Advertising and Price Adjustment for Dropout Consumers

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Abstract

We analyze advertising and price adjustment for consumers who are imperfectly informed about prices and for whom participation in the market is costly. When seller costs are persistent a high price may induce consumers to abstain in the next period, causing a loss of both sales and the ability to demonstrate that prices have fallen. Consumers may then continue to remain pessimistic and abstain thereafter. In equilibrium this tendency of consumers to “drop out” leads to two dynamics: (i) sticky upward price adjustment, with prices more responsive to persistent rather than transitory cost shocks, and (ii) non-uniform advertising dynamics known as “pulse advertising”.

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1 Introduction

When consumers are imperfectly informed about prices or product characteristics, they can become better informed through costly search (e.g., “visiting” firms physically or virtually online). Rational consumers will invest in costly search only if they expect to find affordable prices and/or products which match their taste. Thus consumers who perceive that prices are unaffordably high may not search and abstain from the market, and this harms a seller in two ways. First, when consumers currently do not visit the seller cannot make sales. But in addition, when consumers do not visit the seller cannot demonstrate that prices have dropped, and as a result in the next period consumers may remain pessimistic and continue to abstain. By this mechanism consumers may “drop out” for multiple periods, and perhaps indefinitely, and in this paper we analyze the pricing and advertising dynamics that arise when the seller attempts mitigate this dropout behavior.

We consider a discrete time setting with a single infinitely-lived seller and cohorts of identical buyers, each cohort active for exactly one period. The seller’s cost is uncertain, either high or low, and persistent (though not perfectly) over time. In any given period $t$ the seller observes her current cost $c_t$ and future cost $c_{t+1}$, sets price $p_t$, and decides whether to expend cost $A$ to advertise this price. Consumers in period $t$ observe $p_t$ only if it is advertised, else they form beliefs about $p_t$ based on the history of previous prices, with previous prices observed only if they were advertised or if consumers searched. Based on beliefs consumers then decide whether to spend search cost $\phi$ to visit the seller or to abstain.

To frame the discussion we start with the case in which the seller does not advertise and engages in myopic pricing, setting the stage-optimal prices $p_{\ell}$ and $p_h$ which correspond to the current cost. When in the previous period consumers observe $p_{\ell}$, they infer that yesterday’s cost was low and, because costs are persistent, believe that today’s cost (and price) is also likely to be low and search. However, when in the previous period consumers observe $p_h$, they infer that yesterday’s cost was high, which implies today’s cost and price are likely high, and therefore consumers abstain. Furthermore, due to the consumers abstaining in period $t$, consumers in period $t + 1$ get no new information, observing only that two periods ago the cost was high. Because costs are persistent these consumers are still pessimistic, although not as much as their predecessors, and if the search cost is high enough still abstain. As the period of abstention grows consumers continue to become less pessimistic, the belief that today’s cost is high falling toward one half in the limit. The period of abstention lasts until the belief that today’s cost is high falls below a threshold, and if this threshold is smaller than one half consumers never return.

The seller can help alleviate the effects of dropout consumers by acting strategically rather than myopically. First, the seller can use prices to manage consumers’ beliefs rather than simply maximizing stage payoffs, and we find that in equilibrium the seller’s pricing strategy
is upward sticky. In particular, when consumers are active and the seller’s cost increases to $c_h$, she sets the myopic price $p_h$ only if the next period’s cost is also high. Else if the shock is transitory and tomorrow’s cost is low she sets a lower signaling price $\tilde{p} < p_h$, and next period’s consumers remain active. For the price $\tilde{p}$ to be a credible signal, it must be that the seller is unwilling to set this price when her cost shock is permanent, and we demonstrate that this price must exist because in equilibrium the continuation value of search is higher when the future cost is lower. Thus we find that the threat of dropout consumers results in sticky prices.$^1$

But sticky prices do not fully mitigate dropout, because the seller still induces consumers to drop out when both the current and future cost is high. Given consumers have become inactive, the seller the seller can spend cost $A$ to advertise a low price to get consumers to return, which has the direct benefit of makings sales today and the additional benefit of the option to maintain search in the future. For low values of $A$ the strategy is simple – advertise whenever the current production cost is low – but for intermediate values of $A$ advertising is intermittent, with the seller only spending $A$ when both today’s and tomorrow’s cost is low. Similar to the argument for sticky prices, the continuation value of search decreases in next period’s cost, and therefore there is a range of values of $A$ for which a seller with a persistently low cost is willing to advertise and a seller with a temporarily low cost is not. In this way, we find that that the threat of dropout consumers can lead to “pulse advertising”, that is large infrequent advertising expenditures rather than a consistent level.$^2$

Models in which consumers are informed by both search and advertising include Butters (1977), Anderson and Renault (2006), Robert and Stahl (1993), Janssen and Non (2008, 2009). Those models are static and can not address the dynamic issues on which we focus. Anderson and Renault (2006) in particular focuses on price advertising as way for a seller to commit to provide buyers with enough surplus to overcome the search cost. In our model, we focus on parameters in which when the cost is high there is no profitable price to which the seller can commit to induce consumers to visit. Thus, the role of advertising is not to resolve the commitment problem but rather to signal future returns to search.$^3$ In addition, Tapata (2006), Yang and Ye (2006), Cabral and Fishman (2013) and Lewis (2011) present dynamic search models of sticky price adjustment but without advertising and do not consider the dynamic interaction between advertising and price adjustment which we address.

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$^2$This pattern of advertising is known in the marketing literature and is conventionally attributed to S-shaped demand or sales response to advertising. (e.g. Sasieni, 1971; Simon, 1982; Mahajan and Muller, 1986, Feinberg, 1992).

$^3$In fact, all of our results obtain if the model were changed so that advertising is money burning rather than informative.
In the rest of the paper, we describe the model in Section 2, establish the dropout dynamics when the seller is myopic in Section 3, describe the equilibrium with sticky prices and pulse advertising in Section 1, and conclude thereafter.

2 Model

A monopoly sells a product in discrete periods over an infinite horizon. The seller’s production cost at period $t$ is denoted by $c_t$ and takes one of two possible values: a low cost $c_\ell$ or a high cost $c_h$. Costs evolve over time so that $Pr(c_{t+1} = c_t) = \eta$, with $\eta \in [\frac{1}{2}, 1]$ capturing the degree of persistence. In the beginning of every period the seller observes both the current cost $c_t$ and the future cost $c_{t+1}$, sets price $p_t$, and chooses whether to advertise this price (denoted by $a_t = 1$) at cost $A$ or to leave the price unadvertised (denoted by $a_t = 0$). His period $t$ profit is the markup $p_t - c_t$ times the number of consumers that purchase, and future profits are discounted at rate $\delta \in (0, 1)$ per period.

In each period a unit mass of consumers arrives, interacts with the seller, leaves, and is replaced by an identical group the following period. Each consumer $i$ has unit demand with an uncertain value $\varepsilon_i$ drawn from distribution $F$. A consumer may visit the seller (denoted by $\lambda_t = 1$) at cost $\phi$, where she observes both her value and the price and decides whether to purchase, or can abstain from search (denoted by $\lambda_t = 0$) and receive a payoff of zero.

If the seller advertises then consumers observe the current period price $p_t$ prior to the search decision (but must still expend the search cost to visit). Else consumers must form beliefs about $p_t$, and for this they rely on the game’s history. We assume that period $t$ consumers observe the price and cost from every previous period $\tau < t$ in which either the seller advertised or searchers visited. Otherwise period $t$ consumers do not directly observe anything about period $\tau$. One rationale for this formulation is that consumers share the information available to them. Alternatively, if the model is re-interpreted as that of a single cohort of infinitely lived consumers, then the assumption is simply that each searcher remembers her own history. That consumers can observe contemporaneous costs captures the idea that some information about costs beyond the price can be included in advertising and ascertained through visiting the seller, and the fact that these costs are observed perfectly is a simplification that allows us to tractably express consumer beliefs. Note also that even when a consumer visits and observes $c_t$, the future cost $c_{t+1}$ is still known only to the seller, which allows for the possibility of current price $p_t$ signaling to the future cohort.

\footnote{It is important that the seller has some information about future costs when setting the price; that he knows the next period’s cost perfectly is an assumption to simplify calculations.}

\footnote{An equivalent formulation is that consumers arrive only in the beginning and are infinitely lived, however this introduces the possibility of repeated games equilibria, from which we wish to abstract.}
When consumers search the seller’s expected stage profit is \((p - c)(1 - F(p))\) and we assume this expression is concave,\(^6\) with maximizers \(p_\ell \equiv p(c_\ell)\) and \(p_h \equiv p(c_h)\) and corresponding maximized profits \(\pi_\ell\) and \(\pi_h\). We also denote by \(s(p) \equiv \int_{\varepsilon \geq p} (\varepsilon - p) dF(\varepsilon)\) the consumer’s expected value from searching when the price is \(p\). We impose that \(\varphi \leq s(p_\ell)\) to ensure the market does not collapse.

An equilibrium is an advertising and pricing strategy for the firm and a search strategy for consumers at each history, such that the consumers’ search strategy maximizes their expected utility given the firm’s strategy, and the firm’s strategy maximizes its discounted expected profit given the consumers’ search strategy.

We first establish the phenomenon of dropout consumers by studying their search strategy in a simple setting in which the seller myopically sets prices that match the current cost, showing that once consumers leave they may stay away for multiple periods. We then study equilibria in which the seller strategically sets prices and advertises to deal with dropouts.

### 3 Myopic Pricing and Dropout Consumers

A driving force behind our analysis is the tendency of consumers to drop out of the market, and to see how this occurs we first study the consumer search decision in a simpler environment. In particular, suppose that sellers never advertise and price myopically, with \(p(c_\ell) = p_\ell\) and \(p(c_h) = c_h\). Letting \(\mu\) denote the belief that the current cost is low, consumers search only if the expected benefit \(\mu s(p_\ell) + (1 - \mu) s(p_h)\) outweighs the cost \(\varphi\), implying a threshold belief \(\bar{\mu} = \frac{\varphi - s(p_h)}{s(p_\ell) - s(p_h)}\). Whether the current belief \(\mu\) exceeds \(\bar{\mu}\) depends on the history, and we now show that when acting optimally consumers go through prolonged spells of abstention.

**Lemma 1** With myopic prices \((p(c_\ell) = p_\ell, p(c_h) = c_h)\) and no advertising, when consumers observe \(p_\ell\) they search in the next period and when consumers observe \(p_h\) they abstain for the next \(T\) periods, where

\[
T = \begin{cases} 
\left\lfloor \frac{\ln(1 - 2\bar{\mu})}{\ln(2\eta - 1)} \right\rfloor & \text{if } \bar{\mu} < \frac{1}{2} \\
\infty & \text{if } \bar{\mu} \geq \frac{1}{2}.
\end{cases}
\]

(1)

From the seller’s perspective, the issue with setting price \(p_h\) in the current period is not only that it precludes her from selling in the following period, but also because consumers stop paying attention she cannot persuade them to come in subsequent periods, even when it’s worthwhile for them to do so. To keep consumers active, the seller may wish to keep her price low especially if the current cost increase is transitory, in which case we would observe price stickiness as a response to the dropout threat. Furthermore, she may raise the price

\(^6\)A standard sufficient condition is a log-concave density \(f\).
as costs rise, but then use advertising to entice consumers to search again. In this case we may observe infrequent large advertising expenditures followed by no advertising at all, a phenomenon known in the marketing literature as pulse advertising. We now derive these two equilibrium behaviors formally.

4 Strategic Pricing

We now describe an equilibrium in which the seller mitigates the tendency for consumers to drop out through strategic pricing and advertising. We say prices are upward sticky if when the cost changes from $c_\ell$ to $c_h$ the seller’s price changes from $p_\ell$ to some $\tilde{p} < p_h$ with positive probability, i.e. that the price does not necessarily fully respond to a cost increase. Similarly, we say there is pulse advertising if a seller with a low cost facing inactive consumers does not necessarily advertise, and that there is frequent advertising if in this situation she always advertises. Thus with frequent advertising the seller fully responds to a cost drop, and with pulse advertising she does not.

Proposition 1 There exist values $A$ and $\bar{A}$, with $\pi_\ell \leq A < \bar{A}$, so that in equilibrium prices are upward sticky and there is frequent advertising when $A \in (0, A)$ and pulse advertising when $A \in (A, \bar{A}]$.

We show that prices are upwardly sticky because maintaining search tomorrow is worth more to a seller with a temporarily high cost than a seller with a persistently high cost. To this end, when consumers are actively searching and the seller’s cost increases to $c_h$, the price fully increases to $p_h$ only if the cost shock is persistent, else it increases partially to an intermediate level $\tilde{p} \in (p_\ell, p_h)$. The price $\tilde{p}$ is calibrated so that a seller with a persistent cost shock is indifferent between the stage loss from setting this sub-optimally low price and the increased continuation value from maintaining search. The seller with a temporary shock then strictly prefers to set $\tilde{p}$, and by the standard signaling logic a separating equilibrium exists.

As long as $A > 0$, the continuation value of search is strictly higher when the cost increase is temporary than when it is persistent, and therefore prices are upwardly sticky for the entire range of advertising costs that we consider. On the other hand, whether advertising is sticky, that is whether the seller advertises whenever her cost is low or only if her cost is persistently low, depends on the magnitude of the advertising cost. When consumers are inactive and the seller’s cost is low, advertising is useful both because it enables current profit and provides the continuation value of turning on search thereafter. Active search in the following period is valuable even if the seller knows her cost will be high, both because the stage profit $\pi_h$ is potentially positive and also because the seller has the option to maintain active search for the following period. When the advertising cost $A$ is low then it is worthwhile to advertise even when the low production cost is temporary, however once $A$ is sufficiently high ($A \geq A$)
advertising is worthwhile only if the low production cost is persistent. Not surprisingly, once the advertising cost is high enough \((A \geq \bar{A})\) advertising is never worthwhile.

The formal proof of the proposition is included in the appendix. To ensure that the seller acts optimally we compute the value function for each relevant cost-history state and then check for profitable deviations. Although the exercise is quite involved due to the number of states, the key is a single calculation that demonstrates that the marginal value of search is higher when tomorrow’s cost is low than when it is high. We also then verify that conditional on the beliefs generated by the seller’s strategy the consumer acts optimally.

5 Conclusion

This paper explores the dynamics of advertising and price adjustment when it is costly for consumers to participate in the market. Consumers draw inference from previous prices and, if they become sufficiently pessimistic about the current period price, abstain from visiting. By abstaining consumers not only forego purchasing in the current period, but importantly also remove the seller’s ability to demonstrate that prices have dropped. Consequently, without new information consumers remain pessimistic in the period after the initial abstention, and may remain absent from the market for an extended spell.

We find that the threat of consumer dropout generates two strategic responses from the seller, sticky pricing and pulse advertising, that are often observed empirically. Sticky prices arise when the seller with a temporarily high cost only partially raises her price in order to signal that future prices will be low and thereby induce consumers to remain active. Pulse advertising, that is large infrequent advertising expenditures, arises when consumers are inactive and the seller advertises only when her cost drop is persistent. Both phenomena, sticky prices and pulse advertising, are supported in equilibrium by the fact that the continuation value of keeping consumers active in the next period decreases in next period’s cost.

Bibliography


Appendix

Proof of Lemma 1

Let $\mu_t$ be the public belief that the seller’s cost is low in period $t$, and observe that

$$\mu_t = \mu_{t-1} \eta + (1 - \mu_{t-1})(1 - \eta).$$

(2)

In other words, today’s cost is low either if yesterday’s cost was low and it persisted, or yesterday’s cost was high and it switched. If the period $t - 1$ price is observed to be $p_t$, then $\mu_{t-1} = 1$ and equation (2) implies that $\mu_t = \eta$, which by assumption exceeds $\bar{\mu}$ and induces consumers to search. Else, if the period $t - 1$ price is $p_h$ then $\mu_{t-1} = 0$, therefore $\mu_t = 1 - \eta < \bar{\mu}$ and consumers do not search. Receiving no new price information in period $t$, the consumers’ belief in the following period $t + 1$ is

$$\mu_{t+1} = \eta \mu_t + (1 - \eta)(1 - \mu_t) = 2\eta(1 - \eta) > 1 - \eta.$$

After abstaining consumers are more optimistic in period $t + 1$ than in period $t$, and if $\bar{\mu} \in (\mu_t, \mu_{t+1}) = (1 - \eta, 2\eta(1 - \eta))$ consumers resume search in period $t + 1$. Else they abstain again, and in period $t + 2$ become more optimistic once again and reassess the search decision. We can describe beliefs after any number of periods of abstention $\tau$ by solving the difference equation (2), yielding

$$\mu(\tau) = \frac{1}{2} (1 - (2\eta - 1)^\tau).$$

(3)

It can seen that $\mu(1) = 1 - \eta$, that $\mu(\cdot)$ is strictly increasing, and that $\lim_{\tau \to \infty} \mu(\tau) = \frac{1}{2}$. Therefore, if $\bar{\mu} \in (1 - \eta, \frac{1}{2})$ then there exists a smallest integer $T$ for which $\mu(T) \geq \bar{\mu}$ and consumers resume search, or if $\bar{\mu} \geq \frac{1}{2}$ no such $T$ exists and consumers abstain forever.

Proof of Proposition 1

First to check whether the seller acts optimally we compute her value in each state (to be defined shortly) conditional on the proposed strategy, and then verify that in each state her action is indeed optimal. Then we construct consumer beliefs and verify that the consumers’ proposed strategy is also optimal.

In equilibrium the seller conditions the current price on the state $(c, c', \lambda)$: the current cost $c$, the next period cost $c'$, and whether consumers are actively searching $(\lambda = 1)$ or not $(\lambda = 0)$.\footnote{In principle the seller can condition the price on the entire history, however we only focus on Markov-style equilibria in which the seller conditions actions only on payoff- or signal-relevant information.} Let $V_{c,c',\lambda}$ denote the seller’s value in each state, and let $V_{c,\lambda} \equiv E_{c'}|c [V_{c,c',\lambda}]$ denote...
the expected value given the current cost is \( c \) and the future cost \( c' \) is not yet known. The following table describes the ax-ante value \( V_{c,\lambda} \) for the proposed equilibria in the \((0, A)\) and \((A, \bar{A})\) parameter ranges:

<table>
<thead>
<tr>
<th>Value</th>
<th>( A \in (0, A) )</th>
<th>Sticky p, Pulse a</th>
<th>( A \in (A, \bar{A}) )</th>
<th>Sticky p, Frequent a</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\ell,1} )</td>
<td>( \pi_{\ell} + \delta(\eta V_{\ell,1} + (1 - \eta)V_{h,1}) )</td>
<td>( \pi_{\ell} + \delta(\eta V_{\ell,1} + (1 - \eta)V_{h,1}) )</td>
<td>( \pi_{\ell} + \delta(\eta V_{\ell,1} + (1 - \eta)V_{h,1}) )</td>
<td></td>
</tr>
<tr>
<td>( V_{h,1} )</td>
<td>( \eta(\pi_h + \delta V_{h,0}) + (1 - \eta)(\bar{\pi} + \delta V_{\ell,1}) )</td>
<td>( \eta(\pi_h + \delta V_{h,0}) + (1 - \eta)(\bar{\pi} + \delta V_{\ell,1}) )</td>
<td>( \eta(\pi_h + \delta V_{h,0}) + (1 - \eta)(\bar{\pi} + \delta V_{\ell,1}) )</td>
<td></td>
</tr>
<tr>
<td>( V_{\ell,0} )</td>
<td>( \pi_{\ell} - A + \delta(\eta V_{\ell,1} + (1 - \eta)V_{h,1}) )</td>
<td>( \eta(\pi_{\ell} - A + \delta V_{\ell,1}) + (1 - \eta)\delta V_{h,0} )</td>
<td>( \eta(\pi_{\ell} - A + \delta V_{\ell,1}) + (1 - \eta)\delta V_{h,0} )</td>
<td></td>
</tr>
<tr>
<td>( V_{h,0} )</td>
<td>( \delta(\eta V_{h,0} + (1 - \eta)V_{\ell,0}) )</td>
<td>( \delta(\eta V_{h,0} + (1 - \eta)V_{\ell,0}) )</td>
<td>( \delta(\eta V_{h,0} + (1 - \eta)V_{\ell,0}) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Value function for low and intermediate advertising costs.

In the first line in which the production cost is low and consumers are active, regardless of the advertising cost the seller makes \( \pi_{\ell} \) in the current period and maintains search in the following period, keeping the low cost with probability \( \eta \) and moving to the high cost with probability \( 1 - \eta \). In the second line in which the current cost is high and consumers are active, with probability \( \eta \) the seller’s future cost is also high, in which case she collects \( \pi_h \) today and faces a continuation value \( V_{h,0} \) with a high cost and no search. If instead with probability \( 1 - \eta \) her future cost is low then the seller signals this, obtaining a signaling profit \( \bar{\pi} \) today and a continuation value \( V_{\ell,1} \) with a low cost and active search tomorrow. In the third line, consumers are inactive and the seller’s current cost is low. In the first column in which the advertising cost is low, the seller advertises and obtains \( \pi_{\ell} - A \) regardless of future cost, and her continuation values are \( V_{\ell,1} \) with probability \( \eta \) and \( V_{h,1} \) with probability \( 1 - \eta \). In the second column in which the advertising cost is high the seller advertises only if her future production cost is low. Thus with probability \( \eta \) she advertises, obtains \( \pi_{\ell} - A \) today and a continuation value \( V_{\ell,1} \), and with probability \( 1 - \eta \) she does not advertise, receives zero today and a continuation value \( V_{h,0} \). Finally, in the fourth line in which the seller’s cost is high and consumers are inactive, she abstains regardless of the value of \( A \) or tomorrow’s cost, receives zero payoff in the current stage and a continuation of \( V_{h,0} \) with probability \( \eta \) and \( V_{\ell,0} \) with probability \( 1 - \eta \).

Recall that the signaling stage payoff \( \bar{\pi} \) is set so that when consumers are active (\( \lambda = 1 \)) a seller with persistently high costs is indifferent between \( \bar{\pi} \) which maintains future search or the maximized stage payoff which induces consumers to drop out, that is \( \bar{\pi} + \delta V_{h,1} = \pi_h + \delta V_{h,0} \), and plugging this into the expression for \( V_{h,1} \) and simplifying obtains

\[
V_{h,1} = \pi_h + \delta V_{h,0} + \delta(1 - \eta)(V_{\ell,1} - V_{h,1}).
\]

This reduces Table 1 into a system of four equations with four unknowns, which admits a unique closed form solution. From this we derive the seller’s value of turning on search for next period.
Claim 1 Define $w_c \equiv V_{c,1} - V_{c,0}$ as the seller’s marginal value of turning on search when the cost is $c$. Then $0 < w_h < w_\ell$ for all $A \in (0, \bar{A})$.

Proof of Claim Solving this system of equations in Table 1 and simplifying yields the following expressions for the marginal values of search:

<table>
<thead>
<tr>
<th>Value</th>
<th>$A \in (0, A)$</th>
<th>Sticky $p$, Pulse $a$</th>
<th>$A \in (A, \bar{A})$</th>
<th>Sticky $p$, Frequent $a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_\ell$</td>
<td>$A$</td>
<td>$\frac{\pi_h + \delta(1-\eta)(\pi_\ell + \eta(A-\pi_\ell))}{1 + \delta(1-\eta)(1-\delta(1-\eta))}$</td>
<td>$\frac{\eta(A-\pi_\ell) + \pi_\ell - (1-\delta(1-\eta))\pi_h}{1 + \delta(1-\eta)(1-\delta(1-\eta))}$</td>
<td></td>
</tr>
<tr>
<td>$w_h$</td>
<td>$\frac{\pi_h + \delta(1-\eta)A}{1 + \delta(1-\eta)}$</td>
<td>$\frac{\pi_h + \delta(1-\eta)(\pi_\ell + \eta(A-\pi_\ell))}{1 + \delta(1-\eta)(1-\delta(1-\eta))}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The marginal value of search for low and high production costs and low and intermediate advertising costs.

Inspection of these expressions (in particular using the fact that $\pi_\ell < A$) yields that $0 < w_h < w_\ell$ for all $A \in (0, \bar{A})$.

Having established the preceding claim, we return to check that in every one of the eight $(c, c', \lambda)$ states the seller best responds. To check the seller’s payoff from deviations, we must define consumers’ beliefs off the equilibrium path, and for simplicity we impose that for any off-the-path action consumers believe with certainty that the following period the seller is inactive.\(^8\)

Equilibrium with low advertising costs $A \in (0, \bar{A})$

We start with the equilibrium in the range $A \in (0, \bar{A})$ in which advertising costs are low, the following table summarizes equilibrium and best deviation payoffs for each state: First, in states $(\ell, \ell, 1)$ and $(\ell, h, 1)$ the seller obtains the maximal stage payoff and maintains search in the next period, therefore there is no plausible deviation that can increase the payoff. In states $(h, \ell, 1)$ and $(h, h, 1)$, consumers are active and the seller’s best options are either set the stage-optimal price $p_h$ which turns off search in the next period or the signaling price $\tilde{p}$ which keeps search active in the next period. Because we have established in Claim 1 that $0 < w_h < w_\ell$, there must exist a $\tilde{\pi}$ so that $\delta w_h = \pi_h - \tilde{\pi} < \delta w_\ell$, and therefore the price $\tilde{p}$ which induces the stage profit $\delta \pi$ satisfies the incentive constraints for both $(h, \ell, 1)$ and $(h, h, 1)$.

Next, in states $(\ell, \ell, 0)$ and $(\ell, h, 0)$ in which consumers are inactive but the current cost is low, the seller advertises, obtains a stage profit $\pi_\ell - A$ and turns on search in the next period. Her best alternative is not to advertise, receive zero profit, and enter the following period with inactive consumers. In the $(\ell, h, 0)$ case in particular, the proposed equilibrium

\(^8\)The seller is inactive when she expects no consumers to visit, this is shorthand for there being a small fixed cost to operate. Alternatively, we can impose that in these cases the seller sets a very high price.
action is more profitable whenever $A - \pi_\ell - w_\ell \leq 0$. From the expression for $w_\ell$ derived in the proof of Claim 1, it follows that $A - w_\ell$ increases in $A$ at a constant rate and is negative at $A = 0$. Therefore, there exists a value $A$ such that $A - \pi_\ell - w_\ell < 0$ if and only if $A \in [0, A)$, and for all $A$ in this range the incentive constraint for $(\ell, h, 0)$ is satisfied. Also, because $w_\ell > w_h$, the incentive constraint for $(\ell, \ell, 0)$ must also be satisfied.

Finally, in states $(h, \ell, 0)$ and $(h, h, 0)$ in which consumers are inactive and the current cost is high, the seller also remains inactive in equilibrium but does have the option instead to advertise and set some price $\hat{p}$ that is low enough to encourage consumers to visit this period. By assumption, the search cost $\varphi$ is large enough so that if $\hat{p}$ is the highest price at which consumers choose to visit (i.e., $s(\hat{p}) = \varphi$) then $\hat{p} < c_h$ and consequently the resulting profit $\hat{p} < 0$. But then $A - \hat{\pi} < 0$ and the incentive constraint in both states is met. Intuitively, if the only benefit of advertising today is to turn on search tomorrow, then it is more profitable for the seller to abstain today and advertise tomorrow instead.

We have thus checked that the seller’s strategy is optimal in every state, and now we verify that consumer behavior is also optimal. First note that when the seller advertises a price the consumer is certain this is the price she will see, and it is optimal for her to visit whenever $p \leq \hat{p}$ (as defined in the preceding paragraph). Otherwise without advertising consumers form beliefs about the price they will see when visiting. Starting with the equilibrium path, Table 6 summarizes the consumers’ beliefs for every previous period history in which today’s price is unadvertised. In the first two lines, if either through search or advertising the previously observed price was $p_\ell$, then yesterday’s cost must have been $c_\ell$ and therefore today’s cost is $c_\ell$ with probability $\eta$, in which case today’s price is $p_\ell$. Otherwise with probability $1 - \eta$ today’s cost is $c_h$, and then with probability $\eta$ tomorrow’s cost is also $c_h$ and today’s price is $p_h$, or tomorrow’s cost reverts to $c_\ell$ with probability $1 - \eta$ and then today’s price is $\hat{p}$. Putting these cases together, the consumer expects search benefit of $s(p_\ell)$ with probability $\eta$, $s(\hat{p})$ with probability $(1 - \eta)^2$, and $s(p_h)$ with probability $(1 - \eta)\eta$. In the third line, having observed the signaling price $\hat{p}$ in the previous period the consumer expects with certainty that today’s cost is low, and in turn that today’s price is $p_\ell$. In the fourth

<table>
<thead>
<tr>
<th>State $[(c, c', \lambda)]$</th>
<th>Equilibrium Action $(p, a)$</th>
<th>Equilibrium Equilibrium Deviation Payoff $(V_{c,c',\lambda})$</th>
<th>Deviation Payoff $(p, a)$</th>
<th>Deviation Payoff $\pi - \hat{\pi} \leq \delta w_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\ell, \ell, 1)$</td>
<td>$(p_\ell, 0)$</td>
<td>$\pi_\ell + \delta V_{\ell,1}$</td>
<td>$\pi_\ell + \delta V_{\ell,1}$</td>
<td>$\pi_\ell + \delta V_{\ell,0}$</td>
</tr>
<tr>
<td>$(\ell, h, 1)$</td>
<td>$(p_\ell, 0)$</td>
<td>$\pi_\ell + \delta V_{h,1}$</td>
<td>$\pi_\ell + \delta V_{h,1}$</td>
<td>$\pi_\ell - \pi_\ell \leq \delta w_\ell$</td>
</tr>
<tr>
<td>$(h, \ell, 1)$</td>
<td>$(\hat{p}, 0)$</td>
<td>$\hat{p} + \delta V_{\ell,1}$</td>
<td>$(p_n, 0)$</td>
<td>$\delta V_{\ell,0}$</td>
</tr>
<tr>
<td>$(h, h, 1)$</td>
<td>$(p_h, 0)$</td>
<td>$\pi_h + \delta V_{h,0}$</td>
<td>$(p_e, 0)$</td>
<td>$\delta V_{h,0}$</td>
</tr>
<tr>
<td>$(\ell, 0)$</td>
<td>$(p_\ell, 0)$</td>
<td>$\pi_\ell - \delta V_{\ell,1}$</td>
<td>$(p_\ell, 0)$</td>
<td>$\delta V_{h,0}$</td>
</tr>
<tr>
<td>$(h, \ell, 0)$</td>
<td>$(-, 0)$</td>
<td>$\delta V_{\ell,0}$</td>
<td>$(\hat{p}, 1)$</td>
<td>$\pi_\ell - \pi_\ell \leq \delta w_\ell$</td>
</tr>
<tr>
<td>$(h, h, 0)$</td>
<td>$(-, 0)$</td>
<td>$\delta V_{h,0}$</td>
<td>$(\hat{p}, 1)$</td>
<td>$\pi_\ell - \pi_\ell \leq \delta w_\ell$</td>
</tr>
</tbody>
</table>

Table 3: Seller’s incentive constraints when $A \in (0, A)$
and fifth lines, having either not observed a price in the previous period or a price of $p_h$, the consumer knows the seller expects no search in the current period, and thus in absence of advertising the consumer expects the seller to abstain from carrying the product.

Because we assume $0 \leq \varphi \leq \eta s(p_\ell) + (1 - \eta)s(p_h)$, we then conclude that for histories in the top three lines of Table 4 it is optimal for consumers to search, and in final two histories it is optimal for consumers to abstain. Lastly, we impose for all other histories off the equilibrium path that consumers anticipate the product to be unavailable, and therefore abstain without an advertised price.

**Equilibrium with intermediate advertising costs $A \in (A, \overline{A})$**

Much of the logic from the low range of advertising costs applies to the intermediate range, and here we highlight only where the analysis diverges. First, the seller’s incentive constraints are presented in the table below: The incentive constraints are different only for the $(\ell, h, 0)$ state (highlighted), in which the seller advertised when $A$ was low but now abstains. However, the equilibrium values of $w_\ell$ and $w_h$ are also now different than when $A$ is low (see Table 2) and therefore we must again confirm that all incentive constraints hold. As previously, it is immediate that the incentive constraint is met in the first two lines when search is active and costs are low. In lines 3 and 4 when search is active but current costs are high, given

<table>
<thead>
<tr>
<th>History</th>
<th>Probability of Price</th>
<th>Search Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>searched, $p_{t-1} = p_\ell$</td>
<td>$\eta (1 - \eta)^2 (1 - \eta)\eta$</td>
<td>$\eta s(p_\ell) + (1 - \eta)s(p_h) + (1 - \eta)^2(s(p_\ell) - s(p_h))$</td>
</tr>
<tr>
<td>advertised, $p_{t-1} = p_\ell$</td>
<td>$\eta (1 - \eta)^2 (1 - \eta)\eta$</td>
<td>$\eta s(p_\ell) + (1 - \eta)s(p_h) + (1 - \eta)^2(s(p_\ell) - s(p_h))$</td>
</tr>
<tr>
<td>searched, $p_{t-1} = \overline{p}$</td>
<td>1 0 0 0</td>
<td>$s(p_\ell)$</td>
</tr>
<tr>
<td>searched, $p_{t-1} = p_h$</td>
<td>0 0 0 1</td>
<td>0</td>
</tr>
<tr>
<td>$p_{t-1}$ unobserved</td>
<td>0 0 0 1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Beliefs in the frequent advertising equilibrium with no current advertising.
that Claim 1 demonstrates $0 < w_h < w_\ell$, the same argument as in the case of low values of $A$ applies. In lines 5 and 6 we verify the incentive constraints when consumers are inactive and current cost is low. Similar to the argument given previously, the constraint in line 6 holds whenever $A - \pi_\ell - \delta w_h \geq 0$, and using the expression for $w_h$ in Table 2 we see that the left hand side of this inequality increases at a constant rate with $A$ and is negative at $A = 0$. In fact, it can be verified that at the value of $A$ found earlier the expression holds with equality, and therefore also holds for all $A > A$. The constraint in line 5 is met whenever $A - \pi_\ell - \delta w_\ell \leq 0$; because $w_\ell > w_h$ we know this constraint holds at $A = A$, but because the left hand side of the inequality increases in $A$ at a constant rate, there exists a maximal value $\bar{A} > A$ above which this inequality no longer holds and the equilibrium is not supported. Finally, although the expressions for $w_\ell$ and $w_h$ are different from the equilibrium for low values of $A$, exactly the same argument applies to show that the constraints in lines 7 and 8 hold as well.

Given the seller’s strategy that differs slightly from when advertising costs were low, the table below summarizes beliefs for histories when there is no advertising in the current period: The only difference from the preceding case is in the inference in the second line when

<table>
<thead>
<tr>
<th>History</th>
<th>Probability of Price</th>
<th>Search Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_\ell$</td>
<td>$\bar{p}$</td>
</tr>
<tr>
<td>searched, $p_{t-1} = p_\ell$</td>
<td>$\eta$</td>
<td>$(1 - \eta)^2$</td>
</tr>
<tr>
<td>advertised, $p_{t-1} = p_\ell$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>searched, $p_{t-1} = \bar{p}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>searched, $p_{t-1} = p_h$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_{t-1}$ unobserved</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6: Beliefs in the pulse advertising equilibrium with no current advertising.

in the previous period the seller advertised price $p_\ell$ (highlighted). In the earlier analysis in which the advertising cost was low, the previous period’s seller advertised even if her future cost was high, thus today’s consumer was uncertain about the seller’s current cost. Now, the previous period seller advertises only if today’s cost is low for sure, which means today’s consumers expect a low price for sure. Because now the consumer is even more optimistic, it is a best response for the consumer to search in this case whenever it was a best response to search in the previous case. All other conditions for the consumer are identical, therefore her search strategy is still optimal.