Name-Change Fees, Scalpers and Secondary Markets*

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Abstract

The present paper incorporates name-change fees in a simple two-date, two-state ticket pricing model. A name-change fee is incurred in order to change the name/ownership of a ticket. I show that if a monopolist cannot perfectly adjust her prices in the face of uncertainty (imperfect yield management), then name-change fees can be used to partly make up for the lost revenue. A secondary market can arise, where people who bought early resell their tickets, and, which trades at a different price than the primary market. The name-change fees can be utilized by the monopolist to extract surplus from the secondary market. Conversely, if the monopolist can perfectly adjust her prices (perfect yield management), then an active secondary market generally cannot increase the monopolist’s profit, and the name-change fees should be set such that nobody resells.

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1 Introduction

Name-change fees are incurred by ticket holders when they change the name on their ticket. These fees are associated with the airline industry, and often gather media attention. However, any firm that sells tickets for a service could in principle adopt name-change fees. For example, sellers of theater tickets, sport tickets, hotel rooms, and etc., could in principle require that tickets be registered to specific names, and charge a fee for transfers of ownership. Airline industry insiders suggest that the name-change fees were introduced to eradicate the secondary market. The intuition seems obvious: Airlines face a dynamic and uncertain demand, and selling in multiple dates allows them to intertemporally price discriminate. If name changes are free, then selling early creates future competition as scalpers can buy at a low price, and resell in the secondary market when demand is high which reduces traffic in the primary market.

In the present paper, I examine the interplay between yield management and name-change fees in a simple ticket pricing model. Name-change fees are incorporated in a two-date model where a monopolist sells tickets for a service she provides at the end of date 2. Demand is dynamic, but uncertain: consumers arrive in both dates, but date-2 demand is unknown in date 1. Scalpers arrive in date 1, and do not value the service, but they buy a ticket if they expect to resell in the secondary market for a profit.

The monopolist can set ticket prices, and charge a fee to allow ticket ownership transfers. I find that it is optimal for the monopolist to set the fee such that the secondary market is active in some contingencies. Unsurprisingly, if the monopolist has infinite capacity and sets prices after the resolution of uncertainty (henceforth, perfect yield management), there is no room for the secondary market to increase the monopolist’s profit. If, instead, the monopolist must set prices prior to the resolution of uncertainty (henceforth, imperfect yield management), then, for large enough surplus in the high-demand states, the monopolist wants to use the primary market to maximize profit in the high-demand states, and allow the secondary market to serve the low-demand states, and collect the name-change fees.

1The motivation for this paper was the story of a British man who legally changed his name to avoid paying the fee ($336) since changing his name was cheaper ($103) (http://www.npr.org/sections/thetwo-way/2015/06/05/41295439/man-changes-name-to-adam-west-to-avoid-paying-336-airline-fee).

2Airline spokesmen posit that they fees reflect security check costs, but industry insiders rebuke that claim (http://travelsort.com/blog/airline-ticket-name-change-or-transfer-to-another-person and https://www.usatoday.com/story/travel/flights/2013/12/23/airline-ticket-transfer-name-change/417445/). In principle, certain name changes are officially allowed; spelling mistakes, changing last names due to marriage or divorce, and changes of prefixes (http://travelsort.com/blog/airline-ticket-name-change-or-transfer-to-another-person).

3Dana [1999] considers similar industries, and argues that prices could be set before the demand realization because they might be due to promotions, or advertised etc.
Capacity limits and imperfect yield management can generate excess demand when demand is high. The monopolist benefits by allowing the secondary market to cater to these unserved consumers, and using the name-change fee to extract surplus from the secondary market. The fee can be set such that the secondary market trades at a higher price than the primary market, thus not price competing with it. The monopolist engages in a form of price-discrimination: she receives a higher price, in the form of the fee, from the consumers in the secondary market than the consumers in the primary market. If, instead, the monopolist can practice perfect yield management, then unless date-2 consumers can be sorted according to their valuations, the monopolist prefers to set the fee so the secondary market never emerges. Date-2 consumers can be sorted if higher valuation consumers arrive last. In such case, the monopolist generates excess demand so that consumers with the highest valuations buy from the secondary market at a higher price than the primary market, which again allows the monopolist to extract a higher price from those consumers. A capacity limit and excess demand enables this form of price-discrimination because the marginal valuation in the secondary market is strictly higher than the marginal valuation in the primary market.

The main finding of the paper can be summarized as follows. A monopolist who cannot perfectly adjust prices to the realized demand can utilize the name-change fees, and the secondary market, as a second best solution. The name-change fees can be used as a substitute to partially offset for the surplus the monopolist loses due her inability to adopt perfect yield management. I find cases where an active secondary market allows the monopolist to indirectly receive two different prices: the unit price from the primary market, and the name-change fee from the secondary market. The main result goes through when I allow date-1 consumers to change preferences in date-2. In certain cases the secondary market can be used instead of overbooking, and it might be strictly preferred. This paper’s attitude towards the secondary market is more in the spirit of Broner et al. [2008]. Secondary markets can be used to overcome frictions.

Previous work on ticket pricing and yield management has omitted name-change fees (Courty [2003a,b], Gallego and van Ryzin [1994], McAfee and te Velde [2007]), while previous work on secondary markets has ignored both frictions in yield management and name-change fees. Most of the literature on secondary markets deals with a monopolist selling a durable good. An active secondary market induces a tradeoff. Consumers are willing to pay more for a new good, since they can resell the used version in the future. However, this increases future

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4 Broner et al. [2008] are concerned with allocation and efficiency, which is in not true for the present paper. In their model, the secondary market is used to resolve enforceability problems in bond issuance. Assets are re-traded in secondary market.

5 Courty [2000] provides a nice review of the literature on ticket pricing.
competition (Miller [1974], Anderson and Ginsburgh [1994], Lizzeri and Hendel [1999], Haile [2001], Karp and Perloff [2005], Leslie and Sorensen [2014]). When used goods are valued highly, the optimal transactions costs in the secondary market are zero, since the monopolist can extract more surplus from the consumers buying a new good (Anderson and Ginsburgh). If the monopolist can choose durability, then it is optimal keep the secondary market open, and reduce durability (Lizzeri and Hendel). In durable good models, the secondary market trades a used version of the good, and it is used to segment the consumers. Transactions costs are not a source of revenue for the monopolist. In my framework, the primary and the secondary market trade identical goods (i.e., tickets to a services provided at the end of the game), which makes competition stronger. Moreover, the secondary market transaction costs are the name-change fees, which are imposed by the monopolist. Karp and Perloff [2005] and Courtý [2003a] consider ticket pricing models closer to the one of this paper. A monopolist provides a service, and scalpers can buy and sell in the secondary market. Similar to my results, Karp and Perloff find that the monopolist can utilize the secondary market to increase her profit. The monopolist cannot price-discriminate consumers based on their valuations, whereas scalpers can. Consequently, the monopolist finds it optimal to sell only to scalpers, who then price-discriminate. In Courtý, the consumers learn their valuations over time, which are either high or low. In the subgame perfect equilibrium, it is optimal to sell only in date 1 at a low price. Date-2 consumers with high valuations buy from scalpers. Courtý does not consider charging fees in the secondary market, which leads him to conclude that the monopolist cannot do anything to prevent scalpers from entering, or even share the date-2 profits they make. Courtý [2003b] considers a similar environment with no scalpers or capacity constraints. The monopolist practices perfect yield management, but can ration consumers. Courtý shows that if the monopolist can commit to prices and quantities, then it is never optimal to sell early and allow consumers to resell if their valuations change. Leslie and Sorensen [2014] argue that secondary markets can increase allocative efficiency, but rent-seeking behavior and transaction costs decrease those efficiency gains.

The present paper contributes to the yield management literature, and the ticket pricing literature by explicitly considering the effect of name-change fees. Introducing such fees in a simple yield management model shows that they can partly offset inability to perfectly adjust prices. Moreover, utilizing the name-change fees allows the monopolist to determine the size of the secondary market in contrast to Courtý [2003a]. Similarly, the monopolist can extract surplus from the secondary market through the fees unlike in Karp and Perloff [2005]. This is a crucial difference between our models. In Karp and Perloff, the scalpers are useful because they have superior information, whereas in my model, scalpers are useful

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6Esteban and Shum [2007] consider an interesting empirical model of oligopoly.
because they allow the monopolist to receive two different prices in date 2.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 examines infinite capacity case, and Section 4 complements the analysis by considering the more realistic case of a binding capacity constraint. Section 5 gives some preliminary results for the case of changing preferences, while Section 6 discusses some further extensions. Finally, Section 7 concludes, and the proofs are in the Appendix.

2 Model

I consider a two-date model with a monopolist who provides a service at the end of date 2. The monopolist sells tickets for the service, and only ticket holders can use the service. A *name-change fee* $f$ can be charged by the monopolist in order to allow a transfer of ownership for a ticket already sold.

The demand for the service is dynamic and uncertain. In date 1, mass $m_1 = 1$ of consumers with valuations $v_1 \sim U[0, 1]$ arrive. In date 2, mass $m_2$ of new consumers arrive with valuations $v_2 \sim U[0, Z]$, where $Z = Z_H$ with probability $\gamma_H$, and $Z = Z_L$ with probability $\gamma_L (= 1 - \gamma_H)$, and $1 < Z_L < Z_H < 3$, and $m_2 Z_L > 1$.\(^7\) That is, date-2 consumers are expected to have higher valuations, but the exact distribution of their valuations is unknown in date 1. Consumers value at most one unit of the service, and can browse the secondary market at zero cost. In the main formulation I assume that consumer valuations do not change. Some preliminary results with changing preferences are presented. Date-1 consumers reflect consumers who like to plan ahead.

Mass $m_S = 1$ of *scalpers* arrive in date 1. Scalpers do not value the service, but they buy a ticket from the *primary market* (monopolist) if they expect to make a profit by reselling in the *secondary market*. Accessing the secondary market as a reseller is costly. Scalpers incur a cost $c$, where $c \sim U[0, \bar{c}]$ and $\bar{c} \leq 1$.\(^8\) In contrast, consumers face a high enough cost so that they never resell. Scalpers can buy and resell at most one ticket, but cannot do so within the same date. Thus, a scalper buys a ticket and pays cost $c$ in date 1 in order to resell in date 2. Consequently, a secondary market can only arise in date 2. Reselling a ticket entails a transfer of ownership, and so the name-change fee must to be payed, which I assume is paid by the reseller.

All agents are risk-neutral, there is no discounting between the two dates, and the service

\(^7\)Assuming date-2 demand is not extremely high makes the analysis interesting as the monopolist has an incentive to sell in both dates.

\(^8\)Scalping cost $c$ can be thought of as the cost to set up shop, or a website.
can be provided at zero cost. I consider both the case of infinite capacity, and the case of a monopolist constrained by capacity. Infinite capacity does not accurately reflect reality, however it can be used to understand the mechanics of the model.

**Strategies and Secondary Market**

**Monopolist Strategy** The monopolist chooses \( \sigma^M = \{p, f\} \): price vector \( p \geq 0 \) with a price for each possible date-state, and the name-change fee \( f \geq 0 \) The date-1 price \( p_1 \) and the fee \( f \) are set in date 1 prior to the resolution of demand uncertainty. With perfect yield management, the date-2 price \( p_{2,j} \) (for state \( j = \{L, H\} \)) is set after the resolution of uncertainty, and thus different prices can be set for each state, \( p_{2,L} \neq p_{2,H} \). With imperfect yield management, date-2 price is set before the date-2 demand is observed, thus \( p_{2,L} = p_{2,H} \).

**Secondary Market Price** Denote \( t_j \) the price in the secondary market in state \( j = \{L, H\} \). Price \( t_j \) is determined after the resolution of uncertainty, and it is taken as given by resellers. If the secondary market does not price compete with the primary market in state \( j \), then \( t_j \) adjusts to clear the secondary market; i.e., \( t_j = Z_j - S/m_2 \), where \( S \) is the size of the secondary market.\(^9\) If the two markets price compete, then \( t_j = p_{2,j} \).

**Scalper Strategy** A scalper with cost \( c \) makes two decisions. In date 1, she decides whether to buy a ticket or not, \( \sigma_{S,1}(c) = \{B, N\} \). Then, in date 2, conditional on having bought a ticket, she decides whether to resell or not, \( \sigma_{S,2}(Z_j|\sigma_{S,1} = B) = \{S, N\} \). In date 2, the scalping cost \( c \) is sunk making scalpers ex post identical. By reselling a scalper receives the secondary market price \( t_j \), and incurs the name-change fee \( f \). Thus in state \( j \), the secondary market is active if and only if \( t_j \geq f \). The scalpers anticipate this in date-1, so a scalper with cost \( c \) buys in date 1 if and only if

\[
\sum_{j = \{L, H\}} \gamma_j \rho_j^S (t_j - f) I_{t_j \geq f} - p_1 \geq c, \tag{1}
\]

where \( I_{t_j \geq f} = 1 \) if \( t_j \geq f \) and zero otherwise, and \( \rho_j^S \) is the probability of successfully reselling in state \( j \). I only consider the case with \( \rho_L^S = \rho_H^S = 1 \).

The name-change fee \( f \) allows the monopolist to endogenously determine the size secondary market. Setting \( f \) sufficiently high implies that the secondary market never arises. A low \( f \) can generate positive expected surplus for low cost scalpers in some contingencies. Denote \( \tilde{c} \) the cost of the marginal scaler: condition (1) binds, and scalpers with \( c \leq \tilde{c} \) enter.

\(^9\)Anderson and Ginsburgh \[1994\], Courty \[2003a,b\], Lizzeri and Hendel \[1999\] impose similar assumptions for the secondary market. Assuming \( m_2 Z_L > 1 \), and \( \tilde{c} < c < 1 \) guarantees that \( t_j = Z_j - S/m_2 > 0 \).
Thus, the size of the secondary market is,

\[ S = \frac{\tilde{c}}{\bar{c}}. \]  

(2)

**Consumer Strategy** A date-1 consumer with valuation \( v_1 \) can either buy at date 1 or wait for date 2, \( \sigma_{D1,1}(v_1) = \{B, W\} \). If she buys at date 1, she pays \( p_1 \) giving her a payoff of \( v_1 - p_1 \). If she waits, then in each date-2 state she can buy from the either the primary market (if open), the secondary market (if open), or not buy at all, \( \sigma_{D1,2}(v_1|Z_j, \sigma_{D1,1} = W) = \{P, S, N\} \). Not buying gives zero payoff. Buying from the primary market yields \( v_1 - p_2,j \), and buying from the secondary market gives \( v_1 - t_j \). The consumer wants to buy from the market that offers the lowest price. If the market does not clear at that price, the consumers are rationed. With probabilities \( \rho_j^{P,c} \) and \( \rho_j^{S,c} \), successfully buys a ticket in the primary and secondary market, respectively. Thus, a date-1 consumer buys in date 1, if and only if

\[
\frac{v_1 - p_1}{\text{Payoff from Buying at Date 1}} \geq \sum_{j = (L, H)} \gamma_j \max \left\{ 0, \rho_j^{P,c}(v_1 - p_2,j), \rho_j^{S,c}(v_1 - t_j) \right\}. \tag{3}
\]

Denote \( \tilde{v}_1 \) the valuation of the date-1 indifferent consumer.

A date-2 consumer with valuation \( v_2 \) faces the same problem as a date-1 consumer who waited, and thus, her payoffs are defined analogously. Denote \( \tilde{v}_{2,j} \) the valuation of the date-2 consumer who is indifferent between between buying and not, for \( j = \{L, H\} \). Since date-2 consumers want to buy from the lowest priced market, we have \( \tilde{v}_{2,j} = \min \{p_{2,j}, t_j\} \).

Absent of discounting, if the marginal date-1 consumer expects to buy in both date-2 states, then no sales are made in date 1. To avoid such corner solutions, I assume that \( m_2 \) is large enough so that date-1 consumers do not expect to buy in the high state, or with imperfect yield management. The following assumption is sufficient to eliminate such cases.

**Assumption 1.** \( m_2 > 1/ (\gamma_H Z_H + \gamma_L Z_L - 2) \).

Finally, I solve for the subgame perfect equilibria of the model.

## 3 No capacity constraint

Without any capacity limits the monopolist can serve any level of demand, while by assumption consumers can visit the primary and secondary markets at zero cost. Hence, if, in state \( Z_j \), the primary market operates at price \( p_{2,j} \), then the secondary market must trade
at the same price. That is, the secondary market, whenever it arises, it price competes with
the primary market; \( t_j = p_{2,j} \). Since consumers visit each market randomly, the (residual)
demand in the primary market when secondary market has size \( S \) is
\[
R(Z_j, S) = D_2(Z_j, p_{2,j}) \left( 1 - \frac{S}{D_2(Z_j, p_{2,j})} \right) = D_2(Z_j, p_{2,j}) - S,
\]
where \( D_2(Z_j, p_{2,j}) \) is date-2 total demand for \( j = \{L, H\} \).

With infinite capacity, the secondary market decreases the monopolist’s profits, whenever
the primary and the secondary markets price compete. The scalpers resell if and only if
\( t_j = p_{2,j} \geq f \). The monopolist receives the name-change fee \( f \), but loses \( p_{2,j} \), which is
what the monopolist would have received, had she made the sale. All the transactions in
the secondary market can be accommodated in the primary market. Thus, the monopolist
loses \( p_{2,j} - f \geq 0 \) for each transaction in the secondary market. In addition to the fee, the
monopolist receives the date-1 ticket price \( p_1 \) from each scalper. However, Lemma 1 shows
that if the two markets arise in the same states, then no \( p_1 \) exists such that the monopolist
recuperates date-2 loses and scalpers want to participate. The monopolist and the scalpers
play a zero-sum game: the expected surplus of the scalpers equals the expected loss of the
monopolist, thus, using the date-1 price to extract this surplus implies that the scalpers do
not want to participate.

**Lemma 1.** If the secondary market arises only when it price competes the primary market,
then the secondary market cannot increase the profits of the monopolist.

With perfect yield management, the monopolist can adjust date-2 price to the realized
demand, which means that the primary market can operate in every date-2 state (i.e., \( p_{2,j} <
Z_j \)). By Lemma 1 the secondary market decreases the profits of the monopolist. Proposition
1 follows naturally.

**Proposition 1.** With perfect yield management, the secondary market does not arise. The
fee \( f \) is set high enough such that it is never profitable to resell in the secondary market, and
the monopolist solves a standard monopoly problem. For \( Z_L \geq \left( 1 + \sqrt{1 + m_2} \right) / 2 \), no
date-1 consumers buy in date 2. For \( Z_L < \left( 1 + \sqrt{1 + m_2} \right) / 2 \), some date-1 consumers
buy in the date-2 low state \( Z_L \).

On the equilibrium path it is always optimal to intertemporally separate the markets.
However, if \( Z_L \) is close enough to 1, then the monopolist finds it optimal to deviate in the
low state and include date 1 consumers.
Now consider the case of imperfect yield management. The monopolist can set different prices for date 1 and date 2, but the date-2 price is set prior to observing $Z_t$. Suppose there is no secondary market. Then, expected profit is maximized by setting $p_2 = (\gamma_H Z_H + \gamma_L Z_L)/2$.\footnote{Note that if a date-1 consumer buys at date-2 $p_2$, then she would buy in both states, and by indifference condition, no date-1 consumer would buy in date 1. Assumption rules out this scenario.} If though the surplus in the high state is big enough such that $\gamma_H (Z_H - Z_L) > Z_L$, then $(\gamma_H Z_H + \gamma_L Z_L)/2$ exceeds $Z_L$ and no consumers buy from the primary market in the low state. The monopolist can either lower $p_2$ or targets the high state by setting $p_2 = Z_H/2$. The latter is strictly preferred when the high state is sufficiently likely, i.e., $\gamma_H > (Z_L/(Z_H - Z_L))^2$, and it is equilibrium when $Z_H > 2Z_L$: the primary market is not visited in the low state, but profits are maximized in the high state.

An active secondary market in the low state can benefit the monopolist in such contingencies. The monopolist sets $p_2$ to maximize profits in the high state and foregoes direct sales in the low state. Instead, the secondary market clears at price $t_L$ (since it does not price compete with the primary market), and the monopolist collects the name-change fee $f$ for each transaction. This comes at a cost since allowing a secondary market in the low state necessitates a secondary market in the high state.\footnote{The fee $f$ does not vary with realized demand, so if $t_L \geq f$, then $p_2 > t_L$ implies $p_2 > f$ and thus the scalpers resell in the secondary market in the high state. We know that $p_2 > t_L$ from $p_2 \geq Z_L > t_L \geq f$.} For each transaction in the secondary market, the monopolist loses $p_2$ but again receives $f$. In a sense, the name-change fee allows the monopolist to partly offset her inability to adjust prices by enabling her to charge two prices in date 2.

**Proposition 2.** Assume $Z_H > 2Z_L$, and denote $S = \bar{c}/\bar{c}$ the size of the secondary market. There exists $\bar{\gamma}_H \in (0, (Z_L/(Z_H - Z_L))^2)$, such that for $\bar{\gamma}_H \leq \gamma_H < 1$ the monopolist strictly prefers to set prices and fees such that the primary market does not operate in the low demand state, and the secondary market operates in both states (i.e., $\bar{c} > 0$, and $f < t_L$). No date-1 consumer buys in date 2. The optimal prices are $p_{1}^{sec} = 1/2$, $p_{2}^{sec} = t_H = (m_2 Z_H - S)/2m_2$, $t_L = (m_2 Z_L - S)/m_2 > Z_L/2$, and $f < t_L$. The indifferent consumers are $\bar{v}_{1}^{sec} = 1/2$, $\bar{v}_{2,L}^{sec} = t_L$, $\bar{v}_{2,H}^{sec} = t_H$.\footnote{An equilibrium in which date-1 consumers waited was derived, but it resulted to inconsistent conditions. This is because optimally the monopolist wants to intertemporally separate the consumers. Date-1 consumers will wait if they expect the price in the secondary market to be low enough. This requires a large supply of scalpers, which the monopolist can control through $f$. Thus, setting a low $f$ reduces the revenue she receives from the secondary market, decreases profit in the high state, and deviates the optimal value for the date-1 marginal consumer. Hence, such a contingency cannot arise.}

It is straightforward to check that $t_H > t_L$ such that a secondary market exists in both states. Since the monopolist chooses $p_2$ based on the residual demand $R(Z_H, S)$, $p_{2}^{sec} < Z_H/2$. The monopolist sacrifices some of the high state profit by allowing a secondary market, but
collects revenue through the fees from the low state that she would otherwise have lost, and the high state. In fact, since $\tilde{\gamma}_H < (Z_L/(Z_H - Z_L))^2$ the monopolist is better off with a secondary market even in cases when she would forgo the low state in the absence of a secondary market. In essence, the monopolist is guaranteed $f \cdot S$ in both states, and an additional $p_2 \cdot R(Z_H, S)$ in the high state.

The primary market is visited only in the high state: $Z_H > 2Z_L$ implies $\tilde{p}_2^{\text{sec}}$ exceeds marginal valuation in the low state ($\tilde{v}_2^{\text{sec}} = t_L$), while the secondary market clears at $t_L$. Therefore, no consumer visits the primary market in the low state.

The case where $(Z_H - S)/2 < Z_L < Z_H/2$ cannot arise. This would imply that it is not optimal to sell in the low state when there is no secondary market, but an active secondary market steals enough consumers in the high state such that the resulting primary market price induces consumers to visit it in the low state. Such logic is wrong. The primary market is visited if $t^L \geq \tilde{p}_2^{\text{sec}}$ or if the secondary market does not clear. However, this does not hold, and thus such knife edge cases do not arise.

When $Z_H < 2Z_L$, the optimal price is $p_2 = (\gamma_H Z_H + \gamma_L Z_L)/2$ when there is no secondary market, and the primary market is active in both states. Moreover, it can be shown that $Z_H < 2Z_L$ implies that there is not enough surplus in the high state in order to allow the secondary market to arise.

4 Capacity Constraint

Now suppose there is a capacity limit $k > 0$ on the number of consumers the monopolist can serve. The capacity constraint is non-binding with demand $Z_L$, but binds with $Z_H$. Capacity limits can result to consumers with high valuations being unable to be served in the primary market, which leaves room for the secondary market. Name-change fees can be chosen such that whenever the secondary market arises it does not price compete with the primary market, and consumers visit the secondary market only if they cannot find a ticket in the primary market. The secondary market is used by the monopolist to circumvent her inability to relax her capacity constraint and price-discriminate.

In date 2, the available capacity is $k - q_1 - S$, where $q_1 = (1 - \tilde{v}_1)$ and $S = \tilde{c}/\bar{c}$. With perfect yield management, the monopolist can clear the market in which case Lemma 1 applies, and the fee is set high enough to shut the secondary market. If date-2 consumers’ arrival is inversely related to their valuations then an active secondary market allows the monopolist to sort consumers in the high state when the capacity constraint binds. The secondary market absorbs the highest valuation consumers, and trades at a higher price.
which enables the monopolist to extract a higher fee than the price charged in the primary market. The result goes away if date-2 consumers arrive randomly.

**Proposition 3.** Suppose perfect yield management, and, $\gamma_H (Z_H - p_{2,H}^{\text{sec}}) > p_1^{\text{nosec}}$, where $p_1^{\text{nosec}}, p_2^{\text{sec}}$ are the prices when the secondary market never emerges. If date-2 highest valuation consumers arrive last, the monopolist profit is maximized when the primary market is open in both states and the secondary market is open only in the high state at price $p_{2,H}^{\text{sec}}$. The secondary market prices are $t_L = Z_L - S/m_2 < p_2^{\text{sec}}$, and $t_H = Z_H - S/m_2 > p_{2,H}^{\text{sec}}$. The fee is set at $f \in (p_2^{\text{sec}}, t_H)$. If, instead, date-2 consumers arrival is independent of their valuation, then the secondary market is always closed.

Figure (1a) illustrates the intuition. In the high state with no secondary market, price $p_{2,H}^{\text{sec}} = Z_H - (k - q_1)/m_2$ clears the primary market. If instead $S > 0$, then the available capacity is $k - q_1 - S$, and price $p_{2,H}^{\text{sec}} = p_2^{\text{sec}}$ generates excess demand of size $S$. The monopolist can serve $k - q_1 - S$ consumers in the primary market at price $p_{2,H}^{\text{sec}}$, and let the excess demand to the secondary market. Under parallel rationing the highest valuation consumers buy from the secondary market. Hence, $t_H = Z_H - S/m_2$ since the secondary market is not price-competing the primary market, and $t_H$ is competitively determined. Since $t_H > p_{2,H}^{\text{sec}}$, the monopolist can set $f > p_{2,H}^{\text{sec}}$.

The volume of sales is the same with and without a secondary market. With an active secondary market, the monopolist sells less in the primary market and receives the same price. The sales not made in the primary market, are made in the secondary market, and the monopolist receives a higher price in the form of the fee.

The assumption $\gamma_H (Z_H - p_{2,H}^{\text{sec}}) > p_1^{\text{nosec}}$ guarantees enough surplus for scalpers such that the monopolist and the scalpers can mutually benefit. Note that the optimal prices with an active secondary market are not necessarily the same as with an inactive secondary market. Hence, the monopolist can do even better.\(^{13}\)

Parallel rationing is crucial as it guarantees that the marginal consumer valuation in the secondary market is higher than in the primary market, which results to a higher price in

\(^{13}\)As shown in the Appendix for $Z_L \geq (1 + \sqrt{(1 + m_2)/m_2})$, we actually have $p_1^{\text{sec}} > p_1^{\text{nosec}}$, and $p_{2,H}^{\text{sec}} < p_{2,H}^{\text{nosec}}$. The monopolist prefers to exclude some date-1 consumers that do not resell, and increase the secondary market. She also has more available capacity in date 2, which explains lower date-2 price.

\(^{14}\)The monopolist does not want to charge $Z_H - (k - q_1 - S)/m_2$, the price that clears the market in the high state when available capacity is $k - q_1 - S$. At such price, the scalpers do not resell since the secondary market does not price compete with the primary, and only sells to excess demand. The monopolist does not want to set the clearing price $Z_H - (k - q_1 - S)/m_2$ which exceeds $p_{2,H}^{\text{sec}}$, because the capacity constraint binds, and we are at the increasing portion of the profit function. Even at $f = p_{2,H}^{\text{sec}} (1)$ the monopolist strictly prefers to allow the secondary market to emerge, which would bring total sales to $k - q_1$.
the former \( t_H > p_{2, H}^{se} \). With random arrival, the marginal valuations in the two markets are the same, and so \( t_H = p_{2, H} \). Lemma 1 applies, and there is no secondary market.

With imperfect yield management, the result of Proposition 3 can be extended to the case of random arrival. Without a secondary market, the monopolist can always set date-2 price to clear the primary market in the high state. However, as Peck [1996] suggested, firms often have an incentive to generate excess demand. This is true here if the high state is not too likely. An active secondary market can absorb some of the excess demand, and trade at a higher price. The name-change fee can be set at a higher price enabling the monopolist to "price-discriminate" consumers in the primary and secondary markets.

**Proposition 4.** Suppose imperfect yield management, and, \( \gamma_H (Z_H - p_{2, H}^{nosec}) > p_1^{nosec} \), where \( p_1^{nosec}, p_2^{nosec} \) are the prices when the secondary market never emerges. The monopolist profit is maximized when the primary market is open in both states and the secondary market is open only in the high state at price \( p_{2, H}^{sec} \). The secondary market prices are \( t_L = Z_L - S/m_2 < p_{2, H} \), and \( t_H = Z_H - S/m_2 > p_{2, H}^{sec} \). The fee is set at \( f \in (p_{2, H}^{sec}, t_H) \).

Consider Figure (1b). Absent of a secondary market, the optimal price is \( p_2^{sec} = p_2^{nosec} \) with available capacity \( k - q_1 \). Demand in the two states is \( m_2 (Z_L - p_2^{sec}) \) and \( m_2 (Z_H - p_2^{sec}) \). In the high state there is excess demand, and if consumers arrive randomly, the marginal valuation is \( p_2^{sec} \). Since consumers with valuations exceeding \( p_2^{sec} \) are left unserved, the monopolist can set the fee \( f \) such that the secondary market arises only in the high state, and at a higher price than the primary market. The consumers in the secondary market prefer to buy from the
primary market, but they cannot due to capacity limitations. Note also that the marginal valuation in the secondary market is higher than in the primary market, which is why the secondary market can emerge under random arrival.\footnote{One might wonder why would the monopolist not sell only in date 2. This is because the capacity constraint binds only in one state. Thus, the monopolist wants to sell to date-1 consumers since with probability $\gamma_L = 1 - \gamma_H$ the capacity does not bind.}

5 Extension: Changing Preferences\footnote{I thank Dan Bernhardt and George Deltas for the suggestion to examine this case.}

In the preceding analysis, consumer valuations did not change between dates. However, as Court\`y points out, consumers who buy early might wish not to use the service when time comes.\footnote{In practice, no shows are sufficiently consistent so that airlines are legally allowed to overbook.} I incorporate this insight by assuming that between date 1 and date 2, a date-1 consumer’s valuation goes to zero with probability $\alpha > 0$. A date-1 consumer knows that buying early exposes her to the risk of owing a useless ticket, which decreases the value of buying. Alternatively, if she waits, she still wants to use the service with probability $1 - \alpha$.\footnote{As before no discounting implies that, in order to avoid corner solutions with no consumers buying in date 1, I assume that date-1 consumers do not buy in the high state.}

The monopolist can increase the value of buying early by allowing date-1 consumers to resell when they do not want the service. This insight is identical to the durable good literature, where the monopolist allows buyers to resell their used goods so that she can extract more surplus when they first buy a new good. Instead, I focus on a complementary question. Specifically, I show that Court\`y’s result that the monopolist cannot exclude scalpers goes away if we allow for name-change fees. I also discuss some preliminary results when I allow the monopolist to overbook. I find that the monopolist can avoid overbooking by allowing the secondary market to emerge.

The basic framework needs to be augmented if consumers are to resell in the secondary market. I now assume that accessing the secondary market as a reseller is costless.

Excluding Scalpers I consider whether it is feasible for date-1 consumers trade their tickets only if their valuations change. This is true if $0 \leq t_j - f \leq \hat{v}_1$. Setting $t_j - f \leq \hat{v}_1$ implies that the marginal date-1 consumer does not resell if she still values the service, while $t_j - f \geq 0$ implies that she can resell if she does not value it anymore. Therefore, a date-1 consumer...
consumer buys in date 1 if and only if

\[ (1 - \alpha) v_1 + \alpha \sum_{j \in \{L,H\}} \gamma_j \rho_j^S (t_j - f) I_{t_j \geq f} - p_1 \geq 0 \]  

(5)

\[ (1 - \alpha) \sum_{j \in \{L,H\}} \gamma_j \max \left\{ 0, \rho_j^{P,c} [v_1 - p_{2,j}], \rho_j^S (v_1 - t_j) \right\}. \]

This is equivalent to the earlier condition except for the possibility of changing valuations. The scalper decision whether to participate is identical to before (except that every scalper has zero cost now),

\[ \sum_{j \in \{L,H\}} \gamma_j \rho_j^S (t_j - f) I_{t_j \geq f} - p_1 \geq 0. \]

Proposition 5 shows that it is possible to exclude scalpers while allowing date-1 consumers to resell if their valuation changes. The monopolist can take advantage of the fact that date-1 consumers value the service with positive probability, whereas scalpers do not. Hence, the fee can be set such that expected payoff from scalping is negative.

Proposition 5. Suppose date-1 consumers, who bought in date-1, resell their tickets only if their valuations change: \( 0 \leq t_j - f \leq \tilde{v}_1 \). Then, there exits a fee \( f \geq 0 \) such that it is possible to have a secondary market, where scalpers do not participate.

Proposition 6. An active secondary market (in which the only resellers are date-1 consumers whose preferences changed) can strictly increase the profit of the monopolist with infinity capacity and perfect yield management.

The results of the main analysis apply with changing consumer preferences, where the secondary market has size \( \alpha (1 - \tilde{v}_1) \). It is interesting to examine whether the secondary market can also be populated by scalpers.

Overbooking With changing preferences, the monopolist can overbook in the high state as consumers who do not want to use the service might not show up. As Ely et al. [2017] highlight this is commonplace in the airline industry. When not enough consumers fail to show up, then the airline reimburses consumers up to the point that enough of them concede
their seat. To consider overbooking the model needs to be adapted. With a continuum of consumers, a measure $\alpha$ of date-1 consumers will always not show up, and hence the monopolist never faces a penalty for overbooking. Instead, suppose, for simplicity, that with probability $\alpha > 0$ all date-1 consumers do not show up. I simplify further by assuming that date-1 consumers do not wait in equilibrium, and that the mass of date-2 consumers is 1. The following are sufficient conditions.

**Assumption 2.** $m_2 = 1$, and $Z_L \geq 3 (1 - \alpha) (1 + \gamma_H) / 2$.

Suppose there is no secondary market. At date-2, the available capacity is $k - q_1$, where $q_1 = (1 - \bar{v}_1)$ are the date-1 sales. By overbooking the monopolist sells $k$ tickets in the high state. With probability $(1 - \alpha)$, date-1 consumers show up. In the high state the monopolist needs to reimburse enough consumers such that $q_1$ seats become available. Date-1 consumers are the lowest valuation consumers, and a reimbursement of $r = 1$ is enough for all date-1 consumers to relinquish their seat. No overbooking occurs in the low state, and date-1 consumers use the service. With probability $\alpha$, the valuation of date-1 consumers goes to zero, but there is no secondary market to resell. Marginal date-1 valuation $\bar{v}_1^{\text{Over}}$ solves

$$(1 - \alpha) \left( \gamma_H r + (1 - \gamma_H) \bar{v}_1^{\text{Over}} \right) - p_1 = 0. \tag{6}$$

Rearranging

$$\bar{v}_1^{\text{Over}} = \frac{p_1 - (1 - \alpha) \gamma_H}{(1 - \alpha) (1 - \gamma_H)}$$

The monopolist profit is

$$\Pi^{\text{Over}} = p_1 \left( 1 - \bar{v}_1^{\text{Over}} \right) + \gamma_H \left[ p_{2,H} k - (1 - \alpha) \left( 1 - \bar{v}_1^{\text{Over}} \right) \right] + (1 - \gamma_H) p_{2,L} \left( Z_L - p_{2,L} \right).$$

The optimal prices are $p_1^{\text{over}} = (1 - \alpha) (1 + \gamma_H) / 2$, $p_{2,H}^{\text{over}} = Z_H - k$, and $p_{2,L}^{\text{over}} = Z_L / 2$.

Now suppose the monopolist does not overbook in the high state, but allows the secondary market to emerge. She again receives fee $f$. The date-1 marginal valuation $\bar{v}_1^{\text{sec}}$ solves

$$(1 - \alpha) \bar{v}_1^{\text{sec}} + \alpha \gamma_H (t_H - f) - p_1 = 0.$$

The consumer does not have to concede her seat if she still values it, and can resell in the secondary market at price $t_H$ if her preferences change. Rearranging

$$\bar{v}_1^{\text{sec}} = \frac{p_1 - \alpha \gamma_H (t_H - f)}{(1 - \alpha)}.$$
The profit of the monopolist is then
\[
\Pi^{\text{Sec}} = p_1 (1 - \tilde{v}_1^{\text{sec}}) + \gamma_H [p_{2,H} (k - (1 - \tilde{v}_1^{\text{sec}})) + f \alpha (1 - \tilde{v}_1^{\text{sec}})] + (1 - \gamma_H) p_{2,L} (Z_L - p_{2,L}).
\]

In the high state the monopolist sells \( k - (1 - \tilde{v}_1^{\text{sec}}) \) seats in the primary market. With probability \( \alpha \) the secondary market emerges, and trades \( (1 - \tilde{v}_1^{\text{sec}}) \) tickets, and the monopolist receives \( f \) from each transaction. The optimal prices depend on the relative marginal valuations in the two markets.

**Secondary market or Overbooking** With overbooking the monopolist is guaranteed \( p_{2,H}^{\text{over}} k \), and she is being penalized \( r (1 - \tilde{v}_1^{\text{over}}) \) with probability \( (1 - \alpha) \) where \( r = 1 \left( \leq p_{2,H}^{\text{over}} \right) \). With a secondary market the monopolist earns \( p_{2,H}^{\text{sec}} (k - (1 - \tilde{v}_1^{\text{sec}})) \), and she receives an extra \( f (1 - \tilde{v}_1^{\text{sec}}) \) with probability \( \alpha \). The best the monopolist can do is set \( p_{2,H}^{\text{sec}} = p_{2,H}^{\text{over}} \) so that there is excess demand, and \( t_H \geq f \geq p_{2,H}^{\text{over}} \). Under proportional rationing \( t_H = p_{2,H}^{\text{over}} \), and overbooking strictly dominates the secondary market.

With parallel rationing, the highest valuation consumers arrive last and they buy from the secondary market at price \( t_H = Z_H - S > p_{2,H}^{\text{sec}} \) since \( k > S \). This is almost identical to the case considered with non-changing preferences. However, the monopolist can set \( f = t_H \), since consumers value the service with positive probability. Scalpers did not value the service, and thus we required \( f < t_H \). At the optimal price \( p_1^{\text{sec}} \) one can verify that the option of the secondary market dominates overbooking. The conclusion of the main analysis extends here. If the secondary market can help the monopolist price-discriminate then it dominates the option of overbooking.

### 6 Further Extensions

The basic framework described in Section 2 could be extended in a number of ways. Extending the two-demand distribution to a more generic distribution seems a natural next step. However, other variations might be more interesting.

Two minor changes could have important implications: change when the scalpers incur cost \( c \), and allow scalpers to decide whether to buy in date 1 or wait. In the current framework, the scalpers incur the cost when they buy a ticket instead of when reselling, and they need to buy in date 1. As a result, in date 2 they are ex post identical. Incurring \( c \) when reselling implies that not all scalpers wish to resell in a given state, while allowing scalpers to choose when to buy could lead to some scalpers buying early, and others buying late. If the price rises between the two dates, then buying early means paying a lower price,
but increases the risk of not reselling. Furthermore, buying in date 1 means that the price $p_1$ is sunk. Hence, decisions to resell for scalpers that bought early are different from scalpers that bought in date 2. Moreover, a scalper that buys in date 2 can resell only with capacity constraint, since otherwise no mutually agreement can be found between her and a consumer.

Another interesting extension would be to allow the monopolist to offer the service more than once, where the consumers have different preference over their ideal time of service. Dana [1999] considers a similar static framework, where the distribution of optimal times is unknown, and as a result the monopolist uses yield management to maximize profits. He shows that a price dispersion arises in equilibrium, but ignores the possibility of a secondary market. It would be interesting to examine, whether a secondary market can benefit the monopolist. Courty [2000] discusses some of the issues that arise when a seller provides multiple showings of the same play. He argues that the Coase conjecture might apply (Coase [1972]) since promoters want to lower prices for showings after the peak hours.

Finally, considering the case of oligopoly could provide some interesting strategic interactions. By allowing a secondary market in the next date, a seller increases the competition for the rival as well as for herself. So, it might be optimal to flood the market in date 1. For example, suppose that there are two sellers each offering a service at a specific time. This is a spatial competition and it could be modeled as a linear city model with the two sellers located at opposite ends of the line. In this case, flooding the market in the first date could mean that the marginal consumer moves further away from the seller, thus increasing profits.

7 Conclusion

In this paper, I considered the case of a monopolist who sells tickets to a service provided at the end of date 2. Date 1 demand is known, whereas date 2 demand is known to be higher than date 1 demand, but its exact value is unknown. Selling tickets in date 1 provides scalpers with the opportunity to resell in the secondary market in date 2. Despite the possibility of competition from scalpers, I find that the secondary market can be used as an insurance against the uncertain demand by the monopolist, when the price cannot be adjusted to clear the market. The instrument used is the name-change fees. If the monopolist cannot adjust prices to clear the market, then unserved consumers visit the secondary market, and the monopolist can charge a fee to allow the name on tickets to be changed, which provides a revenue stream to the monopolist. If, though, the monopolist can adjust her prices to the demand realization, then such an insurance mechanism is redundant in most cases. Hence,
the monopolist prefers to shut down the secondary market completely.

The inverse relationship between the ability to adjust prices and the level of the name-change fee could be a reason for the high name-change fees were adopted in the airline industry as suggested by industry insiders. My results suggest that firms that sell tickets, and cannot adjust prices might benefit from introducing name-change fees. Smaller firms might have weaker tools to adjust prices, and can also be subjected to uncertain price changes due to bigger firms. Abstracting from competition concerns, the name-change fees can be used by such small firms to stabilize their revenue.

The insights of the present paper could be adopted in a more complicated model of yield management, where firms face frictions when setting prices. My results suggest that name-change fees have a crucial role to play.

\[19\] I would like to thank George Deltas for this insight.
Appendix

Proof of Lemma 1: The monopolist gains if \( p_1 \geq \sum_{j = \{L, H\}} \gamma_j (p_{2,j} - f) I_{j \geq f} \), whereas scalpers gain if \( \sum_{j = \{L, H\}} \gamma_j \rho_j \delta_j (p_{2,j} - f) I_{j \geq f} - c \geq p_1 \). These are incompatible for \( c > 0 \). □

Proof of Proposition 1: The first part follows from Lemma 1 and the fact that perfect yield management. Suppose the date-1 marginal consumer has valuation \( \tilde{v}_1 \). Since the monopolist can adjust price after she observes the demand realization, she can decide whether to include date-1 consumers that waited. By assumption 1 in the high state she does not want to do that, though she might in the low-state. it.

Excluding date-1 consumers, the optimal prices are \( p_{2,L}^{\text{excl}} = Z_L / 2 \), and \( p_{2,H}^{\text{excl}} = Z_H / 2 \). Conversely, including date-1 consumers, the optimal prices are \( p_{2,L}(0) = p_{2,L}^{\text{incl}} = (\tilde{v}_1 + m_2 Z_L) / 2 (1 + m_2) \), and \( p_{2,H}(0) = p_{2,H}^{\text{incl}} = Z_H / 2 \), where \( p_{2,L}^{\text{incl}} < \tilde{v}_1 \) if and only if \( \tilde{v}_1 (1 + m_2) > m_2 (Z_L - \tilde{v}_1) \). The profit in the low-state is \( \Pi_2^L = (\tilde{v}_1 + Z_L m_2)^2 / 4 (1 + m_2) \).

The value of \( \tilde{v}_1 \) depends on whether the date-1 marginal consumer expects to buy in the low state or not. If not then \( \tilde{v}_1 = \tilde{v}_1^{\text{excl}} = p_1(0, 0) = 1/2 \). If, instead, the date-1 marginal consumer expects to buy in the low state at price \( p_{2,L}^{\text{incl}} \), then

\[
\tilde{v}_1 = \tilde{v}_1^{\text{incl}} = \frac{2 (1 + m_2) p_1 - (1 - \gamma_H) m_2 Z_L}{1 + \gamma_H + 2 \gamma_H m_2}.
\]

The date-1 price is long and unintuitive, and I omit it. It can be solved for by maximizing

\[
\Pi = p_1 \left( 1 - \tilde{v}_1^{\text{incl}} \right) + \gamma_H m_2 (Z_H - p_{2,H}) p_{2,H} + (1 - \gamma_H) \left( \tilde{v}_1^{\text{incl}} + Z_L m_2 \right)^2 / 4 (1 + m_2).
\]

On the equilibrium path the monopolist strictly prefers to intertemporally separate consumers. However, once in date-2, the monopolist can deviate. With date-1 marginal consumer \( \tilde{v}_1 = 1/2 \), she strictly prefers to deviate if \( Z_L < \left( 1 + \sqrt{(1 + m_2) / m_2} \right) / 2 \), while \( p_{2,L}^{\text{incl}} < 1/2 \), when \( \tilde{v}_1 = 1/2 \) and \( Z_L < (1 + 2 m_2) / m_2 \), where \( (1 + 2 m_2) / m_2 > \left( 1 + \sqrt{(1 + m_2) / m_2} \right) / 2 \).

Thus, the monopolist would prefer to intertemporally separate the consumers but she cannot credibly commit to doing it when \( Z_L < \left( 1 + \sqrt{(1 + m_2) / m_2} \right) / 2 \). □

Proof of Proposition 2: If \( \gamma_H = \{0, 1\} \), then there is no uncertainty and the monopolist can set optimal prices, thus, there is no room for the secondary market.

Now consider \( \gamma_H \in (0, 1) \). In the purported equilibrium, the secondary market is the only active market in the low state. Thus, market clearing condition implies \( t_L = (m_2 Z_L - S) / m_2 \). In the high state both markets are open. Then, optimal \( p_2 \) maximizes \( \max p_2 \left( m_2 (Z_H - p_2) - S \right) \) since \( S \) consumers buy from the secondary market. Thus, \( p_2^{\text{sec}} = \)
The marginal scalper has cost \( \tilde{c} \) that solves
\[
\gamma_H \left( \frac{m_2 Z_H - S}{2m_2} \right) + (1 - \gamma_H) \left( \frac{m_2 Z_L - S}{m_2} \right) - f - \bar{p}_{1}^{\text{sec}} - \bar{c} = 0,
\]
where \( S = \tilde{c}/\bar{c} \). Since the scalper resells in both states, she incurs the fee \( f \) with certainty. Solve for \( \bar{c} \) as a function of \( p_1 \) and \( f \) to derive \( \bar{c}(p_1, f) \) and \( S(p_1, f) = \bar{c}(p_1, f)/\bar{c} \). Date-1 consumers do not wait in the posited equilibrium. Therefore, the monopolist solves
\[
\max_{p_1, f} p_1 (1 - p_1) + (p_1 + f) S(p_1, f) + \gamma_H \frac{m_2 Z_H - S(p_1, f))^2}{4m_2}.
\]
The monopolist receives \( (p_1 + f) S \) with certainty from the scalpers, while in the high state she has an additional profit \( \gamma_H = (m_2 Z_H - S(p_1, f))^2 / 4m_2 \). Maximizing
\[
\bar{c} = \frac{2\bar{c}m_2 (1 - \gamma_H)}{4\bar{c}m_2 + 4 - 3\gamma_H} > 0
\]
\[
\bar{p}_{1}^{\text{sec}} = 1/2
\]
\[
\bar{f}^{\text{sec}} = 4 \frac{(1 - \gamma_H)^2 Z_L + (4 - 3\gamma_H) (\gamma_H Z_H - 1) + 4m_2 \bar{c}((1 - \gamma_H) Z_L + \gamma_H Z_H - 1)}{8m_2 \bar{c} + 2 (4 - 3\gamma_H)} < t_L,
\]
and \( t_L > Z_L/2 \) thus date-1 consumers do not buy. Denote the profit as \( \Pi^{\text{sec}} \).

Without a secondary market, the monopolist strategies are described in the main text. For \( \gamma_H \geq (Z_L/(Z_H - Z_L))^2 \), the monopolist prices for the high state. The associated profit is \( (1 + \gamma_H m_2 Z_H^2) / 4 \). By a simple comparison one can check that profit with the secondary market \( \Pi^{\text{sec}} \) is strictly higher. For \( \gamma_H < (Z_L/(Z_H - Z_L))^2 \), the monopolist prices for the average demand, and the associated profit is \( \Pi^{\text{nosec}} = m_2 (\gamma_H Z_H + Z_L (1 - \gamma_H))^2 / 4 \). Define the difference
\[
\Delta (\gamma_H) = \Pi^{\text{sec}} - \Pi^{\text{nosec}},
\]
which equals
\[
m_2 (1 - \gamma_H) \left[ \gamma_H \left[ (4 - 3\gamma_H)(Z_H - 2Z_L) Z_H + 3 (1 - \gamma_H) Z_L^2 \right] + 4m_2 \bar{c} [\gamma_H (Z_H - 2Z_L) Z_H - (1 - \gamma_H) Z_L^2] \right] / (4m_2 \bar{c} + 4 - 3\gamma_H).
\]
Tedious algebra reveals that \( \Delta (\gamma_H) \) has 1 root at \( \gamma_H = 1 \) and one at \( \gamma_H = \bar{\gamma}_H < (Z_L/(Z_H - Z_L))^2 \). Moreover, \( \Delta'(\gamma_H) < 0 \) at \( \gamma_H = 1 \). Therefore, \( \Delta (\gamma_H) \) is decreasing at \( \gamma_H = 1 \), and since \( \Delta (\gamma_H) \) does not cross the \( \gamma_H \)-axis for \( \gamma_H \in (\bar{\gamma}_H, 1) \), we have that it is positive \( \Delta (\gamma_H) > 0 \) for \( \gamma_H \in (\bar{\gamma}_H, 1) \) suggesting that the secondary market increases the monopolist’s profit. \( \square \)

**Proof of Proposition 3:** With perfect yield management, and no secondary market the
optimal prices are \( p_1^{\text{post}} \), \( p_{2,H}^{\text{post}} \), and \( p_{2,L}^{\text{post}} \). In fact, \( p_{2,H}^{\text{post}} = Z_H - (k - (1 - \tilde{v}_1))/m_2 \) which clears the market.

Now, suppose date-2 consumers arrive inversely to their valuation. The monopolist can set the same prices as above, and \( f \geq p_{2,H}^{\text{post}} \). Date-1 consumer and low state consumer behavior does not change. In the high state, the aggregate demand is the same. The secondary market only takes consumers that did not find a ticket in the primary market. Consider a marginal increase in the size of secondary market, from zero to positive. The marginal scalper is \( c = 0 \). If she enters, then she receives \( t_H - f = Z_H - f \) with probability \( \gamma_H \) (since highest valuation consumers enter last), and pays \( p_1 \). Her expected surplus is \( \gamma_H (Z_H - f) - p_1 \). The monopolist is indifferent since \( f = p_{2,H}^{\text{post}} \) and \( p_1^{\text{post}} \). By assumption, \( \gamma_H (Z_H - p_{2,H}^{\text{post}}) - p_1^{\text{post}} > 0 \) so the marginal scalper has positive expected surplus. Thus, the monopolist can increase \( f \) such that \( f > p_{2,H}^{\text{post}} \). This strictly increases profits, and allowing the monopolist to set prices optimally further increases profit.

With random arrival, the residual demand in the secondary market is \( (Z_H - t_H)(1 - (k - (1 - \tilde{v}_1) - S)/(Z_H - p_2)) \), and supply is \( S \). Solving gives \( t_H = Z_H - (k - (1 - \tilde{v}_1))/m_2 \). Then, \( t_H = p_{2,H} \) and the argument collapses since the monopolist must set \( f < p_{2,H} \) in order for the scalpers to have positive surplus. Since secondary market operates only in states which the primary market operates then Lemma \( \text{I} \) applies.

I present the prices for the case \( Z_L \geq \left(1 + \sqrt{(1 + m_2)/m_2}\right) \) (date-1 consumers do not wait). The prices for \( Z_L < \left(1 + \sqrt{(1 + m_2)/m_2}\right) \) are available upon request. The intuition is the same. We have, \( p_{2,L} = Z_L/2 \), and \( p_{2,H} = Z_H - (k - (1 - \tilde{v}_1))/m_2 \), where \( \tilde{v}_1 = p_1 \) with and without a secondary market. Without a secondary market

\[
p_1^{\text{post}} = \frac{m_2 (\gamma_H Z_H + 1) - 2(k - 1)\gamma_H}{2(\gamma_H + m_2)},
\]

with a secondary market in the high state

\[
p_1^{\text{sec}} = \frac{2m_2 \bar{c} (m_2 (\gamma_H Z_H + 1) - 2(k - 1)\gamma_H) + \gamma_H (2m_2 (\gamma_H Z_H + 1) - 3(k - 1)\gamma_H)}{4m_2 \bar{c} (\gamma_H + m_2) + \gamma_H (3\gamma_H + 4m_2)}
\]

and marginal scalper

\[
\bar{c} = \frac{m_2 \bar{c} \gamma_H (\gamma_H Z_H + 2k - 1)}{4m_2 \bar{c} (\gamma_H + m_2) + \gamma_H (3\gamma_H + 4m_2)} > 0.
\]

In the secondary market \( t_H = Z_H - S/m_2 \), where \( S = \bar{c}/\bar{c} \). Also, \( p_1^{\text{sec}} > p_1^{\text{post}} \), and \( p_2^{\text{sec}} <
Proposition 4: Suppose no secondary market. At date 2, the monopolist cannot vary price according to the state, but she can always set the price so that the market clears in the high state. Under Assumption 1, date-1 consumers do not wait, and so the monopolist does not have any new information when setting date-2 price \( p_2 \). As Baylis and Perloff [2008] show the optimization problem of a capacity constraint monopolist can be expressed as a Lagrangian, where the capacity is imposed as a constraint. The capacity constraint binds in the high state, and thus, the Lagrangian is

\[
L = p_1 (1 - p_1) + m_2 (\gamma_H Z_H + (1 - \gamma_H) Z_L - p_2) p_2 - \mu (1 - p_1 + m_2 (Z_H - p_2) - k),
\]

where \( \mu \) is the Lagrangian multiplier. The optimal prices are

\[
p_{\text{clear, nosec}}^1 = \frac{m_2 (1 + 2Z_H - \gamma_H Z_H + (1 - \gamma_H) Z_L) - 2(k - 1)}{2(1 + m_2)}
\]

\[
p_{\text{clear, nosec}}^2 = \frac{1 + \gamma_H Z_H + (1 - \gamma_H) Z_L + 2(m_2 Z_H - k)}{2 (1 + m_2)}.
\]

One can show \( p_{\text{clear, nosec}}^2 = Z_H - \left( k - \left( 1 - p_{\text{clear, nosec}}^1 \right) \right)/m_2 \), and \( p_{\text{clear, nosec}}^2 > p_{\text{clear, nosec}}^1 \). The market clears in the high state. Denote \( \Pi_{\text{clear}} \) the total profit, and \( \Pi_{\text{clear}}^2 (p_1) \) the date-2 profit as a function of \( p_1 \). This strategy profile might not be optimal as it sacrifices sales in the low state. Consider an alternative strategy at date 2. Conditional on \( p_2 \leq Z_H - (k - (1 - p_1)) \) \( /m_2 \) sales in the high state are equal to available capacity \( (k - (1 - p_1)) \). Denote available capacity \( \tilde{k}(p_1) \). The monopolist solves

\[
\max_{p_2} \left( \gamma_H \tilde{k}(p_1) + (1 - \gamma_H) (Z_L - p_2) \right) p_2, \]

which gives

\[
p_{\text{Excess}}^2 (p_1) = \frac{m_2 (1 - \gamma_H) Z_L + \gamma_H (k - (1 - p_1))}{2m_2 (1 - \gamma_H)}
\]

\[
\Pi_{\text{Excess}}^2 (p_1) = \frac{(m_2 (1 - \gamma_H) Z_L + \gamma_H (k - (1 - p_1)))^2}{4m_2 (1 - \gamma_H)}.
\]
and solving back gives

\[
\begin{align*}
p_{1, \text{Excess, nosec}} &= \frac{\gamma_H^2 (k - 1) + m_2 (1 - \gamma_H) (\gamma_H Z_L + 2)}{4m_2 (1 - \gamma_H) - \gamma_H^2} \\
p_{2, \text{Excess, nosec}} &= \frac{\gamma_H (2k - 1) + 2m_2 (1 - \gamma_H) Z_L}{4m_2 (1 - \gamma_H) - \gamma_H^2},
\end{align*}
\]

giving total profit \(\Pi^{\text{Excess}}\). Two requirements need to hold: (i) positive demand in low state, and (ii) \(p_2 \leq Z_H - (k - (1 - p_1)) / m_2\) and \(p_1, p_2 > 0\). The latter holds if

\[
m_2 > \frac{2 (k - 1) + \gamma_H (1 + \gamma_H Z_H) + (\gamma_H Z_L + 2k) (1 - \gamma_H)}{2 (2Z_H - Z_L) (1 - \gamma_H)} \times \frac{\gamma_H^2}{4 (1 - \gamma_H)},
\]

where \(m_2 > \gamma_H^2 / 4 (1 - \gamma_H)\) implies positive prices. The first inequality imposes an upper bound on \(\gamma_H\), which I denote \(\bar{\gamma}_H\). Demand in low state is \(m_2 (Z_L - p_2)\) which equals

\[
\frac{m_2 (\gamma_H (\gamma_H Z_L + 2k - 1) + 2m_2 (\gamma_H - 1) Z_L)}{-(4m_2 (1 - \gamma_H) - \gamma_H^2)}.
\]

The denominator is negative since \(m_2 > \gamma_H^2 / 4 (1 - \gamma_H)\). The numerator increases in \(\gamma_H\) going from negative at \(\gamma_H = 0\) to positive at \(\gamma_H = 1\). By the Intermediate Value Theorem, a root exists at \(\bar{\gamma}_H > 0\) so that for \(\gamma_H < \bar{\gamma}_H\) the numerator is negative, and demand is positive. Taking \(\min \{\bar{\gamma}_H, \bar{\gamma}_H\}\) satisfies all conditions. Then, it is straightforward to show that \(\Pi_{2, \text{Excess}}^\text{Excess} (p_1) > \Pi_{2, \text{clear}}^\text{clear} (p_1)\) for all \(p_1\). For \(m_2 > \gamma_H^2 / 4 (1 - \gamma_H)\), \(\Pi^{\text{Excess}} > \Pi^{\text{clear}}\).

The rest of the proof is identical to the Proposition 3, except that with imperfect yield management and excess demand with proportional rationing, the secondary market clearing condition is

\[
m_2 (Z_H - t_H) \left(1 - \frac{k - (1 - p_1^\text{Excess}) - S}{m_2 (Z_H - p_2^\text{Excess})}\right) = S,
\]

where \(m_2 (Z_2 - p_2^\text{Excess}) > k - (1 - p_1^\text{Excess})\) implies \(t_H \neq p_2^\text{Excess}\). If \(t_H = p_2^\text{Excess}\), the result fails. So, excess demand is needed. Moreover, starting from no secondary market if the marginal scalper \((c = 0)\) enters, the secondary market still has measure 0, which requires \(t = Z_H\) for it to clear. The rest follows the same argument as before. \(\square\)

**Proof of Proposition 5** Date-1 consumers buy now if they have non-negative utility, and
scalpers do not participate if they have non-positive utility. This requires,

\[ (1 - \alpha) \tilde{v}_1 + \alpha \sum_{j=\{L,H\}} \gamma_j \rho_j^S (t_j - f) \mathbb{1}_{t_j \geq f} - p_1 \geq 0 \geq \sum_{j=\{L,H\}} \gamma_j \rho_j^S (t_j - f) \mathbb{1}_{t_j \geq f} - p_1 \]

\[ \iff \tilde{v}_1 \geq \sum_{j=\{L,H\}} \gamma_j \rho_j^S (t_j - f) \mathbb{1}_{t_j \geq f} \]

which holds by the assumption that date-1 consumers want to trade their tickets only if their preferences changed \((0 \leq t_j - f \leq \tilde{v}_1)\). \(\square\)

**Proof of Proposition 6:** Suppose the monopolist allows for the secondary market. The monopolist ex ante gains from each unit exchanged if \(p_1 - \sum_{j=\{L,H\}} \gamma_j (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} \geq 0\), while under perfect yield management and infinite capacity \(t_j = p_{2,j}\). The no scalping condition becomes \(p_1 - \sum_{j=\{L,H\}} \gamma_j \rho_j^S (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} \geq 0\). Proposition \(5\) showed that it is possible to exclude scalpers. Then, re-arranging the date-1 consumer condition, and the monopolist condition, we have

\[ (1 - \alpha) \tilde{v}_1 \geq p_1 - \alpha \sum_{j=\{L,H\}} \gamma_j \rho_j^S (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} \geq p_1 - \sum_{j=\{L,H\}} \gamma_j (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} \geq 0. \]

Hence, \(\tilde{v}_1 \geq 0\). If \(\tilde{v}_1 < 1\) then we can have a secondary market. Suppose \(\tilde{v}_1 \geq 1\) such that no consumers buys in date-1. Then no secondary market exists date-2. Consider \(v_1 = 1\). We have

\[ (1 - \alpha) + \alpha \sum_{j=\{L,H\}} \gamma_j \rho_j^S (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} - p_1 < 1 - \alpha < (1 - \alpha) \sum_{j=\{L,H\}} \gamma_j \max \{0, 1 - p_{2,j}\}, \]

Deviate to buy now

where \(\alpha \sum_{j=\{L,H\}} \gamma_j \rho_j^S (p_{2,j} - f) \mathbb{1}_{p_{2,j} \geq f} - p_1 \leq 0\) as shown above. Then, \(\sum_{j=\{L,H\}} \gamma_j \max \{0, 1 - p_{2,j}\} > 1\), which is a contradiction. Therefore, \(0 \leq \tilde{v}_1 < 1\), and a secondary market can arise. \(\square\)
References


