Targeted advertising and costly consumer search*

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March, 2017

Abstract

We study a market with horizontally differentiated products and sequential consumer search. Firms send ads to consumers to inform about their existence. Advertising may be either random or targeted. Targeting increases search intensity, which intensifies competition. On the other hand, consumers draw higher valuations for products on average under targeted advertising, which creates incentives to raise prices. The first effect dominates when search costs are sufficiently low, and the second may prevail when search costs are high. Then, prices are higher, and consumer surplus lower, under targeting when search costs are high, and lower when search costs are low. A larger cost of advertising helps firms segment the market if firms can target their ads. Prices may then be higher, and consumer surplus lower, under targeting.

Keywords: random advertising, targeted advertising, horizontal differentiation, sequential search

JEL classification: L13, D83

*We are grateful to Simon Anderson, Régis Renault, José Luis Moraga-González, Chris Wilson, Dan Bernhard, the participants of the IXth Workshop on the Economics of Advertising and Marketing, the participants of the seminar in IAE (CSIC) for their invaluable comments and suggestions. The authors acknowledge the financial support of the Spanish Ministry of Economics Grants No. ECO2014–59959-P and No. ECO2015–74328–JIN (AEI/FEDER/UE) and BBVA Fundación

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1 Introduction

Information technologies facilitate firms to target the consumers who are most likely to buy their products after receiving ads.\(^1\) However, in spite of sophisticated data analysis algorithms and the abundance of data, targeting is often imperfect due to the noise in these data\(^2\) or (and) deliberate attempts by consumers to avoid ads.\(^3\) The imperfection of targeting has important bearings on firms’ pricing and market equilibrium.

We study the effects of imperfect targeting in markets with costly sequential consumer search and firms that sell horizontally differentiated products, like in Wolinsky (1986). As in Butters (1977) and Grossman and Shapiro (1984), consumers may visit a firm (learn about its existence) only after receiving an ad from this firm. However, ads do not carry any information about product characteristics and prices.\(^4\)

This type of advertising is common in on-line retail. Consumers often receive promotional e-mails containing nothing but invitations to visit stores and check their new clothing, electronic or grocery products. These ads contain little more than small advertising banners with firms’ logos, and in particular they contain little product information (Honka et al. (2017)). Likewise, advertisement in sports events, on sponsored teams uniforms, or on public transport fall quite comfortably in this category.

We analyze two advertising regimes: random and targeted advertising. In the first regime, firms cannot identify any relevant characteristic of a consumer, and therefore they send their ads randomly. In the second regime, firms are able to identify, and target their ads to, consumers for whom their products rank high in their preferences. Targeting is not perfect, though, and so there is still some residual uncertainty as which is the product that best fits the consumer’s preferences.\(^5\) Thus, in both advertising games, there is room for consumer search.

Targeting is based on observed characteristics of consumers. Some of these characteristics, like average income in the neighborhood, may be imperfect signals of the consumer’s average willingness to pay for some (type of) good. Alternatively, some characteristics, like online search habits, are perhaps more informative about the fit of the characteristics of a firm’s product relative to those of the competitors’. In this paper we focus on this latter type of information by assuming that all consumers have the same average willingness to pay for the goods in the industry, but a firm imperfectly\(^6\) knows how its own variety ranks in terms of match value among the varieties in

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\(^1\)For instance, Lexus and Pandora created special music series with ads of Lexus F Sport. The music tracks were picked by paying attention to the interests of the demographic groups that were most likely to buy this Lexus model. (Castillo, 2015)

\(^2\)During the experiment of Mediasmith with the data of different consumer data vendors, an increase in advertising accuracy due to targeting varied from 5% to 183% (Marshall, 2015). Also, Forbes (2015) notes that 54% of North American companies cite the identification of a target audience as a primary challenge.

\(^3\)According to Loechner (2014), “92% of Americans ignore at least one type of ad seen every day across six different types of media”, and the survey of Reuters shows that 39% and 47% of consumers in the UK and the US respectively use ad blocking software (Austin and Newman, 2015).


\(^5\)In contrast, Athey and Ellison (2011), Chen and He (2011) and Anderson and Renault (2015) assume that consumers know the ranking of products that they search.

\(^6\)According to Nielsen (Ridley, 2014), only 59 per cent of targeted ads reach targeted audiences in Europe. Moreover, the specialists agree that location targeted ads never reach 100 percent success rate even by using the
Advertising targeting has two opposing effects on firms’ pricing incentives. On the one hand, targeting increases the information content of ads. Consumers learn about the existence of the most relevant competitors, given their idiosyncratic tastes, which raises the value of search. This increases competition among firms, and puts a downward pressure on prices. We call this effect a \textit{competition effect}. On the other hand, the products that consumers learn about are more valuable and closer substitutes on average. This reduces the value of search in equilibrium, which encourages sellers to raise their prices. We call this a \textit{demand composition effect}. The strength of both effects varies with the number of firms and the cost of search. We find that, given advertising intensity by firms, if search costs and the number of firms are sufficiently high, the second effect dominates, and the equilibrium price is higher under targeted advertising. On the contrary, if search costs and the number of firms are low, then the first effect dominates, and the equilibrium price is higher under random advertising.

Not surprisingly, whether consumers and firms are better off when advertising is targeted is related to these effects on prices. In particular, if search costs are low, consumer surplus is higher and the profits of firms are lower when advertising is targeted, as consumers gain from both lower prices and lower total search costs. However, if search costs are high, then consumer surplus may be lower and the profits of firms may be higher under targeted advertising.

Certainly, advertising intensity changes with the possibility of targeting. Moreover, the intensity of advertising also changes the information content of ads for the —average— consumer regarding her willingness to pay for the firms’ product. It also affects the information regarding that same variable that a firm may infer from the fact that the consumer visits the firm. The larger the marginal cost of advertising, the lower the incentives for the firm to send ads. This means that, when ads may be targeted, firms will send a larger proportions of their ads to those consumers for whom their products are most likely to be the best fit. That endogenously improves the average quality of targeting and also enhances firms’ market power over their (visiting) consumers. This additional effect, an \textit{endogenous monopolization} effect of the linked to an endogenous market segmentation, results in a strengthened incentive to raise prices.

In what follows, we specify our baseline model and assumptions in Section 2 and characterize symmetric pricing equilibria with random and targeted advertising respectively in Section 3 and Section 4, both for a given level of advertising. In Section 5, we compare equilibrium prices under both advertising regimes. The choice of advertising intensity is first discussed in Section 6 in a simplified form that internalizes intensity but keeps precision of targeting at an exogenous level. Using this simplified form, we present a parametric example and discuss profits, consumer surplus, and welfare in Section 7. In particular, we carry out some pseudo comparative statics exercises to illustrate our points. Then, Section 8 modifies the baseline model to allow for a full-fledged analysis of intensity affecting targeting precision, discuss the endogenous monopolization effect, and illustrate the discussion with our parametric example. Section 9 concludes. Before turning to the model, we now discuss the relation of this work with the literature on targeting in advertising.

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most advanced GPS technologies. (see Kharif (2014)).
1.1 Relation to the literature

The studies of Renault (2016) and Yang (2013) are close to ours in analyzing targeted advertising and costly consumer search. These papers model consumer search among a continuum of horizontally differentiated products. A consumer obtains an ad from a firm if her willingness to pay for the firm’s product is above a specific threshold. Both papers find that targeted advertising increases price competition. In contrast, in our setting a consumer may obtain an ad of the product even if she values it little: it is the ranking of products that drives targeting. Furthermore, by assuming a finite number of firms, so that a consumer receives a finite number of ads, our model allows us to identify the demand composition effect that would be missing in a model with an infinite number of sellers, as Renault (2016) and Yang (2013).\(^7\)

The mechanism behind the potential increase in price associated with targeting in this paper is different from that in Galeotti and Moraga-González (2008). In that paper, firms sell homogeneous products, but differ in their advertising costs. Targeting ads based on consumer characteristics totally unrelated to willingness to pay, firms endogenously segment the market and obtain positive rents —and higher prices—. Thus, targeting allows firms to share the market and sustain a collusive outcome. In our model, the characteristics of a consumer that allow a firm to target its ads to her may also attract the closest competitor. In that sense, in our setting targeting segments the market in the wrong direction from the firms’ point of view.\(^8\)

In Iyer et al. (2005), consumers may either be indifferent between the products offered by two firms or only consider buying one of them. Ads inform the consumer of the existence of the product, but also of the consumer type. That is, consumer search is absent. The authors compare equilibrium with targeted and with non targeted advertising. When firms can target their ads, firms charge on average larger prices. Indeed, to all effects, targeting increases product differentiation, but the higher incentives to search that is a key component of our model is absent here.

Ben Elhadj-Ben Brahim et al. (2011) study a Hotelling-type model where consumers become fully informed about a product after obtaining its ad. Once more, consumer search is absent in the model. When advertising costs are low, firms target their ads to consumers close to them, and so the market is segmented. Prices are higher under targeting, in this case. However, when advertising costs are high, firms do in fact target some of its ads to consumers closer to the rival, as they are not as likely to get ads from that rival. This result follows from a particular cost function of advertising (linear in "length" and convex in "intensity") and the inelasticity of demand implicit in Hotelling-type of models.\(^9\)

\(^7\)de Cornière (2016) also finds that a symmetric equilibrium price may be higher under targeted advertising. However, his finding stems from the advertising fees of a search engine instead of the mean valuation of consumers who buy.

\(^8\)In Bergemann and Bonatti (2011), more precise targeting also decreases the number of competitors in any market segment, reducing competition and increasing prices. In their paper, prices are not a firm’s decision variable, and they focus on the efficiency of advertising intensity as a function of the precision of targeting. This is also the theme in Meurer and Stahl II (1994). Although the latter does not analyze targeting, the authors claim in the conclusions that with targeting, as with no targeting, there could be under or over provision of advertising.

\(^9\)If a firm applies price discrimination and targeted advertising on a Hotelling line, then targeting may lead to lower prices due to competition for the consumers who are in the middle of the line. (see Moorthy and Tehrani (2015) for more details).
When the valuations of products are distributed independently, as in our case, observing one product teaches a consumer relatively very little (under targeted advertising) or nothing (under random advertising) about a different, not yet observed product. This is very different from spatial competition models, e.g. Hotelling lines, spokes,\textsuperscript{10} or Salop circles. Therefore, the effect of targeting and search on equilibrium prices in models a la Perloff and Salop (1985) differs from the findings in location models.\textsuperscript{11}

de Cornière (2016) also considers a spatial model, but in his case consumers differ also in their willingness to pay for their ideal variety. A (large) number of firms located in a circle advertise through a search engine. That means that search precedes advertising, in the sense that the consumer receives one ad at a time, when she clicks on a keyword —and incurs the cost of doing so—. Targeting is imperfect as the engine only directs the consumer to the ideal product with a probability lower than one. Likewise, firms only pay for "ads" that result in a visit. These advertising costs (click rates) are then a fixed cost for a firm —that uses the engine— in the absence of targeting, but will enter the marginal cost when firms decide the scope of their advertising, with targeting. Thus, targeting results in a higher passed-through cost, and this is a reason for higher prices under targeting. This mechanism is completely absent in our model, where the upward pressure on prices comes from what we termed demand composition effect. On the other hand, targeting increases the mark-up of firms, as it increases the value of search for consumers, an effect that, as we have discussed, is also present in our model. An additional feature that we share with this paper, although linked to a different channel, is the endogeneity of precision of targeting that we allow in the generalized model.

With a finite number of firms and targeted advertising, learning the characteristics of —and so, willingness to pay for— a product gives some information about the willingness to pay for non-observed products. In this respect our model also contributes —and is related— to the literature on consumer search with learning. (See, for instance, Dana (1994) and Janssen et al. (2011), where consumers learn about production costs of homogeneous product firms by observing their prices.)

2 Model

In the market with horizontally differentiated products, each of $N > 2$ symmetric firms offers one product to a continuum of consumers of measure one. The marginal production costs of firms are constant, identical, and normalized to zero. Consumers have unit demand and are heterogeneous in their preferences. More precisely, if a consumer $l$ buys product $i$ at price $P_i$, then her utility is

$$u_{li} = z_{li} - P_i,$$

where $z_{li}$ is a consumer-and-product-specific match value. Match values are identically and independently distributed across consumers and products according to a log-concave distribution\textsuperscript{10}\textsuperscript{11}

\textsuperscript{10}See Chen and Riordan (2007); Caminal and Granero (2012)
\textsuperscript{11}For instance, the effect of prominence in a costly search market may differ in Armstrong et al. (2009) and Rhodes (2011).
function $F(z)$ with a log-concave density function $f(z)$ in the interval $[0, 1]$\textsuperscript{12}. We will also assume that $f(z) + zf'(z) > 0$. The condition, that is automatically satisfied if $f$ in increasing or concave, guarantees that the equilibrium price when $k$ firms compete in this framework under consumers’ complete information on prices and match values is decreasing in $k$.

Consumer $l$ observes neither $z_{li}$ nor $P_i$ unless she visits firm $i$ at a cost $s$, identical for all consumers and all firms. Moreover, the consumer learns about the existence of a firm, and so may visit it, only when she receives an ad from the firm\textsuperscript{13}. If a consumer receives several ads, she may visit firms sequentially, with costless recall and costless return. Also, a firm does not distinguish among its visiting customers, in particular whether they have visited another firm before. Thus, firms cannot price discriminate\textsuperscript{14}.

In the following sections, we describe and analyze two games. In the first, firms cannot target their ads to specific consumers. That is, any ad they send will land in the hands of a random consumer. In the second, firm $i, i = 1, 2, ..., N$ knows the identity of those consumers for whom its product is one of the two products with the highest match value. We call these consumers firm $i$’s targetable consumers. In the baseline model, this is all the information that a firm has with respect to consumers, but then firms may target their ads to these consumers. The law of large numbers guarantees that the set of each firm’s targetable consumers has measure $2/N$.

The assumption that targetable consumers are those for whom the firm’s product is among the two highest match values allows for the simplest, yet interesting, model of competition with targeted advertising. If a firm learnt that its product had the top match value for a consumer, then effective competition would likely disappear and monopoly would ensue. Also, assuming less precise rankings (more than two), would make it increasingly complex to compute exact conditional probabilities, but we foresee no new insights that would follow from the effort\textsuperscript{15}.

12 Log-concavity of $F$ is a standard assumption that guarantees that first order conditions well define monopoly pricing, for instance. Together with log-concavity of $F$, log-concavity of $f$ is sufficient for the search rule to be well defined under targeting — i.e., equation (9) below has a unique solution — even when one firm deviates.

13 Differently, Haan and Moraga-González (2011) assume that consumers give priority to the firms whose ads they have obtained and may search other firms too.

14 In Section 8, we will allow firms to have finer information about a consumer, but we will still assumed that price discrimination is not possible.

15 In the following sections, we will allow firms to have finer information about a consumer, but we will still assumed that price discrimination is not possible.

\textsuperscript{6}In the paper, we study the effect of a change of $N$ on equilibrium values of prices. For that exercise, the choice of two is restrictive, because a larger $N$ implicitly implies more precise rankings. Only there we will consider an alternative to the choice of two. (See Section 5.)
the price $P_i$ charged by that firm and decides whether to (a) buy its product; (b) visit another firm, provided she knows of it, i.e. she has more ads from not yet visited firms; (c) return to a previously visited firm and buy its product; or (d) stop searching without buying.

Our main interest is in understanding the effects on prices and welfare of firms’ ability to target. In the targeting regime, the analysis is simplest when firms send ads to all their targetable consumers. Thus, we will assume a cost function $C$ so that, in the targeting case, firms choose $\mu = 2/N$.\textsuperscript{16} We will keep this assumption until Section 8.

3 Pricing under random advertising

In this section, we assume that firms cannot target their ads. We look for symmetric equilibrium prices for a given, symmetric advertising intensity, $\mu$.\textsuperscript{17} We consider equilibria with consumers’ passive beliefs: conjecturing an equilibrium price $P^*$, and independent of the prices observed in previous visits, a consumer expects that the non-visited firms charge $P^*$. We begin with the analysis of consumer behavior.

3.1 Search behavior

Conjecturing a price $P^*$, common to all firms, a consumer who has obtained at least one ad expects positive surplus from visiting a store if

$$\int_{P^*}^{1} (z - P^*) f(z) dz \geq s.$$ \textsuperscript{(1)}

Equilibrium with sales requires (1) to be satisfied. We assume that (1) holds for the monopoly price $P_M$; i.e., the price that a monopolist would set for a consumer whose private valuation $z$ is distributed according to $F(z)$:

$$P_M = \arg \max_P P \left(1 - F(P)\right).$$

This guarantees that there exists an equilibrium with search and sales.

As in Weitzman (1979), a consumer, who has visited some firms but has not visited all of whose existence she knows, expects positive surplus from visiting another store if the largest utility she has found so far, $u_i = z_i - P_i$,$\textsuperscript{18}$ is below the value $\tilde{u}$ that solves:

$$\int_{\tilde{u}}^{1-P^*} (u - \tilde{u}) f(u + P^*) du = s.$$ \textsuperscript{(2)}

Thus, if all firms are charging the same price $P^*$, a consumer keeps searching as long as no match

\textsuperscript{16}For the no-targeting case, we will obtain pricing equilibrium for arbitrary values of $\mu$. Although we will characterize the equilibrium value of $\mu$, the comparisons between the two regimes work just as easily for any value of $\mu$. The effects of targeting will be most transparent with this strategy of analysis.

\textsuperscript{17}We will not need to consider asymmetric advertising strategies as long as the number of ads is not observed by rivals and consumers. Indeed, an unobserved "deviation" in $\mu$ by one firm will not affect the analysis in this section.

\textsuperscript{18}We omit a consumer-specific index unless necessary.
value is found above $w$ that solves:

$$\int_{w}^{1} (z - w) f(z) dz = s.$$  \hspace{1cm} (3)

The value $w$, a function only of $s$, defines the search behavior for consumers. Note that $w$ is decreasing in $s$, and it is 1 for $s = 0$. If the consumer has visited all the firms from which she has received ads, then she buys from whatever firm gives her the highest utility, provided this utility is positive.

No equilibrium (with positive sales) exists with $P^* > w$, since at that price (1) would not hold, and so there would be no search. Also, without even discussing optimal pricing, we may see that for $P^* = w$, it must be true that $P^* = P^M$: a visiting consumer would buy if and only if her willingness to pay is above the price, just as in the monopoly setting. Thus, let $\bar{s}$ be the value of $s$ at which (1) holds with equality when we set $P^* = P^M$; or equivalently, the value at which (3) holds for $w = P^M$. It is then immediate that

\textbf{Lemma 1.} If $s = \bar{s}$, an equilibrium exists where all firms set $P^* = P^M = w$. No pure-strategy, symmetric equilibrium with positive sales exists for $s > \bar{s}$.

We will restrict attention to $s \leq \bar{s}$.

### 3.2 Pricing equilibrium

Suppose that all firms send $\mu$ ads and all of them but firm $i$ set price $P^*$, while firm $i$ sets an arbitrary price $P$. The probability that a consumer who receives an ad from firm $i$ buys from this firm is

$$\Phi(P) = \sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-k-1} \left\{ \int_{P}^{w-P^*+P} f(z) F(z + P^* - P)^k dz + (1 - F(w - P^* + P)) \sum_{l=0}^{N} \frac{1}{k+1} F(w)^l \right\}.$$  \hspace{1cm} (4)

Indeed, the probability that the consumer has received $k$ other ads is the term in the summation outside the curly brackets. In that event, firm $i$ has a chance $1/(k+1)$ of being the potential $l+1^{th}$ visit for that consumer, for $l = 0, ..., k$, and the visit takes place if none of the previous visits has ended in a purchase, $F(w)^l$. Upon the visit, the consumer buys with probability $1 - F(w - P^* + P)$, and continues searching with probability $F(w - P^* + P)$. If the consumer searches beyond firm $i$, she may return and buy after visiting all $k+1$ firms. This happens if the highest positive utility is obtained at firm $i$: if the match value in $z_i$ is between $P$ and $w - P^* + P$ and other match values are bellow $z_i + P^* - P$. The probability of the latter equals $F(z_i + P^* - P)^k$.\textsuperscript{19}

The demand for firm $i$ is simply $\mu \Phi(P)$, and the profits equal $\mu P \Phi(P)$. Ignoring the constant

\textsuperscript{19}The probability (4) resembles the probability of purchase derived by Wolinsky (1986).
\( \mu \), the derivative of the profits with respect to \( P \) is
\[
\left\{ -\frac{f(w - P^* + P)}{k + 1} \sum_{l=0}^{k} F(w)^l + f(w - P^* + P)F(w)^k \right\} \)
\[
- f(P) F(P + P^* - P)^k - \int_{w-P^*+P}^{w} kf(z + P^* - P) f(z) F(z + P^* - P)^{k-1} dz \} + \Phi(P).
\]

After setting \( P = P^* \), integrating by parts the last term of (5) and the integral in \( \Phi(P) \), and noticing that \( \sum_{l=0}^{k} F(w)^l = \frac{1 - F(w)^{k+1}}{1 - F(w)} \) we obtain

**Proposition 1.** For a fixed value of \( \mu \in (0,1) \), a symmetric equilibrium price with random advertising, \( P^* \), satisfies
\[
0 = \sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1 - \mu)^{N-k-1} \left\{ \frac{1}{k + 1} \left( 1 - F(P^*)^{k+1} \right) - P^* \left( \int_{P^*}^{w} F(z)^k f'(z) dz + \frac{f(w) - F(w)^{k+1}}{k + 1} \right) \right\}. \tag{6}
\]

Let us analyze the expression inside the curly brackets in the derivative of the profit function, (6). The first term corresponds to the expected sales of each firm to consumers that obtain the firm’s ad and \( k \) other ads. A unit increase in the price will increase also by one unit the profits of the firm obtained from each of these sales. The second term, i.e., the expression in parenthesis multiplied by \( P^* \), represents the reduction in profits from lost sales due to that unit increase in the price. The first term inside that parenthesis represents the effect on the sales to consumers who search all the firms that they know about before returning to firm \( i \). The second term represents the effect on sales upon consumers’ first visit to firm \( i \). The increase in the price reduces the number of consumers who terminate their search at firm \( i \); it is less likely that a consumer obtains a match value that results in a utility \( \tilde{u} \) or above. But this also increases the number of consumers that may visit all firms and potentially return to firm \( i \). Some of these consumers are more likely to stop at other firms, now that the price is higher at \( i \), but the net effect of an increase in price on returning consumers may be positive.\(^{20}\) Obviously, these consumers would have stopped and bought at firm \( i \) before the increase in price, so the demand of the firm unambiguously shrinks as a consequence of this increase.

The equilibrium condition in Proposition 1 is similar to the ones obtained in Wolinsky (1986) and Anderson and Renault (1999). We will not discuss restrictions on \( F \) that guarantee that (6) has a unique solution, and that indeed a symmetric equilibrium exists. Instead, in Section 7, we will use a parametric family of distributions that satisfies those conditions to discuss (pseudo) comparative static exercises on prices and welfare. Moreover, equilibrium is guaranteed to exist for extreme cases, with \( s \) sufficiently small or \( N \) sufficiently large, for which we have relevant, general results.

\(^{20}\) For instance, if \( f' \geq 0 \), this is always the case.
When firms set indeed $P^*$, the revenue each of them expects, $R^*$, is

$$R^* = \mu \sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-k-1} \frac{P^*}{k+1} \left(1 - F(P^*)^k\right),$$

(7)
as the demand from a consumer who visits $k$ firms is $1 - F(P^*)^k$.

4 Pricing under targeted advertising

We now turn to analyzing the price game where firms may target their ads. For now, assume that firms send ads to all their targetable consumers. Also, in Section 6 we will argue that deviations of both how many ads to send and what prices to set will not be profitable if deviations in the latter are not.\(^{21}\) Hence, each consumer receives two ads, and knows that these are the two best matches among the existing $N$ products. A consumer does not know the relative ranking of these two advertised products, though.

In this section, we will use subscript $t$ for variables, so as to distinguish them from the variables in Section 3. Also, we begin characterizing consumers’ search behavior and then proceed with pricing decisions.

4.1 Search behavior

For the first visit, a consumer randomly chooses one of the two firms of whose existence she knows. After observing the utility at that first sampled seller, the consumer computes the expected gains from visiting the second firm. In what follows, we denote the match value of the first visited seller by $z_1$, and the match value of the second seller by $z_2$.

**Conditional probabilities.** The match value of the first firm may be the highest in the market or the second highest. The conditional probability that the match value $z_1$ is the highest equals

$$\frac{Nf(z_1)F(z_1)^{N-1}}{Nf(z_1)F(z_1)^{N-1} + N(N-1)f(z_1)F(z_1)^{N-2}(1-F(z_1))}.$$ 

Likewise, the conditional probability that $z_1$ is the second highest is

$$\frac{N(N-1)f(z_1)F(z_1)^{N-2}(1-F(z_1))}{NF(z_1)^{N-2}f(z_1)(F(z_1)+N(N-1)(1-F(z_1)))}.$$ 

Thus, the density function of $z_2$ conditional on $z_1$ is

$$g(z_2 | z_1) = \begin{cases} \frac{(N-1)f(z_2)F(z_2)^{N-2}}{F(z_1)^{N-1} + (N-1)(1-F(z_1))} & \text{if } z_2 \leq z_1, \\ \frac{f(z_2)}{1-F(z_1)F(z_2)+(N-1)(1-F(z_1))} & \text{if } z_2 > z_1. \end{cases} \quad (8)$$

\(^{21}\)For that, we will need to define consumer’s beliefs after receiving a number of ads different from 2. In keeping with passive beliefs, we will simply assume that the consumer still thinks that two ads are from the best matches and the other is from a random seller, without being able to tell which is which. Moreover, the consumer still believes that there has been no price deviation.

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Search  After having observed \(z_1\) and \(P_1\), and conjecturing equilibrium price \(P_t^*\), the relevant decision for the consumer is whether to visit a second firm. The consumer is indifferent between visiting the second firm and buying the first product if the expected gain from the visit equals the search cost. That is, if given \(z_1 \geq P_1\),

\[
\int_{z_1 - P_1 + P_t^*}^{1} (z_2 - P_t^* - z_1 + P_1) g(z_2|z_1) \, dz_2 = s. \tag{9}
\]

We denote \(w_t(P_1; P_t^*)\) the value of \(z_1\) that solves (9).\(^{22}\) Also, we let \(w_t = w_t(P; P)\), that is, the value of \(z_1\) that solves

\[
\int_{z_1}^{1} (z_2 - z_1) g(z_2|z_1) \, dz_2 = s, \tag{10}
\]

which is independent of the value of \(P\), and depends only on \(s\).

Note that a consumer who observes \(z_1 < P_t^*\) will never buy from the first visited firm, and will search for a second firm if

\[
\int_{P_t^*}^{1} (z_2 - P_t^*) g(z_2|z_1) \, dz_2 \geq s.
\]

This will be guaranteed if

\[
\int_{P_t^*}^{1} (z_2 - P_t^*) f(z_2) \, dz \geq s. \tag{11}
\]

If (11) is satisfied, then a consumer who visits the first firm will visit the second if and only if \(z_1 < w_t\). Also, under these conditions, and taking into account that the density of \(z_1\) is

\[
g(z_1) = \frac{N f(z_1) F(z_1)^{N-2}}{2} \left[ F(z_1) + (N - 1) (1 - F(z_1)) \right],
\]

it is easy to check that the consumer will indeed visit the first firm.

Suppose that, there exists an equilibrium \(P_t^*\), and that this price is increasing in \(s\). (We will show that this is indeed the case in our parametric example.) Then, since the left hand side of (11) is decreasing in \(P_t^*\), there exists a cutoff value \(\bar{s}_t\) such that (11) is satisfied for any \(s < \bar{s}_t\).

Note that, for \(s = \bar{s}_t\), the equilibrium price must be \(P_t^* = w\) as defined in the previous section. Comparing (10) and (11), since \(G(z|P_t^*) < F(z)\), we have that at this search cost \(\bar{s}_t\) (and corresponding equilibrium price \(P_t^* = w\)) the consumer expected surplus from a second visit when \(z_1 = P_t^*\) is

\[
\int_{P_t^*}^{1} (z_2 - P_t^*) g(z_2|P_t^*) \, dz_2 - s > 0. \tag{12}
\]

Thus, \(w_t > P_t^* (= w)\) for \(s = \bar{s}_t\).

4.2 Pricing equilibrium

Again we consider symmetric equilibria where all \(N\) firms charge the same price \(P_t^*\). Suppose that all firms but firm \(i\) set \(P_t^*\), and firm \(i\) sets a price \(P > P_t^*\).\(^{23}\) Consider a targetable consumer for

---

\(^{22}\)When \(F\) and \(f\) are log-concave, the left hand side of the expression is decreasing in \(z_1\).

\(^{23}\)A downward deviation leads to slightly different demand expression. However, the limit of the demand and its slope when \(P \to P_t^*\) is the same from both directions.
\(i\) who, after receiving its ad, visits firm \(i\) in first place. The probability of this event is \(1/2\). The consumer does not visit any other firm and buys from firm \(i\) if \(z_i \geq w_t(P; P^*_t)\). For \(1/N\) consumers \(z_i\) is the highest, and for \(1/N\) consumers \(z_i\) is the second-highest match value. Then, the number of consumers who visit firm \(i\) first and buy there without visiting other firms is

\[
\frac{1}{2N} \left( \int_{w_t()}^1 NF(z)^{N-1} f(z) \, dz + \int_{w_t()}^1 (N-1) F(z)^{N-2} f(z) (1 - F(z)) \, dz \right).
\]

If \(z_i < w_t(P; P^*_t)\), then the consumer visits the other firm from which she received an ad. In that case, the consumer buys from firm \(i\) only if the utility \(z_i - P\) is higher than \(\max\{z_j - P^*_t, 0\}\), \(j \neq i\). The number of consumers who visit firm \(i\) first, then visit the other firm, and finally return to buy from firm \(i\) is

\[
\frac{1}{2N} \int_{P}^{w_t()}(\int_{0}^{z_i - P + P^*_t} (N-1) F(z) f(z) \, dz) NF(z)^{N-1} f(z) \, dz.
\]

Other consumers, \(1/N\) of them, receive an ad from firm \(i\) but first visit the other firm, \(j\), from which they have received an ad. Firm \(i\) is visited by one of these consumers if \(z_j < w_t\). After the consumer observes \(z_i\), she is fully informed about both products. Thus, firm \(i\) sells to that consumer only if \(z_i - P > \max\{z_j - P^*_t, 0\}\) and \(z_j < w_t\). The number of these consumers is

\[
\frac{1}{2N} \int_{w_t - P^*_t + P}^{w_t} NF(z)^{N-1} f(z) \int_{0}^{z_i - P + P^*_t} (N-1) F(z) f(z) \, dz \, dz + \frac{1}{2N} \int_{P}^{w_t - P^*_t + P} F(z - P + P^*_t)^{N-1} dF(z).
\]

Thus, the total demand for product \(i\) is \(D_i^1(P; P^*_t) / (2N)\), where

\[
D_i^1(P; P^*_t) = \int_{w_t()}^1 NF(z)^{N-1} f(z) \, dz + \int_{w_t()}^1 (N-1) F(z)^{N-2} f(z) (1 - F(z)) \, dz + \int_{P}^{w_t} NF(z - P + P^*_t)^{N-1} dF(z) + \int_{w_t - P^*_t + P}^{w_t} NF(z - P + P^*_t)^{N-1} dF(z).
\]

If firm \(i\) charges \(P = P^*_t\), then the demand becomes

\[
\frac{D_i^1(P^*_t)}{2N} = \frac{1}{N} \left( 1 - F(P^*_t)^N \right),
\]

as the right-hand-side of (13) is \(1/N\) of the total, expected demand for the whole industry.

Thus, the next proposition characterizes symmetric price equilibrium.
Proposition 2. In a symmetric equilibrium, firms charge $P^*_t$ that satisfies

$$\frac{1}{N} \left(1 - F(P^*_t)^N\right) - P^*_t F(P^*_t)^{N-1} f(P^*_t)$$

$$-P^*_t \left[ \int_{P^*_t}^{w_t} (N-1)F(z)^{N-2} f(z)^2 \, dz + \frac{N-1}{2} F(w_t)^{N-2} f(w_t)(1 - F(w_t)) \, dw_t \right] = 0,$$

where $dw_t$ is obtained by implicitly differentiating $w_t()$ with respect to $P$:

$$dw_t = \frac{dw_t(P; P^*_t)}{dP} \bigg|_{P=P^*_t} = \frac{1 - G(w_t|w_t)}{1 - G(w_t|w_t) - \int_{w_t}^{1} (z_2 - w_t) \frac{dg(z_2|w_t)}{dz_1} \, dz_2}$$

$$= \frac{(N-1) (1 - F(w_t))}{(N-1) (1 - F(w_t)) - sf(w_t)(N-2)} = \frac{1}{1 - s \frac{N-2}{N-1} f(w_t)}.$$

If a symmetric equilibrium does exist, then a firm expects revenue

$$R^*_t = \frac{P^*_t}{N} \left[1 - F(P^*_t)^N\right].$$

5 The effects of targeting

In this section, we compare equilibrium prices, firm profits, and consumer surplus in the two advertising regimes described in the previous sections. We will restrict attention to search costs below $\min \{\bar{s}, \bar{s}\}$, $t$. Indeed, we may compare both regimes even without obtaining the equilibrium value of $\mu$ for the non targeting case.

Information is one of the keys to study the difference between $P^*$ and $P^*_t$. When advertising is random and $\mu < 1$, only a share of consumers are informed of all available products, and so can possibly visit all stores. On the contrary, if advertising is targeted, consumers obtain ads from all relevant firms: the one offering the highest match value, and the one offering the product closest—in her preferences—to it. In a symmetric equilibrium, the latter is the relevant competition for the supplier of the former. Moreover, with targeted advertising, two visits are as informative in equilibrium as visiting all $N$ firms.

We may call this effect of targeting a competition effect: all consumers are effectively more informed with targeted advertising, and this increases competition. To illustrate this effect, let $s = 0$. Condition (6) for $P^*$ becomes

$$\sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-k-1} \left\{ \frac{1}{k+1} \left(1 - F(P^*)^{k+1}\right) - P^* \left[f(1) - \int_{P^*}^{1} F(z)^k f'(z) \, dz \right] \right\} = 0.$$ 

$$\sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-k-1} \left\{ \frac{1}{k+1} \left(1 - F(P^*)^{k+1}\right) - P^* \left[f(1) - \int_{P^*}^{1} F(z)^k f'(z) \, dz \right] \right\} = 0.$$ 

(17)

If $\mu = 1$, i.e., if every consumer learns of all $N$ firms, this equation simplifies to

$$\frac{1}{N} \left(1 - F(P^*)^N\right) - P^* \left[f(1) - \int_{P^*}^{1} F(z)^{N-1} f'(z) \, dz \right] = 0,$$

$$\frac{1}{N} \left(1 - F(P^*)^N\right) - P^* \left[f(1) - \int_{P^*}^{1} F(z)^{N-1} f'(z) \, dz \right] = 0,$$

which, integrating by parts, is condition (14) for $P^*_t$ when $s = 0$. 

13
In equilibrium, targeting is indeed a perfect substitute for complete information about the existence of products. Meanwhile, if $\mu < 1$, then there is a positive share of consumers who obtain ads of a firm but do not obtain ads from its closest competitors. Thus, consumers are less informed on average if advertising is random even if the search cost equals zero. We observe that the LHS of (17) is a weighted average of (18) for $N' = 1, 2, \ldots, N$. That is, the conditions that define the equilibrium price with full consumer information with $N' = 1, 2, \ldots, N$ firms in the market. As this equilibrium price is decreasing in $N'$, we conclude:

**Proposition 3.** For $s$ sufficiently close to 0 and $\mu < 1$, $P_t^*$ is lower than $P^*$.  

**Proof.** The left hand side of (18) is decreasing in $P^*$. On the other hand, taking into account that $1 - F(P^*)^N = \int_{P^*}^{1} NF(z)^{N-1} f(z) dz$, this left hand side can be written as

$$\int_{P^*}^{1} F(z)^{N-1} \left[ 1 + P^* f'(z) \frac{f(z)}{f'} \right] f(z) dz - P^* f(1).$$

The expression in the square bracket is positive, since $f(z) + zf'(z) > 0$, and $P^* < z$, by assumption. Thus, the left hand side of (18) is also decreasing in $N$. Thus, the solution in $P^*$ is decreasing in $N$. Since (17) is a linear combination of (18) for values of $k \leq N$, we conclude that $P_t^* < P^*$. $\square$

However, targeting has a second effect, that we call a demand composition effect, with consequences when $s > 0$. Indeed, by selecting the top products in terms of a match value, targeting makes the products offered by the relevant firms, the ones that advertise to one consumer, (more valuable and) less differentiated. This, in turn, reduces the value of search, just as illustrated by the well known Diamond paradox, and so pushes prices up as compared to prices under random advertising.

**Proposition 4.** For any $s \in (0, \bar{s})$ there exists $N(s)$ such that for any $N > N(s)$ with search in both regimes, $P_t^* > P^M > P^*$.  

**Proof.** From the Poisson Limit Theorem,

$$\lim_{N \to \infty} \left( \frac{N - 1}{k} \right) \mu^k (1 - \mu)^{N-k-1} = e^{-M} \frac{M^k}{k!},$$

where $M = \mu N$. Thus, the first-order condition (6), converges to

$$\sum_{k=0}^{N-1} e^{-M} \frac{M^k}{k!} \frac{1}{k+1} \{ (1 - F(P^*)^{k+1}) + P^* \left( \int_{P^*}^{w} (k+1) F(z)^k f'(z) dz - f(w) \frac{1 - F(w)^{k+1}}{1 - F(w)} \right) \} = 0,$$

where $w$ is independent of $N$. Now, as $N, M \to \infty$, $e^{-M} \frac{M^k}{k!}$ vanishes for any value of $k$ different
from $M$. Thus, equation (19) converges to

$$
\lim_{M \to \infty} \left[ 1 - F(P^\ast)^M + P^\ast \left( \int_{P^\ast}^w MF(z)^M f'(z) \, dz - f(w) \frac{1 - F(w)^M}{1 - F(w)} \right) \right]
$$

$$
= \lim_{M \to \infty} \left[ 1 - P^\ast \frac{f(w)}{1 - F(w)} \right] = 0.
$$

That is, $P^\ast_{\infty} = (1 - F(w))/f(w)$. Note that, as $(1 - F(x))/f(x)$ is decreasing in $x$ (by log-concavity) and $w \geq P^M$, we have that $P^\ast_{\infty} \leq (1 - F(P^M))/f(P^M) = P^M$ for any value of $s \leq \bar{s}$. On the other hand, (14) times $N$ converges to 1 as $N \to \infty$ for any $w_t, P^t < 1$. Note that, as $N \to \infty$, the solution to (10) approaches $w_t = 1 - s < 1$. Thus, for sufficiently large $N$, (14) is positive at $P^M$, and so $P^t > P^M$ in any equilibrium with search.

In fact, the intuition of Proposition 4 is simple, and, as we advanced, is nothing but the re-emergence of the Diamond paradox in this model with differentiated products. Indeed, as $N$ approaches infinity, the two highest realizations of the valuation for each consumer approach 1 and each other. (The conditional density of $z_2$ in (8) approaches zero for any value different from 1.) As the expected difference in the valuations of the two products advertised in the targeted ads that each consumer receives shrinks to zero, no search beyond the first visit happens in any symmetric equilibrium, if such equilibrium involves search (first visits). In the limit, conditional on visiting the firm, consumers have valuation 1, and monopoly pricing for this "demand function" ($P^t = 1$) ensues. Such a high price never occurs in the random advertising game: as we have discussed, when consumers never return after a visit, the equilibrium price is $P^M < 1$: conditional on showing up, the "demand function" of a consumer is not affected by $N$.

It is important to stress that Proposition 4 is a result that talks about the effect of an — exogenous— increase in the precision of targeting. With costly search, targeting implies a reduced product differentiation and the selection of the most willing to pay customers visiting a particular store, therefore reducing the elasticity of demand, and the expected gains from searching. This effect, which is absent in the random advertising case, may be strong enough as to result in higher prices under targeted advertising.

To see that it is the quality of information conveyed by ads what is behind Proposition 4, rather than the number of varieties, suppose instead that, as $N$ increases, the share of a firm’s targetable consumers does not change. That is, under targeted advertising, each firm’s targetable consumers are all for which its product is among the $M'$ with highest match value, for a constant proportion $\alpha = M'/N$. Note that this means that indeed the expected differentiation of any two products that the consumer learns about does not change with $N$. That is, the demand composition is not affected by $N$. Each consumer gets more and more ads as $N$ grows, and this leads us to an exercise very similar to the one we have just performed.

Obviously, a full analysis of this alternative model for finite $N$ is beyond practical feasibility, as the search rule is history-dependent and leads to a complex demand for each firm. However,

\[24\] Obviously, for $N$ so large that $P^\ast_{\infty}$ is close to 1, an equilibrium with search could not happen unless $s$ is very close to 0. Nevertheless, the argument in the proposition is more general, and shows that, for any cost below $\bar{s}$ there will be a range for market sizes $N$ where equilibrium with search in both regimes exist and prices are higher with targeting.
for sufficiently large $N$, this optimal choice rule converges to a constant rule. Indeed, as $N$, and so $M'$, gets large, both the horizon and the probability of different realizations in one more visit approach a constant state for any finite number of past realizations. Also, for any given threshold for purchases upon a visit to a store, the probability that the consumer eventually finds a better match later converges to 1. Therefore, and as in the case of non-targeted advertising, the probability that a customer returns to the store after a first visit without a purchase converges to zero.

Thus, search will follow a cutoff rule, defined by (3) where we substitute $\hat{f}(z)$ for $f(z)$, where $\hat{f}$ is the density function of the random variable $z$ truncated below at the upper $M'/N = \alpha$ quantile, so that for any $z \in [1 - \alpha, 1]$,

$$\hat{f}(z) = \frac{f(z)}{\alpha}.$$ 

Obviously, this cutoff point satisfies $\hat{w} > w$. Other than that, the limit value of $P^*_t$, $P^*_t\infty$ will be the same as in the non-targeting case, except that $F$ and $w$ are different,

$$P^*_t\infty = 1 - \hat{F}(\hat{w})\hat{f}(\hat{w}) = 1 - F(\hat{w})f(\hat{w}).$$ 

Since the right hand side is decreasing in $w$, this implies that $P^*_t\infty < P^*_\infty$.

### 6 Equilibrium advertising

So far we have considered pricing decisions for given advertising intensity. We now begin the discussion of the firms’ incentives for advertising under both regimes. As we mentioned in the Introduction, first, we do so while keeping the precision of targeting fixed at the outset. That is, we now make assumptions of $C$ so that firms’ optimal choice is to send $2/N$ ads when advertising is targeted.

Optimally, firms target their ads to their targetable consumers. Suppose every other firm is doing so, and consider firm $i$’s choices of $P$ and $\mu$. That requires, in particular, that

$$C'\left(\frac{2}{N}\right) = \frac{R^*_t}{2/N},$$

(20)

where $R^*_t$ is defined in (16). When this condition holds and the firm sends ads only to its targetable consumers, the marginal cost of doing so is smaller than the marginal revenue until the firm reaches all its targetable consumers. (Note that the marginal revenue from an ad sent to a targetable consumer is indeed $R^*_t$, independent of the number of ads that are sent. Also, note that, independent of this number, the optimal choice of price if ads are only sent to targetable consumers is still $P^*_t$. Thus, $\mu^*_t = 2/NT$ is the equilibrium number of ads if, no matter what consumers are reached by the rest of ads, a firm will always get a larger marginal revenue when sending an ad to a targetable consumer than when sending it to a non-targetable consumer.

\footnote{In fact, the marginal cost could be slightly below the marginal revenue extracted from a targetable consumer, since, as we comment below, there is a discrete difference between this latter number and the revenue that can be extracted from a non-targetable consumer.}
We are assuming that consumers hold passive beliefs, which means that a price deviation does not change the conjecture about other firms’ prices. Let us complement this by assuming that a price deviation, together with the consumer receiving three ads, leads the consumer to infer that the deviating firm is not one of the two offering her most favorite products. Then, the consumer would have less incentives to visit that firm first. Therefore, if a change in price is not in the firm’s interest when the consumer is indeed targetable —so that she receives only one other ad—, it is certainly less so when the consumer is not targetable —and receives two other ads—. Thus, the firm would have no incentive to send an ad to a non targetable consumer unless it can disguise itself by offering the same price, $P^*$. Now, at that price the expected revenue from that consumer is lower, as the consumer visits the firm first with a probability of 1/3, instead of 1/2 and is less likely to have a high willingness to pay for the good.\textsuperscript{26} Thus, if (20) is satisfied, firms will indeed send ads to —all, and only— their targetable consumers.

The random advertising case is even simpler. First, note that all consumers are ex-ante identical from the firm’s point of view. Any deviation in the number of ads by one firm will affect the number of visits, but not the composition of the visiting set of consumers. That is, if every other firm is sending $\mu^*$ ads and charging a price $P^*$, then any ad by firm $i$ has $(\mu^*)^k (1 - \mu^*)^{N-k-1}$ probability of landing in the hands of a consumer with other $k$ ads. Thus, if all other firms are charging $P^*$, then $i$’s best response is to also charge $P^*$, independent of the number of ads that $i$ chooses. Thus, a symmetric equilibrium is characterized by $(P^*$ and $\mu^*)$ that satisfies

$$C''(\mu^*) = \frac{R^*}{\mu^*},$$

where $R^*$ is defined in (7).

If (20) is satisfied, then $(P^*_t, \frac{2}{N})$ constitute a symmetric equilibrium in prices and number of ads when advertising is targeted, and , and $(P^*, \mu^*)$ when it is not. In the next section we illustrate equilibrium outcomes for a parametric example.

7 A parametric example

In the previous sections, we have discussed in depth the consequences of targeting for equilibrium prices. Targeting affects not only the behavior of prices, but also search and advertising by consumers and firms, respectively. All these variables, on the other hand, affect welfare measures. In this section we investigate these other consequences by solving and presenting outcomes of a parametric example.

In particular, we let $F(z) = z^a$, with $a \geq 1$, and $C(\mu) = c\mu^b$, where $c > 0$ and $b > 1$. We still want to keep the precision of targeting exogenous, so that (20) holds. Thus, in this section, as we set different values for $a, b, N$, and $s$, we adjust the value of $c$ to satisfy this equation. (Of course, the same value of $c$ is assigned to each value of $s$ in both the targeting and non-targeting regimes.) In other words, what we present are results of pseudo comparative statics exercises. Figure 1c

\textsuperscript{26}It is true that a consumer who receives three ads has lower reserve utility, so that her cutoff value for search would be lower. However, it is quite straightforward that the probability of meeting that lower reservation utility is lower than it is for a targetable consumer to meet the equilibrium value $w^*_t$.\textsuperscript{17}
shows the equilibrium values of $\mu^*$ for $N = 4$, $a = 4$, and $b = 2$, and for different values of $s$—and so $c$—. Figure 1a presents equilibrium prices in both regimes. Note that, as expected, prices in the non-targeting case are larger than in the targeting case when $s$ is close to zero. However, for large values of $s$, the relationship may be reversed, as we also knew. Profits for both regimes are presented in Figure 1b. Note that prices may be larger with no targeting than with targeting, yet profits (and advertising intensity) may be lower. This is a consequence of the composition demand effect: with no targeting, consumers are not directed to the suppliers of their highest match-value product, so that for the same prices, firms’ demand is lower in the non-targeting case than in the targeting case.

![Graphs showing equilibrium values](image)

Figure 1: Equilibrium for different $s$, when $F(z) = z^4$, $N = 4$, $b = 2$

As for consumer surplus represented in Figure 2a, we observe that for low values of $s$, consumer surplus is higher under targeting. Indeed, for those values, both prices and cost of search are lower under targeting, while the expected value of the product found in a consumer’s visit is higher. However, for sufficiently high search costs, prices are larger in the targeting case and this may more than compensate the positive effects of targeting. (Although not for the particular parameters that we are presenting here, total welfare may also be lower under targeting, as this reduction in consumer surplus may more than compensate the increase in profits.)
Figure 2: The values of consumer surplus and welfare, for different values of $s$. Other parameters: $b = 2$, $N = 4$, $F(z) = z^4$.

8 Finer targeting

Up to this point, we have assumed a crisp information structure so as to more clearly show the effects of imperfect targetability. It is time now to relax this structure and introduce the possibility that firms have more precise information about consumers, which then they may use to target their ads. Suppose that, as before, each firm learns what consumers have their product among the two best matches. In addition, suppose that for each targetable consumer $l$, the firm also observes a signal $\beta_{li}$, interpreted as the probability that in fact the firm’s product is the best match for the consumer. (Naturally, the probability that the firm’s product is the second best match is $1 - \beta_{li}$. Also, for the other firm, $j$, for whom consumer $l$ is targetable, $\beta_{lj} = 1 - \beta_{li}$.) Let $\beta_{li}$ be uniformly distributed on the interval $[\alpha, 1 - \alpha]$ for some $\alpha \in [0, 1/2]$, independent across consumers, $l$, and independent on the realizations of match values, other than their reranking.\footnote{Formally, for every consumer $l$ there is the independent realization $\beta_{li}$, of a random variable uniform in $[\alpha, 1 - \alpha]$. Independently, all $z_{li}$, $i = 1, 2, ..., N$ for consumer $l$ are realized. Then firm $i_1$ and $i_2$ are informed of $\beta_{i1}$. Also, a realization of a Bernoulli random variable with parameter $\beta_{i1}$ determines if $i_1$ is told that $z_{li}$ is highest —if the realization is 1— or second highest —if the realization is 0—. Firm $i_2$ is informed accordingly that $z_{li}$ is second highest —if the realization is 0— or second highest —if the realization is 1—.}

(Of course, consumers do not observe, or need to observe, $\beta$.) Our baseline model is a particular case, with $\alpha = 1/2 = \beta_{li}$ for all $l$.

With this extra information, a firm may decide to send ads to a subset of its targetable consumers, those for whom the ad has a higher probability of resulting in a sale. That is, those for which $\beta_{li} \geq \beta^*_{i1}$ ($\geq \alpha$) for some value of $\beta^*_{i1}$. That means sending a total of $\frac{2^{1-\alpha-\beta^*_{i1}}}{N^{1-2\alpha}}$ ads. Thus, in the targeting advertising case, a consumer may receive only one ad, and the average consumer visiting a firm has this firm as the best match.

This allows us to generalize our baseline model of targeting in the two dimensions that we have mentioned: the quality of information that firms possess and the intensity of advertising. Moreover, this modification of the model endogenizes the quality of targeting, from the consumer point of view. Indeed, firms’ decision on $\beta^*_{i1}$ determine how informative an ad is, on average, for the consumer receiving it. Or, looking at the flip side of it, how high in the ranking of the consumer the product of a firm receiving her visit is, on average —closer to first or to first and second with
equal probability, in our model—.

For a given value of $\beta_i^* = \beta^*$ symmetric for all sellers, let us derive the symmetric equilibrium price $P_i^*$. The seller is a monopolist to the consumers with $\beta_{ii} \geq 1 - \beta^*$, as these consumers will not receive an ad from any other sellers. The expected demand of one such consumer with $\beta_{ii} = \beta$ when the firm sets price $P$ is

$$MD(P; \beta) = \beta \int_{P}^{1} NF(z)^{N-1} dF(z) + (1 - \beta) \int_{P}^{1} N(N - 1) F(z)^{N-2} (1 - F(z)) dF(z).$$

Thus, for a price $P (\geq P_i^*)$, the demand from these consumers will be

$$\frac{2}{N(1 - 2\alpha)} \int_{1-\beta^*}^{1-\alpha} MD(P; \beta) d\beta. \quad (21)$$

Meanwhile, consumers whose $\beta_{ii}$ is in $[\beta^*, 1 - \beta^*)$ obtain two ads. These consumers behave as has been described in Section 4.1.\(^{28}\) Indeed, note that consumers who get only one ad learn that the firm sending that ad is their best shot with probability at least $\beta^*$, but this knowledge is not relevant other than to induce them to visit the firm, as this is their only possibility of purchase. On the other hand, consumers who get two ads learn nothing that consumers did not learn in our basic model. Therefore, the demand from these consumers equals

$$\frac{1}{2} \int_{\beta^*}^{1-\alpha} \left\{ (1 - \beta) \int_{w_t}^{1} N(N - 1) F(z)^{N-2} (1 - F(z)) dF(z) \right. \quad (22)$$

$$+ \beta \left( \int_{w_t}^{1} NF(z)^{N-1} dF(z) + \int_{P}^{w_t} NF(z - P + P_t^*)^{N-1} dF(z) + \int_{w_t - P_t^* + P}^{1} NF(z - P + P_t^*)^{N-1} f(z) dz + \int_{P}^{w_t - P_t^* + P} NF(z - P + P_t^*)^{N-1} dF(z) \right) \right\} d\beta$$

The first term inside the curly brackets represents the probability that the consumer with type $\beta$ has the firm as her second best match $(1 - \beta)$, in which case she buys from the firm only if she visits the firm first (probability 1/2) and her match value is above $w_t()$.\(^{29}\) With probability $\beta$, firm $i$ is the consumer’s best match. Then, if she visits firm $i$ first, she buys from firm $i$ if her match value is above $w_t()$ or if it is between $P$ and $w_t()$ and at least $P - P_t^*$ above the match value for the alternative seller’s product. Finally, if the consumer visits that alternative seller first, then she buys from firm $i$ only if her match value there is below $w_t$ and her match value with $i$ is at least $P - P_t^*$ higher. This is the third line in (22). Firm $i$’s total demand is the sum of (21) and

\(^{28}\)Consistent with our assumption of passive beliefs, we will assume that a consumer does not associate a deviation in price with a deviation in $\beta^*$.

\(^{29}\)Obviously, if the consumer was interested in guessing what the value of $\beta$ was for her, the information on $z_1$ would be relevant. But the consumer is only interested in the probabilities of the realization of $z_2$, and for this, $z_1$ is indeed a sufficient statistic, just as in the baseline model.
inside the first integral (over $\beta$) in (22), we can write this expression as

$$\frac{1}{2} \frac{2}{N(1-2\alpha)} \int_{-\beta}^{1} \left\{ \beta D_t^1 (P; P_t^*) + (1-2\beta) \int_{w_t(\cdot)}^{1} N (N-1) F (z)^{N-2} (1 - F (z)) dF (z) \right\} d\beta,$$

where $D_t^1 (P; P_t^*)$ is as defined in (13). Integrating over $\beta$, the second term vanishes, and we then can write (22) as

$$\frac{2}{N(1-2\alpha)} \left[ D_t^1 (P; P_t^*) \frac{1-2\beta^*}{4} + \int_{-\beta^*}^{1-\alpha} M D_t (P; \beta) d\beta \right].$$

Note that the second term inside the square bracket is independent of $P_t^*$. Moreover, the price that maximizes that second term is larger than $P^M_t$ for large $N$, and so the equilibrium price is larger than $P^M_t$ for any value of $\beta^*$. Therefore, the result of Proposition 4 holds. That is, targeting may result in higher prices.

The value of $\beta^*$ is determined by the —marginal— cost of advertising. Given $\alpha$, a larger value of this marginal cost means a higher value of $\beta^*$, and so a larger proportion of consumers who receive only one ad. This consumers are less responsive to price, but demand is also lower the larger $\beta^*$. Therefore, the sign of the change in demand elasticity, and so on price, as the marginal cost of advertising increases is ambiguous, in general.

However, consider the effect of a larger marginal cost on the price incentives of firms when the search cost $s$ is very small. In that case, consumers will search all firms from which they receive an ad. However, the higher the marginal cost the lower the probability that they do receive more than one ad, that is, the larger $\beta^*$. As $\beta^*$ approaches $1 - \alpha$, then the equilibrium price converges to the monopoly price for a firm facing a demand composed of consumers with willingness to pay equal to the first order statistic of $N$ realizations of $F$. Thus, the equilibrium price must maximize

$$\frac{P}{N} \int_{P}^{1} NF (z)^{N-1} dF (z),$$

or, in other words, should satisfy

$$0 = \frac{1}{N} \left( 1 - F (P_t^*)^N \right) - P_t^* f (P_t^*) F (P_t^*)^{N-1}.$$

This, of course, is nothing but (6) when $\mu = 1$ and $s = 0$, so that $w = 1$. That is, the equilibrium price in the non targeting case when all consumers receive ads from all firms —and search is costless—. But when the marginal cost of advertising is large, firms will not send ads to all consumers, and so the price under non targeted advertising will be lower than the price under
targeted advertising even for very low search costs.

That is, at least when costs of advertising are high, the new effect of targeting, that we can term endogenous monopolization effect, may result in higher prices under targeted advertising whatever search costs are. (This is simply the effect analyzed in, for instance, Iyer et al. (2005).) As a result of endogenously finer targeting, firms may end up drastically increasing market segmentation, and so monopolization of a less elastic demand ensues.

We have computed our parameterized example with this model, for different values of $\alpha$. Figures 3 and 4 show the results of this computations for the same values of $N$, $a$ and $b$ as in Section 7, and for $s = 0.01$ and $\alpha = 0.45$. We let $c$ change this time, and let both $\mu^*$ and $\mu_t^*$ change with $c$. (So, this is now a true comparative static exercise.) As expected, the price with targeting may be large for high advertising costs, where intensity of advertising is low (but still higher than with no targeting) so that more consumers obtain only on ad. The increase in prices (together with the reduced advertising effort) raises profits in the targeting case, but not so much in the case of random advertising, where the reduction in competition and so on prices is much less acute. As a result, profits with targeting are higher than without it for high advertising costs. The different impact on prices (and advertising) implies that consumer surplus is also lower with targeted advertising for large advertising costs, although in this example this lower consumer surplus is more than compensated by the higher profits: welfare is always higher with targeting. (Examples can be constructed where this relationship is also reversed.)

![Figure 3: Equilibrium for different $c$, when $F(z) = z^4$, $N = 4$, $b = 2$, $s = 0.01$](image)

Thus, as anticipated, to the competition and demand composition effects, we now have to add
the endogenous monopolization effect, which pushes prices up in the targeted advertising case especially when the costs of advertising are large.

9 Concluding remarks

Targeting allows firms to reach with their ads consumers who value their products highly. This has two main effects. The first is a competition effect: consumers are effectively more informed of the existence of the relevant alternatives, and this increases competition. The second effect is a demand composition effect: the products offered by the firms whose ads reach a consumer are closer substitutes, and so less differentiated. When consumers pay a search cost to visit a firm, this reduced differentiation is a disincentive to search and may lead to higher prices. Our study shows that indeed, whether targeted advertising leads to higher or lower prices depends, among other things, on the size of consumer search costs. In particular, for a given advertising intensity, the equilibrium price and the profits of firms are lower under targeted advertising when the search costs are low. On the contrary, if the search costs are high, firms charge higher prices and earn higher profits when ads are targeted. These results are still present when we endogenize advertising intensity. Targeting allows firms to avoid wasting ads, and so advertising intensity is lower with targeting when search costs are low. However, as these costs get larger, and in line with our results on profits —per ad—, advertising intensity, may be higher when firms can target their ads that when they cannot.

We have analyzed imperfect targeting, i.e firms cannot exactly identify the consumers who like their products the most, and consumers do not infer the ranking order of products whose ads they have obtained. We have also generalized this model to allow for endogenous precision of targeting by refining the information that firms have on consumers. This additional information increases the ability of firms to target their ads, particularly when advertising costs are high, so that "poaching" a rival's consumers is more costly, this finer information will certainly adds strength to market segmentation obtained thorough advertising, and is an additional incentive for endogenous monopolization.

Our results suggest that imperfect targeting is welfare improving if search and advertising costs
are low, where the informativeness of advertising increases competition among relevant suppliers. However, consumers may be worse off (and welfare may be lower) with targeted advertising if search and advertising are sufficiently costly.
References


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