Market Power and Mergers in Multi-Sided Markets*

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Abstract
Due to the presence of network externalities in multi-sided markets, evaluating the performance of a platform is more difficult than for a firm in a traditional market. In this paper I propose a measure of market power (the Generalized Lerner index) to evaluate platform performance in multi-sided markets. In addition, I develop methods for determining platform marginal costs and post-merger price predictions. I show that with positive network externalities, traditional methods will underestimate unobserved marginal costs and will overstate post-merger pricing predictions. I also find that if a multi-platform seller partially integrates their networks, by allowing network compatibility across its multiple platforms, then a cannibalization effect can occur where some prices decrease relative to their unintegrated counterpart. Finally, I highlight the main results in an application to the video game market.

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JEL Classifications: L11, L14, L40.

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1 Introduction

A firm’s ability to earn profit through superior price-cost markups is a concern for managers, investors, economists, and policy makers. Market power dominance by a firm relative to its competitors often translates into higher markups or market share which result in greater profitability. While the academic community has analyzed the sources and effects of market power in traditional markets, there is little research on market power in multi-sided markets.

When faced with multiple sides, the presence of network externalities distorts platform prices. Network externalities can even cause the price on one side of a platform to be less than marginal cost [Rochet and Tirole (2003), Armstrong (2006), Hagiu (2006), and Weyl (2010)]. Such discounting is observed in many platform markets. For example, when first released, gaming console prices are often less than console marginal costs. Similarly, media platforms like Facebook, Snapchat, YouTube, and Google are free to consumers but generate revenues from advertisers. Hence, considering the price-cost margin of a single side of the market may result in a misspecified measure of platform market power.

To more accurately account for market power across an entire platform, I propose a generalization of the Lerner Index that considers both markups and sales across all sides of a platform. To illustrate this method, I investigate the video game industry where console and game prices provide platforms with revenue generating opportunities. Using the generalized Lerner index, I show that Nintendo obtains the greatest platform market power even though it has the smallest console markup. The greater market power for Nintendo stems from two critical features. First, Nintendo has the highest markup on games. Second, game sales are...

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1In addition, behavioral externalities or search costs can result in some products being sold near marginal cost (home printers) while other complementary products (ink cartridges) are sold with greater markups. This is known as add-on pricing with what Zegers and Kretschmer (2017) refer to as aftermarkets. For further discussion see Ellison (2005), Gabaix and Laibson (2006), Ellison and Ellison (2009) and Grubb (2009). The analysis developed in this paper can also be applied to add-on products, such as printers with ink cartridges, razors with blades, HD-DVD and Blu-ray players, traditional cameras with film, and coffee makers with pods.

2For more information, see Lee (2013) for the sixth generation video game market, “Report: PS3 to sell for $399, cost $494 to make” by Gamespot for the seventh generation, and “Microsoft Xbox One Hardware Cost Comes in Below Retail Price” by IHS for the eighth generation.
significantly larger than console sales — consumers average between 8 and 10 game purchases per console. Thus, by having more market power on the side of the market that generates the majority of sales, Nintendo generates the greatest market power, a result revealed by the generalized Lerner index.

Considering multi-sidedness is also important for correctly assessing the effects of platform mergers. For example, do all platform prices of the merged entity increase post-merger? To analyze the effects of mergers, a general model of platform competition is developed. With this framework, methods for determining unobserved marginal costs and for predicting platform merger outcomes are established for multi-sided markets. By comparing these multi-sided market techniques with those that are typically used for traditional markets, the bias that results from ignoring network externalities is made clear.

Another important distinction between platform mergers and mergers in traditional markets is that a multi-platform seller can integrate networks across platforms. As an example of network integration, Facebook Inc. has integrated user features across its two social media platforms — users can integrate their profiles so that an Instagram post automatically posts to the user’s Facebook page. In a similar fashion, the Nintendo 64 Transfer Pak accessory allows gamers who own a Nintendo 64 console to use Nintendo Gameboy Color games on their Nintendo 64. In this case, network integration between gaming platforms generates a tradeoff. With integration, Nintendo 64 owners can now use Gameboy games which increases the demand for Gameboy games. However, consumers who would have otherwise purchased both the Nintendo 64 and the Gameboy might now only purchase the Nintendo 64, reducing the demand for the Gameboy device. Thus, even if network integration is costless, a multi-platform entity may choose to maintain separation between platforms.

To contrast the multi-sided market methods that are developed in this paper with traditional methods, I apply both to the video game market. I find that the bias from using the traditional formula to estimate console marginal cost is between $59 and $75. These biased

Li and Agarwal (2016) find considerable evidence regarding the benefits that users experience from the integration of the Instagram and Facebook platforms.
console marginal cost estimates result in market power measures that are biased upward by 34-40%. I also find that the traditional approach overestimates post-merger console price predictions by $1-$43. This application highlights the importance of using multi-sided market techniques when examining platform markets. Otherwise, traditional techniques might lead to incorrect measures of markups, market power, or post-merger pricing predictions in platform markets.

Platform mergers have remained largely unstudied in the literature with the exception of a few empirical papers on mergers in media markets. The papers that most closely relate to this study are Filistrucchi et al. (2012) and Affeldt et al. (2013) for newspapers, and Song (2015) for magazines. Taking a more general approach to the platform market, I generalize their post-merger pricing results by showing that positive network effects generate post-merger price increases. I also extend their work by considering platform mergers with network integration which introduces pricing effects that unintegrated mergers cannot explain. In addition, they do not take the generalized Lerner index approach for determining market power; instead, they consider each side individually. Using the generalized Lerner index, I am able to assess platform market power within and across platform industries.

Several other studies take a two-sided approach to certain mergers. Fan (2013) extends the empirical methods developed by Rysman (2004) to consider mergers in the newspaper market. However, she does not observe advertising quantity changes with newspaper mergers, and she uses traditional post-merger price formulas to perform counterfactual exercises. Jeziorski (2014) analyzes the radio market with only a single marginal cost for the platform that lies on the advertising side of the market. He considers how radio mergers affect advertising cost savings, advertising quantities, and welfare, but he does not consider a fully fledged multi-sided approach to platform mergers, pricing, and marginal cost estimation.

\footnote{Chandra and Collard-Wexler (2009) also consider mergers in the two-sided market for newspapers but do not measure costs or use cost data. Instead they find that newspaper pricing below marginal cost is a necessary condition to lower circulation prices post-merger. However, they find no relationship between newspaper concentration measures and platform prices.}

\footnote{Many standard antitrust measures have also been extended to two-sided markets: Emch and Thompson (2006) and Filistrucchi et al. (2014) discuss market definition, Evans and Noel (2008) generalize Critical...}
There are additional studies on platform markets that deserve mention. For example, the literature on two-sided markets using demand estimation techniques includes Rysman (2004) for yellow pages; Nair (2007), Dubé et al. (2010), Lee (2013) and Derdenger (2014) for video games; Gowrisankaran and Rysman (2012) for camcorders; and Gowrisankaran et al. (2014) for DVDs. The video game market has received considerable attention regarding exclusive verses multi-homing game titles and game pricing Shankar and Bayus (2003), Binken and Stremersch (2009), Corts and Lederman (2009), Liu (2010), Landsman and Stremersch (2011), Lee (2013), Derdenger (2014), and Zhou (2016)]. However, this literature generally does not address the issue of market power, platform mergers, or multi-platform network integration. Thus, in addition to providing general platform market power and merger results, this paper contributes to this literature by determining gaming platform market power and by predicting post-merger prices of consoles and games (with and without network integration) for the sixth generation video game market.

The remainder of the paper is organized as follows. Section 2 summarizes the traditional approaches for measuring market power, determining unobserved marginal costs, and predicting post-merger prices. The model of the multi-sided market as well as equilibrium pricing and unobserved marginal cost determination is provided in Section 3. The generalized Lerner index of platform market power is also developed in this section. In Section 4 sellers of multiple platforms and the effect of platform mergers with network integration are considered. To illustrate the main findings in this paper, an application to the sixth generation video game market is presented in Section 5. Finally, Section 6 concludes.

2 A Traditional Market

For comparison purposes, consider first a traditional market. Suppose that there are $N$ sellers of differentiated products that compete in prices. Firm $X$ has demand:

$$q^X = q^X(p^X, p^{-X}),$$

where $p^X$ is the price set by firm $X$, $p^{-X}$ is the vector of prices of the other firms in the market, and $q^X(p^X, p^{-X})$ is the demand function for firm $X$’s product. Demand is continuous with $\frac{dq^X}{dp^X} < 0$ and $\frac{dq^X}{dp^Y} > 0$ for $Y \neq X$. Firm $X$ has profits given by:

$$\Pi^X = (p^X - c^X) \cdot q^X(p^X, p^{-X}),$$

where $c^X$ is the constant marginal cost for firm $X$.

Most measures of market power depend on either price-cost markups or the distribution of market shares. The standard measure of price-cost markups is the Lerner index (Lerner (1934)), which equals the price-cost markup normalized by price. That is, the Lerner index of market power for firm $X$ is given by:

$$L^X = \frac{p^X - c^X}{p^X},$$

which is evaluated at the equilibrium price $p^X$. Market power increases in $L^X$, and $L^X = 0$ indicates no market power.

While this measure provides information on a firm’s ability to price above marginal cost, the cost structure of a firm is often unobserved by practitioners. For example, with administrative cost data provided by a firm, determining the marginal cost for a specific product is likely impossible for firms that are large conglomerates. To overcome the problem of unobserved costs, researchers exploit variation in product characteristics to estimate demand.

\footnote{See Elzinga and Mills (2011) for an overview of the Lerner index.}
With demand elasticities, unobserved marginal costs can be determined. More specifically, in equilibrium we have:

\[ p^X = c^X + \left( -\frac{p^X}{\eta^X} \right) \]  

or  

\[ L^X = \frac{p^X - c^X}{p^X} = -\frac{1}{\eta^X}, \tag{1} \]

where \( \eta^X < 0 \) is the own-price elasticity of demand for firm \( X \) and \( \frac{c^X}{\eta^X} \) characterizes firm \( X \)'s markup due to firm specific market power. From Equation (1):

\[ c^X = p^X \cdot \left( 1 + \frac{1}{\eta^X} \right). \tag{2} \]

Thus, information on prices and elasticities allows a practitioner to estimate unobserved marginal costs and market power in traditional markets. Ideally, a similar approach could be used to measure unobserved marginal costs in multi-sided markets.

The traditional approach can also be used to investigate a merger between firms \( X \) and \( Y \). The profit equation for the merged firms is:

\[ \Pi^{XY} = (p^X - c^X) \cdot q^X(p^X, p^-X) + (p^Y - c^Y) \cdot q^Y(p^Y, p^-Y). \]

The first-order condition of profit maximization with respect to the price of product \( X \) implies that

\[ p^X = c^X + \left( -\frac{p^X}{\eta^X} - D^{XY} \cdot (p^Y - c^Y) \right). \tag{3} \]

where \( D^{XY} \equiv \frac{dq^Y/dp^X}{dq^X/dp^X} < 0 \) denotes the diversion ratio from product \( X \) to product \( Y \). Thus, a merger results in an additional markup, and this diversion markup stems from the lost sales of product \( X \) that are converted to product \( Y \) sales. For multi-sided markets, diversion ratios will also play an important role in determining post-merger pricing.

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7This approach is developed by [Berry, 1994; Berry et al., 1995; and Nevo, 2000].
8If firms sell more than one differentiated product, then information on the full elasticity matrix of own- and cross-price elasticities is required, as discussed in [Berry, 1994; Berry et al., 1995; and Nevo, 2000].
3 A Multi-Sided Market

Now consider a multi-sided market with \( N \) competing platform sellers. Each platform seller charges a price for each “product” or type of “interaction” in their network. I adopt the “product” or “interaction” terminology instead of the “side” terminology that is typically used in the literature. This eases comparison with the traditional market, but also allows for a more complex environment than what are typically considered in the literature of two-sided markets.

Suppose that there are \( K + 1 \) products offered by each platform. In the video game example that I use throughout the paper, I refer to product 0 as the console product with \( K \) game genres. Each of the \( N \) platforms offers one game per genre, and there is game differentiation across platforms within a genre. Naturally, there exists network effects between a platform’s console product and a platform’s \( K \) game products. However, a platform’s adventure game product \( j \) may also be a substitute for the platform’s action game product \( m \). This framework allows not only for network effects across sides, but also product substitutability within a side (e.g., within the game side).

\(^9\) For a media market, an interaction between a consumer and advertiser is a view of an advertisement or a click-through purchase from an ad. In addition, the consumption of news on a media platform is an interaction between a consumer and the platform. Both of these interactions provide the media platform with an opportunity to charge a price. Another example is the video game market. Consumers have demand for the console product and for the many different games offered within the platform network. Thus, where the platform charges a price identifies where a product, with demand, exists.

\(^{10}\) In principle, gaming platforms might differ in terms of the number of games offered. More generally, platforms might differ in terms of the number of sides they support. This can be accounted for by collapsing the supply of product \( i \) to zero for that particular platform.

\(^{11}\) Consoles are differentiated so that playing games within a genre results in a differentiated experience for consumers. That is, console differentiation implies game differentiation even if a game title multi-homes and is available on multiple platforms. A common example of console differentiation resulting in game differentiation for the same game title is the difference in the consumer experience from playing sports games on the Nintendo console versus the other consoles. The Nintendo experience is often worse for these types of games. For more information, see “Madden 2003: Head-to-Head” by IGN, or “XIII Head-to-Head” by IGN.
3.1 Platform Demand

In multi-sided markets, the demand for product $i$ on platform $X$ depends on the prices across platforms for product $i$ and on the supply of other products $k \neq i$ across platforms. For example, the demand for platform $X$’s console depends on the price of each platform’s console and on the supply of each platform’s games. Similarly, the demand for video game $k$ on platform $X$ depends on the prices of the games of type $k$ and on the supply of each platform’s console and other games. Let the quantity of product $i$ on platform $X$ be denoted by $q_i^X$ and let the price of product $i$ on platform $X$ be denoted by $p_i^X$.

The demand for product $i$ on platform $X$ is given by:

$$q_i^X = q_i^X (p_i^X, p_{-i}^X; q_{-i}, q_{-i}^1, q_{-i}^2, \ldots, q_{-i}^N),$$

where $p_{-i}^X$ denotes the vector of prices of other platforms for product $i$, and $q_{-i}$ denotes the vector of quantities of all other products $k \neq i$ on all $N$ platforms (including $X$). Consistent with differentiated price competition in markets with network effects, suppose that $\frac{\partial q_i^X}{\partial p_i^X} < 0$ and $\frac{\partial q_i^Y}{\partial p_i^Y} \geq 0$ for $Y \neq X$. That is, the demand for a platform’s product is decreasing in a platform’s own price and increasing in the price of a competitor. In addition, suppose that $\frac{\partial q_i^X}{\partial q_{-i}^X} \geq 0$. This relates to the presence of network externalities where $\frac{\partial q_i^X}{\partial q_{-i}^X} > 0$ implies a positive network externality. For example, in the case of video games, there is a positive network externality between console sales and game sales ($\frac{\partial q_i^X}{\partial q_i^X} > 0$) for $i = 1, \ldots, K$. Alternatively, $\frac{\partial q_i^X}{\partial q_{-i}^X} < 0$ implies a negative network externality which can occur on media platforms where greater advertisements diminish consumer usage on the media platform.

Given platform $X$ product demands, platform $X$ generates revenues (which can be positive or negative in the case of subsidies) from the sale of each product. Thus, platform $X$’s
profit equation is:

$$\Pi^X = \sum_{i=1}^{K} \left[ (p_i^X - c_i^X) \cdot q_i^X \right] - F^X,$$

(5)

where $c_i^X$ is the marginal cost incurred by platform $X$ for product $i$ and $F^X$ is the platform’s fixed cost.

The model is static and each platform maximizes profit with respect to their prices ($p_i^X$ for $i = 0, 1, ..., K$), taking prices set by the other platforms as given. To ensure that the second-order conditions of profit maximization hold and a unique equilibrium exists, suppose that $\frac{\partial^2 (q_i^X)}{\partial^2 p_i^X} \leq 0$ with $\frac{\partial q_i^X}{\partial p_i^X} \to 0$ as $p_i^X \to 0$. This ensures that network effects are not so large that price reductions that increase demand do not continuously increase profits. In addition, suppose that $\frac{\partial (q_i^X)^2}{\partial^2 q_{-i}^X} < 0$, along with $\frac{\partial q_i^X}{\partial q_{-i}^X} \to 0$ as $q_{-i}^X \to 0$ when $\frac{\partial q_i^X}{\partial q_{-i}^X} > 0$. This ensures that network effects are not so large that demand increases from greater product supply elsewhere on the platform do not persistently increase profits. Combined, these second-order assumptions ensure that the network externalities are sufficiently small so that a platform cannot keep increasing its size continually with a slight price reduction.

When investigating the existence of a solution to the platform’s profit maximization problem, it is important to note that the current demand structure given by Equation (4) contains an infinite feedback loop. That is, in equilibrium, quantity demanded equals quantity supplied which implies that the demand for product $i$ on platform $X$ is a function of both $p_i^X$ and $q_{-i}^X$, but $q_{-i}^X$ is a function of $q_i^X$ which is again a function of both $p_i^X$ and $q_{-i}^X$, and so on. Hence, the derivative assumptions above are partial derivatives (the full or total derivatives do not exist). However, a profit maximizing platform will consider both the direct effect and the indirect feedback loop effect. To solve the platform’s maximization problem, the implicit function theorem is used to generate implicit demand functions where product demand depends on all prices.

**Proposition 1.** If $\frac{\partial q_i^X}{\partial q_j^Y} \neq \left( \frac{\partial q_i^Y}{\partial q_j^X} \right)^{-1}$ for all $X, Y$ and $i \neq j$, then demand functions exist such
that
\[ q^X_i = \tilde{q}^X_i (p^1, p^2, ..., p^N), \]
(6)

where \( p^Y \) for \( Y = 1, ..., N \) is the vector of all prices for platform \( Y \).\(^{13}\)

All proofs are in Appendix A.

The conditions of the implicit function theorem are important because they require asymmetries in the across platform demand dependency derivatives. The term \( \frac{\partial q^X_i}{\partial q^Y_j} \) is the marginal change in demand for platform \( X \)’s product \( i \) from a marginal change in platform \( Y \)’s supply of product \( j \). The requirement implies, for example, that the impact of an additional console sale on game sales cannot equal the inverse of the impact of an additional game sale on console sales: \( \frac{\partial q^X_0}{\partial q^X_1} \neq \left( \frac{\partial q^X_1}{\partial q^X_0} \right)^{-1} \). Most importantly, this result shows that in applications where data may be limited (say \( \frac{\partial q^X_i}{\partial q^Y_j} \) is observed while \( \frac{\partial q^Y_j}{\partial q^X_i} \) is unobserved), one cannot assume that \( \frac{\partial q^X_i}{\partial q^Y_j} = \left( \frac{\partial q^Y_j}{\partial q^X_i} \right)^{-1} \) when estimating unobserved marginal costs or considering platform mergers.

With the implicit demand functions provided in Equation (6), demand with feedback loops is specified by:
\[ q^X_i = \tilde{q}^X_i (p^X_i, p^{X}_i; \tilde{q}^{X}_{-i}, \tilde{q}^{2}_{-i}, ..., \tilde{q}^{N}_{-i}) \]
(7)

Thus, demand depends directly on all platform prices for that product as well as the demand of other products with closed feedback loops. This demand structure allows for a solution to the platform’s problem that considers both the direct effect that prices have on product demand and the feedback effect that prices have on demand through other products.

\(^{13}\)The first-order conditions with respect to demand function \( q^X_i(\cdot) \) implies that \( \frac{\partial q^X_i}{\partial p^Y} < 0 \) and \( \frac{\partial q^X_i}{\partial p^X} \geq 0 \) for \( Y \neq X \). In addition, positive network externalities \( \left( \frac{\partial q^X_i}{\partial q^X_{-i}} > 0 \right) \) imply that \( \frac{\partial q^X_i}{\partial p^{X}_{-i}} < 0 \), while negative externalities \( \left( \frac{\partial q^X_i}{\partial q^{X}_{-i}} < 0 \right) \) imply that \( \frac{\partial q^X_i}{\partial p^{X}_{-i}} > 0 \).

\(^{14}\)Mathematically speaking, one could approach the platform maximization problem using Equation (6). While such an approach also solves the platform’s problem, using the closed loop demand function in Equation (7) provides results that are easier to compare with other results found in the multi-sided market literature and simplifies the notation later in applications.
3.2 Equilibrium

The equilibrium concept is the Nash equilibrium. That is, platform strategies are the collection of prices \((p^1, p^2, \ldots, p^N)\) which constitute an equilibrium if \(\Pi^X(p^X, p^{-X}) \geq \Pi^X(\hat{p}^X, p^{-X})\) for all \(\hat{p}^X\) and for all \(X\). The demand structure produces equilibrium pricing that is standard in multi-sided markets.

**Proposition 2** (Equilibrium Platform Prices). There exist unique prices \((p^1, p^2, \ldots, p^N)\) in equilibrium such that platform \(X\)’s price for product \(i\) is given by:

\[
p^X_i = \frac{c^X_i}{MC} - \frac{p^X_i}{\eta^X_i} \left[ \sum_{k \neq i}^K \frac{\partial q^X_i}{\partial q^X_k} \cdot (p^X_k - c^X_k) \right]
\]  

(8)

The elasticity term in the markup of Equation (8) is the price elasticity of demand for product \(i\) on platform \(X\): \(\eta^X_i = \frac{\partial q^X_i}{\partial p^X_i} \cdot \frac{p^X_i}{q^X_i}\).

Notice that the platform equilibrium pricing result provides a similar expression to the standard pricing result found in the literature. Namely, price equals marginal cost, plus a markup from platform specific market power, minus the marginal gain from the other sides of the market (or in this case, the marginal profits from the other products). This provides a tractable equilibrium pricing function that does not rely on specific knowledge of the consumer network externalities that are required for the standard formulas\(^{15}\).

To better illustrate the platform’s pricing strategy, consider a video game platform’s\(^{16}\)
pricing strategies. The console price is given by:

$$p_0^X = c_0 + \frac{-p_0^X}{\eta_0^X} - \sum_{j=1}^{K} \left[ \frac{\partial q_j^X}{\partial q_0^X} \cdot (p_j^X - c_j^X) \right].$$

Notice that positive network effects between consoles and games \(\frac{\partial q_j^X}{\partial q_0^X} > 0\) reduces the console price closer to its marginal cost. Also note that the interaction between console sales and game sales is often observed in the data. For example, the average game \(j\) purchases for a console owner closely approximates \(\frac{\partial q_j^X}{\partial q_0^X}\), since one less console sale likely implies a decrease in game purchases equal to the average number of game \(j\) sales per console\(^{17}\).

Now consider the platform pricing strategy for its genre 1 game:

$$p_1^X = c_1 + \frac{-p_1^X}{\eta_1^X} - \frac{\partial q_0^X}{\partial q_1^X} \cdot (p_0^X - c_0^X) - \sum_{j=2}^{K} \left[ \frac{\partial q_j^X}{\partial q_1^X} \cdot (p_j^X - c_j^X) \right].$$

For games, the marginal profit elsewhere term can be decomposed into two parts. The first term captures the positive network effects between the game and its console \(\frac{\partial q_j^X}{\partial q_0^X} > 0\), which generates a game price reduction. The second part captures the interactions between the game genres. If genres act as substitutes, then \(\frac{\partial q_j^X}{\partial q_1^X} < 0\), which generates a game price increase. Thus, if we assume that game genres are substitutes, then this pricing decomposition is consistent with the commonly discussed pricing outcome where consoles are priced near marginal costs while games maintain high markups.

The pricing result from Proposition 2 also shows how the markup relates to elasticities. It also allows for comparisons with the traditional formulation regarding product price, marginal cost, and demand elasticity given by Equation (1). In equilibrium, the following relationship holds:

**Corollary 1** (Equilibrium Elasticity). In multi-sided markets, the equilibrium demand elas-

\(^{17}\)Since each game has unit demand, \(\frac{\partial q_j^X}{\partial q_0^X} < 1\). However, for other examples like printers where consumers purchase multiple black ink cartridges over time, \(\frac{\partial q_j^X}{\partial q_0^X} > 1\).
ticity is given by:

$$
\eta^X_i = \frac{-p^X_i}{\sum_{k=1}^K \frac{\partial q^X_k}{\partial q^X_i} \cdot (p^X_k - c^X_k)}.
$$

(9)

In a multi-sided market, the elasticity of product $i$ for platform $X$ is the price divided by the sum of marginal profits across all sides of the market. Thus, if a practitioner observes equilibrium prices, marginal costs, and demand dependency derivatives, then the equilibrium demand elasticities can be determined\(^{18}\).

### 3.3 Platform Market Power

Given the multi-sided market equilibrium, consider platform market power. Market power is often defined as a seller’s ability to price above marginal cost. In addition, a sizable market share suggests that the seller may have a desirable product for consumers. Thus, both a platform’s price-cost markup and the distribution of sales by a platform seller are important for evaluating platform market power.

For example, suppose that there are two platforms that compete in a two-sided market where one side is subsidized by both platforms while the other side faces a positive markup. If sales by each platform are the same on each side but one platform charges higher prices on each side, then the platform with higher prices has greater market power. Alternatively, if prices are the same but one platform is the market share leader on the profitable side of the market while the other platform is the market share leader on the subsidized side, then the platform that dominates the profitable side clearly has greater market power. Thus, both the price-cost margins and the distribution of sales are important determinants of platform market power.

**Definition 1** (The Generalized Lerner Index). Let the market power of platform $X$ in a

\(^{18}\)I state this elasticity result as a corollary because it is used later in the gaming platform application.
multi-sided market be given by:

\[
\tilde{L}^X = \frac{\sum_i \{(p_i^X - c_i^X) \cdot q_i^X\}}{\sum_i \{p_i^X \cdot q_i^X\}}.
\]  

(10)

The generalized Lerner index weights each price-cost margin in the numerator and each price in the denominator by the corresponding amount of sales. The traditional Lerner index is a special case of the generalized index where \(\tilde{L}^X = \frac{(p^X - c^X) \cdot q^X}{p^X \cdot q^X} = L^X\).

One advantage of the generalized Lerner index is that it may be easily computable when administrative cost and revenue data are available. That is, the generalized Lerner index is the ratio of platform variable profits (\(\Pi^X + F^X\)) and variable revenues:

\[
\tilde{L}^X = \frac{\sum_i \{(p_i^X - c_i^X) \cdot q_i^X\}}{\sum_i \{p_i^X \cdot q_i^X\}} = \frac{\Pi^X + F^X}{TR^X + F^X},
\]

where \(TR^X = \sum_i \{p_i^X \cdot q_i^X\}\) is the total revenue generated by platform \(X\).

Determining variable profits and variable revenues for a platform that is owned by a large conglomerate might be difficult since overhead costs are generally shared across other products (e.g., the Google search platform under the Google Corporation, Bing or Xbox under the Microsoft Corporation, the PlayStation console under the Sony Corporation, etc.). This issue is common in the literature and is not specific to platforms. Nevertheless, the relationship between variable profit and variable revenues may prove valuable for smaller independent platforms like Snapchat, Pandora, Twitter, or Hulu where company data are largely associated with a single platform. When this is not the case, however, one can determine platform market power with marginal cost estimates.

### 3.4 Determining Unobserved Marginal Costs

To determine platform market power and to analyze platform mergers, knowledge of each marginal cost is necessary. However, some marginal costs may be difficult to observe. A standard result in industrial organization is that unobserved marginal costs can be estimated
given data on prices, market shares, and cross- and own-price elasticities. Fortunately, a similar result exists for multi-sided markets: given platform seller prices for each product, market shares, all demand dependency derivatives, and all elasticities, then the unobserved marginal costs are determinable.

First consider the determination of a single unobserved marginal cost using the equilibrium pricing result in Proposition 2. By rearranging terms in Equation (8), a single marginal cost is given by:

\[ c^X_i = \left(1 + \frac{1}{\eta^X_i}\right) \cdot p^X_i + \sum_{j \neq i}^K \frac{\partial q^X_j}{\partial q^X_i} \cdot (p^X_j - c^X_j). \]  

Comparing this result to the unobserved marginal cost in the traditional market, Equation (2), it becomes clear that a bias is generated if one uses the traditional approach. For the case of a two-sided market, the direction of bias depends on the sign of the demand dependency (i.e., the network externality). Positive network externalities in multi-sided markets imply that traditional methods generate marginal cost estimates that are biased downward. In addition, the magnitude of the bias depends on the magnitude of markups and network effects. In multi-sided markets with positive network externalities, the magnitude of the bias increases with higher markups or network effects. In Section 5, the bias with respect to game console marginal costs is shown to be considerable as games have high markups and consumers strongly desire video games.

Note that to determine a marginal cost using Equation (11), product demand elasticity, cross-product demand dependency derivatives with respect to \( i \), and all other marginal costs must be known. Thus, this method might not be applicable for when additional marginal costs are also unobserved. However, by using the set of platform first-order conditions, all unobserved marginal costs can be determined.

**Proposition 3 (Determining Unobserved Marginal Costs).** Every unobserved marginal cost for seller \( X \) is determinable if \( p^X, \eta^X_i, \) and \( \frac{\partial q^X}{\partial q_i} \) for all \( i \) are known. That is, \( c^X = \)

---

19This assumes that the platform seller charges positive markups.
Thus, to determine all \( K + 1 \) unobserved marginal costs for each seller, the entire collection of demand elasticities for the \( K + 1 \) products as well as the collection of all demand dependency derivatives in both directions, \( \frac{\partial q^X_i}{\partial q^X_j} \) and \( \frac{\partial q^X_j}{\partial q^X_i} \neq \left( \frac{\partial q^X_j}{\partial q^X_i} \right)^{-1} \), are required. In many applications this might be problematic. However, in some two-sided markets the inherent supply chain structure provides an estimate of one marginal cost. For example, a consumer search on Google or a personal profile on Facebook, Snapchat, or Instagram is nearly costless for the platform.

In the video game market context, the collection of revenue from different stages in the supply chain (retailers, distributors, platform royalties) for a game sale provides a proxy for game marginal costs for a video game platform.\(^{21}\) In these cases where one of the two marginal costs is given, the information that is required for Equation (11) is less restrictive than that of Proposition 3 since only one demand dependency derivative and elasticity are needed rather than two. Thus, exploiting these inherent marginal cost features is useful when data on elasticities or derivatives for one side of the market are lacking. I take advantage of supply chain information to estimate unobserved console marginal costs in the video game application.

4 Multi-Platform Sellers

To this point, the analysis has considered single platform sellers, but many multi-sided markets have multi-platform sellers. For example, Facebook Inc. uses its multiple social media platforms (Facebook, Whatsapp, and Instagram) to compete with other social media platforms like Snapchat, Twitter, and Google Plus. Similarly, in the market for video games,

\[c^X \left( \eta^X; p^X, \frac{\partial q^X_i}{\partial q^X_j}, \ldots, \frac{\partial q^X_K}{\partial q^X_j} \right)\]

\(^{20}\)In a similar manner to the implicit demand functions, the determination of all unobserved marginal costs for seller \( X \) requires that \( \frac{\partial q^X_i}{\partial q^X_j} \neq \left( \frac{\partial q^X_j}{\partial q^X_i} \right)^{-1} \). This requirement is less restrictive, and no additional assumptions are required.

\(^{21}\)For example, in “Anatomy of a $60 video game” by the Los Angeles Times on February 19, 2010, a breakdown of the marginal cost with respect to the supply chain payments from a video game is provided.
Nintendo and Sony each produce two types of gaming platforms (console and handheld) that compete with Microsoft’s console gaming platform and other handheld gaming platforms like smartphones and tablets. Considering a more general environment with multi-platform sellers lays the groundwork for an analysis of platform mergers.

4.1 Pricing by Multi-Platform Sellers

Suppose that there are now \( n \) platform sellers of the \( N \) available platforms, and assume that \( n < N \) so that there are some multi-platform sellers. Suppose that a platform seller sells platforms \( X \) and \( Y \). In this case, platform \( X \)'s prices differ from the single platform seller case provided in Proposition 2.

**Proposition 4** (Multi-Platform Seller Prices). If a platform seller sells platforms \( X \) and \( Y \), then the equilibrium price for product \( i \) on platform \( X \) is given by:

\[
p^X_i = c^X_i + \frac{-p_i^X}{\eta_i^X} - \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) - \frac{D^XY_i}{\partial q_i^X} \cdot (p_i^Y - c_i^Y) - \frac{\partial q_k^Y}{\partial q_i^Y} \cdot (p_k^Y - c_k^Y)
\]

(12)

where \( D^XY_i = \frac{\partial q_Y^i}{\partial p_X^i} / \frac{\partial q_X^i}{\partial p_X^i} < 0 \) is the diversion ratio from platform \( X \) to platform \( Y \) for product \( i \).

The implied post-merger pricing effects from this result are consistent with Filistrucchi et al. (2012), Affeldt et al. (2013), and Song (2015). With a multi-platform seller, the price of a platform’s product consists of the terms in Proposition 2 plus two additional terms. The first additional term is the direct diversion from product \( i \) on platform \( X \) to product \( i \) of the other platform owned by the seller (platform \( Y \)). This direct diversion exists in traditional product markets as well. That is, the platform’s direct diversion term coincides with the
traditional diversion markup term in Equation (3). The second additional term (diversion elsewhere) is the feedback diversion to the other platform for all other products \( k \neq i \). This network externality term is specific to multi-sided markets and does not exist in traditional ones.

Now consider the implications of Proposition 4 with respect to platform mergers. When two platforms merge, the additional diversion terms result in price markups when network externalities are positive \( \left( \frac{\partial q^X}{\partial q^X} > 0 \right) \) and when platforms do not subsidize their products \( (p^X_i > c^X_i) \). Thus, for many platform markets these added terms imply price increases post-merger. However, if platforms use a pricing strategy that mixes markups and subsidies, or if negative network externalities exist, then some platform prices may decrease post-merger. Such a possibility suggests that the post-merger outcome may be more socially efficient. Furthermore, the generalized Lerner index that I propose captures this potential efficiency for platform mergers when some prices increase while other prices decrease. If the collective price changes after the merger are such that the generalized Lerner index decreases for each platform, then the merger is likely welfare improving.

To illustrate the main pricing effects of platform mergers, hypothetical mergers in the sixth generation video game market are analyzed. However, before proceeding to the video game market application, there is an additional issue that is specific to mergers by platforms that must be considered when analyzing post-merger effects in multi-sided markets.

### 4.2 Network Integration by Multi-Platform Sellers

Unlike traditional multi-product sellers, a multi-platform seller can potentially affect the value of each of its platforms by integrating features across its multiple platforms. For example, Facebook Inc. has integrated user features across its social media platforms (e.g., users can set up their profiles so that an Instagram post automatically posts to their Facebook page). For social media advertisers, more extensive user information from integrated data results in a greater willingness to use either platform as an advertising outlet. As another
example, the Nintendo 64 Transfer Pak accessory allows Nintendo 64 console owners to purchase Gameboy games and play them on the Nintendo 64. This Transfer Pak is a partial network integration between Nintendo’s console and portable gaming platforms.

The examination of integrated platforms is especially important for analyzing platform mergers. Does network integration result in greater price markups or do platform markups fall with integration? The answer is not obvious as network integration results in a tradeoff. In the case of network integration between the Nintendo 64 console and the handheld Gameboy, integration increases the demand for Gameboy games due to new demand from console owners; however, console owners now have a reduced incentive to own a handheld Gameboy so the demand for Gameboys decreases. This tradeoff suggests that network integration will not necessarily increase post-merger prices.

To define network integration more formally we say that platform $Y$ is partially integrated with platform $X$ if the supply of the $f_Y \in \{0, 1, ..., K + 1\}$ products can be used on platform $X$:

$$q_i^X(p_i; \tilde{q}_0^X, ..., \tilde{q}_{f_Y - 1}^X, \tilde{q}_{f_Y}^X; q_1^Y, ..., q_{K+1}^Y).$$

If $f_Y = K + 1$, then platform $Y$ is fully integrated into platform $X$. Consider the simple case where platforms are partially integrated along the same products so that $f_Y = f_X = f$.

**Proposition 5** (Network Integration Prices). If a platform seller that sells platforms $X$ and $Y$ decides to $f$-partially integrate the $X$ and $Y$ platforms, then the equilibrium price for product $i$ on platform $X$ is given by:

$$p_i^X = c_i^X + \frac{-p_i^X}{\eta_i^X} - \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) - D_{XY} \cdot (p_i^Y - c_i^Y) - D_{XY} \cdot \sum_{k \neq i} \frac{\partial q_k^Y}{\partial q_i^X} \cdot (p_k^Y - c_k^Y)$$

$$+ \left[ - \sum_{0, k \neq i} f \frac{\partial q_k^Y}{\partial q_i^X} \cdot (p_k^Y - c_k^Y) - D_{XY} \cdot \sum_{0, k \neq i} f \frac{\partial q_k^X}{\partial q_i^Y} \cdot (p_k^X - c_k^X) \right]$$

(13) **Potential Cannibalization Effect**
By comparing network integration prices with their unintegrated counterparts, Equations (12) and (13), prices with network integration have the added term denoted as the potential cannibalization effect. It shows that prices might actually decrease when multi-platform sellers integrate their platforms.

The potential cannibalization effect can be decomposed into two terms. The first term, \( -\sum \frac{\partial q^X_i}{\partial q^Y_k} \cdot (p^Y_k - c^Y_k) \), is generally a price markdown term. This term implies that there is downward pressure on \( p^X_i \) with integration, because there are now additional product demands \((k \neq i)\) on the other platform \( Y \) that are realized from an additional product \( i \) sale on platform \( X \). This provides an incentive to decrease the price of product \( i \) on platform \( X \). In the video game market example, this markdown on the price of console \( X \) stems from the additional platform \( Y \) game sales that derive from owners of console \( X \).

The second term of the potential cannibalization effect, \( -D_i^{XY} \cdot \sum \frac{\partial q^X_k}{\partial q^Y_i} \cdot (p^X_k - c^X_k) \), is generally a markup term. This term implies that there is upward pressure on \( p^X_i \) with integration because demand falls to a lesser extent for products \( k \neq i \) on platform \( X \) from a lost product \( i \) sale. This provides an incentive to increase the price of product \( i \) on platform \( X \). In the video game market, this markup on the price of console \( X \) stems from the reduction in lost platform \( X \) game sales that result from an increase in the price of console \( X \). The reduction in lost game sales for platform \( X \) stems from the new demand from console \( Y \) owners who also purchase platform \( X \) games. The magnitudes of these effects determine whether or not network integration results in a cannibalization effect where the multi-platform seller’s prices decrease with network integration.

From a policy perspective, the welfare effects of network integration are not clear even if integration results in higher markups. Users benefit from integration: improved advertising matching benefits consumers and advertisers; gaming network integration benefits

\footnote{That is, \( -\sum \frac{\partial q^X_i}{\partial q^Y_k} \cdot (p^Y_k - c^Y_k) \) is a price markdown term when platforms charge markups and network externalities are positive.}

\footnote{That is, \( -D_i^{XY} \cdot \sum \frac{\partial q^X_k}{\partial q^Y_i} \cdot (p^X_k - c^X_k) \) is a price markup term when platforms charge markups and network externalities are positive.}
consumers; and social media network integration can save users time when posting on multiple media platforms. These benefits, along with the potential social cost of higher prices, must be considered when analyzing platform mergers and network integration policy.

5 The Sixth Generation Video Game Market

While the video game industry emerged in the 1980s, its growth has increased considerably in recent years as many first generation gamers have become adults with greater disposable income. In 2016, the video game market in the United States was valued at 17.68 billion dollars.\(^{26}\) To illustrate the main results in this paper, I consider the sixth generation video game market. From October 2000 through 2005, there were three platform sellers: Sony’s PlayStation 2 (PS2), Microsoft’s Xbox, and Nintendo’s GameCube (GC). Platform sellers generated revenues from console sales and game sales.\(^{27}\) Over this time period, each platform offered hundreds of different games to consumers. However, I only have information on the aggregated game side of the market for each platform. As a result, I consider the case where \(K = 1\) such that product 0 denotes the console product and product 1 denotes the collective game side. This simplification implies that this application only captures the importance of accounting for network effects in platform markets but does not capture the impact of the substitutability across games.

In terms of game prices and costs with respect to the video game platforms, games are often developed by third parties who sell their games directly to consumers. For this right, these developers pay a royalty to the platform for each game sold. This implies that the royalty can be interpreted as a platform’s price-cost markup on games \((p_1 - c_1)\) where \(p_1\) is the retail price a consumer pays for a game and \(c_1\) is the unobserved amount taken by (i) retailers like BestBuy and (ii) game developers like Ubisoft. Consequently, data on

\(^{26}\)See Statista “Video Game Industry” 2017.

\(^{27}\)While Microsoft’s Xbox Live online subscription program was released in November of 2002, which allows gamers to play collectively through the internet, data limitations prevent the incorporation of the Xbox Live subscription into this analysis.
video game prices and platform royalties identify platform marginal cost for an individual game.\footnote{An alternative approach is to let $p_1$ be the royalty and $c_1$ be the marginal cost of the platform to provide the game. This cost is likely zero for games developed by third party publishers. In this case, $\eta_1$ is measured as the demand elasticity for a video game with respect to the royalty. However, this alternative approach results in $\eta_1$ values (computed using Corollary 1 and the data provided in the next subsection) that are unreasonable for video games: -0.515 for PlayStation 2 games, -0.578 for Xbox games, and -0.676 for GameCube games. Using the royalty as a platform’s price-cost markup for a game sale provides reasonable values of $\eta_1$ (-2.253 for PlayStation 2 games, -2.154 for Xbox games, and -1.939 for GameCube games), which are consistent with the game elasticities determined by Nair (2007) and Lee (2013).} Given this game side assessment, along with the appropriate data and assumptions on demand dependency derivatives, the unobserved marginal costs of platform consoles can be estimated. Furthermore, the existing literature contains sufficient information to consider an analysis of hypothetical platform mergers, with and without network integration.

### 5.1 Data

The primary data source is \cite{Lee2013}. He presents information on console and game sales and prices for each of the three platforms from October 2000 through 2005. Lee also determines the own- and cross- price elasticities for consoles. Table 1 lists this information.\footnote{Lee (2013) uses monthly transaction data for the sixth generation video game market that is provided by the NPD Group, a leading consumer data provider. NPD Group video game data has also been used by Shankar and Bayus (2003), Clements and Ohashi (2005), Nair (2007), Binken and Streemersch (2009), Cortes and Lederman (2009), Dubé et al. (2010) and Landsman and Streemersch (2011).}

Platform royalty information is proprietary, but some information on platform royalties for this generation has been reported. \cite{Evansetal2006} find evidence that Microsoft averaged a $7 royalty for Xbox games and that Sony charged between $3 and $9 in royalties for PlayStation 2 games (\cite{Evansetal2006}, pages 131 and 135). I use the $6 midpoint to proxy the average as displayed in Table 1. Royalty information for Nintendo is more difficult to find. According to \cite{Coughlan2001}, Nintendo had relatively high royalties for its previous generation: $18 for Nintendo compared to Sony’s $9. However, royalties have been

\cite{Lee2013} uses monthly transaction data for the sixth generation video game market that is provided by the NPD Group, a leading consumer data provider. NPD Group video game data has also been used by Shankar and Bayus (2003), Clements and Ohashi (2005), Nair (2007), Binken and Streemersch (2009), Cortes and Lederman (2009), Dubé et al. (2010) and Landsman and Streemersch (2011).

\cite{Lee2013} uses monthly transaction data for the sixth generation video game market that is provided by the NPD Group, a leading consumer data provider. NPD Group video game data has also been used by Shankar and Bayus (2003), Clements and Ohashi (2005), Nair (2007), Binken and Streemersch (2009), Cortes and Lederman (2009), Dubé et al. (2010) and Landsman and Streemersch (2011).

Console sales and elasticities are provided in Tables 1 and 4 in \cite{Lee2013}. The average price is computed using Figures 1(a) and 1(c) where the original prices for the Xbox and PS2 were $300 upon release while the GameCube released at $200. Then in May 2002 the retail price for the Xbox and PS2 were each cut by $100 while the GameCube price was cut by $50. Total game sales are determined by using Table 2 and Lee’s statement that “The top 10 titles on the PS2, XB, and GC (listed in Table 2) accounted for 13%, 16%, and 20% of platform software sales,” and the average game sales price is computed using Figure 1(b).
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>PlayStation 2</th>
<th>Xbox</th>
<th>GameCube</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$ - Console Sales (mil.)</td>
<td>30.07</td>
<td>13.32</td>
<td>9.83</td>
</tr>
<tr>
<td>$p_0$ - Avg. Console Price ($)</td>
<td>226.60</td>
<td>214.26</td>
<td>159.16</td>
</tr>
<tr>
<td>$\eta_0$ - Console Elasticity</td>
<td>-1.973</td>
<td>-2.004</td>
<td>-1.432</td>
</tr>
<tr>
<td>$q_1$ - Total Game Sales (mil.)</td>
<td>296.15</td>
<td>117.5</td>
<td>82</td>
</tr>
<tr>
<td>$p_1$ - Avg. Game Sales Price ($)</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$(p_1 - c_1)$ - Avg. Game Royalty ($)</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

trending downward since that time period. In fact, Nintendo announced that it dropped its royalty rate during the GameCube era. Given the likely higher royalty for Nintendo, I assume that the average royalty for the GameCube is $9. Overall, the distribution of royalties is consistent with reports on the video game industry for this time period (e.g., Takahashi (2002)).

Unlike today, game prices varied considerably during this era. Price changes for consoles also occurred over the five-year period. Hence, average prices are used in the analysis. Little change in console hardware components along with decreasing hardware costs suggests that the marginal cost of consoles declined during this time. Thus, estimates of marginal costs for consoles, platform market power, and post-merger price predictions should be interpreted as averages over this time period.

31 Today, the standard royalty rate is 12% or $6-$8 according to “The Economics of Game Publishing,” (IGN 2006); “Anatomy of a $60 video game,” (Los Angeles Times 2010); and “Video Games and Consoles,” (Encyclopedia.com).


33 The results are largely consistent for other Nintendo royalty rates. A robustness check on platform royalties in the next subsection suggests that the royalties are reasonable.
5.2 Determining Console Marginal Costs

Console production cost information is proprietary. Thus, console marginal costs must be determined using Equation (11). For each platform seller:

\[ c^X_0 = \left( 1 + \frac{1}{\eta_0} \right) \cdot p^X_0 + \frac{\partial q^X_1}{\partial q^X_0} \left( p^X_1 - c^X_1 \right). \]  

(14)

This marginal cost formula shows that by using the traditional formula given by Equation (2), a bias is introduced that distorts console marginal cost estimates. Furthermore, notice how the game markup \((p_1 - c_1)\) affects the implied console marginal cost estimates in Equation (14). That is, an increase in the game markup by $1 results in an increase in the console marginal cost by \(\frac{\partial q^X_1}{\partial q^X_0}\). With no network externalities \(\left( \frac{\partial q^X_1}{\partial q^X_0} = 0 \right)\), the traditional method produces an unbiased estimate of marginal cost. However when present, greater network externalities amplify the bias from the traditional marginal cost measure.

Table 1 contains all of the relevant information except for the demand dependency derivative, \(\frac{\partial q^X_1}{\partial q^X_0}\). For simplicity, let the average number of games per console for each of the platforms serve as proxies for \(\frac{\partial q^X_1}{\partial q^X_0}\). Then, \(\frac{\partial q^{PS2}_1}{\partial q^{PS2}_0} = 9.849\), \(\frac{\partial q^{Xbox}_1}{\partial q^{Xbox}_0} = 8.821\), and \(\frac{\partial q^{GC}_1}{\partial q^{GC}_0} = 8.342\).

Given the information above, console marginal costs are estimated using traditional and multi-sided market formulas as shown in Table 2. These are used to generate traditional and corrected estimates of markups, which are also listed.

<table>
<thead>
<tr>
<th></th>
<th>PlayStation 2</th>
<th>Xbox</th>
<th>GameCube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional MC Estimates ($)</td>
<td>111.75</td>
<td>107.34</td>
<td>48.01</td>
</tr>
<tr>
<td>Corrected MC Estimates ($)</td>
<td>170.84</td>
<td>169.09</td>
<td>123.09</td>
</tr>
<tr>
<td>Bias in Traditional MC Est.</td>
<td>-59.09</td>
<td>-61.75</td>
<td>-75.08</td>
</tr>
<tr>
<td>Traditional Markups ($)</td>
<td>114.85</td>
<td>106.93</td>
<td>111.14</td>
</tr>
<tr>
<td>Corrected Markups ($)</td>
<td>55.76</td>
<td>45.18</td>
<td>36.07</td>
</tr>
</tbody>
</table>

\[^{34}\]The computation of marginal costs is provided in Appendix B.
Notice that the traditional marginal cost formula understates the marginal cost estimates of game consoles. The biased costs generate overstated console markups. In addition, notice that the biased markup estimates are similar (between $106 and $115), but the correct markups have a wider range (between $36 and $55). Thus, the biased marginal cost estimates both overstate console markups and distort the distribution of markups across platforms. Both of these issues might result in inaccurate policy recommendations.

Also notice that the correct price-cost markups for consoles are all positive. Literature and news media outlets have reported that console marginal costs are typically greater than console prices. According to Lee (2013), “most platforms subsidize hardware sales, selling consoles close to or below cost, while charging publishers and developers a royalty for every game sold.” Evans et al. (2006) suggest a cost for the PlayStation 2 of over $400 per unit while the cost of the Xbox was $360 at release. White and Weyl (2016) assume that console marginal costs are $400 in a calibration of their model of the sixth generation video game market. And, Clements and Ohashi (2005) suggest that the marginal production cost of a console may have been greater than or equal to the console price for previous generations as well.

In comparing the literature with the imputed marginal costs in Table 2, the biased estimates fall well short of reported prices compared to the corrected estimates that account for multi-sidedness. Still, the corrected marginal cost estimates remain lower than indicated in published reports. However, note that the marginal cost figures in Table 2 are averages over the five year period while the reported costs are generally quoted at the time of the product release date. Lee (2013) points out how decreasing costs would result in increasing markups over time: “Most platform providers initially sold hardware platforms close to or below cost, with margins increasing over time as production costs fell.” In addition, Sony

\[35\] Liu (2010) documents existing academic literature and news reports which suggest that initial production costs are at or near console prices across platforms and generations.

\[36\] For the more recent Xbox One and PlayStation 4 consoles, an IHS report concludes that the consoles were launched at prices just above costs (“Microsoft Xbox One Hardware Cost Comes in Below Retail Price, IHS Teardown Reveals,” IHS 2013).
earned profits from the sale of PlayStation 2 consoles later in the life cycle of that particular generation of gaming according to Evans et al. (2006). Thus, decreasing costs over time can reconcile differences between the reported costs at the time of release and the average console cost estimates.

5.3 Market Power by Gaming Platforms

Given that platforms can earn profits on both sides of the market, the generalized Lerner index that is defined in Equation (10) may yield a more accurate estimate of platform market power than the traditional Lerner index. Table 3 provides the generalized Lerner index of market power estimates that are derived using biased and unbiased marginal cost estimates.

Table 3: Gaming Platform Market Power

<table>
<thead>
<tr>
<th></th>
<th>Sony</th>
<th>Microsoft</th>
<th>Nintendo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Power using Biased Costs</td>
<td>0.368</td>
<td>0.388</td>
<td>0.506</td>
</tr>
<tr>
<td>Market Power using Unbiased Costs</td>
<td>0.243</td>
<td>0.246</td>
<td>0.302</td>
</tr>
</tbody>
</table>

The platform market power estimates suggest that all three platforms have pricing power. Not surprisingly, market power estimates are greater with biased console marginal cost measures on the order of 34% for Sony, 37% for Microsoft, and 40% for Nintendo. This demonstrates that correctly measuring unobserved marginal costs is important when assessing platform market power.

One important feature of these measures of market power is that they account for both a platform’s ability to charge large price-cost markups and the distribution of sales across consoles and games. From Table 3 we see that Sony has the least amount of market power.

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3. As a robustness check, the average game side markups required for console marginal costs to equal console retail launch price ($300 for the PS2 and Xbox and $200 for the GameCube) are $19.11 for PS2, $21.84 for Xbox, and $18.22 for GameCube, estimates that are all unreasonable given the reported game side royalties.
This is due to Sony’s inability to charge the markups that Microsoft and Nintendo are able to charge for games, which is the significantly more profitable side. In addition, greater market power for Nintendo relative to Microsoft and Sony suggests that Nintendo has successfully differentiated itself from Sony and Microsoft, enabling Nintendo to generate higher game markups. At the same time, Sony and Microsoft are more direct competitors which prevents either of them from obtaining Nintendo’s pricing power even though each commands a larger share of the market.

5.4 Gaming Platform Mergers

To investigate the effects of platform mergers, a calibrated platform merger analysis is conducted for the sixth generation video game market. Three hypothetical mergers are considered: Sony with Microsoft, Sony with Nintendo, and Microsoft with Nintendo. Comparing the predicted outcomes to the original market equilibria allows for a rich investigation of the pricing dilemma that platform sellers face when engaging in a merger. Also considered are the traditional merger predictions when platform network externalities are ignored. By comparing the results from the traditional approach with the results from the techniques developed in this paper, the importance of considering multi-sidedness in platform markets becomes apparent.

To analyze these merger effects, the cross-platform diversion ratios for consoles ($D_{0}^{XY}$ and $D_{0}^{YX}$) and games ($D_{1}^{XY}$ and $D_{1}^{YX}$) are required. A console diversion ratio ($D_{0}^{XY}$) is interpreted as the ratio of console sales that are diverted to platform $Y$ relative to the lost console sales of platform $X$ that result from a marginal increase in the price of platform $X$’s console. The diversion ratios for consoles are computed using the cross-elasticity estimates of Lee (2013) and are listed on Table 4.

The relatively low values of $D_{0}^{PS2/Xbox}$ and $D_{0}^{PS2/GC}$ compared to $D_{0}^{Xbox/PS2}$ and $D_{0}^{GC/PS2}$

39In the last decade, the trend of Nintendo distancing itself from Sony and Microsoft has become even more pronounced. This suggests that such a pricing power relationship continues to exist for Nintendo.

40The computation of the diversion ratios are provided in Appendix B.
Table 4: Console Diversion Ratios

<table>
<thead>
<tr>
<th>Diversion</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0^{PS2/Xbox}$</td>
<td>-0.00718</td>
</tr>
<tr>
<td>$D_0^{PS2/GC}$</td>
<td>-0.00828</td>
</tr>
<tr>
<td>$D_0^{Xbox/PS2}$</td>
<td>-0.16672</td>
</tr>
<tr>
<td>$D_0^{Xbox/GC}$</td>
<td>-0.02504</td>
</tr>
<tr>
<td>$D_0^{GC/PS2}$</td>
<td>-0.13031</td>
</tr>
<tr>
<td>$D_0^{GC/Xbox}$</td>
<td>-0.04542</td>
</tr>
</tbody>
</table>

imply that Microsoft and Nintendo would lose greater console sales to Sony from an increase in their own price than Sony would lose to its competitors from its own price increase. The relatively moderate and similar estimates of $D_0^{Xbox/GC}$ and $D_0^{GC/Xbox}$ suggest that a price increase by either Microsoft or Nintendo would have a similar console diversion to the other platform. Altogether, the estimates indicate that a merger between Sony and Microsoft (or Sony and Nintendo) would result in a price increase of the Xbox (or GameCube) but little or no price increase of the PlayStation 2. This is due to the fact that the majority of Xbox (GameCube) sales would stay within the merged entity.

Lee (2013) provides game price elasticities with respect to each platform’s “hit” game title. These elasticities are $\eta_1^{PS2} = -1.275$, $\eta_1^{Xbox} = -1.144$, and $\eta_1^{GC} = -0.958$. However, these elasticities might serve as poor proxies for the average elasticity of all games because hit game titles likely have relatively less elastic demands than a typical game. Fortunately, Corollary 1 provides a method for determining average game elasticities for each platform. Based on Equation (9) and data from Table 1, the estimated game elasticities are $\eta_1^{PS2} = -2.253$, $\eta_1^{Xbox} = -2.154$, and $\eta_1^{GC} = -1.939$. Not surprisingly, these estimates, which represent an average for all games of a platform, are more elastic than Lee’s estimates for hit games. The accuracy of these estimates is supported by the fact that they lie within the range of game elasticity estimates of Nair (2007), -3.28 to -0.27, for the fifth generation.

41The game titles are Grand Theft Auto III for Sony, Halo for Microsoft, and Super Smash Bros. for Nintendo.
Using the elasticity estimates calculated here, the game diversion ratios can be determined. The necessary derivatives to determine game diversion ratios are derived from
\[ \frac{dq_X^Y}{p_1} = \frac{dq_Y^X}{q_0} \cdot \frac{dq_Y^X}{p_0} \cdot \frac{dq_Y^X}{q_1} \cdot \frac{dq_Y^X}{q_1} \cdot \frac{dq_Y^X}{p_1}. \]
Regarding the demand dependency for consoles with respect to games, \( \frac{\partial q_X^0}{\partial q_X^1} \), suppose that the impact of games on console sales is less influential than the impact of consoles on game sales \( \left( \frac{\partial q_X^0}{\partial q_X^1} \right)^{-1} < \frac{\partial q_X^1}{\partial q_X^0} \). In particular, suppose that \( \left( \frac{\partial q_X^0}{\partial q_X^1} \right)^{-1} = 0.9 \cdot \frac{\partial q_X^1}{\partial q_X^0} \). That is, the demand dependencies are near inverses but games have a slightly reduced impact on consoles relative to a console’s impact on games.\( ^{42} \) This provides the necessary components to derive the game diversion ratios which are presented in Table 5.\( ^{43} \)

### Table 5: Game Diversion Ratios

<table>
<thead>
<tr>
<th>Diversion</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{1}^{PS2/Xbox} )</td>
<td>-0.00579</td>
</tr>
<tr>
<td>( D_{1}^{PS2/GC} )</td>
<td>-0.00632</td>
</tr>
<tr>
<td>( D_{1}^{Xbox/PS2} )</td>
<td>-0.16753</td>
</tr>
<tr>
<td>( D_{1}^{Xbox/GC} )</td>
<td>-0.02131</td>
</tr>
<tr>
<td>( D_{1}^{GC/PS2} )</td>
<td>-0.13846</td>
</tr>
<tr>
<td>( D_{1}^{GC/Xbox} )</td>
<td>-0.04323</td>
</tr>
</tbody>
</table>

The game diversion ratio of platforms \( X \) and \( Y \) corresponds to the amount of game sales from platform \( X \) that are diverted to platform \( Y \) from an increase in platform \( X \)’s game price. The distribution of the game diversion ratios coincides with the distribution of the console diversion ratios that are presented in Table 4. Similar to consoles, the game diversion ratios indicate that in any merger with Sony, the other platform (Microsoft or Nintendo) diverges from \( \frac{\partial q_X^0}{\partial q_X^1} \) in either direction, post-merger price changes become larger and estimates of marginal cost and post-merger predictions become more biased when traditional methods are used. To this point, the importance of the relationship between these demand dependencies should be clear. Differences are necessary to solve the platform’s problem, determine all unobserved platform marginal costs, and predict platform post-merger pricing. The study of the specification of this relationship is left for future research.

\( ^{42} \)As \( \left( \frac{\partial q_X^0}{\partial q_X^1} \right)^{-1} \) diverges from \( \frac{\partial q_X^1}{\partial q_X^0} \) in either direction, post-merger price changes become larger and estimates of marginal cost and post-merger predictions become more biased when traditional methods are used. To this point, the importance of the relationship between these demand dependencies should be clear. Differences are necessary to solve the platform’s problem, determine all unobserved platform marginal costs, and predict platform post-merger pricing. The study of the specification of this relationship is left for future research.

\( ^{43} \)The computations of the cross-derivatives and the diversion ratios are provided in Appendix B.
is more inclined to increase its game price since the majority of the lost game sales will convert to sales for the other member (Sony) of the merged entity. This result also relates to consumer homing decisions. Greater game diversion to Sony suggests that consumers who own more than a single console (i.e., they multi-home) do so with Sony’s PlayStation 2 as one of their consoles.\footnote{44}

Given estimates of diversion ratios on each side of the market, hypothetical platform mergers can now be considered. However, to determine the effects on platform pricing, some structure on product demands is required. Assuming that product demands exhibit constant elasticity of demand allows for the determination of post-merger console and game prices.\footnote{45} Thus, the following analysis assumes constant demand elasticities and focuses on the effect of hypothetical mergers on console and game prices in the video game market. The post-merger price measures are provided in Table\footnote{46} 6 where PS2/Xbox, for example, represents the hypothetical merger between Sony (PS2) and Microsoft (Xbox).\footnote{47} The left panel of the table applies to consoles and the right panel applies to games.

Table 6: Post-Merger Console and Game Prices

<table>
<thead>
<tr>
<th></th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS2/Xbox</td>
<td>$227.06</td>
<td>$224.08</td>
<td>$159.16</td>
<td>25.07</td>
<td>26.64</td>
<td>25.00</td>
</tr>
<tr>
<td></td>
<td>(+0.45)</td>
<td>(+9.81)</td>
<td>(0.00)</td>
<td>(+0.07)</td>
<td>(+1.64)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>PS2/GC</td>
<td>$227.15</td>
<td>$214.26</td>
<td>$166.63</td>
<td>25.08</td>
<td>25.00</td>
<td>26.55</td>
</tr>
<tr>
<td></td>
<td>(+0.55)</td>
<td>(0.00)</td>
<td>(+7.33)</td>
<td>(+0.08)</td>
<td>(0.00)</td>
<td>(+1.55)</td>
</tr>
<tr>
<td>Xbox/GC</td>
<td>$226.60</td>
<td>$215.83</td>
<td>$161.68</td>
<td>25.00</td>
<td>25.25</td>
<td>25.51</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(+1.57)</td>
<td>(+2.53)</td>
<td>(0.00)</td>
<td>(+0.25)</td>
<td>(+0.51)</td>
</tr>
</tbody>
</table>

Terms in parentheses show information on price changes relative to the pre-merger equilibrium prices.

For the platforms that merge\footnote{47} console and game prices increase. This is consistent with

\footnote{44} The consideration of multi-homing by consumers or game developers in this analysis is left for future research.

\footnote{45} Unfortunately, this assumption prevents the determination of the level of post-merger sales.

\footnote{46} The determination of post-merger prices is provided in Appendix B.

\footnote{47} Prices for the non-merging platform do not change post-merger due to the assumption of constant elasticities.
the results from Proposition 4 since gaming platforms charge markups for both products and exhibit positive network externalities. Notice that mergers by either Microsoft or Nintendo with Sony result in negligible price increases for Sony’s console and games. At the same time, the prices for the other platform increase by up to 6%. This is consistent with the previous analysis of the diversion ratios where both Microsoft and Nintendo have high diversion to Sony from price increases. Thus, when either platform merges with Sony, it has a greater incentive to increase prices as shown by the post-merger predictions.

While these price changes might appear minor, consider their effects on markups. For example, when Microsoft or Nintendo merge with Sony, the console markup for the GameCube increases by 20% (17% for games), while the markup on the Xbox increases by 18% (27% for games). In addition, each console owner purchases many games which magnifies the increase in profits from the increase in the game markups.

To assess the bias from ignoring multi-sidedness, the traditional formula is applied to the same platform mergers. Consider the approach of a naive practitioner who does not account for the effect that network externalities have on platform pricing decisions. In this case, the naive practitioner computes the biased console marginal costs provided in Table 2. In addition, the determination of the game elasticities does not account for the multi-sidedness of the platform market which also produces biased game elasticities and game diversion ratios. Finally, the platform merger analysis conducted by a naive practitioner also fails to account for the multi-sided features that affect the merger outcome. The results from this naive approach are provided in Table 7.

Comparing the merger results from the unbiased approach (Table 6) with those from the naive approach (Table 7) reveals that price increases are biased upward with the naive approach. Further, the naive approach generates greater console price variability across platforms. For example, when Microsoft and Nintendo merge, the unbiased console price increases by $1.57 for the Xbox and $2.53 for the GameCube; however, with the naive

48The determination of biased post-merger prices as well as the necessary biased components is provided in Appendix B.
results, the increase in the console price for the Xbox is $6.41 while the GameCube console price increases by $17.06. Thus, (i) the total increase in prices is greater with the naive approach (upward bias), and (ii) the distribution of the price increases across platforms is also distorted (distribution bias). The distribution bias is important as it means that the degree of bias in market power estimates is not consistent across platform sellers.

The reason for the relatively greater price increases from the naive approach is that it fails to account for the lost profit on the other side of the market. In particular, the naive approach supposes that the platform sets console prices without considering the lost game sales that result from each lost console sale. As a result, the console price is biased upward. The same argument applies to game pricing and the potential lost console sales due to high game prices. Thus, accounting for every product that generates profit for a platform is critical in predicting post-merger prices in multi-sided markets. In the case of video games, by ignoring the ties between console sales and game sales (and vice versa), a practitioner obtains post-merger prices that are biased both in magnitude and in distribution. Thus, using the naive approach might lead to an unnecessary injunction of a proposed platform merger by competition authorities.

<table>
<thead>
<tr>
<th></th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS2/Xbox</td>
<td>$228.72</td>
<td>$253.19</td>
<td>$159.16</td>
<td>25.06</td>
<td>26.41</td>
<td>25.00</td>
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<td>(0.00)</td>
<td>(+0.06)</td>
<td>(+1.41)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>PS2/GC</td>
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<td>$214.26</td>
<td>$209.94</td>
<td>25.09</td>
<td>25.00</td>
<td>26.32</td>
</tr>
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<td>(+0.09)</td>
<td>(0.00)</td>
<td>(+1.32)</td>
</tr>
<tr>
<td>Xbox/GC</td>
<td>$226.60</td>
<td>$220.67</td>
<td>$176.22</td>
<td>25.00</td>
<td>25.28</td>
<td>25.49</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(+6.41)</td>
<td>(+17.06)</td>
<td>(0.00)</td>
<td>(+0.28)</td>
<td>(+0.49)</td>
</tr>
</tbody>
</table>

Terms in parentheses provide relative changes with respect to the pre-merger equilibrium prices.
5.5 Network Integration in Gaming Platform Mergers

After a merger, platforms might have the option to integrate. In the market for video games, full network integration implies that a consumer is able to play all games from both platforms on either of the two consoles. Post-merger prices will depend on the presence and extent of integration. In fact, network integration between video game platforms might actually lower prices for the two merged platforms. For example, with integration between Microsoft and Nintendo, a consumer who is interested in both Microsoft’s popular Halo game and Nintendo’s popular Mario games will now only need to purchase one of the two consoles. Integration will result in reduced console sales. However, consumers who only intend to purchase the Xbox console but are marginally interested in a few Nintendo games will now purchase more games than would otherwise be the case. Thus, the effect of a merger with integration on platform prices is unclear, and profits for the two platforms might fall with network integration.

To analyze the tradeoff for platform mergers with and without network integration, the techniques developed in Proposition 5 are applied to mergers between the video game platform pairs. Recall that constant elasticities were assumed for the merger analysis. With the entire game side compressed into product 1, $f$ - partial integration corresponds to an $f \in (0, 1)$ so that $f = 1$ provides full integration where all games can be played across platforms, and $f = 0$ implies no network integration. For illustrative purposes, I consider partial network integration with $f = 0.1$. The results are presented in Table 8.

The results in Table 8 highlight the presence of the cannibalization effect in the video game market. Consider the merger between Sony and Microsoft. In this case, all four prices decrease when the platforms merge with integration relative to their unintegrated counterparts. In particular, the price decrease for the Microsoft Xbox occurs because the

\footnote{Such an assumption becomes unreasonable for extensive network integration, and the pricing results become volatile for large $f$.}

\footnote{The determination of the integrated post-merger prices as well as the necessary components are provided in Appendix B.}
increased demand for Sony games that generates an Xbox price markdown exceeds the reduced Microsoft game demand that generates a Xbox price markup. The greater markdown results in a lower post-merger Xbox price when the merger occurs with integration.

6 Conclusion

In markets with network externalities, platform sellers can obtain market dominance by tipping the market. Yet, many platforms battle for dominance with several competitors that offer similar products that differ in price and market share. To more accurately assess platform market power in this environment, I propose a market power measure that weights markups by sales across all sides of the market that are served by a platform.

To determine platform markups, unobserved marginal costs must be estimated. I extend traditional methods for determining unobserved marginal costs and propose new methods for investigating mergers with and without network integration in platform markets. I show that deriving unobserved marginal costs using the traditional approach results in estimates that are biased downward if a platform exhibits positive network externalities. Furthermore, by comparing results from the multi-sided approach with the traditional approach, it becomes clear that consideration of multi-sidedness is critical to accurately determining platform marginal costs, market power, and post-merger predictions both with and without network integration.

Table 8: Post-Merger Console and Game Prices with Partial Network Integration

<table>
<thead>
<tr>
<th></th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
<th>PS2</th>
<th>Xbox</th>
<th>GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS2/Xbox</td>
<td>$226.63</td>
<td>$216.79</td>
<td>$159.16</td>
<td>24.79</td>
<td>25.59</td>
<td>25.00</td>
</tr>
<tr>
<td></td>
<td>(-0.43)</td>
<td>(-7.47)</td>
<td>(0.00)</td>
<td>(-0.28)</td>
<td>(-1.05)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>PS2/GC</td>
<td>$227.65</td>
<td>$214.26</td>
<td>$156.92</td>
<td>24.81</td>
<td>25.00</td>
<td>24.80</td>
</tr>
<tr>
<td></td>
<td>(+0.50)</td>
<td>(0.00)</td>
<td>(-9.71)</td>
<td>(-0.27)</td>
<td>(0.00)</td>
<td>(-1.75)</td>
</tr>
<tr>
<td>Xbox/GC</td>
<td>$226.60</td>
<td>$216.49</td>
<td>$158.92</td>
<td>25.00</td>
<td>24.60</td>
<td>24.67</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(+0.66)</td>
<td>(-2.76)</td>
<td>(0.00)</td>
<td>(-0.65)</td>
<td>(-0.84)</td>
</tr>
</tbody>
</table>

Terms in parentheses provide relative changes with respect to the post-merger price predictions.
integration.

To illustrate the main findings, the methods derived in this paper are applied to the sixth generation video game market. I identify the extent to which the traditional approach results in distorted estimates of console marginal costs, platform market power, and post-merger outcomes for hypothetical mergers between gaming platforms. For example, using the traditional formula produces console marginal cost estimates that range from $48 to $112 for the three consoles. However, the multi-sided market formulation developed in this paper gives console marginal cost estimates that range from $123 to $170, indicating that the bias from using the traditional formula can be sizable. Traditional merger price predictions are also misleading. For example, the estimated console price increase ranges from $2 to $50 when using the traditional approach but only ranges from $0.50 to $9 with the multi-sided approach. Furthermore, I find that if merged gaming platforms integrate their networks, then a cannibalization effect may occur where prices generally decrease for the merged entity.

While the analysis of the video game market provides an interesting illustration, the methods developed here apply to many other markets. The most obvious are other platform markets: smartphones, media markets, social media platforms, etc. The model also generalizes to add-on products where there are pricing distortions due to other types of externalities. Examples include printers and ink cartridges, razors and blades, Blu-ray players and Blu-ray movies, camcorders and film, and coffee makers with pods.
Appendices

Appendix A: Appendix of Proofs

Proof of Proposition 1: For simplicity, let $K = 2^{[51]}$. In this case, $q_0(p_0, q_1) = q_0$ is the function of all product 0 demands, $q_0 : \mathbb{R}^{N+N} \to \mathbb{R}^N$. Similarly, $q_1(p_1, q_0) = q_1$ is the function of all product 1 demands, $q_1 : \mathbb{R}^{N+N} \to \mathbb{R}^N$. Let $f_0(p_0, q_0, q_1) = q_0 - q_0(p_0, q_1) = 0$ and let $f_1(p_1, q_0, q_1) = q_1 - q_1(p_1, q_0) = 0$. Finally, let $f(p_0, p_1, q_0, q_1) = 0$ be the collection of the $f_0(\cdot)$ and $f_1(\cdot)$ functions so that $f : \mathbb{R}^{N+N+N+N} \to \mathbb{R}^{N+N}$. The implicit function theorem states that there exists $\tilde{q}(p_0, p_1) = q_0 \times q_1$ if the $N + N \times N + N$ Jacobian provided below is invertible.

\[
\begin{bmatrix}
\frac{\partial q_0}{\partial q_0} & \frac{\partial q_0}{\partial q_1} & \cdots & \frac{\partial q_0}{\partial q_N} \\
\frac{\partial q_1}{\partial q_0} & \frac{\partial q_1}{\partial q_1} & \cdots & \frac{\partial q_1}{\partial q_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial q_N}{\partial q_0} & \frac{\partial q_N}{\partial q_1} & \cdots & \frac{\partial q_N}{\partial q_N}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & \cdots & 0 & \frac{\partial q_0}{\partial q_1} & \frac{\partial q_0}{\partial q_2} & \cdots & \frac{\partial q_0}{\partial q_N}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & \cdots & 1 & \frac{\partial q_1}{\partial q_1} & \frac{\partial q_1}{\partial q_2} & \cdots & \frac{\partial q_1}{\partial q_N}
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & \cdots & 1 & \frac{\partial q_1}{\partial q_0} & \frac{\partial q_1}{\partial q_2} & \cdots & \frac{\partial q_1}{\partial q_N}
\end{bmatrix}
\]

This is invertible only if $\frac{\partial q_i}{\partial q_j} \neq \frac{1}{\partial q_j} \frac{\partial q_i}{\partial q_i}$ for all $X, Y$ and $i \neq j$.

Proof of Proposition 2: Each platform $X$ for $X = 1, \ldots, N$ maximizes profits with respect to each available price $p_i^X$ for $i = 1, \ldots, K$. From Equations (7) and (5), platform $X$ has profits given by:

\[
\Pi^X = \sum_{i=1}^{K} \left\{ (p_i^X - c_i^X) \cdot q_i^X (p_i^X, p_i^X; q_i^X(p_1^X, \ldots p_N^X); \bar{q}_{-i}^X, \bar{q}_{-i}^X, \ldots, \bar{q}_{-i}^X) \right\} - F^X.
\]

51The generalization is straightforward but notationally cumbersome.
In maximizing profits, platform \( X \) takes the other platform prices \( (p^Y_k) \) and other platform supplies \( (\tilde{q}^Y_j) \) as given. The first-order conditions of profit maximization imply that for each \( i = 0, 1, \ldots, K \) and each \( X = 1, \ldots, N \) we have that:

\[
0 = q^X_i + (p^X_i - c^X_i) \cdot \frac{\partial q^X_i}{\partial p^X_i} + (p^X_i - c^X_i) \cdot \sum_{k \neq i} \left\{ \frac{\partial q^X_k}{\partial q^X_i} \cdot \frac{d\tilde{q}^X_k}{dp^X_i} \right\} + \sum_{k \neq i} \left\{ (p^X_k - c^X_k) \frac{\partial q^X_k}{\partial q^X_i} \cdot \frac{d\tilde{q}^X_k}{dp^X_i} \right\}.
\]

Note that since \( q^X_i \) is given by Equations (6) and (7), \( \tilde{q}^X_i (p^1, p^2, \ldots, p^N) = q^X_i (p^X_i, \tilde{q}^X_i, \tilde{q}^X_{i+1}, \ldots, \tilde{q}^X_N) \). Taking the derivative with respect to \( p^X_i \) on both sides gives \( \frac{d\tilde{q}^X_i}{dp^X_i} = \frac{\partial q^X_i}{\partial p^X_i} + \sum_{k \neq i} \left\{ \frac{\partial q^X_k}{\partial q^X_i} \cdot \frac{d\tilde{q}^X_k}{dp^X_i} \right\} \). Substituting into the first-order condition yields:

\[
0 = q^X_i + (p^X_i - c^X_i) \cdot \frac{d\tilde{q}^X_i}{dp^X_i} + \sum_{k \neq i} \left\{ (p^X_k - c^X_k) \frac{\partial q^X_k}{\partial q^X_i} \cdot \frac{d\tilde{q}^X_k}{dp^X_i} \right\}.
\]

Lastly, noting that \( \frac{\partial q^X_i}{\partial q^X_i} \) is the change in demand for product \( k \) due to a change in supply of product \( i \), we then have that \( \frac{\partial q^X_i}{\partial q^X_i} = \frac{\partial q^X_i}{\partial q^X_i} \). Solving for \( p^X_i \):

\[
p^X_i = c^X_i + -\frac{q^X_i}{\eta^X_i} - \sum_{k \neq i} \left\{ \frac{\partial q^X_k}{\partial q^X_i} \cdot (p^X_k - c^X_k) \right\}.
\]

Note that the elasticity of demand for product \( i \) on platform \( X \) is given by \( \eta^X_i = \frac{d\tilde{q}^X_i}{dp^X_i} \). This implies that the first-order condition becomes:

\[
p^X_i = c^X_i + \frac{-p^X_i}{\eta^X_i} - \sum_{k \neq i} \left\{ \frac{\partial q^X_k}{\partial q^X_i} \cdot (p^X_k - c^X_k) \right\},
\]

as desired.

To solve for the explicit prices and to show uniqueness, note that by rearranging the above equation we have:

\[
\left( 1 + \frac{1}{\eta^X_i} \right) \cdot p^X_i = c^X_i - \sum_{k \neq i} \left\{ \frac{\partial q^X_k}{\partial q^X_i} \cdot (p^X_k - c^X_k) \right\},
\]

37
or:

\[
(1 + \frac{1}{\eta_i^X}) \cdot p_i^X + \sum_{k \neq i} \frac{\partial q^X_k}{\partial q^X_i} \cdot p_k^X = c_i^X + \sum_{k \neq i} \frac{\partial q^X_k}{\partial q^X_i} \cdot c_k^X.
\]

The collection of the above rearranged $K$ first-order conditions for each platform can be written in vector form:

\[
\begin{bmatrix}
(1 + \frac{1}{\eta_0^X}) & \frac{\partial q^X_0}{\partial q^X_0} & \frac{\partial q^X_0}{\partial q^X_0} & \cdots & \frac{\partial q^X_0}{\partial q^X_K} \\
\frac{\partial q^X_1}{\partial q^X_1} & (1 + \frac{1}{\eta_1^X}) & \frac{\partial q^X_1}{\partial q^X_1} & \cdots & \frac{\partial q^X_1}{\partial q^X_K} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\frac{\partial q^X_K}{\partial q^X_K} & \frac{\partial q^X_K}{\partial q^X_K} & \frac{\partial q^X_K}{\partial q^X_K} & \cdots & (1 + \frac{1}{\eta_K^X})
\end{bmatrix}
\begin{bmatrix}
p_0^X \\
p_1^X \\
p_K^X
\end{bmatrix}
= \begin{bmatrix}
1 & \frac{\partial q^X_0}{\partial q^X_0} & \frac{\partial q^X_0}{\partial q^X_0} & \cdots & \frac{\partial q^X_0}{\partial q^X_K} \\
\frac{\partial q^X_1}{\partial q^X_1} & 1 & \frac{\partial q^X_1}{\partial q^X_1} & \cdots & \frac{\partial q^X_1}{\partial q^X_K} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\frac{\partial q^X_K}{\partial q^X_K} & \frac{\partial q^X_K}{\partial q^X_K} & \frac{\partial q^X_K}{\partial q^X_K} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
c_0^X \\
c_1^X \\
c_K^X
\end{bmatrix}.
\]

The above equation reduces to $A^X X p^X = B^X c^X$. Solving for prices, $p^X$ gives the unique equilibrium prices: $p^X = A^{-1} B^X c^X$. Each platform $X$ takes the prices of all other platforms as given. Thus, equilibrium prices are $p_i^X \left( \eta_i^X, c^X, \frac{\partial q^X_k}{\partial q^X_i} \right)$.

The derivative assumptions, $\frac{\partial q^X_0}{\partial p_i^X} < 0$ and $\frac{\partial q^X_Y}{\partial p_i^X} \geq 0$ for $Y \neq X$, along with the second derivative assumptions, $\frac{\partial^2 q^X_i}{\partial p_i^X} \leq 0$ and $\frac{\partial (q^X)^2}{\partial q^X_i} < 0$, imply that the second-order conditions hold (the second derivative Hessian matrix is negative definite). This implies that the equilibrium prices result in a local maximum. The assumptions that $\frac{\partial q^X_k}{\partial p_i^X} \to 0$ and that $\frac{\partial q^X_k}{\partial q^X_i} \to 0$ when $\frac{\partial q^X_k}{\partial q^X_i} > 0$ imply that the local maximum is indeed a global max so that the platform equilibrium is unique.

**Proof of Corollary**

Rearranging terms in Equation (8) implies:

\[
p_i^X - c_i^X + \sum_{k \neq i} \left( \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) \right) = -\frac{p_i^X}{\eta_i^X}.\]

Given that $\frac{\partial q_k^X}{\partial q_i^X} = 1$ we have that:

\[
\sum_k \left( \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) \right) = -\frac{p_i^X}{\eta_i^X}.
\]
By dividing by $-p_i^X$ and inverting, we have the result:

$$\eta_i^X = \frac{-p_i^X}{\sum_{k=1}^{K} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X)}.$$ 

\[\square\]

**Proof of Proposition 3**: Rearranging terms in the equilibrium price equation in Proposition 2, Equation (8), implies that for all $i$ we have:

$$\left(1 + \frac{1}{\eta_i^X}\right) \cdot p_i^X + \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot p_k^X = c_i^X + \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot c_k^X,$$

where $\eta_i^X$ is the demand elasticity for product $i$ on platform $X$. This implies that there are $K + 1$ equations and $K + 1$ unknown marginal costs ($c_i^X$), when prices, elasticities, and demand dependency derivatives are known. These $K + 1$ equations can be written in matrix form:

$$
\begin{bmatrix}
\left(1 + \frac{1}{\eta_0^X}\right) & \frac{\partial q_1^X}{\partial q_0^X} & \frac{\partial q_K^X}{\partial q_0^X} & \cdots & \frac{\partial q_K^X}{\partial q_{K-1}^X} \\
\frac{\partial q_0^X}{\partial q_i^X} & 1 & \frac{\partial q_1^X}{\partial q_i^X} & \cdots & \frac{\partial q_K^X}{\partial q_i^X} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial q_0^X}{\partial q_K} & \frac{\partial q_1^X}{\partial q_K} & \frac{\partial q_K^X}{\partial q_K} & \cdots & \left(1 + \frac{1}{\eta_K^X}\right)
\end{bmatrix}
\begin{bmatrix}
p_0^X \\
p_1^X \\
\vdots \\
p_K^X
\end{bmatrix}
= 
\begin{bmatrix}
1 & \frac{\partial q_1^X}{\partial q_0^X} & \frac{\partial q_K^X}{\partial q_0^X} & \cdots & \frac{\partial q_K^X}{\partial q_{K-1}^X} \\
\frac{\partial q_0^X}{\partial q_1^X} & 1 & \frac{\partial q_1^X}{\partial q_1^X} & \cdots & \frac{\partial q_K^X}{\partial q_1^X} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial q_0^X}{\partial q_K} & \frac{\partial q_1^X}{\partial q_K} & \frac{\partial q_K^X}{\partial q_K} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
c_0^X \\
c_1^X \\
\vdots \\
c_K^X
\end{bmatrix}
$$

The above equation reduces to $A^X p^X = B^X c^X$ and solving for costs, $c^X$, requires the inverse matrix of $B^X$. The assumptions made using the implicit function theorem imply that $\frac{\partial q_i^X}{\partial q_i^X} \neq \left(\frac{\partial q_i^X}{\partial q_i^X}\right)^{-1}$ which implies that $B^X$ is invertible. Solving for $c^X$ implies that $c^X = c^X (\eta^X, p^X, \frac{\partial q_0^X}{\partial q_0^X}, \ldots, \frac{\partial q_K^X}{\partial q_K^X})$.

\[\square\]

**Proof of Proposition 4**: Each platform seller maximizes their individual profits with respect to platform prices for the platforms that they own, taking the other platforms’ prices and product supplies as given. This results in $K$ first-order conditions for each of the $N$ platforms. For the platform seller that sells platforms $X$ and $Y$, the first-order condition
with respect to price $p_i^X$ is:

$$0 = (p_i^X - c_i^X) \cdot \frac{dq_i^X}{dp_i^X} + q_i^X + \sum_{k \neq i} \left\{ \frac{\partial q_k^X}{\partial q_i^X} \cdot \frac{dq_i^X}{dp_i^X} \cdot (p_k^X - c_k^X) \right\}$$

$$+ (p_i^Y - c_i^Y) \cdot \frac{dq_i^Y}{dp_i^X} + \sum_{k \neq i} \left\{ \frac{\partial q_k^Y}{\partial q_i^X} \cdot \frac{dq_i^Y}{dp_i^X} \cdot (p_k^Y - c_k^Y) \right\}.$$

Rearranging:

$$p_i^X = c_i^X + \frac{q_i^X}{\eta_i^X} - \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) - D_i^{XY} \cdot (p_i^Y - c_i^Y) - D_i^{XY} \cdot \sum_{k \neq i} \frac{\partial q_k^Y}{\partial q_i^X} \cdot (p_k^Y - c_k^Y),$$

where $D_i^{XY} \equiv \frac{\partial q_i^Y/\partial q_i^X}{\partial q_i^X/\partial q_i^X} < 0$ is the diversion ratio between platforms $X$ and $Y$ of product $i$.

Consider the first-order conditions for platform $X$ in matrix form:

$$\begin{bmatrix}
(1 + \frac{1}{\eta_0}) & \frac{\partial q_0^X}{\partial q_0^X} & \ldots & \frac{\partial q_S^X}{\partial q_0^X} \\
\frac{\partial q_0^X}{\partial q_0^X} & (1 + \frac{1}{\eta_1}) & \ldots & \frac{\partial q_S^X}{\partial q_1^X} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial q_0^X}{\partial q_S^X} & \frac{\partial q_{S-1}^X}{\partial q_S^X} & \ldots & (1 + \frac{1}{\eta_S})
\end{bmatrix}
\begin{bmatrix}
p_0^X \\
p_1^X \\
p_S^X
\end{bmatrix}
= 
\begin{bmatrix}
D_0^{XY} & D_0^{XY} \frac{\partial q_0^Y}{\partial q_0^X} & \ldots & D_0^{XY} \frac{\partial q_S^Y}{\partial q_0^X} \\
D_1^{XY} \frac{\partial q_0^Y}{\partial q_1^X} & D_1^{XY} & \ldots & D_1^{XY} \frac{\partial q_S^Y}{\partial q_1^X} \\
\vdots & \vdots & \ddots & \vdots \\
D_S^{XY} \frac{\partial q_0^Y}{\partial q_S^X} & D_S^{XY} \frac{\partial q_{S-1}^Y}{\partial q_S^X} & \ldots & D_S^{XY}
\end{bmatrix}
\begin{bmatrix}
p_0^Y \\
p_1^Y \\
p_S^Y
\end{bmatrix}.$$
Proof of Proposition 5: Each platform seller maximizes their individual profit with respect to platform prices for the platforms that they own, taking the other platforms’ prices and product supplies as given. This results in $K$ first-order conditions for each of the $N$ platforms. For the platform seller that sells platforms $X$ and $Y$, the first-order condition with respect to price $p_i^X$ is:

$$0 = (p_i^X - c_i^X) \cdot \frac{d\hat{q}_i^X}{dp_i^X} + q_i^X + \sum_{k \neq i} \left\{ \frac{\partial q_k^X}{\partial q_i^X} \cdot \frac{d\hat{q}_i^X}{dp_i^X} \cdot (p_k^X - c_k^X) \right\} + \sum_{k \neq i} \left\{ f \cdot \frac{\partial q_k^X}{\partial q_i^X} \cdot \frac{d\hat{q}_i^X}{dp_i^X} \cdot (p_k^X - c_k^X) \right\}$$

$$+ (p_i^Y - c_i^Y) \cdot \frac{d\hat{q}_i^Y}{dp_i^X} + \sum_{k \neq i} \left\{ \frac{\partial q_k^Y}{\partial q_i^Y} \cdot \frac{d\hat{q}_i^Y}{dp_i^X} \cdot (p_k^Y - c_k^Y) \right\} + \sum_{k \neq i} \left\{ f \cdot \frac{\partial q_k^Y}{\partial q_i^X} \cdot \frac{d\hat{q}_i^X}{dp_i^X} \cdot (p_k^Y - c_k^Y) \right\}.$$

Therefore:

$$p_i^X = c_i^X + \frac{-p_i^Y}{\eta_i^X} - \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) - D_i^{XY} \cdot (p_i^Y - c_i^Y) - D_i^{XY} \cdot \sum_{k \neq i} \frac{\partial q_k^Y}{\partial q_i^X} \cdot (p_k^Y - c_k^Y)$$

$$+ f \cdot \left[ -\sum_{k \neq i} \frac{\partial q_k^Y}{\partial q_i^X} \cdot (p_k^Y - c_k^Y) - D_i^{XY} \cdot \sum_{k \neq i} \frac{\partial q_k^X}{\partial q_i^X} \cdot (p_k^X - c_k^X) \right].$$

□
Appendix B: Application to the Video Game Market

This appendix provides the setup and computations for the application of the theoretical results to the video game market that is presented in Section 5. In addition, the R computations, code, and output that generate the main results can be found in the Online Appendix.

To start, first consider the determination of the biased and unbiased marginal cost measures. The biased marginal cost measures are given by:

\[
\begin{bmatrix}
  c^X_0 \\
  c^Y_0 \\
  c^Z_0
\end{bmatrix}
= \begin{bmatrix}
  \left(1 + \frac{1}{\eta_{X0}}\right) \cdot p^X_0 \\
  \left(1 + \frac{1}{\eta_{Y0}}\right) \cdot p^Y_0 \\
  \left(1 + \frac{1}{\eta_{Z0}}\right) \cdot p^Z_0
\end{bmatrix}
= \begin{bmatrix}
  111.75 \\
  107.34 \\
  48.01
\end{bmatrix}
\]

where \(X\) denotes PS2, \(Y\) denotes Xbox, and \(Z\) denotes GC. Similarly, the unbiased marginal cost measures are given by:

\[
\begin{bmatrix}
  c^X_0 \\
  c^Y_0 \\
  c^Z_0
\end{bmatrix}
= \begin{bmatrix}
  \left(1 + \frac{1}{\eta_{X0}}\right) \cdot p^X_0 \\
  \left(1 + \frac{1}{\eta_{Y0}}\right) \cdot p^Y_0 \\
  \left(1 + \frac{1}{\eta_{Z0}}\right) \cdot p^Z_0
\end{bmatrix}
+ \begin{bmatrix}
  p^X_1 - c^X_1 \\
  p^Y_1 - c^Y_1 \\
  p^Z_1 - c^Z_1
\end{bmatrix}
\otimes
\begin{bmatrix}
  \frac{\partial q^X_1}{\partial q^X_0} \\
  \frac{\partial q^Y_1}{\partial q^Y_0} \\
  \frac{\partial q^Z_1}{\partial q^Z_0}
\end{bmatrix}
= \begin{bmatrix}
  170.84 \\
  169.09 \\
  123.09
\end{bmatrix}
\]

These are the results found in Table 2. Given the biased and unbiased marginal costs, some simple algebra provides the results found in Table 3.

To determine console diversion ratios, the full console elasticity matrix from Lee (2013) is required. This is reproduced below:

\[
\begin{bmatrix}
  \eta_{00}^{XX} & \eta_{00}^{XY} & \eta_{00}^{XZ} \\
  \eta_{00}^{YX} & \eta_{00}^{YY} & \eta_{00}^{YZ} \\
  \eta_{00}^{ZX} & \eta_{00}^{ZY} & \eta_{00}^{ZZ}
\end{bmatrix}
= \begin{bmatrix}
  -1.973 & 0.032 & 0.050 \\
  0.148 & -2.004 & 0.068 \\
  0.061 & 0.048 & -1.432
\end{bmatrix}
\]

where \(\eta_{00}^{XY} = \frac{\partial q^Y_1}{\partial q^X_0} \cdot \frac{p^X_0}{q^X_0}\). These elasticities, along with the given prices and consoles sales for each of the three platforms allows for the computation of the matrix of console side
Given the console side derivatives, the console side diversion ratios are:

\[
\begin{bmatrix}
D_{0X}^X & D_{0Y}^X & D_{0Z}^X \\
D_{0X}^Y & D_{0Y}^Y & D_{0Z}^Y \\
D_{0X}^Z & D_{0Y}^Z & D_{0Z}^Z \\
\end{bmatrix} = 
\begin{bmatrix}
1 & -0.00718 & -0.00828 \\
-0.16672 & 1 & -0.02504 \\
-0.13031 & -0.04542 & 1 \\
\end{bmatrix}
\]

This yields the results found in Table 4.

Next, consider the game side of the market. Note that the game side own-price elasticities for each platform are given by Equation (9), which implies that \( \eta_1^X = \frac{-p_1^X}{(p_1^X - c_1^X)(p_0^X - c_0^X)/\eta_1^X} \).

Furthermore, note that the game side derivatives can be found by using \( \frac{d q_1^X}{d q_0} = \frac{d q_1^X}{d p_1} \cdot \frac{d p_1}{d q_0} \), where the \( \frac{d q_1^X}{d p_1} \) and \( \frac{d q_1^X}{d q_1} \) are obtained from Table 1, the \( \frac{d q_0^X}{d p_0} \) and \( \frac{d q_0^X}{d q_0} \) are obtained from the computations in Table 4 above, and the \( \frac{d q_1^X}{d p_1} \) is the own-price elasticity derived from Equation (9). Using these results, the game side derivatives, elasticities, and diversion ratios are given by:

\[
\begin{bmatrix}
\frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} \\
\frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} \\
\frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} & \frac{d q_1^X}{d p_1} \\
\end{bmatrix} = 
\begin{bmatrix}
-26.691 & 1.6957 & 0.88072 \\
0.15458 & -10.122 & 0.27496 \\
0.16856 & 0.21573 & -6.3608 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\eta_1^{XX} & \eta_1^{XY} & \eta_1^{XZ} \\
\eta_1^{YY} & \eta_1^{YZ} & \eta_1^{ZZ} \\
\eta_1^{ZX} & \eta_1^{ZY} & \eta_1^{ZZ} \\
\end{bmatrix} = 
\begin{bmatrix}
-2.253 & 0.033 & 0.051 \\
0.143 & -2.154 & 0.066 \\
0.074 & 0.059 & -1.939 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_1^{XX} & D_1^{XY} & D_1^{XZ} \\
D_1^{YY} & D_1^{YZ} & D_1^{ZZ} \\
D_1^{ZX} & D_1^{ZY} & D_1^{ZZ} \\
\end{bmatrix} = 
\begin{bmatrix}
1 & -0.00579 & -0.00632 \\
-0.16753 & 1 & -0.02131 \\
-0.13846 & -0.04323 & 1 \\
\end{bmatrix}
\]
This provides the results found in Table 5.

Note that for the naive practitioner case, the game side own elasticities that are provided by Equation (9) now become $\eta X_1 = -\frac{p X_1}{p X_1 - c_1}$. This generates biased naive game side derivatives, elasticities, and diversion ratios. For ease of exposition they are not provided in this draft but are available upon request.

Finally, consider the hypothetical mergers between platforms. To consider the pricing strategies and platform first-order conditions in the more general multi-platform seller case, the $3 \times 3$ ownership matrix $O$ is used. Let $D_0$ denote the $3 \times 3$ console side diversion matrix and let $D_1$ denote the $3 \times 3$ game side diversion matrix which are provided above. In addition, the following two matrices are used to construct the general collection of first-order conditions:

$$d_{ij} = \begin{bmatrix} \frac{dq^X_i}{dq^X_j} & \frac{dq^Y_i}{dq^Y_j} & \frac{dq^Z_i}{dq^Z_j} \\ \frac{dq^X_i}{dq^X_j} & \frac{dq^Y_i}{dq^Y_j} & \frac{dq^Z_i}{dq^Z_j} \\ \frac{dq^X_i}{dq^X_j} & \frac{dq^Y_i}{dq^Y_j} & \frac{dq^Z_i}{dq^Z_j} \end{bmatrix} \quad E_k = \begin{bmatrix} \frac{1}{\eta_k} & 0 & 0 \\ 0 & \frac{1}{\eta_k} & 0 \\ 0 & 0 & \frac{1}{\eta_k} \end{bmatrix}$$

Using these matrices, the first-order conditions under general ownership are given by:

$$\begin{bmatrix} [E_0 + D_0] \odot O & [d_{21} \odot D_0] \odot O \\ [d_{12} \odot D_1] \odot O & [E_1 + D_1] \odot O \end{bmatrix} \begin{bmatrix} p_0^X \\ p_0^Y \\ p_0^Z \\ p_1^X \\ p_1^Y \\ p_1^Z \end{bmatrix} = \begin{bmatrix} [D_0] \odot O & [d_{21} \odot D_0] \odot O \\ [d_{12} \odot D_1] \odot O & [D_1] \odot O \end{bmatrix} \begin{bmatrix} c_0^X \\ c_0^Y \\ c_0^Z \\ c_1^X \\ c_1^Y \\ c_1^Z \end{bmatrix}$$

Using the inputs determined in this appendix, the only unknown variables are the six prices. Thus, constant demand elasticities allow for the determination of post-merger prices for all types of platform mergers. By solving the system of equations for the different ownership matrices given by the three types of mergers, the post-merger pricing results found in Table 6 are calculated. To determine the biased merger prices that a naive practitioner
obtains, note that the first-order conditions differ as well as many of the inputs. The biased merger results found in Table 7 are derived from:

\[
\begin{bmatrix}
[E_b^0 + D_b^0] \circ O & \bar{\mathbf{0}} \circ O
\end{bmatrix}
\begin{bmatrix}
E_b^1 + D_b^1 \circ O
\end{bmatrix}
\begin{bmatrix}
p_0^X \\
p_0^Y \\
p_0^Z \\
p_1^X \\
p_1^Y \\
p_1^Z
\end{bmatrix}
= \begin{bmatrix}
[D_b^0] \circ O & \bar{\mathbf{0}} \circ O
\end{bmatrix}
\begin{bmatrix}
d_b^1
\end{bmatrix}
\]

where \(\bar{\mathbf{0}}\) denotes the \(3 \times 3\) matrix with each input as a zero and the superscript \(b\) denotes that the matrix or vector has inputs that are biased and differ from the unbiased measures.

To conduct the merger analysis for when platforms merge with integration, note that Proposition 5 implies that the cross platform network integration derivatives are required. In this case, we have that the cross platform derivatives \(\frac{\partial q_X^1}{\partial q_Y^0}\) are now the ratio of total game sales across platforms \(X\) and \(Y\) and the console sales of the single platform \(Y\). Using inputs from Table 1, we have \(\frac{\partial q_X^{PS}_1}{\partial q_Y^{0}} = 22.233772\), \(\frac{\partial q_Y^{PS}_2}{\partial q_Y^{0}} = 30.127553\), \(\frac{\partial q_X^{box}_1}{\partial q_Y^{0}} = 3.907549\), \(\frac{\partial q_Y^{box}_1}{\partial q_Y^{0}} = 11.953204\), \(\frac{\partial q_Y^{GC}_1}{\partial q_Y^{0}} = 2.726970\), and \(\frac{\partial q_Y^{GC}_1}{\partial q_Y^{0}} = 6.156156\).

Unfortunately, the platform mergers with integration have first-order conditions that do not easily allow for the matrix notation setup with the platform ownership matrix. Thus, the first-order conditions provided by Proposition 5 are used to specify the system of equations required to determine post-merger prices with integration. Suppose that platforms \(X\) and \(Y\) merge with integration while platform \(Z\) remains a single platform seller. For platform
$X$, these conditions are:

$$0 = \left( p_0^{X} - c_0^{X} \right) + \frac{p_0^{X}}{\eta_0^{X}} + \frac{\partial q_1^{X}}{\partial q_0^{X}} \cdot (p_1^{X} - c_1^{X}) + f \cdot \frac{\partial q_1^{Y}}{\partial q_0^{X}} \cdot (p_1^{Y} - c_1^{Y}) + D_0^{XY} \cdot (p_0^Y - c_0^Y)$$

$$+ D_0^{XY} \cdot \frac{\partial q_1^{Y}}{\partial q_0^Y} \cdot (p_1^{Y} - c_1^{Y}) + f \cdot \frac{\partial q_1^{X}}{\partial q_0^Y} \cdot (p_1^{X} - c_1^{X}),$$

$$0 = \left( p_0^{X} - c_0^{X} \right) + \frac{p_0^{X}}{\eta_0^{X}} + \frac{\partial q_0^{X}}{\partial q_1^{X}} \cdot (p_0^{X} - c_0^{X}) + f \cdot \frac{\partial q_0^{Y}}{\partial q_1^{X}} \cdot (p_0^{Y} - c_0^{Y}) + D_0^{XY} \cdot (p_1^{Y} - c_1^{Y})$$

$$+ D_0^{XY} \cdot \frac{\partial q_0^{Y}}{\partial q_1^{Y}} \cdot (p_0^{Y} - c_0^{Y}) + f \cdot \frac{\partial q_0^{X}}{\partial q_1^{Y}} \cdot (p_0^{X} - c_0^{X}).$$

Similarly, for platform $Y$ these conditions are:

$$0 = \left( p_0^{Y} - c_0^{Y} \right) + \frac{p_0^{Y}}{\eta_0^{Y}} + \frac{\partial q_1^{Y}}{\partial q_0^{Y}} \cdot (p_1^{Y} - c_1^{Y}) + f \cdot \frac{\partial q_1^{X}}{\partial q_0^{Y}} \cdot (p_1^{X} - c_1^{X}) + D_0^{XY} \cdot (p_0^X - c_0^X)$$

$$+ D_0^{XY} \cdot \frac{\partial q_1^{X}}{\partial q_0^X} \cdot (p_1^{X} - c_1^{X}) + f \cdot \frac{\partial q_1^{Y}}{\partial q_0^X} \cdot (p_1^{Y} - c_1^{Y}),$$

$$0 = \left( p_0^{Y} - c_0^{Y} \right) + \frac{p_0^{Y}}{\eta_0^{Y}} + \frac{\partial q_0^{Y}}{\partial q_1^{Y}} \cdot (p_0^{Y} - c_0^{Y}) + f \cdot \frac{\partial q_0^{X}}{\partial q_1^{Y}} \cdot (p_0^{X} - c_0^{X}) + D_0^{XY} \cdot (p_1^{X} - c_1^{X})$$

$$+ D_0^{XY} \cdot \frac{\partial q_0^{X}}{\partial q_1^{X}} \cdot (p_0^{X} - c_0^{X}) + f \cdot \frac{\partial q_0^{Y}}{\partial q_1^{X}} \cdot (p_0^{Y} - c_0^{Y}).$$

However, for platform $Z$, the conditions follow the single platform seller conditions that are provided in Proposition \[\Box\]

$$0 = \left( p_0^{Z} - c_0^{Z} \right) + \frac{p_0^{Z}}{\eta_0^{Z}} + \frac{\partial q_1^{Z}}{\partial q_0^{Z}} \cdot (p_1^{Z} - c_1^{Z}),$$

$$0 = \left( p_0^{Z} - c_0^{Z} \right) + \frac{p_0^{Z}}{\eta_0^{Z}} + \frac{\partial q_1^{Z}}{\partial q_0^{Z}} \cdot (p_0^{Z} - c_0^{Z}).$$

Solving the system of equations for each type of merger provides the pricing results found in Table \[\Box\]
References


