Choice of Pricing Rule with Privately Informed Buyers*

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Abstract

If different buyers are aware of the market to different extents, it benefits a seller to price discriminate between them. Buyers, however, have private information on their types, making price discrimination difficult, unless buyers are given a role in self-determining price. This paper explores the incentives a seller has in allowing buyers to determine the price they pay, and investigates whether and which sellers would be able to do so, and to what extent. A comparison of three policies - a posted price for all buyers, an invited bid per buyer, and price matching - leads to propose that price matching is a solution to the information problem, combining elements of the other two policies. A seller’s adoption of it depends on the price distribution in the market and the extent of its adoption by its rivals. On one hand, adopting price matching raises a seller’s posted price; while on the other hand, the lowest price in the market is set with incentives to discourage the adoption of price matching by the rest of the market.

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1 Introduction

In large markets where multiple prices simultaneously co-exist, consider the following: first, that not all sellers must be earning zero profits; and second, that when a seller and buyer face each other, the buyer may be (privately) aware of any subset of prices in the market.

What pricing rule should the seller then adopt, facing this information problem with buyers, while existing in a market with rival sellers? Also, if sellers in the market themselves differ, by their marginal cost of production (or sale), then which sellers will adopt which rule to determine selling price? This paper will try to answer these questions, comparing and contrasting some basic pricing rules, while also highlighting the benefits and problems in each.

First consider the pricing rule that uses one posted price for all buyers, with a take it or leave it (and buy from elsewhere) ultimatum. In a market that has multiple co-existing prices, if a seller earns a positive profit margin at its posted price, the take it or leave it ultimatum is not credible; the seller should be willing to renegotiate the price down, for buyers refusing to purchase at its posted price (assuming, of course, that the seller has no capacity constraints) as long as the seller can verify the credibility of the refusal. A refusal will be considered credible if the buyer in fact has a preferred outside option, and the refusal is not solely motivated with getting a better bargain from the seller. Plus, a single posted price is not optimal for the seller, as different buyers will have different outside options. Both these issues are inter-related because a buyer’s credible refusal of a price depends on her outside option.

The seller might then, do better with price discrimination, although, there may be a possible collective benefit to all sellers from each committing to its posted price [Bester, 1994]. However, because of buyers’ private information, the seller needs a price rule that gives each buyer some initiative in determining the her price, while trying to elicit her private information regarding the rival market or her outside option. Therefore consider the simplest rule that puts the ball in the buyer’s court - inviting a bid per buyer, while the seller can refuse or agree to sell at the bid price. Such a bid from a buyer, is determined by the buyer’s perceived seller cost distribution in the market, rather than by her outside option because the very act of inviting such a bid signals the seller’s ability to do so, thus adversely selecting the buyer’s bid. No seller would then choose this option because we find that with only posted prices, sellers earn non-negative profits. This is essentially a recurrence of the result by Perry [1986], extended to a large market.
The third policy examined, that of price matching, is shown to combine important features of both the posted price and inviting the buyer’s bid, softening the take it or leave it ultimatum associated with the posted price by giving the buyer some leeway in self-determining price, while also using rival prices to censor the adverse selection inherent in buyer’s bid by restricting the bid (match) to existing rival prices. The price discriminatory effect of price matching follows Png and Hirshleifer (1987) directly; we extend their duopoly model of identical sellers to different cost sellers, and investigate questions of how and why sellers are able to adopt the policy and what market equilibrium looks like.

Limiting market entry by profitability, the seller with the highest cost in the market will choose to sell only to the uninformed buyers solely through posted price. Moreover, if all sellers use a posted price each, then their price ranks must perfectly correlate with their cost ranks. However, a seller’s choice to price match raises its posted price, as its price matching policy substitutes (in part) for an actually lower price and it can give private discounts to buyers below its posted price. The ability to do so is a function of the seller’s cost, relative to the market price distribution, especially relative to the lowest price in the market. Only those sellers who expect, on average, to add to their profit with price matching, would do so.

An important finding is that the lowest priced seller, who is unable to price discriminate, chooses its posted price to maximize its expected profit where the probability of purchase by a buyer is a function of its own price. This is because the lowest price in the market defines the extent of adoption of price matching. Therefore, the lower bound on the market price distribution is maintained by this seller’s realization that a higher probability of successful sale comes at the cost of discouraging more rivals from adopting price matching by posting a lower price. The effect of price matching in a market is then both competitive and anti-competitive.

The policy is thus similar to other popular pricing rules that use posted prices without the take-it-or-leave-it ultimatum, and instead use pre-defined deviations from the posted price. Examples of these are quantity discounts, loyalty discounts and redeemable points, rebate coupons, and other forms of price discrimination. In fact, the growing popularity of such price discriminatory policies is probably the result of the diminishing agency costs of monitoring sales agents due to technology improvements in sales collection.
Png and Hirshleifer (1987) illustrated how the policy of price matching can be price discriminatory. Moreover, in their model price matches could actually be invoked and delivered, unlike the rest of the literature that considered no matches actually being invoked or delivered, such that price matching was a strategic threat with the goal of tacit collusion (the literature following Salop, 1986); or was used to discourage market search by consumers in the presence of sequential search costs (Janssen and Parakhonyak, 2013); or a signal of the lowest price in the market (Moorthy and Winter, 2006). We extend the idea of prices actually being invoked and delivered, to the case when the seller has a (private) cost identity such that its ability to adopt the policy may be a function of its own cost. In particular, this study differs from the treatment of the matching policy in Moorthy and Winter (2006), because buyers in this model take the policy at its face value - to actually claim and receive matches of rival prices, rather than finding it attractive because it affects their beliefs about price ranks in the market.

We believe this model therefore contributes both to the wider literature on the optimality, credibility and efficiency of pricing rules in imperfectly informed markets (Bester, 1988; Bester, 1994; Gale, 1988; Wolinsky, 1983; Perry, 1986; Camera and Delacroix, 2004; Wolinsky, 1983) and to the literature on price matching (Salop, 1986; Baye and Kovenock, 1994; Corts, 1995; Corts, 1996; Jain and Srivastava, 2000; Srivastava and Lurie, 2001; Hviid and Shaffer, 1999; Hviid and Shaffer, 2010; Dugar, 2007; Fatas et al., 2005; Janssen and Parakhonyak, 2013; Moorthy and Winter, 2006; Png and Hirshleifer, 1987), while also using a framework of a large market with different cost sellers, similar to (Carlson and McAfee, 1983).

North (1991) proposed that a preference for a single (posted) price may have emerged to avoid the transaction costs of reaching an agreement, and to minimize the agency costs of owners’ monitoring of sales agents. For example sellers who are also owners tend to price discriminate more often (as seen in bazaars and souks) than sellers who are simply sales agents, and this might be because owners do not authorize sales agents this ability. With technology enabling easier monitoring of sales collection, agency costs of monitoring have gone down, and we should thus expect to see an increase in the use of discriminatory pricing rules even by sellers who are not owners.

Chatterjee (2013) is a good summary of similar research in the bargaining literature in large markets with imperfect information.
2 The Model

Consider a large market of sellers selling the same good, but having different marginal costs. Assume that sellers have no fixed cost, and each seller’s marginal cost is drawn from the support \([\bar{s}, \bar{s}]\). A buyer facing a seller does not know the seller’s cost but only the distribution of costs in the market, which is given by the cumulative distribution function \(F(s)\). All buyers have the same value, \(v\), for the good; and each buyer desires to buy at most one unit of the homogeneous good. Each seller’s cost is thus its privately known type, i.e., the buyer is unaware of the size of the trade surplus; whereas the seller is assumed to know each buyer’s value and therefore the size of the available trade surplus. Assume also that sellers are not capacity constrained.

We will exploit the use of the large market to give an upper bound to the support of seller costs. If entry in the market is limited only by the ability to sell and earn profit, the highest cost (seller) in the market is equal to the buyer’s value for the good. That is, \(\bar{s} = v\), or in other words, each seller’s cost is drawn from the support \([s, v]\), with cumulative distribution function, \(F(s)\).

Assume that sellers belong to the side of the market that is immobile, i.e. they wait for buyers, and dictate their trading interactions to buyers; to which each buyer responds by choosing the best deal available to her. A buyer’s type is her private information, and characterizes how well she samples the market. The (private information) type of the buyer will be denoted by \(k\) and must be understood to be the fraction of the sample of prices the buyer can draw from the rest of the market (when facing any one seller). A buyer’s type is assumed to be purely random, and independent of the price distribution in the rest of the market.

A buyer’s type therefore determines her expected outside option (the minimum price known to her, or the first order statistic of her sample of prices) as a function of the market price distribution. Call this the expected minimum price (EMP) of a buyer of a given type; i.e. EMP is a function of \(k\).

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\(^3\)Although even this knowledge may be more than what buyers usually have, it is used only to be able to quantify buyers’ bids if and when they have the option to make them.

\(^4\)That is, buyers are unaware of the true price distribution in the market and have predetermined sample sizes of the market to search.

\(^5\)This was used by Stigler [1961] to propose the determination of optimal level of market search, by a rational buyer. The model here instead uses the EMP to typify each buyer and to feed back into the price determination between buyer and seller.
Let there be $I$ number of sellers in the market, where $I \gg 2$. For a given seller, the support of $k$ for any buyer will be taken to be $[0, \frac{I-1}{I}]$, i.e. the fraction of sellers sampled in the rest of the market excluding the given seller. The lowest type of buyer ($k = 0$) finds no other seller in the market, while the highest type of buyer ($k = \frac{I-1}{I}$) will find all rival sellers and their prices ($Ik = I - 1$). Let the cumulative distribution function of $k$ be given by $\Phi(k)$, and is interpreted as the proportion of buyers of type less than or equal to $k$, and the distribution of $k$ is be continuous\(^6\) and known by sellers.

The model will thus consider bilateral buyer seller interactions in the large market. To every buyer that comes to a seller then, the seller proposes an interaction (with or without a posted price), after which the buyer compares the deal the seller is offering with her outside option (her EMP from the rest of the market is an expectation of this), and finally, the buyer purchases or walks away from the seller. We will assume that the buyer already knows her outside option when she meets any given seller, OR equivalently, that the buyer makes her purchase decision after knowing her sample of prices from the market. Notice that because a buyer’s outside option is her private information, a seller can at best compute the EMP for each buyer type and use it as the expectation of her outside option. Equipped thus, the seller chooses the types of buyers to target selling to, through its chosen price rule and/or posted price.

For any given price rule or price offered by a seller, target buyers are those for whom the seller’s offer is attractive compared to their EMP. We will assume that if a buyer is indifferent between the seller’s offer and her own EMP, then she takes the seller’s offer\(^7\). However, in case of offers to match the buyer’s best known price (either by the seller in question, or by the rest of the market), the buyer is probabilistically split between purchasing from the seller with the lowest price (known) and the seller(s) offering to match.

\(^6\)The number of rival stores known does not have to be a whole number, such that a buyer can know the nth store probabilistically. For example if $k = \frac{3}{2}$, then the buyer is interpreted as finding one rival store for sure, and the second rival store with probability $\frac{1}{2}$.

\(^7\)The reason for assuming a clear choice in one direction of the state of indifference is that because EMP is an expectation of market prices determined by the buyer’s sample size, it may not actually equal the buyer’s realized outside option; it is simply the best means the seller has to convince the buyer to purchase from it without the buyer’s outside option revealed to him.
3 Pricing Rules

The three possible pricing rules that will be considered in this setup are: (1) the seller posts a price and gives the buyer the choice to accept or refuse (take it, or leave it and purchase from elsewhere), (2) the seller invites a price from the buyer and retains the right to accept or refuse to sell at that price, and (3) the seller posts a price and leaves the buyer with the choice of accepting, refusing and walking out, or refusing and invoking an existing (and known) lower rival price at which the seller pre-commits to sell. The above interactions will be respectively referred to as Take-it-or-leave-it (TILI), Buyer Bid (BB), and Price Matching (PM). In BB, a seller need not post a price. In PM on the other hand, the posted price need not be the price at which all sales are made, it is simply an upper bound on the price that will be charged from every buyer, with the lower bound being the lowest of existing market prices. It should be noted that by investigating only these three pricing rules, the claim is not that they exhaust all possibilities of trading interactions, but that other rules - for example: auctions, one-on-one bargaining with back-and-forth bids and offers, etc. - would involve much higher transaction or execution costs of time/effort\(^8\), given the private information in the market.

To understand the nature of the large market in the background, notice first, that the market will necessarily have some dispersion of (variance in) prices (whether bids or posted prices) as long as sellers are not all identical and as long as there are buyers who find more than a few prices (Burdett and Judd, 1983). This is true regardless of whether we consider all sellers simply posting prices or using price matching. We will therefore, focus on the buyer-seller interaction, assuming that price dispersion exists in the rest of the market.

3.1 Take It Or Leave It (TILI)

First consider that the entire market only uses TILI. Let the distribution of market prices in the rest of the market - with respect to a seller with marginal cost \(s_i\) - be given by \(Q(p-i)\). Then for seller \(i\), the EMP of a type of buyer, \(k\), is the first order statistic, given \(k\), from the distribution

\(^8\)Price beating is not considered here although the transaction cost in that would presumably be the same as in PM. It is left out because efficient price beating requires knowing exactly the level of a price cut that makes consumers behaviorally prefer the beating store. This introduces discrete price cuts (refunds) - which may differ between buyers - that lack continuity and can thus make the model cumbersome.
of prices, \( Q(p_{-i}) \). For notational ease, this will be denoted by \( \hat{p}(k, Q(p_{-i})) \).

The assumption of price dispersion gives:

\[
\frac{\partial \hat{p}(k, Q(p_{-i}))}{\partial k} < 0, \quad \forall \ k \in [0, \frac{I-1}{I}).
\]  

(1)

The seller thus, in choosing its list price, is in effect choosing the marginal buyer type (on average) who will purchase from it. That is, \( k \) is the buyer type that is being targeted at the margin with the posted price \( \hat{p}(k, Q(p_{-i})) \).

In other words, it chooses the probability of making a sale given the distribution of buyer types. There is therefore, the usual trade-off between a higher price or a higher probability of making a sale; targeting a more informed buyer type at the margin necessarily implies that the posted price must be lower. That is, by listing price at a certain level, the seller can, on average, target selling to buyers (of types) whose EMP is equal to or higher than its posted price. The seller thus tries to maximize its expected profit from sales to the types of buyers it targets, by choosing its target buyer type, given its cost and the market price distribution.

Notice also that the seller posting the lowest price in the market would be attractive to all buyers who know its price. Whereas for any seller posting any price higher, the prospective pool of buyers would reduce by a bit, depending on where its price stands in rank in the market, precisely because lower priced sellers would be found by its prospective buyers (of all types).

Therefore, the lower a seller’s posted price (or the higher the buyer type it targets to sell to), the higher the proportion of the prospective pool of buyers it has (wherein which, it chooses to target certain buyer types); let this proportion be represented by \( \beta(k) \), such that as described, \( \beta'(k) > 0 \).

The expected profit for the seller in posting price \( \hat{p}(k, Q(p_{-i})) \) then, is as follows, where we use \( G(k) = \beta(k) \Phi(k) \) for the probability of making a successful sale.

\[
\pi_{i}^{TILL} = G(k)[\hat{p}(k, Q(p_{-i})) - s_{i}],
\]  

(2)

with the belief that buyer types less than or equal to \( k \), find the price:

\( p = \hat{p}(k, Q(p_{-i})) \), attractive, on average.

Notice that \( G'(k) > 0 \) because both of its components increase in \( k \). Also notice that from the properties of the first order statistic (without replacement), \( \frac{\partial^2 \hat{p}(k, Q(p_{-i}))}{\partial k^2} \leq |\frac{\partial \hat{p}(k, Q(p_{-i}))}{\partial k}| \).

The first order condition (FOC), is as follows; the solution to which gives
the optimal \( k \) which will be denoted as \( k_{TILLI}^i \).

\[
\hat{p}(k, Q(p_{-i})) - s_i = -\frac{G(k) \partial \hat{p}(k, Q(p_{-i}))}{G'(k)} \frac{\partial}{\partial k}, \text{ at } k_i = k_{TILLI}^i. \tag{3}
\]

The left hand side (LHS) of the FOC is simply the profit from a successful trade (the seller’s price-cost margin), and the right hand side (RHS) is the product of the rate of decrease in price in targeting a higher buyer type and the associated density weighted probability of making a successful sale. The FOC therefore, is analogous to equating marginal benefit and marginal cost of targeting a higher buyer type. The LHS is clearly decreasing in \( k \), from (1). For a unique solution, therefore, the RHS should be increasing in \( k \).

The following assumption ensures this \(^9\) and also ensures the second order condition (SOC) for a maximum \(^10\) at \( k_{TILLI}^i \).

\[
[G'(k)]^2 > G(k)[G''(k)] + G'(k). \tag{4}
\]

Moreover, the FOC, using (1), shows that if \( k_{TILLI}^i \in (0, \frac{I-1}{I}) \), then the optimal price-cost margin must be positive \(^11\):

\[
\hat{p}(k_{TILLI}^i, Q(p_{-i})) - s_i > 0, \forall k_{TILLI}^i \in (0, \frac{I-1}{I}). \tag{5}
\]

The seller’s optimal posted price is thus, a function of its marginal cost, \( s_i \), and although we have isolated the buyer and seller interaction from the rest of the market, in making its optimal choice and considering the buyer’s EMP, the seller is best responding to prices in the rest of the market, \( Q(p_{-i}) \). Optimal price is therefore, \( p_{TILLI}^i = \hat{p}(k_{TILLI}^i, Q(p_{TILLI})) \).

The following lemmas are needed to talk about relative optimal choices

\(^9\)The assumption \(^4\) ensures a positive derivative of the RHS with respect to \( k \), using signs of the first and second derivatives of the first order statistic (EMP), and the fact that the absolute value of its second derivative is no larger than the absolute value of its first derivative. The assumption is sufficient and more than necessary.

\(^10\)The second derivative of the expected profit function is \( G''(k)[\hat{p}(k, Q(p_{-i}))] - s_i + 2G'(k) \frac{\partial \hat{p}(k, Q(p_{-i}))}{\partial k} + G(k) \frac{\partial^2 \hat{p}(k, Q(p_{-i}))}{\partial k^2} \). Using (3), at \( k = k_{TILLI}^i \), this becomes \( \frac{\partial \hat{p}(k, Q(p_{-i}))}{\partial k} [2G'(k) - \frac{G(k)G''(k)}{G'(k)}] + G(k) \frac{\partial^2 \hat{p}(k, Q(p_{-i}))}{\partial k^2} \), which is negative, using (4), and the properties of the first and second derivatives of the EMP.

\(^11\)Only seller with cost equal to \( v \), choosing to sell only to uninformed buyers, earns a zero price-cost margin.
for different sellers, when all choose TILI.

**Lemma 1.** For sellers $a$ and $b$, neither of who define the infimum or supremum of the market price distribution, in TILI equilibrium, $p_a^{\text{TILI}} = p_b^{\text{TILI}}$ iff $k_a^{\text{TILI}} = k_b^{\text{TILI}}$.

*Proof.* See Appendix.

**Lemma 2.** For any sellers $a$ and $b$, neither of who define the infimum or supremum of the market price distribution, in TILI equilibrium, $p_a^{\text{TILI}} > p_b^{\text{TILI}}$ iff $k_a^{\text{TILI}} < k_b^{\text{TILI}}$.

*Proof.* See Appendix.

Notice that Lemmas (1) and (2) do not use the FOC in (3), but are results of the properties of the EMP function, and that in equilibrium each seller chooses a target buyer type at the margin and posts a price equal to the EMP of that target buyer, given the prices that other sellers post. The following proposition uses the FOC and interprets the lemmas with respect to the heterogeneity in seller cost.

**Proposition 1.** In TILI equilibrium, for sellers $a$ and $b$, neither of whose choice defines the minimum/maximum price in the market, if $s_a < s_b$, then it must be that $k_a^{\text{TILI}} > k_b^{\text{TILI}}$ or equivalently, that $p_a^{\text{TILI}} < p_b^{\text{TILI}}$.

*Proof.* See Appendix.

Proposition (1) tells us that in TILI equilibrium, sellers will maintain their cost ranks with their posted prices. This is similar to what Carlson and McAfee (1983) found although theirs is a model of strategic sequential search by consumers. Another important result that follows is that in equilibrium, a lower cost seller must have a higher price-cost margin.

**Corollary 1.** For sellers $a$ and $b$, neither of whose choice defines the minimum/maximum price in the market, if $s_a < s_b$, then in TILI equilibrium it must be that $p_a^{\text{TILI}} - s_a > p_b^{\text{TILI}} - s_b$. 
Proof. See Appendix. □

The seller targeting \( k = 0 \) lists the highest price in the market as it chooses to sell only to the type of buyer who is unaware of all rival prices. These buyers will buy at any price as long as it does not exceed their value for the good. This seller, thus posts price equal to \( v \). At the same time, for the optimal decision that targets \( k = 0 \), notice that \( G(k = 0) = 0 \) because \( \Phi(k = 0) = 0 \), i.e. the RHS of the FOC in (3) is zero. This decision is therefore optimal only when the LHS is zero as well, or in other words, the seller’s price-cost margin is zero. Therefore, it is the marginal entrant in the market, with marginal cost \( \bar{s} = v \), that lists the highest price in the market, \( v \).

Summarizing the above, if all sellers offer TILI, the seller with marginal cost equal to \( v \) posts a price equal to \( v \), thus earning zero profit. All other sellers post prices strictly greater than their marginal cost, but not exceeding \( v \), with positive expected profits. Moreover, price-cost margins are lower for sellers with higher costs.

Because equilibrium behavior as defined above, by the seller’s FOC, only explains optimal choice of \( k < I - 1 \), the model still needs to define the behavior of the seller targeting to sell to all buyer types, such that it targets \( k = I - 1 \). This seller’s price must therefore be the lowest in the market. Given Proposition (1), a market equilibrium exists only if it is the lowest cost seller’s optimal choice to list the lowest price in the market. It hasn’t been established that an equilibrium exists in this market. The question of interest is whether the equilibrium - if it exists - is robust to a wider choice of pricing rules.

3.2 Buyer’s Bid (BB)

\cite{Perry1986} illustrates that if the seller can choose its pricing rule, irrespective of whether its cost is high or low it would not want to invite the buyer to make a bid while reserving for itself the decision of accepting or refusing sale at the price that has been bid. This is because doing so signals low cost, thus adversely selecting the buyer’s bid.

The buyer, in \cite{Perry1986}, was modeled as having private information on her value for the good. Although in our model, buyer types do not have different values but rather different sample sizes of information regarding the rest of the market, this basic result holds here too, as follows.

Consider the seller offering BB to the buyer, while the option of TILI exists for the seller. The buyer, irrespective of type, would expect the seller
to accept her bid as long as its no smaller than the seller’s marginal cost. Believing this, the buyer would make a bid that maximizes her expected consumer surplus from the purchase. The optimal bid of the buyer is therefore $b^*$, such that:

$$b^* = \arg\max_b F(b)(v - b), \text{ with belief that seller type } s \text{ accepts } b \geq s.$$  

The buyer expects $b^*$ to be refused by all sellers with cost, $s_i$, such that $s_i > b^*$, therefore, expecting (correctly) that sellers of these types will be unable to sell through BB. Moreover, comparing with the positive expected profit from TILI, only sellers with cost strictly less than $b^*$ are expected to invite the buyer’s bid. But this implies that only those sellers could prefer BB for who selling at $b^*$ involves at least a positive expected profit. That is, buyers should expect only low cost sellers to offer BB. The observation of BB by a seller therefore signals to a buyer that the seller’s cost is no higher than some threshold, which must be lower than $b^*$; thus truncating the believed distribution of seller cost accordingly, if BB is offered. The buyer’s optimal bid, updating her belief thus, is no longer $b^*$, but rather a lower bid, call it $b^{**}$, such that $b^{**} = \arg\max_{b < b^*} F(b)(v - b)$. This bid is no longer attractive even to some sellers who the buyer earlier believed would prefer BB, truncating the believed cost distribution further, given an offer of BB. Recursively arguing and updating the buyer’s belief, unravels the buyer’s optimal bid all the way to $s$, which also fails to sustain an equilibrium between buyer’s belief and seller’s chosen action, as at this bid no seller prefers to offer BB. This implies that no seller has any incentive to invite the buyer’s bid. A market failure occurs because of the adverse selection of the invited bid. This gives the following as a direct result, and is therefore stated without proof.

**Proposition 2.** No seller will choose to offer BB and deviate from the all-TILI equilibrium.

### 3.3 Price Matching (PM)

The adoption of a price matching policy by a seller is not a decision that is separable from its choice of posted price. A seller choosing PM, when no other seller does so, adds to its TILI expected profit function, such that now it can sell to buyer types higher than the one it is targeting with its posted
price, promising to sell at their best known price which can be approximated by the seller, ex ante, as the buyer’s EMP. The PM expected profit function is therefore a generalization of the TILI expected profit function, adding for expected price matching, and is defined below:

$$\pi_i^{PMdev} = G(k_i)[\hat{p}(k_i, Q(p_{-i})) - s_i] + \delta \int_{k_i}^{k_{i+1}} G'(k)[\hat{p}(k, Q(p_{-i})) - s_i] dk. \quad (6)$$

The first term in this expected profit function is the expected profit from selling to the target and lower types of buyers at the posted price (as in the TILI profit function), and the second term is that from selling to higher types of buyers at their invoked matches (at prices equal to their EMPs). The factor, $\delta$, is a ‘discount factor’ that adjusts for the uncertainty that a match will be invoked by the buyer for whom a case for matching exists. If invoking matches is costless, the match would be expected to be invoked with a probability of $1/2$ if the rest of the market is selling through TILI - so no rival matching offers exist; and with a smaller probability if there are other sellers in the market matching, in which case $\delta$ would become a function of buyer type.

PM can be said to be a combination of TILI and BB, with a buyer type-specific price floor (estimated ex ante by the buyer’s EMP) automatically imposed on the bid (price requested to be matched) by requiring that it exist as a price in the rival market. More specifically, offering PM to a buyer (of type $k_j$), is for seller $i$, equivalent to selling at an expected price equal to:

$$\lambda_j(p_i | p_i \leq EMP_j) + (1 - \lambda_j)(EMP_j | p_i > EMP_j),$$

where $p_i$ is the seller’s posted price; and $\lambda_j$, a decreasing function of $k_j$, is the probability that the buyer’s EMP is no smaller than the seller’s posted price. That is, it is a linear combination of selling at posted price and selling at buyer’s bid as long as the bid exists as a rival price known by the buyer.

More importantly, even if the choice of this rule signals a low cost, the buyer is now restrained from exploiting that information because she can only bid prices that exist in the rest of the market. Thus although PM opens up the possibility for selling at prices bid by buyers in the form of their match requests, the adverse selection is controlled by censoring the prices a buyer can bid. The lowest price known by the buyer, may thus still be greater than the cost the buyer attributes to the seller, given the seller’s
signal of a low cost.

The price discrimination is enabled by eliciting a buyer’s private information regarding the rest of the market (captured by the buyer’s lowest known price or her ‘outside option’), while committing\footnote{Such commitment resembles what Camera and Delacroix (2004) point out as being necessary for buyers to reveal information through a quote/bid. In this case, complying is also the seller’s best response given its offer to price match, because matching interactions in that case give positive expected profit.} to sell at that price, subject to verification that it exists as a rival price. At the same time, it leaves the buyer with a surplus - if any - just large enough to convince her to purchase, given the competition that her best known rival price poses to the seller.

The FOC now is:

\[(1 - \delta)[\hat{p}(k_i, Q(p_{-i})) - s_i] = -\frac{G(k_i)}{G'(k_i)} \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i}.\]  

Comparing with (3), this FOC is different only by the factor \((1 - \delta)\) multiplying the LHS. Intuitively the tradeoff in targeting a marginally higher buyer type is similar to that in TILI except that now higher types are expected to purchase by invoking matches; but may not do so with probability \((1 - \delta)\).

From (3), and given that \(1 - \delta < 1\), it is evident that when the rest of the market uses TILI, a seller’s use of PM, raises its optimal posted price. Let the seller’s optimum buyer type to target in the above FOC be \(k_{i}^{PM_{dev}}\). Then, we have:

\[k_{i}^{PM_{dev}} < k_{i}^{TILI}.\]

The choice of PM as opposed to TILI, when the rest of the market offers TILI is characterized in the following proposition.

**Proposition 3.** In a market where all other sellers offer TILI, seller \(i\) would prefer PM only if

\[\delta \int_{k_{i}^{PM_{dev}}}^{k_{i}^{TILI}} G'(k)[\hat{p}(k, Q(p_{-i})) - s_i] dk > 0.\]

**Proof.** See Appendix. □

Intuitively, the ability to thus price discriminate, exists only as long as doing so leaves the seller with a positive expected profit from the buyer types who will demand matches below the posted price.
PM thus serves to sell at the EMP or outside option of the buyer, rather than lose the buyer. It also solves the incredibility issue inherent in TILI wherein refusal by a buyer gives the seller incentive to renegotiate price. In PM, if a buyer refuses the posted price, she automatically has the option of purchasing at her best known rival price; and the seller has no incentive to lower the price anymore because the buyer has no bargaining power below her best known rival price. However, from Proposition 3, it is likely that the market consists of both - sellers price matching, and those not.

4 TILI or PM

If both TILI and PM exist in the market, then for any given seller, the expected profit from choosing to offer PM is:

\[ \pi_i^{PM} = G(k_i)[\hat{p}(k_i, Q(p_{-i})) - s_i] \int_0^{k_i} \frac{h(p_L, k)G'(k)dk}{G(k)} \]

\[ + \int_{k_i}^{L-1} \delta(p_L, k)G'(k)[\hat{p}(k, Q(p_{-i})) - s_i]dk, \]

where \( p_L \) is the lowest price in the market. Notice we continue to treat EMP as before; i.e. it is still the expectation of the minimum posted price known by a buyer type. That is, a price match offer available to a buyer will be considered over and above her EMP of known (posted) prices.

Because rival sellers may also offer PM, \( \delta(p_L, k) \) is the probability of the buyer (type \( k \)) invoking/requesting a match at the given seller’s store. As the lowest price in the market increases, the number of sellers offering to match lower rival prices would be expected to increase, and therefore, the probability of any one seller receiving a match request decreases. Moreover, a more informed buyer has a higher probability of knowing rivals’ matching policies, and therefore the probability that any one seller gets a price match request from a higher type buyer, is lower. Therefore, we will take the following partial derivatives to hold:

\[ \frac{\partial \delta(p_L, k)}{\partial p_L}, \frac{\partial \delta(p_L, k)}{\partial k} < 0. \]

Similarly, \( h(p_L, k) \) is the probability of the buyer (type \( k \)) purchasing from
the seller when the seller’s posted price equals the buyer’s EMP. This is
less than one, as compared with \( \delta \), because we now consider rival sellers
offering matches. As the lowest price in the market increases, the number
of sellers offering to match lower rival prices would be expected to increase,
and therefore a buyer is more likely to seek matches elsewhere. Moreover,
a more informed buyer has a higher probability of knowing rivals’ matching
policies, and therefore the probability that any one seller gets this buyer’s
purchase (without matching) is lower. Therefore, also:

\[
\frac{\partial h(p_L, k)}{\partial p_L} , \frac{\partial h(p_L, k)}{\partial k} < 0. \tag{10}
\]

The first term on the RHS of (9) is the expected profit from selling to
buyer types equal to and lower than that being targeted \( (k_i) \), at the posted
price \( \hat{p}(k_i, Q(p_{-i})) \), but now the probability of sale is \( G(k_i) \int_0^{k_i} h(p_L, k)G'(k)dk \)
which is simply \( \int_0^{k_i} h(p_L, k)G'(k)dk \), or the average probability of making a
sale without a requested match. And the second term is the expected profit
from buyer types whose EMPs would be lower than the posted price of the
seller, but might ask the seller for a match; the selling price for buyer type
\( k \) would be her EMP in this case. For sellers offering TILI in this market,
only the first term of this expected profit function applies.

I make an important assumption here regarding the probabilities \( h(p_L, k) \),
and \( \delta(p_L, k) \), for a given \( k \) and given \( p_L \). I assume that the probability (for a
given seller) that the buyer will purchase if the posted price equals her EMP
is greater than the probability that she will purchase by invoking a match
(equal to her EMP) when the posted price exceeds her EMP; and that the
difference between the two diminishes as the buyer type increases. That is,
assume that

\[
h(p_L, k) > \delta(p_L, k) \forall k, \text{ and } \tag{11}
\]
\[
\frac{\partial [h(p_L, k) - \delta(p_L, k)]}{\partial k} < 0. \tag{12}
\]

These assumptions imply that a buyer is more likely to purchase (from a
seller) when she does not have to request a match; but this difference in
likelihood diminishes as the type of buyer increases.
The FOC for a seller choosing TILI is then given by:

\[
\hat{p}(k_i, Q(p_{-i})) - s_i = -\frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \int_0^{k_i} h(p_L, k)dk \frac{h(p_{-i}, k_i)}{G'(k_i)h(p_L, k_i)},
\] (13)

while the FOC for choosing PM is:

\[
\hat{p}(k_i, Q(p_{-i})) - s_i = -\frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \int_0^{k_i} h(p_L, k)dk \left[ h(p_{-i}, k_i) - \delta(p_{-i}, k_i) \right] G'(k_i).
\] (14)

The FOC (14) differs from (13) only in the denominator of the RHS. Given our assumptions (11), and (12), the RHS of (14) also increases in \( k_i \). And therefore, an analogy to (8) exists even now, i.e. the target buyer (type) that a seller would choose if offering PM is smaller than that chosen if offering TILI. Equivalently, \( k_i^{PM} < k_i^{TILI} \) even when rival sellers can also choose PM; this gives the next corollary.

**Corollary 2.** The act of offering PM raises any given seller’s posted price, by lowering its target buyer type at the margin.

Also notice that the factor \( \int_0^{k_i} h(p_L, k)dk \) on the RHS of (13) also increases in \( k_i \), by the assumption made above. These FOCs are therefore analogical to the FOC (3). Therefore, a similar result to Proposition (1) is obtained here as well, i.e. amongst all sellers choosing TILI (or amongst all sellers choosing PM), post prices can be ranked by their costs. The following corollary therefore directly results from Proposition (1).

**Corollary 3.** Within the group of sellers choosing to offer TILI, posted prices can be ranked by their costs. This is also true within the group of sellers choosing to offer PM.

Moreover, as before, a seller offers PM only if at its optimal \( k_i^{PM} \), it expects to earn positive profits, on average, from all transactions involving price matching.

Effectively then, in a large market, each seller chooses a strategy - a posted price (by choosing a buyer type to target at the margin) plus the choice between a TILI ultimatum and PM - such that its best response
strategy is a function of $p_L$, and thereby, of the rival price and rule distribution.

**Proposition 4.** An equilibrium exists only if the lowest price is set by a seller with the goal to balance that seller’s incentives to earn a higher price-cost margin, and its incentives to discourage price matching by rivals.

That is, an equilibrium exists only if the seller listing the lowest price in the market, is also playing its best response to rival prices. Notice that the seller listing the lowest price in the market chooses to target selling to all buyer types; while other sellers price according to their FOCs. Also, because rival sellers possibly offer to match, not all prospective buyers of the lowest priced seller necessarily purchase from it, but the probability, $\int_0^{I-1} h(p_L,k)\Phi'(k)dk$, that they do so is itself a function of its price, $p_L$, because that determines the number of rivals that choose PM.

Therefore, in any equilibrium, the seller posting the lowest price, $p_L$, effectively optimizes the following function:

$$\max_{p_L} (p_L - s_1) \int_0^{I-1} h(p_L,k)\Phi'(k)dk. \quad (15)$$

The relevant FOC for this seller is:

$$(p_L - s_1) = -\frac{\int_0^{I-1} h(p_L,k)\Phi'(k)dk}{\int_0^{I-1} \frac{\partial h(p_L,k)}{\partial p_L}\Phi'(k)dk}.$$

Therefore, while sellers who adopt price matching raise their posted prices to extract more surplus from uninformed buyers, and give private discounts to informed buyers demanding matches (thus, price discriminating), the seller posting the lowest price in the market balances the opposing incentives - to post a lower price and discourage the adoption of price matching by the rest of the market, and to post a higher price and earn a higher margin on every unit sold.

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13 This integral uses $\Phi'(k)$ instead of $G'(k)$ because at chosen $k = \frac{I-1}{I}$, $\beta(k) = 1$ which gives that $G'(k) = \Phi'(k)$. 

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19
5 Conclusion

This paper illustrates that in a large market with a variety of buyers and sellers, the opportunity to price discriminate between different types of buyers with differences in information can lead to a mixture of pricing policies being used by different sellers, such that sellers’ choice of pricing rule is an endogenous function of the market price and rule distribution. An important caveat is that the model treats price matching only where an offer must be market wide. A policy to match sub-samples of rival prices is more complicated and the claims in this essay do not extend to that.

The PM policy is shown to correct for the sub-optimality of selling via a single posted price, while also considerably reducing the adverse selection involved in giving the buyer the control to determine the price by making a bid. It also solves the incredible instability of a TILI ultimatum associated with a single posted price, in case of a positive price-cost margin.

Not all sellers, however, can adopt this policy; and therefore, both TILI ultimatums and PM co-exist in such markets. If the marginal entrant’s cost is equal to the buyers’ value for the good, this seller does not find it profit improving to use PM. For sellers who do use it, their conditioning of the policy on consumers invoking it - rather than making it absolute by posting a price equal to the lowest price in the market - makes the decision to adopt price matching endogenous to the market price distribution. Therefore, at least for the seller listing the lowest price, rival price matching policies are not absolute, and are instead dependent on its own price. This seller balances its incentives of a high price-cost margin and a high probability of sale (small probability of rivals’ PM) with its choice of posted price.

On one hand, adoption of PM gives a seller incentives to hike its posted prices because the policy substitutes for an actually lower price; and on the other hand, the availability of PM in a market, motivates the lowest price in the market to discourage some rivals from using the option. Something similar might happen in such markets motivating every (low) price to discourage rival PM adoption to some extent. It would then be that the distribution of prices, especially the mass on the lower tail of prices, endogenously determines the proportion of sellers adopting PM, rather than just the lowest price in the market having such incentives as illustrated here.

Another interesting conjecture that arises from this paper is that there might exist a threshold cost in the market, that is a function of the price and rule distribution, such that sellers with cost above this threshold do not find it profitable to choose PM, and the availability of PM splits the market cost-wise into those who can and cannot offer PM. This is because of the
correlation between cost ranks and price ranks, the fact that when choosing TILI the price-cost margin is higher for lower cost sellers, and because higher cost sellers target lower type buyers with their posted prices. Therefore, a higher cost seller is less likely to be able to offset the losses that may result from matching interactions, while also expecting losses more often in matching interactions because of a larger difference between its cost and the lowest price in the market. Thus, PM would squeeze the profits (and the range of prices) of higher cost sellers because lower cost sellers would adopt PM and raise their posted prices.
6 Appendix: Proofs

Lemma 1 For sellers $a$ and $b$, neither of who define the infimum or supremum of the market price distribution, in TILI equilibrium, $p^{TILI}_a = p^{TILI}_b$ iff $k^{TILI}_a = k^{TILI}_b$.

Proof. Step 1: $p^{TILI}_a = p^{TILI}_b$ is equivalent to $Q(p^{TILI}_a) = Q(p^{TILI}_b)$, i.e. if two sellers list the same price in equilibrium, then the prices in the rest of the market (excluding each of them, but including the price of the other) are the same for both. Similarly, $p^{TILI}_a > p^{TILI}_b$ is equivalent to $Q(p^{TILI}_a)$ first order stochastically dominating (FOSD) $Q(p^{TILI}_b)$, because exactly one price in $Q(p^{TILI}_a)$ exceeds that in $Q(p^{TILI}_b)$, while all other prices are identical in both sets (as long as neither seller’s choice defines the minimum/maximum price in the market).

Step 2: There are exactly three possibilities in comparing the prices of the two sellers: $p^{TILI}_a = p^{TILI}_b$, $p^{TILI}_a < p^{TILI}_b$, and $p^{TILI}_a > p^{TILI}_b$.

Let $k^{TILI}_a = k^{TILI}_b = k^{TILI}$, i.e. the target buyers for both sellers have the same degree of information about the rest of the market. Thus, if these target buyers face the same ‘rival’ market, they should have the same EMP, and the one facing the market with exactly one lower (higher) price should have a strictly lower (higher) EMP. That is,

$$\hat{p}(k^{TILI}, Q(p^{TILI}_a)) = \hat{p}(k^{TILI}, Q(p^{TILI}_b)) \iff Q(p^{TILI}_a) = Q(p^{TILI}_b),$$

and

$$\hat{p}(k^{TILI}, Q(p^{TILI}_a)) > \hat{p}(k^{TILI}, Q(p^{TILI}_b)) \iff Q(p^{TILI}_a) \text{ FOSD } Q(p^{TILI}_b).$$
But, from Step 1 above, \( Q(p_{-a}^{TILI}) = Q(p_{-b}^{TILI}) \) is equivalent to \( p_b^{TILI} = p_a^{TILI} \), and \( Q(p_{-b}^{TILI}) \) FOSD \( Q(p_{-a}^{TILI}) \) is equivalent to \( p_a^{TILI} > p_b^{TILI} \). Therefore,

\[
\hat{p}(k_a^{TILI}, Q(p_{-a}^{TILI})) = \hat{p}(k_b^{TILI}, Q(p_{-b}^{TILI})), \text{ iff } p_a^{TILI} = p_b^{TILI}, \text{ and }
\]

\[
\hat{p}(k_a^{TILI}, Q(p_{-a}^{TILI})) > \hat{p}(k_b^{TILI}, Q(p_{-b}^{TILI})), \text{ iff } p_a^{TILI} > p_b^{TILI}. \quad (16)
\]

But (16) is a contradiction because by definition \( p_i^{TILI} \) is equal to \( \hat{p}(k_i^{TILI}, Q(p_{-i}^{TILI})) \), and therefore \( \hat{p}(k_a^{TILI}, Q(p_{-a}^{TILI})) > \hat{p}(k_b^{TILI}, Q(p_{-b}^{TILI})) \) is by definition equivalent to \( p_b^{TILI} > p_a^{TILI} \).

Therefore, \( k_a^{TILI} = k_b^{TILI} = k^{TILI} \) implies that in equilibrium, neither price can be greater than the other, i.e. they must be equal.

Step 3: For the converse, let \( p_a^{TILI} = p_b^{TILI} \). By definition, this means

\[
\hat{p}(k_a^{TILI}, Q(p_{-a}^{TILI})) = \hat{p}(k_b^{TILI}, Q(p_{-b}^{TILI})). \quad (17)
\]

From Step 1 above, this is equivalent to:

\[
Q(p_{-a}^{TILI}) = Q(p_{-b}^{TILI}). \quad (18)
\]

Equations (17) and (18) are both reconcilable only if \( k_a^{TILI} = k_b^{TILI} \) because different information types would lead to different EMPs from the same distribution of prices.

**Lemma 2.** For any sellers \( a \) and \( b \), neither of who define the infimum or supremum of the market price distribution, in TILI equilibrium, \( p_a^{TILI} > p_b^{TILI} \) iff \( k_a^{TILI} < k_b^{TILI} \).
Proof. Consider $p_a^{TILI} > p_b^{TILI}$. This is equivalent to $\hat{p}(k_a^{TILI}, Q(p_a^{TILI})) > \hat{p}(k_b^{TILI}, Q(p_b^{TILI}))$ and $Q(p_b^{TILI}) \ FOSD \ Q(p_a^{TILI})$. But these are both reconcilable together only if $k_a^{TILI} < k_b^{TILI}$, because the same information types or a $k_a^{TILI}$ higher than $k_b^{TILI}$ would lead to $\hat{p}(k_a^{TILI}, Q(p_a^{TILI})) < \hat{p}(k_b^{TILI}, Q(p_b^{TILI}))$, given $Q(p_b^{TILI}) \ FOSD \ Q(p_a^{TILI})$.

For the converse, let $k_a^{TILI} < k_b^{TILI}$. From the three possibilities: $p_a^{TILI} = p_b^{TILI}$, $p_a^{TILI} < p_b^{TILI}$, and $p_a^{TILI} > p_b^{TILI}$, the first two are ruled out because each gives rival sets of prices from which a lower information type cannot lead to the same or a lower EMP, leading to a contradiction. \hfill $\square$

**Proposition** In TILI equilibrium, for sellers $a$ and $b$, neither of whose choice defines the minimum/maximum price in the market, if $s_a < s_b$, then it must be that $k_a^{TILI} > k_b^{TILI}$ or $p_a^{TILI} < p_b^{TILI}$.

Proof. For sellers $a$ and $b$, let $s_a < s_b$.

First, suppose that in equilibrium, $k_a^{TILI} = k_b^{TILI} = k^{TILI}$. Then from Lemma 1, $p_a^{TILI} = p_b^{TILI}$. From each seller’s FOC as given by (3) and using $p_i^{TILI} = \hat{p}(k_i^{TILI}, Q(p_i^{TILI})), \forall i$; gives $p_a^{TILI} - s_a = p_b^{TILI} - s_b$.

But $p_a^{TILI} = p_b^{TILI}$, while $s_a < s_b$, which is a contradiction.

Next consider, $k_a^{TILI} < k_b^{TILI}$ in equilibrium, which gives $p_a^{TILI} > p_b^{TILI}$ from Lemma 2. But because $p_a^{TILI} > p_b^{TILI}$, seller $a$ faces a first order stochastically dominated rival market price distribution compared with that faced by seller $b$; i.e. for a common $k$, we have $\hat{p}(k, Q(p_a^{TILI})) < \hat{p}(k, Q(p_b^{TILI})), \forall k < \frac{I-1}{T}$, with approximate equality when $k = \frac{I-1}{T}$. An implication of this is that $\hat{p}(k, Q(p_a^{TILI}))$ falls more steeply in $k$ than does $\hat{p}(k, Q(p_{-a}^{TILI}))$.\hfill 24
That is,

\[ \left| \frac{\partial \hat{p}(k, Q(p^\text{TILI}_a))}{\partial k} \right| < \left| \frac{\partial \hat{p}(k, Q(p^\text{TILI}_b))}{\partial k} \right|, \forall k < \frac{I - 1}{I}. \] (19)

On the other hand, from \( s_a < s_b \) and \( p^\text{TILI}_a > p^\text{TILI}_b \), we have \( p^\text{TILI}_a - s_a > p^\text{TILI}_b - s_b \). From (3), the RHS of the FOCs of sellers \( a \) and \( b \) can be related in equilibrium as follows:

\[ \left| \frac{G(k^\text{TILI}_a)}{G'(k^\text{TILI}_a)} \left| \frac{\partial \hat{p}(k^\text{TILI}_a, Q(p^\text{TILI}_a))}{\partial k^\text{TILI}_a} \right| \right| > \left| \frac{G(k^\text{TILI}_b)}{G'(k^\text{TILI}_b)} \left| \frac{\partial \hat{p}(k^\text{TILI}_b, Q(p^\text{TILI}_b))}{\partial k^\text{TILI}_b} \right| \right| . \] (20)

Also from (1), (4), and the properties of the second derivative of the EMP, we know that the RHS of (3) increases in \( k \), which gives for rival prices of seller \( b \):

\[ \left| \frac{G(k^\text{TILI}_b)}{G'(k^\text{TILI}_b)} \left| \frac{\partial \hat{p}(k^\text{TILI}_b, Q(p^\text{TILI}_b))}{\partial k^\text{TILI}_b} \right| \right| > \left| \frac{G(k^\text{TILI}_a)}{G'(k^\text{TILI}_a)} \left| \frac{\partial \hat{p}(k^\text{TILI}_a, Q(p^\text{TILI}_a))}{\partial k^\text{TILI}_a} \right| \right| . \] (21)

Inequalities (20) and (21) together imply

\[ \frac{G(k^\text{TILI}_a)}{G'(k^\text{TILI}_a)} \left| \frac{\partial \hat{p}(k^\text{TILI}_a, Q(p^\text{TILI}_a))}{\partial k^\text{TILI}_a} \right| > \frac{G(k^\text{TILI}_b)}{G'(k^\text{TILI}_b)} \left| \frac{\partial \hat{p}(k^\text{TILI}_b, Q(p^\text{TILI}_b))}{\partial k^\text{TILI}_b} \right| , \] which contradicts (19) because \( \frac{G(k^\text{TILI}_a)}{G'(k^\text{TILI}_a)} > 0 \).

Therefore, if \( s_a < s_b \), in equilibrium, it must be that \( k^\text{TILI}_a > k^\text{TILI}_b \), which is equivalent to \( p^\text{TILI}_a < p^\text{TILI}_b \) from Lemma (2).

Corollary 1: For sellers \( a \) and \( b \), neither of whose choice defines the minimum/maximum price in the market, if \( s_a < s_b \), then in TILI equilibrium it must be that \( p^\text{TILI}_a - s_a > p^\text{TILI}_b - s_b \).

Proof. Let \( s_a < s_b \). From Proposition (1) we therefore have \( k^\text{TILI}_a > k^\text{TILI}_b \) and equivalently \( p^\text{TILI}_a < p^\text{TILI}_b \).
This implies $\hat{p}(k, Q(p_{\text{TILI}}^a)) > \hat{p}(k, Q(p_{\text{TILI}}^b))$, $\forall k < \frac{I-1}{I}$, and the two are approximately equal when $k = \frac{I-1}{I}$. It also implies that $\hat{p}(k, Q(p_{\text{TILI}}^a))$ falls more steeply in $k$ than does $\hat{p}(k, Q(p_{\text{TILI}}^b))$. That is,

$$\left| \frac{\partial \hat{p}(k, Q(p_{\text{TILI}}^a))}{\partial k} \right| > \left| \frac{\partial \hat{p}(k, Q(p_{\text{TILI}}^b))}{\partial k} \right|, \forall k < \frac{I-1}{I}. \quad (22)$$

There are two possibilities regarding the relative price cost margins: $p_{\text{TILI}}^a - s_a < p_{\text{TILI}}^b - s_b$ and $p_{\text{TILI}}^a - s_a > p_{\text{TILI}}^b - s_b$. Consider the first. From the FOC (3) we therefore have

$$G(k_{\text{TILI}}^a) \frac{\partial \hat{p}(k_{\text{TILI}}^a)}{\partial k_{\text{TILI}}^a} |_{Q(p_{\text{TILI}}^a)} < G(k_{\text{TILI}}^b) \frac{\partial \hat{p}(k_{\text{TILI}}^b)}{\partial k_{\text{TILI}}^b} |_{Q(p_{\text{TILI}}^b)}.$$

We also know from (1), (4) that the RHS of the FOC increases in $k$, i.e.

$$\frac{G(k_{\text{TILI}}^a)}{G'(k_{\text{TILI}}^a)} \left| \frac{\partial \hat{p}(k_{\text{TILI}}^a)}{\partial k_{\text{TILI}}^a} \right| < \frac{G(k_{\text{TILI}}^b)}{G'(k_{\text{TILI}}^b)} \left| \frac{\partial \hat{p}(k_{\text{TILI}}^b)}{\partial k_{\text{TILI}}^b} \right|.$$

But this together with (23) is a contradiction to (22). Therefore, it must be that $p_{\text{TILI}}^a - s_a > p_{\text{TILI}}^b - s_b$. \hfill \qed

**Proposition 3:** In a market where all other sellers offer TILI, seller $i$ would prefer PM only if $\delta \int_{k_i^{\text{PMdev}}}^{I-1} G'(k) \left[ \hat{p}(k, Q(p_{-i})) - s_i \right] dk > 0$. 

*Proof.* If $\delta \int_{k_i^{\text{PMdev}}}^{I-1} G'(k) \left[ \hat{p}(k, Q(p_{-i})) - s_i \right] dk \leq 0$, the seller would rather withdraw the offer to match and simply offer TILI without changing its posted price. But this implies that the seller’s profit is, $G(k_i) \left[ \hat{p}(k_i, Q(p_{-i})) - s_i \right]$, with argument maximizer $k_i^{\text{TILI}}$ from (3), rather than $k_i^{\text{PMdev}}$. That is, the seller must then choose $k_i^{\text{TILI}}$ and should only sell at its posted price. Therefore, a deviation to PM when all others offer TILI is profitable for a seller only if $\delta \int_{k_i^{\text{PMdev}}}^{I-1} G'(k) \left[ \hat{p}(k, Q(p_{-i})) - s_i \right] dk > 0$. \hfill \qed

26
References


