Price-Matching Guarantees as a Direct Signal of Low Prices*

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Abstract

If consumers believe that stores offering Price-Matching Guarantees (PMGs) charge low prices, high-search-cost consumers purchase from PMG stores. This leads PMG stores' demand to be less price-sensitive, which drives PMG stores to charge higher prices. The belief that PMG stores charge low prices paradoxically leads them to charge high prices. For this reason, the literature finds that PMGs can only signal low prices when firm heterogeneity is sufficiently large. Because PMGs are offered by retailers who purchase the same product from the same producer, large firm heterogeneity is a strong assumption. The literature predicts that PMGs signal stores' characteristics, such as low marginal-costs or service-quality, which in turn imply that such firms charge low prices. However, experimental findings suggest that consumers use PMGs to draw inferences about prices but not about stores' characteristics. I propose a theory that explains how homogeneous firms may signal their low prices through PMGs: consumers perceive PMG stores to have lower prices not because they expect them to have low marginal-costs or service-quality, but simply because they offer a PMG.

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1 Introduction

A Price-Matching Guarantee (PMG) is a firm's promise to reimburse its consumers if they find a lower price elsewhere. Most PMG promises are advertised together with a statement that the firm offers low prices, such as "in the unlikely event that you find an identical item that you purchased here for a lower price at another store, we promise to refund the difference"[1] and "we are so confident that our prices are the best in the industry we are willing to back them with a price match guarantee".[2]

Empirical evidence finds that, in some markets, PMGs are offered by firms that charge lower prices (e.g. Moorthy and Winter 2006 and Mañez 2006).[3] Moreover, a large body of experimental literature finds that consumers perceive stores that offer PMGs to have lower prices - Jain and Srivastava (2000), Srivastava and Lurie (2001), Biswas et al. (2002), Srivastava and Lurie (2004), Biswas et al. (2006).

Whereas firms change prices frequently, their decision to offer a PMG is more stable. Once a store adopts a PMG policy, it sticks with it for a long period of time. This makes it easier for firms to advertise their PMG policies than to advertise their prices. It is then natural that some consumers may be informed about firms' PMG policies but uninformed about their prices. The aforementioned experimental findings suggest that such consumers infer that PMG stores charge lower prices. Hence, PMGs may act as a signal of low prices, which attracts consumers to stores that offer such promises.

The current models that predict that PMG stores charge low prices (Jain and Srivastava 2000, Moorthy and Winter 2006, and Moorthy and Zhang 2006) make a departure from standard models: they assume that firms are heterogeneous.

[1]This is quoted from Jain and Srivastava (2000). Many stores' descriptions of their PMGs use the words "in the unlikely event".

[2]This is the price matching promise of EnviroTech, described at `http://naturallythebest.com/price-match-guarantee`. Many stores' descriptions of their PMGs start with the words "we are so confident that our prices are the best"

[3]There is also empirical evidence of markets where PMGs are offered by firms that charge higher prices, such as Arbatskaya et al. (2006).

The reason why the current models do not allow for PMGs to signal low prices under firm homogeneity is as follows. If consumers believe that PMG stores charge low prices, high search cost consumers purchase from a PMG store. This leads the demand of PMG stores to be less price sensitive than the demand of non-PMG stores, which drives PMG stores to charge a higher price than non-PMG stores. This is against consumers' belief that PMG stores charge low prices. Hence, the belief that PMG stores charge low prices is not sustainable in a rational expectations equilibrium.

This argument is better understood if we allow for PMGs in the setup of Varian (1980). In his setup, some consumers are informed about prices and shop at the store that offers the lowest price, whereas others are uninformed and can only visit one store. In Varian's model, uninformed consumers purchase from a random store. If we allow for PMGs in this setup, and if consumers believe PMG stores to charge low prices, uninformed consumers will purchase from a PMG store. This then implies that non-PMG stores can only sell to informed consumers, who purchase at the store that lists the lowest price. Non-PMG stores will charge low prices, so that they can sell to informed consumers, whereas PMG stores prefer to charge higher prices to extract a higher surplus from uninformed consumers. But if PMG stores charge higher prices, consumers' belief that PMGs are a signal of low prices are not consistent. The belief that PMG stores charge low prices paradoxically leads them to charge high prices.[4]

Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006) show that firm heterogeneity allows PMGs to act as a signal of low prices because, even in the absence of PMGs, the optimal price that firms want to charge is different. For example, if firms have different marginal costs, the optimal price of a firm with low marginal cost is lower than the optimal price of a firm with high marginal cost. The fact that consumers believe that PMGs are offered by firms with low marginal cost that offer lower prices makes the demand of such stores less price sensitive, so firms with low marginal cost will prefer

[4]I show this result formally in the online appendix.

to charge a higher price (vs the price they would charge if PMGs did not exist). If the difference in marginal costs is high enough, the optimal price for firms with low marginal cost is still lower than the optimal price for firms with high marginal cost. A firm with high marginal cost refrains from offering PMGs because, if it did, it would sell to its consumers at a suboptimal price. In this case, PMGs can signal low prices. If, however, the difference in marginal costs is low, the paradox emerges: if consumers believe that PMG stores charge low prices, firms with low marginal cost will offer PMGs and will optimally charge a price that is higher than the price of firms with high marginal cost that do not offer PMGs. This would be against consumers' expectations and, hence, when the difference in marginal costs is small, PMGs cannot act as a signal of low prices.

The current models in the literature predict that PMGs can only signal low prices when firm heterogeneity is sufficiently large. Moorthy and Winter (2006) assume cost heterogeneity and find that a condition for price-matching as a signal of low prices is that "the cost difference be sufficiently high". Moorthy and Zhang (2006) assume heterogeneity in service levels and find that PMGs signal low prices "when the service differential between the retailers is large enough". Jain and Srivastava (2000) allow for various sources of heterogeneity and find that PMGs signal low prices "when stores are sufficiently asymmetric". Even though firms may be differentiated, it is not easy to quantify whether such differentiation is large enough so that the conditions of the aforementioned models hold. Because PMGs apply to retailers who sell the exact same product and who all purchase from the same producer, large firm heterogeneity may be a strong assumption.

The literature predicts that the signaling role of PMGs crucially depends on firm heterogeneity. PMGs are not a direct signal of prices, they signal prices only indirectly: they are a signal of firms' intrinsic characteristics, such as low marginal cost or low service quality, which in turn imply that such firms charge low prices.

However, experimental findings suggest that consumers use PMGs to draw inferences about prices but not about store characteristics. Jain and Srivastava (2000) conduct studies to understand consumer perceptions of PMG stores and they conclude that "the presence of price-matching policies significantly influences consumer perceptions of store price image", however "consumers do not seem to draw inferences about other aspects of the store [...] on the basis of the presence or absence of a price-matching policy".

The goal of this paper is to provide a model where PMGs are a credible signal of low prices even when firms are homogeneous. In the model proposed here, PMGs act as a direct signal of low prices: consumers perceive PMG stores to have lower prices not because they expect them to have lower costs or low service quality, but simply because they offer a PMG.

Two features of the model in this paper enable PMGs to act as a signal of low prices under firm homogeneity. First, consumers may have different search costs pre and post-purchase. This feature is also present in the model of Jiang et al. (2016). The fact that consumers who have high search cost pre-purchase may have low search cost post-purchase captures the idea that some consumers may be time constrained at the moment of purchase, but may have some free time later to search for a better deal. A PMG allows consumers a grace period in which they can search after purchase. As Jiang et al. (2016) argue, a grace period to exercise the PMG allows consumers to differ search to when their search costs are low.

Second, while some consumers know which firms offer PMGs, others are uninformed about firms' PMG policies. If consumers believe that PMG stores charge low prices, consumers who know which firms offer PMGs go directly to a PMG store, whereas consumers who do not know which firms offer PMGs search at random. For simplicity, I assume that the share of consumers informed about firms' PMG policies is exogenous. However, as I discuss in Section 4, the decision to become informed can be modeled, and the fraction of consumers who choose to learn firms' PMG policies can be endogenously determined.[5]

The existence of these two types of consumers generates a trade-off for firms that offer

[5]The model can also fit other interpretations. For example, it could be assumed that all consumers know which firms offer PMGs, but only some of them are able to choose the first store to visit. The remaining consumers visit a random store (for example, they visit the nearest store). The key assumption is that some consumers can choose to visit a PMG store, whereas others visit a random store.

PMGs. On the one hand, PMGs enable firms to attract consumers who know which firms offer a PMG. Without a PMG, the firm does not sell to these consumers. On the other hand, the purchase decisions of consumers who search at random do not depend on firms' PMG policies. The firm would sell to these consumers even if it did not offer a PMG. By offering a PMG, the firm allows these consumers to search after purchase, and may have to give them refunds in case they find a lower price. The firm would avoid these refunds if it did not offer a PMG.

When the listed price is low, the refunds are also low, and firms benefit from offering PMGs. In particular, the firm that charges the lowest price in the market benefits from offering a PMG, as it allows to attract consumers who know which firms offer PMGs without giving refunds to consumers who search at random. A firm that lists a high price, however, would have to give large refunds to consumers who search at random, and would find it more profitable not to offer PMGs. By not offering PMGs and charging a high price, the firm does not attract consumers who go directly to a PMG store, but extracts a larger surplus from consumers who search at random. The promise to reimburse consumers is, then, a credible signal that the firm offers a low price.

The model predicts that the higher the extent to which consumers search post-purchase, the stronger the role of PMGs as a signal of low prices. This prediction is consistent with experimental evidence from Srivastava and Lurie (2004) who find that "the effectiveness of PMGs as a signal of low store prices depends on individuals' beliefs about the degree to which other consumers in the market engage in price search and enforce PMGs".

In section 3, I allow for firm heterogeneity based on their marginal costs, similar to Moorthy and Winter (2006). I find that the model presented here generates the same results as the previous literature in presence of firm heterogeneity, i.e., firms with low marginal costs offer PMGs more often and charge lower prices. However, contrary to previous literature, in the model presented here firm heterogeneity is not necessary for PMGs to act as a signal of low prices.

1.1 Related Literature

The literature on PMGs is extensive. Besides the signaling purpose of PMGs, other explanations have been proposed, most notably its role as a collusion-facilitating mechanism (Hay 1982, Salop 1986, Doyle 1988, Hviid and Shaffer 1999) and as a price-discrimination device (Png and Hirshleifer 1987, Corts 1996, Chen et al. 2001). Hviid (2010) provides a comprehensive literature review. In this section, I discuss the most recent articles on this topic.

Janssen and Parakhonyak (2013) consider a model where post-purchase consumer search is exogenous. Consumers who purchase from a PMG store may later receive a price quote from another store. Whether or not they receive this information is outside their control. Consumers do not know which firms offer PMGs prior to search, so they visit firms randomly and observe both prices and whether or not a PMG is provided. When a firm offers a PMG, there is a probability that consumers will pay a price lower than the listed price, because they may receive a better price quote afterwards. It follows that PMGs increase consumers' reservation values, which implies that PMG stores charge higher prices than non-PMG stores. Hence, their model is not compatible with the role of PMGs as a signal of low prices. In contrast to Janssen and Parakhonyak (2013), the model presented here endogenizes post-purchase search.

Jiang et al. (2016) develop a model to explain why PMGs are often implemented offline but not online, and why the practice of PMGs vary considerably across retail categories. In their model, consumers differ in their search cost pre and post-purchase, which leads to the existence of three types of consumers: "shoppers", who have zero search costs and go directly to the store with the lowest price; "non-shoppers", who have high search costs and go to the store where they expect prices to be lower (the price expectation depends on whether or not a firm offers PMGs, as consumers know which firms offer them); and "refundees", who have high search cost pre-purchase and zero search cost post-purchase. Refundees purchase at a

PMG-store and search after purchase for a refund. Their model does not allow for PMGs to act as a signal of low prices. If consumers believe PMG stores to offer low prices, both ”non-shoppers” and ”refundees” would purchase from PMG stores. Non-PMG stores would only be able to sell to ”shoppers”, who purchase from the store with the lowest price. This would drive non-PMG stores to charge low prices to attract such consumers, which would be against the belief that PMG stores charge low prices. Instead, in the model of Jiang et al. PMGs act as a signal of high prices: ”non-shoppers” expect PMG stores to charge higher prices and choose to purchase from a store that does not offer PMGs.[6]

Yankelevich and Vaughan (2016) propose a model where all consumers know which firms offer PMGs. Some of them have zero search costs , whereas others have to search sequentially facing a cost for each search they perform. Zero search cost consumers do not necessarily purchase at the firm with the lowest price. Some of them may use a PMG to obtain the lowest price at a firm listing a higher price, if they prefer to shop at that store, as stores may be horizontally differentiated. They find that consumers with high search cost will, in equilibrium, purchase from the first store they visit. In their model, PMGs cannot work as a signal of low prices. If they did, consumers with high search cost, expecting PMG stores to have low prices, would all purchase at a PMG store. PMG stores would sell to all consumers uninformed about prices, whereas non-PMG stores would only sell if they offered the lowest price in the market. This would push non-PMG stores to charge lower prices, which would be against the consumers' belief that PMG stores charge low prices. In their model, the expected price of PMG and non-PMG stores is the same, and high search cost consumers are indifferent between visiting any store, regardless of whether or not they offer a PMG.

Whereas Janssen and Parakhonyak (2013) assume that no consumer knows which firms offer PMGs, Jiang et al. (2016) and Yankelevich and Vaughan (2016) assume exactly the opposite: all consumers know which firms provide a PMG. In either case, PMGs cannot act

[6]When the size of ”refundees” is small, then the expected price of PMG and non-PMG stores is the same and ”non-shoppers” mix between visiting the PMG store and visiting the non-PMG store.

as a signal of low prices. The model presented in this paper is more flexible, and allows for both types of consumers. The coexistence of consumers who know and who do not know which firms offer PMGs is fundamental for PMGs to act as a signal of low prices.

2 Model

Consider a market where n firms compete to supply a homogeneous product. They face the same marginal cost, denoted by c. Firms simultaneously set prices and decide whether or not to provide a Price-Matching Guarantee (PMG).[7]

There is a unit mass of consumers, each demanding at most one unit of the product if their valuation of v is not exceeded. Consumers search stores sequentially and they have perfect recall. To assure full participation, I make the standard assumption that the first search is for free. These assumptions - sequential search, perfect recall, and the first search for free - are common in the consumer search literature (e.g., Stahl 1989, Benabou and Gertner 1993, Kuksov 2004). They have also been used in models that study PMGs, such as Janssen and Parakhonyak (2013), Jiang et al. (2016) and Yankelevich and Vaughan (2016).

Similar to Jiang et al. (2016), I allow for heterogeneity in search costs before and after purchase. A fraction λ of consumers have zero pre-purchase search cost, whereas the remaining consumers have a pre-purchase search cost of $s_H > 0$. Consumers who purchase from a PMG store may search after purchase for a lower price. I assume that the post-purchase search cost is either zero or s_H. A consumer who has a pre-purchase search cost of s_H will have zero post-purchase search cost with probability q.[8] The assumption that consumers are uncertain about their post-purchase search costs is also made by Chen et al. (2005) and Lu and Moorthy (2007) in models to study rebates.

[7] In section 4 I discuss the robustness of the results to an alternative timing, where firms first choose PMG policies, and only choose prices after observing each others' PMG policies.

[8] The post-purchase search cost of consumers who have zero pre-purchase search cost is not relevant, as they search all stores pre-purchase at no cost.

As Jiang et al. (2016) argue, it is realistic to expect that search costs may vary over time and that consumers may have lower post-purchase search costs. If a tire bursts or a TV breaks down, the cost of doing without the product while search is being conducted may be very high. In case of unplanned purchases, these sometimes happen at a moment where the consumer is very time constrained. A PMG allows consumers a grace period in which they can search after purchase. A grace period to exercise the PMG allows consumers to differ search to when their search costs are low. As Lu and Moorthy (2007) argue, a PMG allows consumers to search for several weeks after purchase, and a consumer may experience several draws of low and high search costs during this period. The consumer is able to postpone her search to the nonbusy periods. The probability of having a low post-purchase search cost can be interpreted as the probability of having a low search cost at some point during the grace period of the PMG. Hence, it is not unreasonable that q may be large. In fact, Jiang et al. (2016) assume that some consumers who have high search cost pre-purchase will have zero post-purchase search costs with probability one.

Consumers also differ in the information they have regarding which stores offer PMGs. A fraction ϕ of each consumer-type knows which firms offer PMGs.[9] For simplicity, it is assumed that the fraction of consumers informed about firms' PMG policies is exogenous. However, as I discuss in Section 4, the decision to become informed about firms' PMG policies can be modeled, so that the share of consumers who know which firms offer PMGs is endogenously determined.

Figure 1 depicts consumer types. Notice that whether or not zero search cost consumers know which firms offer PMGs is not relevant because, as they can learn all prices for free, their shopping decisions do not depend on firms' PMG policies. These consumers can go directly to the store that offers the lowest price. I start by analyzing the equilibrium consumer search strategies.

[9]Consumers who do not know which firms offer PMGs hold rational beliefs regarding the probability that each firm offers a PMG.

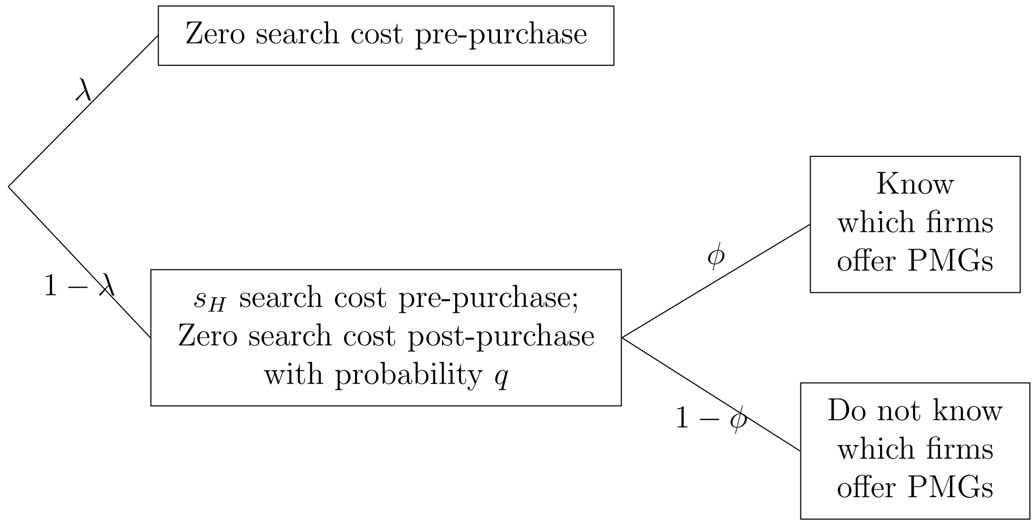

Figure 1: Consumer types

Equilibrium search strategies

Zero search cost consumers

Zero search cost consumers learn all prices at no cost, so they simply purchase the product from the store with the lowest price.

High search cost consumers who know which firms offer PMGs

These consumers hold rational beliefs regarding the price distribution of PMG and non-PMG stores. They visit the store where their expected costs (which include price and search costs) are minimized. Notice that if consumers believe that PMG stores offer lower prices, it is straightforward that these consumers will visit a PMG store. By doing that, consumers will not only be offered a lower price (vs the price of non-PMG stores), but will also have the opportunity to search for a refund after purchase.

High search cost consumers who do not know which firms offer PMGs

As these consumers do not know which firms offer PMGs, they search stores at random. When they visit a PMG store, as long as its price is not higher than v, they always purchase the product. By doing so, they can delay search to after purchase, when they may have zero search costs.

Let $EC_{PMG}(p, i)$ denote the expected total cost (which includes price and search costs) that a consumer incurs, when he purchases from a PMG store, after searching i stores, and the lowest price among those i stores is p. This expected total cost assumes optimal search behavior after purchase, which depends on consumers' post-purchase search costs. Later, I explicitly characterize the optimal post-purchase search strategy.

Suppose that the first (random) store that a high search-cost consumer visits does not offer a PMG, and charges price z. The consumer will search one more store if the benefits of doing so are higher than his search cost, s_H.

Let α denote the equilibrium probability that a store offers a PMG, and let F_{PMG} and F_{NO} denote, respectively, the equilibrium price distributions of PMG and non-PMG stores. Finally, let $F \equiv \alpha F_{PMG}+(1-\alpha)F_{NO}$ denote the equilibrium price distribution, unconditional on firms' PMG policies, and let $\underline{p}$ and $\overline{p}$ denote the lower and upper bound of its support. If a consumer decides to search a second store, after receiving price quote z from a non-PMG store, the expected total cost he incurs is

$$\alpha \int_{\underline{p}}^{\overline{p}} EC_{PMG}\big(min\{x, z\}, 2\big) dF_{PMG}(x) + (1-\alpha) \int_{\underline{p}}^{\overline{p}} min\{x, z\} + s_H dF_{NO}(x)$$

If the consumer purchases at the first store he visits, he pays the listed price z. There exists a unique z such that the expected total cost incurred when searching a second store is equal to z. I denote this unique price threshold as z^{NO}. When a consumer is offered price quote z^{NO} from a non-PMG store, he is indifferent between purchasing or searching one more store. z^{NO} is then the consumer reservation value for non-PMG stores, i.e., he will purchase when he visits a non-PMG store as long as its price is not higher than z^{NO}.

Let $\bar{p}_{NO}$ and $\bar{p}_{PMG}$ denote, respectively, the upper bounds of F_{NO} and F_{PMG}.

Lemma 1 $\bar{p}_{NO} \leq z^{NO}$

Proof. Consider a non-PMG store that charges $\bar{p}_{NO}$. If $\bar{p}_{NO} > z^{NO}$, then consumers with high search cost who do not know which firms offer PMGs do not purchase from this store. This then implies that high search cost consumers who know which firms offer PMGs also do not purchase from this store, even if they believe that non-PMG stores charge lower prices. Once they enter the store and learn its price, these consumers face the same situation: they can either purchase at price $\bar{p}_{NO}$ or search another store.[10] It follows that if consumers with high search cost who do not know which firms offer PMGs do not purchase from this firm, the firm can only sell to zero search cost consumers (in case it has the lowest price in the market). But then a firm that charges price $\bar{p}_{NO}$ would prefer to offer a PMG, as it would be able to sell not only to zero search cost consumers (in case it has the lowest price in the market), but also to high search cost consumers. This contradicts that $\bar{p}_{NO}$ is in the support of F_{NO}. $\square$

Lemma 1 states that, in equilibrium, non-PMG stores never charge a price higher than consumers' reservation value. This implies that, in equilibrium, consumers purchase in case the first (random) store they visit does not offer a PMG. As it was already discussed, consumers also purchase in case the first store they visit offers a PMG. This leads to the following corollary.

Corollary 1 *In equilibrium, high search cost consumers who do not know which firms offer PMGs purchase at the first random store they visit*

I now characterize consumers' optimal post-purchase search strategy. When they purchase from a non-PMG store, there is no scope for post-purchase search, as consumers cannot

[10]Consumers who know which firms offer PMGs even have an advantage, as they can go directly to another store where they expect prices to be lower.

get refunds. If, however, consumers purchase from a PMG store, their optimal post-purchase search strategy depends on their post-purchase search costs.

Trivially, consumers with zero search cost post-purchase optimally search all stores and get a refund for the difference between the price they paid and the lowest price in the market.

When they have high search cost post-purchase, consumers search one more store when the benefits of doing so are higher than their search cost, s_H. In particular, it follows from Kohn and Shavell (1974) that consumers follow a cutoff rule, such that they will stop searching either when they find a price lower than some threshold, which I denote by z^{PMG}, or when they exhaust all stores. z^{PMG} is such that the expected benefit of searching one more store is exactly equal to s_H, i.e.:

$$\int_{\underline{p}}^{z^{PMG}} z^{PMG} - xdF(x) = s_H$$

Lemma 2 $z^{NO} \leq z^{PMG}$

The intuition behind Lemma 2 is that the benefit of searching pre-purchase, after having a price quote from a non-PMG store, is higher than that of searching after purchasing from a PMG store. This follows because, pre-purchase, consumers still have a probability q of having zero search cost post-purchase. Hence, when consumers are searching pre-purchase after getting a price quote from a non-PMG store, they take into account that performing one more search not only may allow them to find a lower price, but also they can find a PMG store, in which case they may be able to search post-purchase at no cost. In contrast, consumers who have high search cost post-purchase search only to find a lower price.

Lemma 3 $\bar{p}_{PMG} \leq z^{PMG}$

To understand the intuition behind Lemma 3 suppose, by contradiction, that $\bar{p}_{PMG} > z^{PMG}$. It then follows from Lemmas 1 and 2 that $\bar{p}_{NO} \leq z^{NO} \leq z^{PMG} < \bar{p}_{PMG}$. It follows that no firm charges a price higher than $\bar{p}_{PMG}$. Hence, a PMG firm that charges $\bar{p}_{PMG}$ never sells at its listed price, because even consumers who have high search cost post-purchase search to get a refund. Because the firm never sells at $\bar{p}_{PMG}$, it benefits from reducing its price slightly. Such price reduction does not decrease the selling price and increases the likelihood that the firm will be the lowest price firm that sells to zero search cost consumers.[11] This contradicts that $\bar{p}_{PMG}$ is in the support of F_{PMG}.

By definition of z^{PMG}, consumers with high search cost post-purchase will not search for a refund if they purchase from a PMG firm that charges a price lower than z^{PMG}. Lemma 3 states that all PMG firms charge a price lower than z^{PMG}. It then follows that consumers with high search cost post-purchase will not search for refunds. The following corollary summarizes consumers' equilibrium post-purchase search strategy.

Corollary 2 *In equilibrium, high search cost consumers who purchase from a PMG store search after purchase if and only if they have zero search cost post-purchase.*

Table 1 summarizes consumers' purchasing strategies.

Pre-purchase search cost	Know firms' PMG policies	Store where they purchase
0	-	Lowest-price store
s_H	Yes	Random PMG store
s_H	No	Random store

Table 1: Consumer search strategies

Note: Whether consumers with high search cost pre-purchase who know which firms offer PMGs visit a PMG store or a non-PMG store depends on their price beliefs. In this table, it is assumed that consumers believe PMG stores to have low prices. This is the assumption maintained throughout the paper.

[11] To be accurate, there is a reduction in the selling price, because if all firms are charging prices between $\bar{p}_{PMG}$ and the slightly lower price that the firm sets, the minimum price in the market decreases. However, this effect is of a lower order of magnitude.

Equilibrium firm strategies

The equilibrium concept I use in this paper is a rational expectations equilibrium, in which consumers' expectations of prices for PMG and non-PMG stores are correct. Because I am interested to show how PMGs can act as a signal of low prices, throughout this paper I focus on equilibria under which consumers believe PMG stores to charge lower prices.

The existence of zero search cost consumers implies that there is price dispersion in equilibrium (e.g., Varian 1980, Narasimhan 1988). I now analyze for what set of prices it is beneficial for firms to offer a PMG.

Consider a firm that is deciding on its pricing strategy. Let k denote the number of the remaining firms offering PMGs. Because PMGs and prices are chosen at the same time, when the firm is making pricing decisions it does not know k, it only knows the equilibrium probability that each store offers PMGs. This induces a probability distribution over k, which I denote by g.

Suppose the firm chooses price p and does not offer a PMG. The firm will sell to zero search cost consumers in case it has the lowest market price. The probability that the firm has the lowest price in the market depends not only on the firm's price, p, but also on the number of remaining firms offering PMGs, k. I denote by $L_k(p)$ the probability that the firm has the lowest price in the market.[12] The firm will also sell to high search cost consumers who do not know which firms offer PMGs and enter its store. Because those consumers search at random, the measure of such consumers who visit the firm is $\frac{(1-\lambda)(1-\phi)}{n}$. The firm will sell to consumers who know which firms offer PMGs only if no store offers a PMG, i.e., if $k = 0$. The expected profit of a firm that charges price p and does not offer a PMG is

$$\pi_{NO}(p) = \sum_{k=1}^{n-1} g(k) \left[\lambda L_k(p)(p-c) + \tfrac{(1-\lambda)(1-\phi)}{n}(p-c) \right] + g(0) \left[\lambda L_0(p)(p-c) + \tfrac{1-\lambda}{n}(p-c) \right]$$

[12] $L_k(p) \equiv [1 - F_{PMG}(p)]^k [1 - F_{NO}(p)]^{n-1-k}$

Now suppose the firm chooses price p and offers a PMG. The firm will sell to zero search cost consumers in case it has the lowest market price, which happens with probability $L_k(p)$. The firm also sells to high search cost consumers who know which firms offer PMGs and enter its store. Those consumers visit a PMG store at random, so the measure of such consumers who visit the firm is $\frac{(1-\lambda)\phi}{k+1}$. Finally, the firm also sells to high search cost consumers who do not know which firms offer PMGs and enter its store. Because those consumers search at random, the measure of such consumers who visit the firm is $\frac{(1-\lambda)(1-\phi)}{n}$. High search cost consumers who purchase from the firm pay the listed price in case they have high post-purchase search cost (which happens with probability $1-q$) and pay the lowest market price in case they have zero post-purchase search cost (which happens with probability q). Let $Emin(p/k)$ denote the expected lowest market price, when the firm charges p and k remaining stores offer PMGs.[13] The expected profit of a firm that charges price p and offers a PMG is

$$\pi_{PMG}(p) = \sum_{k=0}^{n-1} g(k)\left[\lambda L_k(p)(p-c) + \left[\tfrac{(1-\lambda)(1-\phi)}{n} + \tfrac{(1-\lambda)\phi}{k+1}\right]\Big(qEmin(p/k) + (1-q)p - c\Big)\right]$$

It follows that

$$\pi_{PMG}(p) - \pi_{NO}(p) = \sum_{k=1}^{n-1} g(k)\Bigg\{\underbrace{\frac{(1-\lambda)\phi}{k+1}\big(qEmin(p/k)+(1-q)p-c\big)}_{\text{Benefit of offering a PMG}} - \underbrace{\frac{(1-\lambda)(1-\phi)q}{n}[p-Emin(p/k)]}_{\text{Cost of offering a PMG}}\Bigg\}$$
$$+g(0)\Bigg\{\underbrace{(1-\lambda)\phi\frac{n-1}{n}\big(qEmin(p/0)+(1-q)p-c\big)}_{\text{Benefit of offering a PMG}} - \underbrace{\frac{(1-\lambda)q}{n}[p-Emin(p/0)]}_{\text{Cost of offering a PMG}}\Bigg\}$$

Offering a PMG presents a trade-off. On the one hand, a firm that offers a PMG attracts high search cost consumers who know which firms offer PMGs. Selling to these consumers

[13] $Emin(p/k) = \int_0^p \big\{(n-k-1)[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^k f_{NO}(x) + k[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1} f_{PMG}(x)\big\} x dx + [1-F_{NO}(p)]^{n-k-1}[1-F_{PMG}(p)]^k p$

constitutes the upside of offering a PMG.

On the other hand, a firm that offers a PMG may have to give refunds to high search cost consumers who do not know which firms offer PMGs. These consumers purchase at the first random store they visit, so their purchasing decision does not depend on firms' PMG policies. The firm would sell to these consumers even if it did not offer a PMG. However, if the firm offers a PMG, it will have to give refunds to those who search post-purchase. The refunds given to such consumers are the downside of offering a PMG.

Both the benefit and the cost of offering a PMG are increasing in the firm's listed price. However, as $Emin(p/k)$ is concave[14], it follows that the benefit of offering a PMG is concave whereas the cost of offering a PMG is convex. Figure 2 illustrates how the benefits and costs of offering a PMG depend on the firm's listed price.

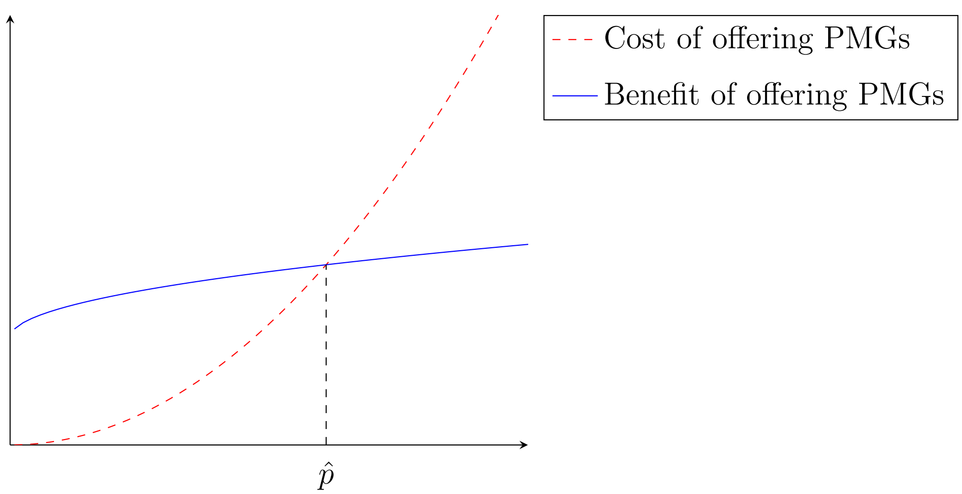

Figure 2: Cost and benefit of offering a PMG

The intuition behind why the cost of offering a PMG is more sensitive to the firm's price than is the benefit of offering the PMG policy is better understood if we consider the extreme case where $q = 1$, i.e., all consumers search post-purchase when they buy from a PMG store. In this case, a firm that offers a PMG attracts consumers who know which firms offer PMGs, but these consumers pay the lowest price in the market. This benefit of offering a PMG is

[14] $\frac{\partial^2 Emin(p/k)}{\partial p^2} = -(n-k-1)[1-F_{NO}(p)]^{n-k-2}[1-F_{PMG}(p)]^k f_{NO}(p) - k[1-F_{NO}(p)]^{n-k-1}[1-F_{PMG}(p)]^{k-1} f_{PMG}(p) < 0.$

almost independent of the firm's listed price.[15] On the other hand, the refunds given to consumers who search at random are the difference between the firm's listed price and the lowest price in the market. Hence, the refunds increase significantly with the firm's listed price.

It follows that the benefit from offering a PMG is higher than its cost only for firms that charge low prices. The following proposition characterizes the equilibrium firm strategies. As firms are identical, I focus on symmetric equilibrium. Let $[\underline{p}, \overline{p}]$ denote the support of the equilibrium price distribution.

Proposition 1 *In equilibrium, firms play mixed strategies over prices. There exists a price threshold, $\hat{p}$, such that firms offer a PMG only when they choose a price below $\hat{p}$. If $\phi \in (0, 1)$ and q is large then $\underline{p} < \hat{p} < \overline{p}$.*

If $\phi \in (0, 1)$ and $q = 0$ it is straightforward to see that $\pi_{PMG}(p) - \pi_{NO}(p) > 0$ and, hence, all firms offer PMGs for any price they choose (in this case, $\hat{p} \geq \overline{p}$). This follows because if $q = 0$ consumers who search at random never come back to collect refunds and therefore there is no downside of offering the PMG policy. When q is large enough, firms that charge high prices find that, if they offer a PMG, the refunds that they must give to consumers who search at random outweigh the profit increase from attracting consumers who know which firms offer PMGs. It follows that, when q is large enough, both PMG and non-PMG stores coexist and PMG stores charge lower prices than non-PMG stores.

The equilibrium is, indeed, a rational expectations equilibrium. Consumers believe that PMG stores charge lower prices and PMG stores find it optimal to charge lower prices. PMGs act as a signal of low prices even though firms are homogeneous. This is in contrast with existing literature that predicts that PMGs can only signal low prices under firm heterogeneity.

In order for PMGs to act as a signal of low prices, the model introduces two features: i)

[15]The benefit is still increasing in the firm's price, because there is a probability that the firm has the lowest price in the market. However, as the firm's price increases, such probability decreases.

consumers with high search cost pre-purchase may have low search cost post-purchase; ii) some consumers are able to go directly to a PMG store and others search at random. The first feature is present when $q > 0$ and the second feature is present when $0 < \phi < 1$.

Lemma 4 *If $q = 0$ or $\phi = 1$ all firms offer a PMG with probability one. If $\phi = 0$ all firms offer PMGs with probability zero.*

Lemma 4 shows that, standing alone, none of the two features allows for PMGs to act as a signal of low prices. Indeed, if we do not allow for consumers with high search cost pre-purchase to have low search cost post-purchase (by setting $q = 0$) the model falls into the same result as the models of Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006) under firm homogeneity, i.e., all firms offer PMGs. If we shutdown the coexistence of consumers who search at random and consumers who can go directly to a PMG store (by setting $\phi = 0$ or $\phi = 1$), the model predicts that either all firms offer PMGs or no firm offers such policy. When all firms choose the same PMG strategy, PMGs do not convey any information about firms' prices. In order for PMGs to act as a signal of low prices under firm homogeneity, both features are necessary.

Whether PMGs can be used to signal low prices depends on the extent to which consumers are informed about firms' PMG policies - measured by ϕ - and on the extent to which consumers search post-purchase - measured by q.

As stated in Proposition 1, there exists a price threshold, $\hat{p}$, such that firms offer PMGs only if they charge a price lower than $\hat{p}$. Suppose that, in equilibrium, a fraction α of stores offer PMGs. Consumers are rational and anticipate that PMG stores charge prices lower than $\hat{p}$ whereas non-PMG stores charge prices higher than $\hat{p}$. A firm that offers PMGs is signaling to consumers that it is among the fraction α of firms that charge lower prices. The lower the equilibrium probability of a firm offering PMGs, the higher the signaling role of this policy. For example, if half of the firms offer PMGs, consumers infer that the price of a PMG firm is below the median. If only one quarter of the firms offer PMGs, consumers infer that the price of a PMG firm is in the first quartile.

Lemma 5 *The equilibrium probability that a store offers a PMG is increasing in ϕ and decreasing in q*

The effect of ϕ on the signaling role of PMGs is ambiguous. The benefit for a firm to offer PMGs is that it can attract consumers who know which firms offer PMGs. The drawback is that the firm has to give refunds to high search cost consumers who do not know which firms offer PMGs. The lower ϕ, the lower the share of consumers that a firm can attract by offering a PMG. When ϕ is low, because the benefit of offering PMGs is small, firms are only willing to offer PMGs in case the expected value of the refunds is also small, which happens when the firm lists a low price. It follows that, as ϕ decreases, PMGs become a better signal of firms' prices. However, ϕ measures the extent to which consumers take into account PMGs before deciding where to purchase. Hence, when ϕ is small PMGs are a good signal of firms' prices, but few consumers are able to use such signal.

In the other extreme case, $\phi = 1$, all high search cost consumers know which firms offer PMGs, and visit one of those stores. Because there are no consumers who search at random, there is no longer a downside of offering PMGs, so all firms offer such policy, regardless of their price. Because all firms offer PMGs, they are no longer a signal of firms' prices. In this case, all consumers are able to decide where to search based on firms' PMG policies, but PMGs are no longer a signal of low prices.

The signaling role of PMGs is strongly related to the extent to which consumers search post-purchase. As it was already discussed, if consumers never search post-purchase ($q = 0$) there is no downside of offering PMGs, because the firm never gives refunds. Hence, all firms offer PMGs, regardless of the price they charge. But, in that case, PMGs do not convey any information regarding firms' prices.

When q is high and a firm offers a PMG, it communicates to its consumers the following message: there are some consumers who purchase at a random store, because they have high search costs and do not know which firms offer PMGs; the firm is willing to give those consumers a refund in case they find a lower price after purchase, even though the firm knows that they will actually search for a lower price, because they are likely to have zero search costs post-purchase. If the firm's price was high, it would not be profitable to give refunds to such consumers. Hence, a PMG is a credible signal that the firm offers a low price.

The higher the extent to which consumers search post-purchase (measured by q), the stronger the role of PMGs as a signal of low prices. This explanation for why PMGs signal low prices is consistent with experimental evidence from Srivastava and Lurie (2004), who find that the effectiveness of PMGs as a signal of low store prices depends on individuals' beliefs about the degree to which other consumers in the market engage in price search and enforce PMGs.

Because PMGs are indeed offered by firms that list low prices, consumers who purchase from PMG stores and search after purchase are unlikely to find a lower price. Hence, PMGs are not frequently redeemed in equilibrium. This prediction is consistent with a survey conducted by Moorthy and Winter (2006) who find that the average redemption rate is around 5%. A natural question is, then, why would consumers search after purchase if they are unlikely to find a lower price? In the model presented here, consumers may have zero post-purchase search costs, so they would search even if the benefits of doing so are marginal. However, as I discuss in Section 4, the results of the model are robust if, instead, the low post-purchase search cost is strictly positive.

3 Heterogeneous Costs

I have assumed that firms are homogeneous in their production cost. This assumption is reasonable, because retailers - not producers - are the ones offering PMGs, so it is plausible that, as they all purchase the product from the same producer, they are paying the same price for it. However, as Moorthy and Winter (2006) point out, in some markets there may be firms that have a higher bargaining power and can purchase the product at a lower price. They find that, when that is the case, firms that have low marginal costs offer PMGs and charge low prices. In this section, I show that the model presented here generates the same results as the previous literature in presence of firm heterogeneity.

Similarly to Moorthy and Winter (2006), I assume that firms face simultaneous, independent draws on unit costs of production: c_L with probability γ and c_H with probability $1 - \gamma$, where $c_L < c_H$.

After privately observing their marginal cost, firms set prices and PMG policies simultaneously. I focus on symmetric equilibrium, in which firms with the same marginal cost play the same strategy.

Proposition 2 *In a symmetric equilibrium, there exists $\underline{p} < \tilde{p} < \overline{p}$ such that firms with marginal cost c_L play prices in $[\underline{p}, \tilde{p}]$ and firms with marginal cost c_H play prices in $[\tilde{p}, \overline{p}]$. In equilibrium, firms with marginal cost c_L offer a PMG with strictly positive probability. If firms with marginal cost c_H offer a PMG with positive probability, then firms with marginal cost c_L offer a PMG with probability one.*

Firms with low marginal cost have a higher margin and are more willing to reduce their price to generate a higher demand. The existence of PMGs does not revert the standard result that firms' prices are increasing in their production cost (e.g., MacMinn 1980, Spulber 1995).

Regarding the incentives to offer a PMG, the intuition is the same as in the model with homogeneous firms. By offering a PMG, a firm is able to sell to more consumers. Notice that firms with cost c_L benefit more from attracting consumers through offering a PMG than do firms with cost c_H, as they have a higher profit margin.

The cost of offering a PMG is the expected value of the refunds that the firm gives to high search cost consumers who do not know which firms offer PMGs. These consumers would purchase from the firm even if it did not offer a PMG. Refunds depend on posted prices only, and not on firms' marginal costs. Hence, the expected value of the refunds is the same for both types of firms.

As in the model with homogeneous firms, there is a price threshold such that firms offer a PMG only if they choose a price lower than the threshold. The two types of firms have different thresholds. Because the cost of offering a PMG is the same for both firms and the benefit of offering it is greater for firms with cost c_L, the threshold of firms with cost c_L is higher. Figure 3 illustrates this point. I denote by $\hat{p}_L$ and $\hat{p}_H$ the threshold of firms with marginal cost c_L and c_H, respectively.

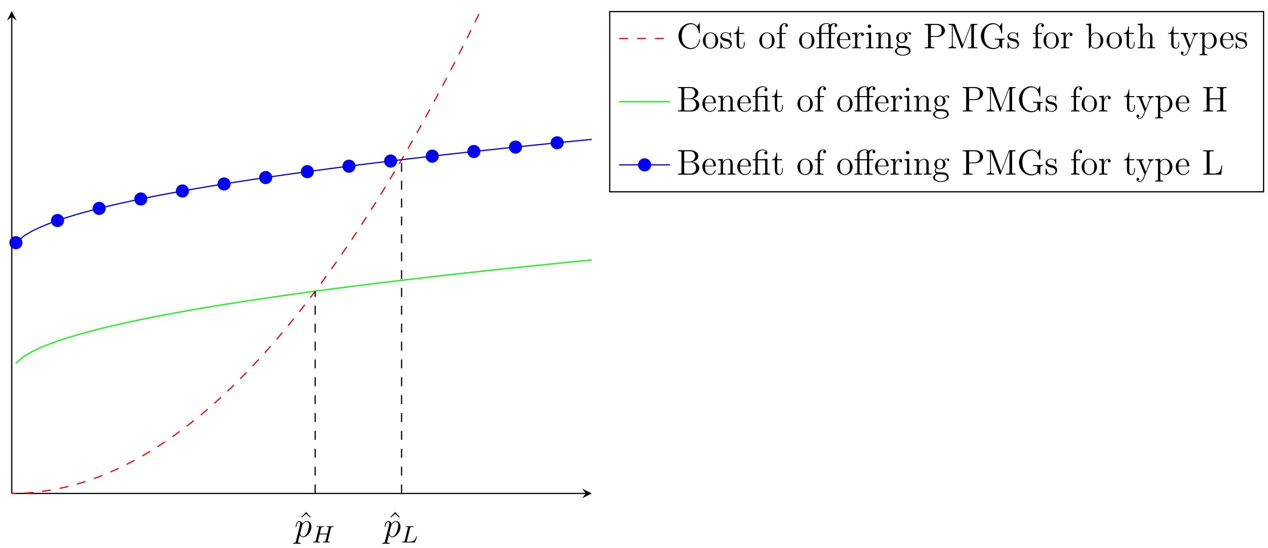

Figure 3: Incentives to offer PMGs for both types

Not only do firms with cost c_L offer lower prices, they also have a higher price threshold for offering the PMG policy. If firms with marginal cost c_H offer a PMG with positive probability, it must be that $\hat{p}_H > \tilde{p}$. Because $\hat{p}_L > \hat{p}_H$, it then follows that $\hat{p}_L > \tilde{p}$. It then follows that firms with marginal cost c_L always offer a PMG.

Summing up, firms with low marginal cost charge lower prices and offer PMGs more frequently. Consumers believe PMG stores to charge low prices, and their expectation is correct. When firms are heterogeneous, the predictions of this model are similar to those of Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006).

However, contrary to existing literature, in this model firm heterogeneity is not necessary for PMGs to act as a signal of low prices.

4 Robustness

4.1 Endogenous decision to learn firms' PMG policies

In order for consumers to rely on PMGs as a signal of firms' prices, they must be uninformed about firms' prices but informed about their PMG policies. The rationale behind this assumption is that a PMG policy is a binary variable: either a firm offers a PMG or it does not. Moreover, the decision to offer a PMG does not change frequently: once a store adopts a PMG policy, it sticks with that policy for a long period of time. In contrast, a retailer carries thousands of products, with corresponding number of prices. As a further matter, firms change prices frequently. This makes it easier for firms to advertise their PMG policies than to advertise their prices.

The literature that explains PMGs as a signal device assumes that all consumers are informed about firms' PMG policies (Jain and Srivastava 2000, Moorthy and Winter 2006, Moorthy and Zhang 2006). This seems too strong an assumption. Even though firms may advertise their PMG policies, consumers who which to learn them still need to incur search costs. Consumer heterogeneity may lead only some of them to choose to become informed. For example, consumers who purchase the product frequently may find it beneficial to become informed, but it may not be worthwhile for occasional buyers to learn which firms offer a PMG.

In the model presented in this paper, I allow for the coexistence of consumers who are informed and uninformed regarding firms' PMG policies. For simplicity, I assume that the fraction of informed consumers is exogenous. However, the decision to become informed about firms' PMG policies can be modeled in a similar way that Varian (1980) models the decision to become informed about prices.

Suppose consumers are heterogeneous in their cost of becoming informed. Let P_i denote the expected price paid by a consumer who is informed about which firms offer PMGs, and let P_u denote the average price paid by consumers who are uninformed about firms' PMG policies. A consumer who becomes informed about firms' PMG policies saves, on average, $P_u - P_i$. Consumers whose cost of becoming informed is lower than $P_u - P_i$ will choose to learn which firms offer PMGs and will go directly to one of those stores to purchase the product. Consumers whose cost of becoming informed is higher than $P_u - P_i$ will prefer to remain uninformed regarding firms' PMG policies and will visit a firm at random.

4.2 Sequential choice of PMG and price

The literature is not consensual regarding the timing that firms use to choose prices and PMG policies. Whereas Png and Hirshleifer (1987), Jain and Srivastava (2000), and Janssen and Parakhonyak (2013) use the same timing as the model in this paper - firms choose PMGs and prices simultaneously -, Moorthy and Winter (2006), Yankelevich and Vaughan (2016), and Jiang et al. (2016) assume that firms first decide on whether or not to offer a PMG, and only choose prices after observing which firms offer PMGs.

The argument to model the decision of PMG policies and prices as sequential is that prices are more flexible tha PMGs, and usually firms stick to their PMG policy for a long period of time, even though they change prices frequently.

Modeling the game as a sequential decision, however, comes with a tractability cost. Firms' prices will depend on the PMG policies of all firms. This implies that, in the pricing stage, there are many subgames, depending on which firms offer PMGs. The models that use this timing overcome the tractability issue by restricting the analysis to a duopoly market.

I make the assumption that firms choose PMG policies and prices simultaneously for tractability reasons, so that I can analyze a general oligopoly model. However, the results presented here are robust to the different timings. In the model presented in this paper, when firms choose prices they do not know the PMG policies of the other firms, but they hold rational beliefs regarding the probability that each firm offers a PMG. Such belief generates a probability distribution over the number of stores that offer PMGs (which I denote by k). Firms choose prices taking into account the probability distribution over k.

If, instead, PMG policies were chosen before prices, firms would observe the actual realization of k before choosing prices. Instead of choosing prices based on a (correct) belief on the distribution of k, firms would choose prices based on the actual realization of k. The underlying intuition of the model and the main results do not change.

In the online appendix I analyze a duopoly model where firms choose PMG policies first and choose prices only after observing each others' PMG policies. It is shown that PMGs also act as a signal of low prices under this alternative timing.

4.3 Strictly positive low post-purchase search costs

Because PMGs are indeed offered by retailers that list low prices, refunds are not frequently redeemed in equilibrium. A consumer who purchases from a PMG store and searches post-purchase is unlikely to find a lower price elsewhere. In the model presented here, consumers may have zero post-purchase search cost, in which case they do not mind searching for a lower price, even if they are very unlikely to find it. In this section, I discuss how the main predictions of the model are robust to an alternative setting, in which post-purchase search costs may be low, but are strictly positive.

In case consumers have a strictly positive post-purchase search cost, they search after purchase only if the benefits of doing so are higher than their search cost. This happens when the price they paid was high. If the price they paid was low, consumers are unlikely to find an even lower price and, even if they do, the refund they will get will be small. Hence, if there is a low but strictly positive post-purchase search cost, consumers optimal post-purchase search strategy will depend on the price they paid. Consumers will search after purchase when they have low post-purchase search cost in case the price they paid was higher than some threshold. A firm that offers a PMG and charges a price lower than such threshold knows that its consumers never search after purchase, even if they have low search costs post-purchase. However, the firm also anticipates that, should it charge a higher price, its consumers would start searching after purchase.

The trade-off that offering a PMG presents to firms is the same as in the model in Section 2. A PMG attracts consumers who know which firms offer PMGs but, if the firm's price is high, consumers will search after purchase and will collect refunds. These refunds will be collected not only by consumers informed about firms' PMG policies (who would not have purchased in the absence of a PMG), but also by consumers who do not know which firms offer PMGs (and would have purchased even if the firm did not offer a PMG).

5 Conclusion

When consumers believe that PMG stores charge lower prices, high search cost consumers purchase at a PMG store. This makes the demand of PMG firms less price sensitive and leads them to charge high prices. Clearly, this cannot happen in a rational expectations equilibrium. There is an apparent paradox in the role of PMGs as a signal of low prices.

In this paper, I propose a model under which PMGs can act as a direct signal of low prices. This is accomplished by adding two features to the standard search models: i) consumers with high pre-purchase search cost may have low post-purchase search cost; ii) consumers are heterogeneous in their cost of learning which firms offer PMGs, so that only a fraction of consumers will become informed about firms' PMG policies.

In this paper, I have focused on Price-Matching Guarantees. However, as Arbatskaya et al. (2004) point out, these promises sometimes take the form of Price-Beating Guarantees, where firms refund more than the difference between the listed price and a lower price found elsewhere. The more common forms of Price-Beating Guarantees are a percentage of the difference (e.g. refund 120% of the difference) and the difference plus a fixed amount (e.g. refund the difference plus $10). In the model presented in this paper, the downside of offering a PMG is measured by the refunds that firms have to give to consumers who do not know which firms offer PMGs and, by chance, end up visiting a PMG store. When firms offer Price-Beating Guarantees, the refunds are larger. Therefore, the firm is only willing to offer such policy if it is likely that, indeed, consumers will not find a lower price elsewhere. Hence, Price-Beating Guarantees may send an even stronger signal of low prices. A complete discussion of the role of Price-Beating Guarantees on equilibrium prices implies solving a model with a larger strategy space, where firms can choose to offer Price-Beating Guarantees. Because this model does not allow for such strategies, the proof of the conjecture that Price-Beating Guarantees may provide an even stronger signal of low prices remains an open question.[16]

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[16] Moorthy and Winter (2006) and Moorthy and Zhang (2006) make a similar conjecture

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A Appendix - Omitted Proofs

Proof of Lemma 2

By definition of EC_{PMG}, it follows that $EC_{PMG}\Big(min\{z^{NO},x\},2\Big) \leq min\{z^{NO},x\} + s_H$. Hence

$$z^{NO} = \alpha \int_{\underline{p}}^{\bar{p}} EC_{PMG}\Big(min\{z^{NO},x\},2\Big) dF_{PMG}(x) + (1-\alpha) \int_{\underline{p}}^{\bar{p}} min\{z^{NO},x\} + s_H dF_{NO}(x)$$
$$\leq \int_{\underline{p}}^{\bar{p}} min\{z^{NO},x\} + s_H dF(x)$$

$$z^{NO} \leq \int_{\underline{p}}^{\bar{p}} min\{z^{NO},x\} + s_H dF(x) \iff s_H \geq \int_{\underline{p}}^{z^{NO}} z^{NO} - x dF(x)$$

It follows that $s_H = \int_{\underline{p}}^{z^{PMG}} z^{PMG} - x dF(x) \geq \int_{\underline{p}}^{z^{NO}} z^{NO} - x dF(x) \iff z^{PMG} \geq z^{NO}$

Proof of Lemma 3

Suppose, by contradiction, that $\bar{p}_{PMG} > z^{PMG}$. It then follows from Lemmas 1 and 2 that $\bar{p}_{NO} \leq z^{NO} \leq z^{PMG} < \bar{p}_{PMG}$. Consider a firm that offers PMGs and charges price $\bar{p}_{PMG}$. All consumers who purchase from such store will search after purchase. It follows from Kohn and Shavell (1974) that they will follow a cutoff rule, such that they will stop searching either when they find a price lower than some threshold, which I denote by τ, or when they exhaust all stores. Let $\varphi(p/k)$ denote the expected price that consumers pay when they follow the cutoff search strategy, given that the firm's price is p and exactly $k+1$ firms (including the first firm they searched) offer PMGs.

I will show that the firm that offers a PMG and charges price $\bar{p}_{PMG}$ can increase its profit by reducing its price. At the time that the firm is choosing its price and PMG policy, the firm does not know how many firms will offer PMGs, it only knows the probability that each firm will offer a PMG. I will show that, for any realization k of the remaining $n-1$ stores offering PMGs, the firm can improve its profit by reducing its price, which implies that it will also want to reduce its price ex-ante.

I denote by $Emin(p/k)$ the expected minimum price in the market, given that exactly $k+1$ firms offer PMGs and one of those firms charges p. Let F_{PMG} and F_{NO} denote the price distributions of PMG and non-PMG firms, respectively.

Claim 1 $Emin(\bar{p}_{PMG}/k) - Emin(\bar{p}_{PMG}-\epsilon/k) \leq [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}\epsilon$

Proof. First notice that

$Emin(p/k) = [1-F_{NO}(p)]^{n-k-1}[1-F_{PMG}(p)]^{k}p +$

$+\int_0^p \{(n-k-1)[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^{k}f_{NO}(x) + k[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1}f_{PMG}(x)\}xdx$

It then follows that

$Emin(\bar{p}_{PMG}/k) - Emin(\bar{p}_{PMG}-\epsilon/k) = -[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}(\bar{p}_{PMG}-\epsilon)+$

$+\int_{\bar{p}_{PMG}-\epsilon}^{\bar{p}_{PMG}} \{(n-k-1)[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^{k}f_{NO}(x) + k[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1}f_{PMG}(x)\}xdx$

$\leq -[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}(\bar{p}_{PMG}-\epsilon)$

$+\int_{\bar{p}_{PMG}-\epsilon}^{\bar{p}_{PMG}} \{(n-k-1)[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^{k}f_{NO}(x) + k[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1}f_{PMG}(x)\}\overline{p}dx$

$= [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}\bar{p}_{PMG} - [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}(\bar{p}_{PMG}-\epsilon)$

$= [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}\epsilon$ □

Claim 2 $\varphi(\bar{p}_{PMG}/k) - \varphi(\bar{p}_{PMG}-\epsilon/k) \leq [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^{k}\epsilon$

Proof. First notice that

$\varphi(p/k) = \Big[\int_\tau^p \{(n-k-1)\frac{[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^{k}}{[1-F(\tau)]^{n-1}}f_{NO}(x) + k\frac{[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1}}{[1-F(\tau)]^{n-1}}f_{PMG}(x)\}xdx+$

$\frac{[1-F_{NO}(p)]^{n-k-1}[1-F_{PMG}(p)]^k}{[1-F(\tau)]^{n-1}}p\Bigg] + \big[1-[1-F(\tau)]^{n-1}\big]E(p/p<\tau)$

It then follows that

$\varphi(\bar{p}_{PMG}/k)-\varphi(\bar{p}_{PMG}-\epsilon/k) = \int_{\bar{p}_{PMG}-\epsilon}^{\bar{p}_{PMG}}\big\{(n-k-1)[1-F_{NO}(x)]^{n-k-2}[1-F_{PMG}(x)]^k f_{NO}(x)+ k[1-F_{NO}(x)]^{n-k-1}[1-F_{PMG}(x)]^{k-1}f_{PMG}(x)\big\}xdx-[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k(\bar{p}_{PMG}-\epsilon)$

It was already shown in the proof of Claim 1 that the above expression is bounded above by $[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k\epsilon$ □

Let $A \equiv (1-\lambda)\big(\frac{1-\phi}{n}+\frac{\phi}{k+1}\big)q$ and $B \equiv (1-\lambda)\big(\frac{1-\phi}{n}+\frac{\phi}{k+1}\big)(1-q)$. Let $\pi(p, PMG; k)$ denote the profit of a firm that offers a PMG and charges p when exactly k remaining stores in the market offer PMGs. WLOG, I normalize $c = 0$.

$$\pi(p, PMG; k) = \lambda[1-F_{NO}(p)]^{n-k-1}[1-F_{PMG}(p)]^k p + AEmin(p/k) + B\varphi(p/k)$$

Let $\epsilon < \frac{\lambda\bar{p}_{PMG}}{\lambda+A+B} = \frac{\lambda\bar{p}_{PMG}}{\lambda+(1-\lambda)\left(\frac{1-\phi}{n}+\frac{\phi}{k+1}\right)}$

I will show that a PMG firm that charges $\bar{p}_{PMG}$ can increase its profit by reducing its price.

$\pi(\bar{p}_{PMG}-\epsilon, PMG) - \pi(\bar{p}_{PMG}, PMG)$

$= \lambda[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k(\bar{p}_{PMG}-\epsilon) - A[Emin(\bar{p}_{PMG}/k) - Emin(\bar{p}_{PMG}-\epsilon/k)] - B[\varphi(\bar{p}_{PMG}/k) - \varphi(\bar{p}_{PMG}-\epsilon/k)]$

$\geq \lambda[1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k(\bar{p}_{PMG}-\epsilon) - [A+B][1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k\epsilon$

$= [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k\Big[\lambda\bar{p}_{PMG} - (\lambda+A+B)\epsilon\Big]$

> 0

Proof of Proposition 1

To show that an equilibrium exists, I use the results in Reny (1999). Since the sum of the players' profits is continuous in prices, it follows from Proposition 5.1 in Reny (1999) that the game is *reciprocally upper semicontinuous.*

I will now show that the game is *payoff secure*, as defined in Reny (1999).

Definition 1 *Player i can secure a payoff $\alpha \in \mathbb{R}$ at $x \in X$ if there exists $\overline{x}_i \in X_i$, such that $\pi_i(\overline{x}_i, x'_{-i}) \geq \alpha$ for all x'_{-i} in some open neighborhood of x_{-i}*

Definition 2 *A game $G = (X_i, \pi_i)_{i=1}^N$ is payoff secure if for every $x \in X$ and every $\epsilon > 0$, each player i can secure a payoff of $\pi_i(x) - \epsilon$ at x*

Let x_A be the price of a firm that plays PMG policy $A \in \{PMG, NO\}$. The firm can secure a payoff of $\pi_A(x) - \epsilon$ at x, by choosing $x_A - \epsilon$. Notice that, by decreasing its price by ϵ, the firm guarantees that the probability that it is the lowest-price firm will not decrease, given that the other firms are playing prices in an ϵ-neighborhood of their prices at x. Since the market size is 1, by decreasing its price by ϵ, profits will fall by less than ϵ.

Since the game is both *reciprocally upper semicontinuous* and *payoff secure*, it follows from Corollary 5.2 in Reny (1999) that there exists a Nash equilibrium.

Now define $\Delta(p) \equiv \pi_{PMG}(p) - \pi_{NO}(p)$ It follows that

$$\Delta(p) = \sum_{k=1}^{n-1} g(k) \left\{ \tfrac{(1-\lambda)\phi}{k+1} \big(qEmin(p,k) + (1-q)p - c\big) - \tfrac{(1-\lambda)(1-\phi)q}{n}[p - Emin(p/k)] \right\}$$

$$+ g(0) \left\{ (1-\lambda)\phi\tfrac{n-1}{n} \big(qEmin(p,0) + (1-q)p - c\big) - \tfrac{(1-\lambda)q}{n}[p - Emin(p/0)] \right\}$$

It is easy to see that $\Delta(\underline{p}) \geq 0$, and Δ is continuous and strictly concave. If $\Delta(p) > 0$ for all $p \in (\underline{p}, \bar{p})$, then the threshold result follows by setting any $\hat{p}$ higher than $\bar{p}$. If there exists $p^* > \underline{p}$ such that $\Delta(p^*) = 0$, concavity of Δ guarantees that $\Delta(p) < 0$ for all $p > p^*$. The threshold result then follows by setting $\hat{p} = p^*$.

Suppose $\phi \in (0, 1)$. To show that, when q is large, $\underline{p} < \hat{p} < \overline{p}$, I first consider the case $q = 1$. First notice that, because $\phi > 0$, $\Delta(\underline{p}) > 0$, which then implies that $\hat{p} > \underline{p}$. The probability that a firm offers PMGs is $F(\hat{p}) > F(\underline{p}) = 0$. Firms offer PMGs with strictly positive probability. I now show that there is no equilibrium under which firms offer PMGs with probability 1. I do this by contradiction. Suppose that in equilibrium all firms offer PMGs with probability 1. Let $[\underline{p}, \overline{p}]$ denote the support of the equilibrium price distribution.

I will show that a firm that charges $\overline{p}$ finds it profitable to reduce its price slightly, which contradicts that $\overline{p}$ is in the support of the price distribution.

Let $Emin(p)$ denote the expected value of the minimum price when the firm charges p. It follows that, when all firms offer PMGs,

$\pi(\overline{p}) = \frac{1-\lambda}{n} Emin(\overline{p})$

$\pi(\overline{p} - \epsilon) = \lambda[1 - F(\overline{p} - \epsilon)]^{n-1}(\overline{p} - \epsilon) + \frac{1-\lambda}{n} Emin(\overline{p} - \epsilon)$

Notice that $Emin(\overline{p}) - Emin(\overline{p} - \epsilon) \leq [1 - F(\overline{p} - \epsilon)]^{n-1}\epsilon$

Let $\epsilon < \frac{\lambda \overline{p}}{\lambda + \frac{1-\lambda}{n}}$. It follows that

$$\begin{aligned}
\pi(\overline{p} - \epsilon) - \pi(\overline{p}) &= \lambda[1 - F(\overline{p} - \epsilon)]^{n-1}(\overline{p} - \epsilon) - \frac{1-\lambda}{n}[Emin(\overline{p}) - Emin(\overline{p} - \epsilon)] \\
&\geq \lambda[1 - F(\overline{p} - \epsilon)]^{n-1}(\overline{p} - \epsilon) - \frac{1-\lambda}{n}[1 - F(\overline{p} - \epsilon)]^{n-1}\epsilon \\
&= [1 - F(\overline{p} - \epsilon)]^{n-1}\left[\lambda \overline{p} - \epsilon\Big(\lambda + \frac{1-\lambda}{n}\Big)\right] \\
&> 0
\end{aligned}$$

Hence, if all firms offer PMGs, the price distribution they play is degenerate. If the price that firms play is higher than marginal cost, it cannot be an equilibrium as each firm would prefer to charge a slightly lower price and sell to all informed consumers. If the price that firms play is equal to marginal cost, firms make zero profit, which cannot be an equilibrium because a firm that charges $c + s_H$ and does not offer a PMG sells to consumers who search at random and makes positive profit, regardless of the prices charged by the other firms. It then follows that, if $q = 1$, firms offer PMGs with a probability strictly between 0 and 1, which implies that $\underline{p} < \hat{p} < \overline{p}$. The argument that this also holds for q large enough follows from continuity.

Proof of Lemma 4

When $q = 0$ it is straightforward that $\pi_{PMG}(p) - \pi_{NO}(p) > 0$ for any p. Hence, firms will choose to offer PMGs for any price they choose. When $\phi = 0$ it is easy to see that $\pi_{PMG}(p) - \pi_{NO}(p) \leq 0$ for any p. Hence, firms will never offer a PMG. Now suppose $\phi = 1$ and let $[\underline{p}, \overline{p}]$ denote the support of the equilibrium price distribution. It follows that $\underline{p} = Emin(\underline{p}/k)$ for any k, which implies that $\Delta(\underline{p}) > 0$, where Δ is defined in the proof of Proposition 1. It then follows that the probability that a firm offers a PMG, which I denote by α, is strictly positive. Suppose there was $p^* < \overline{p}$ such that $\Delta(p^*) = 0$ and $\Delta(p') > 0$ for all $p' < p^*$. Because $\phi = 1$, a firm that chooses not to offer a PMG only sells to uninformed consumers in case no firm offers a PMG, which happens with probability $(1 - \alpha)^{n-1}$. It follows that $\pi(p^*, NO) = \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1 - \alpha)^{n-1}\frac{1-\lambda}{n}(p^* - c)$

Now consider a firm that charges p^* and offers a PMG. The profit of the firm is

$$\begin{aligned}
\pi(p^*, PMG) &= \lambda[1 - F(p^*)]^{n-1}(p^* - c) + \sum_{k=0}^{n-1} g(k)\frac{1-\lambda}{k+1}(Emin(p^*, k) - c) \\
&\geq \lambda[1 - F(p^*)]^{n-1}(p^* - c) + g(0)(1 - \lambda)(Emin(p^*, 0) - c) \\
&= \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1 - \alpha)^{n-1}(1 - \lambda)(p^* - c) \\
&> \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1 - \alpha)^{n-1}\frac{(1-\lambda)}{n}(p^* - c) \\
&= \pi(p^*, NO)
\end{aligned}$$

which contradicts that $\Delta(p^*) = 0$. Notice that I have used the fact that $Emin(p^*, 0) = p^*$. This follows because $\Delta(p') > 0$ for all $p' < p^*$, i.e., all firms that charge prices lower than p^* offer PMGs. Hence, when no firm offers a PMG, the minimum price in the market cannot be lower than p^*.

Proof of Proposition 2

Let L denote the type of a firm that has cost c_L and H denote the type of a firm that has cost c_H. For $i \in \{L, H\}$, let $\pi_i(p, A)$ denote the profit of type i when it charges price p and PMG policy $A \in \{PMG, NO\}$. Let $\pi_i(p) = max\{\pi_i(p, PMG), \pi_i(p, NO)\}$ and let $\Delta_i(p) = \pi_i(p, PMG) - \pi_i(p, NO)$.

Notice that $\Delta_i(p) = \sum_{k=1}^{n-1} g(k)\left\{\frac{(1-\lambda)\phi}{k+1}\big(qEmin(p,k)+(1-q)p\big)-\frac{(1-\lambda)(1-\phi)q}{n}[p-Emin(p/k)]\right\}$
$+ g(0)\left\{(1-\lambda)\phi\frac{n-1}{n}\big(qEmin(p,0)+(1-q)p\big) - \frac{(1-\lambda)q}{n}[p - Emin(p/0)]\right\}$
$- c_i\left[g(0)(1-\lambda)\phi\frac{n-1}{n} + \sum_{k=1}^{n-1} g(k)\frac{(1-\lambda)\phi}{k+1}\right]$

It is straightforward to see that $\Delta_L(p) \geq \Delta_H(p)$. Moreover, $\Delta_i(\underline{p}) \geq 0$ and Δ_i is continuous and strictly concave. Let $\hat{p}_i$ be such that $\Delta_i(\hat{p}_i) = 0$. It follows that $\hat{p}_L \geq \hat{p}_H$.

Let $Q(x, A)$ and $P(x, A)$ denote the average quantity sold and price received by a firm that sets price x and chooses policy $A \in \{PMG, NO\}$.

Lemma 6 *Let x and y be in the support of F such that $x < y$. Let $A \in \{PMG, NO\}$.*

$\pi_L(y, A) \geq \pi_L(x, A) \implies \pi_H(y, A) > \pi_H(x, A)$

$\pi_L(y, NO) \geq \pi_L(x, PMG) \implies \pi_H(y, NO) > \pi_H(x, PMG)$

Proof. For the first part

$\pi_L(y, A) \geq \pi_L(x, A)$

$\iff Q(y, A)[P(y, A) - c_L] \geq Q(x, A)[P(x, A) - c_L]$

$\implies Q(y, A)[P(y, A) - c_H] > Q(x, A)[P(x, A) - c_H]$

$\iff \pi_H(y, A) > \pi_H(x, A)$

Where the implication follows from $Q(x, A) > Q(y, A)$

For the second part

$\pi_L(y, NO) \geq \pi_L(x, PMG)$

$\iff Q(y,NO)[P(y,NO)-c_L] \geq Q(x,PMG)[P(x,PMG)-c_L]$

$\implies Q(y,NO)[P(y,NO)-c_H] > Q(x,PMG)[P(x,PMG)-c_H]$

$\iff \pi_H(y,NO) > \pi_H(x,PMG)$

Where the implication follows from $Q(x,PMG) > Q(y,NO)$ □

Lemma 7 *Let $p' > \tilde{p}$. Let F_i denote the equilibrium price distribution of type i. If $\tilde{p}$ is in the support of both F_L and F_H then p' is not in the support of F_L.*

Proof. I will show that $\pi_L(p') = \pi_L(\tilde{p}) \implies \pi_H(p') > \pi_H(\tilde{p})$, which contradicts that $\tilde{p}$ is in the support of F_H. I split the proof in six cases, that depend on the location of $\tilde{p}$ and p'.

Case 1: $\tilde{p} < \hat{p}_H$ and $p' < \hat{p}_H$

Because in this case both firms find it optimal to offer PMG, it follows from Lemma 6 that $\pi_L(p') = \pi_L(\tilde{p}) \implies \pi_H(p') > \pi_H(\tilde{p})$

Case 2: $\tilde{p} < \hat{p}_H$ and $\hat{p}_H \leq p' \leq \hat{p}_L$

In this case it is optimal for firm L to offer PMG at prices $\tilde{p}$ and p'. Hence, it must be that $\pi_L(\tilde{p},PMG) = \pi_L(p',PMG)$. It then follows from Lemma 6 that $\pi_H(p') \geq \pi_H(p',PMG) > \pi_H(\tilde{p},PMG) = \pi_H(\tilde{p})$

Case 3: $\tilde{p} < \hat{p}_H$ and $p' > \hat{p}_L$

In this case, firm L finds it optimal to offer PMG at price $\tilde{p}$ but not at price p'. Hence, $\pi_L(\tilde{p}) = \pi_L(p') \iff \pi_L(\tilde{p},PMG) = \pi_L(p',NO)$. It then follows from Lemma 6 that $\pi_H(\tilde{p},PMG) < \pi_H(p',NO)$. Moreover, because $\tilde{p} < \hat{p}_H$, it follows that $\pi_H(\tilde{p}) = \pi_H(\tilde{p},PMG)$. We can conclude that $\pi_H(p') > \pi_H(\tilde{p})$

Case 4: $\hat{p}_H \leq \tilde{p} \leq \hat{p}_L$ and $p' < \hat{p}_L$

In this case it is optimal for firm L to offer PMG at prices $\tilde{p}$ and p'. Hence, it must be that $\pi_L(\tilde{p},PMG) = \pi_L(p',PMG)$.

It is straightforward to see that $\frac{\partial \Delta_L}{\partial p} = \frac{\partial \Delta_H}{\partial p}$. Because $\Delta_H(\underline{p}) \geq 0$, $\Delta_H(\hat{p}_H) = 0$ and Δ_H is strictly concave, it follows that Δ_H is decreasing in p after $\hat{p}_H$. It then follows that Δ_L is decreasing in p after $\hat{p}_H$. Hence

$\Delta_L(p') \leq \Delta_L(\tilde{p}) \iff \pi_L(p', PMG) - \pi_L(p', NO) \leq \pi_L(\tilde{p}, PMG) - \pi_L(\tilde{p}, NO)$. Because $\pi_L(\tilde{p}, PMG) = \pi_L(p', PMG)$, it follows that $\pi_L(p', NO) \geq \pi_L(\tilde{p}, NO)$. Lemma 6 then implies that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$.

Because it is optimal for firm H not to offer PMG at prices $\tilde{p}$ and p', we have that $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$

Case 5: $\hat{p}_H \leq \tilde{p} \leq \hat{p}_L$ and $p' \geq \hat{p}_L$

In this case, firm L finds it optimal to offer PMG at price $\tilde{p}$ but not at price p'. Hence $\pi_L(p', NO) = \pi_L(\tilde{p}, PMG) \geq \pi_L(\tilde{p}, NO)$. It then follows from Lemma 6 that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$. Moreover, it is optimal for firm H not to offer PMG at prices $\tilde{p}$ and p'. Hence, $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$

Case 6: $\tilde{p} > \hat{p}_L$ and $p' > \hat{p}_L$

In this case, firm l finds it optimal not to offer PMG at prices $\tilde{p}$ and p'. Hence, $\pi_L(\tilde{p}, NO) = \pi_L(p', NO)$. It then follows from Lemma 6 that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$. Because $\tilde{p} > \hat{p}_L \geq \hat{p}_H$, it follows that firm H finds it optimal not to offer PMG at price $\tilde{p}$. Hence $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$ □

It follows from Lemma 7 that there exists $\underline{p} < \tilde{p} < \overline{p}$ such that type L plays prices in $[\underline{p}, \tilde{p}]$ and type H plays prices in $[\tilde{p}, \overline{p}]$.

For the last part of the proposition, suppose firm H offers PMG with positive probability. Then it must be that $\tilde{p} < \hat{p}_H$ which implies that $\tilde{p} < \hat{p}_L$ which implies that type L offers PMGs for any price in $[\underline{p}, \tilde{p}]$.

Proof of Lemma 5

$$\pi_{PMG}(p) = \sum_{k=0}^{n-1} g(k) \left[\lambda L_k(p)(p-c) + \left[\frac{(1-\lambda)(1-\phi)}{n} + \frac{(1-\lambda)\phi}{k+1} \right] \left(qEmin(p,k) + (1-q)p - c \right) \right]$$

Because $\pi_{PMG}(p) = 0$ for all $p \in (\underline{p}, \hat{p})$, it follows that $\frac{\partial \pi_{PMG}(p)}{\partial p} = 0$ for all $p \in (\underline{p}, \hat{p})$ and

$\lim\limits_{p \to \hat{p}^-} \frac{\partial \pi_{PMG}(p)}{\partial p} = 0$

Notice that, because all PMG firms charge a price lower than $\hat{p}$ and all non-PMG firms charge a price higher than $\hat{p}$, it follows that $L_0(\hat{p}) = 1$ and $L_k(\hat{p}) = 0$ for all $k \geq 1$. Hence, $\lim\limits_{p \to \hat{p}^-} \frac{\partial L_k(p)}{\partial p} = 0$.

Moreover, $\frac{\partial Emin(p,k)}{\partial p} = L_k(p)$. It then follows that

$\lim\limits_{p \to \hat{p}^-} \frac{\partial \pi_{PMG}(p)}{\partial p} = 0 \iff$

$$g(0)\Big[\lambda + \frac{(1-\lambda)(1-\phi)}{n} + (1-\lambda)\phi\Big] + \sum_{k=1}^{n-1} g(k)\Big[\frac{(1-\lambda)(1-\phi)}{n} + \frac{(1-\lambda)\phi}{k+1}\Big](1-q) = 0 \quad (1)$$

equivalently

$$g(0)\Big[\lambda + \frac{(1-\lambda)(1-\phi)}{n} q + (1-\lambda)\phi q\Big] + \sum_{k=0}^{n-1} g(k)\frac{(1-\lambda)\phi}{k+1}(1-q) + \frac{(1-\lambda)(1-\phi)}{n}(1-q) = 0 \quad (2)$$

Let α denote the probability that a firm offers PMG. It follows that

$g(k) = \binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}$

It is straightforward to see that the LHS of (2) is decreasing in α, increasing in ϕ and decreasing in q. (That it is decreasing in q is easier to see by looking at (1)).

It then follows that an increase of ϕ must lead to an increase in α in order for the equality to hold. By the same reasoning, an increase in q must lead to a decrease in α.

B Online Appendix

B.1 Introducing PMGs in the setup of Varian (1980)

In this appendix, I show that, when firms are able to offer PMGs in the setup of Varian (1980), they cannot act as a signal of low prices. In the setup of Varian (1980), a fraction λ of consumers are informed about prices, so they go to the store with the lowest price and purchase the product there. The remaining fraction $1-\lambda$ of consumers are uninformed about firms' prices, and they can only visit one store. They purchase the product at the store they visit provided that the price is not higher than their reservation price, v.

Firms are able to offer PMGs, and uninformed consumers observe firms' PMG strategy. They can then choose whether to visit a PMG store or a store that does not offer such policy.

I will show that there is no rational expectations equilibrium under which PMG stores offer low prices. In order to do that, I assume that consumers believe PMG stores to offer low prices, so uninformed consumers choose to visit a PMG store. I will show that, under this assumption, PMG stores charge higher prices, which is against consumers' belief and, hence, cannot be a rational expectations equilibrium.

Let n denote the number of firms and let k denote the number of firms that offer PMGs. I take k as exogenous. In order for PMGs to signal low prices, both PMG and non-PMG stores must exist. To simplify the analysis, I consider the case where at least 2 stores do not offer PMGs, i.e., $n - k \geq 2$. I consider a symmetric equilibrium, under which all PMG firms play the same strategy and all non-PMG firms play the same strategy. I denote the equilibrium price distributions of PMG and non-PMG firms as F_{PMG} and F_{NO}, respectively.

Because uninformed consumers purchase at PMG stores, non-PMG stores can only sell to informed consumers. Informed consumers purchase from the firm with the lowest price.

First notice that F_{NO} can have no atoms at prices higher than marginal cost. By contradiction, suppose F_{NO} had an atom at some $\tilde{p} > c$. Non-PMG firms would prefer to play a price slightly lower than $\tilde{p}$. This leads to a discontinuous jump in the probability of selling to informed consumers.

Moreover, F_{NO} must be degenerate. If F_{NO} was non-degenerate, a non-PMG firm that charges the upper bound on the support of F_{NO} will never be the firm with the lowest price, and will make zero profit, so it would prefer to charge a lower price.

It then follows that F_{NO} is degenerate at price equal to marginal cost, c.

PMG stores make positive profits in equilibrium, because they can choose to charge any price between c and v in which case they sell to uninformed consumers that enter their store, a fraction $\frac{1-\lambda}{k}$. Hence, it follows that PMG stores charge prices strictly higher than c. But then PMG stores never sell to informed consumers and would prefer to charge v and extract all surplus from uninformed consumers. The following Proposition summarizes the results.

Proposition B.1 *There is no rational expectations equilibrium under which PMGs signal low prices. When consumers believe that PMG stores charge lower prices, non-PMG stores charge c and PMG stores charge v, which is inconsistent with consumers' belief.*

B.2 Sequential choice of PMG policy and price

I make the simplifying assumption that the fraction $1-\lambda$ of consumers with high pre-purchase search cost buy at the first store they visit, as long as the price of such store provides them with nonnegative consumer surplus. Effectively, this is an assumption that the search cost of visiting another retailer is larger than the consumer surplus they expect to get by doing so. Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006) also make similar assumptions.[17] To make the model more tractable, I assume that $q = 1$, i.e., consumers with high search cost pre-purchase have zero search cost post-purchase with probability one. To facilitate the exposition, and without loss of generality, I assume zero marginal costs of production.

Pricing Stage

I start by analyzing the subgames in the pricing stage. Because of symmetry, it is sufficient to consider the following three subgames: i) exactly one retailer offers a PMG; ii) neither retailer offers a PMG; iii) both retailers offer a PMG.

i) exactly one retailer offers a PMG

We assume that consumers believe PMG stores to charge low prices, and we will find the equilibrium price distributions of the PMG and non-PMG store. Those price distributions are an equilibrium if this belief is consistent, i.e., the average price of the PMG store is indeed lower than the average price of the non-PMG store.

Let F_{PMG} and F_{NO} denote the equilibrium price distributions played by the PMG and non-PMG store, respectively. By a standard argument (e.g. Varian (1980)), it can be shown that both distributions are continuous and have the same support. Moreover, the upper

[17] In the model of section 2 high search cost consumers purchase from the first store they visit. However, this was not an assumption, but rather a feature of the equilibrium.

bound of the support is v. Both distributions contain no atoms except, possibly, at v.

Let $\pi_{PMG}(p)$ and $\pi_{NO}(p)$ denote the profit of the PMG and the non-PMG store when they choose price p. Let $Eminp$ denote the expected value of the minimum between p and a random draw from F_{NO}

$\pi_{PMG}(p) = \lambda[1 - F_{NO}(p)]p + \left[\frac{(1-\lambda)(1-\phi)}{2} + (1-\lambda)\phi\right] Emin(p)$

$\pi_{NO}(p) = \lambda[1 - F_{PMG}(p)]p + \frac{(1-\lambda)(1-\phi)}{2} p$

Claim 3 *F_{NO} has a mass point at v*

Proof. Suppose not, i.e, $F_{NO}(v) = 1$. I will show that for $\epsilon < \frac{\lambda v}{\lambda + \frac{(1-\lambda)(1+\phi)}{2}}$, $\pi_{PMG}(v - \epsilon) > \pi_{PMG}(v)$, which contradicts that v is the upper bound of F_{PMG}.

First notice that $Emin(v) - Emin(v - \epsilon) \leq [1 - F_{NO}(v - \epsilon)]\epsilon$. It then follows that

$$\begin{aligned}\pi_{PMG}(v-\epsilon) - \pi_{PMG}(v) &= \lambda[1 - F(v-\epsilon)](v-\epsilon) - \frac{(1-\lambda)(1+\phi)}{2}[Emin(v) - Emin(v-\epsilon)] \\ &\geq [1 - F(v-\epsilon)]\left[\lambda v - \left(\lambda + \frac{(1-\lambda)(1+\phi)}{2}\right)\epsilon\right] \\ &> 0\end{aligned}$$

□

It then follows that F_{PMG} does not have a mass point at v, for if it did, the non-PMG store would never play v and would instead prefer to charge a price slightly lower than v, as that would lead to a discrete jump in the probability of being the lowest priced firm.

Because F_{PMG} has no mass point at v, it follows that when the non-PMG store charges v its profits are $\frac{(1-\lambda)(1-\phi)}{2} v$. We can then find F_{PMG} using the condition that the non-PMG store must be indifferent between all prices in $[\underline{p}, v]$.

$\lambda[1 - F_{PMG}(p)]p + \frac{(1-\lambda)(1-\phi)}{2} p = \frac{(1-\lambda)(1-\phi)}{2} v \iff F_{PMG}(p) = 1 + \frac{(1-\lambda)(1-\phi)}{2\lambda} - \frac{(1-\lambda)(1-\phi)}{2\lambda}\frac{v}{p}$

The condition $F_{PMG}(\underline{p}) = 0$ implies that $\underline{p} = \frac{(1-\lambda)(1-\phi)}{2\lambda + (1-\lambda)(1-\phi)} v$

The expected price of the PMG store is

$E(p_{PMG}) = \int\limits_{\underline{p}}^{v} f_{PMG}(x)xdx = \frac{(1-\lambda)(1-\phi)}{2\lambda} ln\Big(\frac{2\lambda+(1-\lambda)(1-\phi)}{(1-\lambda)(1-\phi)}\Big)v$

The profit of the PMG firm is

$\pi_{PMG} = \Big[\lambda + \frac{(1-\lambda)(1-\phi)}{2} + (1-\lambda)\phi\Big]\underline{p} = \frac{2\lambda+(1-\lambda)(1+\phi)}{2\lambda+(1-\lambda)(1-\phi)}\frac{(1-\lambda)(1-\phi)}{2}v$

Because the PMG store is indifferent between all prices in $(\underline{p}, v)$, it must be that

$\frac{\partial \pi_{PMG}(p)}{\partial p} = 0 \iff \frac{\partial \lambda[1-F_{NO}(p)]p + \frac{(1-\lambda)(1+\phi)}{2}Emin(p)}{\partial p} = 0 \iff$

$$\frac{f_{NO}(p)}{1-F_{NO}(p)} = \frac{K}{p} \tag{3}$$

where $K = \Bigg[1 + \frac{(1-\lambda)(1+\phi)}{2\lambda}\Bigg]$

Denote $H(x) = 1 - F_{NO}(x)$. (3) becomes

$-\frac{H'(x)}{H(x)} = \frac{K}{x}$

Solving the differential equation, $(lnH(x))' = -\frac{K}{x}$ and we find that $H(x) = \frac{c_1}{x^K}$, where c_1 is a constant.

Hence, $F_{NO}(x) = 1 - c_1x^{-K}$ for $x > \underline{p}$. Because $F_{NO}(\underline{p}) = 0$, it must be that $c_1 = \underline{p}^K$. It follows that

$F_{NO}(p) = 1 - \underline{p}^K p^{-K}$ for $p \in (\underline{p}, v)$

The expected value of the price of a non-PMG store, denote by $E(p_{NO})$, is

$E(p_{NO}) = \int\limits_{\underline{p}}^{v} f_{NO}(p)pdp + \Bigg(\underline{p}^K v^{-K}\Bigg)v = \frac{2\lambda(1-\phi)}{[2\lambda+(1-\lambda)(1-\phi)](1+\phi)}\Bigg[\frac{2\lambda+(1-\lambda)(1+\phi)}{2\lambda} - \Big(\frac{(1-\lambda)(1-\phi)}{2\lambda+(1-\lambda)(1-\phi)}\Big)^{\frac{(1-\lambda)(1+\phi)}{2\lambda}}\Bigg]v$

Lemma 8 *For each $\lambda \in (0,1)$, there exists $\tilde{\phi}(\lambda) > 0$ such that, if $\phi < \tilde{\phi}(\lambda)$ then $E(p_{NO}) > E(p_{PMG})$*

Proof. When $\phi = 0$, we have that $E(p_{NO}) = 1 - \frac{2\lambda}{1+\lambda}\Big(\frac{1-\lambda}{1+\lambda}\Big)^{\frac{1-\lambda}{2\lambda}}$ and $E(p_{PMG}) = \frac{1-\lambda}{2\lambda}ln\Big(1 + \frac{2\lambda}{1-\lambda}\Big)$. It can be shown that, for any $\lambda \in (0,1)$ and $\phi = 0$, $E(p_{NO}) > E(p_{PMG})$. The result follows from continuity of $E(p_{NO})$ and $E(p_{PMG})$ in ϕ $\square$

We find that, for the set of parameters described in Lemma 8, the subgame under which exactly one retailer offers a PMG has an equilibrium where PMGs signal low prices. Indeed, we started solving the subgame by assuming that consumers who know which firms offer a PMG choose to visit the PMG store, because they expect it to have low prices. We find that, indeed, the PMG store will choose low prices.

We find that, in this subgame, the PMG store makes profit $\frac{2\lambda+(1-\lambda)(1+\phi)}{2\lambda+(1-\lambda)(1-\phi)}\frac{(1-\lambda)(1-\phi)}{2}v$ and the non-PMG store makes profit $\frac{(1-\lambda)(1-\phi)}{2}v$

ii) neither retailer offers a PMG

The subgame is similar to Varian (1980). Both firms play mixed strategies over prices, and the upper bound of the price distribution is v. Firms make the same profit for each possible price they play. In particular, when a firm plays price v, it sells only to uninformed consumers, and its profit is $\frac{1-\lambda}{2}v$. Hence, in this subgame, each firm makes profit $\frac{1-\lambda}{2}v$.

iii) both retailers offer a PMG

In this subgame, each firms sells to half of high search-cost consumers. These consumers pay the lowest price in the market (because, as $q = 1$, they all search post-purchase to find a lower price and collect a refund). The firm with the lowest price also sells to zero search cost consumers. This puts pressure on both firms to charge low prices. The argument is as follows. Suppose a firm expects its rival to charge price $p > 0$. The best response of the firm is to charge a price slightly lower than p. Indeed, if the firm charges a price higher than p, it makes profit of $\frac{1-\lambda}{2}p$. By charging price $p-\epsilon$ the profit is $\left[\lambda + \frac{1-\lambda}{2}\right](p-\epsilon)$. For ϵ small enough, the profit is larger than when the firm charges p. The fact that firms want to undercut each other leads to an equilibrium where both firms charge marginal cost.

Lemma 9 *When both firms offer a PMG, they choose price equal to marginal cost.*

Proof. Suppose instead, firms made profit in equilibrium. Then both firms will play mixed strategies over prices. A pure strategy equilibrium where the price is higher than marginal cost does not exist, because both firms would want to charge a slightly lower price to attract zero search cost consumers. The upper bound of the price distributions of the two firms must be the same. Let $\overline{p}$ denote such upper bound. Let F_i denote the price distribution of firm $i \in \{1, 2\}$. I will show that a given firm, j, will prefer to charge a price slightly lower than $\overline{p}$, which contradicts that $\overline{p}$ is the upper bound of F_j

Let $Emin_j(p)$ denote the expected minimum market price, given that the price of firm j is p. It follows that

$Emin_j(p) = \int_0^p f_i(x)xdx + [1 - F_i(p)]p.$

For $p < \overline{p}$

$$
\begin{aligned}
Emin_j(\overline{p}) - Emin_j(p) &= \int_0^{\overline{p}} f_i(x)xdx - \int_0^{p} f_i(x)xdx - [1 - F_i(p)]p \\
&= \int_p^{\overline{p}} f_i(x)xdx - [1 - F_i(p)]p \\
&\leq \overline{p} \int_p^{\overline{p}} f_i(x)dx - [1 - F_i(p)]p \\
&= [1 - F_i(p)](\overline{p} - p)
\end{aligned}
$$

In particular, $Emin_j(\overline{p}) - Emin_j(\overline{p} - \epsilon) \leq [1 - F_i(\overline{p} - \epsilon)]\epsilon$. Hence, for $\epsilon < \frac{\lambda \overline{p}}{\lambda + \frac{1-\lambda}{2}}$

$$\begin{aligned}\pi_j(\overline{p}-\epsilon)-\pi_j(\overline{p}) &= \lambda[1-F_j(\overline{p}-\epsilon)](\overline{p}-\epsilon)-\frac{1-\lambda}{2}[Emin_j(\overline{p})-Emin_j(\overline{p}-\epsilon)] \\ &\geq [1-F_j(\overline{p}-\epsilon)]\left[\lambda\overline{p}-\left(\lambda+\frac{1-\lambda}{2}\right)\epsilon\right] \\ &> 0\end{aligned}$$

It then follows that the price distribution is degenerate and, hence, firms must play marginal cost.

□

Notice that the zero profit result follows because it is assumed that $q = 1$. If, instead, $q < 1$ firms would make positive profits, because a profitable deviation would be to charge v and make profit because some consumers do not search post-purchase.

First stage - choice of PMG policy

I now analyze the first stage of the game, where firms decide whether or not to offer a PMG. In particular, I am interested in conditions under which PMG and non-PMG stores coexist.

First notice that both firms offering a PMG is not an equilibrium, as each firm would make higher profits by not offering a PMG, conditional on the fact that the other firm is offering it.

Consider now the case where no firm offers a PMG. It follows from the analysis above that each firm makes profit $\frac{1-\lambda}{2}v$. If one firm deviates and offers a PMG, it will make profit $\frac{2\lambda+(1-\lambda)(1+\phi)}{2\lambda+(1-\lambda)(1-\phi)}\frac{(1-\lambda)(1-\phi)}{2}v$. Such deviation is profitable if $\frac{2\lambda+(1-\lambda)(1+\phi)}{2\lambda+(1-\lambda)(1-\phi)}\frac{(1-\lambda)(1-\phi)}{2}v > \frac{1-\lambda}{2}v \iff \frac{(1-\lambda)(1-\phi)}{2\lambda} > 1$

Notice that $\frac{(1-\lambda)(1-\phi)}{2\lambda} > 1 \implies \lambda < \frac{1}{3}$. This is, then, a necessary condition such that there is no pure strategy equilibria under which both stores play the same PMG policy. Because I am interested in the case in which PMG and non-PMG stores coexist, I will restrict analysis to this condition.

Proposition B.2 *Suppose consumers believe that PMG stores charge low prices. For any $\lambda < \frac{1}{3}$, there exists $\hat{\phi} > 0$ such that if $\phi < \hat{\phi}$ the NE of the first stage are:*

- *Pure strategies: Firm 1 offers PMG and firm 2 does not; Firm 2 offers PMG and firm 1 does not*

- *Mixed strategies: Each firms offers a PMG with probability $\alpha \in (0, 1)$*

When PMG and non-PMG stores coexist, the expected price of the PMG store is lower than the expected price of the non-PMG store

Proof. First notice that $\frac{(1-\lambda)(1-\phi)}{2\lambda} > 1 \iff \phi < 1 - \frac{2\lambda}{1-\lambda}$. Because $\lambda < \frac{1}{3}$ it follows that $1 - \frac{2\lambda}{1-\lambda} > 0$. Now consider $\tilde{\phi}$ as defined in Lemma 8. It follows that, for $\phi < \min\{\tilde{\phi}, 1 - \frac{2\lambda}{1-\lambda}\}$, there exists an equilibrium in the pricing stage under which PMG stores charge lower prices. Because $\frac{(1-\lambda)(1-\phi)}{2\lambda} > 1$, the only pure strategy equilibria are such that exactly one store offers a PMG. There is also a mixed strategy Nash Equilibrium where firms mix on their PMG policy. □

The equilibrium of the sequential game features PMGs as a credible signal of low prices. High search cost consumers who can only visit one store choose to visit the PMG store because they expect it to charge low prices. Their expectation is correct.