Upstream and Downstream Competition in a Search Market.

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Abstract

This paper provides a new model of retail market structure. On the retail level, we study how multi- and single-product retailers compete in the same market when consumers want to buy one good. We find that multi-product retailers charge higher prices and retain all their consumers. Single-product retailers lose consumers to each other, and due to these losses charge lower prices. On the manufacturing level, we find that manufacturers would rather sell at retailers that do not carry competitor’s goods, however, if they can, they prefer to “invade” competitor’s home turf. This means that they over-saturate the retail channel with their products in the following sense: they sell goods in too many common stores, which intensifies price competition between them and leads to lower prices and profits. Consumer benefit in the form of lower prices and lower search costs.

1 Introduction

The fact that consumers do not possess all the product and price information before making their purchases has been well recognized. There is a mounting evidence that consumers acquire costly information before making purchases, and that some consumers have more information than others (see, e.g., De los Santos, Hortaçsu and Wildenbeest (2012)). The theoretical literature on consumer search has two main strands: one concerns itself with homogenous goods markets where consumers know the value of the product, but are a priori uncertain about prices charged by different firms. In many such models, firms randomize prices, and therefore consumers have to search for the best price offer (see Burdett and Judd (1983) and Stahl (1989)). There is also a large literature

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on consumer search with heterogeneous products, starting with Wolinsky (1986). The list of important contributions in the Wolinsky tradition is very long (Anderson and Renault (1999), Anderson and Renault (2006), Armstrong, Vickers and Zhou (2009), Ellison and Wolitzky (2012), Bar-Isaac, Caruana and Cuñat (2011), to name a few). In these models, consumers do not know neither their match with a product nor its price, and search firms sequentially to find out product characteristics and prices. These models effectively study “inspection” goods whose usefulness to a consumer can be verified before purchase (in contrast to “experience” goods). A little understood but a crucial assumption of this strand of the literature is that product characteristics, prices and locations are all the same thing. This “bundling” of distinct features of the model was motivated by interpreting search as physical visits to firms who each manufacture a differentiated good and sell at their premises, and it has served the literature very well due to its simplicity. The main advantage is that if a firm deviates to a price that consumers did not expect to see, the firm does not in any way affect which consumers discover utilities from its product, because consumers who observe the deviation have already discovered their utility from the product. The consumer side of the model is also simple, because consumers do not have to choose what type of information to acquire (price vs product characteristics). The fact that the same product is only sold in one location further simplifies analysis.

In this paper, our aim is to highlight that several of these assumptions are conjointly important for the study of retail competition, and should probably be relaxed. We start with simple observation that retailers often sell substitute products (e.g. TVs) that they do not produce. Furthermore, many retailers sell the same product (the same Samsung TV can be found in many shops). Retailers do provide the venue (and possibly services) to discover product characteristics, but unlike in the standard model, these characteristics are the property of the product not the location where it is purchased. Thus consumers acquire costly product information about products when visiting retailers, but this information is also useful elsewhere. It is therefore clear that retailers are concerned differently about price search from product characteristics search, however Wolinsky model treats them as one and the same.

Consider an extreme situation where all retailers carry the same good. Then once a consumer visits the first one, she knows everything about product characteristics of all the goods in the market. If searching for prices (the only remaining piece of undiscovered information) is costly, the Diamond paradox (Diamond (1971)) prevails. If each retailer carries a unique good, then we are back into the standard Wolinsky model. This paper

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1Exceptions, such as store brands, highlight the norm studied here rather than negate it.
considers the middle ground. The aim is two-fold: one is to understand retail competition where, realistically, retailers have to compete with diverse competitors, some that carry the same goods and some that do not. Second is two take what we have learned about downstream competition, and then look at manufacturers’ decisions where to sell their goods given the competition that unfolds between retailers.

We build a model of vertical relations where manufacturers choose where to sell their goods, and retailers compete for consumers who discover product characteristics and prices when visiting them. In the downstream market we find that there always is a symmetric equilibrium where multi-product retailers charge higher prices than single-product ones and retain all of their consumers. This is because their consumers have nothing to discover about product characteristics (multi-product version of the Diamond paradox), and the retailers make sure that the only remaining motivation to search further - lower prices at single-product retailers - is mute because the price difference does not justify paying the search cost. Single-product retailers have to set lower prices because they try to avoid losing consumers who do not like the product they sell and move on to discover other products. The resulting equilibrium has the following properties - multi-product retailers charge a simple markup over single-product retailers’ prices which is equal to the cost of price discovery (or charge monopoly prices, depending on the search cost). The single-product retailers set prices guided by search cost for both price and product discovery. This means that, as product discovery becomes easier, prices go down at single-product retailers and multi-product retailers’ prices follow one for one. In contrast, when price discovery becomes easier, single-product retailers reduce prices by less than multi-product ones because the latter have to accommodate both lower prices of their competitors, and consumers’ improved ability to leave if their prices are too high.

The downstream market forces guide manufacturer’s decisions on where to sell their goods. We show that they always prefer to not sell at the same store if possible, in the sense that if there are retailers that do not sell their goods, manufacturers would rather sell at such shops than those where competitor is present. However, once manufacturers cover half the market each, they face the following decision: should they put their products into competitor’s stores? The trade-off comes from our results for the downstream market: adding a shop to its retail coverage allows a manufacturer to sell to consumers who would have never discovered its product (who they like better than the competitor’s) otherwise. However, multi-product retailers set higher prices, so the probability that a consumer buys a good is lower at multi-product stores. When price discovery is easy, the latter negative effect is small, and so manufacturers use up all their
retail capacity.

The closest paper to ours is Zhou and Rhodes (2015) who study endogenous retail market structure with consumer search. Crucial difference between their paper and ours is that our focus is on how product characteristics search coexists with price search in the worlds where some retailers carry the same goods but other do not. In Zhou and Rhodes (2015) consumers search only for prices. As we do in our model, Zhou and Rhodes (2015) highlight that multi-product retailers allow to economize on search costs. However, in Zhou and Rhodes (2015) consumers are interested in buying both goods, so multi-product retailers are the only possibility for some of them to fulfill this desire. The latter effect is not present in our model because our retailers sell substitute goods and consumers only wish to purchase one. In this regard, our model is the unique search model that studies multi-product retailers but single-product consumers. Due to this, our predictions and focus are qualitatively different from the literature on multi-product search (in addition to Zhou and Rhodes (2015), the literature includes McAfee, McMillan and Whinston (1989), Lal and Villas-Boas (1998), Hosken and Reiffen (2007), Shelegia (2012), Rhodes (2014) and Zhou (2014).

Our vertical relations model is related to Dudey (1990) who studies location choice in a search model. We do not consider the primary driving force in this paper, locations (e.g. shopping malls) that attract many searchers and thus firms may wish to locate together in spite of intensified competition. In our model all retailers (equivalently all locations) are equally likely to attract first visits, thus the main effect in Dudey (1990) is not present. Even more importantly, we allow manufacturers to sell their goods in multiple locations, a feature that is absent from Dudey (1990), who in this regard is close to standard search models in that one firm has one good, but differs in that not all firms have different locations.

There is a somewhat less related (small) literature on vertical relations in search markets (Janssen and Shelegia (2014), Janssen and Shelegia (2015), Lubensky (2014)). Lubensky (2014) studies the role of manufacturer's suggested retail prices as a mechanism for the manufacturer to inform consumers about his cost. Neither of the listed papers considers the possibility that retailers carry multiple goods, nor the possibility that consumers learn about product characteristics separately from prices, nor that some retailers carry the same good and thus are not differentiated.

The paper is organized as follows: Section 2 introduces the model. In Section 3 we solve the model for downstream market. Section 4 uses results from Section 3 to close the model for the upstream market. Section 5 concludes.
2 The model

Firms

The market consists of two types of firms: retailers and manufacturers. There are two
manufacturers, 1 and 2 (we sometimes refer to them as $m_1$ and $m_2$, respectively). Retail
market is organized as in the spoked model (Chen and Riordan (2007)) with a twist
that there is a continuum of retailers of mass 1. They are therefore located on a circle,
however consumers can only travel to another retailer via the center of the circle and thus
have to travel the distance of two radii. We assume that circumference of the circle is 1.
We call an arbitrary point on the circle origin ($o$), and index retailers in the clockwise
direction by their distance from $o$. Thus retailers are indexed on the interval $[0, 1]$, and
we use $r$ to denote a typical retailer on $[0, 1]$. It is worth repeating, that neighboring
firms on the circle are not “located” close to each other as in the Hotelling model, and
that consumers will have to travel distance $1/\pi$ to reach any other retailer. The spokes
model is convenient in that all firms consider others to be symmetric.

The only time we use exact location of a retailer $r$ is to determine whether manufac-
turers would like retailer $r$ to carry their good.

Manufacturers produce one good each and goods are imperfect substitutes (details
are given when we discuss consumers). Manufacturers have to decide where to sell
their goods. Manufacturer $i = 1, 2$ can choose a finite set of closed intervals called
$R_i \subseteq [0, 1]$. Retailers belonging $R_i$ will carry good $i$. The mass of retailers in $R_i$ is
constrained to be at most $\bar{r} \in (0.5, 1)$. This assumption is a simple way to capture the
idea that covering entire retail channel is very costly, and thus manufacturers can only
sell their goods in a subset of stores.\footnote{A richer model would incorporate a convex cost of increasing market coverage.} Moreover, to simplify analysis, it is convenient
to assume that $\bar{r} > 0.5$, so that in equilibrium all retailers carry at least one good.
Once manufacturers set $R_i$, 4 types of retailers emerge. There are retailers that carry
no goods ($\{r : r \notin R_i, i = 1, 2\}$), only manufacturer 1’s good ($\{r : r \in R_1, r \notin R_2\}$), only
manufacturer 2’s good ($\{r : r \notin R_1, r \in R_2\}$), and both goods ($\{r : r \in R_i, i = 1, 2\}$).
Let the mass (which is endogenous) of these types of retailers be denoted by $r_0$, $r_1$, $r_2$
and $r_3$, respectively. We will abuse notation and use $r_i$ to refer to the type of retailers
in the set $R_i$.

An alternative formulation is where manufacturers offer contracts to retailers and
retailers choose which manufacturer to contract with. Since retailers always gain from
selling more unique varieties, all retailers would try to carry both goods. Because of
this, it is interesting to what degree manufacturers would like to enforce exclusivity by conditioning contracts on whether a retailer sells other goods. We do not pursue this direction and let manufacturers determine who sells their goods.

To drastically simplify analysis we initially assume that wholesale prices are exogenously given, and equal, so that both manufacturers sell their good at at the same unit price $w$.

Treating $w$ as exogenous simplifies treatment of off equilibrium beliefs substantially, and we choose to abstract form these issues in order to concentrate on manufacturer’s retail coverage choices and the analysis of the downstream market. None of the important conclusions change qualitatively when $w$ is endogenized as the symmetric upstream price. The assumption that manufacturers have to set a unit price and cannot charge a fixed fee is more important. As in any standard model of vertical relations, manufacturers would like to set $w = 0$ and extract surplus from retailers by setting fixed fee $f$. We will discuss how this change would affect manufacturers’ retail channel decisions.

Once retailers are informed on which goods they carry (these decisions are imposed by manufacturers), and without observing $R_i$, retailers set their prices simultaneously.

**Consumers**

There is a mass 1 of consumers “located” uniformly on the circle where retailers reside. They would like to purchase at most one unit of the good. Consumer $j$ derives utility $v_j^i - p$ from the product of manufacturer $i$ if she pays $p$ for it. $v_j^i$ are drawn independently across manufacturers and consumers from a cumulative distribution function $F(v)$ on a support $[0,1]$. We assume that $F(v)$ is twice continuously differentiable and log-concave, with a well-behaved density function $f(v)$. Utility from not purchasing anything, $u_i$, is assumed to be 0.

Consumer $j$ is initially uninformed about her idiosyncratic match values with either manufacturer’s good and retail prices. Consumer $j$ can discover these by visiting retailers. We assume that the visit to her “local” retailer is free.\(^3\)

To understand this assumption, it is helpful to think that the first retailer is the closest one to consumer’s home, and all retailers have the same number of local consumers. Endowing retailer with local consumers captures an important value proposition of any retailer - a unique proximity to consumers. If consumers could travel to any retailer for the first visit (an interesting feature of the problem that has been studied by the literature on shop location in shopping malls (CITE)), then in our setting with two unique

\(^3\)Given that the mass of consumers and retailers are equal, there is one consumer per each retailer.
varieties one retailer would be able to provide the same service as all continuum of them. We chose not to do so in our belief that retailers that exist in the model should have a “real” reason to exist. The assumption that all locations (and thus retailers) are created equal is for simplicity, and we will (SOME DAY) study how asymmetry in primary local demand affects our results.

Once at her local retailer, \( j \) discovers her match values with all the goods that are sold at the store, and all the prices charged by that store. E.g. if \( j \) visits \( r \in R_3 \) retailer, she discovers \( v^1_j, v^2_j \), and \( p^1_r \) and \( p^2_r \). Consumers can visit one more retailer.\(^4\) They have two possibilities, at a (transportation) cost \( t \), \( j \) can go to any other retailer and discover her price(s). At a cost \( s \equiv d + t > t \) she can go to another retailer and discover her match value with all the goods sold at that retailer and also discover all the prices. Why would a consumer do one of these things? A consumer who has already discovered both product matches has nothing to learn about them, but may want to visit other retailers to purchase at a lower price. A consumer who has visited a single-good retailer may want to pay \( s \) to discover her match with the other good(s). We emphasize here that the search cost \( s \) consists of two parts, to anticipate different comparative statics with respect to \( t \) and \( d \). We require that a consumer has to pay \( d \) to buy any good, so that no consumer buys a good that she does not know her match value for. The assumption that consumers already know match values at their local store is crucial in that they would have never willingly pay \( d \) if their local store is a single-product one.

Figure 2 illustrate the spacial structure of the model. In this example a consumer local to retailer \( r \) can travel to retailer \( r' \) (over blue path) or \( r'' \) (red path), and even though \( r' \) is closer on the circle to \( r \) than is \( r'' \), consumer has to travel the same distance to reach any of the two.

To drastically simplify analysis we assume that consumers will be able to target the type of the retailer for the second visit. E.g., a consumer wishing to visit \( r_i \) type retailer, can choose to visit a retailer from \( R_i \). Consumer do not know identities of retailers beyond their type, and we will assume that consumers are equally likely to visit any retailer in \( R_i \). This assumption allows to avoid dealing with complications that would arise without targeted second search. This is because in this case the search problem and resulting retailer demand is more complicated. One can think of the manufacturer’s website providing ability to search for shops carrying its good. Finally, as standard in the literature, we will assume that consumers can freely go back to previous stores (free

\(^4\)This can be extended to any number without changing results but at the expense of complicating proofs. This is because in our model there are only two goods and, as stated below, consumer can target a type of the retailer they visit for the second visit.
Figure 1: Spacial structure of the spokes model. Consumer located at $r$ has to travel the same distance to reach $r'$ or $r''$.

The timing of the game is the following. In the first stage, manufacturers simultaneously set $R_i$. In the second stage, retailer $r$ observes goods that she is going to sell and sets her price(s). In the third stage, consumers make first (random) visits, observe price(s) and match value(s), and decide whether to search a second store. After all searches are made, consumers decide whether and which good to buy. After this game ends.

The equilibrium concept we use is weak Perfect Bayesian. We will look for a symmetric equilibrium where retailers of the same type charge identical prices, both products have equal prices, and manufacturers have symmetric coverage, i.e. $r_{1}^{\ast} = r_{2}^{\ast}$. Let $p^{\ast}$ stand for equilibrium prices charged by $r_1$ and $r_2$ retailers, which by symmetry are going to be equal. Let $p^{**}$ stand for price of good 1 and good 2 charged by $r_3$ retailers, which again will be equal by symmetry.\(^5\)

We will solve the game by backwards induction. We thus start solving the game between retailers and consumers downstream, taking their beliefs about $r_{1}^{\ast}$, $r_{2}^{\ast}$ and $r_{12}^{\ast}$ as given.

### 2.1 Equilibrium downstream

The second stage of the model is in itself interesting. To our best knowledge, it is the first search model that accounts for multi-product retailers who sell substitutes.\(^6\)

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\(^5\)In principle, it is possible that prices of the two goods differ at $r_3$ type retailers, but we do not pursue this possibility.

The model highlights that Diamond Paradox (Diamond (1971)) relies on the fact that consumers have nothing to learn by visiting other firms. This is evident in our setting where we show that multi-product retailers behave exactly as in Diamond, but single-product retailers moderate the problem because their consumers are willing to move on the margin because they have yet to discover the other good’s appeal to them. This was Wolinsky’s (Wolinsky (1986)) solution of the paradox.

As is usual, we will assume that upon observing non-equilibrium prices charged by the first retailer they visit, consumers do not change their beliefs about other retailers and continue to believe that single-product retailers carrying good i charge $p^*$ whereas multi-product retailers charge $(p^{**}, p^{**})$ (this assumption is know as “passive” beliefs assumption).

Let us first solve the model for the case where $r_1^*, r_2^*, r_3^* > 0$ and $r_0^* = 0$. This is a situation where all retailers carry at least one good but some carry two. We will eventually impose $r_1^* = r_2^*$, however, it is instructive to understand how unequal numbers of single-product retailers affects pricing downstream. This is irrelevant in the symmetric equilibrium where consumer do not observe $r_i$ and thus have to use the symmetric belief, that has to be correct in equilibrium.

Two types of equilibria may emerge in this setting. In one, $p^{**} > p^*$, and in another $p^{**} < p^*$. The former type of equilibrium always exists, whereas the latter requires sufficiently high $s$ and sufficiently low $r_3$, and may not exist even then.

The first equilibrium type is intuitive to understand. In it, $r_3$ types engage in Diamond paradox pricing and make sure consumers are just indifferent between staying or going to a single-product retailer for a lower price (once at $r_3$, they have no match values to discover). $r_3$ types do not get second visits (they charge higher prices), so single-product retailers are left to compete in an environment which is equivalent to the standard Wolinsky model with two firms (details follow) but unequal numbers of first and second visits (until we impose $r_1^* = r_2^*$). They naturally charge lower prices, justifying $p^{**} > p^*$.

The second the of equilibrium is far less intuitive. To get $p^{**} < p^*$, $r_3$ types’ natural tendency to charge excessive prices has to be moderated by the fact that they get all the second visits. If these visits comprise a big fraction of their consumers (low $r_3$), and the demand from such visits is sufficiently price sensitive (as we shall see, high $s$ is the prerequisite, but is not sufficient), then indeed set $p^{**} < p^*$ and the equilibrium exists.

We will now start characterizing the first type of equilibrium with $p^{**} > p^*$. First,
define \( p^{jm} \) (superscripts \( jm \) stand for joint monopoly) as the solution to
\[
p^{jm} = w + \frac{1 - G(p^{jm})^2}{2g(p^{jm})G(p^{jm})}.
\] (1)

This is the monopoly price for a monopolist with two goods. It is always higher than the single-good monopoly price, denoted by \( p^m \) and defined later. This is because a monopolist selling two substitute goods internalizes the demand externality that one good has on another when its price is raised.

Lemma 1. In equilibrium where \( p^{**} > p^* \), we have \( p^{**} = \min\{p^* + t, p^{jm}\} \). Consumers never visit \( r_3 \) retailers for second visits, and never leave them after the first one (but may drop out of the market).

Proof. It is trivial to see that consumers who make first visits to single-product retailers will never choose to visit \( r_3 \) types for the second visit. This is because such a consumer has already discovered \( v^1 \), and thus can target \( r_{-i} \) retailer for the second visit to find out \( v^1 \), if she wishes to do so. In addition, she expects lower prices at \( r_i \), \( i = 1, 2 \), so is strictly better off visiting a single-product retailer.

A consumer who visits \( r_3 \) type first discovers \( v^1 \) and \( v^2 \), thus she will never search and pay \( s \) to discover her match value. The only possibility is that she visits another single-product retailer in order to pay \( p^* \) instead of \( p^{**} \) for her preferred good, but given that \( \min\{p^* + t, p^{jm}\} \leq p^* + t \), she will never choose to do so.

In order to prove the first part, consider a \( r \in R_3 \) retailer charging \((p_1, p_2)\) such that \( p_i \leq p_i^* + t \), \( i = 1, 2 \) (charging higher prices would result in no demand and is thus not studied). Clearly, no consumer who visits \( r \) for their first visit would leave to another retailer. Without loss, assume \( p_1 \leq p_2 \). Expected profit for \( r_3 \) is
\[
\max_{s.t. \ p_i \leq p_i^* + t} \left\{ (p_1 - w) \int_{p_1}^1 f(v)F(v + p_2 - p_1)dv + (p_2 - w)F(p_1)F(p_2) - \int_{p_1}^1 f(v)F(v + p_2 - p_1)dv \right\}.
\]

We only consider symmetric prices, so we shall impose \( p_1 = p_2 = p^* \). Then the profit simplifies to \( (p - w)(1 - F(p^*)) \). This is the monopoly profit for a two-good retailer. Without the constraint on prices, the maximizer is \( p = p^{jm} \). However, if \( p^{jm} > p^* + t \), then there is a corner solution at \( p^* + t \).

\(^7\)For this, we require that \( F \) is well behaved so that two-product monopoly pricing results in symmetric prices. This is true for all common distribution such as uniform, normal, exponential etc.
First visits account for all the visits to multi-product retailers. All these consumers discover both match values, so the standard incentive to search further - to discover other products - is absent.\(^8\) In the absence of single-product retailers \((r_1 = r_2 = 0)\), a multi-product version of the Diamond paradox would emerge, \(r_3\) types would all charge \(p^{im}\) for both goods.\(^9\) When there are single-product retailers, a milder version of the paradox emerges. Since consumers can always go to single-product retailers and buy their favorite (as discovered during the first visit at \(r_3\) types) good for \(p^\ast\). This puts an upper bound of \(p^\ast + t\) on prices that \(r_3\) types can charge. If \(t\) is high, then the upper bound is not binding and we are back to the Diamond paradox with \(p^{im}\), if not, then \(p^{**} = p^\ast + t\).

Now that we understand pricing by multi-product retailers, we proceed to characterizing \(p^\ast\). We will need to show that \(p^\ast < p^{im}\) to guarantee that indeed in equilibrium \(p^{**} > p^\ast\). Single-Product retailers do not suffer from the Diamond paradox-like problems. Their consumers do not know their match with the other good (the one that is not carried by the retailer), and thus if a single-product retailer increases her price, she encourages some consumers to search and may never get them back.\(^10\) Just like for multiproduct retailers, it is still true for single-good retailers that they never lose consumers to retailers of the same type (provided that a price increase does not exceed \(t\)). However, they lose consumers who have low draws to single-product retailers of the other type.

Multi-product retailers never get second visits, and never lose any consumers who visit them first. This means that single-product retailers lose consumers only to each other. To determine their demand and prices we can analyze a modified model where only single-product retailers exist (in numbers \(r^\ast_1\) and \(r^\ast_2\)).

A consumer who visits retailer carrying good \(i\) and observes \(v^j_i\) and \(p_i\) charged by the retailer. She can purchase there and get utility \(u^j = v^j_i - p_i\). She can also decide to visit another \(r_i\) type, where she will get utility \(v^j_i - p^\ast_i\), but will have to pay search cost \(t\). All consumers would leave if \(p_i - p^\ast_i > t\), thus a price deviation that large can never be profitable and will not be considered by the retailer. Assume from now on that \(p_i - p^\ast_i \leq t\). In this case \(j\) will never choose to go to another single-product retailer selling good \(i\). However, \(j\) can decide to visit a single-product retailer selling \(-i\). She

\(^8\)In search models with mixed strategy price equilibria (e.g., Stahl (1989)), consumers may search for a lower price. This motive is absent in our model.

\(^9\)Rhodes (2014) resolves Diamond paradox by introducing a multi-product monopolist, however, in Rhodes (2014) consumers are interested in purchasing multiple goods. As a result, his solution to the paradox does not work in our setting.

\(^10\)This in fact is Wolinsky’s (Wolinsky (1986)) solution to the Diamond paradox.
would have to pay $s + t$, and would purchase if $v_{-j}^j - p_{-j}^* > v_j^j - p_j$. Otherwise, $j$ will go back to the first retailer and purchase there. Optimal search rule for $j$ is the standard one, and is summarized by reservation utility defined implicitly by

$$
\int_{u^*}^{1} (v - u^*) f(v) dv = c.
$$

Consumer $j$ will accept $p_i$ from the retailer if $v_j^i > u^* + p_j$, or otherwise will visit a retailer selling $-i$, discover her match with good 2 and will then decide where to buy.

In order to derive demand for retailer selling good 1, we first define:

$$
M_1 = 1 - G(u^* + p_1 - p^*) + \int_{p_1}^{u^* + p_1 - p^*} g(u) G(v - p_1 + p^*) dv
$$

$$
M_2 = \frac{r_2^*}{r_1^*} \left[ (1 - G(u^* + p_1 - p^*))G(u^*) + \int_{p_1}^{u^* + p_1 - p^*} g(u) G(v - p_1 + p^*) dv \right]
$$

The first term in $M_1$ is the fraction of consumers who make a first visit to the retailer and purchase immediately because they draw $v_{1j} > u^* + p_1$. In the latter, the price difference is added to the reservation utility. The rest of first visits result in a consumer drawing $v_{1j} < u^* + p_1$ and moving on to a single-product retailer selling good 2. Out of these, consumers who draw $v_{2j} - p^* < v_{1j} - p_1$ come back to the retailer and purchase good 1. The second term in the expression for $M_1$ accounts for these consumers.

Then there are consumers who first visit retailers carrying good 2. Some draw utility $v_{2j} < u^*$ and move on to search retailers selling good 1 (they expect the retailer to charge $p^*$ and thus do not react to the price deviation being considered). Per each retailer selling good 1, there are $r_1^*$ of such consumers (the term in front of the big square brackets). Out of these consumers, all those who draw $v_{1j} > u^* + p_1 - p^*$ will buy good 1 at the retailer (the first term inside square brackets). Out of the rest, those with $v_{1j} - p_1 < v_{2j} - p_2^*$ will still buy good 1 (the second term inside square brackets). Total demand for $r_1$ type retailer is then $M_1 + M_2$.

Even though we consider the symmetric case where $r_2^*/r_1^* = 1$, and this holds even if actual choices of manufacturers are not such (retailers do not know actual choices and use equilibrium belief), it is still interesting to see that lower $r_2^*/r_1^*$ induces a form of prominence (Armstrong, Vickers and Zhou (2009), Armstrong and Zhou (2011)). By this we mean that the proportion of first visits in the total population of consumers is lower than 1. Indeed, when $r_2^*/r_1^* \to 0$, $r_1$ types price as if they get all first visits, whereas the opposite is true when $r_2^*/r_1^* \to \infty$. It is known that this tends to lower prices of
the prominent firms, and increases prices of non-prominent ones. In an extension where retailers observe \( r_2/r_1 \), the manufacturer’s choice of coverage will thus have price effects. Namely, manufacturer with higher coverage will also induce lower retail prices (for the same \( w_i \)), and thus this would further encourage expanding coverage. As mentioned, this effect is mute because retailers do not actually observe coverage decisions and thus cannot react to them with prices.

The pricing equilibrium is then defined by a pair of first order conditions, one for each retailer type. In a symmetric equilibrium with \( r_2^* = r_1^* \), \( p_1^* = p^* \) and the price is the same as in Wolinsky model with 2 firms.

Taking the first order condition, and imposing symmetry, gives the condition for prices:

\[
\hat{p} = w + \frac{1 - G(\hat{p})^2}{2 \int_{p}^{u^*} g(v)^2 dv + 2G(\hat{p})g(\hat{p}) + (1 - G(u^*))g(u^*)}.
\] (2)

Whether \( p^* = \hat{p} \) depends on the search cost \( s \). Namely, if \( \hat{p} \leq u^* \), then indeed \( p^* = \hat{p} \). However, in the opposite case \( p^* = p^m \), where \( p^m \) is defined as the solution to

\[
p^m = w + \frac{1 - G(p^m)}{g(p^m)}.
\] (3)

and is the single-good monopoly price in this model. \( p^* \leq u^* \) holds for relatively low search cost, so we can characterize \( p^* = \min\{\hat{p}, p^m\} \). The threshold \( \bar{c} \) is implicitly defined by \( \hat{p} = u^* \).\(^{11}\)

To summarize:

**Proposition 1.** There exists an equilibrium of the game where all single-product retailers charge \( p^* \) that solves (2) for \( c \leq \bar{c} \) or \( p^* = p^m \) otherwise. Multi-product retailers charge \( p^{**} = \min\{p^* + t, p^m\} > p^* \), consumers first visiting \( r_3 \) never go for second visits, and the rest only go to single-product retailers, if at all.

This equilibrium has a number of interesting properties. In it \( t \) affects prices charged by single-product retailers via \( s \). Namely, an increase in \( t \) increases the total search cost, decreases the reservation utility \( u^* \), and thus leads to higher \( p^* \) (provided that \( s \) is not so high as to have \( u^* < p^* \)). At the same time, it affects \( p^{**} \) directly, whenever \( p^{**} = p^* + t \). Thus an increases in how costly it is to discover prices, increases prices at single-product retailers, and further it increases the price difference between single- and multi-product retailers. As a result, its effect on multi-product retailers is far stronger. To the contrary,\(^{11}\)

\(^{11}\)Wolinsky’s solution to the Diamond paradox by introducing product differentiation eventually fails when \( c > \bar{c} \).
an increase in $s$ affects only $p^*$ and only through it increases $p^{**}$. It thus follows that cost of acquiring product information affects price of single- and multi-product first to the same extent.

**Proposition 2.** [Comparative statics] Assume $t$ and $s$ are sufficiently low so that $p^{**} = p^* + t$ and $p^* < p^m$. Then, $\frac{\partial p^{**}}{\partial t} > \frac{\partial p^*}{\partial t} > 0$ and $\frac{\partial p^{**}}{\partial s} = \frac{\partial p^*}{\partial s} > 0$.

We conclude that when consumers find it easier to discover product values, all prices go down equally. However, when consumer find it easier to discover prices, prices are reduced at multi-product retailer by much more than at single-product retailers.

Now we consider the remaining possibility, that $p^{**} < p^*$. In this case all the consumers make second visits to $r_3$ types because they carry both goods and consumers expect to find lower prices. Also, consumers do not leave $r_3$ types after the first visit, unless $r_3$ type deviates to sufficiently high prices. All this means that single-product retailers only sell to consumers who do not search further, because if they do, they will purchase at $r_3$ retailers.

Let us start with determining demand and prices at single-product retailers. Consumers are facing a non-standard problem in this case. No consumer that moves on to $r_3$ types comes back, because they carry both goods and at lower prices. If a consumer with utility draw $u$ moves on and searches $r_3$ retailers, she expects to get a sure price saving $p - p^{**}$, because she can purchase the same good at a lower price. He can also draw a higher utility for the other good, switch to buying that good and gain that way.

Take a retailer $r_1$ who charges $p$. The reservation utility $u(p)$ solves

$$
\int_{u(p)}^{v_1} (v - u(p))f(v)dv = c - (p - p^{**}).
$$

(4)

As is clear, the expected price difference acts like a search cost reduction, and so $u(p) < u^*$.

Let us assume $u^* \geq p^{**}$, so that consumers who visit single-product retailers continue to search. All the consumers who find $v_1 \geq u(p)$ will stop and purchase provided that $v_1 \geq p$, the rest will search and will never come back. Also, given that even consumers who draw 0 continue to search, it has to be the case that $u(p) > p$, or otherwise that would mean that someone with a negative utility at the firm is indifferent between search
or not. It follow then that the retailer’s expected profit is given by

$$(1 - F(u(p)))(p - w).$$

The optimal price $p^*$ solves

$$p^* = w + \frac{(1 - F(u(p^*)))^2}{f(u(p^*)}}.$$ (5)

The tentative price $p^*$ is determined along with $u(p^*)$ by (4) and (5).

The above equation clearly implies $p^* < p^{**}$ when $u^* \to 1$ (s close to zero), which violates our assumption under which this pricing equations is derived (more on this later).

Let us move on to multi-product retailers and their pricing. They have two types of consumers. First visitors, who never leave to another retailer, but may not purchase if their valuation is not sufficiently high. They also have second visits from single-product retailers who find $v_i^2 < u^* + p^* - p^{**}$ and move on to $r_3$. For what follows, it will be important to define the ratio of first two second visitors, $\lambda \equiv \frac{r_1^2 + r_2^2}{r_3^2}$.

Take $r_3$ who charges $p$ for both goods, $p \leq p^{**} + s_p$ (otherwise retailer gets no demand from first visits, the other case is considered later).

Demand from first visits is $1 - F(p)^2$. Demand from second visits is more involved. Recall that all these consumers have valuations for one of the goods below $u^* + p^* - p^{**}$ (these come in equal numbers from $r_1$ and $r_2$). Demand from these consumers is

$$\lambda \left[ F(u(p^*)) - F(p)^2 \right].$$

Consumers who visited $i$ retailer first and draw valuations below $u(p^*)$ move on, and will purchase at $r_3$ unless both of their valuations are below $p$. This is true for $i = 1, 2$, so everything is multiplied by 2. The number of such consumers per one $r_3$ is $\lambda$.

Putting both parts of demand together gives total profit:

$$(p - w) \left[ 1 - (1 + \lambda)F(p)^2 + \lambda F(u(p^*)) \right].$$

The above equation clearly illustrates that $r_3$ types’ demand is more elastic, the higher is $\lambda$. When $\lambda = 0$, the profit is the same as for a two-product monopoly, and thus the

\footnote{At $p = p^{**}$ we have $u(p) = u^* > p$, and $u'(p) = 1/(1 - F(u)) > 1$ so it has to be the case that for $p > p^{**}$ we have $u(p) > p$.}

\footnote{Considering different prices is irrelevant when determining optimal symmetric price. It is necessary to check that a firm does not want to deviate to asymmetric pricing, however.}
optimal price is also $p^{jm}$.

The first order condition yields the following pricing rule:\textsuperscript{14}

$$p^{**} = w + \frac{1 - (1 + \lambda)F(p^{**})^2 + \lambda F(u(p^{*}))}{2(1 + \lambda)f(p^{**})F(p^{**})}. \quad (6)$$

A crucial feature of this pricing rule is that $p^{**}$ is decreasing in $p^* + u^*$, the latter determining how many second visits $r_3$ types get. The reason is that the derivative of demand is $-2F(p)f(p)$ for both groups of consumers. In the case of second visits this is because all the consumers who arrive have low valuations, thus the marginal ones are around $p$. However, the fraction of marginal consumers is smaller in second visits, and more so the lower is $p^* + u^*$, which determines how many consumers search beyond single-product firms. This means that $r_3$ will charge lower prices as the search cost increases, in contrast to the standard model.

If this equilibrium exists, $p^*$, $p^{**}$ and $u(p^*)$ solve (4), (5) and (6). We still have to establish $p^{**} < p^*$, the condition that is required for our demand derivations to hold. In this regard, the first thing to notice is that when $\lambda$ is close to zero, $p^{**} = p^{jm}$ whereas $p^* < p^{m}$, which given that $p^{m} < p^{jm}$ leads to a contradiction with $p^{**} > p^*$. This makes it clear that the ratio of multi-product to single-product retailers has to be sufficiently low. Now notice that when $s$ is close to 0, $p^*$ has to be below $p^{**}$, or otherwise single-product retailers do not get any demand, leading to contradiction. Therefore, the only possibility is that $s$ is relatively high, and $\lambda$ is low.

We have so far assumed that $u^* \geq p^{**}$. If this condition does not hold, then consumers who first visit single-product retailers do not search further (gains from search are lower than costs). This means that single-product retailers will set $p^* = p^{m}$ and multi-product ones will set $p^{**} = p^{jm}$, a contradiction with $p^{**} < p^*$. It is not immediately clear when this condition holds. Ordinarily, higher $s$ leads to lower $u^*$, thus consumers stop searching when the search cost is higher than a certain threshold. However, here $p^{**}$ may also move with $u^*$ in the same direction, making analysis somewhat more involved.

**Proposition 3.** Equilibrium with $p^{**} < p^*$ never exists when $\lambda$ is sufficiently low or $s$ is sufficiently high.

**Proof.** ADD

Numerical computation for the uniform distribution suggest that the equilibrium with $p^{**} < p^*$ does exist, however, it requires that $s$ is neither too high or too low.

\textsuperscript{14}The sufficient condition for the profit function to be concave is that $f'(v)$ is not too negative.
For the next section we will assume that the downstream market equilibrium is as in Proposition 1 where \( p^{**} > p^* \).

### 2.2 Upstream market equilibrium

Now we turn our attention to the manufacturers’ coverage decisions. We first note that they always strictly prefer to sell at stores where manufacturer’s product is not sold. To address this situation, we need to know what happens when \( r_0 = 0 \). In this case one of three things may happen. If \( s \) is high, then consumer show arrive at \( r_0 \) retailers do not continue further and leave the market. If \( s \) is lower, than depending on the relative magnitudes of \( t \) and \( s \), consumers who arrive at \( r_0 \) would proceed to \( r_3 \) types or split equally between \( r_1 \) and \( r_2 \), the choice deepening on whether \( p^{**} - p^* \) is large enough to erase the benefit of drawing two utilities rather than one. In either case, \( p^{**} \) and \( p^* \) are the same because for the equilibrium what matters is only \( r_1/r_2 \), which is 1 in both cases. If consumers go from \( r_0 \) to \( r_3 \) for second visits, half of them buy from \( m_i \) provided that their valuation is above \( p^{**} \). If, instead, they go to \( r_1 \) and \( r_2 \) in equal numbers, \( m_i \) sells to half of them provided that valuations are above \( p^* \). In either case, “converting” an \( r_0 \) into \( r_i \) strictly increases \( m_i \)’s profits. In the case where consumers from \( r_0 \) go to \( r_3 \), \( m_i \) prefers to sell to all consumers with \( v_i > u^* \) that would have arrived at \( r_0 \), and to half of the remaining ones with valuation above \( p^* \), rather than to sell to half of consumer with valuations above \( p^{**} > p^* \). In the other case where consumers from \( r_0 \) go to \( r_i \), \( i = 1, 2 \), \( m_i \) still prefers to sell to all consumer with \( v_i > u^* \) and half of consumers with valuations above \( p^* \). This means that \( m_i \) will never have \( r_i + r_3 < \bar{r} \) when \( r_0 > 0 \).

In words, if a manufacturer can add a retailer to \( r_i \), it will.

It may still be the case that manufacturers use up all \( \bar{r} \), but it may still be the case that \( r_i + r_3 = \bar{r} \) and \( r_0 \). If consumers who visit \( r_0 \) go to \( r_3 \) types, this means that \( m_i \) can increase her profits by removing on retailer from \( r_3 \) and instead turning \( r_0 \) retailer into \( r_1 \). By doing so, \( m_i \) is more likely to sell to consumers who make first visits to the \( r_0 \) because instead of selling with probability \( (1 - F(p^{**})^2)/2 \), it will sell with a probability higher than \( (1 - F(p^*)^2)/2 \), which is higher than \( (1 - F(p^{**})^2)/2 \). In words, a manufacturer would rather “greet” consumers at a single-product retailer than wait before they end up at a multi-product retailer and pay higher prices.

If consumers who visit \( r_0 \) go to \( r_i \), \( i = 1, 2 \), then manufacturer \( i \) prefers to relocate her retailers from \( r_3 \) to \( r_i \), increasing the probability that consumers purchase form her. Given that \( \bar{r} \geq 1/2 \), we get the following result:

**Lemma 2.** In equilibrium \( r_0 = 0 \).
The main question now is whether manufacturers will choose to “invade” each others’ turf and create multi-product retailers, thus is $r_3 > 0$?

Turns out that this is not a trivial choice. Manufacturer 1 faces the following non-trivial tradeoff. On the one hand, converting a $r_2$ retailer into a $r_3$ retailer allows $m_1$ to capture more first-visit consumers, which results in more of them buying from $i$. In particular, those consumers who would have drawn $v_{-i} > u^*$ do not continue to search and half of them never discover they would have preferred to buy from $i$. On the other hand, $r_3$ retailers charge higher prices, so some consumers who would have visited a $r_2$, continue to $r_1$ and purchase there now will not because they draw $v_1 < p^{**}$. Notice that we do not mention consumers who would have arrive to $r_2$ for the second visit, because if $r_2$ is turned into $r_3$, these consumers arrive at other $r_2$ retailers, and thus the conversion does not change their decisions.

The tradeoff is illustrated below in Figure 2. Areas $A$ and $B$ account for consumers who purchase good 1 after having arrived first to $r_2$. Areas $B$ and $s$ account for consumers who would purchase good 1 had they arrived at $r_3$. Clearly, $m_1$ would profit from converting a retailer $r$ from $r_2$ into $r_3$ (by adding $r$ to $R_1$) if there are more consumers in $s$ than in $A$.

![Figure 2: Lost and gained consumers for $m_1$ from converting $r_2$ into $r_3$. $m_1$ gains $s$ and loses $A$.](image)

Given that $p^{**} - p^*$ is at most $t$, if $t$ is close to zero while $s$ is bounded away, then $s$ always dominates $A$ and thus $m_1$ sets $r_1 + r_3 = \bar{r}$. Intuitively, if the number of consumers
who do not search beyond $r_2$ because of $s$ is large, whereas the price premium consumers have to pay at $r_3$ is low (low $t$), $m_1$ benefits by putting her product into all the retailers which do not carry it.

**Proposition 4.** If $t$ is sufficiently low, and $s$ is intermediate so that $c \leq \bar{c}$, manufacturers use up all their capacity in equilibrium, i.e. $r_1^* = r_2^* = 1 - \bar{r}$ and $r_3^* = 2\bar{r} - 1$.

**Proof.** ADD

The opposite is not necessarily true. To see this note that $A$ is bound in size by $t \leq c$. But large search cost $s$ also results in fewer consumers searching (large area $s$), making converting $r_2$ into $r_3$ also more desirable. Thus it is not clear that, when $s$ is low and $t$ high, manufacturers refrain from invading each other’s turf and set $r_1 = r_2 = 1/2$. In fact, for the uniform distribution this is never the case.

## 3 Discussion

On obvious way to extend the model is to endogenies $w$. In particular, we could study how manufacturers along with $R_i$ choose $w_i$. Immediate thing to note is that, holding eventual symmetric wholesale prices as fixed, the results we have derived for the choice of market coverage remain true. These choices do affect equilibrium $w$. Specifically, if parameters are such that $r_1^* + r_3^* = \bar{r}$, $i = 1, 2$, this means that manufacturers will engage in excessive price competition because in $r_3$ retailers their products are sold next to each other. Thus the choice of coverage is a prisoners’ dilemma type situation where manufacturers would like to commit to $r_3^* = 0$ and $r_1^* = r_2^* = 1/2$. This configuration maximizes competition downstream ($p^*$ vs $p^{**}$), and also maximally relaxes price competition upstream because only a small subset of consumers compares products (in such an equilibrium, the mass is $F(u^*)$).

Another avenue for extending the model is to study non-linear pricing by manufacturers. This would change our analysis of coverage decisions. The new feature of this analysis is that now manufacturers would set zero unit price (for standard reasons), and would try to extract profits from retailers with a fixed fee. The interesting aspect of this is that there is a possibility of equilibrium multiplicity because of multi-product retailers. Their profit from selling two goods is strictly higher than the monopoly profit from selling one. This means that one can construct equilibria where one of the manufacturers charges a higher fee than the other, and neither wants to increase her fee because retailers would reject such an offer.\textsuperscript{15} Beyond this issue, the incentives to expand coverage

\textsuperscript{15}This is reminiscent Cournot’s complements model (Cournot (1838)).

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are also changed. At single-product retailers manufacturers can extract full profit, but may want to poach such retailers by undercutting each other. Multi-product retailers give only fraction (half in the symmetric equilibrium) of profits, but there undercutting is not possible.

4 Conclusion

We have developed a model of vertical relations where manufacturers choose where to sell their goods, and retailers compete for consumers who discover product characteristics and prices when visiting them. In the downstream market we found that there always is a symmetric equilibrium where multi-product retailers charge higher prices than single-product retailers and retain all of their consumers. This is because their consumers have nothing to discover about product characteristics, and the retailers make sure that the only remaining motivation to search further - lower prices at single-product retailers - is mute because the price difference does not justify the search cost. Single-product retailers have to set lower prices because they try to avoid losing consumers who do not like the product they sell and move on to discover other options. The resulting equilibrium has the following properties - multi-product retailers charge a simple markup over single-product retailers’ prices which is equal two the cost of price discovery. The single-product retailers set prices guided by search cost for both price and product discovery. This means that as product discovery becomes easier, prices go down at single-product retailers and multi-product retailers’ prices follow one for one. In contrast, when price discovery becomes easier, single-product retailers reduce prices by less than multi-product ones because the latter have to accommodate both lower prices of their competitors, and consumers’ improved ability to leave if their prices are too high.

The downstream market forces guide manufacturer’s decisions on where to sell their goods. We have shown that they always prefer to not sell at the same store if possible, in the sense that if there are retailers that do not sell their goods, manufacturers would rather sell at such shops than those where the competitor is present. However, once manufacturers cover half the market each, they face the following decision: should they put their products into competitor’s stores? The tradeoff comes from our results for the downstream market: adding a shop to its retail coverage allows a manufacturer to sell to consumers who would have never discovered its product otherwise. However, multi-product retailers set higher prices, so the probability that a consumer buys any good is lower at multi-product stores. When price discovery is easy, the latter negative effect is small, and so manufacturers use up all their retail capacity.
References


Cournot, Antoine-Augustin. 1838. Recherches sur les principes mathématiques de la théorie des richesses par Augustin Cournot. chez L. Hachette. 15


Lubensky, Dmitry. 2014. “A Model of Recommended Retail Prices.” Indiana University, Kelley School of Business, Department of Business Economics and Public Policy Working Papers 2011-06. 1


Rhodes, Andrew. 2014. “Multiproduct retailing.” The Review of Economic Studies, rdu032. 1, 6, 9


