Effects of an Advertising Tax in Two-sided Markets under Imperfect Competition

C. Matthew Shi*

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Abstract Taxation of advertising has been a recurring policy idea in recent U.S. tax reforms and around the globe. In this paper, we examine the incidence and welfare effects of an ad valorem tax on advertising under different market structures and business models using a two-sided market framework. In particular, we present a set of new results showing that for mixed-financed platforms, taxes on advertising have negative effects on all of the advertisers, platforms and ad-averse media consumers in both monopoly and oligopoly with price competition and differentiated products. For (endogenously) solely ad-financed platforms, the tax may benefit consumers while still harming the advertisers and platforms. More broadly, we indicate how and when the conventional wisdom of using a Pigouvian tax to correct for negative externalities does not work in platform industries.

Keywords: Two-sided markets; media market; tax on advertising; imperfect competition

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*Department of Economics, The Chinese University of Hong Kong, Hong Kong SAR. E-mail: shice@cuhk.edu.hk.
1 Introduction

The taxation of advertising has long been a controversial issue. Recently, it has become a subject of heated debate as governments worldwide have increasingly proposed or enacted advertising-related tax policies to seek additional sources of revenue. For instance, in Sweden and Hungary, the governments collect taxes on the revenues from advertising services provided by print, broadcasting, and online media, at rates ranging from 4 to 11 percent. In the United States, advertising - along with many other services and intangible properties - was not subject to sales tax until 1987, when the Florida legislature enacted but soon repealed a state sales tax on all services that include advertising. More states have considered introducing taxes on advertising and other business services thereafter. In the mean time, in Austria, the government taxes advertising expenditure of firms. Proposals in a similar spirit to make advertising expenses less (than fully) deductible at the federal level have also been a recurring idea in U.S. tax reforms since 2014. While the arguments against taxing advertising typically center on its presumed adverse effects on advertisers (which are often small businesses), service-providing firms, and other socio-political outcomes, less is known or discussed about its pass-through effects on consumers in media markets. In fact, as advertising is often a nuisance to media consumers, the conventional wisdom in economics tends to suggest that a tax on advertising externalities is likely to be beneficial overall. In this paper, we examine the effects of an ad valorem tax on advertising on a series of economic variables under different market structures and business models (partial and pure advertising finance) using a model of “two-sided” media markets. Our main results show that under both monopoly and differentiated-product Bertrand oligopoly, when a media platform is mixed-financed, a higher tax on ads raises both advertiser and consumer prices, lowers their participation, and is almost always harmful to all parties, including ad-averse consumers, in media markets.

Unlike traditional markets in which there is a dichotomy of supply and demand, media

\footnote{For an account of the sales tax on services introduced in Florida in 1987, see \textit{Hellerstein (1988)}.}
markets exemplify the so-called “two-sided” markets, where two groups of users interact through an intermediary, or a platform, and each group’s demand and welfare depend not only on one group’s (own) prices but also on the participation of the other. Specifically, in media markets, the two sides are media consumers and advertisers, and while advertisers’ decision depends on the number of consumers that can be reached on a platform, the advertising intensity of a platform in turn affects consumer choice. Facing the interwoven demands of the two sides, a media platform can profit from charging both sides access fees, and thus implicitly coordinating their actions. Hence, a policy intervention on one side of the market, such as a tax on advertising, would generate effects on all two sides and the platform. Nonetheless, the presence of cross-group network externalities in two-sided media markets makes the analysis and quantification of the incidence and welfare effects often difficult. The main contribution of this paper is that we deliver a full set of clear-cut and novel theoretical results on the effects of an \textit{ad valorem} advertising tax on the platform, advertisers, and consumers in media markets under different market structures and business models.

Taking advantage of the recent advances in theory of two-sided markets in industrial organization, we first show in our benchmark monopoly model that a tax on advertising unambiguously reduces the number of advertisers and consumers, increases the prices they pay, and lowers both advertiser and consumer surplus despite that the tax reduces ad nuisance for consumers. Moreover, as the tax decreases a platform’s total profit, it nevertheless creates more profits in the subscription market, resulting in a change in the media business model. The central idea is that a mixed-financed platform internalizes the externalities that the two sides of the market exert on each other, and particularly, it needs to capture consumer attention in order to attract advertising business. When a tax on ads reduces profitability of advertising markets, the tax also weakens the platform’s incentive to engage more consumers by lowering the price. In our paper, this two-sided market incentive is formulated as part of a platform’s effective marginal cost in setting the consumer full price. This formulation also allows us to show similar results in our first extension where we consider a general form
of oligopolistic consumer markets with price competition and product differentiation, that encompasses standard symmetric Bertrand models, including the Hotelling duopoly, Salop’s circular city, Logit, and linear demand models. In our second extension with purely ad-financed platforms, we observe a change in the results: because a tax suppresses advertising activities, consumers receive less ad nuisance. While profit still decreases, if the advertiser demand is log-concave, advertiser surplus also decreases even though there are more consumers that can be reached on a platform. Thus, this paper also relates to the discussion on taxation of internet platforms and the digital economy that often rely exclusively on ad revenues. These issues are particularly important in recent cases in which some European countries have proposed somewhat punitive taxes on internet giants such as Google and Facebook.

This paper builds upon foundational works on theory of two-sided markets (e.g., Rochet and Tirole (2003) and Armstrong (2006)) and especially on standard two-sided media market models (e.g., Anderson and Coate (2005) and Anderson and Jullien (2016)). We focus on the important, policy-relevant question of taxation of advertising activities and present new results regarding the effects of such a tax. We use a flexible framework to analyze the tax effects under different market structures and business models, which can be otherwise difficult to study. In particular, we show new intermediate results in our formulation such that the standard IO toolkits for tax analysis in oligopoly (e.g., Anderson et al. (2001)) all become useful. Our formulation is readily applicable to studies of other forms of taxation and other exogenous market shocks in media industries.

This paper also contributes to the classic tax literature. Since Pigou (1920), economists have advocated taxation as a remedy for the market inefficiencies caused by externality problems. More recently, Weyl and Fabinger (2013) studies tax pass-through in a general model of imperfect competition. This paper examines the welfare effects of a tax on advertising externalities in a two-sided market setting. We specify when and why a Pigouvian tax does not work in media markets with two-sided externalities. We believe our basic results also shed light on the study of other similar non-media settings in which a platform or an
intermediary plays an important role in coordinating market transactions.

Lastly, this paper adds to the small body of literature that focuses specifically on tax issues in media and other two-sided markets. Kind et al. (2008) pioneers this area by first studying tax efficiency questions in a general monopolistic two-sided market. Subsequent theoretical works have used two-sided monopoly frameworks to examine the effects of different forms of tax on various features of the market, such as a tax on the subscription market (Kind et al. (2010)), and the effects of different tax instruments on user privacy (Bloch and Demange (2017) and fiscal revenues (Bourreau et al. (2017)). Rauch (2013) empirically investigates the effects of Austria’s advertising expenditure tax on prices in consumer-product markets. In this paper, we present new results on the effects of an advertising tax in both monopoly and oligopolistic markets. In addition, we examine the mixed-finance business model and the pure ad-finance model, which arises endogenously in our framework as a “corner” situation.

The rest of this paper is organized as follows. In the next section, we develop our benchmark monopoly model with platform mixed-finance. We present the baseline incidence and welfare results in Section 3. In Section 4, we extend our basic results to the case in which the consumer markets are imperfectly competitive with price competition and product differentiation. In Section 5, we consider the pure ad-finance model with oligopolistic platforms. Section 6 concludes.

2 The Model

We begin by considering a two-sided monopoly model, which follows the standard media market settings in Anderson and Jullien (2016). In this model, a single mixed-financed platform sells media content (e.g., news and entertainment) to consumers at a subscription price, $p^s$, and sets the amount of advertising, $a$, to carry. Consumers decide whether to purchase the media product, observing both $p^s$ and $a$. Below we make two common assumptions about media consumers and advertisers in a two-sided market.
Assumption 1 (Additive Ad Nuisance) Consumers have an ad nuisance cost function, \( \gamma(a) \), with \( \gamma > 0 \) and \( \gamma' \equiv d\gamma/da > 0 \), which is additively separable from the subscription price. So, the full consumer price from joining a platform is defined as \( f = p^s + \gamma(a) \).

Specifically, consumers have the same distaste for advertising, which depends on the number of advertisements in a media platform and can be translated into dollar values. Given the full price \( f \), consumers maximize their utility. Their utility maximization generates some demand function for media content, \( N(f) \), which we assume is twice-differentiable and downward sloping (i.e., \( N' \equiv dN/df < 0 \)).

On the other side, there is a continuum of advertisers who derive benefits from reaching consumers, and they pay a unit price, \( P^a \), to buy advertising space from the platform. We make further assumptions about the value of consumers to an advertiser.

Assumption 2 The value to an advertiser from joining a platform is (i) proportional to the number of consumers that can be reached and (ii) independent of other advertisers on the same platform; and (iii) the platform charges an advertising price per consumer; i.e., \( P^a = p^a N \).

As in the standard media economics setting, assumption 2(i) implies constant returns to advertising, and by assumption 2(ii), we assume away any competition among advertisers. Assumption 2(iii) is motivated by the common pricing practices in media and advertising industries. For instance, in both traditional and online media, many companies charge per-consumer or per impression ad rates measured in “CPM” (cost per mille, or cost per thousand impressions) and cost per click.

Given these assumptions, we can rank advertisers’ willingness-to-pay per consumer in descending order, and the resulting inverse demand function for advertising space is given by \( p^a(a) \), with a downward slope: \( dp^a/da < 0 \). When the platform chooses some advertising level \( a \), the corresponding ad price is therefore \( P^a(a) = p^a(a)N \).

2 Although this paper considers ad-averse consumers, our basic results on incidence and welfare effects of an advertising tax also hold with ad-seeking consumers, as noted in Section 3.2.
Without loss of generality, assume that the platform has a constant marginal cost of content production, $c^s$, and a constant marginal cost of advertising per subscriber, $c^a$. Absent of any tax, the platform’s profit maximization problem is

$$\max_{p^s,a} \pi = \frac{(p^s - c^s)N(f)}{\pi^s: \text{subscription profit}} + \frac{(p^a(a)N(f) - c^aN(f))a}{\pi^a: \text{advertising profit}}$$

$$= (p^s - c^s + R(a) - c^a a)N(f);$$

where we write $R(a) = p^a(a)a$, which is ad revenue per subscriber.

Now consider that the government imposes an ad valorem tax on advertising. Under a tax rate of $t$, the platform (producer) price of ads is $p^a(\frac{t}{1+t})$. We define $\tau \equiv \frac{t}{1+t}$, so the platform price of ads can be written as $(1-\tau)p^a$, which we use in the following analysis. With such a tax, the profit function is then

$$\pi = (p^s - c^s + (1-\tau)R(a) - c^a a)N(f). \quad (1)$$

The monopolist platform sets the subscription price and the ad level simultaneously to maximize profit. Here, we use the solution concept introduced in Anderson and Coate (2005). Specifically, we can solve the maximization problem by finding the optimal combination $(p^s^*, a^*)$ while keeping the full consumer price $f$ constant. In other words, for any fixed full price $\bar{f}$, and hence any given number of consumers, $\bar{N} = N(\bar{f})$, we can first solve for $a^*$ that maximizes the platform profit per consumer:

$$a^* = \arg \max_{\frac{\bar{f}}{a} \geq \bar{g}} (\bar{f} - \gamma(a) - c^s + (1-\tau)R(a) - c^a a). \quad (2)$$

Then an interior solution $a^* > 0$ satisfies the following first order condition:

$$(1-\tau)R'(a) = \gamma'(a) + c^a. \quad (3)$$

Condition (3) states that in optimum, marginal ad revenue per consumer should equal
marginal cost of ads plus marginal revenue foregone in the subscription market due to negative advertising externalities on consumers. When marginal ad revenue per consumer is lower, the platform can decrease the level of ads to gain from the reduction of ad nuisance in the subscription market, and *vice versa*. In particular, the two-sided financed platform considers, or internalizes, the marginal negative externalities of ads captured by $\gamma'(a)$. Without the “two-sidedness” of media markets, the right-hand side of (3) would only include the direct cost, $c_a$.

Assume that the second order condition (with respect to $a$) is satisfied: $(1 - \tau)R''(a^*) - \gamma''(a^*) < 0$. We further assume $(1 - \tau)R'(0) > \gamma'(0) + c^a$ to rule out a corner solution, $a^* = 0$, which would lead to the usual one-sided, subscription-only business model.

Note that there can be another solution with $\tilde{f} = \gamma(a^*)$, when $(1 - \tau)R'(a) > \gamma'(a) + c^a$ for all $a$. As noted in the literature (e.g., Anderson and Jullien (2016)), the pure ad-finance business model arises endogenously in this framework when the ad demand is strong and/or the direct cost of ads is low.\(^3\) In Section 5, we analyze the pure ad-finance business model and present some significantly different results from our benchmark model with two-sided pricing.

To obtain the optimal subscription price $p^{**}$, we plug $a^*$ back into the profit function and solve for the optimal consumer full price $f^*$, and subsequently, $p^{**} = f^* - \gamma(a^*)$. (Hereafter we suppress the asterisks for optimality unless emphasis is needed.) So the optimal full price solves the following first order condition with respect to $f$:

$$N(f) + (f - \gamma(a^*) - c^a)N'(f) + ((1 - \tau)R(a^*) - c^aa^*)N'(f) = 0. \tag{4}$$

Rewriting yields

$$N(f) + (f - \tilde{c})N'(f) = 0, \tag{5}$$

where $\tilde{c} \equiv \gamma(a^*) + c^a - (1 - \tau)R(a^*) + c^aa^*$.

In (4), the first two terms are similar to those in the usual (price-setting) monopolist’s

\(^3\)It can also occur when $\gamma < 0$; i.e., consumers are ad-seeking.
problem. The third term is due to the two-sidedness of media markets in that an increase in the consumer full price will lead to a lower subscription level and subsequently lower ad revenues. As in the advertising market, the platform also considers the positive externality of consumer participation on advertisers in choosing the full consumer price. In particular, the platform has an incentive to engage more subscribers for the associated ad revenues \((1 - \tau)R(a^*) - c^a a^*)\). This additional incentive effectively reduces the platform’s marginal cost associated with setting its full price.

Indeed, this first-order condition can be rewritten as a standard (price-setting) monopoly first-order condition with an effective constant marginal cost \(\tilde{c}\), as shown in (5). Therefore, increasing the ad valorem tax on ads would have the same effects in the subscription market as a cost shock to the producer in a traditional one-sided market. We present a novel result in Lemma 1, which provides a surprisingly succinct yet powerful means for establishing our later results. Specifically, a higher tax on ads unambiguously raises the effective marginal cost (of setting the full price) even though the cost term contains different components which may individually increase or decrease with the tax.

**Lemma 1** \(\frac{dc}{d\tau} = R(a^*) > 0\).

**Proof.** Directly, \(\frac{dc}{d\tau} = R(a^*) + [-(1 - \tau)R'(a^*) + \gamma'(a^*) + c^a] \frac{da}{d\tau} = R(a^*) (= \frac{dc}{d\tau})\), where the bracketed term equals zero by the advertising first-order condition (3). □

Lemma 1 is indeed a direct application of the envelope theorem: because the ad level is set optimally to maximize the per-consumer profit with a fixed \(f\), it equivalently minimizes the effective marginal cost. Therefore, by the envelope theorem, the total effect of the advertising tax on the cost term is only the direct effect. Lemma 1 helps to simplify the analysis by transforming the two-sided market problem into a usual one with cost shocks.

Lastly, we assume the second order condition (with respect to \(f\)) is met: \(\Phi \equiv 2N''(a^*) + (f - \tilde{c})N''(a^*) < 0\). Based on optimality conditions (3) and (5), we can obtain comparative static results on the prices and participation in this two-sided market. Both prices and participation are important because, as in any two-sided market, demand and welfare of
either side depend not only on prices but also on the participation of the other side.

3 Tax Incidence and Welfare Effects

3.1 The effects on prices and participation

We first examine the effects of the advertising tax on the advertiser price of ads and advertiser participation. As seen in (3), ceteris paribus, a higher ad valorem tax on advertising decreases marginal ad revenue, which causes the platform to carry fewer ads in optimum. Proposition 1 states the result.

Proposition 1 In a monopolistic advertising market, an increase in the ad valorem tax on advertising reduces the number of ads and raises the advertiser price of ads.

Proof. Applying the implicit function theorem to (3), we obtain

\[
\frac{da^*}{d\tau} = \frac{R'}{(1-\tau)R'' - \gamma''} < 0,
\]

where \( R' > 0 \) by condition (3) and we also use the advertising second-order condition. Immediately, \( \frac{dp^a}{d\tau} = \frac{dp^a}{da} \frac{R'}{(1-\tau)R'' - \gamma''} > 0 \).

Proposition 1 implies that consumer ad nuisance costs, \( \gamma(a^*) \), also decrease with the tax. More importantly, Proposition 1 and its implications for media consumers hold generally for a monopolistic advertising market with any type of subscription market as long as its market structure is consistent with a monopoly on the advertising side. Specifically, they hold for both a two-sided monopoly and competitive bottlenecks, in which a few platforms compete in price and sell differentiated products to single-homing consumers, which is the case we study in Section 4. With the tax effects on the advertiser price and participation known, we now examine how prices and participation change in the subscription market. Since consumer participation depends sufficiently on the full price, the tax effect on the full price is more relevant than the subscription price alone. We find that the tax increases both
Proposition 2  In a two-sided media monopoly, an increase in the ad valorem tax on advertising raises the consumer subscription price and the full price. Thus, the subscription level decreases with the tax.

Proof. Applying the implicit function theorem to (4), we obtain

$$\frac{df^*}{d\tau} = -\frac{\partial(f-\bar{c})}{\partial \tau} N' = \frac{d\bar{c}}{\Phi} \frac{d\Phi}{\Phi} > 0$$

where we use Lemma 1 and that $\Phi < 0$.

It follows directly that $\frac{dp^*}{d\tau} = \frac{df^*}{d\tau} - \gamma' d\bar{a}^* > 0$, and $\frac{dN}{d\tau} = N' \frac{df^*}{d\tau} = \frac{R(N')^2}{\Phi} < 0$. ■

This two-sided market problem is made simple by rewriting it as a (full) price-setting monopolist’s problem with a unique marginal cost term $\bar{c}$. Furthermore, as seen in Lemma 1, the effect of the advertising tax on the consumer full price is equivalent to the effect of an increased effective marginal cost. We illustrate this in Figure 1, which shows that the advertising tax raises the monopolist’s effective marginal cost without affecting the consumer demand; therefore, as marginal cost increases, the monopolist optimally charges a higher full price with lower outputs. 4

From Propositions 1 and 2, we see that although the tax suppresses the advertising activities and thus reduces the ad nuisance to media consumers, consumers nonetheless face a higher full price from joining the platform. It follows that the subscription price must increase more than the reduction in the ad nuisance.

[Figure 1 about here.]

In summary, a higher tax on ads leads to higher prices and lower participation on both sides of the media market, unambiguously. In the following subsection, we explore more comparative statics on the welfare effects of the tax on consumers, advertisers, the platform,

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4 For illustration, we present the effects of a finite tax increase, $\Delta \tau$, instead of an infinitesimal change.
and society as a whole. One important observation is that the direction of the tax effects on prices and participation on both sides is independent of $\gamma(a)$, or how (and whether) consumers dislike ads because in our formulation, the presence of $\gamma(a)$ only affects the effective marginal cost. As we discuss in the following sections, it also turns out that consumer ad preference does not matter for the direction of the effects on welfare and profit, and only plays a role in determining the composition of the platform profit.

3.2 The welfare effects

In a two-sided media market, the welfare of consumers and advertisers depends on both their own prices and the number of participants on the other side. Let $CS(p^s, a)$ denote consumer surplus, which is a function of the subscription price and the advertising level; and let $AS(p^a, N)$ denote (aggregate) advertiser surplus, which is a function of the advertising price and the subscription level. Here, we can write consumer surplus as $CS(p^s, a) = CS(f) = \int_0^\infty N(x)dx$, and advertiser surplus as $AS(p^a(a), N) = N \cdot \bar{AS}(a)$, where $\bar{AS}(a) = \int_0^a (p^a(x) - p^a(a))dx$ is advertiser surplus per subscriber. Note that $dCS/df = -N < 0$ and $d\bar{AS}/da = -a(dp^a/da) > 0$.

In Section 3.1, we conclude that a higher tax on advertising raises both the full consumer price and the advertiser price of ads and reduces the number of participants on both sides. Based on these comparative static results, we see that the tax reduces surplus for all parties in the market. In addition, define social surplus $SS = CS + AS + \pi + T$, where $T = \tau R(a)N(f)$ is the government tax revenue. We find that social surplus also decreases with the tax. Proposition 3 summarizes these results.

**Proposition 3** In a two-sided media monopoly, consumer surplus, advertiser surplus and platform profit all decrease with the ad valorem tax on ads. Thus, social surplus also decreases with the tax.

**Proof.** On the consumer side, consumer surplus depends sufficiently on the full price and by Proposition 2, the full price increases with the tax. Therefore, consumer surplus decreases
with the tax:
\[
\frac{dCS}{d\tau} = -N \frac{df^*}{d\tau} = -N \frac{RN'}{\Phi} < 0.
\] (7)

On the advertiser side, by Proposition 1, advertisers pay a higher price of ads per subscriber, and by Proposition 2, they can only reach fewer subscribers. Therefore, advertiser surplus also decreases:
\[
\frac{dAS}{d\tau} = -Na \frac{dp^a}{da} \frac{da^*}{d\tau} + \frac{dN}{d\tau} \bar{AS} < 0.
\] (8)

For the platform, by the envelope theorem, the platform earns less total profit:
\[
\frac{d\pi^*}{d\tau} = \frac{\partial \pi}{\partial \tau} = -RN < 0.
\] (9)

Finally, for the entire society, because \(\frac{d\pi^*}{d\tau} + \frac{dT}{d\tau} = \tau \left( R \frac{dN}{d\tau} + N R' \frac{da^*}{d\tau} \right) < 0\), we have \(\frac{dSS}{d\tau} = \frac{dCS}{d\tau} + \frac{dAS}{d\tau} + \frac{d\pi}{d\tau} + \frac{dT}{d\tau} < 0\); i.e., a higher tax on ads also reduces social surplus.

In summary, in a two-sided monopoly with mixed finance, a (higher) tax on ads is never good news for either side or the platform. Even if the tax reduces the negative externalities of advertising, media consumers face a higher full price than before, so they are still worse off. Furthermore, because the tax causes the platform to lose more profits than the additional amount of tax revenue that the government is able to collect, the tax on ads is also costly to the entire society.

An important implication of Proposition 3 is that the conventional wisdom of using a Pigouvian tax to correct for negative externality problems in private markets does not work in our context of two-sided media markets. Our benchmark model formalizes this important idea and unravels the underlying mechanism: in a media monopoly with two-sided finance, the platform, to some extent, internalizes the participation externalities that each side imposes on the other. Therefore, a tax on advertising externalities will not help at all but will only weaken the platform’s incentive to lower prices and engage more subscribers whose participation would attract ad revenues that are perhaps more important financially. In Section 4, we show that this two-sided market mechanism is not specific to the monopoly
case by adding imperfect competition for media consumers into our benchmark model. We present a set of new results on the effects of advertising tax in media markets with price competition and differentiated products.

3.3 The effects on the business model

From the above, we note that the direction of the tax incidence and welfare effects is independent of how consumers view advertising. In this section, we analyze the tax effects on the business model - the endogenous profit split between subscription and advertising markets - of a media platform and study the role of consumer ad preference in shaping these effects. The business model of media firms is itself an immensely important subject. For instance, revenue sources are often the determining force that shapes media content and its quality, which in turn have large impacts on a series of socio-economic outcomes, ranging from health behavior and consumption to ideology and public spending.

We see from Proposition 3 that a higher tax on ads reduces the platform profit. To examine the tax effects on $\pi^s$ and $\pi^a$ separately, differentiating yields $\frac{d\pi^a}{d\tau} = N\gamma^t \frac{da^*}{d\tau} + \left[(1 - \tau)R \right]N\frac{df^*}{d\tau} = NR < 0$. By equation (9), $\frac{d\pi^s}{d\tau} = -NR > \frac{d\pi^a}{d\tau}$. In other words, the tax effect on the platform advertising profit is more negative than its effect on total profit. Therefore, it must be that a higher tax actually raises the platform’s subscription profit; i.e., $\frac{d\pi^s}{d\tau} > 0$.

Figure 1 can help us again to visualize these changes. The subscription profit alone is maximized when the price is set at $p_0$ such that at the corresponding level of output, $N(p_0)$, the associated marginal revenue equals marginal cost of content production, $c^s$. This happens to also be the total profit in optimum when the advertising market is financially unimportant, so $p_0 = f_0$. Nonetheless, when there is a relevant advertising market, the optimal full price is chosen such that at the corresponding output level, marginal revenue should equal the effective marginal cost $\tilde{c}$, instead of $c^s$. According to Lemma 1, a higher tax unidirectionally raises $\tilde{c}$, thus weakening platform’s “two-sided market” incentive and pushing
the effective cost higher and closer to the direct cost. With $\tilde{c}$ increased, the platform’s total profit decreases in optimum; however, with $\tilde{c}$ being closer to $c^s$, the subscription profit alone becomes closer to its maximized value and therefore increases.

Noticeably, this mechanism may not apply when consumers are ad-loving instead of being ad-averse. As is also evident in the proof for Proposition 2, with ad-loving consumers, when consumers face a higher full price and receive fewer advertisements, we are unable to assert whether or not the subscription price will increase. For similar reasons, the tax effect on the subscription profit alone becomes ambiguous when consumers prefer more ads.

4 Competitive Bottlenecks

In this section, we add imperfect competition for media consumers to the benchmark model and present new incidence results analogous to those for the monopoly case. Consider $I$ symmetric platforms, indexed by $i$, selling differentiated products to media consumers and compete in price. Consumers consider the full price, namely the subscription price plus the ad nuisance costs, of joining each platform, $f_i$. Among the alternatives, they choose only one platform that will yield the highest level of utility. This is known as consumer “single-homing” in the two-sided market literature. Consumer utility maximization generates some twice differentiable demand function, $N(f_i; f_{-i})$, where $f_{-i}$ is the common price charged by other platforms. For any $i$, $N(f_i; f_{-i})$ is decreasing in Platform $i$’s full price (i.e., $\partial N/\partial f_i < 0$), and increasing in $f_{-i}$ (i.e., $\partial N/\partial f_{-i} > 0$, for gross substitutes). In particular, $N(f_i; f_{-i})$ encompasses demand systems that arise from standard symmetric differentiated-product Bertrand oligopoly models, including spatial models (i.e., Hotelling duopoly and Salop’s circular city), Logit-type models, and linear demand. We assume $\frac{\partial N}{\partial f} \equiv \frac{\partial N}{\partial f_i} + \frac{\partial N}{\partial f_{-i}} \leq 0$.

Since consumers single-home, they can only be reached through a single platform, at most. It follows that each platform has a monopoly position in the advertising market, and that an advertiser’s decision to buy advertising space on a platform is independent of its

\footnote{Armstrong and Vickers (2015) show that linear demand can have a discrete-choice foundation.}
decision to buy from other platforms. Therefore, advertisers can join multiple platforms, or “multi-home,” to reach each platform’s exclusive audience. This market structure with one side single-homing and the other side multi-homing is known as the “competitive bottleneck,” and is a standard setting for the analysis of media markets and other two-sided markets in general. This setting corresponds to a real-life situation, in which, for instance, television audiences can watch only one channel at a time, and advertisers place ads on multiple channels at the same time in order to reach different consumers.

In this case, we attach subscript $i$ to the profit function, which then becomes

$$\pi_i = (p_i^s - c^s)N(f_i; f_{-i}) + ((1 - \tau)R(a_i) - c^a a_i)N(f_i; f_{-i}).$$

(10)

Notice that Platform $i$ is still a monopolist in the advertising market, so the optimal level of advertising, $a_i^*$, is the same as that described in Section 2.2, with all platforms choosing the same ad level: $a_i^* = a_{-i}^* = a^*$. Proposition 1 and all other results based on the advertising first order condition (3) also apply in this case of competitive bottlenecks.

On the subscription side, with given $a^*$, the first order condition with respect to $f_i$ is then

$$f_i - \bar{c} + \frac{N}{\partial N/\partial f_i} = 0,$$

(11)

where we again write $\bar{c} = \gamma(a^*) - c^s + (1 - \tau)R(a^*) - c^a a^*$ as a firm’s effective marginal cost in choosing its full price.

Assume that the second order condition is met and that there is a unique symmetric equilibrium, with $f_i^* = f_{-i}^* = f^*$ (and subsequently, $p_i^{s*} = p_{-i}^{s*} = p^{s*}$).

Similar to our approach in the benchmark monopoly model, here we transform this problem of price competition in competitive bottlenecks to a standard Bertrand model with a modified cost structure. By Lemma 1, we are essentially examining how the market outcomes change in response to a common cost shock. We can then deploy the toolkit for the standard symmetric oligopoly analysis to our model. We begin by noting the following lemma, shown in Anderson et al. (2001) (henceforth AdPK 2001; see details in the Appendix).
Lemma 2 (AdPK 2001) $\Psi \equiv 1 + \frac{\partial}{\partial f} \left( \frac{N}{\partial N/\partial f_i} \right) > 0$.

Proposition 4 below is the competitive bottleneck version of Proposition 2, for it states that the tax on ads increases prices and lowers subscription levels.

**Proposition 4** *In media markets with a competitive bottleneck structure, subscription prices and full consumer prices both increase with the ad valorem tax on advertising. Thus, subscription levels weakly decrease with the tax.*

**Proof.** Applying the implicit function theorem to condition (11) for the symmetric equilibrium, we have

$$\frac{df^*}{d\tau} = \frac{\partial}{\partial \Psi} = \frac{R}{\Psi} > 0$$

where we use Lemmas 1 and 2.

The effect on subscription prices is then $\frac{df^{**}}{d\tau} = \frac{R}{\Psi} - \gamma^* \frac{df^*}{d\tau} > 0$; and that on subscription levels: $\frac{dN}{d\tau} = \frac{\partial N}{\partial f} R \frac{df^*}{d\tau} \leq 0$. ■

To look at the tax effect on profits, we differentiate the profit function (10) at the symmetric equilibrium:

$$\frac{d\pi}{d\tau} = \left( N + (f - \bar{c}) \frac{\partial N}{\partial f_i} \right) \frac{df^*}{d\tau} + (f - \bar{c}) \frac{\partial N}{\partial f_i} \frac{df^*}{d\tau} - NR,$$

where the term in parenthesis is zero by first-order condition (11). In this case, due to the strategic interactions in Bertrand oligopoly, no simple envelope theorem applies. In (13), the second term, which represents the indirect effect on profits through the prices of competing platforms, is positive. The third term represents the direct loss of ad revenue due to a higher tax rate. Overall, in competitive bottlenecks, the tax effect on profits can be ambiguous *ex ante*. As noted by AdPK (2001), imperfect competition introduces the possibility that profits may rise with taxes when the demand curves are “highly convex”. Although this possibility is ruled out by assumption in many common Bertrand models (including the ones we list above), profit-increasing taxes may still be possible in some other formulations. Below we
introduce a few more notations to layout the exact conditions under which a tax on ads can increase platform profits.

As we describe in more detail in the Appendix, we define three elasticity terms: 

$$
\epsilon_m \equiv \frac{\partial}{\partial f} \left( \frac{\partial N}{\partial f} \right) \frac{f^*}{N}, \quad \epsilon_{dd} \equiv \frac{\partial N}{\partial f} \frac{f^*}{N}, \quad \text{and} \quad \epsilon_{DD} \equiv \frac{\partial^2 N}{\partial f^2} \frac{f^*}{N}; \quad \text{and the ratio} \quad E \equiv \frac{\epsilon_m}{\epsilon_{DD}}.
$$

We then proceed by first noting that, at the symmetric equilibrium, 

$$
\pi_i = -\frac{N^2}{2} \frac{\partial N}{\partial f},
$$

where we insert first-order condition (11) into the profit function. Therefore,

$$
d\pi d\tau = -\frac{df^*}{d\tau} \frac{\partial}{\partial f} \left( \frac{N^2}{\partial N/\partial f_i} \right) = -\frac{R}{\Psi} N \left( \frac{2\epsilon_{DD} - \epsilon_m}{\epsilon_{dd}} \right),
$$

which is positive if and only if \( E > 2 \).

**Proposition 5** In media markets with a competitive bottleneck structure, platform profits increase with the ad valorem tax on advertising if and only if \( E > 2 \).

In particular, as AdPK (2001) point out, \( E = 2 \) in Logit models with symmetry. According to Proposition 5, the functional form of the Logit itself implies that a higher tax should have no impact on platform profits. Given the importance of the Logit-type models in structural empirical work, this is especially noteworthy.

In summary, our basic results extend well to mixed-financed platforms in competitive bottlenecks. Specifically, a higher tax on ads raises both advertiser prices and consumer full prices, and reduces participation on both sides. Under the usual circumstances, the tax also reduces all firms’ profits. Only when the consumer demand is too convex, a higher tax may over-shift prices and lead to higher platform profits. However, with consumer demand being highly convex, one would naturally question whether an equilibrium exists at all.

## 5 Purely Ad-Financed Platforms

Until now, we have focused on mixed-financed media platforms that make profits in both the subscription and advertising markets. As we have shown, mixed-financed platforms tend to internalize the participation externalities that both sides exert on each other, so a tax
on ads would in general reduce the platforms’ incentive to engage more subscribers through lower prices. Consequently, higher taxes are indeed harmful to all parties in media markets when media platforms are mixed-financed.

It is then natural to consider an alternative situation in which platforms have no or only limited ability to internalize the external effects through two-sided pricing. In this section, we study media platforms that rely solely on advertising revenues. This business model of pure ad-finance is commonly studied in the literature.\(^6\) In real life, purely ad-financed media include free newspapers and magazines, radio programs, and websites that carry advertisements.

Now, consider a model of purely ad-financed platforms in competitive bottlenecks based on Anderson and Coate (2005), in which \(I\) platforms engage in “price” competition for single-homing subscribers and thus have a monopoly position in relation to the multi-homing advertisers. As discussed in Section 2.2, the pure ad-finance business model arises endogenously in our framework as a corner solution in which \(p^{**}_i = 0\) and \(f^*_i = \gamma(a^*_i)\). This occurs when the demand for advertising space is strong and/or the economic costs of admitting more advertisers are low. We relegate the qualitatively similar results for the monopoly case in the Appendix.

In a pure ad-finance model, the price that consumers face in accessing of each platform only includes the ad nuisance cost: \(f_i = \gamma(a_i)\). Using the same notation and assumptions as in the previous sections, we write a platform’s profit maximization problem as follows:

\[
\max_{a_i} \pi_i = ((1 - \tau)R(a_i) - c^a a_i)N(f_i; f_{-i}); \quad (15)
\]

where, without loss of generality, we assume zero marginal costs of content delivery (i.e., \(c^s = 0\)). As is also noted in Armstrong (2006), equation (15) resembles a Bertrand problem with a special “mark-up” structure.

We assume that a unique symmetric equilibrium exists with \(a^*_i = a^*_{-i} = a^*\),\(^7\) which

\(^6\)See, for example, Anderson and Jullien (2016).

\(^7\)Because (15) is not a standard Bertrand game, in the Appendix, we discuss some sufficient conditions
satisfies the first-order condition for each platform $i$,

$$((1 - \tau)R'(a_i) - c_a)N + \gamma'((1 - \tau)R - c_a a_i)\frac{\partial N}{\partial f_i} = 0; \quad (16)$$

or alternatively,

$$\gamma'(1 - \tau)R - c_a a_i + \frac{N}{(1 - \tau)R' - c_a} = 0. \quad (17)$$

We also assume that the second order condition is met and the equilibrium is stable. Stability implies that $da_i/da_{-i} < 1$ at the symmetric equilibrium. For notational ease, we write $	ilde{R} = (1 - \tau)R(a_i) - c_a a_i$ and subsequently define $	ilde{R}'$ and $	ilde{R}''$.

Based on (16), we see that any platform faces a trade-off between marginal ad revenue per consumer and the subscription level when choosing its optimal quantity of advertising. Although a platform still has to consider the two-sided effects, the externalities no longer enter into the platform’s decision simply as shifting the effective marginal cost. In fact, condition (17) implies that the platforms choose ad levels to equate two elasticity terms when the ad nuisance cost function is linear (e.g., Anderson and Jullien (2016)).

Based on (17), we also see that the left-hand side (LHS) is diminishing in the tax rate $\tau$. The question then is how the LHS changes as the ad levels of all of the platforms change. As we show in the Appendix, in the monopoly case, the LHS must decrease in $a^*$ simply by the associated second order condition. Here, with strategic interactions, we show that with one more assumption of equilibrium stability, the LHS of (17) decreases when the ad levels of all platforms increase (by the same infinitesimal amount) simultaneously. It follows that a higher tax reduces all platforms’ ad levels in equilibrium. We formally state and show the results in Proposition 6.

**Proposition 6** In both monopoly and competitive bottlenecks with pure ad-finance, an increase in the ad valorem tax on advertising reduces the number of ads and raises the advertiser price of ads. Thus, full consumer prices (equal to ad nuisance costs) decrease, and subscription levels weakly increase with the tax.

for existence of the Bertrand-Nash equilibrium.
Proof. For competitive bottlenecks, first apply the implicit function theorem to (16), and the stability condition becomes

\[
\frac{da_i}{da_{-i}} = \frac{\gamma' \left( \tilde{R}' \frac{\partial N}{\partial f_{-i}} + \gamma' \tilde{R} \frac{\partial^2 N}{\partial f_i \partial f_{-i}} \right)}{\left( \tilde{R}'' N_i + 2 \gamma' \tilde{R} \frac{\partial N}{\partial f_i} + \gamma'' \tilde{R} \frac{\partial N}{\partial f_i} + (\gamma')^2 \tilde{R} \frac{\partial^2 N}{\partial f_i^2} \right)} < 1. \tag{18}
\]

where the denominator is positive by the second-order condition.

After substituting the first order condition and using the definitions of the three elasticity terms (the same as those in Section 4), (18) can be readily written as

\[
\epsilon_{DD} - \epsilon_m + \epsilon_{dd}(1 + \frac{\gamma'' \tilde{R}}{\gamma' R'} - \frac{\tilde{R}'' \tilde{R}}{\left(R'\right)^2}) < 0. \tag{19}
\]

Now, applying the implicit function theorem to (17), we obtain

\[
\frac{da^*}{d\tau} = \frac{\epsilon_{dd} \left( \frac{\tilde{R}' R - \tilde{R} R'}{\left(R'\right)^2} \right)}{\epsilon_{DD} - \epsilon_m + \epsilon_{dd}(1 + \frac{\gamma'' \tilde{R}}{\gamma' R'} - \frac{\tilde{R}'' \tilde{R}}{\left(R'\right)^2})} < 0. \tag{20}
\]

So subsequently, \( \frac{dp^a}{d\tau} = \frac{dy^a}{da} \frac{da^*}{d\tau} > 0, \frac{df^*}{d\tau} = \gamma' \frac{da^*}{d\tau} < 0, \frac{dN}{d\tau} = \gamma' \frac{\partial N}{\partial f} \frac{da^*}{d\tau} \geq 0. \)

Other details are included in the Appendix. ■

In this case with pure ad-finance, the consumer full prices equal the ad nuisance costs. So, if a higher tax on ads reduces ad levels on all of the platforms, full prices must decline with the tax. Proposition 6 thus implies that the tax tends to benefit consumers by reducing ad nuisance on all platforms. This is in line with the conventional idea that a tax on negative externalities can enhance consumer welfare. This result differs substantially from our abovementioned results for mixed-financed media. Specifically, in this pure ad-finance business model, the platforms do not have the pricing tools to fully absorb the participation externalities on both sides. It is therefore unsurprising that under imperfect competition, a higher tax will never increase platform profits, unlike the results for mixed-financed platforms.
shown in Section 4. Formally, we have

\[
\frac{d\pi}{d\tau} = \gamma' R \frac{\partial N}{\partial f_i} \frac{d\bar{a}}{d\tau} - RN < 0.
\]  

(21)

Compared to (13) of the mixed-finance model, the first term is the indirect effect on prices from the ad levels chosen by other platforms, and the second term is the direct loss of ad revenue due to a higher tax. Here, since the ad levels, and hence full consumer prices, decrease with the tax, the indirect effect is also negative. Overall, the tax has an unambiguously negative effect on platform profits. In particular, there is no over-shifting of prices in this case, so the tax effect no longer depends on the shape of the demand curves.

On the advertiser side, the tax effect on advertiser surplus (on any platform) can be ambiguous \textit{ex ante}. In reference to equation (8), on the one hand, a higher tax leads to fewer active advertisers (i.e., \(-N_i a^* \frac{\partial \rho}{\partial a} \frac{da^*}{d\tau} < 0\)); but on the other hand, due to the lower ad nuisance costs, more consumers can be reached on each platform (i.e., \(\frac{dN}{d\tau} \bar{A} S_i \geq 0\)). Nonetheless, in the Appendix, we show that the total effect is negative when ad demand is not “too convex” (or log-concave, specifically).

**Proposition 7** \textit{In both the two-sided media monopoly and competitive bottlenecks with pure ad-finance, a higher tax on advertising reduces platform profits. When the ad demand function is log-concave, the tax also reduces advertiser surplus on all platforms.}

In summary, when platforms rely solely on ad revenues, a higher tax tends to enhance consumer welfare but also harm both the platforms and advertisers even though it makes more consumers accessible to advertisers. This echoes the media “see-saw” principle formalized by Anderson and Peitz (2015) in their study of purely ad-financed media with asymmetric Logit consumer demand. Anderson and Peitz (2015) point out that when the ad demand is log-concave, platforms and advertisers always have their interests aligned. As a result, any exogenous shock to the market, such as an entry or merger, that reduces platform profits will also reduce advertiser surplus. Using an aggregative game approach, they show that
consumers are on the other side of the see-saw. Similarly, the “see-saw” relationship is transparent in our comparative static results, for which we are also able to quantify the tax effects on all parties.

6 Concluding Remarks

In this paper, we sign and quantify the effects of an ad valorem tax on advertising on a series of economic variables in media markets. We show that with platform mixed-finance, the tax on ads has overall negative impacts on all parties. However, with pure ad-finance, the tax works in favor of media consumers but still tends to harm the advertisers and platforms. Based on the standard media economics models, our theoretical framework facilitates the analysis of two-sided media markets under monopoly and the less-explored market structure of imperfect competition.

The central ideas that we make transparent in our formulation are the internalization of the participation externalities and cross-group price subsidization by the two-side financed platforms. While a tax such as the one on advertising does not affect whether platforms internalize externalities, it nonetheless reduces their incentives to “subsidize” media subscribers through low prices. From the perspective of the platforms, the two revenue sources serve as “cushions” for each other in that when one side of the market is hit by some shock, a platform can always mitigate the negative impact by seeking alternative sources of revenue in the other market. In contrast, when market primitives only allow a platform to profit from a single side (e.g., the ad market), there is no cushion available. In particular, when a tax is levied on advertising, a platform has to appear less annoying to media consumers, and thus more appealing to advertisers, in order to avoid further losses. Given our assumptions on consumer preference and the pricing and cost structures of the platforms, our main results and the underlying mechanism do not depend on our assumptions about consumers’ attitudes toward advertising, or about market structure. More importantly, our results also should not be specific to the particular form of taxation or policy intervention that we focus
Lastly, we further suggest that the model we study in this paper and our findings are not restricted to media markets. For instance, consider an open-air market that hosts sellers who occasionally pollute the environment (e.g., fish mongers). According to the results of this paper, when there is some entry fee for customers, a tax aimed at the pollution problem would make customers worse off because the market owner may optimally charge a higher fee; and a tax can only reduce pollution and make customers better off overall when the market is free to visit. In the political economy of developing countries, similar comparisons can be used to describe the relationship between revenue-driven local governments (“platforms”), tax-payers (“customers”), and firms (“polluters”): when a central government decides to use taxes to deal with pollution, local governments may respond by raising local income tax rates to compensate for the loss of business tax revenue. Ultimately, our paper indicate the problems of using a Pigouvian tax in marketplaces in which agents are interrelated by some externalities and a profit-seeking intermediary in between.

References


Appendix

Proof of Lemma 2

Lemma 2 is based on Proposition 1 in AdPk (2001). Here, I briefly sketch the proof using this paper’s notation. In essence, the proof only requires satisfaction of the first order condition, the second order condition, and stability of the equilibrium to the Bertrand game described in Section 4.

We follow AdPK (2001) to define three elasticity terms: $\epsilon_{dd} \equiv \frac{\partial N}{\partial f_i f_i}f_i^*$, $\epsilon_m \equiv \frac{\partial}{\partial f_i} \left( \frac{\partial N}{\partial f_i} \right) f_i^*$, and $\epsilon_{DD} \equiv \frac{\partial N}{\partial f} f_i^*$; where $\epsilon_{dd}$ is the elasticity of the curve tracing Platform $i$’s quantity demanded when $i$’s own price changes at the equilibrium price, $\epsilon_m$ is the elasticity of the slope of that curve, and $\epsilon_{DD}$ is the elasticity of the curve tracing Platform $i$’s quantity demanded when all prices change.

By our demand assumptions, $\epsilon_{dd} \leq \epsilon_{DD} \leq 0$. Stability of the equilibrium implies the slope of the reaction function, $df_i/df_{-i}$, should be less than one at the symmetric equilibrium; i.e.,

$$
\frac{df_i}{df_{-i}} = \frac{\frac{\partial N}{\partial f_i} + (f_i - \bar{c}) \frac{\partial^2 N}{\partial f_i \partial f_{-i}}}{\left( \frac{\partial N}{\partial f_i} + (f_i - \bar{c}) \frac{\partial^2 N}{\partial f_i^2} \right)} < 1,
$$

(22)

where $\bar{c} = \gamma(a^*) + c^a - (1 - \tau) R(a^*) + c^a a^*$ is the effective marginal cost (in setting the full price) in our model. The second order condition implies that the denominator is positive,
so (22) becomes
\[-2\frac{\partial N}{\partial f_i} - (f_i - \bar{c})\frac{\partial^2 N}{\partial f_i^2} - (f_i - \bar{c})\frac{\partial^2 N}{\partial f_i \partial f_{-i}} - \frac{\partial N}{\partial f_{-i}} > 0.\]
From (11), we can substitute in \(f_i - \bar{c} = -\frac{N}{\partial N/\partial f_i}\), so stability condition (22) can be written in terms of three elasticities as \(\epsilon_{dd} + \epsilon_{DD} - \epsilon_m < 0\).

Now, using the elasticity definitions, we have
\[
\Psi \equiv 1 + \frac{\partial}{\partial f} \left( \frac{N}{\partial N/\partial f_i} \right) = 1 + \left( \frac{\epsilon_{DD} - \epsilon_m}{\epsilon_{dd}} \right) = \frac{\epsilon_{dd} + \epsilon_{DD} - \epsilon_m}{\epsilon_{dd}} > 0.
\]

**Conditions for Existence of Equilibrium in Section 5**

In competitive bottlenecks with pure ad-finance, the profit function (15) is non-standard. To guarantee existence and uniqueness of equilibrium for this pricing game, we can make additional assumptions on demand function and the ad nuisance cost function. In particular, it suffices to assume that \(N(f_i; f_{-i})\) is log-concave in \(f_i\), \(p^a(a)\) is log-concave, and \(\gamma(a)\) is convex. With these commonly made assumptions, \(\pi_i\) is concave in \(a_i\) for each \(i\), which is sufficient for existence and uniqueness of the Nash equilibrium.

To see that, we first re-write (16) as
\[
\frac{(1 - \tau)R' - c_a}{(1 - \tau)R - c_a} + \gamma' \frac{\partial N}{\partial f_i} = 0.
\]
For concavity of \(\pi_i\) in \(a_i\), we need
\[
\frac{d}{da} \left( \frac{\tilde{R}'}{\tilde{R}} \right) + \gamma' \frac{\partial N}{\partial f_i} + \frac{\partial}{\partial a_i} \left( \frac{\partial N}{\partial f_i} \right) < 0, \text{ for all } a_i.
\]
Since \(R'' < 0\), \(\frac{d}{da} \left( \frac{\tilde{R}'}{\tilde{R}} \right) < 0\). The second term is negative by convexity of \(\gamma(a)\). The third term is also negative since log-concavity of \(N_i\) implies \(\frac{\partial N}{\partial f_i}/N\) is decreasing in \(f_i\) - hence is decreasing in \(a_i\).

Concavity \(\pi_i\) in \(a_i\) implies sanctification of both the second order condition and the equilibrium stability condition to the pricing game (15).

**Details on Proof of Proposition 6**

Recall the stability condition (18):
\[
\frac{da_i}{da_{-i}} = -\frac{\gamma' \left( \tilde{R} \frac{\partial N}{\partial f_i} + \gamma' \tilde{R} \frac{\partial^2 N}{\partial f_i \partial f_{-i}} \right)}{\tilde{R}'' N_i + 2\gamma' \tilde{R} \frac{\partial N}{\partial f_i} + \gamma'' \tilde{R} \frac{\partial N}{\partial f_i} + (\gamma')^2 \tilde{R} \frac{\partial^2 N}{\partial f_i^2}} < 1,
\]
where the denominator is positive by the second-order condition. After rearranging, we have

\[- \left( \tilde{R}'' N_i + 2\gamma' \tilde{R} \frac{\partial N}{\partial f_i} + \gamma'' \tilde{R} \frac{\partial^2 N}{\partial f_i^2} + \left( \gamma' \right)^2 \tilde{R} \frac{\partial^2 N}{\partial f_i^2} \right) - \gamma' \tilde{R} \frac{\partial N}{\partial f_{-i}} - \left( \gamma' \right)^2 \tilde{R} \frac{\partial^2 N}{\partial f_{-i} \partial f_{-i}} > 0.\]

Dividing by \( \tilde{R}' \) and substituting in the first-order condition, we have:

\[- \left( \frac{\tilde{R}''}{\tilde{R}'} N_i + 2\gamma' \frac{\partial N}{\partial f_i} + \gamma'' \frac{\tilde{R}}{\tilde{R}'} \frac{\partial N}{\partial f_i} + \left( \gamma' \right)^2 \tilde{R} \frac{\partial^2 N}{\partial f_i^2} \right) - \gamma' \frac{\partial N}{\partial f_{-i}} - \left( \gamma' \right)^2 \tilde{R} \frac{\partial^2 N}{\partial f_{-i} \partial f_{-i}} > 0.\]

Lastly, multiplying by \(- (\gamma')^{-1} f^*\), we have

\[
\frac{1}{\gamma'} \frac{\tilde{R}''}{\tilde{R}'} f^* + \frac{\partial N}{\partial f_i} f^* - \gamma'' \frac{\tilde{R}}{\tilde{R}'} \frac{\partial N}{\partial f_i} f^* - \frac{f^*}{\partial N/\partial f_i \partial f_i} \frac{\partial^2 N}{\partial f_i^2} < 0.
\]

Or in elasticity terms, this is (19):

\[
\epsilon_{dd} + \epsilon_{DD} - \epsilon_{m} + \epsilon_{dd} \left( \frac{\gamma'' \tilde{R}}{\gamma'} \frac{\tilde{R}'}{(\tilde{R}')^2} - \tilde{R}'' \tilde{R}' - \epsilon_{dd} \right) < 0.
\]

We apply the implicit function theorem to first-order condition (17), so

\[
\frac{da}{d\tau} = \frac{\gamma' \tilde{R}'R - \tilde{R} R'}{(\tilde{R}')^2} \left( \frac{\partial N}{\partial a^*} \right) + \gamma' \left( 1 + \frac{\gamma'' \tilde{R}' \tilde{R}'' - \epsilon_{dd} \tilde{R}'}{(R')^2} \right).
\]

In elasticity terms, this becomes

\[
\frac{da}{d\tau} = \frac{\epsilon_{dd} \left( \frac{\tilde{R}'R - \tilde{R} R'}{(\tilde{R}')^2} \right)}{\epsilon_{DD} - \epsilon_{m} + \epsilon_{dd} \left( 1 + \frac{\gamma'' \tilde{R}'}{(\tilde{R}')^2} \right)} < 0;
\]

where the denominator is negative by (19). To see \( \tilde{R}'R - \tilde{R} R' < 0 \): first note that \( R/a_i = p_i^a > p_i^a + a p_i^a = R' \); i.e., average revenue is greater than marginal revenue. It follows that \(-c_a R/a_i > -c_a R\), and \( R'(1 - \tau)R - c_a R/a_i > R(1 - \tau)R' - c_a R \). After rearranging, the later inequality is just \( \tilde{R} R' > \tilde{R} R \), or \( \tilde{R} R' - \tilde{R} R < 0 \).

**Results for Monopoly with Pure Ad-Finance**

In this section, we show results for two-sided monopoly with pure ad-finance, parallel to those in Section 5 of the main text.

The (two-sided) monopolist’s the first order condition is just (16) without the subscript
i. Using similar notation, \( \tilde{R} \equiv (1 - \tau)R(a) - c_a a \) and \( \tilde{R}' \equiv (1 - \tau)R'(a) - c_a \) with \( \tilde{R} > 0 \) and \( \tilde{R}' > 0 \) at the optimal ad level \( a^* \). The second order condition is \( \phi = (1 - \tau)R''N + 2\gamma\tilde{R}'N' + \gamma^2 \tilde{R}N'' < 0 \).

For the tax effect on ad level, we have

\[
\frac{da^*}{d\tau} = \frac{R'N + \gamma RN'}{\phi} < 0, \tag{25}
\]

To see that the numerator is positive, recall \( \tilde{R}R' > \tilde{R}'R \) (shown above). Rearranging and using the first order condition (25), we have \( \frac{R'}{R} > \frac{\tilde{R}'}{R} = -\gamma \frac{N'}{N} \), or \( R'N + \gamma RN' > 0 \).

Automatically, we see that, with a higher tax on ads, the advertiser price of ad increases (i.e., \( \frac{dp^a}{d\tau} > 0 \)), the consumer full price decreases (i.e., \( \frac{df}{d\tau} = \gamma \frac{da^*}{d\tau} < 0 \)), and the subscription level increases (i.e., \( \frac{dN}{d\tau} = N' \frac{df}{d\tau} > 0 \)).

For the effect on monopoly profit, simply by the envelope theorem,

\[
\frac{d\pi^*}{d\tau} = \frac{\partial \pi}{\partial \tau} = -RN < 0.
\]

**Details on Proof of Proposition 7**

We want to show that \( dAS/d\tau < 0 \) when \( p^a(a) \) is log-concave. The proof is based on a similar result in Anderson and Peitz (2015). For convenience, we suppress the subscript unless necessary. First observe

\[
\frac{dAS}{d\tau} = N \frac{dAS}{da} \frac{da^*}{d\tau} + \frac{dN}{d\tau} AS = \left( N \frac{dAS}{da} + \gamma \frac{\partial N}{\partial f_i} AS \right) \frac{da^*}{d\tau}. \tag{26}
\]

Since \( da^*/d\tau < 0 \) by Proposition 6, it suffices to show the term in parenthesis is positive.

Recall that \( d\bar{A}S/da = -ap^a' \), where \( p^a' \equiv dp^a/da \). Substituting in first-order condition (23), we need to show \( -\frac{R'}{R} ap^a' - \bar{A}S > 0 \).

Since \( \frac{R'}{R} > \frac{\tilde{R}'}{R} \), it suffices to show \( -ap^a' R - ASR' > 0 \). Using definition of \( \bar{A}S \), it becomes \( R \int_0^a p^a(x)dx - p^a R < 0 \).

By definition, log-concavity of \( p^a \) means \( p^a'/p^a \) decreasing in \( a \), which implies \( R'/p^a \) decreasing in \( a \). Thus, we have that, for any \( a, \frac{R'}{R} \int_0^a p^a(x)dx < \int_0^a \frac{R'(x)}{p^a(x)} p^a(x)dx = \int_0^a R'(x)dx = R \). Therefore, in particular, the desired condition holds at the equilibrium ad level, \( a^* \).

Noticeably, Lemma 1 in Anderson and Peitz (2015) is to show that the ratio of advertiser surplus to platform profit, \( AS(a)/\pi(a) \), is (weakly) increasing in \( a \). In context of this paper, given the ratio increasing in \( a \), advertiser surplus decreases with the tax if and only if the tax reduces platform profit. Here, we show directly that the tax reduces advertiser surplus. Nonetheless, these two approaches lead to the exactly same condition to show.
Figure 1: The Tax Effects in Two-sided Monopoly