Green Technology and Patents in the Presence of Green Consumers

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Abstract

We develop a theoretical framework to investigate the impact of patent policies and emission taxes on green innovation and emissions in the presence of environmentally friendly consumers. We analyze the effect of changing patentability requirements and patenting costs when a firm may invest in a green innovation, which reduces the emission output ratio. We show that investment in green technologies reduces emissions only if the fraction of green consumers is sufficiently small, and that the magnitude of this effect decreases as the fraction of green consumers increases. While marginally higher emission tax only increases green investment if the fraction of green consumers is sufficiently small, it always induces less emissions. However, a discrete jump in the tax rate may result in a green paradox, leading to more emissions. A stricter patentability requirement combined with fast-track green patents is only effective at reducing emissions as long as the fraction of green consumers is sufficiently small.

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1 Introduction

We examine the interaction of patent policies and environmental regulation in enhancing innovation in “green”, or less polluting, production technologies, and in reducing the emission of pollutants. Given the increasing environmental consciousness of citizens globally, developing green technologies has become a key policy initiative of international organizations such as the UN and the G8, and of national governments.\(^1\) The analysis of patent policies for the development of green technologies requires a specific analysis due to potential interactions between knowledge and environmental externalities caused by the innovators, as noted by Hall and Helmers (2013). We develop a theoretical framework to investigate the impact of changing patentability requirements and patenting costs in the presence of environmental friendly consumers, and in conjunction with increasing emission taxes.

This paper builds upon existing results in the literatures on the development of green technologies, on patents, and on the environmentally friendly behavior of consumers.

The literature on green technologies is rapidly evolving in response to global environmental problems such as climate change. Climate experts propose that the increase in average global temperature should be restricted to about 2°C to avoid the possibility of catastrophic damage, a goal that may only be reached by developing and implementing “breakthrough” technologies that reduce emissions dramatically (Barrett, 2009; Galiana and Green, 2009). One stream of literature has evolved around the seminal work of Porter (1991) and Porter and van der Linde (1995), referred to as the Porter Hypothesis, and examines whether the implementation of stricter environmental regulations increases firms’ incentives to invest in green R&D. The empirical evidence surrounding the Porter Hypothesis is mixed (Ambec et al., 2013). Our paper is more closely related to a second stream of the literature, which argues that given the combination of envi-

\(^{1}\)In 2016, the Canadian federal government announced that it will invest $200 million annually to create sector specific strategies to support the development of clean technologies and invest $100 million annually in organizations that support clean technology firms such as Sustainable Development Technology Canada. In the U.S., the Department of Energy’s Loan Program Office has more than $40 billion in remaining loans to help finance innovative technologies that can reduce carbon emissions. In Canada and the U.S. respectively, about 2500 and 18500 patents for green technologies are issued annually.
ronmental externalities and knowledge market failures facing regulators, environmental policies are not sufficient to achieve the first best social outcome, and need to be combined with policies addressing the relevant knowledge market failure (Carraro and Siniscalaco, 1994; Carraro and Soubeyran, 1996; Katsoulacos and Xepapadeas, 1996; Popp, 2006; Fischer and Newell, 2008; Acemoglu et al., 2012).

Our focus is on the role played by two different aspects of patent policies, and how they interact with emission taxes in fostering green innovation. Examining patent systems within this context is important due to the lack of consensus among policy-makers and academics regarding the role of patents in promoting R&D in general and green technologies in particular. On the one hand, international organizations advocate royalty-free compulsory licensing of green technologies, excluding green technologies from patenting, and even revoking existing patent rights on them (UNFCCC, 2009). Such provisions are also incorporated in the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) (Derclaye, 2008; Rimmer, 2011). On the other hand, many countries are actively using their patent systems to target green innovations. For example, fast-tracking, or expediting the review process, of green patent applications, is a key policy initiative undertaken by several countries including Australia, Brazil, Canada, China, UK, U.S., Japan and Korea. Given that the time from application to grant has been effectively reduced by up to 75% for patents entering the fast track procedure and that evidence shows that fast-tracking programmes have accelerated the diffusion of knowledge in green technologies in the short run (Dechezleprêtre, 2013), such policies are likely to continue

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2 Patents address the problem due to the externality that results from imperfect appropriability of knowledge by endowing innovators with property rights on their inventions. A patent confers its owner a temporary right to exclude others from exploiting the innovation. In exchange for the exclusionary right, the patent holder must disclose his innovation (Langinier and Moschini, 2002; Scotchmer, 2004; Rockett, 2010).

3 Some studies have illustrated that strong Intellectual Property Rights (IPRs) may not necessarily enhance innovation (Green and Scotchmer, 1995; Gallini, 2002; Bessen and Maskin, 2009). Even though in a static world (single innovation), patents of appropriate scope can encourage innovations (Klemperer, 1990; Gilbert and Shapiro, 1990), this is no longer the case when the cumulative nature of innovation is accounted for. In the case of cumulative innovations, strong patents may even discourage follow-on innovations (Scotchmer, 1991). The prospect of being imitated inhibits inventors in a static world but, in a dynamic world, imitators can benefit both the original inventor and society in general (Bessen and Maskin, 2009).
and proliferate. Hence, the importance to examine their impact. In our model, we capture fast-tracking of green patents by lowering the cost associated with the patent application process for green technologies only.4

We also analyze the impact of changing another aspect of patent policy, that is, patentability requirements. In order to be patentable an innovation must be sufficiently novel (not already in the public domain), non-obvious (to a person with ordinary skills in the particular field), and useful (to have at least one application). The relevant requirements are currently much stricter in the EU than in the U.S. (Eckert and Langinier, 2014). In the spirit of Crampes and Langinier (2009), we model the patentability requirement as a minimum investment threshold level that must be satisfied. We then vary this investment threshold to examine whether a stricter patentability requirement fosters more green innovation.5

We also contribute to the green innovation literature by incorporating environmentally friendly consumers in our model. The increasing environmental consciousness of citizens globally is reflected in widely used eco-labeling schemes internationally.6 A few papers study optimal environmental policies in the presence of environmentally friendly consumers, but they do not address green innovation (Arora and Gangopadhyay, 1995; Cremer and Thisse, 1999; Moraga-Gonzalez and Padron-Fumero, 2002; Bansal and Gangopadhyay, 2003; Lombardini-Riipinen, 2005; Bansal, 2008).7 It is important to include environmentally friendly consumers since this

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4By 2011, Australia, Canada, Israel, Japan, Korea, UK, and U.S. had implemented fast track “green” patent applications, and since then so has Brazil and China. The Canadian Intellectual Property Office (CIPO) launched its fast track program for green patent applications in March 2011. Over the last three years, over 5,000 patent applications have requested accelerated examination, with climate change-related technologies (in particular renewable energy technologies) representing the majority of patents in the fast-tracking programmes.

5Gerlagh et al., (2014) is another paper to examine patent policies for green technologies. It focuses on analyzing the impact of changing the lifetime of patents issued to green innovations.

6For example, in countries like Sweden about 50% of the market share for certain products consists of the environmentally friendly variant. Green marketing is also frequently used to influence consumer behavior in transportation and electricity markets (Kraftborsen, 2001).

7Gil-Moltó and Varvarigos (2013) examine the role of emission taxes in inducing the adoption of cleaner technologies in the presence of environmentally friendly consumers, but abstract away from the R&D decision of firms. Sengupta (2012) examines firms’ investment in green innovation in the presence of emission taxes and environmentally friendly consumers, but does not model patents.
might modify the role of emission taxes in inducing innovation.

We model the market for a product, the production of which causes pollution. We assume that the implementation of a cleaner technology results in a lower emission per unit of output ratio (similar to, for example, Benchekroun and Ray Chaudhuri, 2014, 2015). Moreover, similar to Bansal and Gangopadhyay (2003) and Ibañez and Grolleau (2008), we assume that the product is vertically differentiated in terms of its emission-output ratio with green conscious consumers preferring products with lower emission-output ratios. On the demand side, we allow consumers to be heterogeneous in terms of their degree of environmental friendliness. Our framework has two stages where, in the first stage, an incumbent monopolist decides its level of investment in R&D, and in the second stage, it chooses the price of its product. The monopolist faces potential entry if it does not innovate. We assume that investment by the firm reduces the emission-output ratio. If the innovation is patented, the firm effectively behaves as a monopolist when setting its price in the second stage. If the innovation does not satisfy the patentability requirement, the firm cannot patent it and faces Bertrand competition in the second stage since entry occurs and rival firms are assumed to have free access to the new technology.

Within this setting, we show that investment in green technologies reduces emissions only if the fraction of green consumers is sufficiently small, and that the magnitude of this effect decreases as the fraction of green consumers increases. This is because, as the product becomes cleaner, the green conscious consumers demand more of it. Thus, the higher the fraction of green consumers, the smaller the reduction in emission due to green investment. Moreover, if the fraction of green consumers increases beyond a certain threshold, investment in green technologies results in more emissions. For a sufficiently small fraction of green consumers, we retrieve the expected result found in much of the literature surrounding the Porter hypothesis that a higher emission tax increases green investment. However, this result is reversed if the

\[ \text{This result is related to the “rebound effect” in the literature on energy efficiency policies, by which the decrease in energy consumption caused by increased energy efficiency is mitigated due to behavioral responses. An extreme form of the “rebound effect” may result in an increase in energy consumption caused by increased energy efficiency, referred to in the literature as the “backfire effect”. Papers that find evidence in support of the backfire effect include Semboja (1994), Grepperud and Rasmussen (2004), Glomsrod and Wei (2005), Hanley et al., (2005), and Hanley et al. (2009).} \]
fraction of green consumers rises to a level such that investment in green technologies results in more emissions. Regardless of the impact of investment on emissions, we show that a marginally higher emission tax induces less emissions as long as the tax rate is below some threshold. However, a larger increase in the tax rate pushes the investment level down to the minimum threshold required to obtain a patent. At this investment level, as the tax rate increases further, ceteris paribus, the emission level decreases but at a slower rate. At even higher values of the tax rate, given that the patenting cost is sufficiently high, the firm might decide to not patent, and thus to not invest, in which case the emission level rises to the competitive level. Thus, a sufficiently stricter environmental policy increases emissions, which is an example of the “green paradox” (see, for example, Gerlagh, 2009; Hoel, 2010; Grafton et al., 2012; Van der Ploeg and Withagen, 2012; Sinn, 2012). It follows that a reduction in the cost of patenting (for example, by fast-tracking green patents), may alleviate the green paradox within our context. We also show that a stricter patentability requirement combined with fast-track green patents is only effective at reducing emissions as long as the fraction of green consumers is sufficiently small. Finally, we show that as long as the fraction of green conscious consumers is sufficiently low, the firm underinvests relative to the socially optimal level for a sufficiently low emission tax and overinvests for a sufficiently high emission tax. However, the gap between the socially efficient and privately optimal levels of investment steadily reduces as this fraction increases, until this result is reversed when the fraction of green conscious consumers is sufficiently high.

Our overall conclusion is that the traditional policy tools of increasing green investment and thereby reducing emissions through stricter emission taxes and patent requirements and fast-tracking green patents may become less effective as society becomes more environmentally friendly, with consumers rewarding marginal reductions in emission-output ratios of productions processes. Thus, further research seems warranted regarding the policies to reduce emissions through green innovation and also regarding the type of information to distribute to consumers.

The paper is organized as follows. In Section 2, we present our model. In section 3, we derive the equilibrium and policy implications. Section 4 presents our concluding remarks.
2 The Model

We consider a two-stage model in which a firm sells a final good to consumers in a competitive market, the production of which is polluting and has an initial marginal cost, \( c \). The emission of the pollutant generated per unit of production is given by:

\[
\gamma = \frac{e}{q},
\]

where \( e \) denotes emission and \( q \) denotes output. The firm can invest \( I_G \) to reduce the emission-output ratio, \( \gamma \). Thus, \( \gamma \) is a function of \( I_G \), where \( \gamma(I_G) \) is such that \( \gamma'(I_G) < 0, \gamma''(I_G) > 0 \), \( \gamma(0) = \gamma_H \) and \( \lim_{I_G \to \infty} \gamma(I_G) = \gamma_L > 0 \). The higher is \( I_G \), the greener the product. For notational convenience, henceforth we do not mention the argument of the function \( \gamma \).

2.1 The demand side

The demand side consists of a continuum of \( N \) consumers. Each of them buys either 0 or 1 unit of the good. There exists a fraction \( \lambda \) of ‘green conscious’ consumers, whose utility is increasing in the “greenness” of the product, that is, decreasing in \( \gamma \), and a fraction \( (1 - \lambda) \) of ‘non-green conscious’ consumers, whose utility is independent of the greenness of the product.

Let \( G \) denote the degree of environmental friendliness of a consumer, with \( G \) being uniformly distributed over the interval \([G, \overline{G}]\) with \( \overline{G} > 0 \). We assume that consumers can observe how green a product is. Within this context, this is equivalent to assuming that consumers can observe \( \gamma \).\(^9\) We normalize \( N \) such that \( N = 1 \), and assume that \( \overline{G} - G = 1 \). Let \( P(e) \) denote the pollution damage to each consumer, which is a function of total emissions, \( e \). Following Ibañez and Grolleau (2008), we assume that the pollution level generated by total production is exogenous to each consumer, regardless of his consumption level. Let \( p \) denote the product price.

A green conscious consumer has the following utility function:

\[
U_G = \begin{cases} 
  v - G\gamma - p - P(e) & \text{from buying the product} \\
  -P(e) & \text{from not buying}
\end{cases}
\]  

\(^9\)This is a relevant scenario to consider in the presence of effective eco-labeling programs.
The term \(-G\gamma\) in (2) reflects that the greener the product, that is, the lower is \(\gamma\), the better off the green conscious consumer. Also, \(v\) represents the gross utility of consuming one unit of the good. A green conscious consumer does not buy the product if \(v - G\gamma - p < 0\).

A non-green conscious consumer has the following utility function:

\[
U_{NG} = \begin{cases} 
  v - p - P(e) & \text{from buying the product} \\
  -P(e) & \text{from not buying}
\end{cases}
\]  

(3)

From (3), it follows that a non-green conscious consumer buys the product as long as \(v \geq p\). Therefore, if \(v < p\), neither non-green conscious consumers nor green conscious consumers buy the good. If \(p = v\), only non-green conscious consumers buy the good. Therefore, we focus on the case where \(v > p\), in which case a green conscious consumer with a degree of environmental friendliness \(G\) buys the good as long as \(v - G\gamma - p \geq 0\) and non-green conscious consumers always buy. Henceforth, we assume that \(P(e) = e\).

There exists a green conscious consumer \(\tilde{G}\) who is indifferent between buying the good or not such that \(v - \gamma\tilde{G} - p - P(e) = -P(e)\) or \(\tilde{G} = (v - p)/\gamma\). We focus on the case where \(\underline{G} < \tilde{G} < \overline{G}\), such that some, but not all green conscious consumers buy the product. Thus, from (2) it follows that the demand function, \(D(p)\), is given by:

\[
D(p) = \begin{cases} 
  \lambda\left(\frac{v - p}{\gamma} - \overline{G}\right) + (1 - \lambda) & \text{if } p < v - \gamma\overline{G} \\
  (1 - \lambda) & \text{if } p \leq v \text{ and } p > v - \gamma\overline{G} \\
  0 & \text{if } p > v
\end{cases}
\]  

(4)

2.2 Policy Tools

We consider different policy tools. On the R&D side, we model a patenting policy for green and non-green innovations. On the environmental side, we assume that the firm must pay a tax, \(\tau\), per unit of emission. Thus, the tax bill faced by the firm is given by \(\tau\gamma D(p)\), where, by (1), \(\gamma D(p)\) represents the emissions generated by the firm.\(^{10}\)

The patent policy is such that the firm must discover a sufficiently novel innovation to be able to obtain a patent. In order to reflect this, we assume that the patent is granted only if

\(^{10}\)In several jurisdictions, firms pay a tax only if their emissions exceed a given threshold, \(\bar{e}\). In our model, we are implicitly setting \(\bar{e}\) to zero for simplicity.
the firm reduces the emission-output ratio below a certain threshold \( \gamma_P \), where \( \gamma_L < \gamma_P < \gamma_H \). Therefore, the firm must invest \( I_G \geq I_{PG} \) in order to ensure that \( \gamma \leq \gamma_P \). We consider weak and strong patentability requirements, representing different levels of \( I_{PG} \), as defined in Section 3.1 by Definition 1. In order to obtain a patent, the firm must incur a cost \( C_{PG} \), which includes a monetary fee payable by the firm to the patent office and the opportunity cost in terms of lost profits incurred while waiting for the patent to be granted. We consider that the cost for “green patents”, \( C_{PG} \), is below a given threshold, as defined in Section 3.1 by Definition 2. There are several ways in which policy makers may reduce \( C_{PG} \), including by implementing a fast-track patent system for green technologies that reduces the patent application processing time for green innovations.

If the innovation does not satisfy the relevant patentability requirements, the firm cannot patent it, and faces Bertrand competition in the second stage.

2.3 Assumptions and Timing

We make the following assumptions on the parameters of the model.

**Assumption 1 (A1):** Let \( \frac{\nu-c}{G+\tau} < \gamma_L \) and \( \gamma_H > \frac{\nu-c}{G+\tau} \).

**Assumption 2 (A2):** Let \( \lambda > \frac{1}{G} \), where

\[
\lambda = \frac{\gamma}{\gamma + v - c - \gamma(G + \tau)} > 0
\]

Assumptions (A1) and (A2) together ensure that \( G < \bar{G} < \bar{G} \), as we shall illustrate in the following section. Also, \( \gamma_H < \frac{\nu-c}{\tau(G+\tau)} \) ensures that \( \lambda < 1 \).

**Assumption 3 (A3):** Let

\[
\gamma'' > -\lambda \frac{(\nu-c)^2}{2} \left( \frac{\gamma'}{\gamma} \right)^3.
\]

**Assumption 4 (A4):** Let \( \frac{\nu-c}{2} > 0 \).

As we shall show in the following section, A3 ensures that the second order condition is met.

The timing of the game is as follows. In the first stage of the game, the firm decides the levels of investment in the green technology, \( I_G \). Once an innovation has been discovered, the firm decides whether to patent it.
In the second stage, the firm chooses the price of the product it offers, \( p \), which depends on how green the product is, and on the marginal cost of production.

3 The Equilibrium

We solve for the equilibrium investment levels and price through backward induction. For a given level of investment, we first determine the pricing strategy of the firm. Then, we determine the level of investment at the equilibrium.

We begin by analyzing the case where the firm chooses an investment level \( I_G \) in the first period that satisfies the patentability requirement such that \( \gamma \leq \gamma_P \) or \( I_G > I_{PG} \), and decides to patent its innovation by paying \( C_{PG} \). In this case, the firm effectively chooses the monopoly price.

In the absence of a patent (either because the investment does not satisfy the patentability requirement or because the firm decides not to patent), due to Bertrand competition in the second period, the firm sets its price at marginal cost, and gets zero profit. At the end of the section, we will also analyze the case where \( C_{PG} \) is sufficiently high such that the firm chooses not to invest.

We begin with the second stage, and analyze the pricing strategy of the firm. Assuming that the innovation has been patented, in the second period the firm solves the following:

\[
\max_p \Pi \equiv (p - c) D(p) - \tau \gamma D(p),
\] (5)

where the demand is given by (4). The second order condition is satisfied for the above profit-maximization problem since \( \partial^2 \Pi / \partial p^2 = -2\lambda / \gamma < 0 \). The profit-maximizing price is given by:

\[
p_{m}^{n}(I_G) = \frac{1}{2} (v + c - \gamma (G - \tau)) + \frac{1 - \lambda}{2\lambda} \gamma,
\] (6)

We note that \( \frac{\partial p_{m}^{n}(I_G)}{\partial \gamma} = -\frac{1}{2} (G - \tau) + \frac{1 - \lambda}{2\lambda} \) such that \( \frac{\partial p_{m}^{n}(I_G)}{\partial \gamma} < 0 \) if \( \lambda > \frac{1}{(1+G-\tau)} \), where Assumption (A4) ensures that \( \frac{1}{(1+G-\tau)} < 1 \). That is, only if the fraction of green conscious consumers is sufficiently high, the monopoly price increases as the product becomes greener, as embodied by a lower \( \gamma \).
From (6), it follows that the demand is given by\(^{11}\):

\[
D = \frac{\lambda}{2\gamma}[v - c - \gamma(G + \tau)] + \frac{1}{2}(1 - \lambda).
\]  

(7)

For values of \(\lambda\) that satisfy (A2), the demand function, evaluated at the price \(p^m(I_G)\), is decreasing in the marginal cost, \(c\), and is increasing in the valuation, \(v\). Since \(\partial D/\partial \gamma = -\lambda(v - c)/2\gamma^2 < 0\), we have that the demand is increasing in \(I_G\). As the investment in the green technology increases, the demand increases as the product becomes greener.

Assumptions (A1) and (A2) ensures that the extreme cases of \(D = 0\) and \(D = 1\) are avoided, since these cases would lead to discontinuities.\(^{12}\)

By substituting (6) and (7) into (5), we obtain the net profit of the firm in the second stage as the following:

\[
\Pi^m(I_G) = \frac{\lambda}{4\gamma}[v - c - \gamma(G + \tau)] + \frac{1-\lambda}{\lambda} \gamma^2.
\]  

(8)

From (7), it follows that for any \(I_G\), the emission level of the firm is given by\(^{13}\):

\[
e^m(I_G) = \frac{\lambda}{2}[v - c - \gamma(G + \tau)] + \gamma(1 - \lambda).
\]  

(9)

Let

\[
\lambda_G \equiv \frac{1}{G + \tau + 1}.
\]

As long as (A1) holds, we have that \(\lambda_G > \lambda\). The following Lemma follows directly from (9).

**Lemma 1** The emission level, \(e^m(I_G)\), is decreasing in the emission tax, \(\tau\), and

(i) is decreasing in \(I_G\) for \(\lambda \in (\lambda, \lambda_G]\),

(ii) and increasing in \(I_G\) for \(\lambda \in (\lambda_G, 1]\).

\(^{11}\)We note that for \(\lambda = 0\), that is, in the absence of green conscious consumers, the monopolist would set \(p = v\), and the demand would be given by \(D = 1\).

\(^{12}\)More specifically, the condition \(\gamma_H < (v - c)/(G + \tau)\) implies that the least environmentally friendly consumer has a positive demand for the dirtiest good ensuring that \(D \neq 0\) for \(\lambda = 1\), and together with (A2) ensures that \(D \neq 0\) for all \(\lambda > \lambda\). Moreover, the condition \(\gamma_L > (v - c)/(G + \tau)\) implies that the most environmentally friendly consumer does not buy the cleanest good ensuring that \(D \neq 1\) for \(\lambda = 1\). Also, \(\gamma_L > (v - c)/(G + \tau)\) is a sufficient condition to ensure that \(\bar{G} < \bar{G}\) for all \(\lambda > \lambda\). Thus, Assumptions (A1) and (A2) together ensure that \(\bar{G} < \bar{G} < \bar{G}\).

\(^{13}\)We note that for \(\lambda = 0\), that is, in the absence of green conscious consumers, the emission level would be given by \(e^m(I_G) = \gamma\), since \(D = 1\).
Proof: We have \( \frac{\partial e^m(I_G)}{\partial \tau} = -\frac{\lambda_\gamma}{2} < 0 \). The derivative of \( e^m(I_G) \) with respect to \( I_G \) is given by:

\[
\frac{\partial e^m(I_G)}{\partial I_G} = \frac{1}{2}(1 - \lambda \left(G + \tau + 1\right)) \frac{d\gamma}{dI_G},
\]

where \( d\gamma/dI_G < 0 \). Thus, \( \partial e^m(I_G)/\partial I_G < (>) 0 \) for \( \lambda < (>) \lambda_G \). 

In the absence of many green-conscious consumers, i.e. \( \lambda \in (\lambda, \lambda_G] \), we obtain the standard result that an increase in investment in green technology results in less emissions. However, the rate of decrease of emissions steadily reduces as \( \lambda \) increases (since \( \frac{\partial^2 e^m(I_G)}{\partial I_G \partial \lambda} = -(G + \tau + 1) < 0 \)) until it becomes positive for \( \lambda > \lambda_G \). Thus, Lemma (1) implies that private investment by firms in green technologies may lead to more emissions if the fraction of green conscious consumers is sufficiently large, i.e. \( \lambda > \lambda_G \). This is because, from (7), we have that demand is increasing in \( I_G \) since as the product becomes cleaner, the environmentally friendly consumers demand more of it. Moreover, Lemma 1 states that regardless of the impact of \( I_G \) on the emission level, an increase in the emission tax rate decreases the emission level. Thus, the policy implication of Lemma (1) is as follows. When firms invest in green technologies in the presence of green consumers who increase their demand for cleaner products, it becomes necessary to implement environmental regulation, such as an emission tax to ensure a reduction in the emission level.

Figure 1 illustrates, in more detail, the impact of increasing investment in green technology on emissions.
In Figure 1, in Areas I and II, we have $\frac{\partial e^m(I_G)}{\partial I_G} < 0$, and in Area III, we have $\frac{\partial e^m(I_G)}{\partial I_G} > 0$.

Area I represents the combinations of $\gamma$ and $\lambda$ for which the monopolist chooses to serve only non-green conscious consumers by setting $p = v$. This occurs for $\lambda \in (\underline{\lambda}, \lambda_1)$ where

$$\lambda_1 \equiv \frac{\gamma}{\gamma + v - c + \gamma(G - \tau) - 2(v - c - \gamma \tau)^{\frac{3}{2}} (\gamma G)^{\frac{1}{2}}} > \underline{\lambda}.$$ 

In Area I, the emission level is given by:

$$e^m(I_G) = \gamma (1 - \lambda),$$

and thus,

$$\frac{\partial e^m(I_G)}{\partial I_G} = \gamma' (1 - \lambda) < 0.$$
For $\lambda > \lambda_1$, the monopolist chooses $p^m$ and serves both green and non-green consumers (Areas II and III in Figure 1). The emission level in Areas I and II is given by (9). Area II represents the case where $\lambda \in (\lambda_1, \lambda_G)$ such that $\frac{\partial e^m(I_G)}{\partial I_G} < 0$, as per Lemma 1(i), and Area III represents the case where $\lambda > \lambda_G$ such that $\frac{\partial e^m(I_G)}{\partial I_G} > 0$, as per Lemma 1(ii).

In the first stage of the game, the firm chooses the investment level in green technology $I_G$ that solves the following program:

$$\begin{cases} \text{Max} & \Pi^m(I_G) - I_G \\ \text{s.t.} & \gamma \leq \gamma_P \end{cases}$$

where the profit $\Pi^m(I_G)$ is defined by (8). Let $I_G^m$ be the solution of the unconstrained optimization program (i.e., $I_G^m = \arg \max I_G \Pi^m(I_G) - I_G$) such that it is solution of the following first order condition:

$$-\gamma' \left[ \frac{\lambda}{4} ((\frac{v-c}{\gamma})^2 - (\bar{G} + \tau)^2) + \frac{(1-\lambda)(\bar{G} + \tau)}{2} - \frac{(1-\lambda)^2}{4} \frac{1}{\lambda} \right] = 1 \quad (10)$$

We note that the left-hand side of equation (10) is positive such that (10) is satisfied as long as Assumption (A1) holds.\(^{14}\) The second order condition is satisfied as long as Assumption (A3) holds (see Appendix for details).

**Definition 1** A “weak” patentability requirement is defined to be $I_{PG} \leq I_G^m$ for $\tau = 0$, and a “strong” patentability requirement is defined to be $I_{PG} > I_G^m$ for all $\tau \geq 0$.

When $I_{PG}$ is weak, as per Definition (1), the firm chooses to invest $I_G^m$ in the first period, patents its green innovation and sets the price $p(I_G^m)$ in the second period as long as $\Pi^m(I_G^m) - I_G^m - C_{PG} > 0$, and $I_{PG} \leq I_G^m$.

**Lemma 2** For a weak patentability requirement, investment in green technology, $I_G^m$,

(i) is increasing in the emission tax, $\tau$, and in $\bar{G}$, for $\lambda \in (\lambda_1, \lambda_G]$;

(ii) and decreasing in the emission tax, $\tau$, and in $\bar{G}$, for $\lambda \in (\lambda_G, 1]$.

\(^{14}\)The expression $\frac{\lambda}{4} ((\frac{v-c}{\gamma})^2 - (\bar{G} + \tau)^2) + \frac{(1-\lambda)(\bar{G} + \tau)}{2} - \frac{(1-\lambda)^2}{4} \frac{1}{\lambda}$ is strictly concave for all $0 < \lambda < 1$, with two roots given by $\hat{\lambda} = \frac{\sqrt{3} + c}{c + \gamma + \bar{G} + \tau}$ and $\bar{\lambda} = \frac{1}{c + \gamma + \bar{G} + \tau} > 1$. It can be shown that $\hat{\lambda} < \bar{\lambda} < \lambda_G < 1 < \hat{\lambda}$, as long as Assumption (A1) holds. Therefore, the left-hand side of equation (10) is positive such that (10) is satisfied.
Lemma (2) follows directly from (10) (see Appendix for details of the calculations). In the absence of many green-conscious consumers, i.e. \( \lambda \in (\lambda_1, \lambda_G] \), we obtain the standard result that an increase in emission tax induces greater investment in green technology. However, the rate of increase of investment steadily reduces as \( \lambda \) increases, until it becomes negative for \( \lambda > \lambda_G \). This is because by Lemma (1), an increase in \( I_G \) would be accompanied by an increase in emissions for \( \lambda > \lambda_G \), and thereby, an increase in the tax bill facing the firm. Thus, for \( \lambda > \lambda_G \), the firm chooses to decrease green investment when faced with a higher emission tax. Changes in \( G \) play a similar role to changes in \( \tau \) in our model.

Before proceeding, it is useful to state some other comparative static results in the following Lemma.

**Lemma 3** For a weak patentability requirement, \( I^m_G \) is

(i) decreasing in \( c \), and

(ii) increasing in \( v \).

Lemma (3) follows directly from (10) (see Appendix for details of the calculations). As the marginal cost of production, \( c \), increases, the profitability of the product decreases, which explains why firms have less incentive to invest in the green technology. As \( v \) increases, demand increases, leading to an increase in \( I^m_G \).

For \( \lambda \in (\lambda_G, 1] \), an implication of Lemma (2) is that \( I^m_G \) may keep falling as \( \tau \) increases until we have \( I^m_{PG} > I^m_G \). Let \( \tilde{\tau} \) denote that level of the tax rate where \( I^m_G = I^m_{PG} \). For \( \tau > \tilde{\tau} \), the firm must invest more than \( I^m_G \) in order to satisfy the patentability requirement. That is, the investment level under the weak patentability requirement is given by \( I^m = \text{Max}\{I^m_G, I^m_{PG}\} \).

Thus, the equilibrium price under the weak patentability requirement is given by \( p = p^m(I^m) \). We summarize this finding in the following Lemma.

**Lemma 4** If \( C_{PG} < \Pi^m(I_G) - I_G \) and the firm only invests in the green technology,

(i) it invests \( I^m = \text{Max}\{I^m_G, I^m_{PG}\} \) in the first period, patents its innovation and

(ii) it chooses the price \( p = p^m(I^m) \) as defined by (6) in the second period.
The quantity sold and the emission level in equilibrium are given by:

\[ q^m(I^m) = \frac{\lambda}{2\gamma(I^m)} [v - c - \gamma(I^m)(G + \tau)] + \frac{1}{2}(1 - \lambda), \]

and

\[ e^m(I^m) = \frac{\lambda}{2} [v - c - \gamma(I^m)(G + \tau)] + \frac{1}{2}(1 - \lambda)\gamma(I^m). \]

If \( I_{PG} \) is larger, that is, the patentability requirement is strong as per Definition (1), the firm must invest \( I_{PG} \) for any \( \tau \geq 0 \) in order to satisfy the patentability requirement and will also set a higher price \( p^m(I_{PG}) > p^m(I^m_G) \). It will invest as long as \( \Pi^m(I_{PG}) - I_{PG} - C_{PG} > 0 \).

Thus far, we have presented the case where the firm invests in the green technology and patents it. However, the decision regarding whether to invest depends on the patenting cost, \( C_{PG} \). Indeed, if the patenting cost is relatively small (\( C_{PG} < \Pi^m(I_{PG}) - I_{PG} \)), the firm always invests, no matter how stringent the patent policy. If the patenting cost is very large, (\( C_{PG} > \Pi^m(I^m_G) - I^m_G \)), the firm never invests. For intermediate values of the patenting cost (\( \Pi^m(I_{PG}) - I_{PG} < C_{PG} < \Pi^m(I^m_G) - I^m_G \)), a too stringent patentability requirement discourages the firm from investing, whereas a less strict patentability requirement induces the firm to invest.

We summarize these findings in the following Proposition.

**Proposition 1** The firm invests in the green technology

(i) if the patenting cost is small (\( C_{PG} < \Pi^m(I_{PG}) - I_{PG} \)), or

(ii) if the patenting cost is higher (\( \Pi^m(I_{PG}) - I_{PG} < C_{PG} < \Pi^m(I^m_G) - I^m_G \)), but the patentability requirement is not too strong (\( I_{PG} \leq I^m_G \)).

If the conditions in terms of \( C_{PG} \), as per Proposition (1), are not satisfied, the firm does not invest, such that Bertrand competition occurs in Stage 2 with \( \gamma = \gamma_H \). In this case, the price is given by \( p^c = c + \gamma_H \tau \), the demand is given by \( D(p^c) = \lambda(v - c - \gamma_H(G + \tau))/\gamma_H + (1 - \lambda) \) and the emission level is given by:

\[ e^c = \lambda(v - c - \gamma_H(G + \tau)) + \gamma_H(1 - \lambda). \quad (11) \]

Figure 2 illustrates the equilibrium emission levels as functions of \( \tau \) under a weak patentability requirement for \( \lambda > 3/(3+G) \). The top (red) function represents \( e^c \), the curved one represents \( e^m(I^m) \) when \( I^m = I^m_G \) and the lower (green) one represents \( e^m(I^m) \) when \( I^m = I_{PG} \).
From Figure 2, it follows that, under a weak patentability requirement regime, for \( \tau < \tilde{\tau} \), a sufficiently small increase in \( \tau \) (a stronger environmental policy), reduces the emission level in equilibrium. A larger increase in \( \tau \) pushes the profit-maximizing investment level down to \( I_{PG} \) in which case, in order to be able to patent the firm cannot reduce its investment anymore and, thus, must invest at \( I_{PG} \). At this investment level, as \( \tau \) increases further, the emission level decreases but at a slower rate. At even higher values of \( \tau \), given that the patenting cost is sufficiently high, the firm might decide not to patent, and thus not to invest, in which case the emission level goes up to the competitive level, \( e^c \).

Let us consider the following tax rates, \( \tau_1 \), \( \tau_2 \) and \( \tau_3 \), such that \( \tau_1 < \tilde{\tau} < \tau_2 < \tau_3 \), as shown in Figure 2. We note that the profit level \( \Pi^m(I_G) \) as given by (8) is decreasing in \( \tau \). Moreover, we assume that \( C_{PG} < \min \{\Pi^m(I_G|\tau=\tau_1) - I_G^m|\tau=\tau_1, \Pi^m(I_{PG}|\tau=\tau_2) - I_{PG}|\tau=\tau_2 \} \) and \( C_{PG} > \Pi^m(I_{PG}|\tau=\tau_3) - I_{PG}|\tau=\tau_3 \).

**Proposition 2** The emission level

1. decreases due to marginal increases in the tax rate at any \( \tau_1 < \tilde{\tau} \) and at any \( \tau_2 > \tilde{\tau} \), with a faster rate of decrease at \( \tau_1 \) than at \( \tau_2 \).

2. increases discretely due to an increase in the tax rate from either \( \tau_1 \) or \( \tau_2 \) to \( \tau_3 \).

Proof: The emission level, \( e^m(I^m) \), is decreasing in \( \tau \) since

\[
\frac{\partial e^m(I^m)}{\partial \tau} = \frac{de^m}{d\tau} + \frac{\partial e^m(I^m)}{\partial I^m} \frac{\partial I^m}{\partial \tau} < 0.
\]
For any $\tau_2 > \tau$, the relevant emission level is $e^m(I_{PG})$ since we have that $I_{PG} > I^m_G$ for this range of $\tau$. If $I^m = I_{PG}$, the second term of (12) is null, and we have that $de^m/d\tau = -\lambda \gamma (I^m)/2 < 0$.

For any $\tau_1 < \tau$, the relevant emission level is $e^m(I^m_G)$ since we have that $I_{PG} \leq I^m_G$ for this range of $\tau$. If $I^m = I^m_G$, the second term of (12) is also negative since $\partial e^m/\partial I^m < (>) 0$ for $\lambda < (>) \lambda_G$ by Lemma (1), and $\partial I^m/\partial \tau > (>) 0$ for $\lambda < (>) \lambda_G$ by Lemma (2). It follows that $|\partial e^m(I^m_G)/\partial \tau| > |\partial e^m(I_{PG})/\partial \tau|$. This completes the proof of Proposition 2.1. An increase in $\tau$ from either $\tau_1$ or $\tau_2$ to $\tau_3$ causes $\Pi^m(I^m_G) - I^m_G$ to fall below $C_{PG}$ such that it becomes unprofitable for the firm to invest in the green technology, as per Proposition 1. This causes the emission level to rise from $e^m(I^m)$ to $e^c$ as given by (11). This completes the proof for Proposition 2.2.

An important implication of Proposition 2.2 is that because of the interaction between patent and environmental policies, at sufficiently high levels of the patenting cost, a stricter environmental policy might increase emissions. This is an example of the green paradox, which has been studied in various contexts in the recent literature (see Figure 3 below).

**Figure 3**

**Corollary 1** A sufficient reduction in the cost of patenting, $C_{PG}$, alleviates the green paradox.

Corollary 1 follows directly from Proposition 1. Since the increase in emission occurs due to $\Pi^m(I^m_G) - I^m_G$ falling below $C_{PG}$ due to an increase in $\tau$, this adverse impact could be avoided by decreasing $C_{PG}$, for example, by introducing fast track patents for green technologies.
Under a sufficiently strong patentability requirement such that $I_{PG} > I_{mG}$ for all $\tau \geq 0$, as per Definition 1, the emissions level decreases as $\tau$ increases, and the same mechanism applies as described above, except that the firm is always constrained to invest $I_{PG}$, as shown in Figure 4.

![Figure 4](image)

This leads to Proposition 3.

**Proposition 3** The emission level is

1. higher under a weak patentability requirement, $I_{PG} \leq I_{mG}$, than under a strong patentability requirement, $I_{PG} > I_{mG}$, for $\lambda \in (\lambda_1, \lambda_G]$

2. lower under a weak patentability requirement, $I_{PG} \leq I_{mG}$, than under a strong patentability requirement, $I_{PG} > I_{mG}$, for $\lambda \in (\lambda_G, 1]$

Proof: The proof follows directly from Lemma (1).

Proposition 3 has direct and important policy implications. When the fraction of green conscious consumers is sufficiently small, i.e. $\lambda \in (\lambda_1, \lambda_G]$, moving from a weak to a strong patentability requirements for green innovation, ceteris paribus, decreases the emission level. This corresponds to Areas I and II in Figure 1. However, when the fraction of green conscious consumers is sufficiently large, i.e. $\lambda \in (\lambda_G, 1]$, moving from a weak to a strong patentability requirements for green innovation, ceteris paribus, increases the emission level, corresponding to Area III in Figure 1. Therefore, for $\lambda \in (\lambda_G, 1]$, the stricter the patentability requirement, that
is, the higher that $I_{PG}$ is raised above $I_G^m$, the more urgent it becomes to increase the emission tax simultaneously in order to curb emissions.

**Definition 2** A patent which costs $C_{PG} < \Pi^m(I_G^m) - I_G^m$ is said to be a “green patent”.

**Proposition 4** For weak (strong) patentability requirement, a green patent is sufficient (not sufficient) to induce innovation.

Proof: If $I_{PG} > I_G^m$ for all $\tau$, then unless $C_{PG} < \Pi^m(I_{PG}) - I_{PG} < \Pi^m(I_G^m) - I_G^m$, it is not profitable to invest. ■

Proposition 4 implies that how low $C_{PG}$ needs to be in order to induce innovation depends on the strength of the patentability requirement. The stronger the patentability requirement, the lower the patenting cost must be to induce innovation. Thus, fast-tracking green patents, which would reduce $C_{PG}$ in our model, becomes more important when a strong patentability requirement is implemented. Since, as implied by Proposition 3, a strong patentability requirement is more effective at reducing emissions when $\lambda \in (\lambda_1, \lambda_G]$, fast-tracking green patents also becomes more important when the fraction of green conscious consumers is sufficiently low.

### 3.1 Socially Optimal Investment

The socially optimal level of investment, given that the firm sets the monopoly price in the second period, is the solution of

$$\max_{I_G} W^m(I_G),$$

and is denoted by $I_G^*$, where

$$W^m(I_G) = \Pi^m(I_G) - I_G + CS^m(I_G) + \tau e(I_G) - C_{PG}.$$

Consumer surplus is given by:

$$CS^m(I_G) = \frac{1}{2} \Pi^m(I_G) - e(I_G).$$

Thus, $I_G^*$ must satisfy $dW(I_G)/dI_G = 0$ or, equivalently,

$$\frac{d\Pi^m(I_G)}{dI_G} - 1 + \frac{dCS^m(I_G)}{dI_G} + \tau \frac{de(I_G)}{dI_G} = 0,$$

(13)
Evaluated at $I^n_G$, the left-hand side of (13) above becomes
\[
\frac{1}{2} + (\tau - 1)\frac{de(I_G)}{dI_G}
\]  \hspace{1cm} (14)

By Lemma 1, we have that $de^m(I_G)/dI_G < (>) 0$ for $\lambda < (>) \lambda_G$. Thus, the expression (14) is positive for $\lambda > \lambda_G$ and $\tau > 1$ and for $\lambda < \lambda_G$ and $\tau < 1$. The expression (14) may be negative for $\lambda > \lambda_G$ and $\tau < 1$ and for $\lambda < \lambda_G$ and $\tau > 1$. There exists a threshold $\tilde{\tau}_1 > 1$ such that for $\lambda < \lambda_G$ and $\tau > \tilde{\tau}_2$, we have (14) < 0. There exists a threshold $\tilde{\tau}_2 < 1$ such that for $\lambda > \lambda_G$ and $\tau < \tilde{\tau}_1$, we have (14) < 0. This leads to the following Proposition.

Proposition 5  (i) For $\lambda \in (\lambda_1, \lambda_G]$ and $\tau < 1$, $I^n_G < I^*_G$ (the firm underinvests relative to the socially optimal level) and for $\tau > \tilde{\tau}_1$, $I^n_G > I^*_G$ (the firm overinvests relative to the socially optimal level).

(ii) For $\lambda \in (\lambda_G, 1]$ and $\tau > 1$, $I^n_G < I^*_G$ (the firm underinvests relative to the socially optimal level) and for $\tau < \tilde{\tau}_2$, $I^n_G > I^*_G$ (the firm overinvests relative to the socially optimal level).

Proposition 5(i) states that as long as the fraction of green conscious consumers is sufficiently low, the firm underinvests relative to the socially optimal level for a sufficiently low emission tax and overinvests for a sufficiently high emission tax. This is in line with much of the literature surrounding the Porter hypothesis which predicts that higher taxes induce more investment by firms. However, the gap between the socially efficient and privately optimal levels of investment steadily reduces as $\lambda$ increases, until this result is reversed when the fraction of green conscious consumers is sufficiently high, as stated by Proposition 5(ii).

4 Conclusion

In this paper, we developed a theoretical framework to investigate the impact of patent policies and emission taxes on green innovation and emissions in the presence of environmentally friendly consumers. We analyzed the effect of changing patentability requirements and patenting costs when a firm may invest in a green innovation, which reduces the emission output ratio. We showed that investment in green technologies reduces emissions only if the fraction of green consumers is sufficiently small, and that the magnitude of this effect decreases as the fraction of
green consumers increases. While marginally higher emission tax only increases green investment if the fraction of green consumers is sufficiently small, it always induces less emissions. However, a discrete jump in the tax rate may result in a green paradox, leading to more emissions. A stricter patentability requirement combined with fast-track green patents is only effective at reducing emissions as long as the fraction of green consumers is sufficiently small.

To summarize, this paper showed that the traditional policy tools of increasing green investment and thereby reducing emissions through stricter emission taxes and patent requirements and fast-tracking green patents become less effective as society becomes more environmentally friendly. Thus, further research seems warranted regarding the policies to reduce emissions through green innovation, and also regarding the type of information to distribute to consumers. In particular, if consumers reward marginal reductions in emission-output ratios of productions processes by increasing their demand for the final product, this may inhibit the traditional tax and patent policy tools from working as expected. It may help to make consumers aware of this issue such that they only reward drastic innovations that achieve significant emission reductions.

In future work, we will address the following related questions. How does strategic interaction among multiple firms with market power affect the effectiveness of the patent system? In an open economy setting, where the degree of environmental friendliness of consumers is heterogeneous across countries, and polluting firms with market power are located across different countries, how are governments’ strategies regarding the implementation of IPRs and fast-track patent systems affected? How do these policies affect the distribution of investment, pollution and welfare levels across countries?
References


Appendix

Optimal Green Investment

In the first period, the firm chooses the optimal investment that satisfies

\[
\begin{align*}
\max_{I_G} & \Pi^m(I_G) - I_G \\
\text{s.t.} & \gamma(I_G) \leq \gamma_p
\end{align*}
\]

where the profit \(\Pi^m(I_G)\) is defined by (8). Let \(I_G^m\) be the solution of the unconstrained optimization program (i.e., \(I_G^m = \arg \max \Pi^m(I_G) - I_G\)) such that it is solution of (10) and the second order condition is satisfied as long as

\[
\frac{\lambda}{2} (\gamma')^2 \frac{(v-c)^2}{\tau^3} - \gamma'' \left( \frac{1}{4} \left( \frac{v-c}{\gamma} \right)^2 - (G + \tau)^2 \right) + (1 - \lambda)(G + \tau) - \frac{1}{4} \left( \frac{1-\lambda}{\lambda} \right)^2 < 0.
\]

Using the FOC (10) we can write

\[
-(\frac{1}{4} \left( \frac{v-c}{\gamma} \right)^2 - (G + \tau)^2) + (1 - \lambda)(G + \tau) - \frac{1}{4} \left( \frac{1-\lambda}{\lambda} \right)^2 = \frac{1}{\gamma(I_G)},
\]

that we plug into the SOC to obtain a local condition

\[
\frac{\lambda}{2} (\gamma')^2 \frac{(v-c)^2}{\tau^3} + \frac{\gamma''}{\gamma(I_G)} < 0
\]

or

\[
\gamma''(.) > -\lambda \frac{(v-c)^2}{2} \left( \frac{\gamma'(.)}{\gamma(.)} \right)^3,
\]

which is Assumption (A3). If this local condition is satisfied, locally the profit function is concave.

Proofs of Lemmas 2 and 3: In order to derive the relevant comparative statics results on \(I_G^m\), it is useful to define the following:

\[
F(I_G, y) \equiv -\gamma' \left( \frac{1}{4} \left( \frac{v-c}{\gamma} \right)^2 - (G + \tau)^2 \right) + \left( \frac{1 - \lambda}{2} (G + \tau) - \frac{1}{4} \left( \frac{1-\lambda}{\lambda} \right)^2 \right) - 1,
\]

where \(y\) may represent any of the exogenously given parameters, that is, \(y \in \{\tau, c, v, G, \lambda\}\). By totally differentiating (10), we have the following

\[
\frac{dI_G^m}{dy} = -\frac{\partial F}{\partial y} \frac{\partial F}{\partial I_G}.
\]
Since we have assumed the existence of an interior solution, it follows that a maximum is reached at $I_G^m$. This implies that $\frac{\partial F}{\partial I_G} \leq 0$, and thus

$$\text{sign} \frac{dI_G^m}{dy} = \text{sign} \frac{\partial F}{\partial y}.$$  

For $y \in \{\tau, c, v, G, \lambda\}$, we have

$$\frac{\partial F}{\partial \tau} = \frac{\partial F}{\partial G} = \frac{\gamma'}{2} (\lambda + G) - (1 - \lambda) < 0,$$

if and only if $\lambda \geq \lambda_G$.

$$\frac{\partial F}{\partial c} = \lambda \frac{\gamma'}{2} \frac{v - c}{\gamma^2} < 0,$$

$$\frac{\partial F}{\partial v} = -\lambda \frac{\gamma'}{2} \frac{v - c}{\gamma^2} > 0,$$

$$\frac{\partial F}{\partial \lambda} = -\gamma' \left( \frac{\lambda}{4} \left( \frac{(v - c)^2}{\gamma} \right) - (G + \tau)^2 \right) - \frac{1}{2} (G + \tau) + \frac{1 - \lambda^2}{2}.$$

The following table summarizes the above comparative statics results.

<table>
<thead>
<tr>
<th>TABLE 1: Comparative Statics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter $y$</td>
</tr>
<tr>
<td>$\tau$</td>
</tr>
<tr>
<td>$c$</td>
</tr>
<tr>
<td>$v$</td>
</tr>
<tr>
<td>$G$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

**Figure 2**

The emission level when the price is equal to marginal cost is given by equation (11), $e^c = \lambda (v - c - \gamma_H (G + \tau)) + \gamma_H (1 - \lambda)$ which is decreasing with $\tau$ at the constant rate $-\gamma_H$. The emission level is positive for $\tau < \tilde{\tau} = (v - c)/\gamma_H - G + (1 - \lambda)/\lambda$, which is satisfied according to Assumption 2. It is represented by the top (red) function. The bottom (green) function represents the emission level when the patenting constraint is binding $e^m(I_{PG}) = \lambda (v - c - \gamma_H (G + \tau)) + \gamma_H (1 - \lambda)$. The emission level is positive for $\tau < \tilde{\tau}$.
\[ \gamma_P(G + \tau)/2 + (1 - \lambda)\gamma_P/2. \] Thus, this function is also decreasing at rate \(-\gamma_P/2 > -\gamma_H\). The emission level is positive for \(\tau < (v - c)/\gamma_P - G + (1 - \lambda)/\lambda\) where \((v - c)/\gamma_P - G + 3(1 - \lambda)/\lambda > (v - c)/\gamma_H - G + (1 - \lambda)/\lambda\). The two functions intersect at \(\tau = (v - c)/(2\gamma_H - \gamma_P) - G + (1 - \lambda)/\lambda\).

The middle curve represents the emission level at the profit-maximizing investment level \(I_G^m\), and it intersects with \(e_m(I_{PG})\) at \(\tau\) which is where \(\gamma(I_G^m) = \gamma_P\) or \(I_G^m = I_{PG}\).

Lastly we need to check that \(\tau > \bar{\tau} \equiv (v - c)/\gamma_L - G - 1\) (second part of assumption 2). To do so, we verify that the profit-maximizing emission at the tax level \(\bar{\tau}\) is higher than the emission level at \(\bar{\tau}\).

**Welfare analysis**

The total welfare is

\[ W^m(I_G) = \Pi^m(I_G) - I_G - C_{PG} + \tau e + CS^m(I_G) \]

where \(\Pi^m(I_G)\) is defined by (8) and the consumer surplus is

\[ CS^m(I_G) = \lambda \int_{G}^{v - p - P(e)/\gamma} (v - \gamma G - p - P(e))dF + \lambda \int_{G}^{G} (-P(e))dF + (1 - \lambda)(-P(e)) \]

\[ = \frac{\lambda}{\gamma^2} (v - c - \gamma(G + \tau) - \frac{1 - \lambda}{\lambda})^2 - e. \]

Therefore, the total welfare can be written as

\[ W^m(I_G) = \frac{3}{2}(\Pi^m(I_G) - I_G) + \frac{1}{2}I_G - (1 - \tau)e - C_{PG}. \]

The derivative of the total welfare gives

\[ \frac{dW^m(I_G)}{dI_G} = \frac{3}{2}(\frac{d\Pi^m(I_G)}{dI_G} - 1) + \frac{1}{2} - (1 - \tau)\frac{de}{dI_G}. \]

Evaluated at \(I_G^m\), it becomes

\[ \frac{1}{2} + (\tau - 1)\frac{de}{dI_G}, \]

is positive for \(\lambda > \lambda_G\) and \(\tau > 1\) and for \(\lambda < \lambda_G\) and \(\tau < 1\). The expression (14) may be negative for \(\lambda > \lambda_G\) and \(\tau < 1\) and for \(\lambda < \lambda_G\) and \(\tau > 1\). There exists a threshold \(\tau_1 > 1\) such that for \(\lambda < \lambda_G\) and \(\tau > \tau_1\), we have (14) < 0. There exists a threshold \(\tau_2 < 1\) such that for \(\lambda > \lambda_G\) and \(\tau < \tau_2\), we have (14) < 0.