Of Course Collusion Should be Prosecuted. But Maybe ...
Or (The Case for International Antitrust Agreements)

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Abstract

We study the incentives of competition authorities to prosecute collusive practices of domestic and foreign firms in a multi-market contact model between two firms operating in two countries. In equilibrium, the country of origin of the firms might prefer to delay prosecution to protect profits in foreign markets. This strategic delay is valuable because it limits the information spillover that triggers prosecution in the foreign country. Prosecution delays, however, are not optimal from the point of view of global welfare, something that could be solved by integrating the competition authorities. With multiple industries, both countries can be better off under integration.

JEL classification codes: F23, F53, L41, K21

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1 Introduction

Globalization has increased the importance of anticompetitive conducts in international settings (see, for example, Connor (2004), Connor and Helmers (2007), Barnett (2007)). National competition authorities have reacted to this new environment by paying more attention to collusive practices involving multinational companies. Specifically, large foreign companies seem to be a frequent target of antitrust enforcement. The challenges associated with international antitrust enforcement have also lead to proposals for increasing international cooperation among competition authorities (see, for example, Barnett (2007)). Despite all this, most of the formal literature on the economics of antitrust has been focused on closed economies and completely ignored any interaction between national competition authorities. On the contrary, we study the incentives of competition authorities to prosecute domestic and foreign firms involved in anticompetitive behavior when prosecution in one country generates informational spillovers to other countries.

In the absence of international considerations, collusion is always detrimental to national welfare, usually defined as the sum of profits and consumer surpluses. However, in an open economy, in which firms
can operate in multiple countries, the promotion of national welfare necessarily requires considering the profits that national firms obtained abroad, but excluding the consumer surplus of foreign consumers. A nationalistic bias in antitrust policy can be immediately recognized: A benevolent competition authority with a mandate to maximize national welfare could prefer to do not enforce antitrust policies, delaying the prosecution of domestic firms even after there is sufficient evidence of collusion. This bias becomes evident in the presence of informational spillovers from prosecution between competition authorities of different countries. For instance, if opening an antitrust case in one country induces other countries to scrutinize the involved firms more meticulously.

A necessary condition to generate a nationalistic bias in antitrust enforcement is that competition authorities do not only consider the wellbeing of consumers, but also incorporate producer surpluses in their decisions.\(^1\) There are several ways to justify this assumption, from a positive as well as a normative perspective. First, not all competition authorities are born equal. While in some countries, the mission of the competition authority is narrowly defined as ‘to protect consumers from anticompetitive practices’, in other countries the mission is broader and it includes other goals such as efficiency and the development of a national economy.\(^2\) Second, even when the goal of the competition authority is to maximize consumers surpluses, political economy considerations may dictate that firms’ profits are not neglected.\(^3\) Third, the empirical evidence shows that foreign multinational firms are a common target of antitrust prosecution; possibly suggesting a nationalist bias in antitrust enforcement.\(^4\) Fourth, although in a closed economy employing consumer surplus or total surplus as the normative criteria to evaluate market performance

\(^1\)In the context of a closed economy, it is common to assume that the objective of the antitrust authority is to maximize the well-being of consumers; thus, excluding profits and only considering consumer surpluses in welfare computations. Carlton (2007), however, argues that antitrust analysis should be based on total welfare for several reasons. i) Total welfare favors long run efficiency gains rather than short run price reductions. ii) In a general equilibrium context, households own the firms. iii) Total welfare can deal with buying cartels and monopolies. Farrell and Katz (2006) also favor total surplus as the welfare criteria for antitrust policy. They argue that the consumer surplus induces a redistributive bias in antitrust policies when taxes and subsidies are a more efficient way to address distribution issues. Finally, Bos and Pot (2012) considers a situation in which cartels improve productive efficiency; thus, reducing consumer surplus, but increasing total surplus.

\(^2\)In the US, the Department of Justice enforces antitrust laws and the Federal Trade Commission protects consumers from anticompetitive practices. The mission of the Antitrust Division of the Department of Justice is to “promote economic competition through enforcing and providing guidance on antitrust laws and principles” (see https://www.justice.gov/atr/mission), while the mission of the Federal Trade Commission is to “protect consumers by preventing anticompetitive, deceptive and unfair business practices” (see https://www.ftc.gov/about-ftc). In the European Union, the European Commission has a broader goal as it understands that competition encompasses efficiency, innovation and price reductions (see http://ec.europa.eu/competition/antitrust/overview_en.html). The goal of Japan’s Fair-Trade Commission seems to be even broader as it includes the development of the national economy. Specifically, it’s mission is “to promote the democratic and sound development of the national economy as well as to assure the interest of general consumers by promoting fair and free competition through prohibition of private monopolization, unreasonable restraint of trade (such as cartels and bid riggings) and unfair trade practices” (see http://www.jftc.go.jp/en/about_jftc/role).

\(^3\)There is an extensive literature on regulatory capture in the context of public utilities (see, for example, Peltzman (1976) and Laffont and Tirole (1991)). On capture in the case of mergers see Neven and Röller (2005). To the best of our knowledge there is no formal model of capture in the context of antitrust policy. Nevertheless, the general idea of a supervisor (the competition authority) captured by the agents (the firms) can be applied to antitrust enforcement.

\(^4\)For example, Garrett (2014) documents that between 2001 and 2012, 45% of antitrust prosecutions carried out by the U.S. Department of Justice were to foreign companies (namely 78 out of 175 companies). Data on Sherman Act violations yielding fines over 10 million dollars also shows that, between 1995 and 2017, only 19 of the 139 firms fined were domestic companies (see Antitrust Division U.S. Department of Justice (2017)). Cremieux and Snyder (2016) finds evidence that the U.S. Department of Justice and the European Commission tend to over-prosecute firms based in the rest of the world, i.e., those outside the U.S. and Europe.
usually leads to the same policy recommendation (prosecute collusion), we show that this is not necessarily the case in an open economy in which prosecution has international informational spillovers. Thus, in a global economy a competition authority committed to protect domestic consumers could end up reducing national welfare. Finally, in a multi-industry setting, we show that unilaterally committing to protect domestic consumers could have negative effects on international antitrust cooperation, inducing other countries to behave opportunistically in order to maximize their own national welfare. Instead, what is required to maximize the total surplus of each country and, hence, global surplus, is an international agreement in which all countries simultaneously commit to prosecute collusion regardless of the origin of the firms or their participation in foreign markets.

In order to formally study the strategic enforcement of antitrust policies in a global economy, we develop a simple two-country model of collusion and antitrust policy. In the model there is an industry integrated by two multinational firms which operate in both countries and in each country there is a competition authority in charge of enforcing antitrust laws. In every period firms decide whether to collude or compete and afterwards, each competition authority receives a signal on the conduct of the firms operating in its jurisdiction. Using this information, each competition authority decides whether to prosecute the firms or not. We characterize the equilibrium prosecution and collusion decisions for different scenarios, varying the information that competition authorities have and the collusive schemes of the firms.

In our baseline model we consider a scenario where one country owns multinational firms operating in both countries and the detection ability of the competition authorities is fully aligned with firm ownership. That is, only the competition authority of the country of origin of the firms receives a signal that indicates whether firms are colluding and, given this information, decides to prosecute (and fine) the firms or not. The competition authority of the foreign country does not receive any independent signal of collusion, but prosecution by the country of origin of the firms is informative and can be used as a signal. In other words, the competition authority of the foreign country has no independent detection ability and must rely on the prosecution decision of the country of origin to obtain information about collusion in its own market before it can decide whether or not to prosecute the firms. Finally, we assume that the country of origin of the firms implements its prosecution policy before the foreign country. This sequential structure implies that the competition authority of the country of origin must take into account the reaction of the competition authority of the foreign country.

In the baseline model we show that the foreign country always prosecutes the firms as soon as there is information of prosecution in the country of origin. The reason is that the foreign country does not own the firms and, hence, only takes into account the consumer surplus. Regarding the country of origin, we prove that it only prosecutes the firms when the gain in consumer surplus exceeds the reduction in profits (including fines) from domestic and foreign markets. This condition can be conveniently expressed through a threshold of the discount factor. The subgame perfect equilibrium of the game obtained by combining these two results is such that, for low discount factors, neither competition authority prosecutes the companies, one due to lack of information and the other to protect the foreign profits of domestic firms.

We explore several variations of this baseline model including the scope of collusive agreements (global versus market-specific collusion); the punishment that firms can use to sustain collusion (in each market separately or in both markets); and the information prosecution spillover (prosecution in the country of origin is indefinitely or only temporarily informative for the foreign country). In all these variations of
the baseline model, our main result does not change. For low discount factors (the threshold is always the same), the competition authority of the country of origin never prosecutes the firms if they are colluding in both markets. For discount factors above the threshold, different specifications bring new results. Under market-specific collusion, firms have the option to collude only in the foreign country, avoiding prosecution indefinitely even when the competition authority of the country of origin chooses to prosecute all instances of collusion. This suggests that, when market-specific collusion decisions are possible, multinational companies might be competing in their home-country (usually, a developed country with a professional and well-funded competition authority), but colluding in foreign markets (usually developing countries, which do not have a competition authority capable of independently detecting collusion). Regarding information prosecution spillovers, if prosecution in the country of origin of the firms is only temporarily informative for the foreign country, then firms have strong incentives to organize sequential collusion, first colluding in the country of origin until they are detected and prosecuted; thereafter, colluding in the foreign country, where collusion will never be detected.

We then proceed to study an extension of the baseline model in which the detection ability is partially aligned with firm ownership. That is, the foreign country has the ability to detect collusion independently. In this setting the foreign country has two channels to detect collusion: (i) through the prosecution decision of the country of origin, and (ii) through its own detection signal. This extension brings a more realistic scenario and allows us to formally compute equilibrium delays in prosecution. The country of origin’s decision to refrain from prosecuting is less valuable than in the baseline model, since now foreign collusion profits can no longer be indefinitely protected with a lenient prosecution policy. Therefore, the country of origin is more willing to prosecute the firms as soon as collusion is detected, ameliorating the nationalistic bias of the prosecution decision. Furthermore, even if it prefers not to prosecute the firms when they are colluding in both countries, eventually, the foreign country detects and prosecutes collusion on its own. When this happens, the country of origin will prosecute collusion too, since there are no more collusive profits to protect abroad. In other words, while in the baseline model, prosecution delays can be infinite, in this extension, delays exist but are finite. To the best of our knowledge, this is the first paper that formally models and obtains prosecution delays as an equilibrium.

We also extend the baseline model to allow for endogenous detection probabilities. In particular, competition authorities do not only choose a prosecution policy, but also invest resources to increase their detection ability. We show that, for a given marginal cost of detection, the optimal probability of detection as chosen by the competition authority depends on the ownership distribution of the firms. A competition authority with a mandate to maximize national welfare selects a higher detection probability if the market is served only by foreign firms than if it is served by purely domestic firms, and it selects an even lower probability if the market is served by domestic firms that also operate in foreign markets. Thus, our model augmented with endogenous detection probabilities, generates cross-sector implications on the enforcement of antitrust policies. Sectors operated by foreign companies are expected to be prime targets for antitrust cases, while sectors dominated by domestic companies with large foreign operations are expected to receive a more lenient treatment. To the best of our knowledge, this is the first formal model that generates predictions of antitrust enforcement based on the ownership origin of the firms and its participation in foreign markets.

\footnote{Indeed, export cartels are lawful in several countries. As Levenstein and Suslow (2004) explains this can be either explicit (through statutes that specify that export cartels are exempt of antitrust laws) or implicit when antitrust laws only take into account domestic effects.}
Finally, we discuss the implications of our results for international antitrust policy cooperation. In order to do so, we incorporate a second industry to the model. In this scenario, a country can lead investigations in one industry and be a follower for the other. We can then study cross investigations, an interesting scenario for similar developed countries with multinational companies. More importantly, the multi-industry extension opens the door to mutual gains from international cooperation (for example, through an international agreement or the integration of the competition authorities), something that cannot happen in the one-industry model. Indeed, in a multi-industry world, both countries can be better off under an integrated competition authority that maximizes the combined welfare or, equivalently, signing an international agreement in which they commit to have competition authorities that only care about the wellbeing of domestic consumers. The intuition is that, under integration, each country loses the collusion profits in the foreign market, but it gains the increase in consumer surpluses in both domestic markets. When countries are of different sizes, this might not be enough to avoid winners and losers from integration, but in the case of perfectly symmetric countries, we prove that both countries gain with an integrated competition authority. Finally, we show that in a multi-industry world, if one of the countries is fully committed to only protecting domestic consumers, then the other country could have no incentive to negotiate an international agreement since delaying the prosecution of national firms operating in foreign markets is a dominant strategy for a competition authority with a mandate to maximize national welfare.

1.1 Related Literature

There are several papers related to this work. For example, Barros and Cabral (1994), Head and Ries (1997) and Neven and Röller (2005) study international competition policy, but in the context of mergers and acquisitions. Our paper also belongs to the literature on multi-market collusion. Bernheim and Whinston (1990) were the first to formalize the idea of simultaneous collusion in multiple markets. They show that under asymmetric markets a larger set of collusive outcomes can be sustained in a multi-market setup. Bond (2004) and Bond and Syropoulos (2008) study multi-market collusion in open economies. None of these papers, however, explore antitrust enforcement.

Closer to our work are Choi and Gerlach (2012a,b, 2013). Choi and Gerlach (2013) employ a multi-market model with symmetric countries to study the relationship between demand linkages and antitrust enforcement. They show that antitrust enforcement in one market spills over to other markets in a way that depends on whether the products are complements or substitutes. Choi and Gerlach (2013) serves as the basis for Choi and Gerlach (2012b), where they study the case of firms producing substitute products in different countries when there are trade frictions and local competition authorities have prosecution costs. They show that enforcement is non-monotonic with respect to trade integration. In particular, cartel enforcement is high if the economies are either closely integrated or trade costs are very high. Finally, they compare two regimes, one with local competition authorities and the other with a global competition authority, and identify two sources of inefficiency associated with local prosecution, namely, decentralized information and cross-market externalities. Finally, Choi and Gerlach (2012a) study multi-market collusion with leniency and different information sharing policies between competition authorities. In their model, competition authorities detect collusion and, once an investigation starts,

\textsuperscript{6}In the one-industry model, while the equilibrium does not maximize the combined welfare of the countries (as it involves a strategic prosecution delay), there is not much room for international cooperation to fix the problem as the country of origin of the firms has no incentive to change its antitrust policy.
firms are allowed to apply for leniency. If firms do not apply for leniency, competition authorities face a chance of unsuccessful prosecution. They show that a policy that involves (partial) information sharing between competition authorities increases the probability of detection and successful prosecution in each jurisdiction.

Our work departs from Choi and Gerlach (2012a) and Choi and Gerlach (2012b) in several important ways. First, we assume that competition authorities seek to maximize national welfare instead of the consumer surplus of domestic consumers. This is crucial when we are considering multinational firms because the ownership distribution of the firms matters for how profits are counted in national welfare. An antitrust policy that ignores the profits that national firms obtain in foreign markets prosecutes them more intensively than what is optimal for national welfare. Moreover, in our model, assuming national welfare as the measure of welfare employed by the competition authorities is key to induce strategic prosecution delays. Second, we employ an informational structure that allows us to model a leader and a follower in antitrust enforcement instead of having competition authorities deciding simultaneously. This is a relevant scenario to explore, given that the country of origin of the firms has more accessible information regarding the collusive behavior of the firms. It also captures the asymmetry in resources and capabilities between competition authorities in developed and developing countries. More importantly, this information structure naturally pushes the leading country to internalize the potential information spillovers of its prosecution decisions. Third, in our model the nature of the relationships between products is not relevant for the sustainability of collusive agreements. The linkage in our setting is purely through an information channel. Prosecution in the country of origin of the firms reveals valuable information for the foreign competition authority to also build a solid antitrust case abroad. Finally, in our model integrating antitrust policy has cross-country distributive effects. In particular, we show that in a one-industry model, one country is always worse off under integration.

Antitrust enforcement in several jurisdictions has also been studied from a collective action approach. For example, Feinberg and Husted (2013) empirically explore free riding on antitrust enforcement between U.S. states. They find that the number of states participating in the litigation process promotes the free-riding behavior, while the resources available to the state government ameliorates it. Additionally, they use the number of horizontal conspiracies as a measurement of case complexity and find that it is associated with delays in prosecution efforts. Our model generates equilibrium strategic delays in prosecution through a completely different channel. In our model, it is not the case that a jurisdiction prefers to wait until another jurisdiction pays the cost of dissolving the cartels. The problem is that the jurisdiction of origin of the firms might prefer to delay prosecution to protect the profits of its firms in other jurisdictions.

Finally, the way we approach antitrust enforcement in an international setting has similarities with the literature on the terms of trade approach to international trade agreements (Bagwell and Staiger (1999)). In the context of trade policy, a country that seeks to maximize national welfare has an incentive to impose a tariff in order to improve its terms of trade at the expense of its trade partners. Analogously, the country of origin of the firms organizing collusion has an incentive to postpone prosecution in order to protect the market power of domestic firms in foreign markets. In the context of trade policy, trade agreements could make both countries better off inducing a mutual reduction in tariffs. In our two-industry model, the integration of antitrust policy in a single competition authority could make both countries better off eliminating prosecution delays in both industries.

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7See, for example, Levenstein and Suslow (2003).
The rest of the paper is organized as follows. Section 2 develops the model. Section 3 characterizes the equilibrium under our baseline scenario, that is, when detection ability is fully aligned with ownership. Section 4 characterizes the equilibrium, when detection ability is only partially aligned with ownership. Section 5 endogenizes the detection probabilities. Section 6 extends the model to a multi-industry setting and explores the implications for international antitrust agreements and integration. Section 7 concludes. All proofs can be found in the Appendix.

2 A Simple Model of Antitrust Policy

This section develops a simple model of collusion and antitrust policy in which the firms involved are multinational corporations operating in several countries. In particular, consider 2 multinational firms \((i \in \{1, 2\})\) which operate in 2 countries \((j \in \{A, B\})\) and must decide whether to collude or compete. In each country there is a competition authority which has the power to fine firms if collusion is detected. Time is infinite, discrete and indexed by \(t \in \{0, 1, \ldots\}\). All agents have a common discount factor \(\delta \in (0, 1)\).

The timing of events within each period is as follows:

1. Both firms simultaneously choose to collude or compete in each country.
2. Nature sends a signal to the competition authority of country \(A\) regarding the behavior of the firms in country \(A\). Upon observing the signal, the competition authority of country \(A\) decides whether to prosecute the firms or not.
3. Nature sends a signal to the competition authority of country \(B\) regarding the behavior of the firms in country \(B\). Upon observing the signal, the competition authority of country \(B\) decides whether to prosecute the firms or not.

This timing is compatible with different signalling structures and contents. In particular, note that the signal received by the competition authority of country \(B\) could depend on the prosecution decision selected by the competition authority of country \(A\).

2.1 Firms, Profits and Ownership Distribution

Let \(\left(a_i^{t,A}, a_i^{t,B}\right)\) denote the decision of firm \(i\) in period \(t\), where \(a_i^{t,j} = 1\) indicates that firm \(i\) chooses to collude in country \(j\) and \(a_i^{t,j} = 0\) indicates that firm \(i\) chooses to compete in country \(j\). Let \(\pi_i^{t,j}\) denote the profits earned by firm \(i\) from its operations in country \(j\) in period \(t\) (before paying any fine). Specifically, profits are given by:

<table>
<thead>
<tr>
<th>Firm 1 - Firm 2</th>
<th>(a_t^{2,j} = 1)</th>
<th>(a_t^{2,j} = 0)</th>
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<tbody>
<tr>
<td>(a_t^{1,j} = 1)</td>
<td>(\pi^{c,j}) (\pi^{c,j})</td>
<td>(0, \pi^{d,j})</td>
</tr>
<tr>
<td>(a_t^{1,j} = 0)</td>
<td>(\pi^{d,j}) (0)</td>
<td>(0, 0)</td>
</tr>
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where \(\pi^{d,j} > \pi^{c,j} > 0\). Thus, if both firms collude in country \(j\), each obtains collusion profits \((a_t^{i,j} = a_t^{-i,j} = 1\) implies \(\pi_t^{i,j} = \pi_t^{-i,j} = \pi^{c,j} > 0\)). However, each firm has a short term incentive to deviate from collusion, which leaves no profits for the rival \((a_t^{i,j} = 0\) and \(a_t^{-i,j} = 1\) implies \(\pi_t^{i,j} = \pi^{d,j} > \pi^{c,j}\) and...
Finally, competition fully dissipates profits ($a_{i,j}^{t} = a_{i,j}^{t-1} = 0$ implies $\pi_{i,j}^{t} = \pi_{i,j}^{t-1} = 0$). Let $f_{i,j}^{t}$ denotes the fine imposed to a firm in period $t$ by the competition authority of country $j$. Then, the profits net of fines obtained by firm $i$ in period $t$ are given by:

$$\pi_{i,j}^{t} = \pi_{i,j}^{t-1} - f_{i,j}^{t}$$  \hspace{1cm} (1)

The goal of the firm is to maximize the expected discounted profits net of fines, which are given by:

$$\Pi_{i,j}^{t} = \mathbb{E}_{t} \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_{i,j}^{\tau} \right]$$  \hspace{1cm} (2)

Finally, let $\sigma_{i,j}^{1} \in [0, 1]$ be the share of firm $i$ owned by citizens from country $j$. Naturally, $\sigma_{i,j}^{1} + \sigma_{i,j}^{2} = 1$ for $i = 1, 2$.

### 2.2 Consumers

The consumer surplus obtained by consumers from country $j$ in period $t$ is $CS_{i,j}^{t} = CS_{c,j}^{t}$ when $a_{i,j}^{t} = a_{i,j}^{t-1} = 1$ and $CS_{i,j}^{t} = CS_{com,j}^{t}$, otherwise, where $CS_{c,j}^{t}$ and $CS_{com,j}^{t}$ are the consumer surpluses in country $j$ under collusion and competition, respectively. We assume that $CS_{com,j}^{t} > CS_{c,j}^{t} + 2\pi_{c,j}^{t}$ for all $j$, i.e., in each country, the aggregate surplus under competition ($CS_{com,j}^{t}$) is higher than under collusion ($CS_{c,j}^{t} + 2\pi_{c,j}^{t}$).

### 2.3 Competition Authorities, Collusion Detection and Antitrust Prosecution

In each period, immediately after firms make their decisions, the competition authority of each country receives a signal about the behavior of the firms. Let $s_{i,j}^{t} \in \{0, 1\}$ denote the signal received by the competition authority of country $j$. $s_{i,j}^{t} = 1$ ($s_{i,j}^{t} = 0$) indicates that the competition authority of country $j$ detects (does not detect) collusion in period $t$. The signal conveys information about collusion only if firms are effectively colluding. Otherwise, the competition authority receives a no collusion signal. After observing the signal, the competition authorities decide whether or not to prosecute the firms. Let $p_{i,j}^{t} \in \{0, 1\}$ denote the prosecution decision of the competition authority $j$ in period $t$, where $p_{i,j}^{t} = 1$ indicates prosecution and $p_{i,j}^{t} = 0$ indicates no prosecution. We assume that a competition authority cannot prosecute without detecting evidence of collusion. Thus, when $s_{i,j}^{t} = 0$, it must be the case that $p_{i,j}^{t} = 0$. If collusion is detected, i.e., $s_{i,j}^{t} = 1$, and the competition authority prosecutes the firms, i.e., $p_{i,j}^{t} = 1$, then, each firm must pay a fine $f_{j}^{t} = f_{j}^{t} > \pi_{c,j}^{t}$ and they are forced to compete in all subsequent periods.

The national welfare of country $j$ in period $t$ is given by:

$$w_{i,j}^{t} = CS_{i,j}^{t} + 2f_{i,j}^{t} + \sigma_{i,j}^{1} \pi_{i,j}^{t} + \sigma_{i,j}^{2} \pi_{i,j}^{t}$$  \hspace{1cm} (3)

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8The underlying assumption is that the firms sell homogeneous products and compete in prices. Hence, when reverting to competition upon either a choice of not colluding or a failure of collusion, the profits of the firms are zero. Alternatively, in the case that firms compete in quantities or in prices with differentiated products, we can assume that profits are normalized, i.e., all profits are relative to competitive profits. These alternative assumptions do not affect the analysis.

9Note that we are implicitly assuming that the actions of the firms $a_{i,j}^{t}$ are public information, i.e., known by the firms and also by the competition authorities, but that knowing that there is collusion is not the same as having enough evidence to prosecute the firms. Only when $s_{i,j}^{t} = 1$, the competition authority of country $j$ has the required information to successfully prosecute the firms.
The first term is the consumer surplus in country $j$; the second is the fines collected by the competition authority of country $j$; the third and fourth terms are the shares of the profits (net of fines) of firms 1 and 2 accruing to citizens of country $j$. Alternatively, $w^j_t$ can be rewritten as follows:

$$w^j_t = CS^j_t + PS^j_t + \sum_i \sigma^{i,j} \left( \pi^{i,j}_t - f^j_t \right) - \sum_i (1 - \sigma^{i,j}) \left( \pi^{i,j}_t - f^i_t \right)$$

where $PS^j_t = \pi^{1,j}_t + \pi^{2,j}_t$ is the producer surplus in country $j$. Thus, the national welfare in country $j$ is the total surplus generated in country $j$ ($CS^j_t + PS^j_t$) plus the profits of the firms from their foreign operations accruing to citizens of country $j$ ($\sum_i \sigma^{i,j} \left( \pi^{i,j}_t - f^j_t \right)$) minus the profits of the firms from their domestic operations accruing to foreign citizens ($\sum_i (1 - \sigma^{i,j}) \left( \pi^{i,j}_t - f^i_t \right)$).

The objective of the competition authority of country $j$ is to maximize the expected discounted welfare of the country, which is given by:

$$W^j_t = E_t \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} w^j_{\tau} \right]$$

### 2.4 Definition of Equilibrium

The collusion and antitrust policy model is an infinite dynamic game with four players: the two firms and the two competition authorities. Next, we formally define a strategy and payoff function for each player, a collusion signal function for each competition authority, and an equilibrium for the collusion and antitrust policy game.

Define a t-history as a sequence $h_t = \left\{ (a^1_t, a^2_t), (a^1_\tau, a^2_\tau), (p^A_t, p^B_t) \right\}_{\tau=0}^{t-1}$. That is, $h_t$ contains the decisions of the firms and the competition authorities up to period $t - 1$. Let $H_t$ denote the set of all feasible t(histories. Let $A^j_t (h_t) \subseteq \{0, 1\}$ denote the set of actions available to the firms in country $j$ in period $t$ when the history of the game is $h_t$. Since once firms are prosecuted, they must compete in all future periods, $A^j_t (h_t) = \{0\}$ whenever in history $h_t$ there exists $\tau \leq t - 1$ such that $p^j_t = 1$. Otherwise $A^j_t (h_t) = \{0, 1\}$. Let $P^j_t (s^j_t) \subseteq \{0, 1\}$ be the set of available prosecution decisions for the competition authority of country $j$ in period $t$ when the collusion signal received in period $t$ is $s^j_t \in \{0, 1\}$. Since only when the competition authority receives a positive signal of collusion, it can prosecute the firms, $P^j_t (0) = \{0\}$ and $P^j_t (1) = \{0, 1\}$.

- **A strategy for firm** $i \in \{1, 2\}$ is a sequence of functions $a^i = \{a^i_t\}_{t=0}^{\infty}$, where $a^i_t : H_t \rightarrow \{0, 1\}^2$ and $a^i_t (h_t) = \left( a^1_t (h_t), a^2_t (h_t) \right) \in A^j_t (h_t) \times A^j_t (h_t)$. That is, for each t-history firm $i$ must decide to collude (if such alternative is still available) or compete in each country in period $t$.

- **A strategy for competition authority** $j \in \{A, B\}$ is a sequence of functions $p^j = \{p^j_t\}_{t=0}^{\infty}$, where $p^j_t : H_t \times \{0, 1\}^5 \rightarrow \{0, 1\}$ with $p^j (h_t, a^1_t A, a^1_t B, a^2_t A, a^2_t B, s^j_t) = p^j_t \in P^j_t (s^j_t)$. That is, given a t-history $h_t$, actions of the firms $\left( a^1_t A, a^1_t B \right) \in \{0, 1\}^2$, $\left( a^2_t A, a^2_t B \right) \in \{0, 1\}^2$ and the collusion signal $s^j_t \in \{0, 1\}$ received in period $t$, competition authority $j$ must decide whether to prosecute the firms or not in period $t$. 

• The collusion signal function of competition authority $j$ is a sequence of functions $s^j = \left\{ s^j_t \right\}_{t=0}^\infty$, where $s^j_t : H_t \times \{0,1\}^3 \rightarrow [0,1]$ and $s^j_t \left( h_t, a^1_t, a^2_t, p_t^- \right) = \Pr \left( s^j_t = 1 \right)$. That is, for each $t$-history, actions of the firms in country $j$ in period $t$, and prosecution decision of the other competition authority, nature selects the signal received by competition authority $j$. Since only when the firms are colluding, the competition authority can receive a positive collusion signal, if $\left( a^1_t, a^2_t \right) \neq (1,1)$, then $s^j_t \left( h_t, a^1_t, a^2_t, p_t^- \right) = 0$.

The combination of a pair of collusion strategies for the firms $\left( a^1, a^2 \right)$, a pair of prosecution strategies for the competition authorities $\left( p^A, p^B \right)$, and a pair of collusion signal functions $\left( s^A, s^B \right)$ induce a probability distribution function over $\{ \pi^j_{\tau=t} \}_{\tau=t}^\infty$, the sequence of profits net of fines obtained by firm $i \in \{ 1, 2 \}$, and a probability distribution over $\{ u^j_{\tau=t} \}_{\tau=t}^\infty$, the sequence of welfare obtained by country $j \in \{ A, B \}$. Then, the payoff of firm $i$ induced by $\left( a^1, a^2, p^A, p^B, s^A, s^B \right)$ is given by $\Pi^i_t = \mathbb{E}_t \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi^i_{\tau=\tau} \right] h_t$, where the expectation operator is conditional on $h_t$, the information available to the firms when they make a decision in period $t$. Similarly, the payoff of competition authority $j$ induced by $\left( a^1, a^2, p^A, p^B, s^A, s^B \right)$ is given by $W^j_t = \mathbb{E}_t \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} u^j_{\tau=\tau} \right] \left( h_t, a^1_t, a^2_t, s^1_t \right)$, where the expectation operator is conditional on $\left( h_t, a^1_t, a^2_t, s^1_t \right)$, the information available to the competition authority $j$ when it selects its prosecution decision in period $t$.

Definition 1 An equilibrium of the collusion and antitrust game is a pair of collusion strategies for the firms $\left( a^1, a^2 \right)$ and a pair of prosecution strategies for the competition authorities $\left( p^A, p^B \right)$ that, given the collusion signal functions $\left( s^A, s^B \right)$, form a subgame perfect Nash equilibrium.

3 Detection Ability Fully Aligned with Firm Ownership

This section studies the equilibrium of the model when both firms are owned by citizens of country $A$ and the competition authority of country $B$ is not able to detect collusion on its own, but it can learn from the prosecution process in country $A$. Formally, regarding the ownership distribution of the firms, we assume throughout this section that $\sigma^{1,A} = \sigma^{2,A} = 1$. This implies that the profits of both firms obtained in country $B$ are accounted for in the welfare of country $A$, while country $B$’s welfare only includes the consumer surplus in country $B$ and the fines collected there. With respect to collusion detection, consider the following assumption (which we relax later) regarding the signals received by each competition authority.

Assumption 1. The signals received by the competition authorities are:

$$s^A_t = \begin{cases} 1 & \text{with probability } \alpha^A \quad \text{if } a^1_t = a^2_t = 1, \\ 0 & \text{with probability } \left( 1 - \alpha^A \right) \quad \text{if } a^1_t = a^2_t = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$s^B_t = \begin{cases} 1 & \text{if } a^1_t = a^2_t = 1 \text{ and } p^A_t = 1 \text{ for some } \tau \leq t, \\ 0 & \text{otherwise.} \end{cases}$$
Assumption 1 states that the competition authority of country A can detect collusion on its own, while the competition authority of country B must rely on observing that country A has prosecuted the firms in order to detect collusion in country B.\(^{10}\)

The following lemmas formally characterize the prosecution policies of both countries.

**Lemma 1** Suppose that Assumption 1 holds. Then, the competition authority of country B prosecutes the firms as soon as collusion is detected in country B. **Proof:** See Appendix A.1. ■

The intuition behind Lemma 1 is very simple. Since the competition authority of country B only benefits from the consumer surplus in country B and the fines (which are paid by foreign firms), it immediately prosecutes the firms as soon as collusion is detected.

**Lemma 2** Suppose that Assumption 1 holds. Assume that the competition authority of country A detects collusion, i.e., \(s^A_t = 1\).

1. If the firms are not colluding in country B, then A always prosecutes the firms.
2. If the firms are also colluding in country B, then A prosecutes the firms if and only if
\[
\delta \geq \bar{\delta} = \frac{2f^B}{CS^{\text{com},A} - CS^{c,A} - 2\pi^{c,A} + 2f^B - 2\pi^{c,B}},
\] (5)

Alternatively, (5) can be written as \(\delta (CS^{\text{com},A} - CS^{c,A} - 2\pi^{c,A}) \geq 2 [\delta \pi^{c,B} + (1 - \delta) f^B]\). **Proof:** See Appendix A.1. ■

The intuition behind Lemma 2 is as follows. When firms are colluding in both countries, if the competition authority of country A prosecutes the firms, this will trigger prosecution in country B. As a consequence, firms owned by shareholders from country A will have to pay fines in country B and, in the future, they will be forced to compete, which eliminates their profits from collusion in country B. Thus, prosecution in country A increases the total surplus in country A, but it reduces the profits of the firms in country B. When firms are only colluding in country A, the second effect disappears and, hence, the best policy for the competition authority of country A is to prosecute the firms when collusion is detected.

We now proceed to study the equilibrium collusion decisions of the firms given the antitrust policies selected by the competition authorities of both countries. The following propositions characterize firms' decisions when they can only make global collusion decisions as well as when collusion decisions are market-specific. Since firms play a dynamic game there might be multiple outcomes that can be sustained as a subgame perfect Nash equilibrium. To deal with this multiplicity, we assume that firms always coordinate in their most preferred equilibrium, i.e., the one that generates the highest expected profits for each firm. Note that this does not mean that we ignore other possible equilibria. On the contrary, for each set of parameters we deduce and compare all possible equilibria and select the equilibrium outcome with the highest expected profits.

\(^{10}\)The assumed ownership distribution of the firms together with Assumption 1 could capture the following situation. Consider two multinational firms whose shareholders are from a developed country A, which counts with a professional competition authority with the capacity to detect collusion. The firms also operate in a developing country B, whose competition authority does not have the resources and/or the expertise to detect and prosecute collusion on its own. However, if the competition authority of country B observes country A prosecuting the firms in country A, it will learn how to detect and prosecute collusion in country B.
3.1 Global Collusion Decisions

Proposition 1 explores a simple environment in which firms make global collusion decisions. That is, firms can collude in both countries or in none of them, but they cannot collude in one country and not in the other.

**Proposition 1** Suppose that Assumption 1 holds, firms can either collude in both countries or in none of them and they coordinate in their best equilibrium.

1. Suppose that $\bar{\delta}^1 < \delta < \bar{\delta}$, where $\bar{\delta}^1 = \frac{\pi_{d,A} + \pi_{d,B} - \pi_{c,A} - \pi_{c,B}}{\pi_{d,A} + \pi_{d,B}}$. Then, firms collude in both countries and they are never prosecuted.

2. Suppose that $\delta \geq \bar{\delta}$. Then, there is a threshold $\bar{\alpha}^1$ such that:

   (a) If $\alpha^A \leq \bar{\alpha}^1$, then there is global collusion until the first time $s^A_t = 1$, when firms are prosecuted in both countries. Thereafter, there is competition.

   (b) If $\alpha^A > \bar{\alpha}^1$, then there is always competition. **Proof**: See Appendix A.1.

Proposition 1 Part 1 states that when the discount factor is below some threshold ($\delta < \bar{\delta}$), the expected discounted welfare of country $A$ is higher if collusion in both countries is allowed, and, hence, the competition authority of country $A$ never prosecutes the firms. Then, firms can safely collude in both countries without facing any risk of prosecution. Since firms also need to sustain collusion, we require $\pi^{c,A} + \pi^{c,B} > (1 - \delta)(\pi^{d,A} + \pi^{d,B})$ or, which is equivalent, $\delta > \bar{\delta}^1$. Proposition 1 Part 2 studies the situation in which $\delta \geq \bar{\delta}$ and, hence, the expected discounted welfare of country $A$ is higher if collusion is stopped. In such a situation the competition authority of country $A$ prosecutes the firms as soon as collusion is detected, which also triggers prosecution in country $B$. This prosecution policy might not be enough to dissuade firms to collude when the detection probability is low. Indeed, for $\alpha^A \leq \bar{\alpha}^1$, the expected discounted profits from collusion are high enough to sustain collusion. If this is the case, in equilibrium, there will be collusion until the competition authority of country $A$ detects it and prosecutes the firms. Thereafter, firms will be forced to compete. On the contrary, when $\alpha^A > \bar{\alpha}^1$, the expected discounted profits from collusion are not enough to sustain collusion. The antitrust policy effectively dissuades firms from colluding.

3.2 Market-specific Collusion Decisions

Proposition 2 explores the more general case in which firms can make market-specific collusion decisions. We assume that firms employ the harshest possible punishment, stopping collusion in both countries even when the other firm deviates from collusion in only one market. Similar results can be obtained if firms punish deviations from collusion in each market separately.

**Proposition 2** Suppose that Assumption 1 holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium.

1. Suppose that $\bar{\delta}^2 < \delta < \bar{\delta}$, where $\bar{\delta}^2 = \max\left\{\frac{\pi_{d,A} - \pi^{c,A}}{\pi_{d,A}}, \frac{\pi_{d,B} - \pi^{c,B}}{\pi_{d,B}}\right\}$. Then, firms collude in both countries and they are never prosecuted.
2. Suppose that $\delta \geq \bar{\delta}$. Then, there are thresholds $\bar{\alpha}^2_L$ and $\bar{\alpha}^2_H$ such that:

(a) Suppose that $\alpha^A > \bar{\alpha}^2_H$. Then, firms collude in country $B$ and they are never prosecuted.

(b) Suppose that $\alpha^A < \alpha^A \leq \bar{\alpha}^2_H$. Then, there is a threshold $\bar{\alpha}^2_M$ such that: If $\alpha^A \in \left(\bar{\alpha}^2_L, \bar{\alpha}^2_M\right]$, then firms collude (only in country $A$ or in both countries) until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in \left[\bar{\alpha}^2_M, \bar{\alpha}^2_H\right]$, then firms only collude in country $B$ and they are never prosecuted.

(c) Suppose that $\alpha^A \leq \bar{\alpha}^2_L$. Then, there is a threshold $\bar{\alpha}^2_M$ such that: If $\alpha^A \in \left(0, \bar{\alpha}^2_M\right]$, then firms collude (only in country $A$ or in both countries) until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in \left[\bar{\alpha}^2_M, \bar{\alpha}^2_L\right]$, then firms only collude in country $B$ and they are never prosecuted.

Proposition 2 Part 1 extends the result in Proposition 1 Part 1. Country $A$ does not prosecute the firms when its expected discounted welfare is higher if firms collude in both countries (formally, when $\delta < \bar{\delta}$). In that case, in equilibrium, if firms are patient enough (formally, when $\delta > \bar{\delta}^2$), there is collusion in both countries. Proposition 2 Part 2 is more involved than Proposition 1 Part 2. The reason is that now firms can decide to collude only in country $B$, in which case, they are never detected. In other words, firms can be dissuaded to collude in country $A$, but they will never be dissuaded to engage in collusion in country $B$. This generates three possible equilibrium paths. When the detection probability is high ($\alpha^A \geq \bar{\alpha}^2_H$), the expected discounted profits from collusion in country $A$ are not enough to sustain neither collusion in both countries nor collusion in country $A$ as an equilibrium. Only collusion in country $B$ can be sustained as an equilibrium. In that case, firms collude in country $B$ and they are never detected.

When the detection probability adopts intermediate values ($\bar{\alpha}^2_L < \alpha^A \leq \bar{\alpha}^2_H$) only two types of collusion can be sustained as an equilibrium. First, firms can always sustain collusion in country $B$ because $\delta > \bar{\delta}^2$ implies $\pi^{c,B} > (1 - \delta) \pi^{d,B}$. Second, either collusion in country $A$ or collusion in both countries can be sustained as an equilibrium. The logic is as follows. When $\frac{\pi^{c,A} - (1 - \delta)\pi^{d,A}}{\delta \pi^{d,A} + f^A} > \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{\delta \pi^{d,B} + f^B}$, it is easier to dissuade collusion in both countries than only in country $A$. Then, for intermediate values of the detection probability, firms can sustain collusion in country $A$, but not in both countries. On the contrary, when $\frac{\pi^{c,A} - (1 - \delta)\pi^{d,A}}{\delta \pi^{d,A} + f^A} < \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{\delta \pi^{d,B} + f^B}$, it is easier to dissuade collusion only in country $A$ than in both countries. Then, for intermediate values of the detection probability, firms can sustain collusion in both countries, but not only in country $A$. In any case, firms opt for the type of collusion that generates higher expected discounted profits between the two types that can be sustained as an equilibrium. Note that if they choose collusion in country $A$, eventually, they will be detected, forced to pay a fine and start competing. If they choose to only collude in country $B$, they will be never detected and collusion will last forever.

Finally, when the detection probability is low ($\alpha^A \leq \bar{\alpha}^2_L$), the expected discounted profits from collusion in country $A$ and $B$ are both high enough to sustain every type of collusion as an equilibrium. Then, firms select the equilibrium with the highest expected discounted profits. If in such equilibrium firms are colluding in country $A$, eventually, the competition authority of country $A$ will detect and prosecute the firms. Moreover, if firms are also colluding in country $B$, the competition authority of...
country B will follow the same course of action and, thereafter, firms will be forced to compete in both countries.\footnote{In Appendix A.1 we prove a detailed version of Proposition 2 in which the equilibrium decisions of the firms are fully characterized for any set of values of the model’s parameters. We also explore a variation of Proposition 2 in which firms punish deviations from collusion in each market separately and obtain similar results. In particular, Part 1 remains the same and Part 2 becomes simpler since when deviations are punished in each market separately, if collusion in both countries can be supported as an equilibrium, it automatically induces higher expected profits than collusion in country B.}

### 3.3 Prosecution in A is Only Temporarily Informative for B

Assumption 1 states that when competition authority A prosecutes the firms in country A, competition authority B observes the process and learns how to detect and prosecute collusion in country B in all future periods. A weaker version of this assumption is to assume that prosecution in country A in period \( t \) only allows competition authority B to detect and prosecute collusion in period \( t \), but it is not informative in future periods. Formally:

**Assumption 1.bis.** The signal received by competition authority A is as in Assumption 1 and the signal received by competition authority B is given by:

\[
s_B^t = \begin{cases} 
1 & \text{if } a_1^1, A = a_2^2, A = 1 \text{ and } p_1^A = 1, \\
0 & \text{otherwise}.
\end{cases}
\]

Note that while under Assumption 1, \( s_B^t = 1 \) when \( a_1^1, A = a_2^2, A = 1 \) and \( p_1^A = 1 \) for at least one \( \tau \leq t \), under Assumption 1.bis, \( s_B^t = 1 \) only when \( a_1^1, B = a_2^2, B = 1 \) and \( p_1^A = 1 \). Thus, under Assumption 1, prosecution in country A in one period, allows competition authority B to detect and prosecute collusion in every future period, while under Assumption 1.bis, prosecution in country A is only temporarily informative for competition authority B. The main implication of replacing Assumption 1 by Assumption 1.bis is that now firms have access to new ways of organizing collusion. In particular, firms can collude only in country A until they are detected and, thereafter, start colluding in country B. Collusion in country B will never be detected because it begins after \( p_1^A = 1 \) and, hence, there is no way that competition authority B detects it.

In Appendix A.1 we prove that Lemmas 1 and 2 still hold when Assumption 1 is replaced by Assumption 1.bis. Regarding Lemma 1, it is easy to verify that competition authority B will always prosecute the firms as soon as collusion is detected because the country B does not own the firms. For Lemma 2, the intuition is as follows. When competition authority A detects collusion in A, but there is no collusion in B, it could be that firms will never collude in B or they will start collusion in B once collusion in A is detected. The first case has been already considered in the proof of Lemma 2 under Assumption 1. Competition authority A always prefers to prosecute the firms. In the second case, prosecution in A triggers the start of collusion in B, which only increases the benefits of prosecuting firms in A. In Appendix A.1 we also prove a version of Proposition 2 in which Assumption 1 is replaced by Assumption 1.bis. The main novelty is that the firms strategy of colluding in country A until detected and then switching to country B generates higher expected profits than colluding solely in country A. The reason is that collusion in country B cannot be detected once firms have been prosecuted in country A. This result points toward the idea that if firms are detected in their own market, they will start collusion in foreign markets.
4 Detection Ability Partially Aligned with Firm Ownership

This section studies the equilibrium of the model when both firms are owned by citizens of country A (formally, \( \sigma^1_A = \sigma^2_A = 1 \)) and the competition authority of country B has two channels to detect collusion: observe the prosecution decisions of the competition authority of country A and detect collusion on its own. Thus,

**Assumption 2.** The signal received by competition authority A is as in Assumption 1 and the signal received by competition authority B is given by:

\[
s^B_t = \begin{cases} 
1 & \text{with probability } \alpha^B, \\
0 & \text{with probability } (1 - \alpha^B), \\
1 & \text{if } a^1_t = a^2_t = 1 \text{ and } p^A_T = 0 \text{ for all } \tau \leq t, \\
0 & \text{if } a^1_t = a^2_t = 1 \text{ and } p^A_T = 0 \text{ for all } \tau \leq t, \\
1 & \text{if } a^1_t = a^2_t = 1 \text{ and } p^A_T = 1 \text{ for some } \tau \leq t, \\
0 & \text{otherwise.}
\end{cases}
\]

Several remarks should be made about Assumption 2. First, as in the previous section, it is still the case that prosecution of the firms in country A is the most informative signal of collusion for competition authority B. Indeed, if competition authority A prosecutes the firms in country A, competition authority B immediately learns how to detect collusion in B in the present as well as in all future periods (formally, if \( a^1_t = a^2_t = 1 \) and \( p^A_T = 1 \) for some \( \tau \leq t \), then \( c^B_t = 1 \).) Second, even if competition authority A does not prosecute the firms, it is possible that competition authority B detects collusion in B (formally, if \( a^1_t = a^2_t = 1 \) and \( p^A_T = 0 \) for all \( \tau \leq t \), then \( s^B_t = 1 \) with probability \( \alpha^B \)).

It is easy to verify that Lemma 1 also holds under Assumption 2. Regardless of how competition authority B detects collusion, if \( s^B_t = 1 \), the expected discounted welfare of country B at period \( t \) if firms are prosecuted is \( W^B_t(p^B_T = 1|s^B_t = 1) = CS^{c,B}_t + 2f^B + \frac{\delta}{1-\delta}CS^{com,B}_t \), while if they are not prosecuted it is \( W^B_t(p^B_T = 0|s^B_t = 1) = CS^{c,B}_t \). Since \( CS^{com,B} > CS^{c,B} \), it is always the case that \( W^B_t(p^B_T = 1|s^B_t = 1) > W^B_t(p^B_T = 0|s^B_t = 1) \).\(^{13}\) Thus, as soon as \( s^B_t = 1 \), competition authority B immediately prosecutes the firms.

Next, we turn to the prosecution decision of competition authority A when it detects collusion.

**Lemma 3** Suppose that Assumption 2 holds. Assume that the competition authority of country A detects collusion, i.e., \( c^A_t = 1 \).

1. If the firms are not colluding in country B, then A always prosecutes the firms.

2. If the firms are also colluding in country B, then A prosecutes the firms if and only if

\[
CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{[1 - (1 - \alpha^A)] \delta^2 (1 - \alpha^B) [\delta \pi^{c,B} + (1 - \delta) f^B]}{[1 - (1 - \alpha^B)(1 - \alpha^A)] \delta}.
\]

**Proof:** See Appendix A.2. \( \blacksquare \)

\(^{13}\)Note that since firms are owned by country A, market outcomes in country A are not relevant for the prosecution decision of country B.
The intuition behind Lemma 3 is very similar to the one in Lemma 2. When firms are only colluding in country A, competition authority A prosecutes the firms as soon as collusion is detected because its prosecution decision does not have any impact on the profits of the firms in country B. When firms are colluding in both countries, prosecuting collusion in country A increases aggregate surplus in country A, but it reduces firms’ profits in country B (forcing firms to compete in country B and making them pay fines). The main difference with Lemma 2 is that now competition authority B can detect collusion on its own and, hence, not prosecuting the firms is less valuable for country A. Formally, the right hand side of (6) is decreasing in $\alpha^B$. Thus, as the probability that competition authority B detects collusion on its own increases, it is more likely that condition (6) holds. Finally, note that Lemma 3 is a generalization of Lemma 2. With $\alpha^B = 0$, the prosecution condition (6) becomes (5).

The following proposition studies the equilibrium decisions of the firms given the antitrust policies implemented by the competition authorities of both countries. We focus on the most interesting cases in which firms are willing to collude and the competition authority of country A does not prosecute the firms.

**Proposition 3** Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and $\pi^{c,A} > (1 - \delta) \pi^{d,A}$ and $\pi^{c,B} > (1 - \delta) \pi^{d,B}$. Let

$$
\alpha^B > \frac{2 [\delta \pi^{c,B} + (1 - \delta) f^B] - \delta (\Delta S^A - 2 \pi^{c,A})}{2 [\delta \pi^{c,B} + (1 - \delta) f^B] + \delta (\Delta S^A - 2 \pi^{c,A})},
$$

(7)

$$
\alpha^A \leq \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^A},
$$

(8)

$$
\alpha^B \leq \frac{(\pi^{c,A} + \pi^{c,B}) - (1 - \delta) (\pi^{d,A} + \pi^{d,B})}{\delta (\pi^{d,A} + \pi^{d,B}) - \frac{\delta (\pi^{c,A} - \pi^{c,B})}{1 - \alpha^A} + f^B},
$$

(9)

$$
\alpha^B < \frac{(1 - \delta) \pi^{c,B} + \alpha^A \left[ \delta (\pi^{c,A} + \pi^{c,B}) + (1 - \delta) f^A \right]}{(1 - \alpha^A) \delta f^B}.
$$

(10)

1. Suppose that (7)-(10) hold. Then, firms collude in both countries until the first time $s^B_t = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $s^A_{t+\tau} = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries.

2. Suppose that (7)-(9) hold, but (10) does not hold. Then, firms only collude in country A until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. **Proof:** See Appendix A.2. ■

In order to see the logic behind Proposition 3 it is useful to interpret conditions (7)-(10). (7) is the condition required for competition authority A to do not prosecute the firms when they are colluding in both countries. (8) states that $\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - (1 - \alpha^A) \delta} \geq \pi^{d,A}$, which means that firms are willing to collude in country A even if they know that as soon as collusion is detected, they will be prosecuted. (9) states that $\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^B f^B}{1 - (1 - \alpha^B) \delta} + \frac{\delta \alpha^B (\pi^{c,A} - \alpha^A f^A)}{1 - (1 - \alpha^B) \delta [1 - (1 - \alpha^A) \delta] \delta} \geq \pi^{d,A} + \pi^{d,B}$, which implies that firms are willing
to collude in both countries if they know that competition authority A will prosecute them, but only after competition authority B detects and prosecutes collusion in country B. Finally, (10) means that \( \Pi^{c,AB} > \Pi^{c,A} \), i.e., firms prefer to collude in both countries rather than only in country A.

Proposition 3 Part 1 describes an equilibrium in which firms start colluding in both countries, but they are not prosecuted by competition authority A, even when A detects collusion, i.e., even if \( s^A_t = 1 \). However, competition authority B eventually detects collusion on its own (the first time that \( s^B_t = 1 \)), prosecutes the firms and forces them to compete in country B. This, of course, does not mean that firms stop colluding in country A. Indeed, since \( \Pi^{c,A} \geq \pi^{d,A} \), firms will keep their collusive agreement in country A until they are detected by competition authority A. This time, competition authority A will prosecute the firms, because now there are no profits to protect in country B. Firms can also sustain only colluding in country A, but this collusive agreement generate lower expected profits when \( \Pi^{c,AB} > \Pi^{c,A} \). Finally, note that Proposition 3 Part 1 is a generalization of Proposition 2 Part 1. If \( \alpha^B = 0 \), competition authority B cannot detect collusion on its own and, hence, if competition authority A does not prosecute the firms, there is collusion in both countries forever.

Proposition 3 Part 2 describes an equilibrium in which firms prefer to restrict collusion to country A because the fine that they will have to pay when collusion is detected in country B is too high. Note that this equilibrium is not very likely to occur. Indeed, if colluding in country B generates positive expected profits, firms will always prefer to start colluding in both countries. Formally, \( \Pi^{c-B,A} = \frac{\pi^{c-B} - \alpha^B \pi^A}{1 - (1 - \alpha^B) \delta} > 0 \) is sufficient for \( \Pi^{c,AB} > \Pi^{c,A} \). However, Part 2 is useful to reveal a very interesting mechanism. Even when there is no intrinsic value in colluding in country B, firms might prefer to also collude in country B. Formally, even if \( \Pi^{c,B} < 0 \), it is possible that \( \Pi^{c,AB} > \Pi^{c,A} \). The reason is that colluding in country B postpones prosecution in country A because when firms are colluding in both markets, competition authority A does not prosecute collusion in country A.

4.1 Prosecution Delays

The equilibrium in Proposition 3 Part 1 involves a strategic prosecution delay. In order to see this, the following corollary compares the expected duration of collusion in equilibrium with an hypothetical situation in which both competition authorities prosecute collusion as soon as it is detected.

**Corollary 1** Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and \( \pi^{c,A} > (1 - \delta) \pi^{d,A} \) and \( \pi^{c,B} > (1 - \delta) \pi^{d,B} \).

1. Suppose that competition authority \( j \in \{A, B\} \) prosecutes collusion the first time that \( s^j_t = 1 \) and, under such anti-trust decisions, firms are still willing to collude in both countries. Then, the expected durations of collusion in countries A and B are given by \( \bar{d}^A = \frac{1 - \alpha^A}{\alpha^A} \) and \( \bar{d}^B = \frac{(1 - \alpha^A)(1 - \alpha^B)}{1 - (1 - \alpha^A)(1 - \alpha^B)} \), respectively.

2. Under the assumptions in Proposition 3 Part 1. The expected durations of collusion in countries A and B are given by \( d^A = \frac{\alpha^A + \alpha^B - \alpha^A \alpha^B}{\alpha^B - \alpha^A} \) and \( d^B = \frac{1 - \alpha^B}{\alpha^B} \), respectively.

14This occurs because competition authority A accounts for the profits of the firms in country B and prosecution in country A will trigger prosecution in country B. Additionally, firms are able to collude in both countries because \( \Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B} \).
3. Equilibrium prosecution delays in countries A and B are given by:

\[ d^A - \bar{d}^A = \frac{1}{\alpha^B} \text{ and } d^B - \bar{d}^B = \frac{\alpha^A (1 - \alpha^B)}{\alpha^B [1 - (1 - \alpha^A) (1 - \alpha^B)]} \]

respectively. **Proof:** See Appendix A.2. ■

Two remarks apply to Corollary 1. First, note that the equilibrium in Proposition 3 Part 1 involves a strategic delay in the prosecution of collusion. On average, collusion lasts \( (\alpha^B)^{-1} \) extra periods in country A and \( \frac{\alpha^A (1-\alpha^B)}{\alpha^B[1-(1-\alpha^A)(1-\alpha^B)]} \) extra periods in country B. Second, \( \lim_{\alpha^B \to 0} (d^A - \bar{d}^A) = \infty \) and \( \lim_{\alpha^B \to 0} (d^B - \bar{d}^B) = \infty \). Thus, the equilibrium in Proposition 2 Part 1 generates an infinite prosecution delay.

5 Endogenous Detection Probabilities

Up until this point, we have assumed that the probabilities of detecting collusion are exogenously given. This section incorporates endogenous detection probabilities to the model. Suppose that in period \( t = 0 \) and, before firms make their decisions, each competition authority chooses the probability of detecting collusion.

The cost associated to the probability of detection \( \alpha \) is given by \( C(\alpha) \), assumed smooth, increasing, convex, and satisfying \( C(0) = 0, C'(0) = 0 \) and \( \lim_{\alpha \to 1} C'(\alpha) = \infty \).

We use the augmented model to perform two different exercises. First, taking as given the probability of detection of one competition authority, we study the incentives that the other competition authority has to prosecute collusion depending on the ownership structure and the international presence of the firms. Second, we characterize the Nash equilibrium levels of the probabilities of detection. While the first exercise provides prediction on cross-sector detection probabilities, the second one provides predictions on cross-country detection probabilities.

5.1 Detection Probability as a Function of Ownership and International Presence

We determine the best response detection probability of competition authority A, \( \alpha^A \), given the detection probability of competition authority B, for the following alternative scenarios: 1) The firms are owned by foreign citizens (formally, \( \sigma^1.B = \sigma^2.B = 1 \)). 2) The firms are owned by local citizens and only operate in the domestic market (formally, \( \sigma^1.A = \sigma^2.A = 1 \) and \( \pi^1.B = \pi^2.B = 0 \) for all \( t \)). 3) The firms are owned by local citizens and they also operate in foreign markets (formally, \( \sigma^1.A = \sigma^2.A = 1 \) and \( \pi^1.B = \pi^2.B \geq 0 \) for all \( t \), with strict inequality whenever \( (a^1.B_t, a^2.B_t) \neq (0,0) ) \).

The following proposition characterizes the probability detection selected by competition authority A in each scenario.

**Proposition 4** For a given detection probability of competition authority B, suppose that Assumption 2 holds (with the roles of A and B reversed in Part 1), that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that \( \pi^{c.A} > (1 - \delta) \pi^{d.A} \) and \( \pi^{c.B} > (1 - \delta) \pi^{d.B} \). Then:

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15 The timing is irrelevant because one of the competition authorities has a dominant strategy.

16 See Appendix A.3 for the definitions of \( R^A \) and \( R^B \).
1. Suppose that firms are owned by foreign citizens. Assume that \((a_B, a_H) \in R^B\), where \(a_B\) is the detection probability set by country \(B\) and \(a_H\) is the unique solution to

\[
\frac{\delta (CS^{com,A} - CS^{c,A}) + 2f^A (1 - \delta)}{[1 - (1 - a_H) \delta]^2} = C' (a_H).
\]

Then, the competition authority of country \(A\) selects \(a_A = a_H\).

2. Suppose that firms are owned by local citizens and only operate in the domestic market. Assume that \(a_M \leq \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^A}\), where \(a_M\) is the unique solution to

\[
\frac{\delta (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1 - (1 - a_M) \delta]^2} = C' (a_M).\]

Then, the competition authority of country \(A\) selects \(a_A = a_M\).

3. Suppose that firms are owned by local citizens and they also operate in foreign markets. Assume that \((a_L, a_B) \in R^A\), where \(a_B\) is the detection probability set by country \(B\) and \(a_L\) is the unique solution to

\[
\frac{a_B \delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1 - (1 - a_B) \delta]^2 [1 - (1 - a_L) \delta]} = C' (a_L),\]

Then, the competition authority of country \(A\) selects \(a_A = a_L\). Moreover, \(a_L\) is increasing in \(a_B\).

4. Suppose that \((a_B, a_H) \in R^B\), \(a_M \leq \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^A}\) and \((a_L, a_B) \in R^A\). Then, \(a_H > a_M > a_L\), i.e., the competition authority of country \(A\) is tougher with foreign firms than with purely domestic firms and even less prone to prosecute collusion of domestic firms that operate in foreign markets.

**Proof:** See Appendix A.3.

The intuition behind Proposition 4 is as follows. When firms are owned by foreign citizens, competition authority \(A\) always prosecutes collusion as soon as it is detected. (This is just Lemma 1 with \(A\) playing the role of \(B\).) However, competition authority \(B\) will not prosecute the firms before they are prosecuted in country \(A\). (This is Lemma 3 Part 2 with \(B\) playing the role of \(A\).) As a consequence, the only way in which \(A\) can detect and prosecute collusion in country \(A\) is relying on its own detection efforts. In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country \(A\) is given by

\[
W_0^A = \frac{CS^{c,A} + 2a_A f^A + \frac{a_A^2}{1 - a_A^2} CS^{com,A}}{1 - (1 - a_A) \delta} - C' (a_A),
\]

which has a maximum at \(a_A = a_H\). Note that competition authority \(A\) simply equates the expected marginal benefit of detection \((\frac{\delta (CS^{com,A} - CS^{c,A}) + 2f^A (1 - \delta)}{[1 - (1 - a_H) \delta]^2})\) with the marginal cost of detection \((C' (a_H))\).

Also note that \(a_H\) does not depend on \(a_B\). The reason is that competition authority \(B\) refuses to prosecute the firms until they are prosecuted in country \(A\).
When firms are owned by local citizens and only operate in the domestic market, competition authority \( A \) always prosecutes collusion as soon as it is detected. (This is Lemma 3 Part 1). In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country \( A \) is given by

\[
W_0^A = \frac{CS^{c,A} + 2\pi^{c,A} + \frac{\alpha^A}{1-\delta}CS^{com,A}}{1 - (1 - \alpha^A)\delta} - C(\alpha^A),
\]

which has a maximum at \( \alpha^A = \alpha_M \). Since firms only operate in country \( A \), \( \alpha_M \) is independent of \( \alpha^B \).

When firms are owned by local citizens and they also operate in foreign markets, competition authority \( A \) will prosecute collusion in country \( A \) only after competition authority \( B \) prosecutes collusion in country \( B \). In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country \( A \) is given by

\[
W_0^A = \frac{(CS^{c,A} + 2\pi^{c,A}) + \frac{\alpha^B}{1 - (1 - \alpha^B)\delta}2(\pi^{c,B} - \alpha^B f^B) + \frac{\alpha^A\alpha^B}{[1-\delta][1-(1-\alpha^B)\delta]}CS^{com,A}}{1 - (1 - \alpha^B)\delta} - C(\alpha^A),
\]

which has a maximum at \( \alpha^A = \alpha_L \). Note that \( \alpha_L \) is a function of \( \alpha^B \). Indeed, \( \alpha_L \) is increasing in \( \alpha^B \). The reason is that as competition authority \( B \) is more likely to detect collusion in country \( B \), the sooner collusion will be prosecuted in country \( B \) and, hence, the higher the expected marginal benefit of detecting collusion sooner in country \( A \).

Finally, Part 4 is the most interesting result in Proposition 4. It states that, ceteris paribus the market variables \((CS^{com,A} - CS^{c,A})\) and \(\pi^{c,A}\), the discount factor \(\delta\) and the cost of detection \(C\), competition authority \( A \) is more willing to invest in detection when firms are owned by foreigners than when they are owned by domestic citizens. Furthermore, competition authority \( A \) is less willing to invest in detection when firms owned by domestic citizens are colluding in foreign markets. Proposition 4 Part 4 can also be interpreted as a prediction of cross-sector detection probabilities. Ceteris paribus, \((CS^{com,A} - CS^{c,A})\), \(\pi^{c,A}\), \(\delta\), and \(C\), competition authorities will tend to invest more on detecting collusion in sectors dominated by foreign firms than in those dominated by firms owned by local citizens and, among the last group, competition authorities will tend to be even less harsh with firms that are colluding in foreign markets.

### 5.2 Nash Equilibrium Detection Probabilities

The following proposition characterizes the Nash equilibrium levels of the detection probabilities.

**Proposition 5** Suppose that Assumption 2 holds, that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that \(\pi^{c,A} > (1 - \delta)\pi^{d,A}\) and \(\pi^{c,B} > (1 - \delta)\pi^{d,B}\).\(^{17}\) Let \((\alpha^{A,*}, \alpha^{B,*}) \in R^A\), where \((\alpha^{A,*}, \alpha^{B,*})\) is the unique solution to

\[
\frac{\alpha^B \delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1 - (1 - \alpha^A)\delta]^2 [1 - (1 - \alpha^B)\delta]} = C'(\alpha^{A,*}),
\]

\[
\frac{\delta (CS^{com,B} - CS^{c,B}) + 2f^B (1 - \delta)}{[1 - (1 - \alpha^B)\delta]^2} = C'(\alpha^{B,*}).
\]

\(^{17}\)See Appendix A.3 for the definition of \(R^A\).
Then, the Nash equilibrium probabilities of detection selected by the competition authorities are \((\alpha^A, \alpha^B) = (\alpha^{A*}, \alpha^{B*})\). Moreover, if \(\delta (CS_{\text{com,B}} - CS_{\text{c,B}}) + 2f^B (1 - \delta) > \delta^2 (CS_{\text{com,A}} - CS_{\text{c,A}} - 2p_{\text{c,A}})\), then \(\alpha^{B*} > \alpha^{A*}\). \(^\square\)

Two remarks apply to Proposition 5. First, note that competition authority \(B\) has a dominant strategy \((\alpha^B = \alpha^{B*})\) regardless of \(\alpha^A\), while the best response of competition authority \(A\) depends on \(\alpha^B\). Second, under a relatively mild condition, the country that owns the firms invest less on detection than the country that does not own the firms \((\alpha^{B*} > \alpha^{A*})\). The reason is that for the country that owns the firms, prosecuting collusion has the extra cost of losing collusion profits in the other country.

6 Antitrust Policy, Integration and International Agreements

Up until this point we have studied the equilibrium prosecution decisions of two independent competition authorities. That is, the focus has been on the positive side of the problem. From a normative perspective, it is clear that in order to maximize the aggregate expected welfare of the world \((W_0^W = W_0^A + W_0^B)\), both competition authorities should prosecute collusion as soon as it is detected. Indeed, this would be the policy selected by a globally integrated competition authority with a mandate to maximize \(W_0^W\). Compared with the equilibrium prosecution decisions, full prosecution increases the world’s welfare as well as the welfare of the country that does not own the firms, but it hurts the country of origin of the firms, which would prefer to delay prosecution. Moreover, it would be very complicated to reach an international agreement that implements full prosecution, as the country of origin of the firms would not be willing to participate.\(^{18}\) Note, however, that this logic might not apply if there are several industries and the firms of each country operate and try to organize collusion in different industries. In this section we argue that, by introducing multiple industries, with different countries of origin, there might be room for international agreements to be put in place. In order to formally explore this possibility, we incorporate a second industry to our model.

As in previous sections, assume there are two countries \((A \text{ and } B)\) with their respective competition authorities. In each country there are two industries, denoted by \(x\) and \(y\). Only two companies operate in each industry: companies 1, \(x\) and 2, \(x\) in industry \(x\) and companies 1, \(y\) and 2, \(y\) in industry \(y\). Let \((a^i_{t},z,A, a^i_{t},z,B)\) with \(i \in \{1, 2\}\) and \(z \in \{x, y\}\) denote the decision of firm \(i, z\) in period \(t\), where \(a^i_{t},z,j = 1\) indicates that firm \(i, z\) chooses to collude in country \(j\) and \(a^i_{t},z,j = 0\) indicates that firm \(i, z\) chooses to compete in country \(j\). Collusion can only occur within industry. Let \((\pi^i_{t},z,A, \pi^i_{t},z,B)\) denote the profits that company \(i, z\) obtains in period \(t\) from its operations in countries \(A \text{ and } B\). Assume that profits are given by:

<table>
<thead>
<tr>
<th>Firm 1, (z) - Firm 2, (z)</th>
<th>(a^i_{t},z,j = 1)</th>
<th>(a^i_{t},z,j = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^i_{t},z,j) = 1 (\pi^c_{t},z,j)</td>
<td>(0), (\pi^d_{t},z,j)</td>
<td>(0), (0)</td>
</tr>
<tr>
<td>(\pi^i_{t},z,j) = 0 (\pi^d_{t},z,j)</td>
<td>(0), (\pi^d_{t},z,j)</td>
<td>(0), (0)</td>
</tr>
</tbody>
</table>

where \(\pi^d_{t},z,j > \pi^c_{t},z,j > 0\). Denote by \(\alpha^{i,z,j}\) the share of firm \(i, z\) owned by citizens from country \(j\). Let \(CS_{t}^{i,z,j}\) be the consumer surplus in industry \(z\) and country \(j\). Assume that \(CS_{t}^{i,z,j} = CS_{t}^{c,z,j}\) when \(a^i_{t},z,j = a^i_{t},z,j = 1\) and \(CS_{t}^{i,z,j} = CS_{t}^{\text{com},z,j}\), otherwise, where \(CS_{t}^{\text{com},z,j} > CS_{t}^{c,z,j} + 2\pi^c_{t},z,j\) for all \(z, j\).

\(^{18}\)If forced to do so, it will have a strong incentive to withhold any information of collusion.
Finally, denote by $f^{z,j} > \pi^{c,z,j}$ the fine charged by the competition authority of country $j$ if firms are found organizing collusion in industry $z$.

The timing is as follows.

1. In each industry both firms simultaneously choose to collude or compete in each country.

2. Nature sends a signal to the competition authority of country $j$ ($k$) regarding the behavior of firms in industry $x$ ($y$) in country $j$ ($k$). Upon observing the signal, the competition authority of country $j$ ($k$) decides whether to prosecute the firms or not in industry $x$ ($y$).

3. Nature sends a signal to the competition authority of country $−j$ ($−k$) regarding the behavior of firms in industry $x$ ($y$) in country $−j$ ($−k$). Upon observing the signal, the competition authority of country $−j$ ($−k$) decides whether to prosecute the firms or not in industry $x$ ($y$).

A natural generalization of the ownership distribution studied in the case of one industry is to assume that each country owns the two firms of one of the industries. Formally, let $σ^{1,x,A} = σ^{2,x,A} = 1$ and $σ^{1,y,B} = σ^{2,y,B} = 1$, meaning that only citizens of country $A$ ($B$) own the firms in industry $x$ ($y$). Regarding collusion detection, the following assumption is a generalization of Assumption 2 for an environment with two industries.

**Assumption 3.** The signals received by the competition authorities are as in Assumption 2, with the roles of countries $A$ and $B$ reversed in the case of industry $y$ (see Appendix A.4 for details).

### 6.1 Independent Competition Authorities

Suppose that each country has an independent competition authority with a mandate to maximize the country’s expected aggregate welfare. In such a context, it is straightforward to generalize Proposition 3 Part 1 to the case of two industries (see Appendix A.4 for details). In equilibrium, competition authority $A$ delays prosecuting firms in industry $x$ because it does not want to trigger prosecution in country $B$, while the opposite happens in industry $y$, where competition authority $B$ waits until firms are prosecuted in country $A$ to prosecute collusion in country $B$.

### 6.2 A Globally Integrated Competition Authority

Suppose that the competition authorities of both countries merge and form a globally integrated competition authority with a mandate to maximize the world’s aggregate expected welfare. In such an environment, it is not difficult to show that the global competition authority will always prosecute collusion as soon as it is detected (see Appendix A.4 for details). This, of course, will change the incentives of the firms to organize collusion. In order to keep the analysis as simple as possible and to avoid exaggerating the welfare impact of integration, we focus on a region of the parameter space in which firms are still

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19Note that with this timing the signal received by the competition authority of country $−j$ ($−k$) about industry $x$ ($y$) could depend on the prosecution decision implemented by the competition authority of country $j$ ($k$).

20For example, this could capture the situation of the European Union if part of the integration process includes the consolidation of all the national competition authorities into one competition authority for the whole union. Alternatively, the situation under integration could better approximate the present anti-trust policy in the United States, while independent competition authorities could capture an alternative institutional arrangement in which anti-trust policy is fully delegated to the states.
willing to collude in both countries until detected and, if they are only detected in one country, keep colluding in the other country. Thus, in equilibrium, firms in industry \( x \) (\( y \)) collude in both countries until the first time \( s_{ti}^{x,A} = 1 \) \( (s_{ti}^{y,B} = 1) \), when they are prosecuted in both countries, or until the first time \( s_{ti}^{x,A} = 0 \) and \( s_{ti}^{y,B} = 1 \) \( (s_{ti}^{x,A} = 0 \) and \( s_{ti}^{y,B} = 1) \), when they are prosecuted in country \( B \) \( (A) \). In the latter case, firms keep colluding in country \( A \) \( (B) \) until the first time \( s_{ti+1}^{x,A} = 1 \) \( (s_{ti+1}^{y,B} = 1) \) with \( t \geq 1 \) (see Appendix A.4 for details). In other words, a globally integrated competition authority might not be able to dissuade firms to collude, but, at least, it eliminates the prosecution delays associated with independent competition authorities with a mandate to maximize national welfare.

### 6.3 Welfare Comparison

The following proposition compares the expected welfare of each country when each competition authority maximizes the welfare of its country under integration.

**Proposition 6** Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and \( \pi^{c,z,j} > (1 - \delta) \pi^{d,z,j} \) for \( z \in \{x, y\} \) and \( j \in \{A, B\} \). Assume that \((\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \cap R^{x,A}\) and \((\alpha^{y,B}, \alpha^{y,A}) \in R^{y,B} \cap R^{y,B}\).

Then:

1. **Country A benefits from integration if and only if \( \Delta W_{0}^{x,A} + \Delta W_{0}^{y,A} > 0 \) and (if \( j = B \)), where**

\[
\Delta W_{0}^{x,A} = \frac{\alpha^{x,A}}{1 - (1 - \alpha^{x,B})} \left\{ \delta \left( \Delta CS^{x,A} - 2\pi^{c,x,A} \right) - \frac{2(1 - \alpha^{x,B}) \left[ \delta \pi^{c,x,B} + (1 - \delta) f^{x,B} \right]}{1 - (1 - \alpha^{x,A}) \delta} \right\}
\]

\[
\Delta W_{0}^{y,A} = \frac{\alpha^{y,B}}{1 - (1 - \alpha^{y,A}) \delta} \left\{ \delta \Delta CS^{y,A} + (1 - \delta) 2f^{y,A} \right\}
\]

where \( \Delta CS^{z,j} = CS^{\text{coll},z,j} - CS^{c,z,j} \).

2. **Country B benefits from integration if and only if \( \Delta W_{0}^{x,B} + \Delta W_{0}^{y,B} > 0 \), where**

\[
\Delta W_{0}^{x,B} = \frac{\alpha^{x,A}}{1 - (1 - \alpha^{y,A}) \delta} \left\{ \delta \Delta CS^{x,B} + (1 - \delta) 2f^{x,B} \right\}
\]

\[
\Delta W_{0}^{y,B} = \frac{\alpha^{y,B}}{1 - (1 - \alpha^{y,B})} \left\{ \delta \left( \Delta CS^{y,B} - 2\pi^{c,y,B} \right) - \frac{2(1 - \alpha^{y,A}) \left[ \delta \pi^{c,y,A} + (1 - \delta) f^{y,A} \right]}{1 - (1 - \alpha^{y,B}) \delta} \right\}
\]

3. **Moreover, if \( \pi^{c,z,j} = \pi^{c}, \pi^{d,z,j} = \pi^{d}, \Delta CS^{z,j} = \Delta CS, \alpha^{z,j} = \alpha, \) and \( f^{z,j} = f \) for all \( z \in \{x, y\} \) and \( j \in \{A, B\}, \) it is always the case that both countries are better off under integration.**

**Proof:** See Appendix A.4. ■

The intuition behind Proposition 6 is simple. On the one hand, country \( A \) obtains a lower aggregate welfare in industry \( x \) under integration (\( \Delta W_{0}^{x,A} < 0 \)). The reason is that an independent competition authority will delay prosecution only when the profits from collusion in country \( B \) outweigh the benefit from stopping collusion in country \( A \). On the other hand, country \( A \) obtains a higher aggregate welfare in...
industry $y$ under integration ($\Delta W_{0y,A} > 0$) because an integrated competition authority does not delay the prosecution of firms from country $B$ organizing collusion in country $A$. Note that it is perfectly possible that $\Delta W_{0x,A} + \Delta W_{0y,A} > 0$ and, hence, country $A$ is better off under integration. The welfare comparisons for country $B$ follow the same logic, except that we must reverse the industries. In other words, under integration, country $B$ obtains a higher aggregate welfare in industry $x$, but a lower aggregate welfare in industry $y$ ($\Delta W_{0x,B} > 0$ and $\Delta W_{0y,B} < 0$). Again, if $\Delta W_{0x,B} + \Delta W_{0y,B} > 0$, country $B$ is also better off under integration. Finally, Part 3 considers a particular case in which everything is symmetric (across industries and countries). In such a case, the extra profits that one country is obtaining from collusion in the other country in one industry are perfectly offset by the extra profits that foreign firms are obtaining from collusion in the domestic market in the other industry. Thus, once we aggregate both industries, integration is neutral with respect to collusive profits, but it eliminates the deadweight loss associated with collusion. Proposition 6 shows that, given the incentives that independent national authorities have to delay prosecution, both countries might be better off if they integrate their competition authorities.

6.4 International Agreements and the Goal of Competition Authorities

Is it possible to support the equilibrium under a globally integrated competition authority without actually integrating them? If only countries were willing to commit to do not delay prosecution, then they could replicate the equilibrium under integration. At first sight, a simple way of inducing this is to assume that the goal of each competition authority is to maximize the consumer surplus of domestic consumers (rather than national welfare). In such environment, collusion in both industries will be prosecuted as soon as it is detected because competition authorities pay no attention to profits.

Unfortunately, the problem is not so straightforward. To see this, suppose that one of the competition authorities (say the competition authority of country $A$) has a mandate to maximize the consumer surplus of domestic consumers rather than national welfare. True to its mandate, competition authority $A$ will prosecute collusion as soon as it is detected in each industry. But then, the optimal reaction of country $B$ will be to strategically delay prosecution in industry $y$. In equilibrium, there will be no strategic delay in stopping collusion in industry $x$ (where firms are owned by citizens of country $A$), but, on average, collusion will last more than necessary in industry $y$ (where firms are owned by citizens of country $B$). In terms of national welfare, this is the worst possible outcome for country $A$ and the best possible one for country $B$. In other words, the antitrust prosecution game has the structure of a prisoner’s dilemma. Regardless of what competition authority $A$ (or $B$) does, the best reaction of country $B$ (or $A$) is to instruct its competition authority to delay prosecution in industry $y$ (or $x$). Thus, committing to do not delay prosecution does not change the behavior of the other competition authority.22

The solution is to reach an international agreement in which each country agrees to instruct its competition authority to only consider the wellbeing of domestic consumers as long as the other country follows the same course of action. Thus, in a global economy the mandate of a competition authority should not be defined independently of the mandate of the other competition authority. Moreover, note that a unilaterally commitment to maximize the wellbeing of domestic consumers will not bring the other country to the bargaining table. What is required is a combination of carrots and sticks. The message to the other country should be “there is quid pro quo agreement through which we commit to maximize

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22This follows the same logic of trade agreements (see Bagwell and Staiger (1999)). If a country is fully committed to free trade (that is zero tariff), the best response of the other country is to impose the unilateral optimum tariff.
the wellbeing of domestic consumers, but only if you do exactly the same. Otherwise, we will instruct our competition authority to maximize national welfare”.

7 Conclusions

This paper pushes the frontier of the analysis of antitrust policy in open economies. We develop a political economy model of antitrust enforcement in an open economy and characterize the equilibrium prosecution policies selected by benevolent national competition authorities. In the several scenarios studied in our model, we show that price fixing agreements reduce the world’s aggregate welfare and, therefore, should be prevented. However, we also show that national competition authorities may have biased incentives towards the prosecution of collusive activities. In particular, the country of origin of the firms has weaker incentives to prosecute collusion because domestic prosecution spirals into foreign prosecution, which reduces the profits of domestic firms in foreign markets. This misalignment between the equilibrium prosecution policies and the global welfare maximizing solution could be solved by integrating the competition authorities. Integration would, nevertheless find resistance by the country of origin of the firms, undermining its efficacy. Such a solution is more likely to succeed in a multi-industry world where each country specializes in a different industry. Indeed, we have shown that in a multi-industry world each country could be better off if an internationally integrated competition authority decided on prosecution based on global welfare.

Our results have important implications for the design of antitrust enforcement institutions and agencies. First, although there might be benefits from decentralizing antitrust enforcement to subnational entities, our model suggests that countries should centralize antitrust enforcement in a national competition authority. Second, our results suggest that competition authorities should consider the origin of the firms and their foreign operations when they decide to initiate a collusion case. Political transparency could be a problem. How can the public distinguish a competition authority captured by the firms from one dedicated to maximize national welfare that does not prosecute some firms to protect their foreign profits? For some industries both could be observationally equivalent. There might also exist practical barriers to implement such policy. For example, firms can employ accounting tricks to assign profits to different countries in order to inflate their foreign operations and avoid prosecution.

There are several avenues to expand our analysis. For example, we have developed a two-country model, but the mechanism behind our results should also apply to a setting with multiple countries. More importantly, in such setting there will be room for the strategic formation of coalitions of countries that decide to integrate their competition authorities. Finally, the logic highlighted in this paper can be naturally generalized beyond antitrust enforcement to areas such as international regulation and abuse of dominant position.

References


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Appendix to “Of Course Collusion Should be Prosecuted. But Maybe ... Or (The Case for International Antitrust Agreements)"

This Appendix presents the proofs of all lemmas and propositions.

A.1 Detection Ability Fully Aligned with Firm Ownership

A.1.1 Proof of Lemma 1

Lemma 1. Suppose that Assumption 1 holds. Then, the competition authority of country B prosecutes the firms as soon as collusion is detected in country B.

Proof. Suppose that the competition authority of country B detects collusion in period \( t \), i.e., \( s^B_t = 1 \).

Note that this can only occur if \( a^1_t,B = a^2_t,B = 1 \) and \( p^A_\tau = 1 \) for some \( \tau \leq t \). If the competition authority of country B prosecutes the firms, then the expected discounted welfare of country B at period \( t \) is

\[
W^B_t (p^B_t = 1|s^B_t = 1) = CS^{c,B} + 2f^B + \frac{\delta}{1-\delta}S^{com,B},
\]

while if firms are not prosecuted, it is

\[
W^B_t (p^B_t = 0|s^B_t = 1) = \frac{CS^{c,B}}{1-\delta}.
\]

Since \( CS^{com,B} > CS^{c,B} \), it must be the case that \( W^B_t (p^B_t = 1|s^B_t = 1) > W^B_t (p^B_t = 0|s^B_t = 1) \). Thus, whenever B detects collusion, it immediately prosecutes the firms. ■

A.1.2 Proof of Lemma 2

Lemma 2. Suppose that Assumption 1 holds. Assume that the competition authority of country A detects collusion, i.e., \( s^A_t = 1 \).

1. If the firms are not colluding in country B, then A always prosecutes the firms.

2. If the firms are also colluding in country B, then A prosecutes the firms if and only if

\[
\delta \geq \bar{\delta} = \frac{2f^B}{CS^{com,A} - CS^{c,A} - 2\pi^{c,A} + 2f^B - 2\pi^{c,B}}.
\]

Alternatively, (5) can be written as

\[
\delta (CS^{com,A} - CS^{c,A} - 2\pi^{c,A}) \geq 2 \left[ \delta \pi^{c,B} + (1 - \delta) f^B \right].
\]

Proof. Suppose that firms are only colluding in country A while there is competition in country B and the competition authority of country A detects collusion in period \( t \), i.e., \( s^A_t = 1 \). If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period \( t \) is\(^{23}\)

\[
W^A_t (p^A_t = 1|s^A_t = 1, A) = CS^{c,A} + 2(\pi^{c,A} - f^A) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A},
\]

\(^{23}\)Note that we employ the following notation. \( W^A (p^A_t = 1|s^A_t = 1, A) \) is the discounted expected welfare of country A when firms are only colluding in country A, \( s^A_t = 1 \) and competition authority A choses to prosecute the firms. \( W^A (p^A_t = 1|s^A_t = 1, AB) \) is the discounted expected welfare of country A when firms are colluding in both countries, \( s^A_t = 1 \) and competition authority A choses to prosecute the firms.
while if firms are not prosecuted, it is

\[ W_t^A (p_t^A = 0|s_t^A = 1, A) = \frac{CS_{c,A} + 2\pi_{c,A}}{1 - \delta}. \]

Since \( CS_{com,A} > CS_{c,A} + 2\pi_{c,A} \), it must be the case that \( W_t^A (p_t^A = 1|s_t^A = 1, A) > W_t^A (p_t^A = 0|s_t^A = 1, A) \).

Suppose that firms are colluding in both countries and the competition authority of country A detects collusion in period \( t \), i.e., \( s_t^A = 1 \). If the competition authority of country A decides to prosecute the firms, then the competition authority of country B will detect collusion in country B as well, i.e., \( s_t^B = 1 \). Hence, firms will be also prosecuted in country B. Thus, if firms are prosecuted in country A, the expected discounted welfare of country A at period \( t \) will be

\[ W_t^A (p_t^A = 1|s_t^A = 1, AB) = CS_{c,A} + 2(\pi_{c,A} - f^A + \pi_{c,B} - f^B) + 2f^A + \frac{\delta}{1 - \delta}CS_{com,A}. \]

If the competition authority of country A does not prosecute the firms, then there is no way that the competition authority of country B finds that firms are also colluding in country B. Then, firms will continue colluding in both countries and the expected discounted welfare of country A at period \( t \) will be

\[ W_t^A (p_t^A = 0|s_t^A = 1, AB) = \frac{CS_{c,A} + 2(\pi_{c,A} + \pi_{c,B})}{1 - \delta}. \]

\[ W_t^A (p_t^A = 1|s_t^A = 1, AB) > W_t^A (p_t^A = 0|s_t^A = 1, AB) \] if and only if \( \delta \left( CS_{com,A} - CS_{c,A} - 2\pi_{c,A} \right) \geq 2\left[ \delta\pi_{c,B} + (1 - \delta) f^B \right] \) (equivalently, \( \delta \geq \frac{2f^B}{CS_{com,A} - CS_{c,A} - 2\pi_{c,A} + 2f^B - 2\pi_{c,B}} \)).

A.1.3 Proof of Proposition 1

**Proposition 1.** Suppose that Assumption 1 holds, firms can either collude in both countries or in none of them and they coordinate in their best equilibrium.

1. Suppose that \( \bar{\delta}^1 < \delta < \bar{\delta} \), where \( \bar{\delta}^1 = \frac{\pi_{d,A} + \pi_{d,B} - \pi_{c,A} - \pi_{c,B}}{\pi_{d,A} + \pi_{d,B} - \pi_{c,A} - \pi_{c,B}} \). Then, firms collude in both countries and they are never prosecuted.

2. Suppose that \( \delta \geq \bar{\delta} \). Then, there is a threshold \( \bar{\alpha}^1 \) such that:

   (a) If \( \alpha^A \leq \bar{\alpha}^1 \), then there is global collusion until the first time \( s_t^A = 1 \), when firms are prosecuted in both countries. Thereafter, there is competition.

   (b) If \( \alpha^A > \bar{\alpha}^1 \), then there is always competition.

**Proof.** Suppose that \( \bar{\delta} < \delta \). Then, from Lemmas 1 and 2, if firms collude in both countries they will be never prosecuted. Therefore, the expected profits of a firm under collusion are given by:

\[ \Pi_{c,AB} = \frac{\pi_{c,A} + \pi_{c,B}}{1 - \delta}. \]
(Recall that firms can either collude in both countries or in none of them). Collusion can be sustained as a subgame perfect Nash equilibrium whenever $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which always holds since $\delta^1 < \delta$ implies $\pi^{c,A} + \pi^{c,B} > (1 - \delta)(\pi^{d,A} + \pi^{d,B})$.

Suppose $\delta \geq \delta$. Then, from Lemmas 1 and 2, if firms collude in both countries they will be prosecuted the first time $\alpha^2 = 1$. Therefore, the expected profits of a firm under collusion are given by $\Pi^{c,AB} = \alpha^A (\pi^{c,A} + \pi^{c,B} - f^A - f^B) + (1 - \alpha^A) (\pi^{c,A} + \pi^{c,B} + \delta \Pi^{c,AB})$, which implies

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A (f^A + f^B)}{1 - (1 - \alpha^A) \delta}.$$  

Collusion can be sustained as a subgame perfect Nash equilibrium if and only if $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. (Since firms are not allowed to collude in each market separately, when a firm violates the collusive agreement, it deviates in both countries). Therefore, if $\alpha^A \leq \alpha^1 = \frac{(\pi^{c,A} + \pi^{c,B}) - (1 - \delta)(\pi^{d,A} + \pi^{d,B})}{f^A + f^B + \delta(\pi^{d,A} + \pi^{d,B})}$, firms collude until they are prosecuted, while if $\alpha^A > \alpha^1$ firms do not collude at all. 

A.1.4 Proof of Proposition 2

**Proposition 2.** Suppose that Assumption 1 holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium.

1. Suppose that $\delta^2 < \delta < \bar{\delta}$, where $\delta^2 = \max \left\{ \frac{\pi^{d,A} - \pi^{c,A}}{\pi^{c,A}}, \frac{\pi^{d,B} - \pi^{c,B}}{\pi^{c,B}} \right\}$. Then, firms collude in both countries and they are never prosecuted.

2. Suppose that $\delta \geq \bar{\delta}$. Then, there are thresholds $\tilde{\alpha}_H^2$ and $\bar{\alpha}_H^2$ such that:

   (a) Suppose that $\alpha^A > \bar{\alpha}_H^2$. Then, firms collude in country B and they are never prosecuted.

   (b) Suppose that $\bar{\alpha}_L^2 < \alpha^A \leq \bar{\alpha}_H^2$. Then, there is a threshold $\tilde{\alpha}_M^2$ such that: If $\alpha^A \in (\bar{\alpha}_L^2, \tilde{\alpha}_M^2)$, then firms collude (only in country A or in both countries) until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\tilde{\alpha}_M^2, \bar{\alpha}_H^2]$, then firms only collude in country B and they are never prosecuted.

   (c) Suppose that $\alpha^A \leq \bar{\alpha}_L^2$. Then, there is a threshold $\bar{\alpha}_M^2$ such that: If $\alpha^A \in (0, \bar{\alpha}_M^2]$, then firms collude (only in country A or in both countries) until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\bar{\alpha}_M^2, \bar{\alpha}_H^2]$, then firms only collude in country B and they are never prosecuted.

**Proof.** Suppose that $\delta^2 < \delta < \bar{\delta}$. Then, from Lemmas 1 and 2, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - (1 - \alpha^A) \delta}, \quad \Pi^{c,B} = \frac{\pi^{c,B}}{1 - \delta}, \quad \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1 - \delta},$$

respectively.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$.
For collusion in country $B$ to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Note that, since the punishment for deviation is competition in both countries, once a firm decides to violate the collusive agreement, it deviates in both countries. Since $\pi^{-} > (1 - \delta) \pi^{-j}$ for $j = A, B$, collusion in both countries can always be sustained. Therefore, if $\Pi^{c,A} \geq \pi^{d,A}$, the three types of collusion can be sustained as an equilibrium, while if $\Pi^{c,A} < \pi^{d,A}$, only collusion in country $B$ or collusion in both countries can be sustained as an equilibrium. Finally, note that $\Pi^{c,AB} > \Pi^{c,A}, \Pi^{c,B}$. Thus, firms prefer to coordinate in an equilibrium in which they collude in both countries.

Suppose that $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, the expected profits of a firm under collusion in country $A$, collusion in country $B$ and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^{A} f^{A}}{1 - (1 - \alpha^{A}) \delta}, \quad \Pi^{c,B} = \frac{\pi^{c,B}}{1 - \delta}, \quad \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^{A} (f^{A} + f^{B})}{1 - (1 - \alpha^{A}) \delta},$$

respectively.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country $A$ to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in country $B$ to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in both countries, which requires $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Therefore, we must consider four different cases:

1. Only collusion in country $A$ can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\alpha^{A} > \tilde{\alpha}^{2} = \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^{A}}$ and $\alpha^{A} > \hat{\alpha}^{2} = \frac{(\pi^{c,A} + \pi^{c,B}) - (1 - \delta) \pi^{d,A} + \pi^{d,B}}{\delta (\pi^{d,A} + \pi^{d,B}) + f^{A} + f^{B}}$. Thus, if $\alpha^{A} > \max \{\hat{\alpha}^{2}, \tilde{\alpha}^{2}\}$, firms only collude in country $B$ and they are never detected. 

2. Only collusion in country $B$ or in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\hat{\alpha}^{2} < \alpha^{A} \leq \tilde{\alpha}^{2}$. If $\tilde{\alpha}^{2} > \hat{\alpha}^{2}$, this never holds, while if $\hat{\alpha}^{2} < \tilde{\alpha}^{2}$, firms must decide between collusion in $B$ and collusion in both countries. Firms prefer to collude in both countries when $\Pi^{c,AB} > \Pi^{c,B}$ and to collude in $B$ when $\Pi^{c,AB} < \Pi^{c,B}$. Note that $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\alpha^{A} < \tilde{\alpha}^{2} = \frac{(1 - \delta) \pi^{c,A}}{\delta \pi^{d,A} + f^{A} + f^{B}}$. Thus, if $\tilde{\alpha}^{2} < \hat{\alpha}^{2}$, firms collude in both countries when $\alpha^{A} < \hat{\alpha}^{2}$ and they collude in country $B$ when $\alpha^{A} > \hat{\alpha}^{2}$. 

3. Only collusion in country $B$ or collusion in country $A$ can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\hat{\alpha}^{2} < \alpha^{A} \leq \tilde{\alpha}^{2}$. If $\tilde{\alpha}^{2} > \hat{\alpha}^{2}$, this never holds, while if $\hat{\alpha}^{2} < \tilde{\alpha}^{2}$, firms must decide between collusion in $A$ and collusion in country $B$. Firms prefer to collude in $A$ when $\Pi^{c,A} > \Pi^{c,B}$ and to collude in $B$ when $\Pi^{c,A} < \Pi^{c,B}$. Note that $\Pi^{c,A} > \Pi^{c,B}$ if and only if $\alpha^{A} < \hat{\alpha}^{2} = \frac{(1 - \delta) \pi^{c,A} - \pi^{c,B}}{\delta \pi^{d,A} + f^{A} + f^{B}}$. Thus, if $\hat{\alpha}^{2} < \tilde{\alpha}^{2}$, firms collude in country $A$ when $\alpha^{A} < \hat{\alpha}^{2}$ and they collude in country $B$ when $\alpha^{A} > \hat{\alpha}^{2}$.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\alpha^{A} \leq \min \{\hat{\alpha}^{2}, \tilde{\alpha}^{2}\}$. Firms prefer to collude in $A$ when $\Pi^{c,A} > \max \{\Pi^{c,B}, \Pi^{c,AB}\}$, i.e., when $\pi^{c,B} / f^{B} < \alpha^{A} < \hat{\alpha}^{2}$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max \{\Pi^{c,A}, \Pi^{c,B}\}$, i.e., when $\alpha^{A} < \min \{\pi^{c,B} / f^{B}, \tilde{\alpha}^{2}\}$. Finally, firms prefer to collude in $B$ when $\Pi^{c,B} > \max \{\Pi^{c,A}, \Pi^{c,AB}\}$, i.e., when $\alpha^{A} > \max \{\hat{\alpha}^{2}, \tilde{\alpha}^{2}\}$. Moreover, $\hat{\alpha}^{2} < \tilde{\alpha}^{2}$ if and only if $\pi^{c,B} / f^{B} < \hat{\alpha}^{2}$.

Employing cases 1-4 we can distinguish four different situations:
1. Suppose that \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} - \frac{\pi_c^c,1 - \delta, c}{\omega^c} < \frac{\pi_c^c,1 - \delta, c}{\omega^c} < \frac{\pi_c^c,1 - \delta, c}{\omega^c} \) and \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} < \frac{\pi_c^c,1 - \delta, c}{\omega^c} \). Then, \( \alpha^2 < \bar{\alpha}^2 \) and \( \bar{\alpha}^2 < \bar{\alpha}^2 \). Therefore:

(a) If \( \alpha^2 > \bar{\alpha}^2 \), then firms only collude in country \( B \) and they are never detected.

(b) If \( \bar{\alpha}^2 < \alpha^2 \leq \bar{\alpha}^2 \), then firms collude in country \( A \) when \( \alpha^2 < \bar{\alpha}^2 \) and they collude in country \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

(c) If \( \alpha^2 < \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \pi_c^c,1 - \delta, c \), collude only in country \( A \) when \( \pi_c^c,1 - \delta, c < \alpha^2 < \bar{\alpha}^2 \), and collude only in \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

2. Suppose that \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} > \frac{\pi_c^c,1 - \delta, c}{\omega^c} \) and \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} \). Then, \( \alpha^2 < \bar{\alpha}^2 \) and \( \bar{\alpha}^2 < \bar{\alpha}^2 \). Therefore:

(a) If \( \alpha^2 > \bar{\alpha}^2 \), then firms only collude in country \( B \) and they are never detected.

(b) If \( \bar{\alpha}^2 < \alpha^2 \leq \bar{\alpha}^2 \), then firms collude in country \( A \) when \( \alpha^2 < \bar{\alpha}^2 \) and they collude in country \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

(c) If \( \alpha^2 < \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \pi_c^c,1 - \delta, c \), collude only in country \( A \) when \( \pi_c^c,1 - \delta, c < \alpha^2 < \bar{\alpha}^2 \), and collude only in \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

3. Suppose that \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} \geq \frac{\pi_c^c,1 - \delta, c}{\omega^c} \) and \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} \). Then, \( \alpha^2 \geq \bar{\alpha}^2 \) and \( \bar{\alpha}^2 \). Therefore:

(a) If \( \alpha^2 > \bar{\alpha}^2 \), then firms only collude in country \( B \) and they are never detected.

(b) If \( \bar{\alpha}^2 < \alpha^2 \leq \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \bar{\alpha}^2 \) and they collude in country \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

(c) If \( \alpha^2 < \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \pi_c^c,1 - \delta, c \), collude only in country \( A \) when \( \pi_c^c,1 - \delta, c < \alpha^2 < \bar{\alpha}^2 \), and collude only in \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

4. Suppose that \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} \geq \frac{\pi_c^c,1 - \delta, c}{\omega^c} \) and \( \frac{\pi_c^c,1 - \delta, c}{\omega^c} \). Then, \( \alpha^2 \geq \bar{\alpha}^2 \) and \( \bar{\alpha}^2 \). Therefore:

(a) If \( \alpha^2 > \bar{\alpha}^2 \), then firms only collude in country \( B \) and they are never detected.

(b) If \( \bar{\alpha}^2 < \alpha^2 \leq \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \bar{\alpha}^2 \) and they collude in country \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

(c) If \( \alpha^2 < \bar{\alpha}^2 \), then firms collude in both countries when \( \alpha^2 < \pi_c^c,1 - \delta, c \), collude only in country \( A \) when \( \pi_c^c,1 - \delta, c < \alpha^2 < \bar{\alpha}^2 \), and collude only in \( B \) when \( \alpha^2 > \bar{\alpha}^2 \).

Note that we have fully characterized the equilibrium for any possible values of the parameters. To obtain Proposition 2 define \( \bar{\alpha}^2 = \max \{ \bar{\alpha}^2, \bar{\alpha}^2 \} \), \( \bar{\alpha}^2 = \max \{ \bar{\alpha}^2, \bar{\alpha}^2 \} \), \( \bar{\alpha}^2 = \bar{\alpha}^2 \) if \( \bar{\alpha}^2 \), and \( \bar{\alpha}^2 = \max \{ \bar{\alpha}^2, \bar{\alpha}^2 \} \).
A.1.5 Variations of Proposition 2

Proposition 2 (Alternative). Suppose that Assumption 1 holds, firms punish deviations from collusion in each market separately and they coordinate in their best equilibrium.

1. Suppose that $\delta^2 < \delta < \delta\hat{\delta}$, where $\delta^2 = \max\left\{\frac{\pi^d.A-\pi^c.A}{\pi^d.A}, \frac{\pi^d.B-\pi^c.B}{\pi^d.B}\right\}$. Then firms collude in both countries and they are never prosecuted.

2. Suppose that $\delta \geq \delta\hat{\delta}$. Then, there are thresholds $\alpha_A^2$ and $\alpha_B^2$ such that:

   (a) Suppose that $\alpha^A > \alpha^2_A$. Then, firms collude in country $B$ and they are never prosecuted.

   (b) Suppose that $\alpha^2_M < \alpha^A \leq \alpha^2_H$. Then, there is a threshold $\alpha^2_M = \frac{(1-\delta)(\pi^c.A-\pi^c.B)}{\delta \pi^d.J + (1-\delta)\pi^d.A}$ such that: If $\alpha^A \in (\alpha^2_M, \alpha^2_H]$, firms only collude in country $A$ until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\alpha^2_M, \alpha^2_H]$, firms only collude in country $B$ and they are never prosecuted.

   (c) If $\alpha^A \leq \alpha^2_M$, then firms collude in both countries until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition in both countries.

Proof. Suppose that $\delta^2 < \delta < \delta\hat{\delta}$. Then, from Lemmas 1 and 2, if firms collude in both countries or they only collude in country $A$, they will never be prosecuted, while if they only collude in country $A$, they will be prosecuted the first time $s^A_t = 1$. Therefore, the expected profits of a firm under collusion in country $A$, collusion in country $B$ and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^c.A - \alpha^A f^A}{1 - (1 - \alpha^A) \delta}, \Pi^{c,B} = \frac{\pi^c.B}{1 - \delta}, \Pi^{c,AB} = \frac{\pi^c.A + \pi^c.B}{1 - \delta},$$

respectively. In order to deduce $\Pi^{c,A}$ note that $\Pi^{c,A} = \alpha^A (\pi^c.A - f^A) + (1 - \alpha^A) (\pi^c.A + \delta \Pi^{c,A})$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country $A$ to be an equilibrium, it must be the case $\Pi^{c,A} \geq \pi^d.A$. For collusion in country $B$ to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^d.B$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country $A$, in country $B$, or in both countries, which requires $\Pi^{c,AB} \geq \pi^d.A + \Pi^{c,B}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^d.B$ and $\Pi^{c,AB} \geq \pi^d.A + \pi^d.B$, respectively. Since $\pi^{c,j} > (1 - \delta) \pi^{d,j}$ for $j = A, B$, all these conditions hold. Therefore, if $\Pi^{c,A} \geq \pi^d.A$, the three types of collusion can be sustained as an equilibrium, while if $\Pi^{c,A} < \pi^d.A$, only collusion in country $B$ or collusion in both countries can be sustained as an equilibrium. Finally, note that $\Pi^{c,AB} > \Pi^{c,A}, \Pi^{c,B}$. Thus, firms prefer to coordinate in an equilibrium in which they collude in both countries.

Suppose that $\delta \geq \delta\hat{\delta}$. Then, from Lemmas 1 and 2, if firms only collude in country $B$, they will never be prosecuted, while if they collude in country $A$ or in both countries, they will be prosecuted the first time that $s^A_t = 1$. Therefore, the expected profits of a firm under collusion in country $A$, collusion in country $B$ and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^c.A - \alpha^A f^A}{1 - (1 - \alpha^A) \delta}, \Pi^{c,B} = \frac{\pi^c.B}{1 - \delta}, \Pi^{c,AB} = \frac{\pi^c.A + \pi^c.B - \alpha^A (f^A + f^B)}{1 - (1 - \alpha^A) \delta},$$

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respective. In order to deduce $\Pi^{c,AB}$, note that $\Pi^{c,AB} = \alpha^A (\pi^{c,A} + \pi^{c,B} - f^A - f^B) + (1 - \alpha^A) (\pi^{c,A} + \pi^{c,B} + \delta \pi^{c,AB})$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country $A$ to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in country $B$ to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country $A$, in country $B$, or in both countries. No firm has an incentive to stop colluding in country $A$ if $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$. No firm has an incentive to stop colluding in country $B$ if $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$. No firm has an incentive to deviate in both countries if $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Therefore, we must consider four different cases.

1. Only collusion in country $B$ can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and at least one of the following inequalities holds $\Pi^{c,AB} < \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} < \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$. These conditions hold if and only if $\Pi^{c,A} < \pi^{d,A}$, or, which is equivalent $\alpha^A > \tilde{\alpha}_H^2 = \frac{\pi^{c,AB} - (1-\delta)\pi^{d,A}}{\delta \pi^{c,AB} + f^B}$. Therefore, if $\alpha^A > \tilde{\alpha}_H^2$, firms collude in country $B$ and they are never prosecuted.

2. Only collusion in country $B$ or in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. These conditions lead to $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A (\delta \Pi^{c,B} + f^B)}{1 - (1 - \alpha^A) \delta}$, a contradiction.

3. Only collusion in country $B$ or collusion in country $A$ can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and at least one of the following inequalities holds $\Pi^{c,AB} < \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} < \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$. These conditions simultaneously hold if and only if

$$\pi^{d,A} \leq \Pi^{c,A} < \pi^{d,A} + \frac{\alpha^A (\delta \Pi^{c,B} + f^B)}{1 - (1 - \alpha^A) \delta}$$

or

$$\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A (\delta \Pi^{c,B} + f^B)}{1 - (1 - \alpha^A) \delta} \text{ and } \Pi^{c,B} < \left[1 - \frac{(1 - \alpha^A) \delta}{\delta - \alpha^A} \right] \pi^{d,B} + \alpha^A f^B$$

The first inequality is equivalent to $\frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta \pi^{c,AB} + f^B} < \alpha^A \leq \tilde{\alpha}_H^2$, while the second and third inequalities are equivalent to $\frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta \pi^{c,AB} + f^B} < \alpha^A \leq \tilde{\alpha}_L^2 = \min \left\{ \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta \pi^{c,AB} + f^B}, \frac{\pi^{c,B} - (1-\delta)\pi^{d,B}}{\delta \pi^{c,AB} + f^B} \right\}$. Finally, firms prefer to collude in $A$ when $\Pi^{c,A} > \Pi^{c,B}$ or, which is equivalent, $\alpha^A < \tilde{\alpha}_M^2 = \frac{(1-\delta)\alpha^B}{\delta \pi^{c,AB} + (1-\delta) f^A}$. Otherwise, they prefer to collude in $B$.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$, $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. These conditions hold if and only if

$$\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A (\delta \Pi^{c,B} + f^B)}{1 - (1 - \alpha^A) \delta} \text{ and } \Pi^{c,B} \geq \left[1 - \frac{(1 - \alpha^A) \delta}{\delta - \alpha^A} \right] \pi^{d,B} + \alpha^A f^B$$

These inequalities are equivalent to $\alpha^A \leq \tilde{\alpha}_L^2$. Moreover, note that $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\Pi^{c,A} > \frac{\alpha^A (\delta \Pi^{c,B} + f^B)}{1 - (1 - \alpha^A) \delta}$ while $\Pi^{c,AB} > \Pi^{c,A}$ if and only if $\Pi^{c,B} > \frac{\alpha^A f^B}{1 - \delta}$. Thus, when the three types of collusion can be sustained as an equilibrium, firms prefer to coordinate in an equilibrium in which they collude in both countries.
Proposition 2 (bis). Suppose that Assumption 1.bis holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium.

1. Suppose that $\bar{\delta}^A < \delta < \tilde{\delta}$, where $\bar{\delta} = \max\left\{\frac{\pi^{c,A} - \pi^{d,A}}{\alpha c,M}, \frac{\pi^{d,B} - \pi^{c,B}}{\alpha c,M}\right\}$. Then firms collude in both countries and they are never prosecuted.

2. Suppose that $\delta \geq \bar{\delta}$ and $\delta \pi^{d,A} + f^A > \frac{\delta \pi^{c,B}}{1-\delta}$. Then, there are thresholds $\tilde{\alpha}^A$ and $\tilde{\alpha}^B$ such that:

   (a) Suppose that $\alpha^A > \tilde{\alpha}^B$. Then, firms collude in country $B$ and they are never prosecuted.

   (b) Suppose that $\tilde{\alpha}^A < \alpha^A \leq \tilde{\alpha}^B$. Then, there is a threshold $\tilde{\alpha}_M$ such that: If $\alpha^A \in [\tilde{\alpha}_M, \tilde{\alpha}_B]$, then one of the following two paths will occur. (1) Firms collude in both countries until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition in both countries. (2) Firms collude in country $A$ until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, they collude in country $B$, where they will never be prosecuted. If $\alpha^A \in [\tilde{\alpha}_M, \tilde{\alpha}_B]$, then firms only collude in country $B$ and they are never prosecuted.

   (c) Suppose that $\alpha^A \leq \tilde{\alpha}^A$. Then, there is a threshold $\tilde{\alpha}_M$ such that: If $\alpha^A \in [\tilde{\alpha}_M, \tilde{\alpha}_B]$, then one of the following two paths will occur. (1) Firms collude in both countries until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, there is competition in both countries. (2) Firms collude in country $A$ until the first time $s^A_t = 1$, when they are prosecuted. Thereafter, they collude in country $B$, where they will never be prosecuted. If $\alpha^A \in [\tilde{\alpha}_M, \tilde{\alpha}_B]$, then firms only collude in country $B$ and they are never prosecuted.

Proof. First, we prove that Lemma 2 holds when Assumption 1 is replaced by Assumption 1.bis. Suppose that firms are only colluding in country $A$ while there is and will always be competition in country $B$. Assume that $s^A_t = 1$. If the competition authority of country $A$ prosecutes the firms, then the expected discounted welfare of country $A$ at period $t$ is $W^A_t\left(p^A_t = 1|s^A_t = 1, A\right) = CS^{c,A} + 2\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}$, while if firms are not prosecuted, it is $W^A_t\left(p^A_t = 0|s^A_t = 1, A\right) = \frac{CS^{c,A} + 2\pi^{c,A}}{1-\delta}$. Since $CS^{com,A} > CS^{c,A} + 2\pi^{c,A}$, it must be the case that $W^A_t\left(p^A_t = 1|s^A_t = 1, A\right) > W^A_t\left(p^A_t = 0|s^A_t = 1, A\right)$.

Suppose that firms are only colluding in country $A$, there is competition in country $B$, but as soon as they are prosecuted in country $A$, firms will start colluding in country $B$. Assume that $s^A_t = 1$. If the competition authority of country $A$ prosecutes the firms, then the expected discounted welfare of country $A$ at period $t$ is $W^A_t\left(p^A_t = 1|s^A_t = 1, A\right) = CS^{c,A} + 2\pi^{c,A} + \frac{\delta}{1-\delta}\left(CS^{com,A} + 2\pi^{c,B}\right)$, while if the firms are not prosecuted, it is $W^A_t\left(p^A_t = 0|s^A_t = 1, A\right) = \frac{CS^{c,A} + 2\pi^{c,A}}{1-\delta}$. Since $CS^{com,A} > CS^{c,A} + 2\pi^{c,A}$, it must be the case that $W^A_t\left(p^A_t = 1|s^A_t = 1, A\right) > W^A_t\left(p^A_t = 0|s^A_t = 1, A\right)$.

Suppose that firms are colluding in both countries and $s^A_t = 1$. If the competition authority of country $A$ decides to prosecute the firms, the expected discounted welfare of country $A$ at period $t$ will be $W^A_t\left(p^A_t = 1|s^A_t = 1, AB\right) = CS^{c,A} + 2\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}$. If the competition authority of country $A$ does not prosecute the firms, then the the expected discounted welfare of country $A$ at period $t$ will be $W^A_t\left(p^A_t = 0|s^A_t = 1, AB\right)$ if and only if $\delta \left(CS^{com,A} - CS^{c,A} - 2\pi^{c,A}\right) \geq \frac{\delta 2\pi^{c,B} + (1-\delta) 2f^B}{1-\delta}$. Thus, Lemma 2 holds when Assumption 1 is replaced by Assumption 1.bis.
Suppose that $\bar{\delta}^2 < \delta < \tilde{\delta}$. Then, from Lemmas 1 and 2, if firms are colluding in both countries they will never be prosecuted. Therefore, if firms always collude in both countries, the expected discounted profits of a firm are given by:

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1 - \delta}$$

Note that there is no other form of collusion that will induce higher discounted expected profits. Moreover, always collude in both countries can be sustained as a subgame perfect Nash equilibrium. In order to prove this, note that for collusion in both countries to be an equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Since $\pi^{c,j} > (1 - \delta) \pi^{d,j}$ for $j = A, B$, all these conditions hold.

Suppose that $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, if firms are only colluding in country $B$, they will never be prosecuted, while if they are colluding in country $A$ or in both countries, they will be prosecuted the first time that $s_i^t = 1$. Therefore, there are three types of collusion that firms must consider: 1) collude only in country $A$ until detected, then start colluding in country $B$; 2) always collude only in country $B$; and 3) always collude in both countries. The expected discounted profits of a firm associated with each type of collusion are given by:

$$\Pi^{c.A} = \frac{\pi^{c,A} - \alpha A^f + \alpha A\delta^{\frac{\pi^{c,B}}{1 - \delta}}}{1 - (1 - \alpha A) \delta}, \quad \Pi^{c.B} = \frac{\pi^{c,B}}{1 - \delta}, \quad \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha A (f^A + f^B)}{1 - (1 - \alpha A) \delta},$$

respectively. In order to deduce $\Pi^{c,A}$ and $\Pi^{c,AB}$ note that $\Pi^{c,A} = \alpha A (\pi^{c,A} - f^A + \delta \Pi^{c,B}) + (1 - \alpha A) (\pi^{c,A} + \delta \Pi^{c,A})$ and $\Pi^{c,AB} = \alpha A (\pi^{c,A} + \pi^{c,B} - f^A + f^B) + (1 - \alpha A) (\pi^{c,A} + \pi^{c,B} + \delta \Pi^{c,AB})$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. Collusion in country $B$ is always an equilibrium because $\Pi^{B} \geq \pi^{d,B}$ always holds. For collusion in country $A$ until detected, then collusion in country $B$ to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$.

1. Only collusion in country $B$ can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and either $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta \pi^{d,A} + f^A > \delta \pi^{d,B}$ and $\alpha A > \max\{\hat{\alpha}^2, \tilde{\alpha}^2\}$, where $\hat{\alpha}^2 = \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^A}$ and $\tilde{\alpha}^2 = \frac{(\pi^{c,A} + \pi^{c,B}) - (1 - \delta) (\pi^{d,A} + \pi^{d,B})}{\delta (\pi^{d,A} + \pi^{d,B}) + f^A}$.

2. Only collusion in country $B$ and in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta \pi^{d,A} + f^A > \frac{\delta \pi^{d,B}}{1 - \delta}$ and $\alpha A < \tilde{\alpha}^2$. Firms prefer to collude in both countries if $\Pi^{c,AB} > \Pi^{c,B}$, i.e., when $\alpha A < \tilde{\alpha}^2_M = \frac{(1 - \delta) \pi^{c,A}}{\delta \pi^{c,B} + (1 - \delta) (f^A + f^B)}$.

3. Only collusion in country $B$ and collusion in country $A$ until detected, then collusion in country $B$ can be sustained as an equilibrium when $\Pi^{c.A} \geq \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta \pi^{d,A} + f^A > \frac{\delta \pi^{d,B}}{1 - \delta}$ and $\alpha A \leq \tilde{\alpha}^2$ or $\tilde{\alpha}^2 < \alpha A$. Firms prefer to collude in country $A$ when $\Pi^{c.A} > \Pi^{c,B}$, i.e., when $\alpha A < \hat{\alpha}^2_M = \frac{\pi^{c,A} - \pi^{c,B}}{f^A}$.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c.A} \geq \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta \pi^{d,A} + f^A \leq \frac{\delta \pi^{d,B}}{1 - \delta}$ and $\alpha A \leq \hat{\alpha}^2$ or $\hat{\alpha}^2 < \alpha A$. Firms prefer to collude in $A$ when $\Pi^{c.A} > \max\{\Pi^{c.B}, \Pi^{c,AB}\}$, i.e., when $\frac{f^B + \delta \pi^{d,B}}{\pi^{c,B}} < \alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\{\Pi^{c.A}, \Pi^{c.B}\}$, i.e., when $\alpha A < \hat{\alpha}^2_M$.
\[
\min \left\{ \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}}, \hat{\alpha}_M^2 \right\}. \]
Finally, firms prefer to collude in B when \( \Pi^{c,B} > \max \{ \Pi^{c,A}, \Pi^{c,AB} \} \), i.e., when \( \alpha^A > \max \{ \hat{\alpha}_M^2, \hat{\alpha}_M^2 \} \). Moreover, \( \hat{\alpha}_M^2 < \hat{\alpha}_M^2 \) if and only if \( \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} < \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} \).

Employing cases 1-4 we can distinguish six different situations:

1. Suppose that \( \delta \pi^{d,A} + f^A > \frac{\delta \pi^{c,B}}{1 - \delta}, \pi^{c,A} - (1 - \delta)\pi^{d,A} > \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{1 - \delta}, \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \). Then, \( \hat{\alpha}^2 < \hat{\alpha}^2 \) and \( \hat{\alpha}_M^2 < \hat{\alpha}_M^2 \). Therefore:

(a) If \( \alpha^A > \hat{\alpha}^2 \), then firms only collude in country B and they are never detected.
(b) If \( \hat{\alpha}^2 < \alpha^A \leq \hat{\alpha}^2 \), then firms collude in country A until detected, then collude in country B when \( \alpha^A < \hat{\alpha}_M^2 \) and they only collude in country B when \( \alpha^A > \hat{\alpha}_M^2 \).
(c) If \( \alpha^A < \hat{\alpha}^2 \), then firms collude in both countries when \( \alpha^A < \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} \), collude only in country A until detected, then collude in country B when \( \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} < \alpha^A < \hat{\alpha}_M^2 \), and collude only in B when \( \alpha^A > \hat{\alpha}_M^2 \).

2. Suppose that \( \delta \pi^{d,A} + f^A > \frac{\delta \pi^{c,B}}{1 - \delta}, \pi^{c,A} - (1 - \delta)\pi^{d,A} > \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{1 - \delta}, \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \geq \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \). Then, \( \hat{\alpha}^2 < \hat{\alpha}^2 \) and \( \hat{\alpha}_M^2 \geq \hat{\alpha}_M^2 \). Therefore:

(a) If \( \alpha^A > \hat{\alpha}^2 \), then firms only collude in country B and they are never detected.
(b) If \( \hat{\alpha}^2 < \alpha^A \leq \hat{\alpha}^2 \), then firms collude in country A until detected, then collude in country B when \( \alpha^A < \hat{\alpha}_M^2 \) and they only collude in country B when \( \alpha^A > \hat{\alpha}_M^2 \).
(c) If \( \alpha^A \leq \hat{\alpha}^2 \), then firms collude in both countries when \( \alpha^A < \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} \), collude only in B when \( \alpha^A > \hat{\alpha}_M^2 \).

3. Suppose that \( \delta \pi^{d,A} + f^A > \frac{\delta \pi^{c,B}}{1 - \delta}, \pi^{c,A} - (1 - \delta)\pi^{d,A} \geq \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{1 - \delta}, \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} < \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \). Then, \( \hat{\alpha}^2 \geq \hat{\alpha}^2 \) and \( \hat{\alpha}_M^2 < \hat{\alpha}_M^2 \). Therefore:

(a) If \( \alpha^A > \hat{\alpha}^2 \), then firms only collude in country B and they are never detected.
(b) If \( \hat{\alpha}^2 < \alpha^A \leq \hat{\alpha}^2 \), then firms collude in both countries when \( \alpha^A < \hat{\alpha}_M^2 \) and they collude in country B when \( \alpha^A > \hat{\alpha}_M^2 \).
(c) If \( \alpha^A \leq \hat{\alpha}^2 \), then firms collude in both countries when \( \alpha^A < \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} \), collude in country A until detected, then collude in country B when \( \frac{\pi^{c,B}}{f_B + \delta \bar{\pi}^{c,B}} < \alpha^A < \hat{\alpha}_M^2 \), and collude only in B when \( \alpha^A > \hat{\alpha}_M^2 \).

4. Suppose that \( \delta \pi^{d,A} + f^A > \frac{\delta \pi^{c,B}}{1 - \delta}, \pi^{c,A} - (1 - \delta)\pi^{d,A} \geq \frac{\pi^{c,B} - (1 - \delta)\pi^{d,B}}{1 - \delta}, \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \geq \frac{(1 - \delta)\pi^{c,B}}{(1 - \delta)f_B + \delta \pi^{c,B}} \). Then, \( \hat{\alpha}^2 \geq \hat{\alpha}^2 \) and \( \hat{\alpha}_M^2 \geq \hat{\alpha}_M^2 \). Therefore:
(a) If $\alpha^A > \bar{\alpha}^2$, then firms only collude in country $B$ and they are never detected.

(b) If $\bar{\alpha}^2 < \alpha^A \leq \bar{\alpha}^2$, then firms collude in both countries when $\alpha^A < \bar{\alpha}^2_M$ and they collude in country $B$ when $\alpha^A > \bar{\alpha}_M^2$.

(c) If $\alpha^A \leq \bar{\alpha}^2$, then firms collude in both countries when $\alpha^A < \bar{\alpha}^2_M$ and they collude only in $B$ when $\alpha^A > \bar{\alpha}^2_M$.

5. Suppose that $\delta \pi^{d,A} + f^A \leq \frac{\delta \pi^c_B}{1-\delta} f^B$ and $\frac{(1-\delta) \pi^c_B}{(1-\delta) f^B + \delta \pi^c_B} < \frac{(1-\delta) \pi^{c,A}}{\delta \pi^c_B + (1-\delta) f^A + f^B}$. Then, $\bar{\alpha}_M^2 < \bar{\alpha}^2_M$.

Therefore:

(a) If $\alpha^A > \bar{\alpha}^2$, then firms collude in country $A$ until detected, then collude in country $B$ when $\alpha^A < \bar{\alpha}^2_M$, and collude only in $B$ when $\alpha^A > \bar{\alpha}^2_M$.

(b) If $\alpha^A \leq \bar{\alpha}^2$, then firms collude in both countries when $\alpha^A < \frac{\pi^{c,B}}{f^B + \delta \pi^c_B}$, they collude in country $A$ until detected, then collude in country $B$ when $\frac{\pi^{c,B}}{f^B + \delta \pi^c_B} < \alpha^A < \bar{\alpha}^2_M$, and they collude only in $B$ when $\alpha^A > \bar{\alpha}^2_M$.

6. Suppose that $\delta \pi^{d,A} + f^A \leq \frac{\delta \pi^c_B}{1-\delta} f^B$ and $\frac{(1-\delta) \pi^c_B}{(1-\delta) f^B + \delta \pi^c_B} \geq \frac{(1-\delta) \pi^{c,A}}{\delta \pi^c_B + (1-\delta) f^A + f^B}$. Then, $\bar{\alpha}_M^2 \geq \bar{\alpha}^2_M$.

Therefore:

(a) If $\alpha^A > \bar{\alpha}^2$, then firms collude in country $A$ until detected, then collude in country $B$ when $\alpha^A < \bar{\alpha}^2_M$, and collude only in $B$ when $\alpha^A > \bar{\alpha}^2_M$.

(b) If $\alpha^A \leq \bar{\alpha}^2$, then firms collude in both countries when $\alpha^A < \bar{\alpha}^2_M$ and they collude only in country $B$ when $\alpha^A > \bar{\alpha}^2_M$.

Note that we have fully characterized the equilibrium for any possible values of the parameters. To obtain Proposition 2 define $\bar{\alpha}_M^2 = \max \{ \bar{\alpha}^2_M, \bar{\alpha}_M \}, \bar{\alpha}_H^2 = \max \{ \bar{\alpha}^2, \bar{\alpha}_M^2 \}, \bar{\alpha}_M^2 = \bar{\alpha}_M^2$ if $\bar{\alpha}^2 < \bar{\alpha}^2$, $\bar{\alpha}_M^2 = \bar{\alpha}_M^2$ if $\bar{\alpha}^2 \geq \bar{\alpha}^2$, and $\bar{\alpha}^2_M = \max \{ \bar{\alpha}_M^2, \bar{\alpha}_M^2 \}$.

A.2 Detection Ability Partially Aligned with Firm Ownership

A.2.1 Proof of Lemma 3

Lemma 3. Suppose that Assumption 2 holds. Assume that the competition authority of country $A$ detects collusion, i.e., $c_1^A = 1$.

1. If the firms are not colluding in country $B$, then $A$ always prosecutes the firms.

2. If the firms are also colluding in country $B$, then $A$ prosecutes the firms if and only if

$$CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{\left[ 1 - (1 - \alpha^A) \delta \right] \left[ 1 - (1 - \alpha_B) \right] \left[ \delta \pi^{c,B} + (1 - \delta) f^B \right]}{\left[ 1 - (1 - \alpha_B) \right] \left[ 1 - (1 - \alpha^A) \right] \delta}.$$
Proof. As in Lemma 2 we must distinguish two possible cases: when firms are only colluding in country A and when they are colluding in both countries. Suppose that firms are only colluding in country A and \( s_t^A = 1 \). If competition authority A prosecutes the firms, the expected discounted welfare of country A at period t is \( W_t^A(p_t^A = 1|s_t^A = 1, A) = CS^{c,A} + 2(\pi^{c,A} - f^A) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A} \), while if firms are not prosecuted, it is \( W_t^A(p_t^A = 0|s_t^A = 1, A) = \frac{CS^{c,A} + 2\pi^{c,A}}{1-\delta} \). Since \( CS^{com,A} > CS^{c,A} + 2\pi^{c,A} \), it must be the case that \( W_t^A(p_t^A = 1|s_t^A = 1, A) > W_t^A(p_t^A = 0|s_t^A = 1, A) \). Thus, when firms are only colluding in country A, as soon as \( c_t^A = 1 \), competition authority A immediately prosecutes the firms.

Suppose there is collusion in both countries and \( s_t^A = 1 \). If competition authority A decides to prosecute the firms, then, from Lemma 1, competition authority B will detect collusion in country B as well and, hence, firms will also be prosecuted in country B. Therefore, from period \( t + 1 \) there will be competition in both countries. Then, the expected discounted welfare of country A at period t is:

\[
W_t^A(p_t^A = 1|s_t^A = 1, AB) = CS^{c,A} + 2(\pi^{c,A} - f^A + \pi^{c,B} - f^B) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}.
\]

If competition authority A does not prosecute the firms, then with probability \( \alpha^B \) competition authority B receives \( s_t^B = 1 \) and with probability \( (1 - \alpha^B) \) it receives \( s_t^B = 0 \). If \( s_t^B = 1 \), then competition authority B prosecutes the firms in period t and in period \( t + 1 \) with probability \( \alpha^A \) competition authority A will receive \( s_{t+1}^A = 1 \) and with probability \( (1 - \alpha^A) \) it will receive \( s_{t+1}^A = 0 \). Then, the expected discounted welfare of country A at period t is:

\[
W_t^A(p_t^A = 0|s_t^A = 1, AB) = CS^{c,A} + 2\pi^{c,A} + 2\pi^{c,B} - \alpha^B 2f^B + \alpha^B \delta [\alpha^AW_t^A(p_t^A = 1|s_t^A = 1, A) + (1 - \alpha^A) W_t^A(s_t^A = 0, A)] + (1 - \alpha^B) \delta W_t^A(p_t^A = 0|s_t^A = 1, AB)
\]

where \( W_t^A(p_t^A = 1|s_t^A = 1, A) \) and \( W_t^A(s_t^A = 0, A) \) are given by

\[
W_t^A(p_t^A = 1|s_t^A = 1, A) = CS^{c,A} + 2\pi^{c,A} + \frac{\delta}{1-\delta}CS^{com,A},
\]

\[
W_t^A(s_t^A = 0, A) = CS^{c,A} + 2\pi^{c,A} + \delta [\alpha^AW_t^A(p_t^A = 1|s_t^A = 1, A) + (1 - \alpha^A) W_t^A(s_t^A = 0, A)].
\]

Solving for \( W_t^A(p_t^A = 0|s_t^A = 1, AB) \) we obtain:

\[
W_t^A(p_t^A = 0|s_t^A = 1, AB) = \frac{(1-(1-\alpha^A)\alpha^B)(CS^{c,A} + 2\pi^{c,A}) + 2(\pi^{c,B} - \alpha^B 2f^B) + \alpha^A\alpha^B \delta^2 CS^{com,A}}{1-(1-\alpha^B)\delta}.
\]

Finally, \( W_t^A(p_t^A = 1|s_t^A = 1, AB) > W_t^A(p_t^A = 0|s_t^A = 1, AB) \) if and only \( CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{[1-(1-\alpha^A)\delta][2(1-\alpha^B)][\delta^{c,B} + (1-\delta)f^B]}{[1-(1-\alpha^B)\delta]} \).

A.2.2 Proof of Proposition 3

Proposition 3. Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and \( \pi^{c,A} > (1-\delta) \pi^{d,A} \) and \( \pi^{c,B} > \)
\[(1 - \delta) \pi^{d,B}.\] Let

\[
\alpha^B > \frac{2[\delta \pi^{c,B} + (1 - \delta) f^B] - \delta (CS_{com,A} - CS_{c,A} - 2\pi^{c,A})}{2[\delta \pi^{c,B} + (1 - \delta) f^B] + \delta[1/(1-\alpha^A)] \delta (CS_{com,A} - CS_{c,A} - 2\pi^{c,A})},
\]

\[
\alpha^A < \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{d,A} + f^A},
\]

\[
\alpha^B < \frac{(1 - \delta) \pi^{c,B} + \alpha^A[\delta (\pi^{c,A} + \pi^{c,B}) + (1 - \delta) f^A]}{[1 - (1 - \alpha^A) \delta] f^B}.
\]

1. Suppose that the four inequalities above hold. Then, firms collude in both countries until the first time \(s^B_t = 1\), when they are prosecuted in country \(B\). Thereafter, they collude in country \(A\) until the first time \(s^A_t = 1\) with \(\tau \geq 1\), when they are prosecuted in country \(A\). Thereafter, there is competition in both countries.

2. Suppose that the first three inequalities above hold, but the fourth does not hold. Then, firms only collude in country \(A\) until the first time \(s^A_t = 1\), when they are prosecuted. Thereafter, there is competition.

Proof. Suppose that \(CS_{com,A} - CS_{c,A} - 2\pi^{c,A} \geq \frac{1 - (1 - \alpha^A) \delta}{1 - (1 - \alpha^B)} (\pi^{c,B} + (1 - \delta) f^B)\) or, which is equivalent, \(\alpha^B > \frac{2[\delta \pi^{c,B} + (1 - \delta) f^B] - \delta (CS_{com,A} - CS_{c,A} - 2\pi^{c,A})}{2[\delta \pi^{c,B} + (1 - \delta) f^B] + \delta[1/(1-\alpha^A)] \delta (CS_{com,A} - CS_{c,A} - 2\pi^{c,A})}\). Then, from Lemmas 1 and 3, if firms collude in both countries or they only collude in country \(B\), they will be prosecuted in country \(B\) the first time that \(s^B_t = 1\), while if they only collude in country \(A\), they will be prosecuted the first time \(s^A_t = 1\). Therefore, the expected profits of a firm under collusion in country \(A\), collusion in country \(B\) and collusion in both countries are given by:

\[
\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - (1 - \alpha^A) \delta}, \quad \Pi^{c,B} = \frac{\pi^{c,B} - \alpha^B f^B}{1 - (1 - \alpha^B) \delta}, \quad \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A f^B - \alpha^B f^A + \delta \alpha^B (\pi^{c,A} - \alpha^A f^A)}{1 - (1 - \alpha^B) \delta},
\]

respectively. In order to deduce \(\Pi^{c,AB}\) note that \(\Pi^{c,AB} = \pi^{c,A} + \pi^{c,B} + \alpha^B (-f^B + \delta \Pi^{c,A}) + (1 - \alpha^B)(\pi^{c,A} + \pi^{c,B} + \delta \Pi^{c,AB})\).

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country \(A\) to be an equilibrium it must be the case that \(\Pi^{c,A} \geq \pi^{d,A}\). For collusion in country \(B\) to be an equilibrium it must be the case that \(\Pi^{c,B} \geq \pi^{d,B}\). For collusion in both countries to be an equilibrium it must be the case that \(\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}\). \(\Pi^{c,AB} \geq \pi^{d,A}\) if and only if \(\alpha^A \geq \frac{\pi^{c,A} - (1 - \delta) \pi^{d,A}}{\delta \pi^{c,A} + f^A}\). \(\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}\) if and only if \(\alpha^B \leq \frac{(\pi^{c,A} + \pi^{c,B}) - (1 - \delta)(\pi^{d,A} + \pi^{d,B})}{\delta (\pi^{d,A} + \pi^{d,B}) + f^B}\). Note that if \(\Pi^{c,A} \geq \pi^{d,A}\), then \(\Pi^{c,AB} \geq \Pi^{c,B}\). Hence, when collusion in country \(A\) and collusion in both countries can be sustained as an equilibrium, firms always prefer to collude in both countries rather than only in country \(B\). Finally, \(\Pi^{c,AB} \geq \Pi^{c,A}\) if and only if \(\alpha^B < \frac{(1 - \delta)(\pi^{c,A} + \pi^{c,B}) + (1 - \delta) f^A}{[1 - (1 - \alpha^A) \delta] f^B}\). ◼
A.2.3 Proof of Corollary 1

Corollary 1. Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and \( \pi^{c,A} > (1 - \delta) \pi^{d,A} \) and \( \pi^{c,B} > (1 - \delta) \pi^{d,B} \).

1. Suppose that competition authority \( j \in \{A, B\} \) prosecutes collusion the first time that \( s_j^t = 1 \) and, under such anti-trust decisions, firms are still willing to collude in both countries. Then, the expected durations of collusion in countries \( A \) and \( B \) are given by \( d^A = \frac{1 - \alpha^A}{\alpha^A} \) and \( d^B = \frac{(1 - \alpha^A)(1 - \alpha^B)}{1 - (1 - \alpha^A)(1 - \alpha^B)} \), respectively.

2. Under the assumptions in Proposition 3 Part 1. The expected durations of collusion in countries \( A \) and \( B \) are given by \( d^A = \frac{\alpha^A + \alpha^B - \alpha^A \alpha^B}{\alpha^A \alpha^B} \) and \( d^B = \frac{1 - \alpha^B}{\alpha^B} \), respectively.

3. Equilibrium prosecution delays in countries \( A \) and \( B \) are given by:

\[
d^A - \bar{d}^A = \frac{1}{\alpha^B} \quad \text{and} \quad d^B - \bar{d}^B = \frac{\alpha^A (1 - \alpha^B)}{\alpha^B [1 - (1 - \alpha^A)(1 - \alpha^B)]},
\]

respectively.

**Proof.** Suppose that both competition authorities always prosecute collusion as soon as they detect it and firms are still willing to collude in both countries. In such environment, the expected duration of collusion will be given by:

\[
d^A = \sum_{t=0}^{\infty} t (1 - \alpha^A)^t \alpha^A = \frac{1 - \alpha^A}{\alpha^A},
\]

for country \( A \) and by:

\[
d^B = \sum_{k=0}^{\infty} k \left[ (1 - \alpha^A) (1 - \alpha^B) \right]^k \left[ 1 - (1 - \alpha^A)(1 - \alpha^B) \right] = \frac{(1 - \alpha^A)(1 - \alpha^B)}{1 - (1 - \alpha^A)(1 - \alpha^B)}
\]

for country \( B \). The logic behind these formulas is straightforward. When competition authority \( A \) prosecutes the firms as soon as collusion in country \( A \) is detected, the probability that the firms are detected in period \( k \) is given by \( (1 - \alpha^A)^k \alpha^A \), i.e., the probability that firms are not detected from \( t = 0 \) to \( t = k - 1 \) times the probability that firms are detected in period \( t = k \). Analogously, the probability that collusion in country \( B \) is detected in period \( k \) is given by \( [(1 - \alpha^A)(1 - \alpha^B)]^k \left[ 1 - (1 - \alpha^A)(1 - \alpha^B) \right] \), i.e., the probability that collusion is not detected neither in country \( A \) nor in country \( B \) from \( t = 0 \) to \( t = k - 1 \) times the probability that collusion is detected either in country \( A \) or in country \( B \) in period \( t = k \). Finally, using the properties of the geometric distribution, it is easy to compute \( \bar{d}^A \) and \( \bar{d}^B \).

The expected duration of collusion associated with the equilibrium in Proposition 3 Part 1 is given by:

\[
d^A = \sum_{k=0}^{\infty} \sum_{j=0}^{k-1} \left( 1 - \alpha^B \right)^j \alpha^B (1 - \alpha^A)^{k-1-j} \alpha^A = \frac{\alpha^A + \alpha^B - \alpha^A \alpha^B}{\alpha^B \alpha^A}
\]
for country $A$ and by:

$$d^B = \sum_{k=0}^{\infty} k (1 - \alpha^B)^k \alpha^B = \frac{1 - \alpha^B}{\alpha^B}.$$  

for country $B$. The probability that competition authority $A$ prosecutes collusion in period $k$ is given by

$$\sum_{j=0}^{k-1} (1 - \alpha^B)^j \alpha^B (1 - \alpha^A)^{k-1-j} \alpha^A,$$

where $(1 - \alpha^B)^j \alpha^B$ is the probability that competition authority $B$ detects collusion in period $j \leq k - 1$ and $(1 - \alpha^A)^{k-1-j} \alpha^A$ is the probability that competition authority

$A$ detects collusion $k - j$ periods after collusion was detected and prosecuted in country $B$. Thus, $(1 - \alpha^B)^j \alpha^B (1 - \alpha^A)^{k-1-j} \alpha^A$ is the probability that $B$ prosecutes in period $j$ and $A$ prosecutes $k - j$ periods after. (Recall that in the equilibrium described in Proposition 3 Part 1, firms start colluding in both countries, but competition authority $A$ only prosecutes the firms after competition authority $B$ detects and prosecutes collusion in country $B$). Regarding country $B$, in equilibrium, the probability that competition authority $B$ detects collusion in period $k$ is $(1 - \alpha^B)^k \alpha^B$. Again, using the properties of the geometric distribution, it is simple to compute $d^A$ and $d^B$. $\blacksquare$

A.3 Endogenous Detection Probabilities

A.3.1 Proof of Proposition 4

**Proposition 4.** For a given detection probability of competition authority $B$. Suppose that Assumption 2 holds (with the roles of $A$ and $B$ reversed in Part 1), that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that $\pi^{c,A} > (1 - \delta) \pi^{d,A}$ and $\pi^{c,B} > (1 - \delta) \pi^{d,B}$. Then:

1. Suppose that firms are owned by foreign citizens. Assume that $\left(\alpha^B, \alpha_H^B\right) \in R^B$, where $\alpha^B$ is the detection probability set by country $B$ and $\alpha_H$ is the unique solution to

$$\frac{\delta \left(C^{\text{com},A} - C^{\text{c},A}\right) + 2f^A (1 - \delta)}{[1 - (1 - \alpha_H) \delta]^2} = C' (\alpha_H).$$

Then, the competition authority of country $A$ selects $\alpha^A = \alpha_H$.

2. Suppose that firms are owned by local citizens and only operate in the domestic market. Assume that $\alpha_M \leq \frac{\pi^{c,A}}{\delta \pi^{c,A} + f^A}$, where $\alpha_M$ is the unique solution to

$$\frac{\delta \left(C^{\text{com},A} - C^{\text{c},A} - 2\pi^{c,A}\right)}{[1 - (1 - \alpha_M) \delta]^2} = C' (\alpha_M).$$

Then, the competition authority of country $A$ selects $\alpha^A = \alpha_M$.

3. Suppose that firms are owned by local citizens and they also operate in foreign markets. Assume that $\left(\alpha_L, \alpha^B\right) \in R^A$, where $\alpha^B$ is the detection probability set by country $B$ and $\alpha_L$ is the unique solution to

$$\alpha^B \delta^2 \left(C^{\text{com},A} - C^{\text{c},A} - 2\pi^{c,A}\right) \frac{1}{[1 - (1 - \alpha_L) \delta] [1 - (1 - \alpha^B) \delta]} = C' (\alpha_L).$$
Then, the competition authority of country $A$ selects $\alpha^A = \alpha_L$. Moreover, $\alpha_L$ is increasing in $\alpha^B$.

4. Suppose that $(\alpha^B, \alpha_H) \in R^B$, $\alpha_M \leq \frac{\pi^A - (1-\delta)\pi^A}{\delta \pi^A + f^A}$ and $(\alpha_L, \alpha^B) \in R^A$. Then, $\alpha_H > \alpha_M > \alpha_L$, i.e., the competition authority of country $A$ is tougher with foreign firms than with purely domestic firms and even less prone to prosecute collusion of domestic firms that operate in foreign markets.

**Proof.** Define $R^j = \{ (\alpha^j, \alpha^{-j}) \in [0,1] \times [0,1] : (11)-(14) \text{ hold} \}$, where $j \in \{A, B\}$ and

\[
\alpha^{-j} > \frac{2 \left[ \delta \pi^{c,-j} + (1-\delta) f^{-j} \right] - \delta \left( CS^{com,j} - CS^{c,j} - 2\pi^{c,j} \right)}{2 \left[ \delta \pi^{c,-j} + (1-\delta) f^{-j} \right] + \frac{\delta (1-\alpha^j)}{1-(1-\alpha^j)} \delta \left( CS^{com,j} - CS^{c,j} - 2\pi^{c,j} \right)}, \tag{11}
\]
\[
\alpha^j \leq \frac{\pi^{c,j} - (1-\delta) \pi^{d,j}}{\delta \pi^{d,j} + f^j}, \tag{12}
\]
\[
\alpha^{-j} \leq \frac{\left( \pi^{c,A} + \pi^{c,B} \right) - (1-\delta) \left( \pi^{d,A} + \pi^{d,B} \right)}{\delta \left( \pi^{d,A} + \pi^{d,B} \right) - \frac{\delta (1-\alpha^j)}{1-(1-\alpha^j)} f^j + f^{-j}}, \tag{13}
\]
\[
\alpha^{-j} < \frac{(1-\delta) \pi^{c,-j} + \alpha^j \left[ \delta \left( \pi^{c,j} + \pi^{c,-j} \right) + (1-\delta) f^j \right]}{[1 - (1-\alpha^j) \delta] f^{-j}}. \tag{14}
\]

Note that $R^A$ is identical to set of conditions in Proposition 3 Part 1, while $R^B$ would be the set of conditions in Proposition 3 Part 1 when the roles of $A$ and $B$ are reversed.

Part 1. Suppose that the firms are owned by foreign citizens. Fix $\alpha^B$ and consider $P^A = \{ \alpha^A \in [0,1] : (\alpha^B, \alpha^A) \in R^B \}$, i.e., the set of all $\alpha^A$ for which $(\alpha^B, \alpha^A) \in R^B$. From Proposition 3 Part 1 (note that the roles of $A$ and $B$ are reversed), we know that $(\alpha^B, \alpha^A) \in R^B$ implies that, in equilibrium, firms collude in both countries until the first time $s^A_t = 1$, when they are prosecuted in country $A$. Thereafter, they collude in country $B$ until the first time $s^B_{t^B+t} = 1$ with $t \geq 1$, when they are prosecuted in country $B$. Thereafter, there is competition in both countries. Thus, for $\alpha^A \in P^A$, the expected discounted welfare of country $A$ at period $t$ if $s^A_t = 1$ is given by

\[
W^A_t (s^A_t = 1) = CS^{c,A} + 2 f^A + \frac{\delta}{1-\delta} CS^{com,A},
\]

while if $s^A_t = 1$, it is

\[
W^A_t (s^A_t = 0) = CS^{c,A} + \alpha^A \delta W^A_t (s^A_t = 1) + (1 - \alpha^A) \delta W^A_t (s^A_t = 0).
\]

At $t = 0$, before $A$ selects $\alpha^A$, the discounted expected welfare of country is given by $W^A_0 = \alpha^A W^A_0 (s^A_t = 1) + (1 - \alpha^A) W^A_0 (s^A_t = 0) - C (\alpha^A)$. Therefore

\[
W^A_0 = \frac{CS^{c,A} + 2 \alpha^A f^A + \frac{\alpha^A \delta}{1-\alpha^A} CS^{com,A}}{1 - (1-\alpha^A) \delta} - C (\alpha^A).
\]

Note that $W^A_0$ is a $C^2$ and strictly concave function of $\alpha^A$, while $P^A$ is an interval because it is just the intersection of linear inequalities. Let $\alpha_H$ be the unique solution to $(W^A_0)' = 0$, i.e.,

\[
\frac{\delta (CS^{com,A} - CS^{c,A}) + 2 f^A (1-\delta)}{[1 - (1-\alpha_H) \delta]^2} = C' (\alpha_H).
\]

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Suppose that \( \alpha_H \in P^A \). Then, \( \alpha_H = \arg \max_{\alpha \in P^A} W^A_0 \).

Part 2. Suppose that the firms are owned by local citizens and only operate in the domestic market. Assume that \( \alpha^A \leq \frac{\pi^c,A-(1-\delta)\pi^d,A}{\delta \pi^d,A+f^A} \). Then, there is competition until the first time \( s^A_t = 1 \), when they will be prosecuted. Thereafter, there will be competition. Then, the expected discounted welfare of country \( A \) at period \( t \) if \( s^A_t = 1 \) is given by

\[
W^A_t(s^A_t = 1) = CS^c,A + 2\pi^c,A + \frac{\delta}{1-\delta}CS^{com,A},
\]

while if \( s^A_t = 0 \) it is

\[
W^A_t(s^A_t = 0) = CS^c,A + 2\pi^c,A + \alpha^A\delta W^A_t(s^A_t = 1) + (1 - \alpha^A)\delta W^A_t(s^A_t = 0).
\]

At \( t = 0 \), before \( A \) selects \( \alpha^A \), the discounted expected welfare of country is given by

\[
W^A_0 = \frac{CS^c,A + 2\pi^c,A + \alpha^A\delta CS^{com,A}}{1 - (1 - \alpha^A)\delta} - C(\alpha^A).
\]

Note that \( W^A_0 \) is a \( C^2 \) and strictly concave function of \( \alpha^A \), while \( \alpha^A \leq \frac{\pi^c,A-(1-\delta)\pi^d,A}{\delta \pi^d,A+f^A} \) is an interval. Let \( \alpha_M \) be the unique solution to \( (W^A_0)' = 0 \), i.e.,

\[
\frac{\delta (CS^{com,A} - CS^c,A - 2\pi^c,A)}{[1 - (1 - \alpha^A)\delta]^2} = C'(\alpha_M)
\]

Suppose that \( \alpha_M \leq \frac{\pi^c,A-(1-\delta)\pi^d,A}{\delta \pi^d,A+f^A} \). Then, \( \alpha_M = \arg \max_{\alpha^A \leq \frac{\pi^c,A-(1-\delta)\pi^d,A}{\delta \pi^d,A+f^A}} W^A_0 \).

Part 3. Suppose that the firms are owned by local citizens and they also operate in foreign markets. Fix \( \alpha^B \) and consider \( P^A = \{ \alpha^A \in [0, 1] : (\alpha^A, \alpha^B) \in R^A \} \), i.e., the set of all \( \alpha^A \) for which \( (\alpha^A, \alpha^B) \in R^A \). From Proposition 3 Part 1, we know that \( (\alpha^A, \alpha^B) \in R^B \) implies that, in equilibrium, firms collude in both countries until the first time \( s^B_t = 1 \), when they are prosecuted in country \( B \). Thereafter, they collude in country \( A \) until the first time \( s^A_{t+\tau} = 1 \) with \( \tau \geq 1 \), when they are prosecuted in country \( A \). Thereafter, there is competition in both countries. Thus, for \( \alpha^A \in P^A \), the expected discounted welfare of country \( A \) is given by:

\[
W^A_0 = (CS^c,A + 2\pi^c,A + 2\pi^c,B) + \alpha^B \left(-2f^B + \delta \bar{W}_0^A\right) + (1 - \alpha^B)\delta W^A_0,
\]

where \( \bar{W}_0^A = \frac{CS^c,A + 2\pi^c,A + \alpha^B\delta CS^{com,A}}{1 - (1 - \alpha^B)\delta} \). Hence:

\[
W^A_0 = \frac{\left[\frac{1-\delta+\alpha^B\delta^2\delta}{1-(1-\alpha^A)\delta}\right] \left(CS^c,A + 2\pi^c,A\right) + 2 \left(\pi^c,B - \alpha^B f^B\right) + \alpha^A \alpha^B \delta^2 CS^{com,A}}{1 - (1 - \alpha^B)\delta}.\]

Note that \( W^A_0 \) is a \( C^2 \) and strictly concave function of \( \alpha^A \), while \( P^A \) is an interval because it is just the intersection of linear inequalities. Let \( \alpha_L \) be the unique solution to \( (W^A_0)' = 0 \), i.e.,

\[
\frac{\alpha^B \delta^2 \left(CS^{com,A} - CS^c,A - 2\pi^c,A\right)}{[1 - (1 - \alpha_L)\delta]^2} = C'(\alpha_L).
\]
Suppose that $\alpha_L \in P^A$. Then, $\alpha_L = \arg \max_{\alpha \in P^A} W^A_0$.

Part 4. Note that $\frac{\delta (CS^{com,A} - CS^{c,A}) + 2f^A(1-\delta)}{[1-(1-\alpha)\delta]^2}$, $\frac{\delta (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha)\delta]^2}$, and $\frac{\alpha^B\delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha)\delta][1-(1-\alpha\beta)\delta]}$ are decreasing in $\alpha$, while $C'(\alpha)$ is increasing in $\alpha$. Moreover,

$$\frac{\delta (CS^{com,A} - CS^{c,A}) + 2f^A(1-\delta)}{[1-(1-\alpha)\delta]^2} > \frac{\delta (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha)\delta]^2} > \frac{\alpha^B\delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha)\delta]^2[1-(1-\alpha\beta)\delta]}$$

where the last inequality holds for all $\alpha^B$. Therefore, $\alpha_H > \alpha_M > \alpha_L$.

Finally, from the implicit function theorem, the derivative of $\alpha_L$ with respect to $\alpha^B$ is given by:

$$\frac{d\alpha_L}{d\alpha^B} = \frac{\delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A}) (1-\delta)}{[1-(1-\alpha_L)\delta]^2[1-(1-\alpha^B)\delta]} C''(\alpha_L) + \frac{1-(1-\alpha\beta)\delta(2\alpha^B \delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A}))}{[1-(1-\alpha_L)\delta]}$$

Since $C''(\alpha_L) \geq 0$, we have $\frac{d\alpha_L}{d\alpha^B} > 0$. □

A.3.2 Proof of Proposition 5

Proposition 5. Suppose that Assumption 2 holds, that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that $\pi^{c,A} > (1-\delta)\pi^{d,A}$ and $\pi^{c,B} > (1-\delta)\pi^{d,B}$. Let $(\alpha^{A,*}, \alpha^{B,*}) \in R^A$, where $(\alpha^{A,*}, \alpha^{B,*})$ is the unique solution to

$$\frac{\alpha^{B,*}\delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha^{A,*})\delta]^2[1-(1-\alpha^{B,*})\delta]} = C'(\alpha^{A,*})$$

Then, the Nash equilibrium probabilities of detection selected by the competition authorities are $(\alpha^{A}, \alpha^{B}) = (\alpha^{A,*}, \alpha^{B,*})$. Moreover, if $\delta (CS^{com,B} - CS^{c,B}) + 2f^B(1-\delta) > \delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})$, then $\alpha^{B,*} > \alpha^{A,*}$.

Proof. Let $\alpha^{B,*}$ be the unique solution to

$$\frac{\delta (CS^{com,B} - CS^{c,B}) + 2f^B(1-\delta)}{[1-(1-\alpha^{B,*})\delta]^2} = C'(\alpha^{B,*})$$

Assume that $\alpha^{B,*} \in P^B = \{\alpha^B \in [0,1] : (\alpha^{A},\alpha^{B}) \in R^A\}$. Then, from Proposition 4 Part 1, $\alpha^{B,*} = \arg \max_{\alpha^B \in P^B} W^B_0$. Thus, $\alpha^{B,*}$ is a dominant strategy for competition authority $B$.

Let $\alpha^{A,*}$ be the unique solution to

$$\frac{\alpha^{B,*}\delta^2 (CS^{com,A} - CS^{c,A} - 2\pi^{c,A})}{[1-(1-\alpha^{A,*})\delta]^2[1-(1-\alpha^{B,*})\delta]} = C'(\alpha^{A,*})$$

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Assume that \( \alpha^{A,*} \in P^A = \{ \alpha^A \in [0,1] : (\alpha^A, \alpha^{B,*}) \in R^A \} \). Then, from Proposition 4 Part 3, \( \alpha^{A,*} = \arg \max_{\alpha^A \in P^A} W_0^A \). Thus, \( \alpha^A = \alpha^{A,*} \) is the best response of competition authority A to \( \alpha^{B,*} \). Therefore, \( (\alpha^A, \alpha^B) = (\alpha^{A,*}, \alpha^{B,*}) \) is the unique Nash equilibrium.

Since \( \frac{\delta (CS_{com,B} - CS_{c,B}) + 2 f^B(1-\delta)}{[1-(1-\alpha)p^A][1-(1-\alpha p^B)]} \) and \( \frac{\alpha^B \delta^2 (CS_{com,A} - CS_{c,A} - 2 \pi^c_{A})}{(1-\delta + \alpha p^{B,*})} \) are decreasing in \( \alpha \), while \( C' (\alpha) \) is increasing in \( \alpha \), a sufficient condition for \( \alpha^{B,*} > \alpha^{A,*} \) is

\[
\delta (CS_{com,B} - CS_{c,B}) + 2 f^B(1-\delta) > \frac{\alpha^{B,*} \delta^2 (CS_{com,A} - CS_{c,A} - 2 \pi^c_{A})}{(1-\delta + \alpha^{B,*} \delta)}
\]

The right hand side of this inequality is increasing in \( \alpha^{B,*} \). Hence, if \( \delta CS_{com,B} - CS_{c,B} + 2 f^B(1-\delta) > \delta^2 (CS_{com,A} - CS_{c,A} - 2 \pi^c_{A}) \), it must be the case that the inequality holds for any \( \alpha^B \).

**A.4 Antitrust Policy, Integration and International Agreements**

**Assumption 3.** The signals received by the competition authorities are:

1. **Industry x:**

\[
s_{t}^{A,x} = \begin{cases} 
1 & \text{with probability } \alpha^{x,A} \text{ if } a_{t}^{1,x,A} = a_{t}^{2,x,A} = 1, \\
0 & \text{with probability } (1 - \alpha^{x,A}) \text{ if } a_{t}^{1,x,A} = a_{t}^{2,x,A} = 1, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
s_{t}^{B,x} = \begin{cases} 
1 & \text{with probability } \alpha^{x,B} \text{ if } a_{t}^{1,x,B} = a_{t}^{2,x,B} = 1 \text{ and } p_{t}^{x,A} = 0 \text{ for all } \tau \leq t, \\
1 & \text{if } a_{t}^{1,x,B} = a_{t}^{2,x,B} = 1 \text{ and } p_{t}^{x,A} = 1 \text{ for some } \tau \leq t, \\
0 & \text{with probability } (1 - \alpha^{x,B}) \text{ if } a_{t}^{1,x,B} = a_{t}^{2,x,B} = 1 \text{ and } p_{t}^{x,A} = 0 \text{ for all } \tau \leq t, \\
0 & \text{otherwise.}
\end{cases}
\]

2. **Industry y:**

\[
s_{t}^{A,y} = \begin{cases} 
1 & \text{with probability } \alpha^{y,B} \text{ if } a_{t}^{1,y,B} = a_{t}^{2,y,B} = 1, \\
0 & \text{with probability } (1 - \alpha^{y,B}) \text{ if } a_{t}^{1,y,B} = a_{t}^{2,y,B} = 1, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
s_{t}^{B,y} = \begin{cases} 
1 & \text{with probability } \alpha^{y,A} \text{ if } a_{t}^{1,y,A} = a_{t}^{2,y,A} = 1 \text{ and } p_{t}^{y,B} = 0 \text{ for all } \tau \leq t, \\
1 & \text{if } a_{t}^{1,y,A} = a_{t}^{2,y,A} = 1 \text{ and } p_{t}^{y,B} = 1 \text{ for some } \tau \leq t, \\
0 & \text{with probability } (1 - \alpha^{y,A}) \text{ if } a_{t}^{1,y,A} = a_{t}^{2,y,A} = 1 \text{ and } p_{t}^{y,B} = 0 \text{ for all } \tau \leq t, \\
0 & \text{otherwise.}
\end{cases}
\]

**A.4.1 Independent Competition Authorities**

**Proposition 3 (Two-industries and Independent Competition Authorities).** Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and \( \pi^{x,z,j} > (1-\delta) \pi^{d.x,j} \) for \( z \in \{x,y\} \) and \( j \in \{A,B\} \).
1. Industry $x$. Suppose that $(\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A}$. Then, in industry $x$ firms collude in both countries until the first time $s_t^{x,B} = 1$, when they are prosecuted in country $B$. Thereafter, they collude in country $A$ until the first time $s_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$, when they are prosecuted in country $A$. Thereafter, there is competition in both countries.

Hence:

$$
\left(\begin{array}{l}
\end{array}\right)
$$

This completes the proof of Proposition 3 (Two-industries and Independent Competition Authorities).

Proof. The proof is identical to the proof of Proposition 3 Part 1. The only required change in notation is to add the industry superindex $z \in \{x,y\}$ and define $R^{z,j} = \{(\alpha^{x-j}, \alpha^{x-z-j}) \in [0,1] \times [0,1]: (15)-(18) \text{ hold}\}$, where $z \in \{x,y\}, j \in \{A,B\}$ and

$$
\alpha^{x-z,j} > \frac{2 \left[ \delta \pi^{c,z-j} + (1 - \delta) f^{z,j} \right] - \delta \left( \Delta S^{z,j} - 2 \pi^{c,z,j} \right)}{2 \left[ \delta \pi^{c,z-j} + (1 - \delta) f^{z,j} \right] + \frac{\delta(1-\alpha^{x-j})}{1-(1-\alpha^{x-j})} \delta \left( \Delta S^{z,j} - 2 \pi^{c,z,j} \right)},
$$

(15)

$$
\alpha^{z-j} \leq \frac{\pi^{c,z-j} - (1 - \delta) \pi^{d,z,j}}{\delta \pi^{d,z,j} + f^{z,j}},
$$

(16)

$$
\alpha^{z-j} \leq \frac{(\pi^{c,z,j,A} + \pi^{c,z,j,B}) - (1 - \delta) (\pi^{d,z,j,A} + \pi^{d,z,j,B})}{\delta (\pi^{d,z,j,A} + \pi^{d,z,j,B}) - \frac{\delta(1-\alpha^{x,j})}{1-(1-\alpha^{x,j})} \delta (\pi^{c,z,j,A} + \pi^{c,z,j,B}) + f^{z,j}},
$$

(17)

$$
\alpha^{z-j} < \frac{(1 - \delta) \pi^{c,z-j} + \alpha^{z-j} \left[ \delta \left( \pi^{c,z+j} + \pi^{c,z-j} \right) + (1 - \delta) f^{z,j} \right]}{1 - (1 - \alpha^{z,j}) \delta f^{z,j}}.
$$

(18)

This completes the proof of Proposition 3 (Two-industries and Independent Competition Authorities).

Welfare under no integration. Consider industry $x$ under no integration. Suppose that $(\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A}$. Then, in equilibrium, firms collude in both countries until the first time $s_t^{x,B} = 1$, when they are prosecuted in country $B$. Thereafter, they collude in country $A$ until the first time $s_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$, when they are prosecuted in country $A$. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country $A$ in industry $x$ is:

$$
W_0^{x,A} = \left(\begin{array}{l}
\end{array}\right)
$$

where

$$
\tilde{W}_0^{x,A} = \frac{CS^{c,x,A} + 2\pi^{c,x,A} + \pi^{c,\cdot,B} + \alpha^{x,B} \left( 2f^{x,B} + \delta W_0^{x,A} \right) + (1 - \alpha^{x,B}) \delta W_0^{x,A},}
$$

Hence:

$$
W_0^{x,A} = \frac{1 - \delta + \alpha^{x,A}\delta + \alpha^{x,B}\delta}{1 - (1 - \alpha^{x,A})\delta} \left(\begin{array}{l}
\end{array}\right).
$$

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The expected discounted welfare of country $B$ in industry $x$ is

$$W_{0,x,B} = \frac{CS_{c,x,B} + 2\alpha_{x,B}f_{x,B} + \alpha_{x,B}^2 CS_{com,x,B}}{1 - (1 - \alpha_{x,B})\delta}. $$

Consider industry $y$ under no integration. Suppose that $(\alpha_{y,B}, \alpha_{y,A}) \in R_{y,B}$. Then, in equilibrium, firms collude in both countries until the first time $s_{y,A}^{t+\tau} = 1$, when they are prosecuted in country $A$. Thereafter, they collude in country $B$ until the first time $s_{y,B}^{t+\tau} = 1$ with $\tau \geq 1$, when they are prosecuted in country $B$. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country $B$ in industry $y$ is:

$$W_{0,y,B} = \left[1 - \delta + \alpha_{y,B}\delta + \alpha_{y,A}\delta\right] \left(CS_{c,y,B} + 2\pi_{c,y,B} + 2\pi_{c,y,A} - \alpha_{y,B}f_{y,A} + \frac{\alpha_{y,B}\alpha_{y,A}\delta^2 CS_{com,y,B}}{1 - (1 - \alpha_{y,B})\delta(1 - \alpha_{y,B})}\right),$$

while the expected discounted welfare of country $A$ in industry $y$ is

$$W_{0,y,A} = \frac{CS_{c,y,A} + 2\alpha_{y,A}f_{y,A} + \alpha_{y,A}^2 CS_{com,y,A}}{1 - (1 - \alpha_{y,A})\delta}. $$

### A.4.2 A Globally Integrated Competition Authority

**Proposition 3 (Two-industries and Globally Integrated Competition Authority).** Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and $\pi_{c,z,j} > (1 - \delta)\pi_{d,z,j}$ for $z \in \{x,y\}$ and $j \in \{A,B\}$.

1. **Industry $x$.** Suppose that $(\alpha_{x,A}, \alpha_{x,B}) \in R_{x,A}$. Then, firms collude in both countries until the first time $s_{x,A}^t = 1$, when they are prosecuted in both countries, or until the first time $s_{x,A}^t = 0$ and $s_{x,B}^t = 1$, when they are prosecuted in country $B$. In the later case, firms keep colluding in country $A$ until the first time $s_{x,B}^{t+\tau} = 1$ with $\tau \geq 1$.

2. **Industry $y$.** Suppose that $(\alpha_{y,B}, \alpha_{y,A}) \in R_{y,B}$. Then, firms collude in both countries until the first time $s_{y,B}^t = 1$, when they are prosecuted in both countries, or until the first time $s_{y,B}^t = 0$ and $s_{y,A}^t = 1$, when they are prosecuted in country $A$. In the later case, firms keep colluding in country $B$ until the first time $s_{y,B}^{t+\tau} = 1$ with $\tau \geq 1$.

**Proof.** The world’s aggregate welfare in period $t$ is $w_t = (CS_t^x + PS_t^x) + (CS_t^y + PS_t^y)$, where $CS_t^z$ and $PS_t^z$ are the consumer and producer surpluses in period $t$ in industry $z \in \{x,y\}$, respectively. The world’s aggregate expected discounted welfare is $W_t^W = E_t \left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} w_{\tau}\right]$. Since $CS_{com,z,j} > CS_{c,z,j} + 2\pi_{c,z,j}$, $CS_t^x + PS_t^x$ adopts a maximum when firms operating in industry $z$ compete. Thus, in order to maximize $W_t^W$ collusion must be prosecuted as soon as it is detected.
Consider industry $x$. Then, the expected profits of a firm under collusion in country $A$, collusion in country $B$ and collusion in both countries are given by:

$$
\Pi^{c,x,A} = \frac{\pi^{c,x,A} - \alpha^{x,A}fx,A}{1 - (1 - \alpha^{x,A})\delta} \\
\Pi^{c,x,B} = \frac{\pi^{c,x,B} - \alpha^{x,B}fx,B}{1 - (1 - \alpha^{x,B})\delta}, \\
\Pi^{c,x,AB} = \Pi^{c,x,A} + \frac{\pi^{c,x,B} - [1 - (1 - \alpha^{x,B})(1 - \alpha^{x,A})]fx,B}{1 - (1 - \alpha^{x,A})(1 - \alpha^{x,B})\delta}.
$$

respectively. For collusion in country $A$ to be an equilibrium, it must be the case that $\Pi^{c,x,A} \geq \pi^{d,x,A}$. For collusion in country $B$ to be an equilibrium it must be the case that $\Pi^{c,x,B} \geq \pi^{d,x,B}$. For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,x,AB} \geq \pi^{d,x,A} + \pi^{d,x,B}$. $\Pi^{c,x,A} \geq \pi^{d,x,A}$ if and only if $\alpha^{x,A} \leq \frac{\pi^{c,x,A} - \alpha^{x,A}fx,A}{fx,A + \delta\pi^{x,A}}$. $\Pi^{c,x,B} \geq \pi^{d,x,B}$ if and only if $\alpha^{x,B} \leq \frac{\pi^{c,x,B} - \alpha^{x,B}fx,B}{fx,B + \delta\pi^{x,B}}$. $\Pi^{c,x,AB} \geq \pi^{d,x,A} + \pi^{d,x,B}$ if and only if

$$
\sum_{j=A,B} \frac{\pi^{c,x,j} - \alpha^{x,A}fx,j}{1 - (1 - \alpha^{x,A})\delta} \geq \frac{\pi^{c,x,j} - \alpha^{x,A}fx,j}{1 - (1 - \alpha^{x,A})\delta} \geq \frac{\pi^{c,x,B} - (1 - \alpha^{x,B})(1 - \alpha^{x,A})fx,B}{1 - (1 - \alpha^{x,A})(1 - \alpha^{x,B})\delta}.
$$

The left hand side of this inequality is positive if and only if $\alpha^{x,A} < \alpha_H = \frac{\pi^{c,x,A} + \pi^{c,x,B} - \pi^{d,x,A} - \pi^{d,x,B}}{fx,A + fx,B - \pi^{d,x,A} - \pi^{d,x,B}}$, while the right hand side is positive if and only if $\alpha^{x,A} > \alpha_L = \frac{\pi^{c,x,A} - \pi^{c,x,B} + \pi^{d,x,A} + \pi^{d,x,B}}{fx,A + fx,B - \pi^{d,x,A} - \pi^{d,x,B}}$. Moreover, note that $\alpha_L < \alpha_H$. Hence, the inequality holds when $\alpha^{x,A} \leq \alpha_L$ or $\alpha_L < \alpha^{x,A} < \alpha_H$ and $\alpha^{x,B} \leq \frac{\sum_{j=A,B} \pi^{c,x,j} - \alpha^{x,A}fx,j - [1 - (1 - \alpha^{x,A})\delta]\pi^{x,j}}{(1 - \alpha^{x,A})[\delta\pi^{x,B} + \delta\pi^{x,B} - \delta(\Pi_{x,A} - \pi^{d,x,A})]}$. Finally, $\Pi^{c,x,AB} > \Pi^{c,x,A}$ if and only if $\alpha^{x,B} < \frac{\pi^{c,x,B} - \alpha^{x,A}fx,A}{(1 - \alpha^{x,A})fx,B}$, while $\Pi^{c,x,AB} > \Pi^{c,x,B}$ if and only if $\frac{\pi^{c,x,A} - \alpha^{x,A}fx,A}{1 - (1 - \alpha^{x,A})\delta} > \alpha^{x,A}(1 - \alpha^{x,B})[\delta\pi^{x,B} + \pi^{d,x,B}] > \frac{\pi^{c,x,B} - \alpha^{x,A}fx,A}{1 - (1 - \alpha^{x,A})\delta}$.

Summing up, suppose that $(\alpha^{x,A}, \alpha^{x,B}) \in \{ (\alpha^{x,j}, \alpha^{x,j}) \in [0,1] \times [0,1] : (19)-(21) \text{ hold} \}$, where

$$
\alpha^{x,j} = \frac{\pi^{c,x,j} - (1 - \delta)\pi^{d,x,j}}{fx,j + \delta\pi^{d,x,j}}, \quad (19) \\
\left[ \frac{\pi^{c,x,j} - (1 - \delta)fx,j}{fx,j + \delta\pi^{d,x,j}} \right] < \alpha^{x,j} < \frac{\pi^{c,x,A} + \pi^{c,x,B} - (1 - \delta)(\pi^{d,x,A} + \pi^{d,x,B})}{fx,A + fx,B + \delta(\pi^{d,x,A} + \pi^{d,x,B})} \quad \text{and} \quad (20) \\
\left[ \frac{\pi^{c,x,j} - (1 - \delta)fx,j}{fx,j + \delta\pi^{d,x,j}} \right] < \alpha^{x,j} < \frac{\pi^{c,x,A} + \pi^{c,x,B} - (1 - \delta)(\pi^{d,x,A} + \pi^{d,x,B})}{fx,A + fx,B + \delta(\pi^{d,x,A} + \pi^{d,x,B})} \quad \text{and} \quad (21)
$$

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Then, when competition authorities are integrated, firms in industry $x$ collude in both countries until they are detected and prosecuted. Following exactly the same steps, but reversing the roles of $A$ and $B$ and replacing $x$ for $y$, we obtain that $\bar{R}^y_B$. Then, if $(\alpha^y_B, \alpha^y_A) \in \bar{R}^y_B$ and competition authorities are integrated, firms in industry $y$ collude in both countries until they are detected and prosecuted.

**Welfare under integration.** Consider industry $x$ under integration. Suppose that $(\alpha^x_A, \alpha^x_B) \in \bar{R}^x_A$. Then, firms collude in both countries until the first time $s_1^{x,A} = 1$, when they are prosecuted in both countries, or until the first time $s_1^{x,B} = 0$ and $s_1^{x,B} = 1$, when they are prosecuted in country $B$. In the later case, firms keep colluding in country $A$ until the first time $s_{t+τ}^{x,A} = 1$. Then, the expected discounted welfare of country $A$ in industry $x$ is:

$$W_0^{x,A} = (CS_{c,x,A} + 2π_{c,x,A} + 2π_{c,x,B}) + α^{x,A} \left( -2fx^B + δ\frac{CS_{com,x,A}}{1 - δ} \right) + (1 - α^{x,A}) α^{x,B} \left( -2fx^B + δ\tilde{W}_0^{x,A} \right) + (1 - α^{x,A}) (1 - α^{x,B}) δ\tilde{W}_0^{x,A},$$

where

$$\tilde{W}_0^{x,A} = (CS_{c,x,A} + 2π_{c,x,A}) + α^{x,A} δ\frac{CS_{com,x,A}}{1 - δ} + (1 - α^{x,A}) δ\tilde{W}_0^{x,A}.$$

Hence,

$$W_0^{x,A} = \frac{CS_{c,x,A} + 2π_{c,x,A}}{1 - (1 - α^{x,A}) δ} + \frac{2π_{c,x,B} - [1 - (1 - α^{x,A})(1 - α^{x,B})]}{1 - (1 - α^{x,A})(1 - α^{x,B}) δ} \left( 2fx^B + δ\frac{CS_{com,x,A}}{1 - δ} \right) + \frac{α^{x,A} δCS_{com,x,A}}{1 - (1 - α^{x,A}) δ}.$$

The expected discounted welfare of country $B$ in industry $x$ is:

$$W_0^{x,B} = \frac{CS_{c,x,B} + [1 - (1 - α^{x,A})(1 - α^{x,B})]}{1 - (1 - α^{x,A})(1 - α^{x,B}) δ} \left( 2fx^B + δ\frac{CS_{com,x,B}}{1 - δ} \right).$$

Consider industry $y$ under integration. $(α^y_B, α^y_A) \in \bar{R}^y_B$. Then, firms collude in both countries until the first time $s_1^{y,B} = 1$, when they are prosecuted in both countries, or until the first time $s_1^{y,B} = 0$ and $s_1^{y,A} = 1$, when they are prosecuted in country $A$. In the later case, firms keep colluding in country $B$ until the first time $s_{t+τ}^{y,B} = 1$. Then, the expected discounted welfare of country $B$ in industry $y$ is:

$$W_0^{y,B} = \frac{CS_{c,y,B} + 2π_{c,y,B}}{1 - (1 - α^{y,B}) δ} + \frac{2π_{c,y,A} - [1 - (1 - α^{y,A})(1 - α^{y,B})]}{1 - (1 - α^{y,A})(1 - α^{y,B}) δ} \left( 2fy^A + δ\frac{CS_{com,y,B}}{1 - δ} \right) + \frac{α^{y,B} δCS_{com,y,B}}{1 - (1 - α^{y,B}) δ}.$$

while the expected discounted welfare of country $A$ in industry $y$ is:

$$W_0^{y,A} = \frac{CS_{c,y,A} + [1 - (1 - α^{y,A})(1 - α^{y,B})]}{1 - (1 - α^{y,A})(1 - α^{y,B}) δ} \left( 2fy^A + δ\frac{CS_{com,y,A}}{1 - δ} \right).$$
A.4.3 Proof of Proposition 6

Proposition 6. Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and \( \pi^{c,z,j} > (1 - \delta) \pi^{d,z,j} \) for \( z \in \{x, y\} \) and \( j \in \{A, B\} \). Assume that \((\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \cap R^{x,B} \) and \((\alpha^{y,B}, \alpha^{y,A}) \in R^{y,B} \cap R^{y,B} \). Then:

1. Country A benefits from integration if and only if \( \Delta W_{0}^{x,A} + \Delta W_{0}^{y,A} > 0 \) and \( (j = B) \), where
\[
\Delta W_{0}^{x,A} = \frac{\alpha^{x,A}}{1 - (1 - \alpha^{x,B}) \delta} \left\{ \frac{\delta (\Delta CS^{x,A} - 2\pi^{c,x,A})}{[1 - (1 - \alpha^{x,A}) \delta]} - \frac{2 (1 - \alpha^{x,B}) [\delta \pi^{c,x,B} + (1 - \delta) f^{x,B}]}{[1 - (1 - \alpha^{x,A}) (1 - \alpha^{x,B}) \delta]} \right\},
\]
\[
\Delta W_{0}^{y,A} = \frac{\alpha^{y,B} (1 - \alpha^{y,A})}{[1 - (1 - \alpha^{y,A}) \delta]} \left\{ \frac{\delta \Delta CS^{y,A} + (1 - \delta) 2 f^{y,A}}{[1 - (1 - \alpha^{y,B}) \delta]} \right\},
\]

2. Country B benefits from integration if and only if \( \Delta W_{0}^{x,B} + \Delta W_{0}^{y,B} > 0 \), where
\[
\Delta W_{0}^{x,B} = \frac{\alpha^{x,A} (1 - \alpha^{x,B})}{[1 - (1 - \alpha^{y,A}) (1 - \alpha^{y,B}) \delta]} \left\{ \frac{\delta \Delta CS^{x,B} + (1 - \delta) 2 f^{x,B}}{[1 - (1 - \alpha^{x,A}) (1 - \alpha^{x,B}) \delta]} \right\},
\]
\[
\Delta W_{0}^{y,B} = \frac{\alpha^{x,B}}{1 - (1 - \alpha^{y,A}) \delta} \left\{ \frac{\delta (\Delta CS^{y,B} - 2\pi^{c,y,B})}{[1 - (1 - \alpha^{y,B}) \delta]} - \frac{2 (1 - \alpha^{y,A}) [\delta \pi^{c,y,A} + (1 - \delta) f^{y,A}]}{[1 - (1 - \alpha^{y,A}) (1 - \alpha^{y,B}) \delta]} \right\},
\]

3. Moreover, if \( \pi^{c,z,j} = \pi^{c}, \pi^{d,z,j} = \pi^{d}, \Delta CS^{z,j} = \Delta CS, \alpha^{z,j} = \alpha \), and \( f^{z,j} = f \) for all \( z \in \{x, y\} \) and \( j \in \{A, B\} \), it is always the case that both countries are better off under integration.

Proof. Suppose that \((\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \cap R^{x,B} \) and \((\alpha^{y,B}, \alpha^{y,A}) \in R^{y,B} \cap R^{y,B} \). Then, the aggregate expected welfare of country \( j \in \{A, B\} \) under no integration is \( W_{0}^{x,j} + W_{0}^{y,j} \), while under integration it is \( W_{0}^{x,j} + W_{0}^{y,j} \). Thus, country \( j \) is better off under integration than under no integration if and only if \( W_{0}^{x,j} - W_{0}^{x,j} + W_{0}^{y,j} - W_{0}^{y,j} > 0 \). Define
\[
\Delta W_{0}^{x,A} = \bar{W}_{0}^{x,A} - W_{0}^{x,A} = \frac{\alpha^{x,A}}{1 - (1 - \alpha^{x,B}) \delta} \left\{ \frac{\delta (\Delta CS^{x,A} - 2\pi^{c,x,A})}{[1 - (1 - \alpha^{x,A}) \delta]} - \frac{2 (1 - \alpha^{x,B}) [\delta \pi^{c,x,B} + (1 - \delta) f^{x,B}]}{[1 - (1 - \alpha^{x,A}) (1 - \alpha^{x,B}) \delta]} \right\},
\]
\[
\Delta W_{0}^{y,A} = \bar{W}_{0}^{y,A} - W_{0}^{y,A} = \frac{\alpha^{y,B} (1 - \alpha^{y,A})}{[1 - (1 - \alpha^{y,A}) \delta]} \left\{ \frac{\delta \Delta CS^{y,A} + (1 - \delta) 2 f^{y,A}}{[1 - (1 - \alpha^{y,B}) \delta]} \right\},
\]

Then, country A is better off under integration if and only if \( \Delta W_{0}^{x,A} + \Delta W_{0}^{y,A} > 0 \). Moreover, note that \((\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \) implies \( \Delta W_{0}^{x,A} < 0 \), while it is clear that \( \Delta W_{0}^{y,A} > 0 \). Define
\[
\Delta W_{0}^{x,B} = \frac{\alpha^{x,A} (1 - \alpha^{x,B})}{[1 - (1 - \alpha^{y,A}) (1 - \alpha^{y,B}) \delta]} \left\{ \frac{\delta \Delta CS^{x,B} + (1 - \delta) 2 f^{x,B}}{[1 - (1 - \alpha^{x,A}) (1 - \alpha^{x,B}) \delta]} \right\},
\]
\[
\Delta W_{0}^{y,B} = \frac{\alpha^{x,B}}{1 - (1 - \alpha^{y,A}) \delta} \left\{ \frac{\delta (\Delta CS^{y,B} - 2\pi^{c,y,B})}{[1 - (1 - \alpha^{y,B}) \delta]} - \frac{2 (1 - \alpha^{y,A}) [\delta \pi^{c,y,A} + (1 - \delta) f^{y,A}]}{[1 - (1 - \alpha^{y,A}) (1 - \alpha^{y,B}) \delta]} \right\}.
\]
Then, country $B$ is better off under integration if and only if $\Delta W^x_B + \Delta W^y_B > 0$. Moreover, note that $\Delta W^x_B > 0$, while $(\alpha_{y,B}, \alpha_{y,A}) \in R^{y,B}$ implies $\Delta W^y_B < 0$.

Finally, assume that $\pi^{c,z,j} = \pi^c$, $\pi^{d,z,j} = \pi^d$, $\Delta CS^z_j = \Delta CS$, $\alpha^z_j = \alpha$, and $f^z_j = f$ for all $z \in \{x, y\}$ and $j \in \{A, B\}$. Then

$$
\begin{align*}
\Delta W^{x,A} &= \Delta W^{y,B} = \frac{\alpha}{[1 - (1 - \alpha) \delta]} \left\{ \frac{\delta (\Delta CS - 2 \pi^c)}{[1 - (1 - \alpha) \delta]} - \frac{2 (1 - \alpha) [\delta \pi^c + (1 - \delta) f]}{[1 - (1 - \alpha) \delta]} \right\}, \\
\Delta W^{y,A} &= \Delta W^{x,B} = \frac{\alpha (1 - \alpha) [\delta \Delta CS + (1 - \delta) 2 f]}{[1 - (1 - \alpha)^2 \delta] [1 - (1 - \alpha) \delta]}
\end{align*}
$$

Therefore:

$$
\Delta W^{x,A} + \Delta W^{y,A} = \Delta W^{x,B} + \Delta W^{y,B} = \frac{2 - \alpha - (1 - \alpha)^2 \delta}{[1 - (1 - \alpha) \delta]^2} \frac{\alpha \delta (\Delta CS - 2 \pi^c)}{[1 - (1 - \alpha) \delta]^2 [1 - (1 - \alpha)^2 \delta]} > 0.
$$

Hence, it is always the case that both countries are better off under integration. ■