Abstract

This paper studies the effect of forward trading on the sustainability of collusion when spot market strategies take the form of supply functions. It is found that the existence of forward markets can enlarge the range of discount factors for which collusion can be sustained, whereas the opposite effect prevails when a potentially deviating firm holds a significant amount of forward contracts. The results are indifferent to the type of contract fulfillment (financial settlement or physical delivery), and compare to the case of Cournot competition in the spot market. Demand uncertainty has an ambiguous effect on the sustainability of collusion. It is found that the existence of liquid forward markets with anonymous trading are incompatible with the prevalence of collusion.

1 Introduction

Many commodities are traded forward such that the time of closing the contract significantly pre-dates the time of delivery. Forward trading allows producers to hedge price risks by locking-in future revenues well ahead of actual production. But
by selling forward, firms not only hedge risk, but also change their strategic position in the following spot market. In a classic paper, Allaz and Vila (1993) show how forward trading increases competition in a Cournot oligopoly. The contribution of the following analysis is to provide insights on the effect of forward trading on the stability of collusion when the spot market clears in supply functions, a setting that is specifically close to, but not restricted to the design of electricity wholesale markets.

The empirical relevance of collusion in such a market is evidenced by Sweeting (2007) who shows that two dominating electricity producers in the UK power market exerted substantial market power in a period when the general market conditions became more competitive. He concludes that this is in line with collusive conduct. The same UK power sector has served as a major example for a market where firms bid supply functions in a short term market and can close forward contracts several months ahead (see e.g. Green, 1999). In fact, most electric power exchanges worldwide impose some kind of supply function bidding for spot sales, and allow to hedge the price risk through forward contracts. The theoretical literature, however, has not yet addressed the question of collusion in such a setting.

The competitive effect of forwards in Allaz and Vila’s model is due to the repeated interaction of rival suppliers before the market finally clears. Each firm has an incentive to sell forward, locking-in its quantity for the spot market and thus to achieve a kind of Stackelberg advantage over its competitor. Thus, total supply in the spot market is larger with forward trading and the equilibrium price is lower. Since both firms are allowed to trade forward, they both face the same incentive to expand their output through forward contracting although they would be better off if there was no contracting at all. Mahenc and Salanie (2004) find the reverse result when studying price competition in the spot market. By buying forward, the producer commits to have an interest in raising prices in the spot market. A similar result is obtained by Ferreira (2003) for Cournot competition with infinitely many openings of the forward market. Green (1999) and Newbery (1998) show that the pro-competitive results of Allaz and Vila (1993) carry over to the case of supply function bidding in the spot market. Holmberg (2011) generalises these models to non-linear supply functions. Holmberg and Willems (2015) moreover consider supply functions and the strategic use of option contracts.

\footnote{see e.g. Wolak (2003) for California, Hortaçsu and Puller (2008) for Texas, Wolak (2007) for Australia, Green and Newbery (1992) for the UK, or Fabra and Reguant (2014) for Spain.}
None of these contributions yet addresses the possibility of collusion. Liski and Montero (2006) in contrast model a repeated game with several openings of the forward and the spot market. They show that the possibility of forward trading facilitates collusion for both, price as well as quantity competition in the spot market. But, somewhat parallel to the dichotomy of Allaz and Vila (1993), and Mahenc and Salanie (2004), the strategic effect of forward positions differs fundamentally between both cases. For a price-setting oligopoly, selling forward locks in the corresponding quantities and thus reduces the size of the market which could be captured by a deviating firm. This contrasts with competition in quantities where a deviating firm can never capture the complete spot market because other firms set their quantities in advance. So Liski and Montero (2006) conclude that when spot market competition is in prices, the critical discount factor for collusion is decreasing in the forward sales of firms. On the contrary, when firms compete in quantities, collusion is harder to sustain when firms have sold forward. Although the authors motivate their analysis with reference to electricity wholesale, neither Cournot nor Bertrand models provide an appropriate description of the supply function bidding which characterises these markets. Ciarretta and Gutiérrez-Hita (2006, 2012) model collusion in supply functions, but do not consider forwards.

Since the fundamental work of Klemperer and Meyer (1989), it is well known that a Supply Function Equilibrium (SFE) is located between the theoretical Cournot and Bertrand prices and quantities. One might therefore expect that the effect of forward trading on collusion in supply functions will be somehow contained between the results for price and quantity competition. But in light of the contrary results for both cases in the literature, there is no evident intuition what effect forward positions should have.

The following analysis sheds light on the effect of forward trading on the stability of tacit collusion, integrating the models of Green (1999), Liski and Montero (2006) and Ciarretta and Gutiérrez-Hita (2006). In Section 2, I present the model and the spot market strategies for the one-shot game and for joint profit maximisation. The repeated game with several forward and spot market openings is studied in Section 3. On one hand, the mere existence of a forward market helps to sustain collusion when firms fear to find themselves in a situation with substantial forward sales once the collusive agreement breaks. On the other hand, the feasible amount of forward sales during collusion depends on observability of firms forward positions. Forward trading ceases completely when colluding firms
forward positions are completely unobservable. Demand uncertainty and its effect on the sustainability of collusion is addressed in Section 4. In Section 5, I discuss the generalisation to non-linear supply functions in the spot market, and show the equivalence of financial versus physical settlement of forward contracts. Section 6 concludes.

2 The Static Game

Two firms $i, j$ produce a homogeneous good and compete on a spot market where they bid supply functions $q_i(p) = \alpha_i + \beta_i p$ and $q_j(p) = \alpha_j + \beta_j p$, with $p$ denoting the spot market price and $\beta_i, \beta_j > 0$. Demand is given by $D(p) = A - bp + \varepsilon$, where $\varepsilon$ is a zero mean random shift that realises after the firms made their decisions, and $p$ is determined by the market clearing condition $D(p) = q_i(p) + q_j(p)$. Finally, each firm produces at a cost of $C(q) = c_1 q + c_2 q^2$, the quantity that corresponds to its chosen supply function at the equilibrium price. It is assumed that $(A + \varepsilon)/b > c_1$ which ensures that there is always positive demand when the price equals the intercept of firms’ marginal costs.

Before bidding on the spot market, firms can close ‘contracts-for-difference’ (financial forwards), specifying that once the spot market cleared, the seller of the contract receives the forward price $f$ times the contracted quantity from the buyer, and pays to the buyer the realised spot price $p$ times the contracted quantity respectively. The forward market follows the description of Allaz and Vila (1993): Both firms simultaneously disclose the quantities $x_i, x_j$ of contracts they intend to sell. There are at least two risk neutral and not liquidity constraint speculators $s = 1, 2, \ldots$, which observe the total amount of open interest $x_i + x_j$, and choose simultaneously their individual price $f_s$ at which they are willing to buy the offered contracts. The highest price bid wins the total amount of contracts.\footnote{For details see Appendix A of Allaz and Vila, 1993. Equivalently, one could assume a large number $N$ of individually liquidity constrained speculators, such that each of them behaves competitively and with $N$ large enough such that there is no relevant liquidity constraint on the market.} This competitive forward market imposes a no arbitrage condition $f = p$ in expectation, although demand shocks might result in $f \neq p$ ex post. In line with Green (1999), I do not consider producers going long in the forward market, e.g. buying forward their own output.
The ex-post realised profit of firm $i$ is

$$\pi_i = (f - p)x_i + pq_i(p) - C(q_i(p)),$$

and the spot market clearing price can be expressed as

$$p = \frac{A + \varepsilon - \alpha_i - \alpha_j}{b + \beta_i + \beta_j}$$

### Spot Market Strategies

In the spot market, firms choose their supply function, taking the supply function of the rival and the forward positions of both firms as given. In a general setting with flexible supply functions, the first order condition would involve a complex functional derivative: $\frac{d\pi_i}{dq_i(p)} = 0$ subject to the market clearing condition. Klemperer and Meyer (1989) show how the flexibility of the supply function allows firms to choose an optimal price for any realisation of their residual demand, which leads to a more tractable first order condition:

$$\text{for all } p \quad \frac{d\pi_i}{dp} = 0 \quad \text{s.t. } q_i(p) = D(p) - q_j(p)$$

The complexity of the problem is somewhat reduced for the linear case because firms just choose the slope $\alpha$ and the intercept $\beta$ of their supply function. However, the direct first order conditions do not allow identifying the optimal slope and intercept parameters because $\frac{d\pi_i}{dq_i} = p \cdot \frac{d\pi_i}{d\alpha_i}$. As already shown by Turnbull (1983), solving for unique best response supply functions requires to take some price variation into account. Following Klemperer and Meyer (1989) (and more specifically Newbery, 1998, and Green, 1999) we evaluate what price firm $i$ would choose optimally for an arbitrary demand realisation, and then derive the corresponding values for $\alpha_i$ and $\beta_i$.

Replace firm $i$’s quantity by its residual demand $q_i = D(p) - q_j(p)$ to rewrite the profit of firm $i$.

$$\pi_i = (f - p)x_i + p [D(p) - q_j(p)] - C(D(p) - q_j(p))$$

The first order condition for an optimal price at an arbitrary realisation of $\varepsilon$ is

$$\frac{d\pi_i}{dp} = -x_i + D(p) - q_j(p) + (D' - q_j') (p - c_1 - c_2[D(p) - q_j(p)]) = 0.$$
We can again use \( q_i(p) = D(p) - q_j(p) = \alpha_i + \beta_i p \) to obtain
\[
\frac{d\pi_i}{dp} = -x_i + \alpha_i + \beta_i p - (b + \beta_j) \left( p - c_1 - c_2(\alpha_i + \beta_i p) \right) = 0. \tag{4}
\]

The first order condition should hold for any realisation of demand and the corresponding spot prices \( p \). Thus, it must hold as an identity, which allows to equal all factors multiplying \( p \) at the same power.\(^3\) The optimal supply function parameters are derived by Green (1999, pp. 111-112):
\[
\beta_i^* = BR(\beta_j) \equiv \frac{b + \beta_j}{1 + c_2(b + \beta_j)} \tag{5}
\]
\[
\alpha_i^* = \frac{x_i - c_1(b + \beta_j)}{1 + c_2(b + \beta_j)} = \beta_i^* \left( \frac{x_i}{b + \beta_j} - c_1 \right) \tag{6}
\]

Equations (5) and (6) (equivalent to Green, 1999, Equ. 7) allow to express the best response supply function of \( i \) given \( \beta_j \) as:
\[
q_i^* = \beta_i^* \left( p + \frac{x_i}{b + \beta_j} - c_1 \right)
\]

The inverse of this supply function is such that the price is equal to marginal costs at the level of forward contracts (Proposition 1 Green, 1999). This effect is illustrated in Figure 1: For the case without forward sales, the inverse supply function coincides with marginal costs at the intercept but exhibits a higher slope. With positive forward sales, the supply function shifts outwards such that it crosses marginal costs at the level of the contracted quantity. Larger quantities are sold above marginal costs, smaller quantities even below marginal costs.

The definition in (5) represents the well known slope of a linear supply function duopoly (see e.g. Green, 1996). It can be solved for the symmetric non-cooperative equilibrium slope which will be denoted by \( \beta_n \) in the remainder of the paper:
\[
\beta_n = BR(\beta_n) > 0
\]

Note that \( \beta_i^* \) and therefore \( \beta_n \) do not depend on firms forward positions.

\(^3\)e.g. assume \( p = 0 \) to obtain the expression for \( \alpha_i \). Then, for (4) to hold at any positive price, it is clear that all the factors multiplying \( p \) must sum to zero as well, which in turn allows to derive an expression for \( \beta_i \).
Forward Positions

Firms choose their forward positions, anticipating the following spot market equilibrium. The spot market best response in (5) and (6), however, shows that firms only react to the rivals strategy through the slopes $\beta_i, \beta_j$, and that these slope parameters are unaffected by forward positions $x_i$ and $x_j$.

Phrased differently: In the linear SFE model, forward contracting of firm $i$ shifts out its own supply function through the intercept $\alpha_i$, therefore increases its sales in the spot market, and decreases the spot market price. But the rivals strategy is unaffected. Inherently, there is no strategic effect of selling forward in the linear model, and therefore no incentive to do so. That is why Newbery (1998) and Green (1999) model the choice of forward positions through conjectural variations which can be calibrated.

This paper at first follows the approach of Green (1999) and Newbery (1998) to focus on linear supply functions and treating forward positions as parameters to calibrate. On one hand, this allows to determine if firms could adopt forward positions which would facilitate or hinder collusion. On the other hand, the linear SFE model with exogenous forward positions provides a reference case for the discussion of non-linear models with endogenous forward contracting, which is provided in Section 5.2.

Joint profit maximisation

I will study joint profit maximisation, first abstracting from incentive compatibility which is addressed in Section 3. While a cartel can control all supply in the spot market, there still are competitive speculators in the forward market. Knowing that $f = p$ in expectation, forward positions completely cancel from the profit function (1). The optimal strategies for joint profit maximisation are therefore equivalent to the situation without forward trading and can be adopted from Ciarreta and Gutiérrez-Hita (2006)$^4$: 

$$\beta_c \equiv \frac{b}{2 + c_2 b}$$ (7)

$$\alpha_c \equiv -\frac{-bc_1}{2 + c_2 b} = -\beta_c c_1$$ (8)

$^4$Note that Ciarreta and Gutiérrez-Hita (2006) normalise $c_1$ to zero and omit the supply function intercept $\alpha$. 

7
Note that in principle, a monopolist which has contracted forward at a monopolistic price would have the incentive to decrease the actual price in the following spot market, thus widening the gap \( f - p \) for to profit from its forward positions. But this can never be an equilibrium as speculators would not be willing to buy at the high price in the first place. The monopolist in contrast has no incentive to sell forward below the optimal monopoly price that she can implement on the spot market. So forward trading will never occur in a one shot monopoly game. (For a detailed discussion of forward trading in monopoly see Muermann and Shore, 2005.) In a repeated game setting, however, a situation where the monopolists sells forward and sticks to the monopolistic supply function given by (8) and (7) on the spot market can be an equilibrium when the monopolist can credibly commit to this strategy. We will see later on that a collusive agreement provides such a commitment.

Figure 1 illustrates the different spot market strategies.

\[
\begin{array}{c}
\text{price, cost} \\
\end{array}
\]

\[
C'(x_1) \quad \text{Nash supply fct. (no forwards)} \\
\text{Nash supply fct. (financial forwards)} \\
collusive supply fct. (forward has no effect) \\
\text{marginal costs}
\]

\[
\text{spot market supply, output}
\]

Figure 1: Collusive and Nash spot market supply with and without financial contracting (see Ciarreta and Gutiérrez-Hita (2006) and Green (1999))

Consider \( q_i = \alpha_c + \beta_c p = 0 \) and observe that this implies \( p = c_1 \). So the very first unit of collusive supply is priced at marginal cost. The same is true for non-collusive supply when firms do not hold any forwards. Larger quantities are then supplied with an increasing mark-up. Obviously, the collusive supply curve exhibits increasingly higher mark-ups on marginal cost compared to its non-collusive counterpart: \( \beta^{-1}_c > \beta^{-1}_n > c_2 \).
3 The Repeated Game

Consider now the case of infinitely many interactions of the same two firms on, first, the forward market for the coming spot market, then the corresponding spot market, then the forward market for the next period, and so on. I will focus on trigger strategies where firms revert permanently to the one-shot Nash equilibrium once the collusive agreement breaks. A deviating firm can thus at best earn deviation profits once. I am moreover abstracting from the effect of uncertainty on expected profits which is addressed in Section 4.

Sustainability of collusion in the spot market

Suppose first, that deviation only occurs in the spot market. Spot market profits are maximised when the deviating firm bids its best response given by (5) and (6) to the collusive supply function of its competitor as defined in (7) and (8). Let \( \alpha_d \) and \( \beta_d \) denote the deviating strategy parameters with \( \beta_d = BR(\beta_c) \) and note that \( \beta_n > \beta_d > \beta_c \). Furthermore, \( \pi^d \) designates the one period profit of deviation. The incentive constraint in it’s standard form is

\[
\frac{1}{1-\delta}\pi^c > \pi^d + \frac{\delta}{1-\delta}\pi^n
\]

where \( \delta \) is the discount factor, and \( \pi^c, \pi^n \) are the profits which arise when both firms choose supply functions with intercept \( \alpha_c \) and slope \( \beta_c \) or intercept \( \alpha_n \) and slope \( \beta_n \) respectively. (The same subscripts are used for prices \( f, p \), and quantities \( q \) later on.) Again, it is obvious that \( \pi^d > \pi^c > \pi^n \). One can thus determine the minimum discount factor \( \hat{\delta} \) for which the incentive constraint just binds as an equality.

\[
\hat{\delta} = \frac{\pi^d - \pi^c}{\pi^d - \pi^n}
\]

So how does forward trading affects the possibility to sustain collusion? Let \((x_i^c, x_j^c)\) denote the forward positions of firms during collusion and \((x_i^n, x_j^n)\) the forward position of firms during the punishment phase. The key result is summarised in Lemma 1.
Lemma 1. Collusion is less easy to sustain when firms have sold forward during collusion. Precisely, the critical discount factor $\delta$ is increasing in $\max(x^c_i, x^c_j)$. Collusion is easier to sustain when firms expect significant forward sales during the punishment phase, meaning, the critical discount factor is decreasing in a joint variation of $x^n_i$ and $x^n_j$.

Whereas Liski and Montero (2006) have found two diametrical effects of forwards for price versus quantity competition, Lemma 1 states that the result for competition in supply functions – which is indeed a generalisation to the concepts of competition merely in prices or quantities – resembles the case of quantity competition. Collusion is easier to sustain if firms do not hold any forwards, but fear to be trapped in a situation with large forward sales and consequently in a more competitive spot market. This clearly contrasts with the result of Liski and Montero (2006) for price competition in the spot market where a reverse argument applies.

A sufficient condition for Lemma 1 is the following proposition which is also needed for the further discussion.

Proposition 1.

(a) Collusive profits $\pi^c$ are unaffected by the forward positions $x_i, x_j$ held by firms.

(b) Profits $\pi^d_i$ of a deviating firm $i$ are convex and increasing in the number of its forward positions $x_i$ irrespective of the forward position of its rival. Precisely,

$$\frac{d\pi^d_i}{dx_i} = f_c - p_d.$$

(c) Non-cooperative Nash profits $\pi^n_i$ of a firm $i$ are concave and decreasing in the number of forwards $x_i = x_j = x$ hold by both firms.

Proof. The proof of Proposition 1.a is trivial. Collusive profits are unaffected by the forward positions of firms because the spot strategies (7) and (8) are constant in $x_i$ and the forward price equals the expected spot price.

For the proof of proposition 1.b, note that a firm which deviates in the spot market earns additional revenues for the difference of the monopoly price, which is contracted in advance, and the spot price, which is lower than the contracted
price because of deviation. The formal proof for Proposition 1.b is in Appendix A.1.1.

For the proof of Proposition 1.c, note that non-cooperative profits are decreasing the more the firms contract forward, because the spot market becomes more competitive as shown by Green (1999). A detailed proof of Proposition 1.c is in Appendix A.1.2.

Consider the definition in (9). Proposition 1 implies that the critical discount factor $\delta$ is increasing in the level of contracts held by a firm during collusion and decreasing in the number of contracts which firms expect in the punishment phase, which is equivalent to Lemma 1.

Figure 2 illustrates this argument by depicting collusive, non-cooperative and deviative profits over different levels of contracted quantities. The critical discount factor $\delta$ has a direct graphical interpretation: at a given level of contracting during collusion, $\delta$ is equivalent to the distance between deviation profits and collusive profits, divided by the distance between deviation profits and Nash profits at the expected level of contracting during Nash reversion.

![Figure 2: Profits over different levels of forward contracting (symmetric in case of Nash).](image)

Proposition 1 moreover has an implication which is needed for the discussion in Section 4:

**Corollary 1.** There are infinitely many combinations $(x_d, x_n)$ of forward positions $x_d$, held by a deviating firm, and the forward positions $x_n$ held by both firms.
during the punishment phase, such that the critical discount factor $\delta$ is equivalent to the case without forward markets (e.g. $x_d = x_n = 0$). This constant level of $\delta$ is in the following denoted $\delta_0$.

**Proof.** Immediate from definition (9), Proposition 1, and the fact that profits are continuous in the level of forward sales. \hfill $\Box$

**Observability and deviation in forward markets**

Deviation has only been discussed for the spot market strategies yet. The definition of ‘deviation in the forward market’, however, is not evident when the spot market clears in supply functions. Liski and Montero (2006) consider the case where a deviating firm sells more than the monopoly quantity forward. In the model studied here, there is no fixed monopoly quantity which can be surpassed by a deviating firm, but just a collusive supply function which is independent of the forward positions of firms.

So, what can a deviating firm achieve in the forward market? At best, it can cheat on speculators or customers by selling forwards at the collusive price, and then depress the spot price to cash the difference between spot and forward prices. Proposition 1.b specifies, that there is no optimal level of forward sales for the deviating firm, rather, a deviator has the incentive to sell as many forwards as possible. Speculators, in contrast, will not accept to buy forwards at the monopolistic price when the sustainability of collusion is in doubt. Consider three possible cases:

1. Firms individual forward positions are fully observable and the sustainability of collusion, given the forward sales $x_i$ and $x_j$, is straightforward to detect by the contracting counterparts of a potential deviator. Risk neutral speculators will accept to buy forwards at the monopolistic price as long as the incentive constraint holds. This puts a restriction on the maximum number of forwards a single firm can sell.

2. When total open interest $x_i + x_j$ is observable but individual positions are not, speculators cannot distinguish what firm seeks to sell forward which quantity. The worst case for the sustainability of collusion would be that all offered quantities are from one firm. So the restriction on the maximum number of forwards mentioned above now applies to the total volume on the market.

3. Finally, suppose that forward positions are not observable at all. A speculator receives an offer of a firm to buy forwards, but cannot verify if the same firm
has also sold to other parties. Speculators infer that firms have an incentive to sell forwards at a monopolistic price and then to depress the spot price later on. With this expectation, they will not accept any price higher than the expected spot price in case of deviation $p_d$. But the latter is continuously decreasing in the number of forwards sold, and the incentive of the deviator to sell forward is unbounded. So no rational agent will buy any contract and forward trading ceases completely.

Taken together, a potentially deviating firm cannot make any additional profit from selling forward. So a firm will either sustain collusion if the incentive constraint holds, or deviate in the spot market. In either case, ‘deviation in the forward market’ is not a relevant or attractive strategy. This implies that a stable collusive agreement provides a credible commitment of the producers not to depress the spot price, once they have sold forward a moderate amount of contracts. The latter is in contrast to the case of a single monopolist discussed in Muermann and Shore (2005).

Moreover, we can now conclude that collusion is unlikely to prevail in markets where significant volumes are traded forward and where firms’ positions are unobservable.

4 Uncertainty of Demand

The preceding analysis just considered firms profits ($\pi_i, \pi_j$) at a fixed level of demand ($\varepsilon = 0$), which is in line with the majority of the literature, where some demand variation is necessary to identify the optimal slope of supply functions, but the profit effect of that variation is largely ignored. Ciarreta and Gutiérrez-Hita (2006), for example, do not discuss the effect of demand shocks on expected profits although the authors assume demand uncertainty for the definition of the equilibrium strategies. This is surprising at first glance because the range of possible price-cost margins increases with higher variability of demand. The critical discount factor $\delta$ is defined by the ratio of additional profits from deviation compared to sustained collusion, and it is not obvious that this ratio would be constant in the variance of demand shocks. The following lemma therefore complements the results from the preceding section and those of Ciarreta and Gutiérrez-Hita (2006).

**Lemma 2.**
a. When there are no forward sales, neither during collusion nor during the punishment phase, the critical level of the discount factor $\delta = \delta_0$ is the same for any level of variance $\sigma^2$ of the demand shock $\varepsilon$:

$$\frac{d\delta_0}{d\sigma^2} = 0$$

b. When firms sell forward during collusion or during the punishment phase such that $\delta \neq \delta_0$, then

$$\delta > \delta_0 \Rightarrow \frac{d\delta}{d\sigma^2} < 0$$

and

$$\delta < \delta_0 \Rightarrow \frac{d\delta}{d\sigma^2} > 0$$

Lemma 2.a implies that Ciarreta and Gutiérrez-Hita (2006) are safe to ignore the effect of uncertainty because it completely cancels from the incentive constraint when there are no forward markets. Lemma 2.b can be resumed less accurate but more intuitively as: the larger is the variability of demand, the smaller is the effect of forward positions on the critical discount factor.

Proof. Recall the definition from Equation (1) of profits $\pi_i$ at a given realisation of demand, and use $q_i = \alpha_i + \beta_i p$ to rewrite $\pi_i$ in terms of powers of $p$.

$$\pi_i = (fx - c_1 \alpha_i) + p \left( -x + \alpha_i - c_1 \alpha_i - \frac{c_2}{2} \beta_i^2 \right) + p^2 \left( \beta_i - \frac{c_2}{2} \beta_i^2 \right)$$

Demand shocks affect the firms profit through the market clearing price. Let $\hat{\pi}_i$ denote the ex-ante expected profit of firm $i$,

$$\hat{\pi}_i = (fx - c_1 \alpha_i) + E(p) \left( -x + \alpha_i - c_1 \alpha_i - \frac{c_2}{2} \beta_i^2 \right) + E(p^2) \left( \beta_i - \frac{c_2}{2} \beta_i^2 \right),$$

and note from Equation (2) that the market clearing price is linear in the shock $\varepsilon$. Because $E(\varepsilon) = 0$ and the expected value function is linear in its arguments, the variance of $\varepsilon$ affects $\hat{\pi}_i$ only through the quadratic term $p^2$. This allows to write expected profits as $\hat{\pi}_i = \pi_i + Var(p)(\beta_i - \frac{c_2}{2} \beta_i^2)$. From the equilibrium price in (2) follows $Var(p) = \sigma^2(b + \beta_i + \beta_j)^{-2}$. Expected profits therefore become:

$$\hat{\pi}_i = \pi_i + \sigma^2 \left( \frac{\beta_i - \frac{c_2}{2} \beta_i^2}{(b + \beta_i + \beta_j)^2} \right).$$

(10)
which allows to obtain a handy proposition for the proof of Lemma 2:

**Proposition 2.** The derivatives of expected profits of deviation, collusion and Nash-reversion with respect to $\sigma^2$ are all positive, constant in $\sigma^2$, and can be ordered as follows:

$$\frac{d\hat{\pi}_d}{d\sigma^2} > \frac{d\hat{\pi}_c}{d\sigma^2} > \frac{d\hat{\pi}_n}{d\sigma^2}$$

A full proof of Proposition 2 is in Appendix A.2.

For an illustration, note that Proposition 2 states that the ordering of derivatives is the same as the order of the levels: profits of deviation are larger than collusive profits, which again are larger than non-cooperative equilibrium profits. Figure 3 depicts expected profits over different levels of $\sigma^2$ in a similar manner as Figure 2 depicts profits over different levels of forward contracting. Again, the critical discount factor is expressed in terms of profits as $\hat{\delta} = (\hat{\pi}_d - \hat{\pi}_c)/(\hat{\pi}_d - \hat{\pi}_n)$. In Figure 3 these differences in profits are visualised for the case without any forward trading by the distances $BC/BD$ at some arbitrary level of $\sigma^2$.

![Figure 3: Expected profits over variance of demand shocks $\sigma^2$](image)

Due to linearity in $\sigma^2$, we can use the intercept theorem to analyse the evolution of profit relations for different levels of $\sigma^2$. When the three lines have a common point of intersection, the intercept theorem applies and the fraction $(\hat{\pi}_d - \hat{\pi}_c)/(\hat{\pi}_d - \hat{\pi}_n)$ will be constant for any level of $\sigma^2$ located on the same side of the point of intersection. In Figure 3, this intersection occurs at $\sigma^2 = -(A - bc_1)^2$. A negative
value for the variance obviously has no reasonable interpretation. Here, it just provides the joint zero of the three lines.

The following paragraphs show that the intercept theorem applies generally for the case without forward trading. Consider the case without forward sales, \( x_i = 0 \), and rewrite profits from Equation (1) as

\[
\pi_{i,x_i=0} = q_i(p) \left( p - c_1 + \frac{c_2}{2} q_i(p) \right).
\] (11)

Note that the supply function intercept \( \alpha_i \) reduces to \( \alpha_i = -\beta_i c_1 \), irrespective of firm \( i \) being engaged in collusion (8) or playing it’s best response (6). Thus the firms quantity can be written as \( q_i = \beta_i (p - c_1) \). Substituting \( q_i \) and the spot market clearing price \( p \) from Equation (2) into (11) gives

\[
\pi_{i,x_i=0} = \left( \frac{A - bc_1}{b + \beta_i + \beta_j} \right)^2 \left( \beta_i - \frac{c_2}{2} \beta_i^2 \right).
\]

Reconsider the expression for \( \hat{\pi}_i \) from Equation (10), and replace \( \pi_i \) with \( \pi_{i,x_i=0} \) as defined above.

\[
\hat{\pi}_{i,x_i=0} = \frac{\beta_i - \frac{c_2}{2} \beta_i^2}{(b + \beta_i + \beta_j)^2} \left( (A - bc_1)^2 + \sigma^2 \right)
\]

It is obvious that \( \hat{\pi}_{i,x_i=0} \) has a null for \( \sigma^2 = -(A - bc_1)^2 \), no matter what strategies \( \beta_i, \beta_j \) the firms play. Therefore, without forward trading, \( \hat{\pi}^d, \hat{\pi}^c \) and \( \hat{\pi}^n \) have a common point of intersection in the plane spanned by \( \hat{\pi}_i \) and \( \sigma^2 \). So the intercept theorem applies and the fraction \( (\hat{\pi}^d - \hat{\pi}^c) / (\hat{\pi}^d - \hat{\pi}^n) \) is constant in \( \sigma^2 \). This fraction is equivalent to \( \delta \), which ends the proof of Lemma 2.a.

Now, suppose a deviating firm has sold forward a quantity \( x_d > 0 \). By Proposition 1, its ex-post profits will be higher than without forward contracting. But by Proposition 2, the derivative of profits with respect to \( \sigma^2 \) is unchanged. Therefore, the line depicting its expected profits \( \hat{\pi}^d \) over \( \sigma^2 \) is shifted upwards as shown in Figure 3 by the line through point E. The three lines for collusive, non-cooperative and deviation profits do not intersect in a common point. Expected profits of deviation instead intersect non-cooperative profits at some level of \( \sigma^2 \) which is larger (less negative) compared to the intersection with collusive profits. Therefore, \( \hat{\pi}^d - \hat{\pi}^n \) increases in larger proportion with \( \sigma^2 \) than \( \hat{\pi}^d - \hat{\pi}^c \) for any \( \sigma^2 \geq 0 \). This implies that the critical discount factor \( \delta \) is now decreasing with \( \sigma^2 \).

Conversely, consider the case when colluding firms and a possible deviator do not sell forward, but firms expect positive forward sales in the punishment phase:
\( x_d = 0 \) and \( x_n > 0 \). By Proposition 1, the profits of Nash-reversion \( \pi^n \) are lower compared to the situation without forwards. The corresponding line in Figure 3 shifts downwards. A reverse argument to the one before above applies. Now, \( \hat{\pi}^d - \hat{\pi}^c \) will increase in larger proportion with \( \sigma^2 \) compared to \( \hat{\pi}^d - \hat{\pi}^n \).

So it is evident, that there are infinitely many combinations of \( x_d \) and \( x_n \) which shift deviation profits upwards and punishment profits downwards such that the three lines still keep a common point of intersection. These combinations of \( x_d \) and \( x_n \) therefore imply that the ratio of \( \hat{\pi}^d - \hat{\pi}^n \) and \( \hat{\pi}^d - \hat{\pi}^c \) is the same for all non-negative levels of \( \sigma^2 \). Take \( \sigma^2 = 0 \), and we are back to the case without uncertainty which is described in Corollary 1. Therefore,

\[
(x_d, x_n) \text{ such that } \delta = \delta_0 \iff \frac{d\delta}{d\sigma^2} = 0
\]

\[
(x_d, x_n) \text{ such that } \delta > \delta_0 \iff \frac{d\delta}{d\sigma^2} < 0
\]

\[
(x_d, x_n) \text{ such that } \delta < \delta_0 \iff \frac{d\delta}{d\sigma^2} > 0
\]

where \( \delta_0 \) is the critical discount factor from Corollary 1. This ends the proof of Lemma 2.b and completes the proof of Lemma 2.

\[ \Box \]

5 Generalisations

5.1 Physical vs. Financial Forwards

Most commodities nowadays are traded forward with both physical and financial delivery. Liski and Montero (2006, footnote 5) point out that in homogeneous goods and with price competition, financial contracts might have substantially different effects on collusion compared to physical contracts. The following paragraphs show that considering physical forward contracts is equivalent in terms of equilibrium price and revenues to the case of only financial forward contracting which has been studied before.

In the following, \( x_{i,f}, x_{j,f} \) denote financially contracted quantities which are cleared by a balancing payment of the (positive or negative) difference \( f - p \) from the buyer to the seller at the time when the spot market clears. Physical forward sales \( x_{i,\phi}, x_{j,\phi} \) oblige the seller to produce the corresponding amount of electric power at the contracted time and allow the buyer to consume this energy without bidding for it on the corresponding spot market. The buyer pays the contracted
price to the seller upon delivery. A basic no-arbitrage condition imposes equality of the forward price for physically and financially settled contracts which is jointly denoted by \( f \). Spot market demand is now \( D(p) - x_{i,\phi} - x_{j,\phi} \) because some demand has already been contracted in advance. \( q_i, q_j \) still denote total output of firms, but this is not necessarily equal to spot market supply any more. Let \( s_i(p), s_j(p) \) denote the spot market supply functions of the linear form, \( s_i = \alpha_i + \beta_i p \), and note that \( q_i = x_{i,\phi} + s_i(p) \). While the firms output must be non-negative\(^5\), supply in the spot market can be positive or negative; in other words, firms might buy back on the spot market what they have sold forward before. The profit function becomes

\[
\pi_i = (f - p)x_{i,f} + fx_{i,\phi} + ps_i(p) - c_1(s_i(p) + x_{i,\phi}) - \frac{c_2}{2}(s_i(p) + x_{i,\phi})^2
\]

The derivation of optimal strategies follows the same procedure as in Section 2. The first order condition, equivalent to Equ. (4), is now

\[
\frac{d\pi_i}{dp} = s_i - x_{i,f} - (b + \beta_j)(p - c_1 - c_2(x_{i,\phi} + s_i)) = 0,
\]

Using \( s_i = s_i(p) = \alpha_i + \beta_i p \) and equating all factors multiplying \( p \) yields the same result for the optimal slope \( \beta_i \) as in (5) which is independent of the firms forward positions. The intercept, however, now accounts for the physically forward contracted quantities. Definition (6) is generalised as follows:

\[
\alpha_i^* \equiv \beta_i \left( \frac{x_{i,f}}{b + \beta_j} - c_1 - c_2x_{i,\phi} \right) \tag{6'}
\]

The market clearing condition determines the spot market price:

\[
p = \frac{A - \alpha_i - \alpha_j - x_{i,\phi} - x_{i,\phi} + \varepsilon}{b - \beta_i - \beta_j},
\]

Take the definition in (6') and note that by Equ. (5), \( \beta_i/(b + \beta_j) \) can be expressed as \( 1 - \beta_i c_2 \). The market clearing price becomes:

\[
p = \frac{A + \varepsilon - (x_{i,\phi} + x_{j,\phi} + x_{i,f} + x_{j,f})(1 - \beta_n) + 2\beta_n c_2}{b - 2\beta_n},
\]

which shows that physical \((x_{i,\phi}, x_{j,\phi})\) and financial forward positions \((x_{i,f}, x_{j,f})\)

\(^5\)Non-negativity of firms output is still secured by the assumption, \((A + \varepsilon)/b > c_1) \forall \varepsilon\), but the notation makes it less evident here.
have an identical effect on the non-cooperative equilibrium price in the spot market, and therefore on total output. It is easy to verify that also individual quantities $q_i$, $q_j$, are the same for a given level of forward sales, independent if these are financial or physical forward contracts. Moreover, the optimal collusive strategy obviously does not change through the mere possibility to sell physically forward, and the same applies for the incentives to deviate. The strategic effect of either form of contracting is equivalent.

The only difference between the effect of physical and financial forwards is that physical forward sales substitute trades in the spot market, thus, they reduce the observed volumes. Figure 4 shows how firms best response supply functions vary for the case of physical and financial contracting respectively. This might be of relevance when studying real world situations where a-priori firms can be financially or physically contracted. The ‘supply function’ now extends to the range of negative quantities. In other words, firms which have sold physically forward their output will buy on the spot market for prices below the marginal cost of their contracted output and sell additional units when the realised price exceeds marginal costs.

Figure 4: Spot supply of firms with physical vs. financial forward positions
5.2 Non-linear supply functions and the strategic effect of forward sales

The linear SFE model is widely used and has proven to work well in empirical applications (Green, 1996). But it also represents a very special case with respect to forward trading, where firms with Nash conjectures have no strategic incentive to sell forward at all. Real world markets suggest rather that the majority of sales are forward. Moreover, empirically observed supply functions are typically non-linear e.g. when supply is limited by production capacity. Klemperer and Meyer (1989) have shown that there generally exists a multitude of non-linear supply function equilibria which are bound from above by the supply function which finally becomes inelastic and hits the Cournot price and quantity for the maximum realisation of demand. A lower bound is given by the Bertrand solution. Electricity markets are typically characterised by some form of convex marginal costs, e.g. due to capacity constraints, which makes increasingly inelastic supply functions bending towards the Cournot solutions for higher levels of demand plausible. Indeed, Green and Newbery (1992), Holmberg (2008), and Genc and Reynolds (2011) show how capacity constraints can narrow the range of equilibria found by Klemperer and Meyer (1989) towards the upper bending supply functions. Brandts et al. (2014) provide supporting experimental evidence. In summary, this literature suggests that non-linear supply function equilibria that become increasingly inelastic for higher levels of demand are likely to prevail in markets where firms total production capacity is limited.

So what are the effects of forwards in such a non-linear setting? Consider a situation with convex marginal costs. Without forwards, firms fix the lower end of their supply function \((q = 0)\) at a price equal to marginal costs and bid above marginal costs for all positive quantities. When firm \(i\) sells \(x_i\) forward contracts, its optimal supply function shifts outwards such that it crosses marginal costs at the forward contracted quantity. But contrary to the linear case, the slope of marginal costs is not constant and also the slope of the supply function changes with the level of forward sales. This implies that by contracting forward, firm \(i\) can commit to alter the slope of its supply function in the spot market, and because the supply function of firm \(j\) depends on the slope of firm \(i\)’s supply function, \(i\) has now an instrument to affect firm \(j\)’s strategy (see Green, 1999, pp. 115-116 for a more detailed discussion). Holmberg (2011) proves existence of a unique Nash equilibrium with positive forward sales, less elastic supply functions,
and larger quantities compared to the case without forward trading under fairly general conditions.

**Non-linear SFE and collusion**

So how would considering such non-linear supply functions affect the incentives for forward trading and collusion in a repeated game? While the results from Section 3 are obtained from the linear model, an equivalent reasoning applies when non-linear SFE are possible.

Consider Proposition 1.a: Collusive profits are unaffected by the possibility to sell forward. This is also true when supply functions can be non-linear. The monopoly solution maximises joint profits in the spot market. Additional profits from selling forward could only be due to a difference between spot and forward prices, which contradicts the assumption of a competitive forward market. Thus, colluding firms always implement the joint profit maximising supply function, linear or not, independent of their forward positions.

Consider Proposition 1.b: A deviating firm earns both on the expense of its rivals and on the expense of speculators. After the forward price is locked in at the collusive level, the deviator plays its spot market best response \( q_i^*(p) \) to the collusive supply function \( q_j^*(p) \) of its rival. Both supply functions are restricted to be non-decreasing and fully differentiable. The best response of \( i \) fulfils the first order condition

\[
\frac{d\pi_i}{d q_i^*(p)} = 0 \quad \text{for all } p.
\]

The derivative of profits of deviation with respect to the number of forwards sold by the deviating firm can be decomposed as follows (with \( f \) fixed in advance):

\[
\frac{d\pi_i^d}{dx_i} = \frac{\partial \pi_i^d}{\partial x_i} + \left( \frac{\partial \pi_i^d}{\partial p_i} \frac{dp}{dq_i(p)} + \frac{\partial \pi_i^d}{\partial q_i^*(p)} \frac{dq_i^*(p)}{dx_i} \right) \frac{dq_i^*(p)}{dx_i}.
\]

Note that in principal, the partial derivative of price to \( q_i(p) \) involves the difficult to handle functional derivative \( dq_j(p)/dq_i(p) \) which captures the strategic interaction of spot market strategies. Here, we can ignore this complication for a simple reason: In the decomposition of profits given above, The whole term in parentheses multiplying \( q_i^*(p)/x_i \) equals zero by the definition of \( q_i^d(p) \) as firm \( i \)'s best response. This allows to simplify:

\[
\frac{d\pi_i^d}{dx_i} = \frac{\partial \pi_i^d}{\partial x_i} = f^c - p^d
\]

21
which is to equivalent to the finding in 1.b for the linear case.

Finally, consider Proposition 1.c: the profits of Nash-reversion decrease with increased forward sales of firms because contracting increases competition in the spot market as discussed by Green (1999, pp.115-116) and Holmberg (2011).

In summary, Proposition 1 and therefore Lemma 1 appears to hold also for non-linear supply functions in the spot market. Moreover, Holmberg (2011) shows that non-linear supply function equilibria provide endogenous incentives to sell forward. For the one-shot game, Green (1999, page 116) concludes: “The ‘upper limit’ of supply function competition in a spot market gives the same price-quantity results as Cournot competition, and adding a contract market (with zero conjectural variations) would be equivalent to Allaz and Vila’s model.” But with the Allaz and Vila (1993) model as the limiting case, we are back to the case of Cournot competition discussed in Liski and Montero (2006). This reinforces the generality of Lemma 1, showing that the case of supply function bidding compares rather to the result of Liski and Montero (2006) for Cournot competition than to the reverse case of price competition in the spot market.

Lemma 2 is less straight-forward to generalise. When Nash-reversion results in non-linear SFE, the price-cost margin evolves non-linearly along the supply function and therefore, profits will be non-linear in the demand shock. Depending on the functional form of collusive, deviation, and Nash profits, increasing uncertainty might potentially stabilise or destabilise a collusive agreement.

In summary, non-linear convex SFE provide an endogenous incentive for forward contracting which is not present in the linear case discussed in Sections 2 to 4. With this endogenous incentive to sell forward, the possibility to be trapped in a prisoner’s dilemma with massive forward contracting and intensified competition becomes a credible punishment for deviation. Therefore, the core result of the linear model survives: Forward markets can increase the range of discount factors for which collusion can be sustained.

6 Conclusion

The preceding analysis reveals how forward sales of firms can alter the stability of a collusive agreement when firms compete in supply functions on the spot market. Forward markets can ease collusion because they increase competition during the punishment phase. On the contrary, a forwards can destabilise collusion when firms sell forward while being engaged in a collusive agreement. A deviating firm
would seek to sell forward at a monopolistic price, and profit on the expense of rivals and speculators by depressing the price in the spot market. Firms which engage in collusion should therefore not sell forward at all, or carefully control the volumes of forward sales in relation to the overall output of firms. This compares well to the comparable setting with quantity competition and contrasts with price competition in the spot market.

The result is informative for regulatory authorities because the liquidity of the forward market can serve as an indicator for effective competition. With a collusive agreement in place, the liquidity of the forward market would be limited because selling forward when firms’ positions are observable would endanger collusion. When forward positions are not perfectly observable, firms might have an interest to sell forward, but speculators and customers are not willing to buy forward at any price, expecting that producers have an incentive to depress the spot price afterwards. Either way, liquid and anonymous forward markets are not compatible with sustained collusion.

The findings are robust to a number of generalisations. For example, the strict distinction between physical versus financial forward contracts which is made in the literature (Allaz and Vila, 1993; Ferreira, 2003; Liski and Montero, 2006; Green and Le Coq, 2010) is irrelevant in a setting where firms bid supply functions in the spot market. Moreover, it is shown that uncertainty of demand does not necessarily work against collusion. Instead, it has an ambiguous effect such that increasing demand uncertainty just reduces the impact of forward positions on the critical discount factor. Finally, the core result about the effect of forward contracts on collusion does not change when considering non-linear supply functions in the spot market.
References


A Appendix – supplemental material

A.1 Proof of Proposition 1

A.1.1 The effect of forward positions on deviation profits (Proposition 1.b)

Forward sales have a direct effect on firms profits as well as an indirect effect which works through the firm’s spot market strategy parameter $\alpha$. Consider the following decomposition of $d\pi_i/dx_i$:

$$
\frac{d\pi_i}{dx_i} = \frac{\partial\pi_i}{\partial x_i} + \left( \frac{\partial\pi_i}{\partial f} \frac{df}{d\alpha_i} + \frac{\partial\pi_i}{\partial p} \frac{dp}{d\alpha_i} + \frac{\partial\pi_i}{\partial q_i} \frac{dq_i}{d\alpha_i} \right) \frac{d\alpha_i}{dx_i} \tag{A.1}
$$

We study the effect of $i$’s forward positions $x_i$ on the incentive to deviate in the spot market. The preceding contract market is still characterised by the expectation that collusion holds, so the deviating firm can sell forward at a collusive price $f_c$ with $\frac{df}{d\alpha_i} = 0$.

A deviating firm plays its (spot market) best response to the collusive strategy of its rival $j$. By the definition of the best response, the derivative of profits (with fixed forward prices) to the firms strategy $d\pi_i/d\alpha_i^*$ is zero. Therefore, the above given decomposition of $d\pi_i/dx_i$ reduces for the deviating firm to:

$$
\frac{d\pi_i^d}{dx_i} = \frac{\partial\pi_i^d}{\partial x_i} = f_c - p_d
$$

which is positive, because the collusive forward price $f_c$ is larger than the realised spot price $p_d$ when $i$ deviates.

Knowing that $f$ is fixed, it is sufficient for the proof of convexity to show that $p_d$ is decreasing in $x_i^d$.

$$
\frac{d^2\pi_i^d}{dx_i^2} = -\frac{dp_d}{d\alpha_i^d} \frac{d\alpha_i^d}{dx_i}
$$

At hand of Equation (2) and (6) it is straight forward to see that $\frac{dp_d}{d\alpha_i^d} < 1$ and $\frac{d\alpha_i^d}{dx_i} > 0$, and therefore profits of a deviating firm are convex in its forward positions, independent of the forward position of the rival firm.
A.1.2 The effect of forward positions on non-cooperative profits (Proposition 1.c)

Reconsider the decomposition in Equation (A.1). In Nash equilibrium, both firms play their best response, therefore the same argument as in Section A.1.1 implies that \( \frac{d\pi_i}{d\alpha_i^*} = 0 \) for a fixed level of forward prices. However, in the repeated non-cooperative game, other than for the case of deviation, the price dampening effect of forward positions also works through market expectations which materialise in the forward price \( f \). Competition among speculators will impose \( f_n = p_n \) in expectation, so \( \frac{\partial\pi^n_i}{\partial x^n_i} = f_n - p_n = 0 \) (in expectation), as well as \( \frac{df_n}{d\alpha^n_i} = \frac{dp_n}{d\alpha^n_i} \).

With \( \frac{\partial\pi^n_i}{\partial f_n} = x^n_i \), (A.1) therefore reduces to

\[
\frac{d\pi^n_i}{dx^n_i} = x^n_i \frac{dp_n}{d\alpha^n_i} \frac{d\alpha^n_i}{dx^n_i}
\]

which is negative for any \( x^n_i > 0 \).

For the joint variation of \( x_i = x_j = x \)

\[
\frac{d\pi_i^n}{dx} = \frac{d\pi_i^n}{dx_i} + \frac{d\pi_i^n}{dx_j} = \frac{d\pi_i^n}{dx_i} + \frac{d\pi_i^n}{dp_n} \frac{dp_n}{d\alpha_j} \frac{d\alpha_j}{dx_j} = x \frac{dp_n}{d\alpha_i} \frac{d\alpha_i}{dx_i} + \left( q_i \frac{dp_n}{dp_n} (p_n - c_1 - c_2 q_i) \right) \frac{dp_n}{d\alpha_j} \frac{d\alpha_j}{dx_j}
\]

Using \( \alpha_i = \alpha_j = \alpha_n \), the expression above can be reduced to

\[
\frac{d\pi_i^n}{dx} = \frac{dp_n}{d\alpha_i} \frac{d\alpha_i}{dx} \left( x + q_i + \beta_n (p_n - c_1 - c_2 q_i) \right).
\]

Refer to Equation (4) and use \( q_i = \alpha_i + \beta_i p \) to express the profit margin \( (p_n - c_1 - c_2 q_n) \) as \( (q_i - x)/(b + \beta_n) \). Take the derivatives of \( \alpha_i^* \) defined in (6), and the equilibrium price defined in (2) to obtain

\[
\frac{d\pi_i^n}{dx} = \frac{-1}{b + 2\beta_n} \frac{\beta_n}{b + \beta_n} \left( x + q_i + \beta_n \left( \frac{q_i - x}{b + \beta_n} \right) \right).
\]

which can be reduced to

\[
\frac{d\pi_i^n}{dx} = \frac{-\beta_n}{(b + \beta_n)^2} \left( q_i + x \frac{b}{b + 2\beta_n} \right)
\]

which is obviously negative for any positive level of contracting.
The second derivative of profits reads
\[
\frac{d^2 \pi^n_i}{dx^2} = \frac{-\beta_n}{(b + \beta_n)^2} \left( \frac{dq_i}{dx} + \frac{b}{b + 2\beta_n} \right).
\]

Use the definition of the best response \( q_i = \beta^*_i \left( p + \frac{x_i}{b + \beta_i} - c_1 \right) \) to obtain
\[
\frac{dq_i}{dx} = \beta_i \left( \frac{1}{b + \beta_j} \frac{dp}{dx} \frac{d\alpha_i}{dx_i} + \frac{dp}{dx} \frac{d\alpha_j}{dx_j} \right)
\]
\[
= \beta_n \left( \frac{1}{b + \beta_n} + \frac{-2\beta_n^2}{b \beta_n} \right)
\]
\[
= \frac{(b + \beta_n)(b + 2\beta_n)}{(b + \beta_n)(b + 2\beta_n)}.
\]

which yields
\[
\frac{d^2 \pi^n_i}{dx^2} = \frac{-\beta_n}{(b + \beta_n)^2} \left( \frac{b\beta_n}{(b + \beta_n)(b + 2\beta_n)} + \frac{b}{b + 2\beta_n} \right)
\]
\[
= \frac{-b\beta_n}{(b + \beta_n)^2(b + 2\beta_n)} \left( \frac{\beta_n}{(b + \beta_n)} + 1 \right).
\]

This last term is negative and therefore \( \pi^n \) is proved to be concave and decreasing in \( x \).
A.2 Proof of Proposition 2

Consider the definition in Equation (10). The derivative of expected profits with respect to \( \sigma^2 \) is

\[
\frac{d\hat{\pi}_i}{d\sigma^2} = \frac{\beta_i - \frac{c_i^2}{2} \beta_i^2}{(b + \beta_i + \beta_j)^2}
\]

This derivative is positive for \( \beta_i < 2/c^2 \) which is true for every profit maximising supply function, irrespective of whether it is a collusive or a best response bid. (Note that not bidding below marginal costs already implies \( \beta_i \leq 1/c \), so \( \beta_i < 2/c \) is no relevant constraint.) The difference between collusive, deviation, and non-cooperative profits is most conveniently studied in terms of the strategy parameters \( \beta_i \) and \( \beta_j \). Take the following cross derivative:

\[
\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta_i} = \frac{(1 - c_2\beta_i)(b + \beta_i + \beta_j)^2 - 2(b + \beta_i + \beta_j)(\beta_i - \frac{c_i^2}{2} \beta_i^2)}{(b + \beta_i + \beta_j)^4}
\]

The latter expression is exactly zero whenever \( \beta_i \) is the best response \( \beta_i^* \) given in Equation (5). It is strictly positive when \( \beta_i \) is smaller than the best response. We know that \( \beta_d = \beta^*(\beta_c) > \beta_c \), therefore

\[
\frac{d\hat{\pi}_i}{d\sigma^2} > \frac{d\hat{\pi}_c}{d\sigma^2}.
\]

Now, consider the effect of a joint variation of slopes \( \beta_i = \beta_j = \beta \).

\[
\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta} = \frac{(1 - c_2\beta)(b + 2\beta)^2 - 4(b + 2\beta)(\beta - \frac{c^2}{2} \beta^2)}{(b + 2\beta)^4}
\]

This is zero for \( \beta = \beta_c \) given in Equation (7) and negative for all \( \beta > \beta_c \). Knowing that in the punishment phase \( \beta_i = \beta_j = \beta_n \) and \( \beta_n > \beta_c \), one obtains

\[
\frac{d\hat{\pi}_c}{d\sigma^2} > \frac{d\hat{\pi}_n}{d\sigma^2}.
\]

This ends the proof of Proposition 2. \( \square \)