Abstract

Production rationalization, the process of re-allocating production across facilities so as to reduce total costs, results in firms equating marginal cost across markets. This results in higher marginal costs, and hence higher prices, in some markets. We study how fleet re-optimization following the US Airways/American Airlines merger increased US Airways’ marginal cost from that of their pre-merger fuel efficient aircraft to that of American Airlines’ pre-merger costly aircraft. The merged firms’ prices increased 10% more on US Airways’ routes than on American Airlines’ routes, and increased 5% more than rival firms’ prices on US Airways’ routes. Price-cost regressions confirm that such price increases were most likely due to fleet re-optimization and not other merger effects.

Keywords: merger effects, cost synergies, American Airlines

JEL Classification: D40, L11, L40, L93
1 Introduction

Firms constantly re-allocate production across facilities and across technologies in an effort to improve profitability, a process known as production rationalization. There are many sources of such efficiencies, from increasing economies of scale at certain facilities to better utilizing more efficient technologies. For example, cement manufacturers need to balance being close to consumers and utilizing fully production facilities. This trade-off induces them to, at times, use two geographically disperse facilities at 50% utilization, and at other times, to use only one of the two facilities at full utilization. Although firms constantly re-optimize production according to changes in supply and demand conditions, these re-allocations tend to be largest after a merger, as the merging parties re-optimize production across the newly merged entity.

The production re-allocation may increase marginal cost despite decreasing average cost, resulting in cost savings and price increases for the re-allocating firm. While profitable for the firm, and possibly for competing firms, this re-allocation may be detrimental to consumers. For example, if a firm with a high constant marginal cost merges with another which has low marginal costs up to certain capacity and high marginal cost thereafter, the resulting marginal cost of the joint-firm may be the high cost of the first firm, despite the second firm operating below capacity prior to the merger. Hence, the joint-firms’ average cost decreased but the marginal cost for operations of the second firm increased. Because of this cost dynamic, regulators should use caution when approving mergers even when such mergers induce no changes in market power and present significant and verifiable cost efficiencies. Firms should also bear caution when suggesting the profitability of such cost efficiencies, as the resulting higher marginal cost results in higher prices and lower revenue, which partially offsets the gains from lower average cost.

In essence, when a firm’s output can be produced across multiple facilities, each with their own dis-economies of scale, firms optimally allocate production to equalize marginal cost across facilities, in effect, operating on a cost curve that averages all facilities’ cost curves. In this paper we develop a simple model of competition that formalizes this intuition and we investigate if such intuition can be applied to the American Airlines - US Airways merger. Specifically, prior to their merger, US Airways utilized a fleet of Airbus A320s for domestic operations, while American Airlines utilized McDonald Douglas MD80s, a more costly aircraft to operate. As US Airways had some slack in the operation of its A320s, the merger allowed the parties to re-optimize fleet utilization, increasing utilization of the A320s and decreasing that of MD80s. However, it is possible that, due to this re-optimization, the marginal cost of serving a US Airways customer increased from that of an A320 to that of an MD80, resulting in higher prices.

In the paper we test if prices of the merging firms increased after the merger, and, more specifically, on routes which prior to the merger were served predominantly by US Airways, as opposed to American Airlines. In effect, we find that prices of US Airways/American Airlines increased 10% more on US Airways’ routes than on American Airlines routes (i.e. routes served predominantly by American Airlines prior to the merger). We also find that the merging parties’ prices increased, on
average, at least 5% more than rivals' prices on US Airways’ routes. Finally, using cost estimates for both MD80s and A320s, we find prices of the merging parties on US Airways routes track the costs of the A320 more closely than those of the MD80 prior to the merger, and vice-verse after the merger. In contrast, prices on American Airlines routes track the costs of the MD80 more closely than those of the A320 both pre- and post- merger. This evidence suggests that marginal costs for US Airways’ operations increased after the merger, consistent with rationalizing production in a way that reduced average cost but increased marginal cost.

This paper ‘sits’ in-between the literature on merger efficiencies and that on multi-market competition. Most literature on merger efficiencies[1] delves on the type and size of efficiencies that mitigate consumer welfare loss from reductions in competition. The literature on multi-market competition delves on how cross-market supply spillovers (e.g. Bulow et al. (1985)) or conduct spillovers (e.g. Bernheim and Whinston (1990)) affect market outcomes. This paper argues how a merger generates the opportunity for rationalizing production across markets. Even if there are no reductions in competition, this rationalization has effects on the marginal cost of production in every market, altering prices, rivals’ responses, and consumer welfare. [[other literature on dis-economies of scope and multi-plant operations??]]

The paper also adds to the growing empirical literature that evaluates merger effects post facto, and particularly in the airline industry[2] [Peters (2006)] shows how price changes for five different airline mergers were mostly due to changes in unobserved costs (inferred from pricing decisions), and not so much as changes in market structure. [Werden et al. (1991)] specifically compares price changes following the Northwest-Republic and TWA-Ozark mergers across routes that suffered loss of competition relative to those that did not. They find prices increased on select routes that did not suffer loss of competition. Unfortunately, as the main focus of their paper were routes that did suffer loss of competition, the authors do not explore further plausible causes for such price increases.

Finally, the airline industry has been vastly used to illustrate various cross-market effects, including multi-market contact (e.g. Evans and Kessides (1994) and Ciliberto and Williams (2012)), density economies (e.g. Brueckner and Spiller (1994)), hub dominance (e.g. Borenstein (1989)), and endogenous network formation (e.g. Bamberger and Carlton (1996)). We add to this literature by detailing how production rationalization affects prices.

In the following section we propose a simple theoretical model that formalizes how a multi-facility firm equates marginal cost across production facilities and the effects a merger has on costs and prices. Section 3 explores evidence of price changes in the US airline industry following the American Airlines - US Airways merger and relates such price changes to the aforementioned theoretical model. Section 4 discusses briefly the implications for policy and firm behavior. Conclusions follow.

---

[1] See, for example, Farrell and Shapiro (1990) for the size of marginal cost efficiencies; Werden and Froeb (1998) on how merger efficiencies constrain future entry; Stennek (2001) on information transfer as a form of efficiencies.

[2] Articles that investigate, within the airline industry, price changes due to lost competitors include Luo (2014), Goolsbee and Syverson (2008), Kwoka and Schumilkina (2010), Gayle (2008), and Kim and Singal (1993).
2 A Simplified Model of Competition

Assume two distinct markets, $A$ and $B$, each with $J_m$ identical firms, $m \in \{A, B\}$. Firms face a smooth, downward sloping, (weakly) concave, inverse demand in each market that is a function of total market output in such market: $P_m(Q_m)$ where $Q_m \equiv \sum_{j \in J_m} q_{jm}$ and $q_{jm}$ is firm $j$’s output in market $m$. In what follows, denote with $Q_{-jm}$ the aggregate sales in market $m$ of all firms but firm $j$.

Firms incur a cost of manufacturing $q$ goods, given by $C_m(q)$, with $C'_m > 0$ and $C''_m \geq 0$. As with traditional Cournot analysis, assume firms choose output that maximizes profits simultaneously with rivals:

$$\pi = \max_q P_m(q + Q_{-im})q - C_m(q)$$

Equilibrium output is given by the usual FOCs that equate marginal revenue to marginal cost. As much of the analysis that follows depends directly on marginal revenue curves, let $\eta(q, Q)$ be shorthand for such function:

$$\eta_m(q, Q_{-j}) \equiv qP'_m(q + Q_{-j}) + P_m(q + Q_{-j})$$

The Merger We model a merger between firms operating in different markets. As such, there are no market power effects in this model and all merger effects operate through supply side dynamics. Specifically, a merger between one firm in market $A$ with another firm in market $B$ allows the merging parties to produce output in any of the two markets and to freely re-allocate such output across markets. To accommodate such flexibility, let $\vartheta_m$ be the production amount in market $m$ and let $q_m$ be the allocation to market $m$. To distinguish between the merging parties and the non-merging parties, let ‘$i$’ and ‘$j$’ index a merging and a non-merging party, respectively. The merged firm’s problem is

$$\tilde{\pi}_i = \max_{(q_m, \vartheta_m)_{m \in \{A, B\}}} \sum_{m \in \{A, B\}} P_m(q_m + Q_{-im})q_m - C_m(\vartheta_m)$$

subject to

$$\sum_m q_m \leq \sum_m \vartheta_m \quad (q_m, \vartheta_m) \geq 0 \quad m \in \{A, B\}$$

Proposition 1 characterizes the equilibrium post-merger. Proposition 2 develops the comparative statics used to compare equilibrium outcomes pre- and post- merger. In essence, the merger allows the firm to reshuffle production from the high-cost facilities to the low-cost facilities. While this is

---

3 The model can easily be extended to allow for heterogeneity among firms, including differential cost structures and differentiated demand.

4 A similar analysis can be constructed using demand curves and having firms choose both prices and output simultaneously, but such analysis would have to specify rationing rules (as in Kreps & Schenikman [198X] and Davidson & Deneckere [19XX]). To avoid the complications that arise from defining alternative rationing rules, we chose to model the game in strategic substitutes.

5 Results are qualitatively similar for costly re-allocation as long as such re-allocation is not too expensive, specifically, less than the difference in the merging parties’ pre-merger marginal costs.
highly beneficial to the merging parties, it may be detrimental to consumers and beneficial to rivals.

By reshuffling production, the merged firm (weakly) increases the marginal cost of production in markets that used to be served by the low-cost facility. This increase in marginal cost results in decreased output by the focal firm and increased output by rival firms. In contrast, in the market with the high-cost facility the merged firm’s marginal cost may have decreased, resulting in higher industry output and higher consumer surplus.

**Proposition 1.** Equilibrium output in the above game, post-merger, is characterized by the following equations:

\[
\eta_A(q^*_{iA}, Q^*_{-iA}) = \eta_B(q^*_{iB}, Q^*_{-iB}) = C'_A(\vartheta^*_{iA}) = C_B(\vartheta^*_{iB}) \\
\eta_m(q^*_{jm}, Q^*_{-jm}) = C'_m(q^*_{jm}) \quad j \in J_m \setminus \{i\}
\]

The merged firm equates both marginal revenue, and marginal cost, across markets.

All proofs are provided in the appendix. The intuition behind equating marginal revenue is straightforward: if the merged firm wants to increase output in a given market, instead of producing an additional good, it can always reduce a unit of output in the alternative market. As such, marginal revenue in the alternative market serves as the opportunity cost of increasing output in the focal market. As for why the firm equates marginal cost across markets, if this were not so, the firm could always decrease production in the expensive facility (i.e. one market) and produce it in the cheap facility, keeping total output production constant but decreasing production costs.

Proposition 2 explores how the merger affects market allocations (of both the merging parties and of rival firms) and its impact on welfare.

**Proposition 2.** If, prior to the merger, the merging firms’ marginal cost is higher in market A than in market B, then after the merger:

1. The merging firms’ output increases in market A and decreases in market B
2. Rival firms’ output decreases in market A and increases in market B
3. Consumer surplus increases in market A and decreases in market B.

The opposite effects occur if, prior to the merger, the merging firms’ marginal cost is lower in market A than in market B.

As the proposition states, a merger that has no unilateral effects and has significant variable cost efficiencies can result in net consumer welfare loss. The rationale is simple: by reshuffling production across facilities the merged firm now faces a marginal cost in both markets that is an ‘average’ of prior marginal costs. As this ‘average’ marginal cost is, in at least one market, necessarily higher than pre-merger marginal costs, the firm retreats from such market. Rivals react in the usual way...
and mitigate some of the losses by increasing output in such markets, but the reactions are not large enough to overcome the loss of output, one-for-one.

The two key assumptions that drive Proposition 2 are the dis-economies of scale and the cheap transfer of output across markets. The validity of these assumptions will depend on the specific setting under study. Capacity constraints and scarcity in non-transferable inputs are two common sources of such dis-economies of scale. As firms’ output adds value over inputs, it is not uncommon that re-allocating inputs across markets may be prohibitive, but not so much outputs, resulting in scarce inputs and cheap transfer of outputs.

[[ Can we say something on consumer welfare across the whole industry?? I’m sure there is something we can say that under some conditions, industry CS drops, as firm profits are misaligned with consumer surplus ]]

The following example shows just how pervasive such effects can be. The example is a very simplified model of the airline industry, which more than modeling such industry, the intent is to clearly show how the mechanisms described above can significantly hurt consumers.

### 2.1 An Example Applied to the Airline Industry

Consider a setting with two firms, firm $a$ and $b$, operating as monopolist in two distinct markets, $A$ and $B$. Each market is characterized by a linear demand, that differs across markets solely due to scale: $D_A(p) = \alpha - \beta p$ and $D_B(p) = \tau(\alpha - \beta p)$. Each firm has a constant marginal cost of production, $c_a$ and $c_b$, where $c_a < c_b$. Finally, assume firm $a$ has a capacity constraint, $\kappa$, large enough such that it is not-binding given firm $a$’s optimal monopoly pricing, but not as large so as to supply both markets, even if firm $a$ priced at firm $b$’s higher price.

In a pre-merger setting, each firm chooses prices to maximize profits in its own market and the capacity constraint doesn’t bind. A merger between the these two firms grants no market power, but allows firm $b$ to utilize firm $a$’s excess capacity, which produces at a lower cost. As described above, the merged firm’s problem is

$$\max_{(p_a, p_b, q_a, q_b) \geq 0} D_A(p_a)p_a + D_B(p_b)p_b - c_a q_a - c_b q_b$$

$$s.t. \quad q_a + q_b \geq D_A(p_a) + D_B(p_b) \quad q_a \leq \kappa$$

The setting is very similar to the main theoretical model above, except that the convexity of cost functions is built-in through the capacity constraint. As with the theoretical model above, firms will choose prices such that marginal revenue across markets equals each other and, since the capacity constraint will bind, equal to the higher marginal cost. Hence, firm $b$’s price remain unchanged with the merger, but firm $a$’s post-merger price rises to firm $b$’s pre-merger price level.

To ground this example in a setting which the reader can more easily relate too, let firm $a$ be
US Airways and firm $b$ be American Airlines\textsuperscript{[6]} Pre-merger prices are $223$ and $232$, respectively, corresponding to 2014 average prices for a one-way ticket\textsuperscript{[6]} $c_a$ is $206$, implying a margin of 7.7\%, same as US Airways’ 2014 operating margin. Given $p_a$ and $c_a$, optimal pricing implies ‘$\alpha/\beta$’ is 240. And given ‘$\alpha/\beta$’, $p_b$, and optimal pricing for American Airlines, $c_b$ is calculated to be $224$. $\tau$ is derived from American Airlines sales relative to US Airways, which for 2014 were 1.49 times more. Accounting for price differences, these sales differences imply $\tau$ is 3.1. Finally, US Airways’ capacity, $k$, is set at ‘$23/\beta$’, derived from a pre-merger utilization rate of 73\%.\textsuperscript{[8]}

Given these values, the merger would increase profits of the joint-firm by 39\% and decrease consumer surplus in market $a$ by 73\%. Consumer surplus in market $b$ would remain unaffected. Details are given in the appendix.

The merger has very negative effects on consumers and should not be approved if consumer welfare is the standard for merger approval. However, the merging parties could credibly argue that the merger does not increase market power and generates substantial, merger-specific, \textit{variable} cost efficiencies: i.e. cognizable efficiencies. Ignoring the results presented here would likely result in authorities approving such merger.

In what follows we document evidence that the US Airways - American Airlines merger did indeed result in price increases for pre-merger US Airways customers. We then estimate a demand and supply model to quantify the welfare effects of the merger in light of aircraft reallocation.

3 US Airways - American Airlines Merger Effect on Prices

3.1 A Brief on the US Airways - American Airlines Merger

US Airways was a major American airline that merged with American Airlines in December of 2013. Initially announced in February 2013, the merger allowed American Airlines to emerge from bankruptcy. The Department of Justice did issue a complaint against the merger on the grounds that it would facilitate coordinated effects among industry players and reduce competition at select airports (e.g. Washington Reagan, LaGuardia, etc.). However, the DOJ reached a settlement with the merging parties in November 2013 in which the merging parties would divest landing slots and gates at select airports as a way of decreasing barriers to entry for low cost carriers (cf. United States

\textsuperscript{[6]}The example builds on US Airways having lower operating costs than American Airlines due to their more fuel efficient domestic fleet: A320s and A319s compared to American Airlines’ MD80s.

\textsuperscript{[7]}Average prices are obtained from the Bureau of Transportation Statistics’ (BTS) Origin and Destination Survey. Operating margins were obtained from US Airways’ 2013 10-K report (US Airways Group, Inc. (2013)). Total passenger-miles were obtained from the BTS’ Airline Statistics’ T-100 Segment report. Both prices and sales are based on US domestic operations for passenger service only.

\textsuperscript{[8]}US Airways’ 2014 utilization rate of 73\% is calculated as US Airway’s productivity of its A320 fleet relative to the 75th percentile productivity value of rivals’ A320s. A carrier’s A320 productivity is given as yearly passenger-miles flown on A320s over total days of assigned service for A320s. Days of assigned service are obtained from the BTS’ Schedule P.5.2 and yearly passenger-miles are obtained from the T-100 database.
District Court for the District of Columbia (2014)). The merging parties finalized consolidating operations in April 2015, when they obtained a Single Operating Certificate from the FAA.

Prior to the merger, US Airways’ domestic fleet was conformed by more fuel efficient aircraft than American Airlines. Specifically, US Airways’ mainline, narrow-body, fleet was conformed by 93 Airbus A319s, 75 A321s, 72 A320s, and 56 other aircraft, with an average age of 11 years. In contrast, American Airlines’ mainline, narrow-body, fleet was conformed by 195 Boeing 737s, 191 McDonnell Douglas MD-80s, and 106 Boeing 757s, with an average age of 14 years. The merging parties did view the merger as a way of optimizing fleets, calling for the “Right Aircraft in the Right Place at the Right Time” (AMR Corporation and US Airways Group, Inc (2013)) creating $550M in cost synergies, a 21% increase over the combined firms’ 2013 operating profits (cf. American Airlines Group, Inc (2014)).

We interpret the above synergies to imply the joint-firm would be able to increase utilization of less expensive aircraft and decrease utilization of expensive aircraft. It is likely that such fleet re-optimizing changed the marginal aircraft for US Airways operations from a low-cost aircraft (e.g. A320) to a high-cost aircraft (e.g. MD80). This increase in marginal cost would be likely reflected in an increase in price. The following subsection explores this hypothesis.

3.2 Data

Sources We use data from the Bureau of Transportation Statistic’s (BTS) Airline Origin and Destination Survey (DB1B database), which contains a 10% sample of all itineraries sold for domestic flights in the US. The data is reported quarterly and we obtain data from the first quarter of 2010 up to the second quarter of 2016. We aggregate up the itineraries to the carrier-route-quarter level, and define a route as a unidirectional city pair. The unit of observation is a carrier-route-quarter triplet: e.g. American Airlines’ service between Tucson and Miami on the third quarter of 2014. The data contains information on passengers served, the average price paid per passenger, and the flight structure: non-stop or connecting.

We additionally use the Air Carrier Statistics database (T-100 Segment). This data contains, at the monthly level, the number of seats assigned and passengers flown by each carrier’s aircraft class on each (directional) airport pair. An aircraft class is a manufacturer-model pair: e.g. 737-300, 737-800, A320-100/200, etc., which we categorize into one of three categories: regional jets, mainline jets,

---

9US Airways Corp, including US Express, also operated 26 twin-aisle aircraft, 67 regional jets, and 44 turbo-props. AMR, including American Eagle, also operated 122 twin-aisle aircraft, 245 regional jets, and 9 turbo-props. Cf. US Airways Group, Inc. (2013) and AMR Corporation (2013))

10As a carrier can service a route through multiple flight structures (e.g. non-stop flight, one-stop flight connecting in X hub, one-stop flight connecting in Y hub, two-stop flights, etc.), we aggregate only the itineraries that were serviced with the modal flight structure. We do, however, take total sales regardless of flight structures when calculating shares. Also, we drop the observations, i.e. carrier-route-quarters, in which the modal flight structures involved two or more connections.

11We drop observations for which average prices are outrageous—below $25 dollars or above $2,500 dollars.
and other aircraft. The Air Carrier Statistics data is aggregated to a carrier-route-quarter and merged onto the DB1B data. In doing so we record the modal aircraft category used to service each leg of each route and the average load factor on those legs. As connecting service has two flight legs, we retain the maximum load-factor across the two legs.

Lastly, we incorporate cost information using the BTS’ Form 41 - Financial Data, Schedule P.5.2. This data reports, at the quarterly level, domestic operating costs and statistics for each carrier and for each aircraft class. From this data we retain operating costs per hour of airtime, by quarter, carrier, and aircraft class, where operating costs include costs of crews, fuel, maintenance, depreciation, and rental equipment. As the T-100 Segment data has information on air time and on flying distance, we merge the Schedule P.5.2 cost data with the T-100 data and obtain quarterly operating costs per passenger-seat mile, for each carrier and each aircraft class.

As the DB1B is an extensive data set with many charter carriers, executive jets, and freight carriers, we drop all carriers with less than 1% national market share, calculated yearly. We drop small routes, routes to small cities, routes within Alaska or Hawaii, and routes to, from, or within US territories. We consider a carrier to be active on a route at a certain quarter if the carrier services at least a thousand passengers or has at least 15% market share in that quarter.

**Route Classification** In the analysis that follows, we would like to distinguish how the American Airlines / US Airways merger affected prices for US Airways relative to American Airlines. However, in the years that follow the merger US Airways ceases to exist and its operations are taken over by American Airlines. So as to have a ‘hold’ on how prices of the merged firm changed on its US Airways operations, we classify routes according to which of the two merging parties had a more dominant position on that route prior to the merger. Specifically, we use 2013 yearly sales to calculate the Herfindahl index for each route, and the projected change in such index that a merger between US Airways and American Airlines would have implied. Any route in which this projected change is larger than 100 points is classified as a *Joint* route. For the remaining routes, any route in which US Airways’ share is larger than the average share (i.e. $s_{US} > 1/N$) and American Airlines’ share is less than the average share is classified as a *US Airways* route. Conversely, routes in which American Airlines’ share is larger than the average share and US Airways’ share is smaller than the average share is an *American Airlines* route. Routes in which neither carrier had shares larger than the average share is classified as a *Non-Served* route. With this route classification we are

---

12 Classified as a mainline jet are all of Airbus’s A319, A320, A321, and A330 class jets, all of Boeing’s 737, 757, and 767 class jets, as well as the MD80/90 family. Regional jets are all of Embraer’s and Bombardier’s jet aircraft and Boeing’s 717. Other aircraft is any other aircraft not mentioned here.

13 For each itinerary coupon, the DB1B data reports two carriers: an *operating carrier*, responsible for operating the flight, and a *ticketing carrier*, responsible for sales of the ticket. The DB1B is aggregated to the carrier-route-quarter level utilizing the *ticketing carrier*. However, the identity of the modal *operating carrier* is retained, and it is on this modal carrier with which the match with the T-100 data is made.

14 Small routes are those with less than 100 quarterly passengers. Small cities are those with less than 100 daily enplanements.

15 1,000 passengers over a quarter is equivalent to 80 weekly passengers, barely enough to justify a single weekly flight on a regional jet. Note that we do take these offerings into account when calculating market share.
able to follow how the merging firms’ prices changed after the merger on routes that were typical US Airways routes (e.g. Phoenix-Vegas), compared to prices on routes that were typical American Airlines routes (e.g. Chicago-Miami).

**Measures of Price** We use two different measures of ‘price’. The first, yield, is the average revenue per passenger-mile. It is commonly used in the industry and accounts for how longer routes cost more. The second, log of the average price per passenger, is used in other academic papers and allows for a straightforward interpretation of elasticities and semi-elasticities.

**AA/US Prices** We refer to the merging firms’ prices as AA/US prices, after the carriers IATA abbreviation. When referring to AA/US prices on US Airways routes, however, we refer solely to US Airways’ prices pre-merger, and American Airlines’ prices post-merger. Although American Airlines’ may have had, on these routes, some sales pre-merger, American Airlines’ prices are excluded. Similarly, when referring to AA/US prices on American Airlines routes, US Airways’ pre-merger prices are excluded. This strategy removes potential noise from sporadic service when analyzing AA/US prices. For Joint routes, pre-merger prices of both carriers are retained.

### 3.3 US Airway Prices vs American Airlines Prices

In this first analysis we explore if the merging firms’ prices on US Airways routes increased after the merger relative to prices on American Airlines routes. Figure\[1\] shows yield of the merging parties over time, by route type. So as to reduce noise from seasonality variation and persistent carrier differences, shown in figure\[1\] are the residuals of a regression of yield on quarter and carrier fixed effects, centered at average values. The grayed area shows the merging period (2014Q1 - 2015Q1). As is apparent from the figure, yields on US Airways’ routes increased significantly after the merger relative to yields on American Airline routes.

We formalize the above graph using a difference-in-difference estimation. The first difference compares prices before and after the merger. The second difference compares prices across route types: Joint routes (J), US Airways’ routes (U), and American Airlines’ routes (omitted category). All Non-Served routes are excluded from the estimation sample, as neither carrier was a competitive player on those routes the year prior to the merger and, hence, it is uncertain how they are pricing those routes, neither before nor after the merger. The merging period is also excluded from the sample, as it is unclear to what extent the merging parties were coordinating operations during this time.

The exact empirical specification is

$$yield_{irt} = \beta^U d^U_{t} d^U_{q} + \beta^J d^J_{t} d^J_{q} + \alpha^U d^U_{t} + \alpha^J d^J_{t} + \alpha^{\text{post}} d^{\text{post}}_{t} + \alpha x_{irt}^{(1)} + \epsilon_{irt}$$
US Airways’ routes are those in which US Airways had a dominant position in 2013 and American Airlines did not. American Airlines routes are those in which American Airlines had a dominant position in 2013 and US Airways did not. Joint routes are those in which both carriers had a dominant position in 2013. See text for details on the exact constructs.

where \( \text{yield}_{irt} \) is carrier \( i \)’s yield, American Airlines’ or US Airways’, on route \( r \) in quarter \( t \). \( d_{U}^{r} \) and \( d_{J}^{r} \) are dummy variables indicating US Airways routes and Joint routes, respectively. \( d_{t}^{pst} \) is a time dummy for all quarters after the merger: 2015Q2 and onward. \( x_{ir}^{(1)} \) are a set of controls for costs and market power: a dummy for non-stop service, a dummy of US Airways’ prices on joint routes, a dummy for American Airlines’ prices after declaring bankruptcy (2012Q1), load-factor, the Herfindahl index on the route, the number of carriers, of non-stop carriers, of low-cost carriers, and of potential entrants\(^{16}\) market share at endpoint cities—averaged across both cities—, and the number of non-stop routes the carrier services at the endpoint cities—averaged across both cities—.

A positive value on \( \beta_{U} \) is indicative that prices of the merged firm increased on US Airways routes relative to American Airlines routes following the merger. Table 1 shows the results from this estimation. Depending on the specification, following the merger yields on US Airways routes increased between 0.7 and 4.9 cents/passenger-mile more than what yields on American Airlines routes increased. The results are statistically significant across specifications II-V, which control for long-run differences across routes and changes in competition within routes. They are also large: given an average yield of 26 cents/passenger-mile, a price increase of 2.5 cents/passenger-mi (specification III) is almost a 10% increase, twice the 5% SSNIP benchmark used in antitrust to flag a merger as potentially harmful to consumers (cf. U.S. Department of Justice and Federal Trade Commission)

\(^{16}\)Low cost carriers are all carriers excluding United Airlines, Delta Airlines, Continental Airlines, and Alaska Airlines. Potential entrants are defined as carriers with flights at both end-points of a route but no service on the route itself.
Table 1: Change in AA/US Yields following the merger, by route type

<table>
<thead>
<tr>
<th></th>
<th>Yield (¢/seat-mi)</th>
<th>Ln[p]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>US Airways’ routes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-merger</td>
<td>0.67</td>
<td>4.86*</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(1.63)</td>
</tr>
<tr>
<td>Overall</td>
<td>8.19*</td>
<td>4.20*</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(1.06)</td>
</tr>
<tr>
<td>Joint routes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post Merger</td>
<td>0.47</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(1.14)</td>
</tr>
<tr>
<td>Overall</td>
<td>-8.97*</td>
<td>-6.89*</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>Post-Merger Dummy</td>
<td>0.86†</td>
<td>2.35*</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.85)</td>
</tr>
</tbody>
</table>

Control Variables  Y  Y  Y  Y
Route and Quarter F.E.  Y  Y  Y
Only mainline jet routes  Y
Adj. R-Sq  0.146  0.305  0.886  0.885  0.643
N  73,831  73,831  73,831  46,067  73,831

Standard errors, in parenthesis, are clustered by quarter-route type groups.
American Airlines’ routes are the omitted category. Statistically different than zero at a 5% (*) and at a 10% (†) p-value. See text for list of control variables. Mainline jet routes are those which, in 2013, AA/US utilized mainline jets, as opposed to regional jets and turbo-props, to predominantly service any leg of the route.

(1997), section 4.1.2). Specification (V) uses log-price as the dependent variable and there too does the merger appear to induce a 12% increase in prices on US Airways’ routes over American Airlines’ routes.

The estimates for Joint routes appear to reflect a positive, but small, price increase following the merger. One would have expected prices to increase on these routes as the merger increased the merging firm’s market power. However, the remedies requested by the DOJ for merger approval should have mitigated, and possibly even decreased, this market power. The largest price increase estimated, relative to the American Airlines’ routes, is of 5% (spec. V). Other specifications estimate a price increase of 3% (specification III), or even no statistically meaningful price changes (specification II and IV). This is suggestive that the DOJ remedies were effective at restricting increases in market power due to the merger.

3.4 US Airway Prices vs Rival Prices

The above analysis showed that AA/US prices on US Airways’ routes increased after the merger relative to prices on American Airlines’ routes. However, such price changes could have resulted
from prices decreasing for the American Airlines operations of the joint firm, as American Airlines was emerging from bankruptcy with renewed cost structures (e.g. new union contracts). To explore if prices did in fact increase for the US Airways operations of the joint firm, we compare post-merger changes in prices of the joint firm to those of rival firms, focusing on US Airways’ routes.

Figure 2 shows the merging firm’s yield over time on US Airways’ routes. It also shows the average yield of rival carriers, segmented by low-cost-carriers and rival legacy carriers, on those same routes. As with figure 1, we plot the residuals from a regression of yield on quarter and carrier class fixed effects to remove noise from seasonality and long-run carrier differences. The figure shows how US Airways/American Airlines went from being, prior to the merger, the lowest priced competitor to, post-merger, the highest priced competitor.

As before, we formalize the above analysis with a difference-in-difference estimation. Our DID specification is:

\[
yield_{irt} = \beta M d_i^M d_t^{\text{non-stop}} + \gamma M d_i^M + \gamma \text{nst} d_t^{\text{non-stop}} + \gamma x_{x(2)} + \epsilon_{irt}
\]

where \(d_i^M\) is a dummy equal to one for US Airways/American Airlines. All rival carriers form the omitted category (i.e. control group). \(x_{x(2)}\) are control variables for costs and market power: a non-stop service dummy, load factor, market share on the route, the Herfindahl index, the number of carriers, non-stop carriers, low-cost carriers, and potential entrants; market share at end-point cities, and the number of non-stop routes available at end-point cities.

Table 2 shows the estimates from this DID analysis. Following the merger, yields on US Airways’ routes increased between 1.60 and 4.50 ¢/passenger-mile more for AA/US than for rivals. These increases, which are statistically significant, are economically large. US Airways’ average yield on
Table 2: AA/US Prices relative to Rivals’ prices on US Airways’ routes

<table>
<thead>
<tr>
<th></th>
<th>Yield (¢/pass-mi)</th>
<th>Ln[p]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>AA/US * Post-merger 1</td>
<td>1.62†</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>(0.84)</td>
<td>(1.42)</td>
</tr>
<tr>
<td>AA/US</td>
<td>6.79*</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.85)</td>
</tr>
<tr>
<td>Post-Merger Dummy</td>
<td>-0.05</td>
<td>3.08*</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(0.78)</td>
</tr>
</tbody>
</table>

Control Variables

| Route, Quarter, and Carrier F.E. | Y | Y | Y | Y |
| Only mainline jet routes         | Y | Y | Y |
| Adj. R-Sq                        | 0.04 | 0.20 | 0.778 | 0.804 | 0.620 |
| N                                | 54,217 | 54,217 | 54,217 | 26,406 | 54,217 |

Standard errors, in parenthesis, are clustered by quarter-carrier type groups. Statistically different than zero at a 5% (*) and at a 10 % (†) p-value. Sample includes only US Airways’ routes. All rival carriers are the omitted category. See text for list of control variables. Mainline jet routes are those which, in 2013, AA/US utilized mainline jets, as opposed to regional jets and turbo-props, to service any leg of the route.

These routes had been, prior to the merger, 35 ¢/passenger-mile. As such, the estimated yield increase was of at least 5%, and possibly even 13%, over and beyond what rivals’ yields changed.

3.5 Price-Cost Correlations

As is clear from the past two analysis, prices of AA/US on US Airways’ routes increased after the merger: they increased relative to the merging firms’ prices on non-US Airways’ routes and they increased relative to competitors’ prices. The rationale for this behavior that we postulate in this paper is that marginal costs for US Airways’ operations increased due to the re-optimization of fleets, in which the marginal cost of adding an additional flight changed from being that of an A320 to that of an MD80. Of course, one can also conceive alternative reasons on why prices would have behaved as such. For example, prices could have increased if the American Airlines’ frequent flyer program creates more value for customers than US Airways’ program, and the merged firm adjusted prices to account for such. Similarly, if US Airways’ operations inherited the American Airlines union contracts, and these union contracts increased costs for US Airways operations.

In order to show favor for the fleet re-optimization rationale over alternative hypothesis, we provide an additional empirical test. We use the Schedule P.5.2 cost information to test whether the merging firms’ prices on US Airways’ routes follow more the operating costs of the A320 over the operating...
Table 3: Relation Between AA/US Prices and A320 and MD80 Costs, by Route Type

<table>
<thead>
<tr>
<th></th>
<th>US Airways’ routes</th>
<th>American Airlines’ routes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yield (I)</td>
<td>Ln[p] on Ln[c] (II)</td>
</tr>
<tr>
<td></td>
<td>(III)</td>
<td>(IV)</td>
</tr>
<tr>
<td>A320 x Pre-merger</td>
<td>1.64</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>MD80 x Pre-merger</td>
<td>-0.73*</td>
<td>-0.27*</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>A320 x Post-merger</td>
<td>1.32*</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>MD80 x Post-merger</td>
<td>4.01*</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.86)</td>
<td>(0.07)</td>
</tr>
<tr>
<td></td>
<td>-36.1*</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(8.35)</td>
<td>(0.17)</td>
</tr>
</tbody>
</table>

| Controls                       | Y        | Y                     |
| Route F.E.                    | Y        | Y                     |
| Adj. R-Sq                      | 0.006    | 0.015                 |
| N                               | 12,140   | 12,140                |

Standard errors in parenthesis. Statistically different than zero at a 5% (*) and at a 10 % (†) p-value. Sample excludes the five quarters of the merger period and the routes which did not have at least one flight segment predominantly served with mainline jets in 2013. Yield and costs are measured in ¢/passenger-mile. Ln[p] on Ln[c] regress log of average price ($/passenger) on log of aircraft operating costs. See text for list of control variables.

costs of the MD80, and how that relationship changed after the merger. Specifically, we estimate:

\[
\text{yield}_{rt} = \beta^{A320,\text{pre}} c^{A320}_{t} d^{\text{pre}}_{t} + \beta^{MD80,\text{pre}} c^{MD80}_{t} d^{\text{pre}}_{t} + \beta^{A320,\text{post}} c^{A320}_{t} d^{\text{post}}_{t} + \beta^{MD80,\text{post}} c^{MD80}_{t} d^{\text{post}}_{t} + \beta^{\text{D}} c^{\text{D}}_{t} + \omega^{x^{(3)}} x^{(3)}_{rt} + \epsilon_{rt}
\]

where \( c^{A320}_{t} \) and \( c^{MD80}_{t} \) are the merging firms’ quarterly average operating costs per passenger-seat-mile for the respective aircraft class. \( d^{\text{pre}}_{t} \) and \( d^{\text{post}}_{t} \) are dummies for the time periods prior-to and post-merger. \( x^{(3)}_{rt} \) controls for market power and costs by including a non-stop service dummy, number of carriers, number of non-stop carriers, number of low-cost carriers, potential entrants, market share at endpoint cities, the number of non-stop destinations at endpoint cities. The estimation sample includes only US Airways’ routes and excludes routes that in 2013 did not have at least one flight segment predominantly serviced with mainline jets: Airbus A319 and larger, Boeing 737 and larger, and MD80/90.\(^{17}\) This excludes routes supplied exclusively by regional jets and turbo-props: e.g. Phoenix-Tucson.

The results show, presented in Table 3, show how US Airways’ prices prior to the merger followed

\(^{17}\) To be precise, a mainline jet is any jet in the Airbus’s A319, A320, A321 and A330 families; in Boeing’s 737, 757, and 767 families; and jets in the McDonald Douglas MD80 and MD90 families.
more closely costs of the A320 (i.e. US Airways’ work-horse aircraft) than those of the MD80 (i.e. American Airlines’ work-horse aircraft). As shown in Table 3 - Specification (II), a one cent-per-passenger-mile increase in the cost of operating an A320 is correlated with a statistically significant 1.33 cent/passenger-mile increase in yield. In contrast, a similar increase in the cost of operating an MD80 is associated with a decreases in yield of 0.46 cents/passenger-mile. Interestingly, after the merger period, the merging firm’s prices on US Airways routes are positively correlated with the MD80’s cost and completely uncorrelated, statistically and economically, with the costs of the A320. Columns (III) and (IV) replicates columns (I) and (II) using a log-log specification instead of a linear-linear specification and the results are qualitatively the same: pre-merger, a one percent increase in the costs of the A320 is associated with a 0.47 percent increase in yield; a pass-through of 47 percent. Post-merger, the pass through for the A320 costs is zero, and that for the MD80 costs is of 13 percent. In summary, prior to the merger, prices on US Airways routes tracked closely the costs of the A320, but switched to tracking the costs of the MD80s post-merger.

So as to provide a falsification exercise, to test whether the above finding was not due to some spurious correlation between the costs of the different aircraft and prices on US Airways’ routes, we repeat the exact same analysis except that we change the estimation sample to be prices of AA/US on American Airlines’ routes[18]. The results, also shown in Table 3, show that on American Airlines’ routes, AA/US prices follow more closely the costs of the MD80, both before and after the merger, than the costs of the A320. We interpret this as evidence that the merging firms’ prices post-merger respond to the merging firms’ marginal costs: those of the more expensive MD80.

The costs of the MD80 and of the A320 vary over time, but not across routes. Hence, the above regressions can be subject to col-linearity issues, as there are three variables that are identified off the five quarters post-merger (2015Q2-2016Q2): MD80 x Post-merger, A320 x Post-merger, and Post-merger. To address this concern we build four different cost variables, and have non-nested tests dictate which of these four cost variables best fit the pricing data. The first two cost variables are simply the cost of the A320 over time and the cost of the MD80 over time: $c^\text{A320}_t$, $c^\text{MD80}_t$. The third cost variable follows the cost of the A320 up to the merger, and that of the MD80 after the merger. The fourth does the opposite:

\[
\begin{align*}
\text{A320/MD80}^{\text{A320}} & = c^\text{A320}_t d^\text{pre}_t + c^\text{MD80}_t d^\text{post}_t \\
\text{A320/MD80}^{\text{MD80}} & = c^\text{MD80}_t d^\text{pre}_t + c^\text{A320}_t d^\text{post}_t
\end{align*}
\]

For each one of these variables, and for each of the two route types studied (i.e. US Airways’ routes and American Airlines’ routes), we run the OLS regression:

\[
yield_{rt} = \beta^\nu c^\nu_t + \beta^\text{post} d^\text{post}_t + \varphi_x x^{(3)}_{rt} + \epsilon_{rt}
\]

where $c^\nu_t$ is one of the four cost variables. $x^{(3)}_{rt}$ contains the same controls as in Table 3, a dummy

\[18\text{We also add a dummy variable for the bankruptcy period.}\]
Table 4: Regression of US/AA Yields on Alternative Cost Variables and Corresponding Likelihood Ratio Tests

<table>
<thead>
<tr>
<th>Cost Variable</th>
<th>US Airways’ routes</th>
<th>American Airlines’ routes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>Pre / Post</td>
<td>MD80</td>
<td>A320</td>
</tr>
<tr>
<td>Estimate</td>
<td>0.78*</td>
<td>0.62*</td>
</tr>
<tr>
<td>Std. Error</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Adj. R-sq</td>
<td>0.9181</td>
<td>0.9178</td>
</tr>
<tr>
<td>Log-LH</td>
<td>-33,968</td>
<td>-33,985</td>
</tr>
<tr>
<td>LR-test: model (I) outperforms column model?</td>
<td>N/A</td>
<td>2.51</td>
</tr>
<tr>
<td>Z-statistic</td>
<td>N/A</td>
<td>0.994</td>
</tr>
<tr>
<td>P-value</td>
<td>-6.70</td>
<td>-5.50</td>
</tr>
<tr>
<td>LR-test: model (III) outperforms column model?</td>
<td>N/A</td>
<td>0.901</td>
</tr>
</tbody>
</table>

Sample includes only those routes who, in 2013, AA/US predominantly serviced at least one flight segment with mainline jets: Airbus A-319/320/321/330, Boeing 737/757/767 and MD-80/90. Sample excludes the five quarters of the merger prior: 2014Q1-2015Q1. See text for list of control variables.

For non-stop service, number of carriers, number of non-stop carriers, number of low-cost carriers, potential entrants, market share at endpoint cities, the number of non-stop destinations at endpoint cities, a dummy for American Airlines during their bankruptcy period, and route fixed effects.

Using the regression results, we test whether the model with $c_{t}^{A320/MD80}$ outperforms the other models in terms of fit. Specifically, we use Vuong (1989)’s likelihood ratio test for non-nested models. As the tests assume log-likelihoods, we assume the error term is distributed normal and iid and calculate the corresponding log-likelihoods. We also test if the model with $c_{t}^{MD80/MD80}$ outperforms the other models. Results are presented in Table 4.

As suggested, when fitting yields on US Airways’ routes, the model in which costs are those of the A320 prior to the merger and those of the MD80 after the merger outperforms all other models. It is the model with the highest R-sq and likelihood ratio test confirms that it outperforms all other models. Interestingly, when fitting yields on American Airlines’ routes the model in which costs are those of the MD80 both before and after the merger is the one which best fits that data.

4 Discussion

These tests confirm our hypothesis that the merged firm’s prices on US Airways routes increased after the merger, mostly due to the reallocation of aircraft, in which the marginal aircraft for US Airways’ operations became the MD80. The price increases are large, between five and fifteen percent, and consumers are likely to have been affected. A formal estimate of the potential consumer welfare loss
is beyond the scope of this paper. However, the price increases estimated are significantly above the FTC’s suggested five percent increase for SSNIP tests. We do not suggest that welfare losses are as large as suggested by our (extremely) simplified model of competition in section 2.1, as consumers may have responded to the price increase by switching to more affordable rival carriers, and not by leaving the industry as the simplified model suggests. More importantly, American Airlines has initiated a massive fleet renewal project in an effort to retire its MD80 fleet. As American Airlines renews its fleet and its marginal aircraft becomes either a Boeing-737 or an Airbus A320, we should expect prices to decrease, potentially to pre-merger levels.

The purpose of this exercise was to show how production reshuffling across merging parties may result in lower average costs for the merging parties but in higher marginal costs for a subset of the merging firms. Many proposed mergers present, to both investors and regulators, production reshuffling synergies as one of the key benefits of the merger. We caution both investors and regulators that while such production reshuffling can decrease average cost, it can increase marginal cost and this increase in marginal cost may place the merging firm at a disadvantage relative to rivals, resulting in higher prices and lower sales.

This merger effects proposed here are a subset of cross-market dynamics that have become common as firms globalize. For example, over the past three decades global cement manufacturing has become concentrated, with global manufacturers shipping production across continents in an effort to hedge demand and supply swings. It is unfortunate that the FTC’s merger guidelines state so little about the benefits and harms that may come to consumers from such cross-market dynamics, including dis-economies of scope (Bulow et al. (1985)), multi-market contact (Bernheim and Whinston (1990), Arie et al. (2016)), and the effects presented here.

5 Conclusion

This paper contains a simple idea: production rationalization averages cost curves across facilities, decreasing costs in some markets and increasing costs in others. Mergers whose efficiencies (a.k.a. synergies) are based on such rationalizations may be beneficial to the merging parties but detrimental to consumers, even if there are no shifts in market power.

The US Airways/American Airlines merger provides a great example of how such rationalization can negatively affect consumers while positively affecting the merging parties. As US Airways operated more efficient aircraft than American Airlines prior to their merger, and US Airways had slack in the utilization of such aircraft, the merger allowed the merging parties to fully utilize such efficient aircraft but altered US Airways’ marginal cost from a low-cost aircraft (the A320) to a high-cost aircraft.

To obtain formal estimates on changes in consumer surplus one needs, at a minimum, a model of demand, and, preferably, a model of supply, so as to include how rivals’ react to the merged firm’s pricing strategies. These models involve complex data processing and, more importantly, strong modeling and identifying assumptions. As welfare calculations are susceptible to such assumptions, we prefer to leave that exercise to future research so as to give the exercise the proper attention it requires.
aircraft (the MD80). This resulted in prices of the merging firm rising 10% more on US Airways’
routes than on American Airlines’ routes and 5% more than rivals’ prices on US Airways’ routes.
Such price hikes are higher than the 5% threshold commonly utilized by the antitrust agencies in
determining potentially anti-competitive mergers.

When promoting mergers, ‘synergies’ tend to be highly invoked but much less understood. This
paper takes on one such synergy, production re-allocation, and illustrates how it affects both firm
profits and consumer welfare. The paper pin-points how such synergy could have come about in
the US Airways/American Airlines merger and the effect it had on prices. It would be of great
value to businessmen and regulators alike to see clear examples of other synergies at play, including
increasing economies of scale, know-how diffusion, and information benefits.

References


AMR Corporation, “Annual Report (Form 10-K),” Retrieved from SEC EDGAR website


Arie, Guy, Sarit Markovich, and Mauricio Varela, “The Competitive Effect of Multimarket
Contact,” working paper, 2016.

Bamberger, Gustavo E. and Dennis W. Carlton, “Airline Networks and Fares,” November
1996.

Bernheim, B. Douglas and Michael D. Whinston, “Multimarket Contact and Collusive Be-

Borenstein, Severin, “Hubs and High Fares: Dominance and Market Power in the U.S. Airline


Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer, “Multimarket Oligopoly:

Ciliberto, Federico and Jonathan W. Williams, “Does Multimarket Contact Facilitate Tacit


A Proof of Propositions

For clarity, for each proposition found in the main text, we re-state the proposition and develop a formal proof.

Proposition. Equilibrium output in the above game when the focal firms have merged is characterized by the following equations:

\[ \eta_A(q_{1A}^*, Q_{1A}^*) = \eta_B(q_{1B}^*, Q_{1B}^*) = C_A'(\vartheta_{1A}) = C_B'(\vartheta_{1B})\eta_m(q_{im}^*, Q_{im}^*) = C_m'(q_{im}^*) \quad i \in J_m \{1\} \]

The merged firm equates both marginal revenue, and marginal cost, across markets.

Proof. Let \( \mu \) be the Lagrangian multiplier on the constraint \( \sum_m q_m \leq \sum_m \vartheta_m \) and \( (\mu_{q_A}, \mu_{q_B}, \mu_{\vartheta_A}, \mu_{\vartheta_B}) \) be the Lagrangian multipliers on the non-negativity constraints. Firm 1’s problem can be restated as

\[
\max_{(q_m, \vartheta_m)_{m \in \{A, B\}}} \sum_m P_m(q_m + Q-m)q_m - C_m(\vartheta_m) + \mu \left( \sum_m \vartheta_m - q_m \right) + \sum_m \mu_{q_m} q_m + \sum_m \mu_{\vartheta_m} \vartheta_m
\]

and for any other firm, the problem is

\[
\max_{q_i} P_m(q_i + Q_{im}^*)q_i - C_m(q_i) + \mu_{q_i} q_i
\]

As all firms but firm 1 are identical, the equilibrium output of a rival firm (a firm that is not firm 1) is the same. For simplicity, let this value be \( \hat{q}_m \) and it is distinguished from firm 1’s output in market \( m \): \( q_m \). Total market output is then \( Q_m = q_m + n_m \hat{q}_m \) where \( n_m \) is the number of rival firms in market \( m \). Finally, define firm 1’s marginal revenue and marginal cost in market \( m \) as \( \eta_m \equiv \eta_m(q_m, n_m \hat{q}_m) \) and \( C'_m \equiv C'_m(\vartheta_m) \), and, similarly, define rivals’ marginal revenue and marginal cost as \( \hat{\eta}_m \equiv \eta_m(\hat{q}_m, q_m + (n_m - 1)\hat{q}_m) \) and \( \hat{C}'_m \equiv C'_m(\hat{q}_m) \).

FOCs from the above problems are

\[
\eta_m = \mu - \mu_{q_m} \quad C'_m = \mu + \mu_{\vartheta_m} \quad \hat{\eta}_m - \hat{C}'_m = \mu_{\hat{q}_m}
\]

If \( q_m, \hat{q}_m, \) and \( \vartheta_m \) are all positive in both markets then all the non-negativity constraint multipliers are zero and \( \eta_A = \mu = \eta_B = C'_A = C'_B \) and \( \hat{\eta}_m = \hat{C}'_m \). Hence, what is left to show is that all decision variables are non-zero. We do this in steps. [NEED TO FINISH]

1. Firm 1 has no excess production: \( q_A + q_B = \vartheta_A + \vartheta_B \). The argument follows in the lines of Spence [1977]: if firm 1 had excess production, it could cut the excess by half and continue to hold the same allocation. Rival firms’ profits are unaffected, and hence their best responses are also unaffected, but firm 1’s cost are reduced, resulting in higher profits.
2. Firm 1’s sales in both markets are strictly positive. To show this, assume they are zero in at least one market, and wlog, let this market be market $A$. $\mu_{QA} > 0$ and $\eta_A > \overline{p} \geq \max \{ C^r_A, C^r_B \}$. 

\[\square\]

**Proposition.** If, prior to the merger, firm 1’s marginal cost is higher in market $A$ than in market $B$, then after the merger:

1. Firm 1’s output increases in market $A$ and decreases in market $B$
2. Firm 1’s production decreases in market $A$ and increases in market $B$
3. Consumer surplus increases in market $A$ and decreases in market $B$.

The opposite effects occur if, prior to the merger, firm 1’s marginal cost is lower in market $A$ than in market $B$.

**Proof.** There are two market structures to consider and compare. The first market structure, labeled the “pre-merger” structure, has firm 1 acting independently across both markets with no possibility of reshuffling production across markets. The second market structure, labeled “post-merger”, has firm 1 working jointly across both markets and reshuffling production as needed. So as to make the two market structures comparable, we expand the decision variables in the pre-merger market structure such that firm 1 chooses, in each market, how much to produce and how much to sell, separately. However, we do not allow firm 1 to sell in market $A$ what it produced in market $B$.

As we assume all rivals firms (i.e. all firms that are not firm 1) are symmetric, in the sense that they have the same cost and demand functions, under each market structure there is an equilibrium in which all rivals choose the same output decisions. Focusing on this equilibrium, and taking into account the four decision variables of firm 1, an equilibrium is characterized by six variables: firm 1’s sales in each market ($q_A, q_B$), firm 1’s production decisions in each market ($\vartheta_A, \vartheta_B$), and rival firms’ output decisions in each market ($\hat{q}_A, \hat{q}_B$). We group these six variables into the vector $x$: $x \equiv (q_A, \hat{q}_A, q_B, \hat{q}_B, \vartheta_A, \vartheta_B)$.

Let $\mathcal{H}^{\text{pre}}(x)$ and be $\mathcal{H}^{\text{post}}(x)$ be the vector of equations that characterize the equilibrium in the pre-merger and post-merger market structures:

\[
\mathcal{H}^{\text{pre}}(x) \equiv \begin{bmatrix}
P^r_A(Q_A)q_A + P_A(Q_A) - C^r_A(\vartheta_A) \\
Q_A(q_A)\hat{q}_A + P_A(Q_A) - C^r_A(\hat{q}_A) \\
P^r_B(Q_B)q_B + P_B(Q_B) - C^r_B(\vartheta_B) \\
Q_B(q_B)\hat{q}_B + P_B(Q_B) - C^r_B(\hat{q}_B) \\
q_A - \vartheta_A \\
q_B - \vartheta_B
\end{bmatrix}
\]

\[
\mathcal{H}^{\text{post}}(x) \equiv \begin{bmatrix}
P^r_A(Q_A)q_A + P_A(Q_A) - C^r_A(\vartheta_A) \\
P^r_A(Q_A)\hat{q}_A + P_A(Q_A) - C^r_A(\hat{q}_A) \\
P^r_B(Q_B)q_B + P_B(Q_B) - C^r_B(\vartheta_B) \\
P^r_B(Q_B)\hat{q}_B + P_B(Q_B) - C^r_B(\hat{q}_B)
\end{bmatrix}
\]

where, given $n_m$ ‘rival’ firms in market $m$, $Q_m = q_m + n_m\hat{q}_m$. 

22
The equilibrium variables are given by the fixed points: \( H^{\text{pre}}(x^{\text{pre}}) = 0 \) and \( H^{\text{pst}}(x^{\text{pst}}) = 0 \). To show how these equilibrium outcomes differ across the two market structures, we invoke the mean value theorem: there exists a \( t \in [0, 1] \) such that, for \( \tilde{x} \equiv t \cdot x^{\text{pst}} - (1 - t) \cdot x^{\text{pre}} \), 
\[ H^{\text{pst}}(x^{\text{pre}}) + \nabla_x H^{\text{pst}}(\tilde{x}) \cdot (x^{\text{pst}} - x^{\text{pre}}) \]. As \( H^{\text{pst}}(x^{\text{pst}}) = 0 \), then
\[ x^{\text{pst}} - x^{\text{pre}} = -\left(\nabla_x H^{\text{pst}}(\tilde{x})\right)^{-1} H^{\text{pst}}(x^{\text{pre}}) \]

The term \( H^{\text{pst}}(x^{\text{pre}}) \) has zeros at all elements but the last element. Hence, only the last column of \( \left(\nabla_x H^{\text{pst}}(\tilde{x})\right)^{-1} \) is of interest in determining how equilibrium allocations differ between the pre-merger and post-merger market structures.

In order to simplify the calculations of \( \left(\nabla_x H^{\text{pst}}(\tilde{x})\right)^{-1} \), it is useful to introduce the following notation:

1. Denote with \( c_m \equiv C_m''(\vartheta_m) \)
2. Define \( \pi_{m,1} \) the slope firm 1’s profits with respect to firm 1’s output (including within profits the cost of producing additional output) and \( \pi_{m,11} \) as the concavity of these profits and \( \pi_{m,12} \) as the change in the slope of firm 1’s profits as \( \hat{q}_m \) increases:
   \[ \pi_{m,1}(x) \equiv P_m'(Q_m)q_m + P_m(Q_m) - C_m'(\vartheta_m) \]
   \[ \pi_{m,11}(x) \equiv P_m''(Q_m)q_m + 2P_m'(Q_m) - C_m''(\vartheta_m) \]
   \[ \pi_{m,12}(x) \equiv n_m \left( P_m''(Q_m)q_m + P_m'(Q_m) \right) \]

All these are functions of the decision variables \( x \).

3. Similarly, define \( \hat{\pi}_{m,1} \) as the slope of a given rival firm’s profits in market \( m \) with respect to the same firm’s output (\( \hat{q}_m \)), and \( \hat{\pi}_{m,11} \) and \( \hat{\pi}_{m,12} \) as the slope of \( \hat{\pi}_{m,1} \) with respect to the decision variables \( \hat{q}_m \) and \( q_m \)
   \[ \hat{\pi}_{m,1}(x) \equiv P_m'(Q_m)\hat{q}_m + P_m(Q_m) - C_m'(\hat{\vartheta}_m) \]
   \[ \hat{\pi}_{m,11}(x) \equiv n_m \left( P_m''(Q_m)\hat{q}_m + \frac{n_m + 1}{n_m} P_m'(Q_m) \right) - C_m''(\hat{\vartheta}_m) \]
   \[ \hat{\pi}_{m,12}(x) \equiv P_m''(Q_m)\hat{q}_m + P_m'(Q_m) \]

4. Finally, define \( E_m \) as the Jacobian of the FOCs of a single-market Cournot game in which firm 1 is allowed to behave differently than the rest of the firms:

\[ E_m \equiv \begin{bmatrix} \pi_{m,11} & \pi_{m,12} \\ \hat{\pi}_{m,12} & \hat{\pi}_{m,11} \end{bmatrix} \]
With these definitions, \( \nabla_x \mathcal{H}^\text{inst}(\bar{x}) \) is:

\[
\nabla_x \mathcal{H}^\text{inst}(\bar{x}) = \begin{bmatrix}
\pi_{A,11} + c_A & \pi_{A,12} & 0 & 0 & -c_A & 0 \\
\hat{\pi}_{A,12} & \hat{\pi}_{A,11} & 0 & 0 & 0 & 0 \\
0 & 0 & \pi_{B,11} + c_B & \pi_{B,12} & 0 & -c_B \\
0 & 0 & \hat{\pi}_{B,12} & \hat{\pi}_{B,11} & 0 & 0 \\
1 & 0 & 1 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & c_A & -c_B
\end{bmatrix}
\]

The last column of the inverse of \( \nabla_x \mathcal{H} \) is given by (using a symbolic solver):

\[
[\nabla_x \mathcal{H}]^{-1}_{(6)} = \frac{1}{\det[\nabla_x \mathcal{H}]} \begin{bmatrix}
c_A \hat{\pi}_{A,11} \det[\mathcal{E}_B] \\
-c_A \pi_{A,11} \det[\mathcal{E}_B] \frac{\hat{\pi}_{A,12}}{\pi_{A,11}} \\
-c_B \hat{\pi}_{B,11} \det[\mathcal{E}_A] \\
c_B \hat{\pi}_{B,11} \cdot \det[\mathcal{E}_A] \frac{\hat{\pi}_{B,12}}{\pi_{B,11}} \\
\det[\mathcal{E}_B] (\det[\mathcal{E}_A] + c_A \hat{\pi}_{A,11}) \\
-\det[\mathcal{E}_A] (\det[\mathcal{E}_B] + c_B \hat{\pi}_{B,11})
\end{bmatrix}
\]

We now prove no. 1 of the proposition in steps.

1. \( \hat{\pi}_{m,11} < 0 \) everywhere (omitting the market subscript): \( \hat{\pi}_{11} = n \left( P''q + \frac{n+1}{n} P'(Q) \right) - C''(\hat{q}) < n \left( P''q + \frac{n+1}{n} P'(Q) \right) \). By concavity\(^\text{20}\), \( P'' \leq 0 \), hence \( \hat{\pi}_{11} < 0 \).

2. \( \det[\mathcal{E}_m] > 0 \) for any value of \( x \). Specifically, \( \det[\mathcal{E}_m] = \pi_{m,11} \hat{\pi}_{m,11} - \pi_{m,12} \hat{\pi}_{m,12} \). We show \( \det[\mathcal{E}_m] > -c_m \hat{\pi}_{m,11} > 0 \) (omitting the market subscript):
   
   (a) Separate \( \pi_{11} \) and \( \hat{\pi}_{11} \) into two parts:
   
   \[
   \pi_{11} = \eta_1 - c \quad \eta_1 \equiv (P''q + 2P')
   \]
   
   \[
   \hat{\pi}_{11} = \hat{\eta}_1 - \hat{c} \quad \hat{\eta}_1 \equiv n \left( P''q + \frac{n+1}{n} P' \right)
   \]
   
   (b) \( \det[\mathcal{E}] \) is then:
   
   \[
   \det[\mathcal{E}] = (\eta_1 - c) \hat{\pi}_{11} - \pi_{12} \hat{\pi}_{12} = \eta_1 \hat{\pi}_{11} - c \hat{\pi}_{11} - \pi_{12} \hat{\pi}_{12} > \eta_1 \hat{\eta}_1 - c \hat{\eta}_1 - \pi_{12} \hat{\pi}_{12}
   \]
   
   \[
   \det[\mathcal{E}] + c \hat{\pi}_{11} > \eta_1 \hat{\eta}_1 - \pi_{12} \hat{\pi}_{12}
   \]
   
   (c) \( |\eta_1| > \frac{1}{n} |\pi_{12}| \) and \( \frac{1}{n} |\hat{\eta}_1| > |\hat{\pi}_{12}| \): since demand is downward\(^\text{21}\) sloping, \( P''q + 2P' < P''q + P' \) and \( P''q + \frac{n+1}{n} P'(Q) < P''q + P' \). Jointly, these conditions imply \( \det[\mathcal{E}] + c \hat{\pi}_{11} > 0 \).

\(^{20}\)A weaker assumption may be used. What is needed is that revenue be concave at any output between the pre-merger allocation and the post-merger allocation.

\(^{21}\)The proof relies on reaction functions being bounded between \(-1\) and \(0\) everywhere: \( |\pi_{11}| > |\pi_{12}| \), where \( \pi \) includes costs and is not just revenue. In the homogeneous good case presented here, that is achieved by assuming demand is downward sloping everywhere.
3. \( \det [\nabla_x H] > 0 \) for any value of \( x \):

\[
\det [\nabla_x H] = c_A \det [\mathcal{E}_B] (\det [\mathcal{E}_A] + c_A \hat{\pi}_{A,11}) + c_B \det [\mathcal{E}_A] (\det [\mathcal{E}_B] + c_B \hat{\pi}_{B,11})
\]

As \( \det [\mathcal{E}_m] + c_m \hat{\pi}_{m,11} > 0 \), as shown above, and \( c_m > 0 \), then \( \det [\nabla_x H] > 0 \).

Assume, wlog, that \( C'_A(\varphi^\text{pre}_A) > C'_B(\varphi^\text{pre}_B) \), such that in the pre-merger equilibrium, marginal revenue and marginal cost are higher in market \( A \) than in market \( B \). Given that \( \hat{\pi}_{A,11} < 0 \), \( \det [\nabla_x H] > 0 \) and \( \det [\mathcal{E}_m] > 0 \) as shown above, and that \( c_A \geq 0 \) by convexity of costs, the signs of the change in equilibrium outcomes are given as follows

\[
\text{sign} [x^{\text{pst}} - x^{\text{pre}}] = (+, -, -, +, +)'
\]

The total change in output in market \( A \) is given by

\[
\frac{-c_A \hat{\pi}_{A,11} \det [\mathcal{E}_B]}{\det [\nabla_x H]} \left( 1 - \frac{\hat{\pi}_{A,12}}{\hat{\pi}_{A,11}} \right)
\]

As we’ve already shown that \( \hat{\pi}_{m,12}/\hat{\pi}_{m,11} < 1 \), the total change in output in market \( A \) is positive. As consumer surplus is increasing in total output, consumer surplus increases. The opposite effects occur in market \( B \).

As for production, the merger results in decreased production in market \( A \) and increased production in market \( B \).