Non-price Discrimination by a Prejudiced Platform*  

Ming Gao  
Tsinghua University  
Email: gaom@sem.tsinghua.edu.cn  

Travis Ng  
The Chinese University of Hong Kong  
Email: TravisNg@cuhk.edu.hk  

January 19, 2017  

Abstract  
Once a platform market has tipped, the dominant platform may act on its prejudice free from competitive pressure. Should regulators tighten anti-discrimination policies to platform markets, especially those with a tendency to tip? We build a two-sided market model to address this issue. We find that a prejudiced platform monopoly does not always act on its prejudice to discriminate. Even when it does, social surplus does not necessarily decrease. In order to remain fairly attractive to users from both sides, the platform may have to sufficiently lower prices when it discriminates, which eventually raises user surpluses on both sides. We derive a condition under which even the group of users that are being discriminated against may be better off. The policy implication is that it is not always socially efficient to ban discrimination by platforms even when the market has tipped. Our model also suggests that regulating platform through well-designed mandatory disclosure requirements can reduce discrimination.  

Key Words: discrimination; two-sided market; non-price strategies; prejudice; regulation; policies on platforms.  
JEL Classification: D42, L11, L12  

*Part of this article has previously been circulated under the title “The Economics of App Review.” We thank Bruno Jullien, Martin Osborne, Andrew Rhodes, Alexander White, Chiu Yu Ko, Junjie Zhou and the seminar participants at the Toulouse 2016 E-commerce, Digital Economy and Delivery Services Conference, the Nanjing University 2016 International Conference on Innovation and Industrial Economics, Shanghai Jiao Tong University and Shanghai University of Finance and Economics. Ming Gao acknowledges the financial assistance of Project 20151080389 supported by the Tsinghua University Initiative Scientific Research Program and Project 71203113 supported by the National Natural Science Foundation of China.
1 Introduction

While platform businesses are becoming all the more important, they are also attracting increasingly more regulatory attention. Many platforms defy traditional business models by using new ways to connect sellers with buyers, men with women, borrowers and lenders, etc. Their radically new business models almost always challenge existing regulatory frameworks.\(^1\)

Many interesting papers study the condition for a platform market to tip, that is, one platform emerges as the clearly dominant player while others are marginalized, if not disappear.\(^2\) Antitrust regulators often keep a close watch at these markets. It is because once a market has tipped, it lacks competition as network effects make the dominant platform highly defensible.\(^3\) It seems reasonable for regulators to step in to protect users from potential abuses.

An issue that the public has expressed concern about is discrimination by platforms. Discrimination allegations abound. Some borrowers have complained that Lending Club has discriminated against them by requesting them to submit more documents than what others are required to submit.\(^4\) Some app developers have complained Apple Store of either rejecting their apps or approving them with substantial delays without being given clear reasons. Shopping malls connecting businesses and shoppers have also been alleged of discrimination by both sides. In Radek v. Henderson Development (Canada), an aboriginal woman successfully sued a mall for discriminating against her.\(^5\) A Singaporean mall has to issue an open apology for denying the rental application of a Malaysian businesswoman, which appeared to be due to racial bias.\(^6\) Match.com published an article suggesting that older people should not state in their online dating profile that they are

\(^{1}\)There are many high-profile regulatory challenges for platforms. Should the hosts and the guests of Airbnb pay hotel taxes? Are Uber drivers independent contractors or employees? How would the SEC regulate the peer-to-peer lending platforms such as Prosper and Lending Club? A managerial implication articulated best by Hagiu and Rothman (2016) is that not only is a platform hard to build, but once it is built, regulatory risks can kick in at any moment.

\(^{2}\)Hossain, Minor, and Morgan (2011) and Hossain and Morgan (2013), for instance, conduct illuminating experiments to single out the forces for a platform market to tip. Sun and Tse (2007) find the tendency of users to “multi-home” is critical in determining whether a market will tip.

\(^{3}\)The U.S. Department of Justice, for fear of a dominant search platform, inform Google and Yahoo! that it would file an antitrust lawsuit to block the implementation of a search agreement between the two companies. Link: U.S. Department of Justice’s announcement.

\(^{4}\)Here are the two links: 1 and 2

\(^{5}\)The Canadian case was Radek v. Henderson Development (Canada) Ltd. (No. 3) (2005), 52 C.H.R.R. D/430, 2005 BCHRT 302. Here is the link.

\(^{6}\)A Singaporean mall has been alleged of discriminating against a Malaysian businesswomen. Here is the link.
“young for my age,” irritating many older women.\textsuperscript{7} An atheist has alleged that eHarmony has discriminated against him by not turning up any match for him.\textsuperscript{8} In \textit{Fair Housing Council of San Fernando Valley v. Roommates.com}, Roommates.com was convicted of extracting information from potential customers as a condition of accepting them as clients.\textsuperscript{9} Some customers find it offensive for Roommates.com to ask about gender, sexual orientation, number of children, and whether the children lived with the customer. It is hard to tell whether the platforms did discriminate, and whether they did it because of prejudice.

Discrimination by a platform can look especially bad when a market has tipped. If a market is competitive enough, Becker (1957) reassures us that even if a firm is not prohibited from acting on its prejudice, the market will make it pay for it.\textsuperscript{10} It is because those being discriminated against would self-select to interact with those firms that are not discriminatory. In addition, a discriminatory firm will be losing market shares to its rivals who are not.\textsuperscript{11}

If a platform market has tipped, it is conceivable that the dominant platform can freely indulge in whatever prejudices it has against certain users based on age, sex, race, nationality, political view, look, disability, etc. Those against whom the platform discriminates cannot opt for another platform. Therefore such a platform does not lose market share even if it is discriminatory. What should regulators do in this situation? Should anti-discrimination laws be more strictly applied to platform markets showing signs of tipping?

\textbf{Model} \quad Is it socially efficient to tighten any anti-discrimination regulations on platforms, especially to those that have emerged as the dominant platforms in their respective markets? We develop a model to address this question. Our strategy is to look into the worst case scenario: a platform market that has already tipped. In addition, the platform discriminates against certain users \textit{because of} prejudice.\textsuperscript{12} Such a scenario stacks the cards

\textsuperscript{7}The article can be seen on Match.com \textit{here.}
\textsuperscript{8}The article can be seen \textit{here.}
\textsuperscript{9}\textit{Fair Housing Council of San Fernando Valley v. Roommates.com, LLC}, 521 F.3d 1157 (9th Cir. 2008)
\textsuperscript{10}However, see Sunstein (1991) for an alternative view on the limit for the market to eliminate discrimination.
\textsuperscript{11}Analyzing a one-sided monopolized market, Epstein (1992) remarks that “The use of the anti-discrimination provision, therefore, has powerful justification whenever practical or legal circumstances prevent the emergence of a competitive market. (page 85)” On the other hand, Alchian and Kessell (1962), in response to Becker (1957), highlighted the likelihood that a monopoly would engage in non-wealth maximizing discriminatory practices from a property right economics perspective. Becker (1962) was the discussant of their paper.
\textsuperscript{12}All these cases mentioned above have a common feature: it is hard to tell whether the platform discriminates against certain users because of prejudice, or because of valid reasons concerning the positioning, the quality, or the operating cost of their platform business.
against us by making it difficult to conceive no government intervention.

We formalize this idea as a modified model of Armstrong (2006) featuring a platform that facilitates transactions between buyers and sellers (as the two sides). Since there is no competing platform after a market has tipped, users on both sides have no other choices but to either join the platform or opt out.\textsuperscript{13} The platform incurs an additional cost of serving a certain subset of sellers.\textsuperscript{14} Such an additional cost can be due to the platform’s prejudice against them, i.e., the platform owner just dislikes interacting with them. However, these sellers have not the slightest difference from any other sellers in the eyes of the buyers. Therefore, when the previously mentioned additional cost is due to prejudice alone, discrimination will be purely taste-based.\textsuperscript{15}

We assume the platform monopoly cannot charge different users on the same side differently. To act on the prejudice, the platform has to use non-price discrimination practices.\textsuperscript{16} We do not address price discrimination for three reasons.

First, not only that the real world allegations above involved no price discrimination, but also outright price discrimination is prohibited by several antitrust laws.\textsuperscript{17} Second, the literature on price discrimination by platforms is comprehensive. We do not intend to contribute to this literature. Rather, our focus is on more subtle forms of discrimination that are usually non-price, harder to be detected but can be commonly used by those who are afraid to be sued for outright price discrimination. Third, our aim is to focus on the anti-discrimination policy implications in a platform market, rather than the optimal pricing of a platform. Eliminating the possibility of the platform of choosing prices allows us to focus on analyzing anti-discrimination policies. Non-price discrimination practices do not transfer utilities, they involve outright destruction of utilities, making it harder for

\textsuperscript{13}We assume away the possibility of user disintermediation, i.e., buyers and sellers conduct businesses outside of the platform. However, Hagiu and Rothman (2016) suggest that it can be a serious concern for platforms.

\textsuperscript{14}To fix idea, throughout the paper we refer a platform as connecting buyers and sellers. Of course, a platform can connect men with women, travelers with hosts, drivers with riders, etc.

\textsuperscript{15}Such a setup avoids all the complications of detecting discrimination, or sorting out whether a discriminatory practice is statistical or taste-based. Heckman (1998) discusses the empirical challenges associated with sorting out the two types of discrimination.

\textsuperscript{16}There is a literature on non-price discrimination by an upstream monopolist who is competing in its downstream industry too. Economides (1998) shows that such an upstream monopolist has incentives to engage in non-price discrimination against other competing downstream rivals, raising the concerns that there can be anti-competitive effects of non-price discrimination. Our paper is related to this literature but there is no upstream and downstream industry. Sand (2004) discusses the effects of regulation on the upstream monopolist’s incentives to non-price discriminate. The platform in our paper serves users in both sides. Therefore, the results in our paper does not contradict with those in Economides (1998) and Sand (2004).

Discrimination must be annoying. Consistent with our examples, to discriminate in our model means the platform imposes some disutility to the users it discriminates against,\(^{18}\) in addition to setting a price for each side of the market.

Before we talk about regulations, we first make it clear that there can be many policy objectives concerning discrimination. Our model allows us to look into one particular objective: efficiency. However, it does not allow us to address concerns about fairness, morality, friendship, and other values judgment.\(^{19}\)

**Result**  First, when the platform has a prejudice against certain users, it does not necessarily act on its prejudice even if there is no competition. Discrimination enables the platform to discourage participation by the users it finds “uncomfortable” to serve, and thereby avoid some psychological cost due to its prejudice. However, because a two-sided platform’s business depends crucially on the network externalities across its two sides, discrimination against certain users also reduces the platform’s appeal to users on the opposite side who possess no prejudices. Therefore it only has an incentive to discriminate either when its prejudice is significant enough, or when the proportion of the users who would be discriminated against is not too large. The tradeoff determining the platform’s incentive for discrimination is characterized in Proposition 3.

Second, when the platform chooses to act on its prejudice and discriminates, social welfare does not necessarily decrease, even if we ignore the platform’s interests and only consider those of its users. Discrimination is a tool that the platform uses to adjust the composition of users, in an effort to lower its perceived average cost. Because the size of either side of the platform is of utmost importance to attract business from the opposite side, even when the platform finds it desirable to discourage participation by a subset of participants on one side, it still has the same incentive to encourage precipitation by other participants on both sides, and this can only be achieved by price cuts.

We find that the platform may accompany discrimination with a lower total price for two sides in order to remain attractive to both sides, which implies that the price for at least one side - sometimes both - will be optimally reduced by the platform when it chooses to discriminate (see Proposition 7). This mechanism of “self-correcting” pricing is the fundamental source of welfare improvement from discrimination, and is present only

---

\(^{18}\)For instance, aboriginal peoples are asked to leave the mall and they feel hurt. Malay businesswomen are denied rental application. Homosexual individuals would feel especially offended by Roommates.com’s questions related to sexual orientation.

\(^{19}\)It is socially inefficient to force two individuals who do not like each other to be roommates. But educating can imply children would learn to accept others. Education purpose can be important, but our paper cannot address objectives like this.
due to the two-sided nature of the market (and therefore absent in one-sided markets).

Third, anti-discrimination policy against platforms can backfire. We show that making sure discrimination is socially efficient requires us to know a lot about the market. We ran simulations under rather simple uniform distributions, but the amount of information needed is still huge (see Proposition 5). It is unlikely that the court will be competent enough to obtain and analyze all the necessary information to determine whether discrimination increases social welfare. This is so even if regulatory enforcement costs are assumed to be zero.

Forth, and perhaps the most counter-intuitive result we find, is that it is even possible for discrimination to benefit the group of users that are being discriminated against by the platform, which may happen when the “self-correcting” pricing mechanism mentioned previously is strong enough (see Lemma 3).

Two policy implications follow. First, it is not always socially efficient to prohibit discrimination even if a two-sided market has tipped. Therefore, it makes it less compelling to apply stricter anti-discrimination policies to any two-sided markets that may tip.

The second has to do with antitrust policies towards platform businesses. There can be other reasons to concern about mergers between platforms, exclusionary practices, or coordinated practices among competing platforms. Fearing that a platform, having monopolized a two-sided market, can better discriminate appears a legitimate reason against any anti-competitive practices. Our results, however, suggest that it is not.

2 Related literature

Discrimination among platform users. While our paper studies discrimination by a platform, many interesting papers have studied a related kind of discrimination - discrimination among the users. An growing set of exciting new research on discrimination among users of digital platforms. Does discrimination persist in online platforms? Do online platforms even exacerbate discrimination?

Fisman and Luca (2016) highlight some papers addressing these questions and draw many insightful managerial implications. While Edelman, Luca, and Svirsky (forthcoming) also suggest several managerial implications faced by online platforms, they conduct experiments on Airbnb and find that applications from guests with distinctively

---

20 Evans and Schmalensee (2014) For an excellent survey of the literature.
21 Platforms such as Airbnb and Uber have published formal statements and policies addressing these types of discrimination. For AirBnB, please read: http://www.inc.com/tess-townsend/airbnb-hires-eric-holder.html. For Uber, please read: http://flavorwire.com/581580/ubers-evolving-relationship-with-discrimination.
African-American names are significantly less likely to be accepted by hosts relative to their control groups with distinctively White names. On the other side of the Airbnb market, Edelman and Luca (2014) find that African-American hosts ask and get significantly lower prices than otherwise similar White hosts. Pope and Sydnor (2011) look into peer-to-peer lending at Prosper.com and find that loan listings with blacks in the attached picture are significantly less likely to receive funding than those of whites with similar credit profiles. Duarte, Siegel and Young (2012) also examine Prosper.com; they rate borrowers’ trustworthiness only by viewing their photos and find that those “look trustworthy” are significantly more likely to have their loan requests granted. However, they are also more likely to eventually repay their loans. Experimenting with Uber and Lyft, Ge, Knittel, MacKenzie, and Zoepf (2016) find that African American passengers suffer longer waiting times and their orders are more likely to be canceled by the drivers. Doleac and Stein (2013) examine Craigslist and find that the same iPod receives significantly fewer responses from potential buyers if it is held by a Black hand than a White hand.

**Discrimination by platforms.** There is a growing interest on discrimination by digital platforms, especially on addressing the question of whether the use of big data and algorithms by digital platforms exacerbate discrimination. For instance, would whites be shown ads of more expensive items using a search engine because its opaque algorithms suggest whites are more likely to buy expensive purchases? Whites may feel discriminated against by the platform for it is harder for them to locate bargains.

Computer scientists have developed various tools and methodologies that help regulators and researchers to examine black-box algorithms to detect discrimination (Sandvig, Hamilton, Karahalios and Langbort, 2014). Sweeney (2013) find that “Googling” for common African-American names significantly more likely to result in ads offering criminal background checks than “Googling” for names common among whites. Datta, Tschantz and Datta (2015) also find that Google did not show as many ads for high-paying jobs if the Google profile’s setting is female rather than male.

Certainly, discrimination can be made by platforms that are non-digital. And there remains a need to clearly delineate whether any discrimination by digital platforms are taste-based or statistical. It is also useful to analyze whether regulators should step in, and what they should do about it. However, while computer scientists have been accumulating interesting research, as far as we know, there has not been any economics research on this area. Our paper fills this gap by offering a theoretical model within which a platform can discriminate some users because of prejudice and allow us to examine the welfare
implications under different legal environments.

3 The model

Basic set-up

Our model builds on the monopoly model by Armstrong (2006).\footnote{Rochet and Tirole (2003 and 2006) and Armstrong (2006) provide some of the canonical models for two-sided markets. In some two-sided markets, platforms use quality as a criterion to exclude some agents on one or both sides from entering the platforms. For instance, night clubs are likely to forbid guys and girls looking lousy from entering. Hagiu (2009) provides a model of using quality as an exclusion criterion and investigates the conditions for the imposition of such an exclusion system.} A platform facilitates interaction between two groups (i.e. sides) of users, group-1 (buyers) and group-2 (sellers). To each group i, suppose the platform can only charge a price $p_i$ for access to the platform. Each group has a total mass normalized to 1. Denote $n_i$ the number of users eventually joining the platform on side $i \in \{1, 2\}$. Users on either side directly benefit from a larger number of users on the other side. In particular, each side-i user derives a total utility $u_i$ from joining the platform, given by

$$u_i = \alpha_i n_j - p_i, \quad (1)$$

where $\alpha_i (\geq 0)$ represents the network externality that each side-i user enjoys from interacting with everyone on the opposite side, and is assumed to be exogenous.\footnote{We choose not to model any user’s pricing decisions, but focus on the platform’s incentives. If seller’s pricing decisions are explicitly modeled, since there is no different between one or the other seller in the eyes of the buyers, in equilibrium, all sellers should price buyers the same price. In reality, some platforms, such as Uber, do not allow users to set their own prices, while some do, such as AirBnB.} Denote

$$\alpha \equiv \alpha_1 + \alpha_2.$$

Following Armstrong (2006), we assume that the number of buyers depends solely on their utilities, denoted

$$n_1 = \phi_1(u_1).$$

where $\phi_1$ is increasing.\footnote{We provide a specific functional form of $\phi_1$ in section 5.} Assume it costs the platform $f_1$ to serve each buyer on side 1.

Two seller types

There are two types of sellers, L and H. The fraction of type-L sellers is $\lambda \in (0, 1)$ and that of type-H sellers is $(1 - \lambda)$. In the eyes of any buyer, sellers are equally valuable. This
is, however, not true for the platform. Specifically, it costs the platform $f_L$ to serve each type-L seller and $f_H$ to serve each type-H seller, where $f_H \geq f_L$. The fact that type-H sellers are more costly to the platform may give the latter an incentive to discriminate against them.

Denote $n_k$ the number of type-$k$ sellers joining the platform ($k \in \{L, H\}$). Without discrimination, the numbers of different types of sellers are determined by the utility each seller derives from joining the platform, $u_2$, in the following way

- type-L sellers: $n_L = \lambda \cdot \phi_2(u_2)$,
- type-H sellers (without discrimination): $n_H = (1 - \lambda) \cdot \phi_2(u_2)$,

where $\phi_2$ is increasing. The total number of sellers on the platform becomes

$$n_2 = n_L + n_H.$$ 

Denote $\Delta f$ the additional cost that type-H sellers impose on the platform compared to type-L sellers:

$$\Delta f \equiv f_H - f_L.$$ 

Such a difference has several interpretations.

- **Prejudice (non-pecuniary):** First, it can be the case that the operational cost of serving both types of sellers are the same. But the platform owner dislikes interacting with type-H seller. When the platform interacts with one, there is a psychological cost of $\Delta f$, which also measures the prejudice the platform has against type-H sellers.\(^{25}\)

- **Appeal (non-pecuniary):** Second, it can be the case that the operational cost of serving both types of sellers are the same. But the platform owner prefers interacting with type-L seller. When the platform interacts with one, there is a psychological benefit of $\Delta f$, which also measures the appeal type-L sellers have to the platform.\(^{26}\)

- **Additional operation cost (pecuniary):** Third, it can be just additional operational cost of serving type-H sellers.

\(^{25}\)Note that the platform monopoly does not believe that type-H sellers are less valuable in the eyes of the buyers too. Neither does type-H sellers have a chance to underinvest. Therefore, the kind of discrimination in our model is not driven by the possibility that the beliefs of the platform can be self-fulfilling, as in Phelps (1972) and Arrow (1973).

\(^{26}\)An example is that some shopping mall owners are environmentalists (health-conscious) and even though the operational costs of serving different shops are the same, those that have zero carbon footprints (selling organic food) really appeal to them more than other that have high carbon footprints (selling non-organic food).
Our model addresses discriminatory concerns due to a platform’s prejudice; we adopt the first interpretation. But the analysis below would not change even if we adopt the second or the third interpretations. It turns out that comparing these different interpretations would allow us to better understand our theoretical results. We will come back to this remark.

Without loss of generality, assume \( f_1, f_L, f_H, \Delta f \geq 0 \). For tractability, we also assume that both \( \phi_1 \) and \( \phi_2 \) are twice differentiable.

**Non-price discrimination**

Suppose for legal or practical reasons, the platform cannot price discriminate against any sellers. However, because type-H sellers are more costly to serve, the platform may still try to discourage them from joining.\(^{27}\) To formalize this notion, suppose the platform can use some non-price discriminatory practices to impose a disutility of \( D \in [0, \bar{D}] \) on each type-H seller who joins the platform.\(^{28}\) The notion \( D \) measures the extent of discrimination.

Therefore, while each type-L seller derives \( u_2 \) from the platform, the utility that each type-H seller derives is only \( (u_2 - D) \) whenever the platform discriminates. Under discrimination, the number of type-H sellers who join the platform becomes

\[
\text{type-H sellers (with discrimination): } n_H = (1 - \lambda) \cdot \phi_2(u_2 - D),
\]

where \( D \in [0, \bar{D}] \) is the disutility from discrimination imposed by the platform.

The platform’s utility is then given by

\[
\pi = (p_1 - f_1)n_1 + (p_2 - f_L)n_L + (p_2 - f_H)n_H, \tag{2}
\]

### 4 Private incentive to discriminate

The platform discriminates only if doing so increases its own profits (including all benefits and costs, psychological or otherwise).\(^{29}\) This section characterize the condition under

---

\(^{27}\)A platform can selectively expedite the services (such as help desk support) for some users but delay those of others. A mall can order its security guards to ask minority customers to show their id cards before entering the mall. A peer-to-peer lending website can ask certain group of users for more documents than normally are needed by other users.

\(^{28}\)When \( D = 0 \), there is no discrimination. \( D \) is clearly bounded from above as any seller has the outside option to leave the platform if the latter imposes too much disutility through discrimination.

\(^{29}\)Alchian and Kessel (1962) give reasons to support the notion that a monopoly is not profit-maximizing but utility-maximizing. If one is not comfortable with the notion that the monopoly is utility-maximizing, the model can adopt the third interpretation of \( \Delta f \), and the platform is profit-maximizing instead of utility-maximizing.
which the platform will have an incentive to act on its prejudice and discriminate against type-H sellers. To facilitate the analysis, we consider the utilities that the platform provides for different sides, \( u_1 \) and \( u_2 \), and the extent of discrimination, \( D \), as its choice variables.

Rewrite expression (2) as a function of \( u_1, u_2 \) and \( D \), substituting \( f_L = f_H - \Delta f \), and we get:

\[
\pi(u_1, u_2, D) \equiv (p_1 - f_1)n_1 + (p_2 - f_H + \Delta f)n_L + (p_2 - f_H)n_H,
\]

(3)

where demand from different groups and prices are also denoted as functions of \( u_1, u_2 \) and \( D \):

\[
\begin{align*}
n_1(u_1) &= \phi_1(u_1); \\
n_L(u_2) &= \lambda \phi_2(u_2); \\
n_H(u_2, D) &= (1 - \lambda)\phi_2(u_2 - D); \\
n_2(u_2, D) &= n_L(u_2) + n_H(u_2, D); \\
p_1(u_1, u_2, D) &= \alpha_1 n_2(u_2, D) - u_1; \\
p_2(u_1, u_2) &= \alpha_2 n_1(u_1) - u_2.
\end{align*}
\]

(4)

The first-order condition of (3) with respect to \( u_1 \) is

\[
\left( \frac{\alpha n_2 - u_1 - f_1}{n_1} \right) \cdot \frac{dn_1}{du_1} = \frac{n_1}{\text{rise in } n_1} \quad \text{loss in revenue}
\]

(5)

where \( \frac{dn_1}{du_1} = \phi'_1(u_1) \).

The first-order condition of (3) with respect to \( u_2 \) is

\[
\left( \frac{\alpha n_1 - u_2 - f_H}{n_2} \right) \cdot \frac{dn_2}{du_2} = \frac{n_2}{\text{rise in # of sellers}} - \frac{\Delta f}{\text{loss in revenue saved by each type-L}} \cdot \frac{dn_1}{du_2} = \frac{\lambda \phi'_2(u_2) + (1 - \lambda)\phi'_2(u_2 - D)}{\text{rise in # of type-L}}
\]

(6)

where \( \frac{dn_2}{du_2} = \lambda \phi'_2(u_2) + (1 - \lambda)\phi'_2(u_2 - D) \) and \( \frac{dn_1}{du_2} = \lambda \phi'_2(u_2) \).

Each seller creates a total value of \( \alpha n_1 \) for the platform, and therefore the economic profit that a type-H seller generates is \( \alpha n_1 - u_2 - f_H \). Raising \( u_2 \) by one unit, the platform can attract a total of \( \frac{dn_2}{du_2} \) more new sellers, and the left-hand side of (6) represents the associated increase in economic profit if all new sellers were of type-H. On the right-hand
side, the gross loss in revenue due to the additional utility offered to all sellers equals their number $n_2$. However, as a fraction of $\frac{dn_1}{dn_2}$ new sellers are of type-L from whom the platform saves its psychological cost of $\Delta f$ each, the total cost saving from them is equal to $\Delta f \cdot \frac{dn_1}{dn_2}$. This saving needs to be deducted from the gross loss in revenue.

Given some $D \in [0, \bar{D}]$, denote the $u_1$ that satisfies (5) and $u_2$ that satisfies (6) as the following

$$ (u_1^*(D), u_2^*(D)) \equiv \arg \max_{(u_1, u_2)} \pi(u_1, u_2, D) \tag{7} $$

and denote the maximized utility as a function of $D$ alone as follows

$$ \Pi(D) \equiv \pi(u_1^*(D), u_2^*(D), D) \tag{8} $$

Assume that the profit function (3) is well-behaved such that $u_1^*(D)$ and $u_2^*(D)$ are differentiable for $D \in [0, \bar{D}]$. This implies that $\Pi(D)$ is also differentiable. We have the following useful property of $\Pi(D)$.

**Lemma 1** $\Pi(D)$ is quasiconvex on $[0, \bar{D}]$.

(All omitted proofs are provided in the Appendix.) Lemma 1 implies that the platform’s maximized utility as a function of $D$ must have no peak. Therefore, no intermediate level of discrimination $D \in (0, \bar{D})$ will be optimal. We immediately have the following result.

**Proposition 1 (Binary Discrimination Choice)** If the platform can choose the extent of discrimination $D \in [0, \bar{D}]$, it will either choose not to discriminate at all ($D = 0$), or choose to fully discriminate ($D = \bar{D}$).

Since the maximized utility without discrimination is equal to $\Pi(0)$, we can use $\Pi'(0)$ and Lemma 1 to determine the platform’s incentive to introduce discrimination. For $D \in [0, \bar{D}]$, apply an envelope argument for (8) and we must have

$$ \Pi'(D) = \frac{d}{dD} \pi(u_1^*(D), u_2^*(D), D) = \frac{d}{dD} \pi(u_1^*(D), u_2^*(D), D) \tag{9} $$

Therefore, we can find $\Pi'(0)$ by evaluating $\frac{d\pi}{dD}$ at the optimal utilities without discrimination.

---

30 The property in Lemma 1 results from the fact that non-price discrimination represents pure value destruction - no market participant directly benefits from it.
Proposition 2 (Optimal Pricing without Discrimination) Without discrimination, the optimal prices for two sides, \((p_0^1, p_0^2)\), are given by

\[
\begin{align*}
p_0^1 &= f_1 - \alpha_2 n_2 + \frac{n_1}{n_1'} \\
p_0^2 &= (f_H - \lambda \Delta f) - \alpha_1 n_1 + \frac{n_2}{n_2'}
\end{align*}
\]

and the optimal utilities that the platform provides to two sides, \((u_0^1, u_0^2)\), are given by

\[
\begin{align*}
\phi_1(u_0^1) &= \phi_1'(u_0^1)(\alpha \phi_2(u_0^2) - u_0^1 - f_1) \\
\phi_2(u_0^2) &= \phi_2'(u_0^2)(\alpha \phi_1(u_0^1) - u_0^2 - f_H + \lambda \Delta f)
\end{align*}
\]

Without discrimination, the platform cannot treat the two types of sellers differently. Therefore, the utility it offers to them all is determined by the average cost of serving each seller, \((f_H - \lambda \Delta f)\).

The partial derivative of (3) with respect to \(D\) gives

\[
\frac{\partial}{\partial D} \pi(u_1, u_2, D) = \frac{\partial n_H}{\partial D}(\alpha n_1 - u_2 - f_H)
\]

where \(\frac{\partial n_H}{\partial D} = -(1 - \lambda)\phi_2'(u_2 - D)\). Intuitively, raising \(D\) has the exact opposite effect on type-H sellers as raising \(u_2\) does. The negative impact on the platform’s utility is the product between the decrease in the number of type-H sellers and the economic profit each of them creates.

Without discrimination, \(n_2 = \phi_2(u_0^2)\) and \(\frac{\partial n_2}{\partial u_2} = \phi_2'(u_0^2)\). By the first-order condition (6), the platform will choose the optimal price (and utility) for the sellers to balance the economic profit from type-H sellers and the net loss it incurs per seller when raising \(u_2\), that is,

\[
(\alpha n_1 - u_2^0 - f_H) = \frac{n_2 - \Delta f \cdot \frac{dn_1}{du_2}}{\frac{dn_2}{du_2}}
\]

Substitute in (12) and by (9) we have

\[
\Pi'(0) = (1 - \lambda)[\lambda \Delta f \phi_2'(u_0^2) - \phi_2(u_0^2)].
\]

Therefore by Lemma 1 and Proposition 1 we have the following result.

Proposition 3 (Incentive to Discriminate) The platform will optimally choose to discriminate if

\[
\lambda \Delta f \geq \frac{\phi_2(u_0^2)}{\phi_2'(u_0^2)}
\]
where $u^0_2$ is the utility that each seller obtains from the platform without discrimination, as given by (11).

**Intuition:** Compared to a type-$H$ seller, each type-$L$ seller saves the platform’s psychological cost of $\Delta f$, whilst creating the same network benefits for the buyers on the opposite side. Recall that $\lambda$ is the proportion of type-$L$ sellers. Therefore, $\lambda \cdot \Delta f$ represents the total psychological cost saved due to existing type-$L$ sellers on the platform, compared to the “worst” situation in which all sellers are of type $H$. Proposition 3 implies that the platform will discriminate if the total cost saving from existing type-$L$ sellers is significant enough.

For a given level of $\Delta f > 0$, when the proportion of type-$L$ sellers, $\lambda$, is sufficiently large, the platform has an incentive to discriminate against type-$H$ sellers. Doing so discourages some type-$H$ sellers, resulting in savings of the psychological cost. However, it also reduces the appeal of the platform to buyers because fewer the total number of sellers is decreased. Discrimination only makes sense when the proportion of type-$H$ sellers is sufficiently small such that the platform will still remain fairly attractive to buyers even after losing some of them.

Similarly, for a given proportion $\lambda > 0$ of type-$L$ sellers, the platform has an incentive to discriminate against type-$H$ sellers when $\Delta f$ is big enough, i.e. if the platform’s prejudice towards type-$H$ sellers is big enough.

The platform discriminates against type-$H$ sellers to discourage them from joining the platform. When the platform’s prejudice is minor (i.e., $\Delta f$ being small), the problem created by type-$H$ sellers is negligible; when their proportion $(1 - \lambda)$ is too large, they become crucial to the platform’s business. In either case, the cost saved due to existing type-$L$ sellers will be small, dwarfing the platform’s incentive to discriminate type-$H$ sellers.

**Caveat:** Note that (13) is a sufficient condition for the platform to optimally choose to discriminate. Whenever (13) holds with strict inequality, we know $\Pi'(0) > 0$. Then because of quasiconvexity (Lemma 1), we know that $\Pi(D)$ must be increasing throughout $[0, \bar{D}]$, that is $\Pi(\bar{D}) > \Pi(0)$, and therefore discrimination is optimal. In case (13) holds with equality, $D = 0$ is a local minimizer of $\Pi(D)$, and therefore some discrimination is always better.$^{31}$ However, when the converse of (13) is true, i.e. $\Pi'(0) < 0$, it may be possible that $\Pi(D)$ is U-shaped on $[0, \tilde{D}]$ (i.e. it first decreases and then increases, which does not violate quasiconvexity). Without knowing exactly how big the upper bound $\tilde{D}$ is, we cannot rule out the possibility that $\Pi(\bar{D})$ may eventually exceed $\Pi(0)$, and therefore condition (13) may not be necessary. Because we cannot think of a good criterion for

$^{31}$It can be proved that $\Pi''(D) > 0$ whenever $\Pi'(D) = 0$. See the proof of Lemma 1 in the Appendix for more technical details.
selecting a general upper bound $\bar{D}$ that applies to all the examples we discuss in section 1, there is no general conclusion. Nonetheless, whenever the converse of (13) holds, there must exist $D_0 \in (0, \bar{D})$ such that $\Pi(D)$ is strictly decreasing everywhere within $[0, D_0]$ - which means introducing one unit of discrimination strictly lowers the platform’s profit - and therefore the incentive to introduce discrimination is still negative in that case.

**Remark**: Proposition 3 characterize the condition (i.e., inequality (13)) under which the platform would act on its prejudice to discriminate against type-$H$ sellers. When inequality (13) fails to hold, even if the platform is a monopoly, it has no incentive to act on its prejudice. Therefore, prejudice do not always lead to discrimination even if the platform market has tipped.

**Proposition 4 (Optimal Pricing with Discrimination)** When the platform discriminates (with $\bar{D}$), its optimal prices for two sides are given by

\[
\begin{align*}
p_1^* &= f_1 - \alpha_2 n_2 + \frac{n_1}{\frac{dn_1}{du_1}}, \\
p_2^* &= f_H - \Delta f \frac{dn_1}{du_2} - \alpha_1 n_1 + \frac{n_2}{\frac{dn_2}{du_2}},
\end{align*}
\]  

(14)

where $\frac{dn_1}{du_1} = \phi_1'(u_1^*(\bar{D}))$, $\frac{dn_1}{du_2} = \lambda \phi_2'(u_2^*(\bar{D}))$ and $\frac{dn_2}{du_2} = \lambda \phi_2'(u_2^*(\bar{D})) + (1 - \lambda) \phi_2'(u_2^*(\bar{D}) - \bar{D})$.

## 5 Social incentive to discriminate

In this section we will measure the surpluses of different groups of participants in the market, and gauge how they change under discrimination. For expository clarity, we first provide some more “micro-foundations” of our basic model.

### 5.1 The general framework

Suppose that a buyer derives an idiosyncratic value $t_1$ if she joins the platform, while her outside option yields zero utility. Given the utility that the platform offers to each buyer, $u_1$, she joins the platform if and only if

\[ u_1 + t_1 \geq 0. \]

Denote $F_1$ on $\mathbb{R}$ as the cumulative distribution of buyers, and the accumulation process of buyers is

\[ \phi_1(u_1) = \Pr[u_1 + t_1 \geq 0] = 1 - F_1(-u_1) \]
Consistent with our previous specification of $\phi_1$, the function $1 - F_1(-u_1)$ is strictly increasing in $u_1$. We can now characterize the buyers’ total surplus as

$$v_1(u_1) \equiv \mathbb{E}_{t_1}[\max(u_1 + t_1, 0)] = \int_{-u_1}^{+\infty} (u_1 + t) dF_1(t),$$

(15)

which implies

$$v'_1(u_1) = \phi_1(u_1) = n_1$$

Similarly, assume that a seller derives an idiosyncratic value $t_2$ if he joins the platform, while his outside option yields zero utility. Given the utility that the platform provides to each seller, $u_2$, he joins the platform if and only if

$$u_2 + t_2 \geq 0.$$  

Denote $F_2$ on $\mathbb{R}$ as the cumulative distribution of sellers, and the accumulation process of sellers is

$$\phi_2(u_2) = \Pr[u_2 + t_2 \geq 0] = 1 - F_2(-u_2)$$

The function $1 - F_2(-u_2)$ is also consistent with our previous specification of $\phi_2$.

Recall that a proportion of $\lambda$ sellers are of type-L whereas the remainder are of type-H. The platform discriminates against type-H sellers by imposing a disutility of $D$ to them. Therefore, the sellers’ total surplus depends on both $u_2$ and $D$ as follows:

$$v_2(u_2, D) \equiv \lambda \mathbb{E}_{t_2}[\max(u_2 + t_2, 0)] + (1 - \lambda) \mathbb{E}_{t_2}[\max(u_2 - D + t_2, 0)],$$

(16)

$$= \lambda \int_{-u_2}^{+\infty} (u_2 + t) dF_2(t) + (1 - \lambda) \int_{D-u_2}^{+\infty} (u_2 - D + t) dF_2(t),$$

which implies

$$\frac{\partial}{\partial u_2} v_2(u_2, D) = \lambda \phi_2(u_2) + (1 - \lambda) \phi_2(u_2 - D) = n_2;$$

$$\frac{\partial}{\partial D} v_2(u_2, D) = -(1 - \lambda) \phi_2(u_2 - D) = -n_H$$

The social surplus is

$$w(u_1, u_2, D) = \pi(u_1, u_2, D) + v_1(u_1) + v_2(u_2, D).$$

We can now compare the levels of surpluses when the platform discriminates or not, given that the platform maximizes its utility in either case. We use the optimal $u_1^*(D)$ and
\[ u_2^*(D) \] as characterized previously in (7).

Denote \( V_1(D) \equiv v_1(u_1^*(D)) \) the aggregate buyer surplus in (15) as a function of \( D \) alone, and \( V_2(D) \equiv v_2(u_2^*(D), D) \) the aggregate seller surplus in (16) as a function of \( D \) alone. Finally, denote the social surplus when the platform has maximized its own utility given \( D \) as

\[
W(D) \equiv \Pi(D) + V_1(D) + V_2(D) = \pi(u_1^*(D), u_2^*(D), D) + v_1(u_1^*(D)) + v_2(u_2^*(D), D).
\]

By examining \( W(D) \), we can determine whether the platform’s discrimination results in a net loss to the society. In particular, the derivative of \( W(D) \) is

\[
W'(D) = \Pi'(D) + V_1'(D) + V_2'(D) = \Pi'(D) + v_1'(u_1^*(D)) \cdot u_1''(D) + \frac{\partial}{\partial u_2} v_2(u_2^*(D), D) \cdot u_2''(D) + \frac{\partial}{\partial D} v_2(u_2^*(D), D),
\]

\[
= \Pi'(D) + n_1 \cdot u_1''(D) + n_2 \cdot u_2''(D) - n_H.
\]

Clearly, the private discrimination incentive of the platform does not always align with the interest of the society because we have

\[
W'(0) - \Pi'(0) = V_1'(0) + V_2'(0) = n_1 \cdot u_1''(0) + n_2 \cdot u_2''(0) - n_H. \tag{17}
\]

The extent of misalignment between the social and private incentives to discriminate is represented by the difference between \( W'(0) \) and \( \Pi'(0) \), or equivalently, by \( V_1'(0) + V_2'(0) \), which are in the calculation of the society but not in that of the platform.

If \( V_1'(0) + V_2'(0) \) is equal to zero, then the platform’s interest is perfectly aligned with that of the society. However, there is no reason to expect that \( V_1'(0) + V_2'(0) \) is zero. If \( V_1'(0) + V_2'(0) < 0 \), the platform’s incentive to discriminate is too large from the society’s perspective. On the other hand, if \( V_1'(0) + V_2'(0) > 0 \), the society may actually want the platform to discriminate while the platform lacks the incentive to do so.

One implication of this analysis, therefore, is that when a platform discriminates against certain groups of users, there is no reason to believe that doing so always goes against the interest of the society. As such, it may not be in the interest of the society to set up anti-discrimination laws to ban a platform from discrimination.
5.2 Welfare improvement under discrimination

As (17) shows, we need to know $u_1'(0)$ and $u_2'(0)$, or at least some of their properties to tell the sign of $V'_1(0) + V'_2(0)$, $W'(0)$ or $W'(D)$. This is impossible without specifying the distributions of buyers and sellers (i.e. the $F_1$ and $F_2$ defined previously).

In order to show through what channels social welfare may be improved when the platform discriminates, in this section we assume that the accumulation processes of participants on both sides of the market follow the same uniform distribution.

5.2.1 Simulation results

We ran 503,119 times of simulation, where we used different sets of parameter values for the model. In addition to the parameters for the distributions of buyers and sellers, the other parameters include the strength of the cross-group network effects, the cost of serving the buyers and sellers, the proportion of sellers that are prejudiced by the platform, and the extent of prejudice. These results can be summarized as follows. The Appendix shows the details.

**Proposition 5 (Welfare Simulation)** Simulation results show that the platform’s incentive to discriminate may be aligned or misaligned with the interest of the society, depending on the distributions of buyers and sellers, as well as other parameters in the model.

In particular, we check if there exist parameter values such that $\Pi(0) > 0$ (it makes sense for the platform to exist), and the following 4 cases: (a) $W'(0) < 0 < \Pi'(0)$; (b) both $\Pi'(0)$ and $W'(0)$ are positive; (c) both $\Pi'(0)$ and $W'(0)$ are negative; and (d) $\Pi'(0) < 0 < W'(0)$.

In both (b) and (c), there are no incentive misalignment in a sense that when the platform has an incentive to discriminate, the society does not get hurt, and vice versa. In case (a), the platform has an incentive to discriminate but the society gets hurt. In case (d), the society benefits from discrimination but the platform has no incentive to do so. Only (a) justifies banning discrimination.

Our simulation results show that all these four cases are possible under a rather simple assumption of uniform distribution. To argue that anti-discrimination laws are socially efficient in a two-sided market is equivalent to making sure that the parameter values of that two-sided market fall exactly within the set that generates case (a). However, that requires the rather daunting task of carrying out empirical estimation of all the parameters in the model.
5.2.2 The channels of welfare improvement

We now show that, when the platform optimally chooses to discriminate, in equilibrium it may optimally lower the prices for both sellers and buyers, and hence raise aggregate seller surplus, aggregate buyer surplus, and social surplus.

For a two-sided platform, non-price discrimination is actually a way for the platform to adjust seller composition, in order to save the psychological cost it incurs due to type-H sellers. Because its business depends on the indirect network effects across the two market sides, it still needs to attract sufficiently large numbers of participants from both sides, even when it chooses to discriminate against some sellers.

When the platform discriminates against type-H sellers, the total number of sellers would decrease if the platform does not make any price adjustments. Because the platform would still hope to remain fairly attractive to buyers, it will have an incentive to lower the price for all sellers such that more type-L sellers would join. If lowering seller price itself is not sufficiently effective to persuade buyers to stay, the platform may also have an incentive to lower the buyer price. As long as the total savings in its psychological cost due to type-H sellers is large enough to compensate for the losses from lower prices, the platform will optimally lower its prices.

We now derive the formulas for the model under uniform distribution, and characterize the circumstances in which discrimination by the platform leads to welfare improvements. Suppose both $t_1$ and $t_2$ follow the same uniform distribution on $[a, b]$. Then we have

$$F_1(x) = F_2(x) = \frac{x - a}{b - a}, \text{ for } x \in [a, b].$$

Denote $A \equiv \frac{1}{b-a}$, $B \equiv \frac{b}{b-a}$, and therefore the accumulation processes on both market sides are given by

$$\phi_1(x) = \phi_2(x) = 1 - F_1(-x) = Ax + B, \text{ for } x \in [-b, -a].$$

Therefore, given $u_1, u_2, (u_2 - D) \in [-b, -a]$, we have the following expressions.

$$n_1 = \phi_1(u_1) = Au_1 + B,$$

$$n_L = \lambda \phi_2(u_2) = \lambda (Au_2 + B),$$

$$n_H = (1 - \lambda) \phi_2(u_2 - D) = (1 - \lambda) (Au_2 + B - AD),$$

$$n_2 = n_L + n_H = Au_2 + B - (1 - \lambda) AD,$$

$$p_1 = \alpha_1 n_2 - u_1,$$

$$p_2 = \alpha_2 n_1 - u_2.$$
Given \( D \in [0, \bar{D}] \), we can solve the first-order conditions for the (interior) optimal utilities offered to two sides \( u^*_1 \) and \( u^*_2 \), and then derive their derivatives with respect to \( D \).\(^{32}\) The signs of \( u'_1(D) \) and \( u'_2(D) \) in turn help us find the following result.

**Proposition 6 (Welfare Improvement)** Suppose \( t_1 \) and \( t_2 \) follow the same uniform distribution on \([a, b]\), and the optimal utilities that the platform offers to two sides, with or without discrimination, are all interior solutions. Then we have

i) There exists a social incentive to discriminate whenever there exists a private incentive to discriminate, if and only if \( \frac{\alpha}{b-a} > 2 \); or equivalently

ii) The aggregate surpluses of all buyers and sellers in the market increase when the platform introduces discrimination, if and only if \( \frac{\alpha}{b-a} > 2 \).

Note again that \( \alpha \) represents the total network benefits created when each pair of buyer and seller interact, which is the main source of value creation in this two-sided market. And \( \frac{\alpha}{b-a} = \alpha A \) as defined previously. The condition \( \frac{\alpha}{b-a} > 2 \) is satisfied in the numerical solution we will provide shortly in section 5.2.3, where the aggregate surpluses of both sellers and buyers are increased under discrimination. And because the platform optimally chooses to discriminate, its profits are also higher. Therefore all parties benefit from discrimination.

Given \( D \in [0, \bar{D}] \), rewrite (4e) and (4f) and denote the platform’s optimal prices as functions of \( D \) alone:

\[
\begin{align*}
p^*_1(D) &\equiv \alpha_1 \cdot n_2(u^*_2(D), D) - u^*_1(D), \\
p^*_2(D) &\equiv \alpha_2 \cdot n_1(u^*_1(D)) - u^*_2(D),
\end{align*}
\]

and we have the following conclusion.

**Proposition 7 (Price Adjustment Under Discrimination)** Suppose \( t_1 \) and \( t_2 \) follow the same uniform distribution on \([a, b]\), and the optimal utilities that the platform offers to two sides, with or without discrimination, are all interior solutions. Then for \( D \in [0, \bar{D}] \), we have

i) \( p'_1(D) + p'_2(D) < 0 \);

ii) \( p'_1(D) < 0 \) if \( \frac{\alpha}{b-a} > 2 \) and \( \frac{\alpha_1}{\alpha} < \frac{1}{2} \);

iii) \( p'_2(D) < 0 \) if \( \frac{\alpha}{b-a} > 2 \) and \( \frac{\alpha_1}{\alpha} > \frac{2(b-a)^2}{\alpha^2} \).

Under uniform distribution, the total equilibrium price that the platform charges to both sides always decreases when the platform discriminates. The equilibrium price for buyers decreases under discrimination if buyers enjoy smaller network externalities than

---

\(^{32}\)These conditions and solutions are provided in the proof of Proposition 6 in the Appendix.
sellers do. When sellers enjoy too small a fraction of total network externalities, the equilibrium price for sellers will decrease under discrimination. Therefore, depending on the model parameters, it is possible for the equilibrium price for both sides to decrease under discrimination, i.e. when $\frac{\alpha}{b-a} > 2$ and $\frac{2(b-a)^2}{a^2} < \frac{\alpha_1}{\alpha} < \frac{1}{2}$, which is indeed true in the numerical example we present in section 5.2.3.

**Proposition 8 (Welfare Result in One-Sided Market)** i) Suppose $t_1$ and $t_2$ follow the same uniform distribution on $[a, b]$. When $\alpha_1 = \alpha_2 = 0$, such that there exist no network effects whatsoever, and thus the buyer side and seller side are separated as if the platform simply operates in two distinct one-sided markets, then we must have

i) $p_1^*(0) = 0, V_1'(0) = 0$;  
ii) $p_2^*(0) < 0, V_2'(0) < 0$;  
iii) $W'(0) - \Pi'(0) < 0$.

That is, in a one-sided market, the social incentive to introduce discrimination is never aligned with the platform’s private incentive. For the buyer side, neither the buyers’ aggregate surplus nor the platform’s equilibrium price for them is affected by discrimination of type-$H$ sellers. On the other hand, discrimination is always accompanied by a price cut for the seller side, but sellers’ aggregate surplus never increases under discrimination.

When the market is one-sided, under uniform distribution the social incentive is never aligned with the private incentive to introduce discrimination. Discrimination is always accompanied by a price cut for sellers, but buyer price never adjusts at all. Even so, sellers’ total welfare never increases under discrimination, and therefore introducing discrimination never benefits any other market participants besides the platform itself.

The stark contrast between Propositions 6 and 8 therefore shows that the alignment of social and private incentives to discriminate that Proposition 5 shows is exactly due to the two-sidedness of the market.

### 5.2.3 A numerical example

Let $t_1$ and $t_2$ both follow the uniform distribution on $[-10, 5]$, and let the parameters in the model take the following values.

\[
\begin{align*}
f_1 &= 8, & \alpha_1 &= 20, & \lambda &= 0.8, \\
f_H &= 12, & \alpha_2 &= 60, & \bar{D} &= 0.5, \\
\Delta f &= 10, & \alpha &= 80, \\
\end{align*}
\]

We find the following interior numerical solution.\(^{33}\)

\(^{33}\)We use Scientific WorkPlace to generate our numerical solutions.
Condition (13) in Proposition 3 holds, and therefore the platform indeed earns more profits when it discriminates. It is clear from the comparison that the platform chooses to lower its prices for both buyers and sellers, when it chooses to discriminate.

5.2.4 Type-H sellers

The proportion of type-H sellers will definitely decrease when the platform discriminates against them, no matter how it adjusts its prices. This is because type-H sellers receive strictly less net utility than type-L sellers, when there is discrimination.

**Lemma 2 (Proportion of type-H sellers)** Given \( u_2 \) and \( D \), denote \( \tau(u_2, D) \) the proportion of type-H sellers among all sellers, i.e.

\[
\tau(u_2, D) = \frac{n_H}{n_2},
\]

then when \( t_1 \) and \( t_2 \) follow the same uniform distribution on \([a, b]\), we have

\[
\frac{d}{dD}\tau(u_2^*(0), 0) < 0,
\]

such that the proportion of type-H sellers among all sellers always decreases when the platform introduces discrimination.

Proof. \( \frac{d}{du_2}\tau(u_2, D) = \frac{\lambda(1-\lambda)A^2D}{(n_2)^2} \), which implies \( \frac{d}{du_2}\tau(u_2^*(0), 0) = 0. \) And \( \frac{d\tau}{dD} = -\frac{\lambda(1-\lambda)A(u_2+B)}{(n_2)^2} < 0. \]

Nonetheless, it is potentially possible for type-H sellers’ total number and aggregate surplus to rise in equilibrium, as shown by the following result.
Lemma 3 (Welfare of type-H sellers) Let $v_H(u_2, D) \equiv (1 - \lambda) \int_{D-u_2}^{+\infty} (u_2 - D + t) dF_2(t)$ denote type-H sellers’ aggregate surplus given $(u_2, D)$, and let $V_H(D) \equiv v_H(u_2^*(D), D)$ denote their aggregate surplus in equilibrium. Then when $t_1$ and $t_2$ follow the same uniform distribution on $[a, b]$, we have

$$V_H'(D) > 0 \text{ and } \frac{\partial n_H}{\partial D} > 0 \text{ if and only if } \frac{2 - \lambda (\alpha^2A^2 - 2)}{\alpha^2A^2 - 4} > 0.$$ 

Proof. $V_H'(D) = n_H(u_2''(D) - 1)$, and $\frac{\partial n_H}{\partial D} = A(1 - \lambda)(u_2''(D) - 1)$. Therefore they are both positive if and only if $u_2''(D) - 1 = \frac{2 - \lambda (\alpha^2A^2 - 2)}{\alpha^2A^2 - 4} > 0$. ■

Intuitively, if $u_2''(D) > 1$, the utility that the platform offers to all sellers $u_2^*$ increases faster than the disutility from discrimination does, and therefore the platform’s price cut for all sellers more than compensates for the disutility it imposes on type-H sellers through discrimination. Potentially, given the uniform distribution on $[a, b]$ and when $\alpha A = \frac{\alpha}{b-a} > 2$, the condition in Lemma 3 reduces to $\lambda < \frac{2}{\alpha^2A^2 - 2}$, such that the proportion of type-H sellers $(1 - \lambda)$ needs to be sufficiently large, or the total network benefit $\alpha$ needs to be sufficiently small, for type-H sellers to benefit from discrimination.

6 Alternative policy: imperfect discrimination

Some may argue that when it comes to discrimination, there are a lot more other concerns. One looks heartless if she does not feel for those being discriminated against. What if not banning discrimination conveys the wrong idea to kids that discriminating others are okay? Therefore, even if discrimination can increase social surplus, the policy objective could be to eliminate them as much as we can. To address this, the model may allow us to explore alternative ways to eliminate discrimination in a less costly fashion.

One potential indirect way to eliminate discrimination without banning it is to make it more costly for the platform to discriminate. One way to deliver such a goal is to ensure whatever policies the platform uses to treat any particular users should also be made transparent to other users within the same group. Instead of anti-discrimination laws, one may think of this practice as a form of disclosure requirement. It may work in reducing the incentives for the platform to discriminate by making sure that those who are not discriminated also know how those who are discriminated are treated, with an expectation of raising their sympathy or concern. As such, it is a policy not to directly ban discrimination, but to make any discriminatory practices to certain group of users on one side of the market known to other users on the same side too. For instance, law
can mandate that platforms disclosure the complaints from users if the users want their complaints to be displayed somewhere for a set period of time.

To formalize this idea, suppose whenever the platform discriminates against type-H sellers by imposing a disutility $D$, each type-L seller also experience a disutility $\theta D$, where $\theta \in [0, 1]$ represents either the “degree of sympathy” that type-L sellers hold towards type-H sellers, or the “degree of imprecision” in the platform’s targeting when carrying out discrimination. Therefore, when type-L sellers are more sympathetic, or when the discrimination targeting is less precise, type-L sellers experience more utility losses. In either case, we say the discrimination is imperfect.

Therefore, under imperfect discrimination, when a type-H seller obtains $u_2 - D$ from the platform, a type-L developer expects to obtain a utility of $u_2 - \theta D$ (instead of $u_2$). This changes how the number of type-L sellers under discrimination is determined, which now also depends on $D$ and becomes

$$n_L = \lambda \phi_2 (u_2 - \theta D). \quad (18)$$

Denote $\frac{\partial n_L}{\partial D} = -\theta \lambda \phi'_2 (u_2 - \theta D)$.

The first order conditions for the optimal $u_1$ and $u_2$ are still given by (5) and (6), and the platform’s incentive to discriminate still depends on $\Pi'(0)$ derived from (8), except that because $n_L$ also depends on $D$ according to (18), we now have the following result.

**Proposition 9 (Incentive to Discrimination with Imperfection)** When discriminate is imperfect, the platform has an incentive to introduce it if and only if

$$\frac{(1 - \theta)\lambda(1 - \lambda)}{1 - \lambda + \theta \lambda} \Delta f \geq \frac{\phi_2(u_2)}{\phi'_2(u_2)} \quad (19)$$

where $\theta$ is the fraction of type-H’s disutility imposed on a type-L seller, and $u_2^0$ is the utility that each seller obtains from the platform without discrimination, as given by (11).

Because the left-hand side of (19) is strictly decreasing in $\theta$, we immediately have the following result.

**Proposition 10** The more imperfect the discriminatory strategy is (i.e. the larger $\theta$ is), the less likely that the platform will have an incentive to discriminate.

To identify what is discrimination and what not is potentially very difficult. On top of it, identifying whether any discriminatory practices are based on prejudice or based on actual business calculation is extremely difficult (Heckman, 1998). Court may not be in
the right place to judge these difficult issues. It is possible that certain types of disclosure requirements, instead of directly prohibiting discrimination, makes it less costly for the regulator to administer. And it also has the effects of reducing discrimination, if not completely eliminating it. Exactly how to design these disclosure requirements remain an open question and are likely to be specific to the business nature of the platforms concerned.

7 Conclusion

We find that even if a platform market has tipped, the prejudice of the platform does not necessarily transform to actual discriminatory practices. The platform may not always find it beneficial to act on its own prejudice. When a platform does act on it, it does not necessarily reduce social surplus. It is possible that when the platform discriminates, social surplus increases. Therefore, tightening anti-discrimination policies on platforms, especially those in which the market has tipped is not always socially efficient even if we ignore the platform’s profitability.

When the platform discriminates, it has to lower its prices charged to the users in order to maintain its attractiveness. Doing so makes it possible for the users alone to be better off when the platform discriminates. We also derive a condition under which even those users who are discriminated against by the platform can be better off.

If social efficiency is not the only concern, our model suggests that mandatory disclosure requirements that are well crafted out can potentially make discrimination more difficult to the platform, indirectly reducing discrimination. It is conceivable that these disclosure requirements can be less costly to administer than policies that prohibit discriminatory practices, for distinguishing what is and what is not discrimination is extremely difficult, especially for non-price practices. In addition, separating taste-based discrimination from other types of discrimination is also challenging (Heckman, 1998). Not to mention the potential litigation costs, the burden to the court, and the enforcement costs, identifying other forms of regulations to address discrimination is a fruitful area for future research.

8 Proofs and Derivations

Lemma 1 Using (4), the partial derivative of (3) with respect to \( D \) gives

\[
\frac{\partial}{\partial D} \pi(u_1, u_2, D) = \frac{\partial n_H}{\partial D} (\alpha n_1 - u_2 - f_H)
\]
where
\[
\frac{\partial n_H}{\partial D} = -(1 - \lambda)\phi'_2(u_2 - D)
\]

At the optimal utilities \((u_1^*(D), u_2^*(D))\), by (6) we have
\[
(\alpha n_1 - u_2^*(D) - f_H) = \frac{n_2 - \Delta f \cdot \frac{\partial n_1}{\partial u_2}}{\frac{\partial n_2}{\partial u_2}}
\]
which implies
\[
\Pi'(D) = \frac{\partial}{\partial D} \pi(u_1^*(D), u_2^*(D), D)
\]
\[
= \frac{\partial n_H}{\partial D} \cdot \frac{n_2 - \Delta f \cdot \frac{\partial n_1}{\partial u_2}}{\frac{\partial n_2}{\partial u_2}}
\]
\[
= -\frac{\partial n_H}{\partial D} \left( \Delta f \cdot \frac{\partial n_1}{\partial u_2} - n_2 \right)
\]

Therefore
\[
\Delta f \cdot \frac{\partial n_1}{\partial u_2} - n_2 = (-\frac{\partial n_2}{\partial u_2}) \cdot \Pi'(D)
\]
and
\[
\Pi''(D) = \frac{\partial^2}{\partial D^2} \pi(u_1^*(D), u_2^*(D), D)
\]
\[
= \frac{\partial n_1}{\partial u_2} \cdot \left( \frac{\partial^2 n_H}{\partial D^2} \right) \cdot \left( \Delta f \cdot \frac{\partial n_1}{\partial u_2} - n_2 \right) + \left( \frac{\partial n_H}{\partial D} \right)^2
\]
\[
= \frac{\partial n_1}{\partial u_2} \cdot \left( \frac{\partial^2 n_H}{\partial D^2} \right) \cdot \left( \frac{\partial n_2}{\partial u_2} \right) \cdot \Pi'(D) + \left( \frac{\partial n_H}{\partial D} \right)^2
\]
\[
= \frac{\partial n_1}{\partial u_2} \cdot \left( \frac{\partial^2 n_H}{\partial D^2} \right) \cdot \left( \frac{1}{\frac{\partial n_1}{\partial u_2}} \right) \cdot \Pi'(D) + \left( \frac{\partial n_H}{\partial D} \right)^2
\]
where
\[
\frac{\partial^2 n_H}{\partial D^2} = (1 - \lambda)\phi''_2(u_2 - D)
\]

Because \(\frac{\partial n_2}{\partial u_2} > 0\), and \(\frac{\partial n_H}{\partial D} \neq 0\), we have the following conclusion:

**Lemma 4** \(\Pi''(D) > 0\) whenever \(\Pi'(D) = 0\).

Now we prove that this implies quasiconvexity.
Suppose \( \Pi(D) \) is not quasiconvex, then there exist \( [x, y] \subseteq [0, \bar{D}] \), and \( k \in [0, 1] \) such that
\[
\Pi(kx + (1 - k)y) > \max\{\Pi(x), \Pi(y)\}.
\]
Because \( kx + (1 - k)y \in [x, y] \), and twice-differentiability implies that \( \Pi(D) \) is continuous and differentiable on \([0, \bar{D}]\), there must exist \( z \in [x, y] \) such that \( z = \arg \max_{D \in [x, y]} \Pi(D) \), which in turn implies that \( \Pi'(z) = 0 \) and \( \Pi''(z) < 0 \), a contradiction. Therefore \( \Pi(D) \) must be quasiconvex on \([0, \bar{D}]\).\( \blacksquare \)

**Proposition 1**  This is immediately implied by quasiconvexity of \( \Pi(D) \) on \([0, \bar{D}]\).\( \blacksquare \)

**Proposition 2**  (10) and (11) are found by letting \( D = 0 \) in (5) and (6), and inverting (1).\( \blacksquare \)

**Proposition 3**  Let \( D = 0 \) in (20) and we have
\[
\Pi'(0) = (1 - \lambda)\lambda \Delta f \phi_2'(u_0^0) - \phi_2(u_0^0)]
\]
As \( \phi_2'(u_0^0) > 0 \), we know \( \frac{\phi_2(u_0^0)}{\phi_2'(u_0^0)} > 0 \). Therefore we have
\[
\Pi'(0) > 0 \text{ if and only if } \lambda \Delta f > \frac{\phi_2(u_0^0)}{\phi_2'(u_0^0)}.
\]
Now consider the case when \( \lambda \Delta f = \frac{\phi_2(u_0^0)}{\phi_2'(u_0^0)} \), that is, when \( \Pi'(0) = 0 \). By Lemma 4 we know \( \Pi''(0) > 0 \), therefore \( D = 0 \) is a minimizer of \( \Pi(D) \). Because introducing discrimination raises \( D \) from 0, it will make \( \Pi'(D) > 0 \) and is hence profitable.
Then by Lemma 1 we know, \( \Pi(\bar{D}) > \Pi(0) \) must hold when (13) holds.\( \blacksquare \)

**Proposition 4**  (14) is found by solving (5) and (6), and inverting (1).\( \blacksquare \)

**Proposition 5**  We ran 503,119 times of simulation of the special case of uniform distribution, and found that all four cases (a) through (d) exist. As examples, the four sets of parameter values that yield the four different cases are provided respectively as follows, where \( t_1 \) and \( t_2 \) follow the same uniform distribution on \([a, b]\), \( A \equiv \frac{1}{b-a} \), \( B \equiv \frac{b}{b-a} \), and more details are provided in section 5.2.2.

*Example 1. Misalignment:* \( \Pi'(0) > 0 \) and \( W''(0) < 0 \).
\[
\{A, B, \alpha, f_1, f_{H}, \Delta f, \lambda\} = \{0.5, 0.8, 2.4, 1, 2, 1.5, 0.1\}
\]
\( \Pi'(0) = 0.0288281; W''(0) = -0.075542; \Pi(0) = 0.0473633. \)
Example 2. Alignment: $\Pi'(0) > 0$ and $W'(0) > 0$.
\[\{A, B, \alpha, f_1, f_H, \Delta f, \lambda\} = \{1/15, 1/3, 80, 8, 12, 10, 0.8\}\]
$\Pi'(0) = 0.09903$; $W'(0) = 0.09973$; $\Pi(0) = 0.016364$.

Example 3. Misalignment: $\Pi'(0) < 0$ and $W'(0) < 0$.
\[\{A, B, \alpha, f_1, f_H, \Delta f, \lambda\} = \{0.5, 0.8, 3, 1, 2, 1.5, 0.1\}\]
$\Pi'(0) = -0.0353571$; $W'(0) = -0.334745$; $\Pi(0) = 0.0564286$.

Example 4. Misalignment: $\Pi'(0) < 0$ and $W'(0) > 0$.
\[\{A, B, \alpha, f_1, f_H, \Delta f, \lambda\} = \{1/15, 1/3, 80, 8, 5, 4, 0.15\}\]
$\Pi'(0) = -0.00031$; $W'(0) = 0.00391$; $\Pi(0) = 0.00065$.

**Proposition 6** With uniform distribution on $[a, b]$, given $D \in [0, \bar{D}]$, the first-order conditions for the interior optimal utilities offered to two sides $u_1^{*}$ and $u_2^{*}$ are
\[
\begin{align*}
\alpha(Au_1^{*} + B) - \alpha(1 - \lambda)AD - 2u_1^{*} - f_1 - \frac{B}{A} &= 0, \\
\alpha(Au_2^{*} + B) - 2u_2^{*} - f_H - \frac{B}{A} + (1 - \lambda)D + f_3\lambda &= 0,
\end{align*}
\]
whose solution is
\[
\begin{align*}
u_1^{*}(D) &= \frac{2B - 2AD - AB\alpha + 2AF_1 + A^2 D\alpha - A^2 \lambda - A^2 \lambda F_1 - A^2 \lambda F_3 - A^2 B\alpha^2}{A^2 \alpha^2 - 4A}, \\
u_2^{*}(D) &= \frac{2B - 2AD - AB\alpha + 2AF_1 + A^2 D\alpha - A^2 \lambda - A^2 \lambda F_1 - A^2 B\alpha^2 + A^3 \lambda D - A^3 \lambda D\alpha^2}{A^3 \alpha^2 - 4A}.
\end{align*}
\]
And therefore we have
\[
\begin{align*}
u_1''(D) &= \frac{(1 - \lambda)\alpha A}{\alpha^2 A^2 - 4}, \\
u_2''(D) &= \frac{(1 - \lambda)(\alpha^2 A^2 - 2)}{\alpha^2 A^2 - 4},
\end{align*}
\]
which immediately imply

**Lemma 5** 
\[\text{i)} \text{ If } \alpha A > 2, \text{ we have } \nu_1''(D) > 0 \text{ and } \nu_2''(D) > 0;\]
\[\text{ii)} \text{ if } \sqrt{2} < \alpha A < 2, \text{ we have } \nu_1''(D) < 0 \text{ and } \nu_2''(D) < 0; \text{ and}\]
\[\text{iii)} \text{ if } \alpha A \leq \sqrt{2}, \text{ we have } \nu_1''(D) < 0 \text{ and } \nu_2''(D) \geq 0.\]

With uniform distribution on $[a, b]$, because we have
\[V_1'(0) = n_1 \cdot \frac{(1 - \lambda)\alpha A}{\alpha^2 A^2 - 4},\]
and
\[V_2'(0) = n_2 \cdot u_2''(0) - n_H = \frac{2(1 - \lambda)(Au_2 + B)}{\alpha^2 A^2 - 4}.
\]
And finally by (17), we have $V'_1(0) + V'_1(0) > 0$ if and only if $\alpha A > 2$. ■

**Proposition 7** Because $p_1 = \alpha_1 n_2 - u_1$, and $p_2 = \alpha_2 n_1 - u_2$, by (22), we have

$$
\frac{dp_1^*}{dD} = \frac{(1-\lambda)A}{\alpha^2A^2 - 4} \cdot (2\alpha_1 - \alpha),
$$

$$
\frac{dp_2^*}{dD} = \frac{(1-\lambda)}{\alpha^2A^2 - 4} \cdot (2 - \alpha A^2 \alpha_1),
$$

which immediately implies

$$
\frac{dp_1^*}{dD} + \frac{dp_2^*}{dD} = -\frac{(1-\lambda)(\alpha_1 A + 1)}{\alpha A + 2} < 0;
$$

$$
\frac{dp_1^*}{dD} < 0 \text{ iff } \frac{\alpha_1}{\alpha} < \frac{1}{2};
$$

$$
\frac{dp_2^*}{dD} < 0 \text{ iff } \frac{\alpha_1}{\alpha} > \frac{2(b-a)^2}{\alpha^2}.
$$

**Proposition 8** Let $\alpha_1 = \alpha_2 = 0$ in the basic model and we immediately get all these results. ■

**Proposition 9** The first order conditions for the optimal $u_1$ and $u_2$ are still given by (5) and (6), and the platform's incentive to discriminate still depends on $\Pi'(0)$ derived from (8), except that because $n_L$ also depends on $D$ according to (18), we now have

$$
\Pi'(D) = \frac{\partial}{\partial D} \pi(u_1^*(D), u_2^*(D), D)
$$

$$
= \frac{\partial n_L}{\partial D} \cdot (\alpha n_1 - u_2 - f_L) + \frac{\partial n_H}{\partial D} \cdot (\alpha n_1 - u_2 - f_H).
$$

By (5) and (6), and let $D = 0$, we have

$$
\Pi'(0) = (1-\theta)\lambda(1-\lambda)\Delta f \cdot \phi'_2(u_2^0) - (1-\lambda + \theta \lambda) \cdot \phi_2(u_2^0),
$$

which implies (19). ■

**Proposition 10** We can take the first order derivative of the left-hand side of (19) with respect to $\theta$ to find the impact of imperfection on the incentive to discriminate:

$$
\frac{\partial}{\partial \theta} \frac{(1-\lambda)\lambda(1-\theta)}{(1-\lambda + \theta \lambda)^2} = \frac{\lambda(\lambda - 1)}{(1-\lambda + \theta \lambda)^2} < 0.
$$
which immediately implies Propositions 10.

References


