \text{ wins}] 
+ q \cdot \alpha \cdot b^{II} \cdot \Pr (i's \text{ bid is 2nd}) + q \cdot \alpha \cdot \mathbb{E}_{v,i}[b^{II}, i's \text{ bid is } < 2^{nd}]
$$

Denote $\Gamma_2(F(\cdot), \alpha, \beta, q)$ the second price auction defined by $F(\cdot), \alpha, \beta$ and $q$. Remember that Charitystars never publishes the reserve price, which is secret. Following (Bajari and Hortacşu, 2003), we analyze only auctions that terminated in a sale and treat the reserve price as an additional bidder. The next proposition provides the bidding function in a symmetric perfect Bayesian Nash equilibrium.

**Proposition 1.** In the symmetric equilibrium of $\Gamma_2(F(\cdot), \alpha, \beta, q)$ bidders bid according to:

$$
b^*(v; \alpha, \beta, q) = \begin{cases} 
\frac{1}{1 + q \cdot (\alpha - \beta)} \left( v + \int_{v}^{\bar{v}} \left( \frac{1 - F(x)}{1 - F(v)} \right) \frac{1 - q \cdot \beta}{q \cdot \alpha + 1} \, dx \right) & \text{if } \alpha > 0 \land q > 0 \\
\frac{v}{1 - q \cdot \beta} & \text{if } \alpha = 0 \lor q = 0
\end{cases}
$$

**Proof.** See Appendix D.2.

This bid function is also optimal in an ascending auction. In this case, $b^*(v; \alpha, \beta, q)$ describes the highest value at which winning is worthwhile. In equilibrium a bidder stays in the auction as

---

18Engers and McManus (2007) demonstrate that the optimal strategy in a second-price charity auction is also optimal in the analogous button auction version (Milgrom and Weber, 1982). They also notice that the observation of bidders’ exiting times is not required for the result to hold. Therefore the equivalence can be extended to more general ascending auctions, like online auctions. In such an auction, the FOC are required to hold only for the second highest bidder at the price at which he or she drops out.
long as the standing price, $p$, is below her composite value, $b(v; \alpha, \beta, q)$, and the winner pays the second-highest bid, $b^{II}$. Because the parallel between the button auction and the online auction may not hold for all bidders, in taking this model to the data it is only required that the button auction assumption holds for the second highest bidder.\footnote{This assumption is commonly accepted in the empirical literature on English auctions (Song, 2004, il Kim and Lee, 2014).}

When bidders are not charitable, $\alpha = \beta = 0$, or when the proceeds of the auction are not used to finance any public good, $q = 0$, bidders bid their valuation $b^*(v; 0, 0, q) = b^*(v; \alpha, \beta, 0) = v$. It can be proved that (3.2) describes a global maximum if Assumption 1.3 holds. The model describes a legitimate bidding function because $b^*(v; \cdot)$ is increasing in $v$. The limit of the function in the top row of (3.2) converges to that in the bottom row as $\alpha$ goes to 0. Finally, Proposition 8 in Engers and McManus (2007) demonstrates that revenues in charity auctions are bounded, meaning that under no combination of $\alpha$ and $\beta$ bidders make unlimited transfers to the auctioneer. Their proof holds also in this scenario where $q < 1$.

### 3.1 Charitable motives

The charitable parameters $\alpha$ and $\beta$ describe different models of giving. Table 5 summarizes the most common theories of altruism, which are briefly exposed here in relation to changes of the bid function. The null hypothesis, rejected by the structural estimation in Section 5, is that bidders are not interested in giving. This corresponds to $\alpha = \beta = 0$. In this case, the classic textbook equilibrium applies, and bidding is in no way shaped by altruistic behaviors.

We use different labels for own contribution ($\beta$) and for that from other consumers ($\alpha$), but they do not need to be so. This is the classic case called pure altruism ($\alpha = \beta > 0$), where agents are moved by their compassionate concern for others, as a psychological gain that is independent from the identity of the donor (Halfpenny, 1999). The treatment of these auctions first appeared in Engelbrecht-Wiggans (1994), who analyzed auctions with price-proportional benefits to bidders (all bidders receive an equal share of the final price).

Although donating is an altruistic behavior, it may be the result of selfish motives due to the pride (or reduced guilt) inherent in pro-social behaviors (Fisher et al., 2008). This is studied in the impure altruism literature, championed by Andreoni (1988, 1989) who proposed a model of warm-glow where utility flows from the mere act of being the giver. In this case, we would expect that $\beta > \alpha > 0$. An extreme case of impure altruism is the see-and-be-seen model, where $\beta > 0$ but $\alpha = 0$, that happens when bidders receive utility only from their own contributions. This model captures the role of social status or prestige in donation (e.g., Harbaugh, 1998): donating increases a donor’s prestige in the society, eventually increasing her utility. Situations, where the name of the donors is given to the public, belong to this category.\footnote{Social pressure is another reason for people to donate, but it is not directly related to online auctions. It arises when subjects do not want to donate, but they do so because they are asked by solicitors (e.g., DellaVigna et al.,}
Table 5: Overview of the most common models of giving

<table>
<thead>
<tr>
<th>Model</th>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noncharity ((\alpha = \beta = 0))</td>
<td>Bidders do not pay a premium in charity auctions.</td>
</tr>
<tr>
<td>Pure altruism ((\alpha = \beta &gt; 0))</td>
<td>Bidders obtain extra utility from donating, and are willing to pay a premium. They do not distinguish across sources of donation.</td>
</tr>
<tr>
<td>Warm glow ((\beta &gt; \alpha &gt; 0))</td>
<td>Bidders derive greater satisfaction from their own donation (impure altruism).</td>
</tr>
<tr>
<td>See-and-be-seen ((\beta &gt; \alpha = 0))</td>
<td>Bidders derive utility only from their own donation. Limiting case of warm-glow ((\alpha = 0)).</td>
</tr>
<tr>
<td>Volunteer shill ((\alpha &gt; \beta &gt; 0))</td>
<td>Bidders obtain greater utility from giving by others.</td>
</tr>
</tbody>
</table>

Source: Leszczyc and Rothkopf (2010).

Outside of the fundraising literature, a model where \(\beta\) is the sole positive parameter is similar to a setting where bidders receive subsidies from the auctioneer for each dollar spent. Set-asides and subsidies are commonly used in procurement auctions for natural resources. For example, in their analyses of Californian timber auctions, Athey et al. (2013) find that policies such as subsidies and entry restrictions are welfare improving over set-asides. In this case, the government would promote a \(\beta > 0\) by instituting subsidies.

Notice that the opposite situation where \(\alpha > \beta > 0\) is also possible. This model is called volunteer shill and is characterized by the fact that bidders receive more utility by free-riding than winning the auction. In a second-price auction, this leads low value bidders to increase their bids in order to affect the transaction price (Leszczyc and Rothkopf, 2010).

3.2 Comparative statics

Understanding how individual choices are affected by those of the others is a fundamental factor for successful fundraising campaigns. For example, Shang and Croson (2009) find a positive association between somebody else’s contributions and once own in the voluntary contribution mechanism. However, auctions are different in that only the winner donates.

To address the same question one should look at how bidders affect each others. We do so by evaluating the bidding function after marginal changes in the parameters \((\alpha, \beta, \text{and} \ q)\). While one would expect different models of altruism to result in sharply different predictions for the outcomes accruing to both bidders and auctioneer, this is not necessarily the case. In fact, the

2012, Huck et al., 2015).
extent of bidding above or below one’s private value does not only depend on the relative size of $\alpha$ and $\beta$, but it is also related with the probability that all the opponents have smaller values, $F(v)$.

Figure 2: Comparative statics

(a) Derivative with respect to $\alpha$

(b) Derivative with respect to $q$

Note: Panel (a). The effect of a marginal increase in $\alpha$. Panel (b). The effect of a marginal increase in $q$. In both graphs $q = 85\%$ and $F(\cdot)$ is a truncated normal in $[0, 100]$ with mean 50 and standard deviation $\sigma$ (see the legend).

It is easy to show that the bid functions in (3.2) are unambiguously increasing in the own donation parameter, $\beta$: the greater the positive feeling from being the donor, the more bidders are willing to pay for the item. Instead, the comparative statics are ambiguous for $\alpha$.

Figure 2a illustrates the bid derivative with respect to $\alpha$ over the support of $v$. The figure shows that after a marginal change in $\alpha$ most high-value bidders decrease their bids. This implies that bidders substitute high bids with lower ones the more they expect that their final utility will depend on the bids of the others. Yet, not all bidders choose lower bids. In particular, having low private values is associated with a positive derivative. These bidders are confident that they are not likely to win the auction and thus increase their bids in an attempt to push up the final price. Since they know that other bidders will outbid them, their sole goal is that to increase the standing price of the auction in order to extract surplus from bidders with higher valuations. This intuition can be formalized with the following proposition:

**Proposition 2.** If $\alpha > 0$ bids are decreasing in $\alpha$ for high value bidders.

**Proof.** See Appendix D.3.

Therefore, subjects with low valuations increase their bids, while bids drop optimally for all

---

21In the plot the distribution of values is a truncated normal in $[0, 100]$, and 85% of the proceeds are donated.
the others to offset the higher expected prices. Also, note the stark difference between the two dotted lines: when there is more uncertainty (high variance) the “likely losers” raise their bids further. This happens because uncertainty gives them more chances to set the price.

Next, we address how \( q \) affects \( b(\cdot) \). Of course, if \( \alpha = 0 \) all bidders increase their bid. In fact, as more money goes to charity for each dollar paid, all bidders derive more surplus from winning.\(^{22}\) The same reasoning explains the fact that bids increase in \( q \) in the warm glow case \((\beta \geq \alpha > 0)\) The case \( \alpha > 0 \) is depicted in Figure 2b which shows the derivative of the bid function (with respect to \( q \)) for different values of \( \alpha \).\(^{23}\) Notice that the bid derivative for \( \alpha \) large are strikingly similar to those in Figure 2a, where bidders either post higher or lower bids. This intuition can be formalized too:

**Proposition 3.** Bids are increasing in \( q \) if \( \beta \geq \alpha \). If \( \alpha > \beta \) bids are decreasing in \( q \) for high value bidders.

**Proof.** See Appendix D.4.

A few points are worth highlighting from the proof. First, bids decrease in the fraction donated only for those bidders who are already willing to pay less than what they value the jersey. An even larger \( q \) shifts their demand curve downwards as they rather lose than pay for the price and donate. Second, bidders attaching a small value to the item always raise their bids as they gain from extracting surplus from the winners. Third, because of the monotonicity of the bidding function in \( v \) only bidders with high valuation contemplate decreasing their bids.

Overall, this indicates that if we observe two second-price auctions differing only for the proportion donated, simply comparing changes in the final price across auctions could lead the econometrician to believe that bids are decreasing in the amount donated. In reality, it could be that high-value bidders are shedding their bids in the auction with the greater \( q \) (see Figure 2b and Proposition 3). This could lead the researcher to conclude that bidders do not react to charitable incentives, or that unobservables play a major role in the estimation. Hence, in order to demonstrate the impact of impure public goods on market prices structural methods accounting for these strategic concerns are required.

### 3.3 Surplus

How do bidders fare in charity auctions? Despite the several models of giving, in a second-price charity auction the consumer surplus is at least as large as that from a similar non-charity auction

\(^{22}\)This case is observationally equivalent with an increase in \( \beta \).

\(^{23}\)In this plot, the values are normally distributed in \([0, 100]\), \( q \) is 85\% and \( \beta = 46\% \). These parameters were chosen because they are close to the estimated ones (\( \beta = 46\% \) and \( \alpha = 19\% \)).
Proposition 4. When $\alpha = 0$, the expected consumer surplus in a charity auction is equal to the consumer surplus in a non-charity auction. It is greater when $\alpha > 0$.

Proof. See Appendix D.5. The proof is constructive and reflects the fact that the boost in utility from losing the auction (through the public good) is greater than the (possible) higher price paid when winning it. In fact, notice that in the $\alpha = 0$ case bidders extract the same utility as in the non-charity case. When $\alpha > 0$, the positive externality flowing from the winner to the losers increases bidders’ expected utility further, making bidders in charity auctions better-off.

Finally, does supplying impure public goods boosts a firms net revenues? This is a central interrogative to understand the increasing presence of these arrangements (for example, environmentally friendly goods, such as electric cars or fare-trade coffee brands). In the absence of externality the higher bid paid does not result in higher net revenues to the auctioneer, as bidders account only for a $\beta$ part of the public good funded ($\beta < 1$). Firms should not hold charity auctions in this case.

Small cross-bidders externalities can improve revenues with higher $q$. However, if $\alpha$ is not small enough the transaction prices gets hurt as more high-value bidders benefit by free-riding (Proposition 2). This highlights the peculiar way public goods affect markets creating complex strategic interactions across agents. In a charity auction bids do not only balance the intensive margin (the surplus) with the extensive margin (the probability of winning), but also an additional margin represented by the free-riding. When the latter is kept at bay firms may find charity auctions profitable.

Proposition 5. The producer surplus in a charity auction is below that in a similar non-charity auction when $\alpha = 0$. When $\alpha > 0$ it can be greater or smaller than that in a comparable non-charity auction.

Proof. See Appendix D.6.

To provide an intuition of how bids are affected by $\alpha$ and $\beta$, Figure 3 plots the distribution of bids and private values. In Panel (a) $\beta > \alpha$ and all bidders overbid relative to their private values as the return from winning is much higher than free-riding. Of course in this case the auctioneer earns gross revenues beyond the non-charity auction.

However, charity auctions cannot always guarantee revenues beyond non-charity ones. Figure

\[ q = 0 \] or an auction with selfish bidders ($\alpha = \beta = 0$).
Figure 3: Comparing the bids and private values under different models of giving

(a) Warm glow ($\beta > \alpha > 0$)

(b) Volunteer shill ($\alpha > \beta > 0$)

Note: Distributions of private values (solid line) and bids (dotted line). 50 simulations, 12 bidders. While bidders bid more than their private values in Panel (a), most of the bidders bid below their private values in Panel (b). The primitives of the model are $q = 85\%$, $F(\cdot) \sim \mathcal{N}(50, 25)$ on $[0, 100]$.

3b shows that for certain parameters ($\beta > \alpha > 0$) the auctioneer would improve his records by announcing a non-charity auction instead.\textsuperscript{25} This case mirrors the idea portrayed in Propositions 2 and 3 that high value bidders shade their bids more than the others. When this happens the effect on revenues is uncertain.

The previous results should be interpreted in light of the lack of agreement in the empirical literature assessing the effect of donation on prices and revenues in charity auctions (e.g., Carpenter et al., 2008, Schram and Onderstal, 2009, Isaac et al., 2010, Elfenbein and McManus, 2010a, Leszczyc and Rothkopf, 2010, Elfenbein et al., 2012). For example, Leszczyc and Rothkopf (2010) found in a series of controlled experiments that the volunteer shill model (i.e., $\alpha < \beta$) closely describes certain aspects of bidding in their data. They attributed the high revenues realized by the auctioneer to the desire of bidders with low valuation to affect the transaction price, rather than to the warm-glow effect.\textsuperscript{26}

In contrast, another field experiment found charitable prices to sink below non-charity ones (Carpenter et al., 2008). The goal of the experiment was to compare the revenues of first-price, second-price and all-pay auctions in real markets. The authors found that these auctions recovered only 98%, 66% and between 72% and 52% of the retail value of the auctioned items, respectively. Moreover, goods in first-price auctions were 30% more likely to be sold to the highest value bidders compared to the other formats. These findings can be interpreted in light of

\textsuperscript{25}See Figure 14b in Appendix C.4 for a revenue comparison across charitable and non-charity auctions for this case. Figure 14a compares revenues when $\alpha$ and $\beta$ are equal to those estimated in the structural model.

\textsuperscript{26}Table 5 compares models of altruism.
the model previously exposed, and they suggest the presence of large externalities. First, revenues below non-charity auctions can be explained in terms of Figure 3b, where externalities are large. Second, the authors report poor allocative efficiency in second-price auction compared to winner-pay auctions. Allocative efficiency is higher in first-price auctions because losing bidders do not have the same incentives as in a second-price auction (Engers and McManus, 2007). In fact, in first-price auctions a bidder pays his or her own bid and cannot affect the payment of the others. This further corroborates the suspicion of large externalities because as a result of bid shedding by high-value bidders, lower value bidders may end up winning the auction contest, thus reducing efficiency.27

In conclusion, charitable incentives may affect bids and profits in surprising ways. These comments highlight the importance of properly understanding bidders’ preferences in markets of impure public goods in order to design mechanisms to simultaneously increase profits while raising funds.

4 Nonparametric identification

This section establishes the nonparametric identification of the primitives, $\alpha, \beta$ and $F(v)$, given the observed bids, $b(v; \alpha, \beta, q)$, and the percentage donated, $q$. However, first we state a nonidentification result which indicates which source of variation is needed to identify the model.

To set the notation, let $G(b)$ be the observed distribution of bids, and $\lambda(b)$ be its inverse hazard rate.28 Following the seminal work of Guerre et al. (2000), the bids can be inverted, which implies that the distribution of bids is equal to the distribution of private values.29 Therefore, $G(b(v)) = F(v)$. Replacing $F(v)$ and $f(v)$ with the bid distribution and density respectively in the FOCs reduces the number of unknowns and gives us an equation that must hold for all bidders in second-price auctions (and second highest bidders in online auctions).

$$v = \xi(b, \alpha, \beta, q) = (1 + q \cdot (\alpha - \beta)) \cdot b - q \cdot \alpha \cdot \lambda(b)$$

(4.1)

Obviously, without knowledge of the vector of valuations on the left-hand-side (and a rank

27 In addition, the authors report that allocative efficiency drops with the number of bidders, which is also evidence of large externalities.

28 For simplicity, denote the bid function by $b(v)$ and its realization by $b$. In the remainder of the paper a bid will be denoted by $b(v; \alpha, \beta, q)$ only when it would be confusing otherwise. The inverse hazard rate of the bid distribution is $\lambda(b) = \frac{1-G(b)}{g(b)}$, while the inverse hazard rate of the distribution of private values is $\lambda(v) = \frac{1-F(v)}{f(v)}$.

29 Under Assumption 1 the following relations hold: $G(B) = Pr(b(v) < B) \equiv Pr(v < b^{-1}(B)) = F(b^{-1}(B))$. An analogous relation holds for the pdf $g(b(v))b'(v) = f(v)$.
condition) it is not possible to identify the charitable parameters.

**Proposition 6.** $\alpha$, $\beta$ and $F(v)$ are not identified without additional restrictions.

**Proof.** See Appendix D.7.

The proof shows that two different bids placed under different models are observationally equivalent. In fact, without additional restrictions, the parameters and the distribution can be combined in different ways yielding the same bids distributions.

A similar nonidentification result holds in the estimation of risk aversion in first-price auctions. In this case the econometrician relies on quantile restrictions and on cross-auction variation on the number of bidders to identify the risk parameters (e.g., Guerre et al., 2009, Campo et al., 2011). However, placing restrictions on the number of bidders cannot be a source of identification in the second-price charity auctions because the number of bidders cancels out in the first order conditions.

Let’s turn to (4.1) to figure out the most opportune restriction leading to identification of the primitives. First, notice that $\zeta(b, \alpha, \beta, q)$ is strictly increasing in $b$. In fact, the multiplicative term in front of the bid is positive ($\alpha$, $\beta$ and $q$ are percentages), while $\lambda(b)$ is decreasing in $b$ because of the increasing hazard rate property (Assumption 1.3). Second, the hazard rate and $b$ are nonlinearly related. Thus, there are no two different combinations of $\alpha$ and $\beta$ yielding the same vector of pseudo-private values $v$.

These two observations are key because they ensure that every distribution $F(\cdot|\hat{\alpha}, \hat{\beta}, q)$ (one for each $\hat{\alpha}$, $\hat{\beta}$ combination) is unique by Theorem 1 in Guerre et al. (2000). This theorem relies on the independent private value framework, and on the fact that $\zeta(\cdot)$ is strictly monotonically increasing and differentiable in $b$ (Assumption 1). Therefore, using variation in the percentage donated across auctions, the primitives are identified by distribution equality

$$F(\cdot|\hat{\alpha}, \hat{\beta}, q_A) = F(\cdot|\hat{\alpha}, \hat{\beta}, q_B)$$

with $q_A \neq q_B$. However, this strategy fails if, for example, higher valued bidders self-select in auctions with higher (or lower) amount donated. We require an additional assumption:

**Assumption 2.** Identification: $F(\cdot)$ and $q$ are independent.

The identification restriction is an orthogonality condition between the distribution of private values and the percentage donated. It holds if bidders do no select into different auctions (e.g., auctions with different amount donated) based on their valuations. The model exposed

---

30Theorem 1 in Guerre et al. (2000) is based on a set of regularity conditions: values $v$ are private and independent, the bidding function $b(v; \cdot)$ is increasing and $\zeta(\cdot)$ is strictly increasing in $[\underline{b}, \overline{b}]$ and differentiable. These properties follow from Assumption 1.
in Section 3 automatically satisfies this assumption, because it defines \( F(v) \) as the unconditional distribution of private values. In the model, \( q \) modulates the net utility to a bidder through the origination of a public good: in no way it affects \( v \). Moreover, Section 2.2 provided supporting empirical evidence showing that Charitystars’ bidders do in fact bid in multiple auctions without regard for \( q \). Also, the same section showed no correlation between the number of bidders and \( q \).\(^{31}\) All these stylized facts corroborate the idea that \( q \) does not affect the valuation bidders hold for the private good (the auctioned item), but only the composite valuation for the impure good. In addition, we maintain the optimization condition (in Assumption 1) which requires that bidders’ valuation come from the same distribution and respect some regularity conditions.\(^{32}\) Upon estimation of the model, the validity of these conditions can be tested (see Section 5.3).

**Proposition 7.** In second-price auctions, under Assumption 1 the parameters \( \alpha \) and \( \beta \) and the distribution of values \( F(v) \) are identified by variation in \( q \) across auctions.

**Proof.** See Appendix D.8.

The intuition behind this proposition is rather simple. Assume that the econometrician observes two auctions, \( A \) and \( B \), conforming to the assumption of Proposition 7. In a symmetric equilibrium, two bidders with equal valuations (e.g., \( v^A = v^B \)) taking part in two auctions that differ only with regard to the percentage donated (i.e., \( q^A \neq q^B \)) will place different bids (i.e., \( b(v; \cdot, q^A) \neq b(v; \cdot, q^B) \)) according to (3.2), but the ranking of their bids on their respective bid distribution will be the same (i.e., \( G^A(b(v; \cdot, q^A)) = G^B(b(v; \cdot, q^B)) \)). This is because for any bidder whose private value is at the \( \tau \)-quantile of the distribution of private values, also their bids will be at the \( \tau \)-quantile of the bid distribution (by monotonicity of \( b(v) \)). This observation allows us to simplify further (4.1) by taking difference across the two sets of FOCs along the quantiles of the bid distributions. \( \alpha \) and \( \beta \) are identified under the full-rank condition of the resulting matrix, while \( F(v) \) is identified by plugging the identified parameters in the RHS of (4.1).

This approach is related to Lu and Perrigne (2008) who identify risk-averse utility functions nonparametrically using a combination of first-price and English auctions. Since risk-aversion does not affect equilibrium bidding in the latter, they first recovered the distribution of values from the open auctions, and then plugged its quantiles in the FOC for the bidders in first-price auctions. This is equivalent to solving (4.1) knowing the private value on the left-hand side. Shifters similar to \( q \) have also been used to study correlated private values in English auctions.

\(^{31}\)The Spearman correlation between the number of bidders and \( q \) in Charitystars’ auctions is also very low. It is 0.0756 and 0.0564 in the large and small sample respectively.

\(^{32}\)We quickly discuss this assumption here. First, the auctions used originate from the same website, Charitystars.com, and the same bidders bid in all types of auctions (see Section 2.2). This support the assumption that bidders’ valuation are equally distributed across different auctions. Second, Section 2.2 failed at singling out a clear pattern of asymmetric behavior across bidders. Therefore, the assumption of constant \( \alpha \), \( \beta \) and \( F(\cdot) \) across auctions and bidders appear reasonable, especially after controlling for observables and unobservable heterogeneity.
Proposition 7 can also be extended to include a finite number of auction types (e.g., \( q \in \{q_1, q_2, ..., q_K\} \)), as shown in the following corollary. The proof uses the panel structure of the data to create a projection matrix that cancels out the left-hand side of (4.1) in a way akin to the previous proposition.

**Corollary 1.** In second-price auctions, \( \alpha, \beta \) and \( F(v) \) are nonparametrically identified also when the dataset includes more than 2 types of auctions.

**Proof.** See Appendix D.9.

Despite these identification results, these procedures do not cover Charitystars’s auctions for two reasons. First, to win an English auction bidders bid the second-highest bid plus an increment. While minimum increments on Charitystars are negligible, and thus constitute no harm for identification (Haile and Tamer, 2003, Chesher and Rosen, 2017), this statement means that the winner’s bid does not solve (4.1). Second, the equivalence between English auctions and button auctions may not hold for most bidders. Thus, the willingness to pay for all bidders but the second-highest one is not identified.\(^{33}\) As a result, most bids do not reflect the private valuations, and one cannot use the monotonicity of the bidding function to determine the distribution of private values from the observation of the distribution of bids.\(^ {34}\) The next proposition amends Proposition 7 and ensure identification of the primitives under this circumstances.

**Proposition 8.** In English auctions, \( \alpha, \beta \) and \( F(v) \) are nonparametrically identified by first deriving the distribution of bids that would have been observed in parallel second-price auctions, and then by applying Proposition 7.

**Proof.** See Appendix D.10.

The precise meaning of “parallel second-price auction” is a second price auction with the same primitives as those in the English auction. The distribution of values in a parallel auction can be found by observing only the winning bids. In fact, this bid identifies the price at which the second-highest bidder opts-out of the auction. Therefore, the FOC (4.1) must hold at this price for this bidder.\(^ {35}\) In addition, the existence of a one-to-one mapping between the distribution of bids and that of the second-highest bids (following the theory of order statistics) ensures that the former is identified. This would be equal to the distribution of bids observed in a parallel auction.

---

\(^{33}\)The bidding process in online auctions may incorrectly rank bidders. For instance, if Bob values the good $100 but he bids only $40 at first, while Claudia and Alice bid immediately their true values of $80 and $120, Bob will be mistakenly associated with a private value of $40 instead of $100.

\(^{34}\)Formally, from the observed bids one cannot guarantee that \( b^{-1}(v) = v \), and thus that \( F(V) = \Pr(v \leq V) = \Pr(b(v) \leq b(V)) = G(b(V)). \)

\(^{35}\)Note that the minimum increment in Charitystars is $1 for most auctions.
second-price auction\textsuperscript{36}. At this point, we have all ingredients to identify the primitives following Proposition 7 (or Corollary 1).

Symmetric bidding is a crucial assumption. Theorem 6 in Athey and Haile (2002) shows that the primitives of an asymmetric auction model are identified from the winning bids only if the identity of the bidders is known. Still, the assumption of symmetric bidding in charity auctions may not be so restrictive. First, there does not seem strong evidence suggesting asymmetric behavior in Charitystars (see Section 2.1). Second, Elfenbein and McManus (2010a) provide empirical evidence of symmetric bidding for the highest bidders in a study of eBay charity auctions\textsuperscript{37}.

5 Estimation method and results

The estimation closely follows the identification in Proposition 8 exposed in the previous section. However, additional difficulties comes from the need to deal with auction heterogeneity. Controlling for observable and unobservable heterogeneity is important as exposed in Section 2.1. The estimation procedure includes three steps and is described in the next section. Sections 5.2 reports the results and 5.3 performs out-of-sample validation using additional data and simulations.

5.1 Structural estimation

The estimation is based on comparing two types of auctions with different $q$. Figure 1 indicates that in most auctions the auctioneer donates either 85%, 78% or 10%. In fact these auctions make up almost 85% of the whole dataset, and therefore we focus on these auctions. Clearly, as $q^A \rightarrow q^B$ the necessary rank condition fails to hold and the model is not identified\textsuperscript{38}. Thus, the primitives are estimated using the sample of auctions with $q \in \{10\%, 85\%\}$. The remaining auctions ($q = 78\%$) will be helpful to validate the estimates with an out-of-sample analysis.

Given that the samples are not random, an important empirical issue is whether auctions at 10% systematically differ from the others. This would complicate the comparison between the two sets of auctions. A logistic regression in Appendix C.2 explores this further by analyzing the probability that a listing is chosen to be at 10% or at 85%. Table 16 reports the results. The most

\textsuperscript{36}Denote with a subscript $w$ the winning bid and its distribution. The distribution of bids from a “parallel second-price auction”, $G(b) = \Pr(b(v) \leq b)$, is found by a reparametrization in $n$ of the distribution of the winning bid $G_w(b)$ which is obtained from the data. In fact, the latter is equal the distribution of the second highest bid among $n$ bidders $G_2^{(n)}(b)$. That is $G_w(b) = n \cdot G(b)^{n-1} + (n-1) \cdot G(b)^n$.

\textsuperscript{37}Furthermore, the notion of symmetry adopted in this paper does not imply that bidders cannot bid asymmetrically across auctions. For example, given a large dataset, symmetry can be relaxed by allowing the giving parameters to change across charity characteristics (each auction is linked to a given charity), or other observables, using a logit construction for the parameters $\alpha$ and $\beta$.

\textsuperscript{38}This is confirmed by a number of Monte Carlo simulations based on a small difference in the percentage donated ($q^A - q^B < 10\%$). See Table 22 in Appendix E.
important regressors to be accounted for are the reserve price, the number of bidders and some charity dummy variables. It will be therefore necessary to account for these observables in the estimation.

The first step of the estimation procedure deals with auction heterogeneity. A common approach to deal with observables is to perform a hedonic regression of the bids on covariates, and use the error terms as pseudo-winning bids (Haile et al., 2003). The advantage is that it pools all the data together by homogenizing bids into residuals ($\epsilon = b - x'\gamma$), ultimately offering a very tractable way to analyze the data.

One of the major shortcomings of this approach is that it fails to properly account for unobservables. However, if unobservables and reserve prices are positively affiliated, Roberts (2013) shows how a control function approach can be used to account for them. In fact, in case the reserve price is not set optimally by the auctioneer, oscillations in the reserve price may reflect different market conditions (e.g., news, or charitable events) or object characteristics that affect bids but are not explicitly accounted for in the data (e.g., the jersey was worn and sold right after the player won a particular individual award which is not captured by the covariates). According to Engers and McManus (2007) if the reserve price is set optimally it should be a function of the charitable motives, and thus a function of $q$. However, $q$ and the reserve price are not correlated in Charitystars’ data, suggesting that the latter are not set optimally.

We proceed as follows. The first step performs an OLS regression of the logarithm of the reserve price on the covariates (i.e., $\log(\text{reserve price}) = \delta_0 + x \delta$). Denote the generated regressor by $\hat{U}H$ (i.e., $\hat{U}H = \log(\text{reserve price}) - \hat{\delta}_0 - x \hat{\delta}$). Successively, the following hedonic regression

$$\log(price) = \gamma_0 + x \gamma + \hat{U}H \gamma_{UH}$$

(5.1)

yields the homogenized bids. (5.1) regresses the logarithm of the transaction price on the observed ($X$) and the unobserved heterogeneity ($\hat{U}H$). The error term in (5.1) are the homogenized pseudo-winning bids, $b_w$, which will be used in the next two steps of the estimation. Table 15 in Appendix C.2 displays the estimated coefficients from (5.1) for both samples.

In Section 2.1 it was entertained the possibility of asymmetry among bidders who won multiple auctions and the other bidders. As an additional check, we would expect that the distributions of the pseudo-winning bids from (5.1) conditional on the winner being an object collector to be much more skewed to the right compared to that of the other bidders, as collectors should bid higher values on average. The data fail to support this thesis. Instead, the Kolmogorov-Smirnov test does not reject equality of the distributions at 0.1422 in the large sample and 0.1520.

---

39. This method is widely used and appears also in Krasnokutskaya (2011) and il Kim and Lee (2014) among others.

40. Figure 9b in Appendix B plots the pdf of the reserve price for the two auctions.

41. $x$ includes all the variables labelled Main Variables in Appendix B. This set of covariates corresponds to column (II) in Table 2, which has the lowest BIC. Note that the percentage donated (and the reserve price which is used in the control function approach) is not used in either OLS regressions, as variation over $q$ is key for identification.
in the small sample.\footnote{Figure 13 in Appendix C.4 plots the pdfs.} This result further confirms that the alleged weak asymmetry vanishes once accounting for unobserved heterogeneity, either by using a more homogeneous sample (as in the last two columns of Table 14) or by a control function approach (as in 5.1).

The second step is concerned with the derivation of the distribution of bids in the “parallel second-price auction” as by Proposition 8. Let the superscript \( a \in \{10\%, 85\%\} \) indicate that a variable belongs to auctions at 10\% and 85\%, respectively. Following the proof, the distribution of bids, \( G^a(b) \), is obtained by observation of the winning bids \( b^a_w \). Denote the distribution of the winning bids by \( G^a_{w}(\cdot) \). Then \( G^a(b) \) is found as the solution in \([0, 1]\) of

\[
G^a_{w}(b) = nG^a(b)^n - (n - 1)G^a(b)^n
\]

(5.2)

where the second term in brackets is the distribution of the second-highest order statistic expressed in terms of the primitive \( G^a(b) \). Solving, we obtain the distribution of bids that would have been observed in the parallel sealed-bid auction. The density \( g^a(b) \) is found similarly. The derivative with respect to \( b^a \) of equation (5.2) is:

\[
g^a_{w}(b) = n(n - 1)G^a(b)^{n - 2}[1 - G^a(b)]g^a(b),
\]

(5.3)

which uniquely pinpoints the density of the bids, \( g^a(b) \). To solve (5.2) and (5.3) we need to estimate first \( G^a_{w}(b) \) and \( g^a_{w}(b) \). This is done by a Gaussian kernel whose bandwidths are chosen according to Li \textit{et al.} (2002).\footnote{The bandwidth of the kernel estimators are \( h^a_{G} = c^a \cdot T_a^{-1/5} \) for each pdf and \( h^a_{G} = c^a \cdot T_a^{-1/4} \) for each CDF, where \( c^a = 1.06 \cdot \min\{\sigma^a, IQR^a / 1.349\} \), \( T_a \) is the number of auctions of type \( a \in \{l, h\} \), \( \sigma^a \) is the standard deviation and \( IQR^a \) is the interquantile range of the transaction prices of auction \( a \). Trimming is employed to account for the bias at the extreme of the support of the bids.}

In the last step, \( G^a(b) \) and \( g^a(b) \) form the inverse hazard rates, \( \lambda^a(b) \), for each auction format \( a \). The objective is to match the FOCs for the two types of auctions along the quantiles of the bid distribution. According to the model, at the \textit{true} parameters the LHS of (4.1) for the 85\% auctions at the \( \tau \)-quantile, \( \hat{\lambda}^{85\%}_\tau \), is equal to the LHS of the same equation for the 10\% auctions at the same \( \tau \)-quantile, \( \hat{\lambda}^{10\%}_\tau \). This delivers the moment condition

\[
\hat{\lambda}^{10\%}_\tau - \hat{\lambda}^{85\%}_\tau = 0.
\]

The number of possible moment conditions is theoretically infinite. This issue is solved by matching the quantiles of the bid data for the 10\% auctions with the smoothed version of the 85\% auction data, for which more observations are available (and therefore we can approximate of the distribution of bids with more confidence). Define \( \Theta = \{\alpha, \beta\} \); the criterion function to be
minimized is:

\[
\Theta^* = \arg \min_{\Theta} \frac{1}{T} \sum_{\tau} \left( \hat{v}_{10\%}(\Theta) - \hat{v}_{85\%}(\Theta) \right)^2
\]  

(5.4)

where \( T \) is the number of 10% auctions. The minimization algorithm searches for the values of \( \alpha \) and \( \beta \) minimizing the criterion function in the admissible region \( (\alpha, \beta \in [0,1]) \). Finally the distribution of private values \( F(v) \) is found as the empirical distribution of the left-hand side of (4.1).

The performance of the estimator is good even in small samples as shown by the Monte Carlo simulations reported in Table 23 in Appendix E.\(^{44}\) The simulations suggest asymptotic normality of the estimator as the RMSE decreases at a rate close to \( \sqrt{n} \) when the number of auctions increases.

### 5.2 Estimation results

A requirement of the structural estimation is to fix the number of potential bidders to compute the distributions of the pseudo-bids from the pseudo-winning bids (as shown in equation 5.2). Setting the number of bidders at the 99th-quantile of the distribution of bidders is a good choice as it avoids concerns with outliers. In fact, extremely high number of bidders may depend more on the item characteristics rather than on the distribution of values and parameters. Additional estimations of \( \alpha \) and \( \beta \) for lower quantiles of \( n \) are to be considered as robustness checks.

Table 6 reveals that \( \beta > \alpha \), coherent with the warm-glow model of altruism (Andreoni, 1988, 1989). The estimates hardly vary with the number of potential bidders and are always significant. The 5% confidence interval are also reported in square brackets and are obtained by bootstrap. Finally, notice that the CI for \( \alpha \) are smaller than those for \( \beta \) across all rows.

The same exercise is reported in Table 17 in Appendix C.2, where the dataset is restricted to observations with prices greater than \( €100 \) and smaller than \( €400 \). The \( \alpha \) parameter stays roughly unchanged, while its counterpart \( \beta \) has slightly dropped (ca. 38% instead of 46%), but still greater than \( \alpha \). The confidence intervals are much larger than in the previous estimates which do not rule out a greater \( \beta \) than that estimated in the smaller sample. This is probably due to the smaller number of observations which is only ca. 60% of those in the larger database (470 auctions in total). Overall, the smaller sample confirms that bidders take into account the public good according to warm-glow.\(^{45}\)

Finally, Tables 18 and 19 in Appendix C.2 report estimation results when the three steps of the estimation routine are applied to the auctions with \( q = 10\% \) and \( q = 78\% \). The estimated \( \alpha \) and \( \beta \) are close to those in Tables 6 and 17 and the confidence intervals largely overlap.

\(^{44}\)A look at the median columns shows that the Gaussian kernel outperforms the Triweigh kernel in small samples.

\(^{45}\)Changing the covariates does not qualitatively affect these estimates.
Table 6: Estimation of $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>Number of bidders</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantile</td>
<td>[5% CI]</td>
<td>[5% CI]</td>
</tr>
<tr>
<td>99%</td>
<td>16</td>
<td>19.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[10.0%, 29.1%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>19.3%</td>
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<tr>
<td></td>
<td></td>
<td>[10.3%, 28.6%]</td>
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<tr>
<td>90%</td>
<td>12</td>
<td>19.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[10.4%, 27.1%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>18.6%</td>
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<tr>
<td></td>
<td></td>
<td>[9.2%, 27.3%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>17.4%</td>
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<tr>
<td></td>
<td></td>
<td>[7.9%, 27.1%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). Dataset restricted to all auctions such that $q \in \{10\%, 85\%\}$ and price between €100 and €1000. 731 observations in total.

5.3 Model fit

A simple way to validate the estimation routine is to apply the estimates to the 78% auctions and check whether the implied distribution of values differ from that estimated in Section 5.2. This can be done in multiple ways. The most challenging is to apply the full three-step procedure to the 78% data. This consists in (i) manipulating the winning bids and covariates with the coefficients estimated in the two regressions at the first-step of the estimation procedure (using the coefficient in equation 5.1). Then, (ii) the distribution of bids are computed following (5.2) (and (5.3) for the density). Finally, (iii) the private values are determined assuming the same $\hat{\alpha}$ and $\hat{\beta}$ exposed in Table 6.

Figure 4a plots the densities from auctions at 10% (solid line) and those at 78% (dotted line) for $n = 16$. The two pdfs have similar shape, displaying a slight bimodality at the same values. The Kolmogorov-Smirnov test does not reject the null hypothesis that the two distributions are equal at 0.1154 level. The outcome does not change if we modify the first step and perform the regression analysis on the pooled auction database consisting of all observations with $q \in \{10\%, 78\%, 85\%\}$.

The same exercise can also be repeated on a different set of covariates. For example, Appendix C.3.1 report estimates of $\alpha$ and $\beta$ when the Total Number Of Bids Placed is added as a dependent variable. First, notice that the estimates do not vary substantially from those in Table 6. Second, Figure 21 show that the pdf from a similar out-of-sample analysis does not reject the

An analogous plot is found for other values of $n$ and the KS test always fails to reject identity of the distributions.
null hypothesis of equality of densities.\textsuperscript{47}

This is a remarkable result: the distribution of private values computed using the external data cannot be distinguished from that found by matching the FOCs of two types of auctions. This is a direct test of Assumption 2 because dependence between $q$ and $F(\cdot)$ would immediately return an incongruence in the estimates. A similar result would be expected in case of heterogeneous $\alpha$ and $\beta$. Furthermore, this test clearly suffer from auction heterogeneity as the auctions ($\{10\%, 78\%, 85\%\}$) vary both in terms of timing (e.g., period of the year) and the object characteristics (both observable and unobservable). Despite these difficulties, the methodology well control these issues. In addition, this also implies that the model predicted prices are accurate.

Another way to validate the estimates in Table 6 is to simulate bids and try to match them with some stylized facts highlighted in Section 2.1. A striking result from that section was the linearity between the logarithm of the price and the percentage donated. Figure 4b replicates this finding by plotting the simulated (y axis) bids for different percentages donated (x axis). The three curves (corresponding to the bids placed by bidders with values at the first three quartiles of the estimated $F(\cdot)$) are remarkably linear and increasing in $q$, as previously highlighted (compare with Table 3 and Figure 10)

\textsuperscript{47}The p-value of the Kolmogorov-Smirnov test is 0.1874.
These checks confirm the goodness of the estimates of $\alpha$ and $\beta$. These tests provide indirect support for the way nonpecuniary motives are modelled in the theoretical literature of auctions with externalities and, more generally, revenue-dependent profits (e.g., Engelbrecht-Wiggans, 1994, Goeree et al., 2005, Engers and McManus, 2007, Lu, 2012). To simplify derivations and the presentation of the results most of these models assume constant marginal return to charity revenues. Yet this (strong) assumption has never been tested. An additional contribution of the analysis of this paper is to provide empirical evidence that does not oppose this modeling shortcut. Ultimately, this analysis delivers additional credibility to the theoretical results in the literature.

6 Counterfactuals and discussion

The estimation of the structural model suggests that bidders hold warm-glow motives. However, how do the estimated parameters affect market outcomes? This section discusses how bids are shaped by (i) the charitable motives and (ii) the distribution of values. The next section (Section 6.1) investigates the optimality of the charity auctions in terms of revenues and compares it with non-charity auctions.

A first concern in charity auction is to understand whether there exists a premium to giving. For example Elfenbein and McManus (2010a) finds a 5-7% price increase in charity auctions. The structural estimates in the previous section allows us to observe the full distribution of bids and not just the transaction prices. Figure 5 plots the distributions of bids (dotted line) and private values (solid line), suggesting stochastic dominance of bids on values. Thus all bidders increase their bids beyond their private values. This means that (for any given number of bidders) the expected second-highest bid from the second-price auction is larger than the second-highest private value. Hence, gross revenues (i.e., including the donation) to Charitystars are greater than those from a comparable non-charity auction.

Since all bidders place bids greater than their marginal values, free-riding seems to be a marginal concern in Charitystars data. This could indicate the presence of some sort of punishment mechanism to lessen the free-riding incentives (e.g., Fehr and Gächter, 2000, Andreoni et al., 2003). In particular, the decision of low-value bidders to raise their bids can be interpreted as a punishment for their high-value counterparts for not contributing as much as they should. In fact in such a situation, due to the large $\beta$, the last group of bidders can still obtain a considerable utility by increasing their bids and winning the contest. Therefore the relation between $\alpha$ and $\beta$ highlights an interesting duality. When $\alpha$ is higher free-riding may be an issue that could cost the auctioneer lower revenues, while when $\beta$ is higher punishment ensures higher

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48 For example, Goeree et al. (2005) at page 906 assert to “keep the constant marginal benefit assumption because it provides a tractable model [...]”. Outside of the charity auction literature, models of direct donation prefer instead to assume a diminishing utility on the marginal dollar donated (e.g., Andreoni, 1990).
Having estimated $F(v)$, we can experiment with different $\alpha$ and $\beta$ parameters to assess the elasticity of bids to changes in the charitable parameters. For example, we showed in Figure 3b that a harsh version of the volunteer-shill model of giving ($\alpha = 0.5$ and $\beta = 0.1$) may result in lower gross revenues to the auctioneer in charity auctions than non-charity auctions. Figure 6b performs the same analysis with the estimated distribution of values. The CDF shows that all bidders below the 0.8 quantile of the value distribution increase their bids substantially, while the remaining bidders shade their bids below valuation. Additional computations show that even though 20% of the highest value bidders submit smaller bids, expected revenues are still above non-charity auctions. Similarly, when $\beta > \alpha$, revenues in charity auctions dominate those in non-charity auctions (Figure 6a).\(^{49}\) This analysis shows graphically that the outcome to the auctioneer does not only depend on $\alpha$ and $\beta$, but also on how these two parameters relate with the distribution of values. In fact, it is the latter to establish the likelihood of a bidder to win or lose the auction, which in turns triggers the willingness of the bidder to raise the price or free-ride based on the charitable motives.

\(^{49}\)Note that as $q \to 0\%$ the effect decreases until the solid and dashed curves match. Therefore, little difference is to be expected both in optimal strategies as well as in expected revenues when only a small fraction of the final price is donated. This is also mirrored in the regression of prices on $q$. 

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Note: Bids are computed drawing 200 values from the estimated distribution of private values $F(v)$ and the estimated $\alpha$ and $\beta$. 

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Figure 6: Counterfactual scenario – different charity parameters

(a) Estimated $F(v); q = 1$

Warm glow ($\beta > \alpha > 0$)

(b) Estimated $F(v); q = 1$

Volunteer shill ($\alpha > \beta > 0$)

Note: Bids are computed drawing 200 values from the estimated distribution of private values $F(v)$ and the selected $\alpha$ and $\beta$. Panel (a) assumes warm-glow bidding while Panel (b) assumes volunteer shill bidding. Only auctions with price between €100 and €1000.

6.1 Revenues

Before discussing the revenues to Charitystars we present an additional robustness check for the estimates. Table 7 compares the simulated and realized median and average revenues. These prices are computed as the expectation of the second-highest bid using the estimated primitives. To account for auction heterogeneity, these quantities are transformed in euros by adding back either the average or the median value of the prices from (5.1). The table displays a good fit of the estimates with values within 10% of the realized ones.\footnote{The computation of the expected revenues is standard (e.g., il Kim and Lee, 2014). First, to find the expected (or median) revenues from the homogenized auctions we integrate the bidding function (equation 3.2) with respect to the distribution of the second highest-bid, yielding $\eta_b = \int b(t; \hat{\alpha}, \hat{\beta}) dF^{(2)}(t)$. Second, we evaluate the entity of the cross-auctions heterogeneity as $\eta_X = \log(price) - \hat{\gamma} \cdot \log(bidders)$, where $\hat{\alpha}$ indicated the fitted (or estimated in case of $\gamma$) value of each covariate $x$ in (5.1). Finally, the expected (or median) revenues is the sum of $\eta_b$ and the average (or median) value of $\eta_X$. In the second step, the effect of the number of bidders is subtracted from the fitted prices because this variable already affects the first step through the distribution of the second highest bid. Since the number of bidders may correlate with other unobservables, the results reported in Table 7 are obtained by including also the log(Total Number of Bids Placed) among the covariates in the first step of the estimation routine (equation 5.1). Additional robustness checks in the appendix show that adding this variable does not qualitatively change the estimated $\hat{\alpha}$ and $\hat{\beta}$ (see Appendix C.3.1).}
Table 7: Estimated revenues vs realized revenues

<table>
<thead>
<tr>
<th></th>
<th>$q = 85%$</th>
<th></th>
<th>$q = 10%$</th>
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<tbody>
<tr>
<td></td>
<td>Estimated</td>
<td>Realized</td>
<td>Estimated</td>
<td>Realized</td>
</tr>
<tr>
<td>Median revenues</td>
<td>353.58</td>
<td>347.50</td>
<td>299.27</td>
<td>300.50</td>
</tr>
<tr>
<td></td>
<td>(+1.74%)</td>
<td>(-0.41%)</td>
<td>(-4.89%)</td>
<td>(-10.09%)</td>
</tr>
<tr>
<td>Average revenues</td>
<td>355.78</td>
<td>374.09</td>
<td>301.14</td>
<td>334.92</td>
</tr>
<tr>
<td></td>
<td>(-4.89%)</td>
<td>(-10.09%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimated median and average unitary revenues for Charitystars. Revenues in euros are computed in multiple steps. (i) We subtract the median number of bidders times its estimated coefficient in the OLS regression (5.1) from the fitted values of the same regression. (ii) We compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 5.1 ($F(\cdot, a, \beta)$). (iii) We sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. Realized revenues are determined at the median number of bidders for each auction type. The covariates used in (5.1) include the total number of bids as in Appendix C.3.1.

Second-price auction (dashed lines), and two alternative mechanisms: the all-pay auction (dotted lines) and a non-charity second-price auction (solid line).\(^51\) Engers and McManus (2007) theoretically show that first-price charity auctions perform worst than the other two charitable formats studied in this section and are not reported in the figure to simplify the exposition.\(^52\)

In both figures, the second price format outperforms the all-pay auction for all number of bidders in the graph. Charitystars cannot raise more funds by switching to all-pay auctions, a method that was highlighted in the theoretical literature as the best mechanism from a revenue standpoint, especially when the pool of bidders is large (e.g., Goeree \textit{et al.}, 2005, Engers and McManus, 2007).\(^53\)

Importantly, the role of $\beta$ should not be undermined. Goeree \textit{et al.} (2005) studied revenues equivalence across different mechanisms when $\alpha = \beta > 0$ and concluded that all-pay auctions are the revenue maximizing mechanism.\(^54\) Therefore, the optimality of the second-price auction

\(^51\)Revenues are computed as described in the note to Table 7. The optimal bid in an all-pay auction is $b^A(v) = (vF(v)^{n-1} - \int_v^\infty F(x)^{n-1}dx) / (1 - q \cdot \beta)$, similar to the equilibrium studied in Engers and McManus (2007) for the $q = 1$ case.

\(^52\)This is also true for Charitystars’ data. The intuition for this outcome is that in a first-price auction losing bidders do not have an incentive to increase their bids. This happens because these bidders cannot affect the transaction price as they do in second-price auctions.

\(^53\)In the charity all-pay format each bidder gains from the sum of the contributions of the others and can accept to bid a value equal to her own private profit. This cannot happen in winner-pay formats because such a bid would be suboptimal to surely loosing the auction and earning the value of the externality. This reasoning does not apply when the number of bidders is low. In this case the total contribution in the all-pay format is not high enough and bidders shades their bids as a result, making the second-price auction the optimal choice for the auctioneer. It can be shown numerically that the difference in Figure 7 between all-pay and second-price auctions goes asymptotically to 0 with the number of bidders.

\(^54\)They prove that the lowest-price all-pay auction augmented with reserve price and entry fee is the optimal mechanism across competing charity auctions. An analysis of reserve prices and entry fees is outside the scope of this paper.
over the all-pay auction for CharityStars depends directly on the higher satisfaction by winning and donating.

An additional reason to prefer second-price auctions over all-pay auctions is the variance of the expected revenues. Figure 15 in Appendix C.4 shows that the variance of the all-pay auctions sharply increases with the number of bidders, while it decreases with $n$ for sealed bid formats. Therefore, English auctions do not only maximize CharityStars’ expected revenues, but also reduce the volatility of its cash flows: a key objective for any start-up. This fact could explain the preference for English auctions among most online charity marketplaces (e.g., CharityStars.com, eBay for Charity, CharityBuzz.com, BiddingForGood.com and many others).

To properly understand the optimality of charity auctions, it is important to compare their revenues with those from standard non-charity auctions. In fact, despite the philanthropic mission, CharityStars is a for-profit start-up, which has undergone five funding rounds since 2013 totalling to $3.9$ million. The firm counts important venture capital funds among its investors such as 360 Capital Partners, which invested $2$ million in 2016.\footnote{More information on the corporate nature of CharityStars are available at https://www.crunchbase.com/organization/charitystars.}

The comparison across charity and non-charity formats is also in Figure 7a. The revenue dif-

\begin{figure}[h]
\centering
\begin{minipage}[c]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{chart1a}
\caption{Expected gross revenues per auction}
\end{minipage}\hspace{0.5cm}
\begin{minipage}[c]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{chart1b}
\caption{Expected gross revenues; $q = 10\%$}
\end{minipage}
\end{figure}

Note: Revenue comparison across mechanisms for different number of bidders. The percentage donated is set to 85% and 10% in Panel (a) and Panel (b) respectively. The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Revenues in euros are computed in multiple steps. (i) We subtract the median number of bidders times its estimated coefficient in the OLS regression (5.1) from the fitted values of the same regression. (ii) We compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 5.1 ($F(\cdot), \alpha, \beta$). (iii) We sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. Realized revenues are determined at the median number of bidders for each auction type. The covariates used in (5.1) include the total number of bids as in Appendix C.3.1.
ference between the two type of auctions appear to be rather small, reaching little above $50 for high \( n \). Still, this implies a \( \sim 14\% \) premium over the non-charity auctions, that is twice as much as that estimated for eBay for Charity (Elfenbein and McManus, 2010a). Since the auctioneer forfeits 85\% of the transaction price (10\% on panel b), replacing the charity auctions with a non-charity one is a profitable deviation for the auctioneer (under same marginal costs). For example, donating 85\% of the revenues implies forgoing about €300 on average, at the median number of bidders (7). Instead, with a non-charity auction the auctioneer would make about €260, without donating a cent. The current policy results in over €200 deficit. Moreover, the deviation seems so large that even if setting non-charity auctions was much more expensive for the company (e.g., the donation is a form of publicity to the VIPs, which could make them more inclined to provide the items), it would still be more profitable for Charitystars to keep for itself a much larger portion of the final unit price.\(^{56}\)

An alternative explanation is that the auctioneer could enjoy contributing to the public good itself. This could explain the preference for charity auctions with such a large \( q \) (remember that for over half of the dataset \( q = 85\% \), as seen in Figure 2b). This is consistent with recent studies on the motivation of private investments in socially responsible firms. For example, using a combination of administrative data, surveys and experiments, Riedl and Smeets (2017) find that investors in socially responsible firms are satisfied with more modest financial performance and explain this finding in terms of prosocial preferences.

It should be noted that alternative charitable mechanisms exist. In particular, lotteries and raffles are widely studied (Morgan, 2000, Morgan and Sefton, 2000) and used in practice, while new mechanisms have been recently proposed (Carpenter et al., 2014, Duffy and Matros, 2016). Despite this, the efficiency of auctions in awarding the bidder with the highest value makes it the best way to raise funds, and especially so when bidders share a preference for being the donor (Goeree et al., 2005). However, there are several limits with the comparisons across auctions just mentioned. Differential bidder-entry across mechanisms is one of them. This may be problematic if Charitystars users had a stronger taste for certain auctions than others. For instance, according to Figure 7b when 17 bidders are expected in an all-pay auction but only 6 in a second-price auction, switching to the all-pay auction would grant Charitystars revenues in excess of €50 over the second-price model.\(^{57}\)

Overall, the results reported in this section are corroborated by the empirical and theoretical

\(^{56}\): To make this point clear, notice that on average the price of a soccer jersey is €100. Thus, factoring in the expenditure for the jersey (instead of obtaining one for free) and the costs to get them signed by the players, setting \( q = 0 \) would still yield higher profits.

\(^{57}\): A field experiment by Carpenter et al. (2008) reported a similar result: first-price charity auctions delivered higher revenues than all-pay auctions, a surprising finding. They explain this result by greater entry in the former format. Limited familiarity with certain mechanism may similarly sway away bidders or reduce their ability of optimizing their payoffs. In this regard, deviations from theory are confirmed by lab experiments as bid below the Nash equilibrium are systematically recorded for those bidders with little confidence with this format (e.g, Schram and Onderstal, 2009, Corazzini et al., 2010).
evidence in the previous sections. On one hand, the OLS regressions argue in favor of a positive association between donations and prices. This provides first evidence that bidders consider the public good when bidding. On the other hand, the structurally estimated distribution of bids stochastically dominates the distribution of private values. Warm-glow motives can explain this in terms of a higher return from winning the auction and being the donor than from the mere private benefits from somebody else’s contribution toward a public good. This results in revenues above those from comparable non-charity auctions. Moreover, revenues are increasing in the amount donated (Figure 4b) as free riding in this case is minimal, supporting the reduced form results.

Our results contrasts the conclusions of Lesczycz and Rothkopf (2010) who provided experimental and anecdotal evidence suggesting a dominant role played by the externality. According to their analysis, this leads low-value bidders to bid more aggressively in an attempt to gain by undercutting the winner’s surplus (consistent with the volunteer-shill model of altruism based on $\alpha > \beta > 0$). However, if $\beta$ is not large enough, this paper shows that a high externality may result in reduced transaction prices. Therefore understanding and assessing bidders’ charitable motives, and more broadly externalities among bidders is an economically important problem which deserves attention to further improve our comprehension of how markets function.

7 Conclusion

The “business of business” may not be just business, contrary to what Friedman (1970) famously argued. Firms can increase their revenues by offering goods that indirectly contribute to welfare by creating public goods (or reducing public bads as in Bartling et al., 2014). However, this prosocial behavior does not necessarily reward a firm with higher profits, as the Charitystars example demonstrates.

The first result of the paper is that charitable motives affect bids, and that bidders display warm-glow type utilities (e.g., Andreoni, 1990, 1995). They receive a large psychological or emotional benefit by either winning the auction or by driving up the transaction price when losing the auction. Observing this effect on Charitystars data, the paper documents bids in excess of the private valuation for all bidders, boosting the firm’s gross revenues beyond those attainable by non-charity auctions.

While consumers are better off with charity rather than non-charity auctions, the same may not be true for producers. In fact, the price increase due to the charitable donations can fall short of the money transferred to the charity ultimately reducing a firm’s profit. The Charitystars’ auction data confirm this hypothesis showing that the firm could largely increase revenues by

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58 See for example Figure 3b, where the auctioneer ends up losing money respect to a non-charity auctioneer in a similar set-up. Such a scenario is plotted in 14b in Appendix C.4 where revenues from non-charity second price auctions are greater than those obtained by similar charitable second-price auctions.
switching to standard English auctions. Overall, the production of *impure public goods* (Kotchen, 2006) may not be the most efficient method to maximize profits and this example highlights how markets may fail to provide firms with the right incentives for supplying public goods.

The paper also contributes to the identification and estimation of structural auction models in the presence of cross-bidders externalities (e.g., Kuehn, 2016) by building on results from auctions with risk averse bidders (e.g., Guerre et al., 2009). The econometric methods can be easily extended to auction formats other than second-price auctions.

Finally, the empirical analysis in this paper can be translated to other markets where there are subsidies paid to bidders (e.g., Athey et al., 2013) or where an agent’s utility is affected by the actions of the others (e.g., Bresnahan, 1982, Sullivan, 2017). These conditions hold in several environments. For instance, Lu (2012) draws a parallel between charity auctions and auctions where bidders are financially constrained. In corporate buyouts, Singh (1998) illustrates how owning shares of the target company (called “toehold”) pushes a bidder to bid more aggressively as a high sale price increases the value of her toehold. Similar externalities are also present in bidding rings and cartels, as bidders internalize the outcome of their coalition partners. In this case, estimating the externality is also interesting to understand the sustainability of the coalition (e.g. McAfee and McMillan, 1992, Marmer et al., 2016, Schurter, 2017).
References


Supplementary material

A  Auction webpage

Figure 8: A screenshot of the webpage for a listing at the time of data collection (2016-2017)

Note: Screenshot of a webpage of a running auction on Charitystars.com for an AC Milan jersey worn and signed by the player Giacomo Bonaventura. The standing price is GBP 110: this bid was placed by an Italian bidder with username “Supermanfra”. A total of five bids are placed at this point. Although at the current highest bid the reserve price is not met, this can change by the end of the auction. The auction will be active for other 3 days and 18 hours and will expire on June 7th at 7AM. 85% of the proceeds will be donated to “Play for Change”.
B Data description

The data was collected using a Python script that searched a list of keywords across the pages dedicated to soccer items on the company website, available at http://www.charitystars.com. The script would gather all the available information regarding the auction, the charity, the listing and the bid history. This information was augmented with data from other sources. For example, a similar python script was used to recover footballers’ quality scores from a renowned videogame (FIFA). Information on each charity’s mission was obtained from both Charitystars as well as each charity’s website.

The analysis considers only a subset of the available auctions (see Section 2), according to the following conditions: (i) transaction prices higher than the reserve price, (ii) reserve prices greater than zero, (iii) two or more bidders, (iv) minimum increment is within €25 and (v) maximum donation of 85% of the final price.

B.1 Description of the variables

The regressions in Section 2 and in the Appendix display only some of the actual variables used in the analyses due to space limitations. These variables were distinguished in four groups based on their relevance.

1. Main Variables: these are the variables used in all regression tables and in the structural model. They are listed in Table 8 and their meaning is described by their label. Some variables whose meaning is not immediately clear are described in the following list:

   • The variable Length counts the number of days between the first bid and the closing date (the listing date of the auction is unknown);
   • The dummy Extended time is 1 if two or more bidders placed a bid in the last minutes of the auction. In this case the time is extended until all but 1 bidders drop out
   • Auctions within 3 weeks (same team) counts the number of auctions listing jerseys of the same team as the one of the auctioned item. It only includes auctions within a 3 week window from the end of the auction. It considers all auctions not only those with final price larger than €100 and smaller than €1000;
   • Auctions up to 2 weeks ago (same player/team) counts the number of auctions for a jersey worn by the same player playing with the same team in the same year as the match of the jersey that is auctioned. It considers all the listings up to 2 weeks from the end of the auction (Charitystars’s auctions last between 1 and 2 weeks). It considers all auctions not only those with final price larger than €100 and smaller than €1000;
   • Count auctions same charity is a progressive count of the number of listings for each charity;
   • The dummy Player belongs to FIFA 100 list is 1 if the player is in the FIFA 100 list (the list of the best soccer players ever);
   • The variable Number of goals scored is equal to the number of goals scored by the player with the auctioned jersey in a particular match if this number is mentioned in the content of the listing. It is zero otherwise.
   • The dummy Player belongs to an important team is 1 if the player plays for one of the following teams (alphabetic order): AC Milan, Argentina, Arsenal FC, AS Roma, Atletico de Madrid, Barcelona FC, Bayern Munich, Belgium, Borussia Dortmund, Brazil, Chelsea FC, Colombia, England, FC Internazionale, France, Germany, Italia, Juventus FC, Liverpool FC, Manchester
2. **Add. Charity Dummies:** this group includes dummies for heterogeneity across charities based on the mission of each charity. These dummies are not exclusive bins as most charities do more than only one activity. There are 101 different charities in total. The dummy variables used

- **Helping disables** for charities involved in assistance to disable subjects;
- **Infrastructures in developing countries** for charities building infrastructures in developing countries;
- **Healthcare** for charities dealing with healthcare and health research;
- **Humanitarian scopes in developing countries** for charities helping people in situation of poverty and undernourishment;
- **Children’s wellbeing** for charities providing activities (e.g., education and sport) to the youth;
- **Neurodegenerative disorders** for charities helping those suffering from neurodegenerative disorders;
- **Charity belongs to the soccer team** are charities linked to a football team;
- **Improving access to sport** charities give opportunity of integration through sport activities;
- **Italian charity**
- **English charity**

3. **League/Match Dummies:** these are dummies for soccer league and match heterogeneity. They include:

- Dummies for each major competition (*Champions League*, *Europa League*, *Serie A*, *Italian Cup*, *Premier League*, *La Liga*, *Copa del Rey*, *European Supercup*, *Italian Supercup*, *Spanish Supercup*, *UEFA European Championship*, *Qualifications to UEFA European Championship*, *World Cup*, *Qualification to the World Cup*);
- Dummies weather the listings mentions that the match was won (*Won*) and for unofficial/replia jerseys which are not worn but only signed (*Unofficial*).

4. **Time Dummies:** this group include *Month* dummies (12 variables) and *Year* dummies (2 variables).

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1 In unreported analysis player quality was accounted also using the players’ evaluation from the videogame FIFA. These variables do not affect prices significantly and are dropped from this.

2 All remaining competitions are treated as friendly matches.
Table 8: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Q(25%)</th>
<th>Q(50%)</th>
<th>Q(75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auction characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage donated (q)</td>
<td>0.70</td>
<td>0.27</td>
<td>0.78</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Transaction price in €</td>
<td>364.25</td>
<td>187.50</td>
<td>222.00</td>
<td>315.00</td>
<td>452.50</td>
</tr>
<tr>
<td>Reserve price in €</td>
<td>179.03</td>
<td>132.02</td>
<td>100.00</td>
<td>145.00</td>
<td>210.00</td>
</tr>
<tr>
<td>Minimum increment in €</td>
<td>1.71</td>
<td>3.15</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Number of bidders</td>
<td>7.83</td>
<td>3.27</td>
<td>5.00</td>
<td>7.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Sold at reserve price (d)</td>
<td>0.04</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Length (in # days)</td>
<td>8.08</td>
<td>3.07</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Extended time (d)</td>
<td>0.43</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Web-listing details</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of description (in # words)</td>
<td>141.89</td>
<td>42.11</td>
<td>123.00</td>
<td>140.00</td>
<td>161.25</td>
</tr>
<tr>
<td>Content in English (d)</td>
<td>0.30</td>
<td>0.47</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Content in Spanish (d)</td>
<td>0.00</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Length of charity description (in # words)</td>
<td>123.47</td>
<td>56.62</td>
<td>107.00</td>
<td>107.00</td>
<td>120.00</td>
</tr>
<tr>
<td>Number of pictures</td>
<td>5.67</td>
<td>1.98</td>
<td>5.00</td>
<td>6.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Auctions within 3 weeks (same team)</td>
<td>1.46</td>
<td>4.08</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Auctions up to 2 weeks ago (same player/team)</td>
<td>5.06</td>
<td>7.92</td>
<td>0.00</td>
<td>1.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Count auctions same charity</td>
<td>128.35</td>
<td>151.94</td>
<td>14.00</td>
<td>54.00</td>
<td>216.50</td>
</tr>
<tr>
<td><strong>Player and match characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player belongs to FIFA 100 list (d)</td>
<td>0.11</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Unwashed jersey (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Jersey is signed (d)</td>
<td>0.52</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Jersey is signed by the team players/coach (d)</td>
<td>0.06</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Jersey worn during a final (d)</td>
<td>0.03</td>
<td>0.17</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Number of goals scored</td>
<td>0.03</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Player belongs to an important team (d)</td>
<td>0.88</td>
<td>0.32</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Charity provenience</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charity is Italian (d)</td>
<td>0.90</td>
<td>0.29</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Charity is English (d)</td>
<td>0.08</td>
<td>0.28</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Charity’s activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helping disables (d)</td>
<td>0.35</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Infrastructures in developing countries (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Healthcare (d)</td>
<td>0.23</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Humanitarian scopes in developing countries (d)</td>
<td>0.14</td>
<td>0.34</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Children‘s wellbeing (d)</td>
<td>0.84</td>
<td>0.36</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Neurodegenerative disorders (d)</td>
<td>0.06</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Charity belongs to the soccer team (d)</td>
<td>0.10</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Improving access to sport (d)</td>
<td>0.63</td>
<td>0.48</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Overview of the main covariates used in all specifications in the reduced form analysis and in the structural model. (d) stands for dummies. Only auctions with price between €100 and €1000. Prices are in euros. If the listing was in GBP the final price was converted in euros using the exchange rate of the last auction day.
B.2 Prices and revenues

The plots in Figure 9 reports the density plots for transaction prices and reserve prices for the most common auction types.

Figure 9: Density of key variables by percentage donated

(a) Transaction Price
\[ p \in (100, 1000) \]

(b) Reserve Price
\[ p \in (100, 1000) \]

Note: Panel (a) and Panel (b) show the density of the transaction price and reserve price respectively for selected auctions. The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidths (Silverman, 1986).
### Table 9: Relation between log($Price$) and percentage donated (small dataset)

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Bidders)</td>
<td>0.158***</td>
<td>0.182***</td>
<td>0.182***</td>
<td>0.180***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>q</td>
<td>0.050</td>
<td>0.127***</td>
<td>0.146***</td>
<td>0.157***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.046)</td>
<td>(0.052)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td></td>
<td></td>
<td>0.261***</td>
<td>0.254***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.027)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.584***</td>
<td>3.159***</td>
<td>3.390***</td>
<td>3.330***</td>
</tr>
<tr>
<td></td>
<td>(0.207)</td>
<td>(0.222)</td>
<td>(0.225)</td>
<td>(0.267)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Add. Charity Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>League/Match Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time Dummies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>18.59%</td>
<td>29.31%</td>
<td>31.43%</td>
<td>35.15%</td>
</tr>
<tr>
<td>BIC</td>
<td>510</td>
<td>415</td>
<td>507</td>
<td>559</td>
</tr>
<tr>
<td>N</td>
<td>713</td>
<td>713</td>
<td>713</td>
<td>713</td>
</tr>
</tbody>
</table>

Note: OLS regression of log of the transaction price on covariates. Only auctions with price between €100 and €400. Control variables are defined in Appendix B.  
* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 

---

C  Tables and figures
Table 10: Regression of log($Price$) on $q$ and $q^2$

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Reserve Price)</td>
<td>0.356***</td>
<td>0.344***</td>
<td>0.347***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.295***</td>
<td>0.283***</td>
<td>0.276***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.881</td>
<td>0.624</td>
<td>1.152**</td>
</tr>
<tr>
<td></td>
<td>(0.535)</td>
<td>(0.566)</td>
<td>(0.578)</td>
</tr>
<tr>
<td>$q^2$</td>
<td>–0.686</td>
<td>–0.419</td>
<td>–0.952</td>
</tr>
<tr>
<td></td>
<td>(0.569)</td>
<td>(0.599)</td>
<td>(0.616)</td>
</tr>
</tbody>
</table>

Main Variables: Y Y Y
Charity Dummies: Y Y
League/Match Dummies: Y Y
Time Dummies: Y

$q + q^2$ | 0.195*** | 0.206*** | 0.201*** |
|          | (0.059)  | (0.062)  | (0.068)  |

Adjusted R-squared | 45.91% | 47.17% | 49.28% |
BIC | 1,168 | 1,276 | 1,337 |
$N$ | 1,108 | 1,108 | 1,108 |

Note: OLS regression of log of the transaction price on covariates and $q^2$ to test the linearity of donation. Only auctions with price between €100 and €1000. Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 
Figure 10: Plot of the coefficient for $q$ from a quantile regression

Note: Coefficients from a series of quantile regressions of the log of the transaction price on $q$ and covariates as in the first column of Table 10 in the main text. The shaded regions is the confidence interval. The dashed (dotted) line reports the coefficient of the OLS regression (5% confidence interval). Only auctions with price between €100 and €1000. Boostrapped standard errors with 400 repetitions.
Figure 11: Plot of the coefficient for $q$ from a quantile regression (small dataset)

Note: Coefficients from a series of quantile regressions of the log of the transaction price on $q$ and covariates as in the first column of Table 10 in the main text. The shaded regions is the confidence interval. The dashed (dotted) line reports the coefficient of the OLS regression (5% confidence interval). Only auctions with price between €100 and €400. Boostrapped standard errors with 400 repetitions.
Table 11: Linearity of the relation between log($Price$) and percentage donated (small dataset)

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Q(0.25)</td>
<td>Q(0.50)</td>
<td>Q(0.75)</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.261***</td>
<td>0.372***</td>
<td>0.288***</td>
<td>0.182***</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.049)</td>
<td>(0.044)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.182***</td>
<td>0.188***</td>
<td>0.171***</td>
<td>0.124***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.034)</td>
<td>(0.045)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.127***</td>
<td>0.164***</td>
<td>0.171**</td>
<td>0.146**</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.056)</td>
<td>(0.071)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Constant</td>
<td>3.159***</td>
<td>2.294***</td>
<td>2.881***</td>
<td>4.057***</td>
</tr>
<tr>
<td></td>
<td>(0.222)</td>
<td>(0.311)</td>
<td>(0.350)</td>
<td>(0.346)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Adjusted/Pseudo R-squared</td>
<td>29.31%</td>
<td>26.51%</td>
<td>18.11%</td>
<td>14.70%</td>
</tr>
<tr>
<td>$N$</td>
<td>713</td>
<td>713</td>
<td>713</td>
<td>713</td>
</tr>
</tbody>
</table>

Note: OLS Regression and quantile regressions of log of the transaction price on covariates to test the linearity of donation. Only auctions with price between €100 and €400. Boostrapped standard errors with 400 repetitions. The null hypothesis that $q$ is the same in column (II), (III) and (IV) is not rejected beyond 90% level. Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 
### C.1 Asymmetric behavior

#### C.1.1 Nationalities

Table 12: Regression of log($Price$) on bidder nationality

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>0.230***</td>
<td>0.230***</td>
<td>0.235***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.048)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>Win: Italy</td>
<td>-0.043*</td>
<td>-0.038</td>
<td>-0.058</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Italian Team</td>
<td></td>
<td>-0.068**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.031)</td>
<td></td>
</tr>
<tr>
<td>Win: North America</td>
<td></td>
<td>0.058</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.077)</td>
<td></td>
</tr>
<tr>
<td>Win: France</td>
<td></td>
<td>-0.095</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
<td></td>
</tr>
<tr>
<td>Win: European Union</td>
<td></td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.071)</td>
<td></td>
</tr>
<tr>
<td>Win: China</td>
<td></td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.070)</td>
<td></td>
</tr>
<tr>
<td>Win: UK</td>
<td></td>
<td>-0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.076)</td>
<td></td>
</tr>
<tr>
<td>Win: Unknown</td>
<td></td>
<td>-0.154</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.134)</td>
<td></td>
</tr>
<tr>
<td>Win: Asia</td>
<td></td>
<td>-0.079</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>Win: South-East Asia</td>
<td></td>
<td>-0.121</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.386***</td>
<td>2.413***</td>
<td>2.368***</td>
</tr>
<tr>
<td></td>
<td>(0.229)</td>
<td>(0.230)</td>
<td>(0.238)</td>
</tr>
</tbody>
</table>

Note: OLS regressions of log of the transaction price on covariates to test the symmetry assumption over different nationalities of the winner. The first column tests Italian vs Non-Italian winners. The coefficient shows that Italian winners bid less than others, however Column (II) reveals that this correlation vanishes when a dummy variable for whether the football jersey is from an Italian team is also present. No geographic dummy variable is significant in Column (III). Only auctions with price between €100 and €1000. Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1,108</td>
<td>1,108</td>
<td>1,108</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>46.02%</td>
<td>46.24%</td>
<td>46.11%</td>
</tr>
</tbody>
</table>
Table 13: Nationalities of the bidders

<table>
<thead>
<tr>
<th>Italian</th>
<th>UK</th>
<th>France</th>
<th>Other EU</th>
<th>North Am</th>
<th>China</th>
<th>Asia</th>
<th>East Asia</th>
<th>Rest World</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winner</td>
<td>560</td>
<td>74</td>
<td>41</td>
<td>119</td>
<td>62</td>
<td>130</td>
<td>35</td>
<td>28</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>50.54%</td>
<td>6.68%</td>
<td>3.70%</td>
<td>10.74%</td>
<td>5.60%</td>
<td>11.73%</td>
<td>3.16%</td>
<td>2.53%</td>
<td>5.32%</td>
</tr>
<tr>
<td>Second</td>
<td>603</td>
<td>79</td>
<td>28</td>
<td>113</td>
<td>51</td>
<td>117</td>
<td>33</td>
<td>26</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>54.42%</td>
<td>7.13%</td>
<td>2.53%</td>
<td>10.20%</td>
<td>4.60%</td>
<td>10.56%</td>
<td>2.98%</td>
<td>2.35%</td>
<td>5.23%</td>
</tr>
<tr>
<td>Total</td>
<td>1,163</td>
<td>153</td>
<td>69</td>
<td>232</td>
<td>113</td>
<td>247</td>
<td>68</td>
<td>54</td>
<td>54</td>
</tr>
</tbody>
</table>

Note: Nationalities of the bidders by geographic area. “Rest of the World” includes Eastern Europe, Middle East, Africa, Oceania, Latina America and Unknown nationalities (which comprises 12 winners and highest losers).

C.1.2 Collectors

Table 14: Regression of log(Price) on recurrent winners

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 &lt; p &lt; 1000</td>
<td>100 &lt; p &lt; 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.351***</td>
<td>0.372***</td>
<td>0.262***</td>
<td>0.271***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.295***</td>
<td>0.063*</td>
<td>0.181***</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.037)</td>
<td>(0.028)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>q</td>
<td>0.227***</td>
<td>0.207***</td>
<td>0.120***</td>
<td>0.110**</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Recurrent Winner</td>
<td>0.047**</td>
<td>0.030</td>
<td>0.035</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.022)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Total Number of Bids Placed</td>
<td>0.218***</td>
<td>0.143***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.022)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.324***</td>
<td>2.127***</td>
<td>3.135***</td>
<td>3.010***</td>
</tr>
<tr>
<td></td>
<td>(0.225)</td>
<td>(0.220)</td>
<td>(0.221)</td>
<td>(0.213)</td>
</tr>
</tbody>
</table>

Main Variables        | Y       | Y       | Y       | Y       |

Adjusted R-squared    | 0.4605  | 0.5025  | 0.2947  | 0.3342  |
N                     | 1108    | 1108    | 713     | 713     |

Note: OLS regressions of the log of the transaction price on covariates to test the symmetry assumption. The dummy variable Recurrent Winner is 1 if the winner of the auction won more than 3 auctions (the median in the data). Control variables are defined in Appendix B.

* – p < 0.1; ** – p < 0.05; *** – p < 0.01.
## C.2 Additional tables from the structural model

Table 15: First step of the structural estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>(I) 100 &lt; p &lt; 1000</th>
<th>(II) 100 &lt; p &lt; 500</th>
<th>(III) 100 &lt; p &lt; 470</th>
<th>(IV) 100 &lt; p &lt; 470</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{UH}$</td>
<td>0.341*** (0.025)</td>
<td>0.267*** (0.028)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Increment</td>
<td>-0.010 (0.026)</td>
<td>0.003 (0.015)</td>
<td>-0.036 (0.028)</td>
<td>-0.018 (0.044)</td>
</tr>
<tr>
<td>Minimum Increment$^2$</td>
<td>0.0001 (0.0005)</td>
<td>0.0004 (0.001)</td>
<td>0.002 (0.001)</td>
<td>0.001*** (0.0005)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>-0.014 (0.055)</td>
<td>0.312*** (0.033)</td>
<td>-0.136*** (0.058)</td>
<td>0.150*** (0.031)</td>
</tr>
<tr>
<td>Sold at Reserve Price</td>
<td>0.555*** (0.044)</td>
<td>0.042 (0.074)</td>
<td>0.490*** (0.074)</td>
<td>0.056 (0.041)</td>
</tr>
<tr>
<td>Length</td>
<td>0.045 (0.055)</td>
<td>0.071*** (0.020)</td>
<td>0.005 (0.056)</td>
<td>0.055*** (0.020)</td>
</tr>
<tr>
<td>Length$^2$</td>
<td>-0.003 (0.002)</td>
<td>0.003*** (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.002*** (0.001)</td>
</tr>
<tr>
<td>Extended Time</td>
<td>0.100 (0.005)</td>
<td>0.072** (0.003)</td>
<td>0.091 (0.005)</td>
<td>0.073*** (0.002)</td>
</tr>
<tr>
<td>Number of Pictures</td>
<td>-0.074 (0.053)</td>
<td>-0.060*** (0.045)</td>
<td>0.010 (0.024)</td>
<td>-0.002 (0.021)</td>
</tr>
<tr>
<td>Number of Pictures$^2$</td>
<td>0.009 (0.005)</td>
<td>0.006*** (0.004)</td>
<td>0.002 (0.003)</td>
<td>0.001 (0.002)</td>
</tr>
<tr>
<td>Auctions within 3 weeks</td>
<td>0.013 (0.009)</td>
<td>0.016*** (0.004)</td>
<td>-0.016*** (0.007)</td>
<td></td>
</tr>
<tr>
<td>Auctions up to 2 weeks ago</td>
<td>0.004 (0.008)</td>
<td>0.011*** (0.004)</td>
<td>-0.005 (0.004)</td>
<td>0.008*** (0.002)</td>
</tr>
<tr>
<td>Count Auctions Same Charity</td>
<td>0.0004 (-0.0003)</td>
<td>-0.001*** (0.0003)</td>
<td>-0.0004 (0.0003)</td>
<td>-0.001*** (0.0002)</td>
</tr>
<tr>
<td>Player belongs to FIFA 100 list</td>
<td>0.216*** (0.052)</td>
<td>0.201** (0.051)</td>
<td>0.168 (0.091)</td>
<td>0.098*** (0.054)</td>
</tr>
<tr>
<td>Unwashed Jersey</td>
<td>0.143 (0.076)</td>
<td>0.282*** (0.050)</td>
<td>-0.029 (0.096)</td>
<td>0.140*** (0.054)</td>
</tr>
<tr>
<td>Jersey is Signed</td>
<td>-0.405*** (0.052)</td>
<td>-0.054 (0.037)</td>
<td>-0.320*** (0.054)</td>
<td>-0.042 (0.035)</td>
</tr>
<tr>
<td>Jersey is signed by the team players/coach</td>
<td>-0.592*** (0.101)</td>
<td>-0.172*** (0.075)</td>
<td>-0.493*** (0.096)</td>
<td>-0.041 (0.069)</td>
</tr>
<tr>
<td>Jersey Worn During a Final</td>
<td>0.348 (0.196)</td>
<td>0.411*** (0.095)</td>
<td>-0.280*** (0.099)</td>
<td>0.204*** (0.055)</td>
</tr>
<tr>
<td>Number of Goals Scored</td>
<td>0.192 (0.114)</td>
<td>0.247*** (0.062)</td>
<td>0.012 (0.266)</td>
<td>0.336*** (0.087)</td>
</tr>
<tr>
<td>Player Belongs to an Important Team</td>
<td>0.230*** (0.069)</td>
<td>0.291*** (0.048)</td>
<td>0.144*** (0.071)</td>
<td>0.169*** (0.041)</td>
</tr>
<tr>
<td>Charity is Italian</td>
<td>-0.086 (0.126)</td>
<td>0.143 (0.137)</td>
<td>-0.168 (0.269)</td>
<td>0.113 (0.080)</td>
</tr>
<tr>
<td>Charity is English</td>
<td>0.304 (0.175)</td>
<td>0.233** (0.133)</td>
<td>0.034 (0.292)</td>
<td>0.230*** (0.082)</td>
</tr>
<tr>
<td>Helping Disables</td>
<td>-0.142 (0.068)</td>
<td>0.019 (0.040)</td>
<td>-0.214*** (0.074)</td>
<td>-0.006 (0.076)</td>
</tr>
<tr>
<td>Infrastructures in Developing Countries</td>
<td>-0.069 (0.136)</td>
<td>0.152** (0.077)</td>
<td>-0.482*** (0.175)</td>
<td>-0.138** (0.063)</td>
</tr>
<tr>
<td>Healthcare</td>
<td>-0.152 (0.104)</td>
<td>-0.232*** (0.080)</td>
<td>-0.116 (0.088)</td>
<td>-0.057 (0.061)</td>
</tr>
<tr>
<td>Humanitarian Scopes in Developing Countries</td>
<td>0.159 (0.119)</td>
<td>0.165** (0.068)</td>
<td>0.037 (0.178)</td>
<td>-0.027 (0.066)</td>
</tr>
<tr>
<td>Children’s Wellbeing</td>
<td>0.117 (0.104)</td>
<td>0.121 (0.067)</td>
<td>-0.056 (0.109)</td>
<td>0.038 (0.055)</td>
</tr>
<tr>
<td>Neurodegenerative Disorders</td>
<td>0.489*** (0.145)</td>
<td>0.273*** (0.089)</td>
<td>-0.020 (0.151)</td>
<td>0.094 (0.084)</td>
</tr>
<tr>
<td>Charity Belongs to the Soccer Team</td>
<td>0.037 (0.153)</td>
<td>-0.239*** (0.089)</td>
<td>0.375*** (0.185)</td>
<td>0.016 (0.069)</td>
</tr>
<tr>
<td>Improving Access to Sport</td>
<td>-0.041 (0.115)</td>
<td>-0.026 (0.073)</td>
<td>-0.124 (0.145)</td>
<td>0.043 (0.065)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.699*** (0.380)</td>
<td>4.175*** (0.230)</td>
<td>5.333*** (0.357)</td>
<td>4.538*** (0.183)</td>
</tr>
</tbody>
</table>

Adjusted $R^2$                                      | 28.0%               | 46.8%              | 31.3%               | 36.4%               |

Note: The table reports the two regressions in the first step of the structural model. Columns (I) and (III) regress log(Reserve Price) on covariates, while Columns (II) and (IV) regress log(Price) on the unobserved heterogeneity ($\hat{UH}$) and covariates. Control variables are defined in Appendix B. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. 

---

13
Table 16: Logit regression of $q$ on covariates.

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>SE</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.846***</td>
<td>(0.254)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>-0.973***</td>
<td>(0.353)</td>
</tr>
<tr>
<td>Length</td>
<td>-0.577***</td>
<td>(0.215)</td>
</tr>
<tr>
<td>Length$^2$</td>
<td>0.016**</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Minimum Increment</td>
<td>-0.669***</td>
<td>(0.203)</td>
</tr>
<tr>
<td>Minimum Increment$^2$</td>
<td>0.026***</td>
<td>(0.007)</td>
</tr>
<tr>
<td>PriceEqualRes</td>
<td>0.905 (0.583)</td>
<td>1.174 (0.869)</td>
</tr>
<tr>
<td>Lenght of description</td>
<td>0.026***</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Content in English</td>
<td>-0.446 (0.453)</td>
<td>-0.556 (0.666)</td>
</tr>
<tr>
<td>Content in Spanish</td>
<td>0.000 (.)</td>
<td>0.000 (.)</td>
</tr>
<tr>
<td>Length of Charity Desc.</td>
<td>0.006*</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Number of Pictures</td>
<td>0.319 (0.356)</td>
<td>0.887 (0.578)</td>
</tr>
<tr>
<td>Number of Pictures$^2$</td>
<td>-0.043</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Within 3 Weeks</td>
<td>-0.933***</td>
<td>(0.222)</td>
</tr>
<tr>
<td>Count 2 weeks</td>
<td>-0.011</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Count Auctions Same Charity</td>
<td>0.025***</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Extended Time</td>
<td>-0.339</td>
<td>(0.310)</td>
</tr>
<tr>
<td>FIFA 100</td>
<td>0.408 (0.390)</td>
<td>0.958* (0.534)</td>
</tr>
<tr>
<td>Jersey not Washed</td>
<td>-1.096 (0.834)</td>
<td>0.000 (.)</td>
</tr>
<tr>
<td>Signed</td>
<td>-0.854**</td>
<td>(0.396)</td>
</tr>
<tr>
<td>Signed Team</td>
<td>-0.876</td>
<td>(0.746)</td>
</tr>
<tr>
<td>Goals scored</td>
<td>0.447</td>
<td>(0.579)</td>
</tr>
<tr>
<td>Italian Charity</td>
<td>-1.539</td>
<td>(1.391)</td>
</tr>
<tr>
<td>English Charity</td>
<td>5.195***</td>
<td>(1.458)</td>
</tr>
<tr>
<td>Disability</td>
<td>1.840***</td>
<td>(0.425)</td>
</tr>
<tr>
<td>Development</td>
<td>-0.901</td>
<td>(0.914)</td>
</tr>
<tr>
<td>Health</td>
<td>-1.624***</td>
<td>(0.621)</td>
</tr>
<tr>
<td>Humanitarian</td>
<td>-0.387</td>
<td>(1.057)</td>
</tr>
<tr>
<td>Children and Youth</td>
<td>-2.270***</td>
<td>(0.470)</td>
</tr>
<tr>
<td>Neurodegenerative dis.</td>
<td>-1.681**</td>
<td>(0.747)</td>
</tr>
<tr>
<td>Team Related</td>
<td>-0.602</td>
<td>(1.071)</td>
</tr>
<tr>
<td>Sport</td>
<td>-1.883***</td>
<td>(0.504)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.948</td>
<td>(2.418)</td>
</tr>
</tbody>
</table>

Note: Logit regression of $q$ on covariates. The dependent variable is 1 if $q = 0.1$. Only auctions with $q \in \{10\%, 85\%\}$. Column (I) refers to auctions whose transaction price is in the interval €(100,1000), while the interval is €(100,400) in Column (II). Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 

14
C.3 Additional structural estimations

This section reports structural estimates for different versions of the models. In particular, the following tables replicate the results in the main text (i) by using only data in the €100 - €400 interval, (ii) by comparing auctions at 10% and 78% (instead of 85%),\(^3\) and (iii) by using a larger set of covariates. Overall, the results highlights the robustness of the estimates in the main text (Table 6).

Table 17: Structural estimation when \(q \in \{10\%, 85\%\}\) and price €100 and €400

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Number of bidders</th>
<th>(\alpha) [5% CI]</th>
<th>(\beta) [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>17.2% [5.3%, 32.1%]</td>
<td>37.6% [8.0%, 61.8%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>17.0% [4.6%, 29.7%]</td>
<td>37.5% [8.5%, 60.2%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>16.7% [4.1%, 28.9%]</td>
<td>37.5% [12.8%, 60.9%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>16.2% [3.6%, 27.5%]</td>
<td>37.4% [5.3%, 61.4%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>15.2% [3.8%, 27.3%]</td>
<td>37.1% [8.9%, 62.1%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of \(\alpha\) and \(\beta\) for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that \(q \in \{10\%, 85\%\}\) and the price between €100 and €400. 470 observations in total.

\(^3\)Notice that using auctions at 78% and 85% do not yield consistent estimates because the full rank condition assumption break down as the two \(q\) are almost identical. See the simulations in Appendix E for a numerical example.
Table 18: Structural estimation when $q \in \{10\%, 78\%\}$ and price €100 and €1000

<table>
<thead>
<tr>
<th>Quantile</th>
<th>$n$</th>
<th>$\alpha$ [5% CI]</th>
<th>$\beta$ [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>22.6% [11.1%, 30.6%]</td>
<td>51.8% [32.4% 68.8%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>24.0% [11.0%, 33.6%]</td>
<td>52.1% [34.3% 69.2%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>24.5% [13.3%, 33.7%]</td>
<td>52.2% [33.3% 71.7%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>24.8% [10.7%, 32.8%]</td>
<td>52.3% [34.2% 68.2%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>25.1% [12.7%, 33.7%]</td>
<td>52.4% [35.1% 69.4%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €1000. 366 observations in total.

Table 19: Structural estimation when $q \in \{10\%, 78\%\}$ and price €100 and €400

<table>
<thead>
<tr>
<th>Quantile</th>
<th>$n$</th>
<th>$\alpha$ [5% CI]</th>
<th>$\beta$ [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>24.5% [9.9%, 39.4%]</td>
<td>52.0% [36.7% 82.2%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>24.2% [9.3%, 34.4%]</td>
<td>51.9% [34.7% 81.4%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>23.8% [9.5%, 34.3%]</td>
<td>51.8% [35.3% 81.2%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>23.3% [9.0%, 35.8%]</td>
<td>51.7% [30.4% 79.2%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>21.8% [6.5%, 32.6%]</td>
<td>51.3% [28.6% 83.7%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €400. 258 observations in total.
C.3.1 Different set of covariates

In this section the log(Total Number of Bids Placed) is added to the covariates. This variable helped explaining differences in competition in Table 14. In fact, the more bids are submitted by a given number of bidders (we also control for the number of unique bidders), the more intense is the competition in the auction. The estimates of the parameters $\alpha$ and $\beta$ do not change substantially from those reported in the main text (Table 6).

In addition, Figure 12 reports the same out-of-sample validation exercise proposed in the main text (see Section 5.3). It is based on the comparison between the pdf of the density obtained from estimating the model using auctions with $q \in \{10\%, 85\%\}$ (the $\alpha$ and $\beta$ parameters are shown in Table 20) with that derived by projecting the full three-step estimation procedure onto the auctions with $q = 78\%$.

Table 20: Structural estimation when $q \in \{10\%, 85\%\}$ and price €100 and €1000

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Number of bidders</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[5% CI]</td>
<td>[5% CI]</td>
</tr>
<tr>
<td>99%</td>
<td>16</td>
<td>17.4%</td>
<td>42.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[7.6%, 26.8%]</td>
<td>[23.2% 57.7%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>17.2%</td>
<td>42.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[7.5%, 25.2%]</td>
<td>[18.7% 56.1%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>16.9%</td>
<td>42.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[7.3%, 25.5%]</td>
<td>[20.7% 57.6%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>16.5%</td>
<td>42.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[6.8%, 25.7%]</td>
<td>[21.4% 60.3%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>15.5%</td>
<td>42.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[6.0%, 23.5%]</td>
<td>[19.8% 57.5%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €1000. 731 observations in total.
Table 21: Structural estimation when \( q \in \{10\%, 85\% \} \) and price €100 and €400

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Number of bidders</th>
<th>( \alpha ) [5% CI]</th>
<th>( \beta ) [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>14.5% [3.0%, 29.4%]</td>
<td>33.8% [3.6%, 57.0%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>14.3% [2.7%, 27.7%]</td>
<td>33.7% [8.0%, 57.1%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>14.1% [3.9%, 26.8%]</td>
<td>33.7% [3.6%, 56.2%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>13.7% [2.2%, 26.0%]</td>
<td>33.6% [2.4%, 56.6%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>12.8% [2.1%, 22.5%]</td>
<td>33.4% [6.4%, 53.0%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of \( \alpha \) and \( \beta \) for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that \( q \in \{10\%, 78\% \} \) and the price between €100 and €400. 470 observations in total.

Figure 12: Out-of-sample fit of the estimated density (10% and 78% auction)

Note: Replication of the out-of-sample validation in Section 5.3 adding the variable \( \log(\text{Total Number of Bids}) \) to the covariates in the first step. The plot shows the comparison of the density of the private values estimated from the structural model employing data from auctions with \( q = \{10\%, 85\% \} \) and the density of the private values estimated by projecting the three-step estimation on the \( q = 78\% \) auctions (as in Figure 4a). The null hypothesis (equality) cannot be rejected at 0.1874 level. The computation assumes \( n = 16 \), but the same result can be replicated with different \( n \). The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidth (Silverman, 1986).
C.4 Plots from the structural model and revenue comparison

Figure 13: Density of the homogenized transaction prices by returning winners

(a) Returning vs Non-Returning winners

\[ p \in (100, 1000) \]

(b) Returning vs Non-Returning winners

\[ p \in (100, 400) \]

Note: Density of the pseudo-winning bids in the structural model by returning winners. Returning winners (Collectors) are auction winners who won at least 4 bids (the median is 3). The Kolmogorov-Smirnov test does not reject the null hypothesis (equality) at 0.1554 level in Panel (a) and at 0.1423 in Panel (b). The plot is obtained using only auctions with \( q \in \{10\%, 85\%\} \). The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidths (Silverman, 1986).
Figure 14: Expected revenues across different auction formats

(a) Warm glow (α = 20%, β = 46%)

(b) Volunteer shill (α = 50% β = 10%)

Note: Revenue comparison across different auction mechanisms. Panel (a) uses the estimated α and β in the structural model. Panel (b) shows that the distribution of bids and private value in 3b result in lower revenues than non-charity auctions. The other primitives for both panels are q = 85%, F(·) ∼ N(50, 25) on [0, 100].

Figure 15: Variance of the revenues across different auction formats

(a) Variance gross revenues; q = 85%

(b) Variance gross revenues; q = 10%

Note: The figure compares the variance of the revenues across mechanisms for different number of bidders. The percentage donated is set to 85% and 10% in Panel (a) and Panel (b) respectively. The density f(v) is approximated using a cubic spline. Only auctions with price between €100 and €1000. The variances are computed without accounting for covariates.
D Proofs of propositions and corollaries

This section reports the proofs to the proposition and corollaries presented in the main text. The following lemma will be used multiple times in the proofs.

D.1 Lemma 1

Lemma 1. \( \lim_{x \to 0} x \log x = 0. \)

Proof: \( \lim_{x \to 0} x \log x = \lim_{x \to 0} \frac{\log x}{1/x} . \) Applying L’Hospital’s Rule \( \lim_{x \to 0} \frac{1/x}{-1/x^2} = \lim_{x \to 0} -x = 0. \)

D.2 Proof of Proposition 1

In the symmetric equilibrium of \( \Gamma_2(F(\cdot), \alpha, \beta, q) \) bidders bid according to:

\[
b^*(v; \alpha, \beta, q) = \begin{cases} 
\frac{1}{1 + q/\alpha - \beta} \left\{ v + \int_v^\infty \left( \frac{1-F(x)}{1-F(x)} \right)^{1/q/\alpha + 1} \, dx \right\} & \text{if } \alpha > 0 \land q > 0 \\
\frac{v}{1-q/\beta} & \text{if } \alpha = 0 \lor q = 0 
\end{cases}
\]

Proof: A similar proof can also be found in Engers and McManus (2007). This proof is only reported for completeness. All the following results hold for \( 0 < q \leq 1 \). Let \( F_{(n)}^k(v) \) be distribution of the \( k \)-th highest element out of \( n \); the problem in (3.1) can be rewritten as

\[
\mathbb{E}[u(v; \alpha, \beta, q)] = \int_v^\infty \left[ v - (1 - qb_i) b(u) \right] dF(u)^{n-1} \\
+ q\alpha_i b(s)(n-1)F(s)^{n-2}[1-F(s)] + q\alpha_i \int_s^\infty b(u) dF_{(n-1)}^2(u) \\
\bigg. \text{if } \text{winner, pays } b^{(i)} \\
\bigg. \text{if } \text{loser, } b_i = b^{(i)} \\
\bigg. \text{if } \text{loser, } b^{(i)} \text{ is paid}
\]

Here \( F_{(n-1)}^2(u) = F(u)^{n-1} + (n-1)F(u)^{n-2}(1 - F(u)) \) is the distribution of the second highest value, given that the number of agents is \( n - 1 \) because bidder \( i \) is counted out as his bid is below \( b^{(i)} \). The equilibrium bidding function is found where \( \frac{dE[u(v; \alpha, \beta, q)]}{ds} \bigg|_{s=v} = 0. \)

\[
\frac{dE[u(v; \alpha, \beta, q)]}{ds} \bigg|_{s=v} = 0 \\
= v \frac{\partial F(s)^{n-1}}{\partial s} + b(q\beta - 1) \frac{\partial F(s)^{n-1}}{\partial s} + qab(n-2) \frac{\partial F(s)^{n-1}}{\partial s} \frac{1 - F(s)}{F(s)} \\
+ qab'(n-1)[1-F(s)]F(s)^{n-2} - qab \frac{\partial F(s)^{n-1}}{\partial s} \\
- qab(n-2) \frac{\partial F(s)^{n-1}}{\partial s} \frac{1 - F(s)}{F(s)} = 0
\]
After deleting and moving terms, we get the FOC:

\[vf(v) = (1 + qa - q\beta)b(v)f(v) - qab'(v)(1 - F(v))\]

This differential equation can be finally solved by multiplying both sides of (D.1) by \(-\frac{1 - F(s)}{q\alpha}\) and integrating.

\[\frac{1}{q\alpha} \int_v^s t[1 - F(t)]^{-q\beta}dF(t) = \int_v^s \frac{\partial}{\partial t} [1 - F(t)]^{-q\beta}dt\]

This leads to the symmetric bidding function:

\[b^*(v; \alpha, \beta, q) = \begin{cases} 
\frac{1}{q\alpha} \int_v^s [1 - F(t)]^{-q\beta}dF(t) & \text{if } q\alpha > 0 \land q > 0 \\
\frac{1}{1 - q\beta} & \text{if } \alpha = 0 \lor q = 0 
\end{cases}\]

Note that the constant of integration that arises after integrating the RHS of the FOC is 0 (to see this, check the bidding function when \(v = \bar{v}\)). Further integrating the integral in the top row by parts we get (3.2):

\[b^*(v; \alpha, \beta, q) = \frac{1}{1 + q \cdot (\alpha - \beta)} \left\{ v + \int_v^s \left( \frac{1 - F(x)}{1 - F(v)} \right)^{-q\beta + 1} dx \right\} \]

which collapses to \(\frac{v}{1 - q\beta}\) when \(\alpha\) or \(q\) are zero. Finally, notice that this is a legitimate bidding function because it is increasing in \(v\):

\[\frac{\partial b^*(v; \alpha, \beta, q)}{\partial v} = \begin{cases} 
\frac{1}{q\alpha} \int_v^s \left( \frac{1 - F(x)}{1 - F(v)} \right)^{-q\beta + 1} dx \frac{f(v)}{1 - F(v)} & \text{if } \alpha > 0 \land q > 0 \\
\frac{1}{1 - q\beta} & \text{if } \alpha = 0 \lor q = 0 
\end{cases}\]

\[\text{D.3 Proof of Proposition 2}\]

If \(\alpha > 0\) bids are decreasing in \(\alpha\) for high value bidders.

**Proof:** We take derivative of (3.2) w.r.t. \(q\) for \(\alpha > 0\) is

\[\frac{\partial b^*(v; \alpha > 0, \beta, q)}{\partial q} = -q \int_v^s \left( \frac{1 - F(x)}{1 - F(v)} \right)^{-\frac{1 + \beta}{\alpha}} \left( 1 + \frac{(1 - \beta)(1 + \alpha - \beta)}{\alpha^2} \log \frac{1 - F(x)}{1 - F(v)} \right) dx + v\]

The integral is continuous and finite everywhere with respect to \(x\) by Lemma 1. The derivative will cross the x-axis at \(v\) where the numerator is zero, or:

\[-\frac{1 - \beta}{\alpha^2} \int_v^s \left( \frac{1 - F(x)}{1 - F(v)} \right)^{-\frac{1 + \beta}{\alpha}} \log \frac{1 - F(x)}{1 - F(v)} dx = v^* + \int_v^{v^*} \left( \frac{1 - F(x)}{1 - F(v^*)} \right)^{-\frac{1 + \beta}{\alpha}} dx\]

\[= b(v^*)\]
At \( v \neq v^* \), (D.4) is positive (negative) if the LHS is greater (smaller) than the RHS. Next, we focus on the uniqueness of \( v^* \). To limit some cumbersome notation, we denote \( \int_{v^*}^{\pi} \left( 1 - \frac{F(x)}{1 - F(v^*)} \right)^{\frac{1+\beta-\delta}{\delta}} \frac{1}{\phi} \quad \text{dx by \( \phi \).} \) The derivative of (D.4) w.r.t. \( v \) is negative if

\[
- \frac{1 - \beta}{\bar{\alpha}^2} \phi - \frac{1 - \bar{\beta}}{\bar{\alpha}^2} \frac{1 + \bar{\alpha} - \bar{\beta}}{\bar{\alpha}} \int_{v^*}^{\pi} \left( \frac{1 - F(x)}{1 - F(v^*)} \right)^{\frac{1+\beta-\delta}{\delta}} \log \frac{1 - F(x)}{1 - F(v^*)} \quad \text{dx} \leq \frac{1}{\bar{\alpha}} \phi \tag{D.6}
\]

where the RHS is the derivative of \( b(v) \) w.r.t. \( v \) as in (D.3). At \( v^* \) we can replace the second term on the LHS with the RHS of (D.5). Solving this inequality it yields the following condition

\[
- \frac{1 - \beta}{\bar{\alpha}^2} \phi + \frac{v^* + \phi}{\bar{\alpha}} \leq \frac{1}{\bar{\alpha}} \phi \quad \Rightarrow \quad \frac{(1 + \bar{\alpha} - \bar{\beta})}{\bar{\alpha}} \phi \geq \bar{\alpha}(v^* + \phi) \tag{D.7}
\]

where we in the third row we divided by \((1 + \bar{\alpha} - \bar{\beta})\) both sides of the inequality and expressed \( \phi \) in terms of bids and values using the optimal bid function (3.2).

Therefore, if the condition in (D.7) is respected, (D.4) is decreasing at \( v^* \). This condition is always satisfied if \( v^* \) exists. To show this we rely on two results: (i) the limit of (D.4) is negative at the upperbound of the \( v \)'s support, and (ii) there is no region on the support of \( v \) such that \( b(v) > (1 - \beta)v \) to its left and right and \( b(v) < (1 - \beta)v \) within this region. Therefore, once (D.4) becomes negative, it cannot switch sign from negative to positive and back to negative again, implying that if \( v^* \) exists, it is unique.

First, we show that the limit of (D.4) is negative. In fact, repeatedly applying L'Hospital's rule to the the limit of the first term

\[
\lim_{v \to \pi} -q \int_{v}^{\pi} \left( 1 - \frac{F(x)}{1 - F(v)} \right)^{\frac{1+\beta-\delta}{\delta}} \frac{1}{\phi} \quad \text{dx} \quad \text{and} \quad \text{while the limit of the last term (at the numerator is} \lim_{v \to \pi} -qv = -q\pi < 0.
\]

Second, because of the monotonicity of \( b(v) \), and because \( b(\pi) > \pi \) there exists at most only one \( v \in [\pi, \pi] \) such that \( b(\pi) = \pi \). By the same logic, there exists at most one value of \( v \) such that \( b(v)(1 - \beta) = \pi \). Call this value \( v** \). We already know that if \( v^* < v** \), (D.5) holds with equality and (D.4) is decreasing at \( v^* \). Moreover, by (D.7), there cannot be another cut-off value at a \( v \) greater than \( v** \) such that (D.4) is decreasing at that value. Therefore, because of the continuity of (D.4) and given that its upper limit is negative, (D.4) cannot switch sign multiple times. In turn, this also implies that if (D.4) is negative at \( \pi \) it will be negative for all \( v \).

Hence, if \( v^* \) exists it is unique because the function passing by this point can only be decreasing. Bidders with values below \( v^* \) increase their bids, while bidders with values above \( v^* \) decrease it after an increase in \( \alpha \).

\[\Box\]

**D.4 Proof of Proposition 3**

Bids are increasing in \( q \) if \( \beta \geq \alpha \). If \( \alpha > \beta \) bids are decreasing in \( q \) for high value bidders.

**Proof:** Assume \( q > 0 \). To reduce the notation set \( \bar{\alpha} = q \cdot \alpha \) and \( \bar{\beta} = q \cdot \beta \). First we analyze the derivative of
Step 1. We take derivative of (3.2) w.r.t. q for \( \alpha > 0 \) is
\[
\frac{\partial b^*(v; \alpha > 0, \beta, q)}{\partial q} = \int_v^\infty \left( \frac{1-F(x)}{1-F(v)} \right) \frac{1+\tilde{\beta}}{\tilde{x}} \left( -\frac{a(1+\tilde{\beta})}{\tilde{x}^2} \log \frac{1-F(x)}{1-F(v)} - (\alpha - \tilde{\beta}) \right) dx - (\alpha - \tilde{\beta})v
\]
\[(D.8)\]
The integral is continuous and finite everywhere with respect to \( x \) by Lemma 1. Inspection of this equation reveals that is positive if \( \beta \geq \alpha \) (these refers to warm glow or pure altruism models). Therefore, bids are increasing in \( q \) if \( \beta > \alpha \) for all \( v \).

Step 2. We turn to the remaining case \((0 < \alpha < \beta \text{ or shill bidders model})\) and we want to show that (D.8) crosses the x-axis at most ones at \( v^* \). By rewriting (D.8) we see that at \( v^* \)
\[
- \int_v^{v^*} \left( \frac{1-F(x)}{1-F(v)} \right) \frac{1+\tilde{\beta}}{\tilde{x}} \frac{a(1+\tilde{\alpha} - \tilde{\beta})}{\tilde{\alpha}^2} \log \frac{1-F(x)}{1-F(v)} dx = (\alpha - \tilde{\beta}) \left( \left. v + \int_v^{v^*} \left( \frac{1-F(x)}{1-F(v)} \right)^{1-\tilde{\beta}+1} \right. \right) \bigg|_{v=v^*} \\
= (\alpha - \tilde{\beta})(1+\tilde{\alpha} - \tilde{\beta})b(v^*)
\]
\[(D.9)\]
where the second line replaces the expression to the RHS with the optimal bid function in (3.2).

Step 3. This step produces an intermediary result that will be helpful in the following stop: the derivative is decreasing for \( b(v^*) \geq v^* \). Given that the right hand side is increasing in \( v \) everywhere, it will also be increasing in \( v \) at \( v^* \). To show that there is at most one \( v^* \), it suffices to show that the LHS is a decreasing function of \( v \), at \( v^* \). The derivative of the LHS w.r.t. \( v \) is:
\[
- \int_{v^*}^{v} \left( \frac{1-F(x)}{1-F(v)} \right)^{\frac{1-\tilde{\beta}}{\tilde{\alpha}}} \frac{1+\tilde{\alpha} - \tilde{\beta}}{\tilde{\alpha}^2} \left( 1 + \frac{1+\tilde{\alpha} - \tilde{\beta}}{\tilde{\alpha}} \log \frac{1-F(x)}{1-F(v)} \right) dx \frac{f(v^*)}{1-F(v^*)}
\]
\[(D.10)\]
while the derivative of the RHS correspond to (D.3) and can be rewritten as:
\[
\frac{(\alpha - \tilde{\beta})(1+\tilde{\alpha} - \tilde{\beta})}{\tilde{\alpha}} \frac{b(v^*)(1+\tilde{\alpha} - \tilde{\beta}) - v^*}{1-F(v^*)} f(v^*)
\]
\[(D.11)\]
We can put the last two expressions together and study the sign

\[
-(b(v^*) (1+\bar{\alpha} - \bar{\beta}) - v^*) \frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} + (\alpha - \beta)(1+\bar{\alpha} - \bar{\beta})^2 b(v^*) \leq \frac{(\alpha - \beta) (1+\bar{\alpha} - \bar{\beta})}{\bar{\alpha}} (b(v^*) (1+\bar{\alpha} - \bar{\beta}) - v^*)
\]

\[
\Rightarrow \alpha \frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} b(v^*) \geq \frac{\alpha - \beta}{\bar{\alpha}^2} v^* = \alpha \frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} v^*
\]

\[
\Rightarrow b(v^*) \geq v^*
\]

(D.12)

where we rewrote \( \int_{v^*}^{\bar{\alpha}} \left( \frac{1-F(x)}{1-F(v^*)} \right)^{\frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}}} dx \) in terms of bid minus values. Therefore, as long as the equilibrium bid at the cut-off value \( v^* \) is greater than the cut-off itself, bids will be decreasing in \( q \) for all \( v \), for \( \alpha > \beta \).

**Step 4** To show the uniqueness of \( v^* \) we merge two results. First, we show that (D.8) is negative at the greatest private value. Second, the monotonicity of the bid function implies that there is no region on the support of \( v \) such that \( b(v) > v \) to both its left and right and \( b(v) < v \) inside. Therefore, once (D.8) becomes negative, it cannot switch sign from negative to positive and back to negative again, implying that if \( v^* \) exists, it is unique.

When \( \alpha > \beta \), the limit of (D.8) for \( v \to \bar{\alpha} \) is negative. In fact, under the increasing hazard rate condition (Assumption 1.3) applying L’Hospital’s rule to the first term of (D.8) yields

\[
\lim_{v \to \bar{\alpha}} \int_{v^*}^{\bar{\alpha}} \left( 1-F(x) \right)^{\frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}}} \left( -\frac{\alpha (1+\bar{\alpha} - \bar{\beta})}{\bar{\alpha}^2} \log \frac{1-F(x)}{1-F(v^*)} - (\alpha - \beta) \right) dx = 0
\]

while the limit of the (numerator of the) remaining part is \( \lim_{v \to \bar{\alpha}} - (\alpha - \beta) v = -(\alpha - \beta) \bar{\alpha} < 0 \).

Next, \( b(v) \) intersects the 45° line only once, at \( v^{**} \), such that \( b(v) > v \) for \( v < v^{**} \) and \( b(v) < v \) for \( v > v^{**} \). To see this, notice that at \( \bar{\alpha} \) the bid of a charitable bidder is greater than \( \bar{\alpha} \), while at \( \bar{\alpha} \) it is smaller than \( \bar{\alpha} \) for \( \alpha > \beta \). Since the bid function is strictly increasing, a bidder’s bid will be equal to her value for no more than one \( v \).

The requirement that (D.8) is negative when evaluated at the upper bound, coupled with its continuity on the support of \( v \) and the monotonicity of \( b(v) \), necessarily means that (D.8) cannot switch sign more than once. This means that for \( v > v^{**} \) (D.8) cannot be positive. Moreover, this also implies that \( v^* \) does not exist if (D.8) is negative at \( \bar{\alpha} \). In this case, the derivative of the bid w.r.t \( q \) will always be negative. Thus, (D.8) cannot be increasing for \( b(v^*) < v^* \), and at \( v^* \) the bid must be greater than the private value. Therefore, if \( v^* \) exists, it is unique and separates those who increase their bid from those who decrease it.

**D.5 Proof of Proposition 4**

When \( \alpha = 0 \), the expected consumer surplus in a charity auction is equal to the consumer surplus in a non-charity auction. It is greater when \( \alpha > 0 \).

**Proof:** In a second-price non-charity auction a bidder’s dominant strategy is to bid her private value (i.e.

\[
\text{That is } b(\bar{\alpha}) = \frac{1}{1+\bar{\alpha} - \bar{\beta}} \left( \bar{\alpha} + \int_{\bar{\alpha}}^{\bar{\alpha}} (1-F(x)) \frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} dx \right) > \int_{\bar{\alpha}}^{\bar{\alpha}} (1-F(x)) \frac{1+\bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} dx > 0 \text{ and } b(\bar{\alpha}) = \frac{\bar{\alpha}}{1+\bar{\alpha} - \bar{\beta}} \text{ implying that } b(v) < \bar{\alpha}. \]
The ex-ante consumer surplus is therefore:

\[
CS^{NC} = \int_{\mathbb{V}} \int_{\mathbb{V}_x} v - b^{NC}(x) dF(x)^{n-1} dF(v)
\]

\[
= \int_{\mathbb{V}} v \cdot (1 - F(v)^{n-1}) + \int_{\mathbb{V}} F(x)^{n-1} dx \, dF(v)
\]

where the second equality is derived integrating by parts. Showing that $CS^{NC}$ is identical to the consumer surplus in an analogous charity auction where bidders do not gain from externalities (i.e., $\alpha = 0$) is trivial because the expected payment in the former is equal to the expected payment minus warm glow of donating in the latter.\(^5\)

We turn now to the consumer surplus of a charitable second-price auction when $\alpha > 0$, which is

\[
CS^C = \int_{\mathbb{V}} \int_{\mathbb{V}_x} v - (1 - \beta q) \cdot b^C(x) dF(x)^{n-1} dF(v) + q\alpha \cdot b^C(v) F(v)^{n-2}(1 - F(v)) + q\alpha \cdot \int_{\mathbb{V}} b^C(x) dF^{(2)}_{(n-1)}(x)
\]

where $b^C(x)$ is defined as in (3.2) and $F^{(2)}_{(n-1)}(x) = F(x)^{n-1} + (n - 1)F(x)^{n-2}(1 - F(x))$. Plugging in the bidding function in (D.14), and after some algebra, one obtains:

\[
CS^C = \int_{\mathbb{V}} \left\{ v \cdot (1 - F(v)^{n-1}) + \int_{\mathbb{V}} F(x)^{n-1} dx \right. \\
- \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} dF(v)^{n-1} - \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F(x)^{n-1} dx \\
+ \frac{q\alpha}{1 - q\beta + q\alpha} \left[ v + \int_{\mathbb{V}} (1 - F^{(2)}_{(n-1)}(x)) dx + \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F^{(2)}_{(n-1)}(x) dx - \int_{\mathbb{V}} F(x)^{n-1} dx \right] \right\} dF(v)
\]

Importantly, the first line in this equation is $CS^{NC}$ in (D.13). Therefore, in order to prove the theorem we only need to show that the remaining two lines are positive.

In order to simplify the computations, we approximate $CS^C$ with $\hat{CS}^C$ where the expression in the second line $- \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} dF(v)^{n-1}$ is substituted with $- \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F(x)^{n-1} dx$ as follows

\[
\hat{CS}^C = CS^{NC} \\
\int_{\mathbb{V}} \left\{ - \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F(x)^{n-1} dx - \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F(x)^{n-1} dx \\
+ \frac{q\alpha}{1 - q\beta + q\alpha} \left[ v + \int_{\mathbb{V}} (1 - F^{(2)}_{(n-1)}(x)) dx + \int_{\mathbb{V}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - \frac{q\beta + \alpha}{\alpha}} F^{(2)}_{(n-1)}(x) dx - \int_{\mathbb{V}} F(x)^{n-1} dx \right] \right\} dF(v)
\]

Because $F(x)^{n-1} > F(v)^{n-1}$ for $x > v$, the last expression is smaller than the former. Hence we can think of $\hat{CS}^C$ as a lower bound for $CS^C$. Therefore if $\hat{CS}^C > CS^{NC}$ then also $CS^C > CS^{NC}$. The remainder of this proof heavily relies on L’Hospital’s rule and integration by parts to show that this ordering holds. In particular we will show that the third line (which involves sums of positive terms) is greater than the second line.

To save on notation, denote $\left( \frac{1 - F(x)}{1 - F(v)} \right)$ by $\Phi$ and $\frac{1 - q\beta + \alpha}{q\alpha}$ by $\psi$. Integrating by part the first term in the

\(^5\)That is, if the expected payment is $x$, then $x = \frac{x}{1 - pq}(1 - \beta q)$.
second line of (D.16) (i.e., $\int_\varpi^\sigma - \int_\varpi^\sigma \Phi^\theta F(x)^{n-1}dxF(v)$) gives
\[
- \left[ \int_\varpi^\sigma \Phi^\theta F(x)^{n-1}dxF(v) \right]_{\varpi}^{\sigma} - \int_\varpi^\sigma F(v)^n + \int_\varpi^\sigma \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}F(x)^{n-1}dxF(v)dv
\]
(D.17)

The first term in brackets is 0, while the second term (negative) and the third term (positive) will be cancelled out using a combination of the second expression in the second line and the term $+ \frac{1}{\psi} \int_\varpi^\sigma \Phi^\theta [F(x)^{n-1} + (n-1)F(x)^{n-2}(1-F(x))]dx$ in the middle of the third line of (D.16). Starting from the former, and integrating it by parts:
\[
\int_\varpi^\sigma - \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_\varpi^\sigma \Phi^\theta F(x)^{n-1}dx dF(v) = - \left[ \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_\varpi^\sigma \Phi^\theta F(x)^{n-1}dxF(v) \right]_{\varpi}^{\sigma} \\
- \int_\varpi^\sigma \frac{1 - q\beta}{1 - q\beta + q\alpha}F(v)^n + \int_\varpi^\sigma \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}F(x)^{n-1}dxF(v)dv
\]
(D.18)

The first term in the square brackets is zero because of the L’Hospital’s rule. In addition, the second two terms in (D.17) and (D.18) are similar, with the only difference being the multiplicative constant and the integration regions. However, because:
\[
\int_\varpi^\sigma A(x)dx = \int_\varpi^\sigma A(x)dx - \int_\varpi^\sigma A(x)dx
\]
where $A(x)$ is a continuous function, we can rewrite the last term in (D.18) as
\[
\int_\varpi^\sigma \int_\varpi^\sigma - \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}F(x)^{n-1}dxF(v)dv = \\
\int_\varpi^\sigma \int_\varpi^\sigma - \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}F(x)^{n-1}dxF(v)dv \\
- \int_\varpi^\sigma \int_\varpi^\sigma - \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}F(x)^{n-1}dxF(v)dv
\]
(D.19)

where the term in the second line is positive while the remaining term is negative. Given this algebra, we can cancel out the last term in (D.17) by summing the last term in (D.19) with a similar one but with $1/\psi$ as a multiplicative factor (i.e. $1/\psi + (1 - q\beta)/(1 - q\beta + q\alpha) = 1$). This term is found by integrating by parts the term in the second line of (D.16) that was previously mentioned:
\[
\int_\varpi^\sigma \frac{1}{\psi} \int_\varpi^\sigma \Phi^\theta [F(x)^{n-1} + (n-1)F(x)^{n-2}(1-F(x))]dx dF(v)
\]
\[
= \frac{1}{\psi} \left[ \int_\varpi^\sigma \Phi^\theta F(v)^{n-1} + (n-1)F(v)^{n-2}(1-F(v))]dxF(v) \right]_{\varpi}^{\sigma} - \int_\varpi^\sigma [F(x)^n + (n-1)F(x)^{n-1}(1-F(x))]dv \\
- \int_\varpi^\sigma \int_\varpi^\sigma \Phi^\theta \cdot \psi \cdot \frac{f(v)}{1-F(v)}[F(x)^{n-1} + (n-1)F(x)^{n-2}(1-F(x))]dxF(v)dv
\]
(D.20)

Once again the first term in brackets goes to zero. Moreover, a portion of the last term (i.e., $- \int_\varpi^\sigma \Phi^\theta \cdot \psi \cdot$

---

6In fact the $\lim_{v \to \varpi} \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_\varpi^\sigma \Phi^\theta F(x)^{n-1}dx = 0$, while the expression is also zero for $v \to \varpi$ because $F(\varpi) = 0$. 

27
\[
\frac{f(v)}{1-F(v)} F(x)^{n-1} dx F(v) dv \]
can be summed with the last term in (D.19) to cancel out the last term in (D.17). Similarly, the second term in (D.17) cancels out with the sum of the second term in (D.18) and a portion of the second term in (D.20) (i.e., \( \int_0^v F(x)^n dv \)).

To conclude the proof, we must make sure that the sum of the remaining terms in (D.20) and (D.16) is positive. To show that the remaining part of (D.20) is positive, rewrite it as:

\[
\int_0^v \left( (n-1) \cdot F(v)^{n-2} (1-F(v)) - \int_0^v (1-F(x))^\psi (n-1) F(x)^{n-2} (1-F(x)) dx \right) \psi (1-F(v))^{-\psi-1} f(v) F(v) dv
\]

Integration by parts of the second line of (D.21) yields

\[
- \left[ \int_0^v (1-F(x))^\psi (n-1) F(x)^{n-2} (1-F(x)) F(v) dx \right]_{v=0}^{v=v} - \int_0^v (n-1) \cdot F(v)^{n-2} (1-F(v)) F(v) dv + \int_0^v (n-1) \cdot F(v)^{n-2} (1-F(v)) F(v) dv
\]

The first term evaluated at the limits of the support of \( v \) is zero.\(^7\) In addition, the term in the second line of (D.22) is equal to the first term in (D.21) but with opposite signs so they cancel out, while the last term is positive. Finally, the sum of the first and last term in the third line of (D.16) is positive.

We conclude that \( \widehat{CS}^C > CS^{NC} \) implying also that \( CS^C > CS^{NC} \).

**D.6 Proof of Proposition 5**

The producer surplus in a charity auction is below that in a similar non-charity auction when \( \alpha = 0 \). When \( \alpha > 0 \) it can be greater or smaller than that in a comparable non-charity auction.

**Proof:** Assuming 0 marginal cost, the producer surplus is equal to the net revenue to the auctioneer for each object sold:

\[
PS(\alpha, \beta, q) = \begin{cases} 
\int_0^\beta (1-q) \cdot \frac{1}{1+q(\alpha-\beta)} \left\{ v + \int_v^\beta \left( \frac{1-F(x)}{1-F(v)} \right)^{1+\frac{1}{\gamma} q} dx \right\} dF^{(2)}(v) & \text{if } \alpha > 0 \land q > 0 \\
\int_0^\beta (1-q) \cdot \frac{v}{1-\beta q} dF^{(2)}(v) & \text{if } \alpha = 0 \lor q = 0 
\end{cases}
\]

When \( \alpha = 0 \) the derivative of the producer surplus w.r.t. \( q \) is negative as \( \beta \in (0,1) \). Therefore, the auctioneer is always better off by setting \( q = 0 \).

Next we turn to the other case (\( \alpha > 0 \)). First consider the \( q = 1 \) case: this cannot be optimal because it would leave the auctioneer with zero profits. In this case, setting \( q = 0 \) would be a profitable deviation, proving that \( q = 1 \) cannot be a solution for a profit maximizer auctioneer. Taking derivative with respect

\[^7\text{This result follows from L'Hospital's rule. In fact}
\]

\[
\lim_{v \to \beta} \frac{(1-F(v))^{\gamma+1}(n-1)F(v)^{n-1}}{\psi(1-F(v))^{\gamma-1}f(v)} + \int_0^\beta (1-F(v))^{\gamma+1}(n-1)F(v)^{n-2} df(v) = 0
\]

The first term goes to zero, while to prove that the second term goes to zero as well we can divide numerator and denominator by \( f(v) \) and apply L'Hospital’s rule again.
to \( q \), an interior solution for \( q \) is to be preferred if

\[
\left( \frac{1}{1 + q (\alpha - \beta)} \right)^2 \int_{\overline{v}} \left\{ v + \int_{v} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1-q\beta}{q\alpha}+1} dx \right\} (\beta - \alpha - 1)
\]

\[
- \frac{1-q}{q} \int_{v} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1-q\beta}{q\alpha}+1} + (1 + q \cdot (\alpha - \beta)) \frac{1}{q \cdot \alpha} \log \left( \frac{1 - F(x)}{1 - F(v)} \right) dx dF(\overline{v}^2(v)) = 0
\]

for \( 0 < q < 1 \).

Figure 16a demonstrates numerically that interior solutions for \( q \) are possible: if \( \alpha = 5\% \) and \( \beta = 90\% \) the auctioneer makes more net revenues with a charity auction (solid line) than with a non-charity one. To the contrary, net revenues are below non-charity revenues in the other case (the dashed line is computed using the estimated \( \alpha \) and \( \beta \) of the structural model). Moreover, the number of expected bidders is an important factor to be taken into account as shown in Figure 16b.

We provide a discussion of this empirical result here. The first term in brackets is negative and converges to \(-v(1 - \beta + \alpha)\) as \( v \to \overline{v} \) which is increasing in \( \beta \) and decreasing in \( \alpha \). The term in the second row is positive and converges to 0 from above by Lemma 1. Importantly, when \( \alpha \) is large the convergence rate is slow. Therefore, the sum of these two terms can be larger or smaller than zero for \( v \) in a neighbourhood of \( \overline{v} \), depending on the parameters and the distribution of private values. Given that the expectation is taken with respect to the distribution of the second highest out of \( n \) draws from \( F(\cdot) \), the previous equation puts a greater probability mass on events towards the upper tail of the distribution (e.g. close to \( \overline{v} \)). Hence, it can hold for positive \( q \).

Figure 16: Difference between net expected revenues from charity and non-charity auction

(a) The difference can be positive or negative

(b) The difference depends on \( n \)

Note: Both plots show the difference in net revenues to the auctioneer between a second-price charity auction (with \( q = 85\% \)) and a second-price non-charity auction (e.g., \( q = 0\% \)) for different number of bidders. The private values are normally distributed with mean 50 and standard deviation 25 on \([0, 100]\). Plot (a) depicts two combinations of the charitable parameters \( \alpha \) and \( \beta \). The case with low \( \alpha \) and high \( \beta \) results in revenues to the auctioneer beyond the non-charity case (solid line). The opposite is true in the other case which uses the parameters estimated in the structural model in Section 5 (dashed line). Plot (b) shows that the number of bidders play an important role in the decision of the auction format.
D.7 Proof of Proposition 6

\(\alpha, \beta\) and \(F(v)\) are not identified without additional restrictions.

**Proof:** The model is not identified without (i) auxiliary data, or (ii) additional distributional assumptions. For simplicity, assume that \(q = 1\). It is easy to see that the model is not identified when \(\alpha = 0\). In fact, it is not possible to tell apart the following two expected utility models:

\[
\begin{align*}
    u(v; \alpha, \beta, q) &= \begin{cases} 
        v - b^{II} + \beta b^{II} & \text{if } i \text{ wins} \\
        0 & \text{otherwise}
    \end{cases} \\
\end{align*}
\]

implying that the model \(\Gamma_2(F, \alpha = 0, \beta, q = 1)\) is observationally equivalent to the model \(\tilde{\Gamma}_2(\tilde{F}, \alpha = 0, \beta, q = 1)\), with \(\tilde{\beta} = 0\) and \(\tilde{F} = F((1 - \beta)v_i)\). In both models bidder \(i\) bids \(b(v) = \frac{v}{1 - \beta} = \tilde{b}\), where \(\tilde{b} = \tilde{b}(\tilde{v})\).

Setting \(\alpha > 0\) does not improve identification. In this case the model \(\Gamma_2(F, \alpha, \beta, q = 1)\) is observationally equivalent to \(\Gamma_2(F, \pi, \beta, q = 1)\), where \(\tilde{\beta} = 0, \tilde{\pi} = \frac{\alpha}{1 - \beta}\) and \(\tilde{F}(v) = F((1 - \beta)v)\). Therefore, it is impossible to determine the tuple \((\alpha, \beta, F)\).

D.8 Proof of Proposition 7

In second-price auctions, under Assumption 1 the parameters \(\alpha\) and \(\beta\) and the distribution of values \(F(v)\) are identified by variation in \(q\) across auctions.

**Proof:** The researcher observes two types of auction, \(A\) and \(B\), and that the only difference among them is the fraction of the transaction price that will be donated, i.e. \(q^A \neq q^B\). Because the distribution of values \(F(\cdot)\) is the same across auctions \(A\) and \(B\), then for each value corresponding to the \(\tau\)-quantile of the value distribution, \(v^\tau\), the distribution of bids computed at that \(\tau\)-quantile must be equal, i.e. \(G(b^\tau | q^A) = G(b^\tau | q^B)\). Therefore, after rewriting the first order condition for each set of auctions (D.1) as in (4.1) we can match the FOCs from the two auctions (D.1) based on the quantiles of the bid distribution. This gives us the following equation for each quantile of \(v^\tau\):

\[
\begin{align*}
    v_x^A - v_x^B &= b_x^A - b_x^B + (\alpha - \beta) \cdot (q^A \cdot b_x^A - q^B \cdot b_x^B) - \alpha \cdot (q^A \cdot \lambda(b_x^A) - q^B \cdot \lambda(b_x^B)) \quad (D.24)
    \end{align*}
\]

where \(G(b^\tau) = G(b_x^A) = G(b_x^B)\). Since \(v_x^A - v_x^B = 0\), eq. (D.24) can be rewritten in matrix notation as

\[
\Delta(b_x) = B_x \times \begin{bmatrix} \alpha - \beta \\ -\alpha \end{bmatrix}, \quad (D.25)
\]

where \(\Delta(b_x) = -(b_x^A - b_x^B)\) and \(B_x\) is the matrix \([q^A b_x^A - q^B b_x^B; q^A \cdot \lambda(b_x^A) - q^B \cdot \lambda(b_x^B)]\).

It can be shown that the matrix \(B_x\) has full rank (the two columns are linearly independent). To prove this assume that \(B_x\) is not invertible and therefore that its columns are linearly dependent (i.e. \(b_x = k \cdot \lambda(b_x)\) for both auctions \(A\) and \(B\)). Dependence leads to \(G(b_x) = 1 - b_x \cdot g(b_x) / k\) for a constant \(k > 0\). Note that \(k\) must be positive because otherwise (i) \(G(b_x) > 1\) as \(g(b_x) \geq 0, \forall b_x\) and (ii) the FOC equation \(\xi(b_x, \alpha, \beta, q)\), which was defined in (4.1), may not be monotonically increasing in \(b_x\). The previous differential equation admits a solution \(g(b_x) = c \cdot (b_x)^{-(k+1)}\), where \(c\) is an integration constant. Thus, \(G(b_x) = 1 - c \cdot (b_x)^{-k}/k\). Evaluating the CDF at \(b_x = 0\) yields \(G(0) = -\infty, \forall k > 0\). Moreover, the inverse hazard rate

\[
\lambda(b_x) = \frac{1 - 1 + c \cdot (b_x)^{-k}/k}{c \cdot (b_x)^{-(k+1)}} = \frac{b_x}{k} \quad \text{for } k > 0
\]

\[8\]I would like to thank Jorge Balat for these examples.
is increasing in \( b_\pi \). This implies that
\[
\frac{1 - F(v_\pi)}{f(v_\pi)} = \frac{1 - G(b(v_\pi))}{g(b(v_\pi)) \cdot b'(v_\pi)} = \frac{1}{k b'(v_\pi)}
\]
which is an increasing function because (i) the optimal bidding function \( b(v) \) is increasing in the private value \( v_\pi \) and (ii) the bidding function \( b(v_\pi) \) maximizes a bidder’s utility \( b''(v_\pi) \leq 0 \). This means that the inverse hazard rate is not decreasing and that therefore \( b(v_\pi) \) is not a best response for \( v_\pi \). This is a contradiction that violates Proposition 1 proving that the columns in \( B \), are not linearly dependent and that \( B_x \) is invertible.\(^9\) Given that \( B_x \) has full rank, \( \alpha, \beta \) and \( F(v) \) are nonparametrically identified. \( \blacksquare \)

### D.9 Proof of Corollary 1

In second-price auctions, \( \alpha, \beta \) and \( F(v) \) are nonparametrically identified also when the dataset includes more than 2 types of auctions.

**Proof:** Assume that \( q \) has dimension \( K_C \) and that \( K_\pi \) quantiles of the distribution of values are observed. The FOC (4.1) in matrix notation becomes
\[
V_{K_\pi \times K_C} = (\alpha - \beta) \times B_{K_\pi \times K_C} \times Q_C + B_{K_\pi \times K_C} - \alpha \times \Lambda_{K_\pi \times K_C} \times Q_C_{K_\pi \times K_C}
\]
where \( V \) is a matrix of dimension \( K_\pi \times K_C \) displaying the value \( v_\pi \) for the \( \pi - \) quantile (row) in auction of type \( C \) (column), and 0 otherwise. Similarly, \( Q_C \) is a diagonal matrix with entries equal to the percentage donated \( q_j^C \) and 0. The other matrices are defined as:
\[
B = \begin{bmatrix} b^1_0 & b^2_0 & \ldots & b^K C \vspace{0.2cm} b^1_1 & b^2_1 & \ldots & b^K C \vspace{0.2cm} \vdots & \vdots & \ddots & \vdots \vspace{0.2cm} b^1 K_C & b^2 K_C & \ldots & b^K C \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda^1(b^1_0) & \lambda^2(b^2_0) & \ldots & \lambda^K C(b^K C) \vspace{0.2cm} \vdots & \vdots & \ddots & \vdots \vspace{0.2cm} \lambda^1(b^1_1) & \lambda^2(b^2_1) & \ldots & \lambda^K C(b^K C) \end{bmatrix}
\]
where superscripts indicate that the amount donated is equal to \( q_j \) for \( j \in [1, K_C] \) and subscripts indicate the \( \pi - \) quantiles of the distribution of values or bids.

There exists a projection \( M \) (with rank \( K_C - 1 \)) such that \( V \times M = 0 \). Postmultiplying (D.26) by \( M \) and moving terms, the following equation represents the FOC where the dependent variable is a known object
\[
-B \times M = (\alpha - \beta) \times B \times Q_C \times M - \alpha \times \Lambda \times Q_C \times M
\]
After stacking the matrices in vectors, the last equation can be represented by the system of equations
\[
y = [b \quad l] \times \begin{bmatrix} \alpha - \beta \\ -\alpha \end{bmatrix}
\]
where \( y = \text{vec}(-B \times M), b = \text{vec}(B \times Q_C \times M), l = \text{vec}(\Lambda \times Q_C \times M) \), and \( \text{vec}(\cdot) \) indicates the vectorization of the matrices in parenthesis.

Nonparametric identification requires showing that \( b \) and \( l \) are not proportional to each other. Non-proportionality follows directly from the argument in the proof of Proposition 7 in Appendix D.8. Hence, \( b \) and \( l \) are not linearly dependent, establishing identification of \( \alpha, \beta \) and \( F(v) \). \( \blacksquare \)

---

\(^9\)If \( k = 1 \), then \( g(b_{\pi}) = 0 \) for \( b_{\pi} < 0 \) and \( g(b_{\pi}) > 0 \) for \( b_{\pi} \geq 0 \). Therefore \( g(\cdot) \) is the Dirac delta function, which is not differentiable and does not admit a decreasing inverse hazard rate. In turn, given that \( F(v) = G(b(v)) \), the non-differentiability of \( G(\cdot) \) implies that also the distribution of values \( F(\cdot) \) is not differentiable, a contradiction.
D.10 Proof of Proposition 8

In English auctions, \( \alpha, \beta \) and \( F(v) \) are nonparametrically identified by first deriving the distribution of bids that would have been observed in parallel second-price auctions, and then by applying Proposition 7.

**Proof:** Assume that we observe two kind of auctions \( A \) and \( B \), characterized by \( q_A \) and \( q_B \) respectively (with \( q^A \neq q^B \)). The starting point is to note that \( G(b) = F(v) \), and that also in charity auctions the distribution of the winning bid is equal to the distribution of the second-highest bid: \( G_w(b) = G_{(n)}(b) \). Therefore, the distribution of bids that would have been realized in an equivalent second-price auction is found using the classic inversion of the latter relation. In particular, \( G(b) \) is found as the root (in \([0, 1]\)) of

\[
G_w(b) - nG(b)^{n-1} + (n-1)G(b)^n.
\]

We can now write the FOCs (4.1) for each \( \alpha \) and \( \beta \). Therefore, we can apply the same logic in Proposition 7, and identify \( \alpha, \beta \) and \( F(v) \).

\( \blacksquare \)
E Monte Carlo simulations

Objectives. The Monte Carlo simulations in this section fulfil two goals. First, we want to show that the estimation routine described in Section 5 return consistent estimates of the parameters. Second, we need to support with some empirical evidence the claim that the estimates are not consistent when the amount donated in the two auction types is very close.

Design of the experiments. There are two auction types (A and B) such that \( q^A = .10 \) and \( q^B = .85 \). Private values are generated for all bidders drawing from a uniform distribution in \([0, 1]\) in Tables 22 and 23 and in \([-1, 1]\) in Tables 24. There are 10 bidders in each auctions. They bid according to the bid function in (3.2). The true charitable parameters are \( \alpha_0 = .25 \) and \( \beta_0 = .75 \).

The steps of the estimation procedure are outlined below:

1. Draw values from the distribution \( F(v) \) for each bidder in the two auctions. In total 20 values.
2. Compute the bids for each bidder in the two auctions. Save the winning bid in each auction.
3. Nonparametrically estimate the density of the winning bids (either by Triweight, or Gaussian Kernel). The bandwidth is chosen using the rule-of-thumb. Trimming follows Guerre et al. (2000) who suggested trimming observations close to \( 0.5 \times \) bandwidth to the boundary.\(^{10}\)
4. Given the number of bidders \( (n = 10) \) invert the distribution of the winning bids to determine the distribution and density of the bids as in (5.2) and (5.3).
5. Compute the distribution and density of auctions of type A for each losing bids in the interval between the smallest winning bid and the largest winning bid of type A.
6. Compute the distribution and density of type B \( (q = .85) \) over 100,000 points.
7. Match the quantile of the distribution of type B with those of the distribution of type A through (4.1).
8. Find the couple \( (\alpha, \beta) \) that minimizes the objective function (5.4) starting from a random seed. The search algorithm constraints the parameters in the unit interval.
9. Save the estimates and restart from 1.

These steps are repeated 401 times. The tables below report the mean, median, quantiles and root mean squared errors for \( \alpha \) and \( \beta \) for each combination of parameters.

Results. First, let’s asses the consistency of the estimates. Different experiments are reported in Tables 23 and 24, showing that the estimates are close to the true parameters. In particular, even with a small number of observations (the first line in each panel), the mean and medians are always within 3% of the true parameters.

These tables are composed by different panels: each panel refers to a different kernel used to estimate the distributions (and densities) of the winning bids. The Gaussian and Triweigh kernels give similar results. Within each panel, the rows differ on the number of auctions used to estimate the primitives. The number of bidders in each auction is always constant and equal to 10. Since for each auction only the winning bid is used, we empirically consider the asymptotic properties of the estimator by looking at

\(^{10}\)For the Gaussian case the \( h_{pdf} = 1.06\sigma n^{-1/5} \) and \( h_{CDF} = 1.06\sigma n^{-1/3} \) where \( \sigma = \min\{\text{s.d.}(b^k_{\text{w}}), \text{IQR}/1.349\} \), where \( b^k_{\text{w}} \) is the vector of winning bid for auction of type \( k \), and \( h_{CDF} = 1.587\sigma n^{-1/3} \). For the triweigh case \( h_{pdf} = h_{CDF} = 2.978\sigma n^{-1/5} \) (Härdle, 1991, Li et al., 2002, Li and Racine, 2007, Lu and Perrigne, 2008).
the rate at which the root mean squared error (RMSE) decreases as the number of auctions grows (i.e., comparing RMSE across columns).\textsuperscript{11} Comfortingly, this rate is close to $\sqrt{n}$ for all experiments.

To study the consistency of the estimates when there is only limited variation over $q$ across auctions we run similar experiments varying $q$ instead of the nonparametric kernel. From Table 22 it is clear that $\alpha$ and $\beta$ cannot be estimated consistently when $q_A \simeq q_B$ as the mean and median of the estimated parameters are about 0 and .50 instead of .25 and .75 for $\alpha$ and $\beta$ respectively.

Table 22: Distance between $q^A$ and $q^B$ – Monte Carlo simulations

<table>
<thead>
<tr>
<th>$T^A$</th>
<th>$T^B$</th>
<th>$\mu_\alpha$</th>
<th>$\mu_\beta$</th>
<th>Med$\alpha$</th>
<th>Med$\beta$</th>
<th>25$%_\alpha$</th>
<th>75$%_\alpha$</th>
<th>25$%_\beta$</th>
<th>75$%_\beta$</th>
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<tbody>
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<td>0.0069</td>
<td>0.5333</td>
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<td>0.7398</td>
<td>0.1984</td>
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</table>

Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by $A$ and $B$. Each panel shows the estimated parameters for different percentage donated. The bandwidths in step 2 are computed with a Gaussian Kernel. The data is generated according to $\alpha = 25\%$, $\beta = 75\%$ and $F(v)$ is assumed uniform in $[0,1]$. Each auction has 10 bidders. 401 repetitions.

\textsuperscript{11}I am analyzing the asymptotic properties of this class of estimators theoretically in another project, which is still a work in progress.
Table 23: Monte Carlo simulation 1

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<tr>
<th>$T^A$</th>
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<th>$\mu_\alpha$</th>
<th>$\mu_\beta$</th>
<th>Med$_\alpha$</th>
<th>Med$_\beta$</th>
<th>25%$_\alpha$</th>
<th>75%$_\alpha$</th>
<th>25%$_\beta$</th>
<th>75%$_\beta$</th>
<th>10%$_\alpha$</th>
<th>90%$_\alpha$</th>
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<th>90%$_\beta$</th>
<th>RMSE$_\alpha$</th>
<th>RMSE$_\beta$</th>
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</table>

Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by $A$ and $B$. The bandwidths in step 2 are computed either with a Gaussian Kernel (top panel) or with a Triweight kernel (bottom panel). The data is generated according to $\alpha = 25\%$, $\beta = 75\%$, $q^A = 10\%$, $q^B = 85\%$ and $F(v)$ is assumed uniform in $[0, 1]$. Each auction has 10 bidders. 401 repetitions.
Table 24: Monte Carlo simulation 2

<table>
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<th>(T^A)</th>
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<th>(\mu_\beta)</th>
<th>(\text{Med}_\alpha)</th>
<th>(\text{Med}_\beta)</th>
<th>25%_\alpha</th>
<th>75%_\alpha</th>
<th>25%_\beta</th>
<th>75%_\beta</th>
<th>10%_\alpha</th>
<th>90%_\alpha</th>
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</tbody>
</table>

Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by \(A\) and \(B\). The bandwidths in step 2 are computed either with a Gaussian Kernel (top panel) or with a Triweigh kernel (bottom panel). The data is generated according to \(\alpha = 25\%\), \(\beta = 75\%\), \(q^A = 10\%\), \(q^B = 85\%\) and \(F(v)\) is assumed uniform in \([-1, 1]\). Each auction has 10 bidders. 401 repetitions.
References


