1. Introduction

We consider the vertical relationship between upstream manufacturers and retailers in various institutional environments found in practice. One typical motivation for such analysis is given by the market structure and conduct for grocery and household products.

Our model (Section 1 below) pushes forward two key aspects of the vertical relationships between upstream manufacturers and downstream retailers.

1. Moral hazard. First, manufacturers value the effort of retailers to promote their products through advertising campaigns, adequate disposal (the so called “têtes de gondoles”) and the like. Although, those dimensions of the retailing activities can be somewhat described contractually, other dimensions remain non-verifiable. These non-verifiable dimensions of efforts are moral hazard variables that must be induced by manufacturers through a convenient design of the wholesale contracts that run their relationship with their retailers (see Section 4 for a characterization of those incentive constraints). These variables are actions chosen by the retailers but not directly observable by the manufacturers or a third-party. Wholesale contracts have not only an allocative role in such contexts, determining how prices are “passed through” along the supply chain; a distributive role in determining how the overall profit of the vertical structure is shared between manufacturers and retailers; and, lastly, they also have an incentive role; inducing the right choices of actions from the point of view of the vertical structure.

2. Limited Vertical Control. Wholesale contracts are generally constrained and cannot allocate freely surplus between manufacturers and retailers. To illustrate, fixed fees are generally absent or limited in size and, in our context, wholesale contracts can thus rely only on two instruments. First, a wholesale price that specifies the cost for the retailer of supplying the manufacturer’s good. This wholesale price
affect downstream margins and profits. It is through this channel that the exercise of upstream market power if any is passed through to final consumers. Second, a backwards rebates that applies in case the demand has been particularly high, thanks to the retailer’s promotional effort. This aspect of contracting is more specific to the moral hazard context that characterizes the vertical relationship under scrutiny. The incentive role of such backwards rebates is pretty clear; the retailer exerts enough promotional effort to boost demand and enjoy these rebates whenever large demand accrues to the shop.

**Fixed Fees and Collusive Upstream Manufacturers.** One of the main lessons of the moral hazard literature is that, had fixed fees been available, upstream manufacturers if they had all bargaining power, for instance because they collude, manufacturers could reap all of the retailer’s profits by means of such fixed fee while selling their good at cost to their retailers and giving no specific bill-back allowances (also often-called backwards rebates or retrospective payments or back margin). Indeed, by selling at cost, upstream manufacturers can make retailers residual claimant for the maximization of the overall profit of the vertical structure (which is presented in Section 3.1). In particular, retailers charge the monopoly price to final consumers. With the addition of fixed fees, manufacturers could reap all that profit of the vertical structure (see Section 3.2). The drawbacks of such scheme is that the optimal fixed fee may not be covered by the retailer’s profit unless his promotional effort has been successful.

It is important to stress that, whenever the retailer is made residual for the profit of the vertically integrated structure, moral hazard is costless for that structure and effort is provided at its efficient level. Of course, contractual limits, and more specifically the absence of fixed fee in the analysis that follows, makes it impossible to make the retailer residual claimant. Wholesale contracts are now designed not only with an eye on how profits are distributed along the supply chain but also with the goal of inducing effort.

**The Distributive Role of Wholesale Contracts.** Let suppose that fixed fees are not available but that there is no moral hazard; or more specifically, collusive upstream manufacturers can exert promotional effort by themselves (see Section 3.3). This of course describes a hypothetical setting where the sole role of retailers is restricted to sell the manufacturers’ product. Without any fixed fee, charging a wholesale price equal to the the marginal cost of the upstream manufacturer allows retailer to perfectly internalize the pricing decision of the vertical structure. They would then charge a retail price that replicates the vertically integrated solution as far as pricing is concerned. Yet, this solution is certainly not optimal because upstream manufacturers want to capture more of the retailing profit and, to do so, they charge a wholesale price above cost. This increase is passed through final consumers by an increase in retail price. This setting depicts the familiar *double marginalization* stressed by Spengler (1950). Each firm along the supply chain charges a price above his own marginal cost and allocative distortions are piled up along this chain.

What is remarkable is that the overall profit of the vertically integrated structure could also be achieved had upstream manufacturers sold homogenous goods and competed fiercely to serve retailers (Section 5). With such “head-to-head” competition, manufacturers make zero profit, selling at cost and offering no backwards rebate. Yet, retailers are
again residual claimant for the overall profit of the vertical structure with such scheme. In particular, they have the right incentives to exert promotional effort and they still charge the monopoly price. The only difference comes now from the fact that retailers keep all the profit of the vertical structure. In other words, with the use of fixed fees, competition and upstream collusion are similar from an allocative viewpoint; they entail the same price to final consumers. Yet, they strongly differ in terms of the distribution of the vertically integrated profit along the supply chain.

Importantly, because the retail price remains above marginal cost in that competition scenario, a ban on pricing below cost would have no bite. In other words, Resale-Below-Cost Laws (as the Loi Galland in France) can only impact market conduct when upstream manufacturers have more collusive behavior.

Collusion between upstream manufacturers reverses the bargaining power in comparison with the competition scenario. Now those manufacturers want to design their wholesale contracts to extract more of the downstream profit while preserving incentives to exert promotional effort (Section 6).

The important issue is to determine how, in the absence of fixed-fees, upstream manufacturers design wholesale contracts so as to reconcile the objectives of extracting the retailer’s profit and inducing promotional effort. We show that an optimal wholesale contract would induce below-cost pricing at the retail level by imposing a high wholesale price, raising the retailer’s cost. At the same time, this optimal wholesale contract also rewards significantly the retailer for his promotional effort by means of significant backwards rebates. The logic of the argument is simple. Below-cost pricing acts as a bonding device, forcing negative base profits which in turn obliges retailers to exert enough promotional effort to boost demand and cover their loss with the windfall profits that come when this effort has been successful. Upstream manufacturers play on “stick and carrots” to induce effort and reap downstream profits. The “stick” is the threat of only keeping negative base profits when the retail price is below the wholesale price. The “carrots” is the possibility to be rewarded with backwards rebates only when effort has been successful in boosting demand.

Of course, such collusive strategies, because they entail below-cost pricing, are no longer available when so called resale-below-cost laws regulate the market. Those laws have a long history throughout the world with mixed views. Some countries have strongly opposed to such laws while others have been more favorable. Ireland competition authorities, for instance, releases any prohibitions on RBC sales through the so called Groceries Order Act. France has instead been a proponent of such laws over the years. The first such act goes back to 1963. With such act further refined in 1986, retail prices below the effective wholesale price were banned. The so called Loi Galland which was enacted in 1996 defined more clearly the retail price to which the ban applies and, especially does not include rebates that may be paid at later date.1 RBC laws are often promoted explicitly to protect low volume, high price suppliers, notably small retailers. In the sequel, we will leave aside the reasons why such laws are enacted. We will take those laws as given and will analyze how they impact on market conduct, especially with an eye on

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1We refer to Biscourp, Boutin and Vergé (2013) for more details description of those laws and their consequences on retail pricing.
how upstream collusive manufacturers react those laws and how those reactions trigger changes in retail prices and consumption patterns (Section 7).

*Resale-below-cost Laws* (thereafter *RBC* laws) are obstacle to the best collusive strategies of upstream manufacturers. This means that those laws are likely to impose binding constraints on retail prices under upstream collusion. In those circumstances, any shift in wholesale prices is thus completely passed to final consumers.

Yet, collusion under the shadow of such laws has not only an impact on the level of retail prices and efforts but also on the distribution of profits. Under weak specifications of preferences and technologies, a *RBC* law drives retail prices up and effort down. A naive view of *RBC* laws could argue that, when those laws are binding, retail prices are forced to be equal to wholesale prices and thus the double marginalization that was stressed above could be avoided, leading to more allocative efficiency. This is an erroneous view. In fact, imposing such a law can only change market conduct when, absent the law, retail prices would be below wholesale prices. This is precisely the circumstances stressed above to feature a scenario of upstream collusion with no regulation. In other words, and to insist, *RBC* laws have only an impact when upstream collusion is a concern.

Banning below-cost pricing significantly limits the ability of collusive manufacturers to extract downstream profits. An optimal wholesale contract can no longer play on “sticks and carrots” to induce effort, forcing the retailer to recover the loss on base profits with extra promotional effort so as to boost demand. Because base profit can no longer be negative, collusive manufacturers have to increase backwards rebates to induce effort. This means that they also have to concede some (so called in the jargon of theory of incentives) liability rent to retailers. When “sticks” are no longer feasible, only “carrots” remains but this is financially costly for upstream manufacturers.

In a rough sense, the benefits of a *RBC* law is to shift profits away from the collusive manufacturers towards their retailers; although such shift is far from reestablishing the optimistic scenario that prevails when manufacturers compete and leave retailers enjoy the whole profit of the vertical structure. In other words, our analysis offers an estimate of the damages that upstream collusion under the shadow of a *RBC* law imposes on retailers. Remedies for the harm caused by collusion to the retailers should be equal to the profit of the vertically integrated structure that accrues to retailers under perfect competition minus the liability rent they enjoy under a *RBC* law.

Under rather weak conditions on demand and technologies (linear demand and quadratic disutility of effort), we are able to give an explicit formula for these remedies and link them to the value of demand after and before collusion has been banned by an Antitrust Authority. Remedies correspond to a percentage of after-collusion profits that only depends on the ratio of those demands (although in a highly nonlinear way).

A basic tenet of the incentives literature, is that the liability rent that accrues to retailers is costly for manufacturers and that cost increases as manufacturers are willing to implement higher effort downstream. Backwards rebates would have to be significantly increase to provide the retailer with enough incentives. An optimal wholesale contract must now reduce that rent and, to do so, implements lower level of effort downstream.

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2See Laffont and Martimort (2002, Chapter 4).
At the same time, a RBC law limits the ability of upstream manufacturers to reap downstream profits. It thus moves retail prices up towards those obtained in the “double-marginalization” scenario; at least under the weak conditions on demand and technologies.

2. Model

We consider a bilateral relationship between an upstream manufacturer and a downstream retailer. Given a retail price $p$, the demand for the good is denoted by $D(p)$, with $D'(\cdot) < 0$ and $D''(\cdot) \leq 0$.\(^3\) Let $w$ denote the wholesale price paid by the retailer to the manufacturer.

By exerting promotional services, the retailer can boost the demand for the manufacturer’s product. The demand for familiar services and products can be increased by improving promotional services while, for more complex products, retailers can improve customers’ information and reduce search costs. In return for these retailing services, the retailer receives a “backwards rebate” (the so-called “marge arrière”) $z \geq 0$ on every additional unit sold. This margin can be viewed as a rebate offered to the retailer at the end of the accounting period when a particularly high demand for the manufacturer’s product has realized.

We assume that the retailer’s effort $e \in [0, 1]$ to boost demand is non verifiable so that the vertical relationship between the manufacturer and his retailer is plagued by moral hazard. More specifically, with probability $e$, consumer demand becomes $(1 + \theta)D(p)$ where $\theta \geq 0$ is a scale parameter, while with the complementary probability $1 - e$ the demand remains equal to its base value $D(p)$.

Exerting such promotional services is costly to the retailer. We denote by $\psi(e)$ the corresponding disutility of effort. We assume that $\psi(\cdot)$ is increasing convex and satisfies the usual Inada conditions ($\psi'(\cdot) \geq 0$, $\psi''(\cdot) > 0$, $\psi'(0) = 0$ and $\psi'(1) = +\infty$). For technical reasons ensuring quasi-concavity of some of the optimization problems below, we shall also sometimes assume that $\psi'''(\cdot) \geq 0$.

To give explicit (quasi-)expressions of prices and effort, we will sometimes refer to the quadratic case where $\psi(e) = \frac{e^2}{2}$ and $D(p) = 1 - p$. In that scenario, we suppose that parameter values (especially for $\theta$) ensure that optimal effort remain interior.

REMARK: For future references, it is useful to stress that a wholesale contract is a pair $(w, z)$ and that fixed fees are not included as part of those arrangements. We comment below in due place on the role of this important contractual restriction for our analysis. Consistently with RBC laws found in practice, our model distinguishes between wholesale prices and rebates; “backwards rebates’ remain out of the scope of the law.

\(^3\)Our results actually hold more generally provided that the demand is not too convex.
3. Benchmarks

This section develops a few benchmarks which are useful to understand the rest of our analysis. To make the analysis as simple as possible, we assume that there is a single manufacturer-retailer pair taken in isolation. Later, we will consider more complex scenarios.

3.1. The Vertically Integrated Structure

Suppose that the upstream manufacturer vertically integrates with the downstream retailer. As a by-product of this integration, the manufacturer is able to dictate the choice of promotional effort that is exerted by the retailer. The integrated outcome, which maximizes the industry profit, is obtained as a solution to the following problem

\[(P_i) : \max_{(p,e)} (p - c)D(p)(1 + \theta e) - \psi(e),\]

where \(c \geq 0\) is the manufacturer’s marginal cost.

The solution to this problem is readily obtained as

\[(3.1)\]

\[p_m^m - c = - \frac{D(p_m^m)}{D'(p_m^m)}\]

and

\[(3.2)\]

\[\psi'(e_m^m) = \theta \pi_m^m\]

where the monopoly profit satisfies

\[(3.3)\]

\[\pi_m^m = (p_m^m - c)D(p_m^m).\]

Since the promotional effort boosts demand multiplicatively, the monopoly price always maximizes profit whether the demand has been scaled up or not. The optimal effort trades off a marginal benefit coming from enjoying some extra monopoly profit \(\theta \pi_m^m\) beyond the “business as usual” level against the retailer’s marginal disutility of effort.

3.2. Two-Part Tariff and Residual Claimancy of the Retailer

The above solution is also achieved when the manufacturer and the retailer are two different units but the manufacturer can rely on a tow-part tariff to regulate his relationship with the retailer. To see how, suppose that the manufacturer charges a wholesale price equal to its marginal cost, that is, i.e., \(w = c\). Then, the retailer becomes the residual claimant for the choice of the retail price and the promotional effort. If the backwards rebate is \(z = 0\), the retailer would maximize

\[(P_m) : \max_{(p,e)} (p - c)D(p)(1 + \theta e) - \psi(e) - F\]
where $F$ is a fixed fee that can now be used by the manufacturer to extract all the retailer’s profit.

The solution is again given by the monopoly outcome $(p^m, e^m)$. The optimal fee is worth

$$F^m = (1 + \theta e^m)\pi^m - \psi(e^m)$$

It is important to stress that whenever the retailer fails in increasing demand and only reaps $\pi^m$ downstream, he incurs a loss which is worth:

$$\pi^m - F^m = -R(e^m) < 0.$$  

The quantity $R(e) = e\psi'(e) - \psi(e)$ is the non-negative retailer’s liability rent; that is, the retailer’s amount of rent that must be given up by the manufacturer to induce a downstream effort $e$ in a moral hazard setting.\(^4\) From our assumptions on $\psi(\cdot)$, we immediately get $R(e) \geq 0$ with $R'(e) \geq 0$.

### 3.3. Double Marginalization Along the Supply Chain

Another benchmark that proves useful later on is obtained when the promotional effort is undertaken by the manufacturer himself. In those circumstances, moral hazard is no longer a concern. Consider, besides, that the manufacturer cannot use any fixed fee.

Since there is no moral hazard, there is no need to provide the retailer with an extra compensation when demand turns out to be high, i.e., $z = 0$. Given an effort $e$ made by the manufacturer, the retailer’s problem writes now as

$$\max_p (p - w)D(p)(1 + \theta e).$$

This yields the following first-order condition characterizing the retailer’s best response

$$p - w = -\frac{D(p)}{D'(p)}.$$  

Given the wholesale price set by the manufacturer, the retailer exerts its monopoly position over final customers to set the corresponding monopoly price.\(^5\)

For future reference, it is useful to introduce the following non-negative function

$$\varphi(p) = -\frac{D^2(p)}{D'(p)}.$$  

\(^4\)See Laffont and Martimort (2002, Chapter 4).

\(^5\)To illustrate, observe that, had demand elasticity $\varepsilon = -\frac{pD'(p)}{D(p)}$ been constant, any marginal change $\Delta w$ in the value of the wholesale price is then passed through to final consumers by means of a change in retail price $\Delta p$ which is worth:

$$\Delta p = \frac{\Delta w}{1 - \varepsilon}.$$  

As demand becomes more elastic more of the change in cost is passed through to final consumers.
This function is the retailer’s downstream profit when the upstream manufacturer chooses a wholesale price \( w \) that implements the price \( p \) through the passed-through condition \((3.5)\). Observe that \( \varphi'(p) < 0 \) under the assumptions made on the demand function. In other words, as the retail price to be implemented increases, the manufacturer also increases the wholesale price but the profit of the retailer falls.

The manufacturer now has to decide of a wholesale price and a promotional effort that maximize its profit, anticipating the price-setting rule of the retailer, that is,

\[
\max_{w,e} (w - c)D(p)(1 + \theta e) - \psi(e) \text{ subject to } (3.5).
\]

Through the passed-through condition \((3.5)\), the choice of the wholesale price implements a retail price \( p \). Using this one-to-one mapping between wholesale and retail prices, a perhaps more illuminating way to write the above maximization problem is as follows:

\[
\max_{p,e} ((p - c)D(p) - \varphi(p))(1 + \theta e) - \psi(e).
\]

This maximization illustrates the standard double marginalization problem when each firm along the supply chain applies its own markup. Indeed, the first-order condition with respect to the retail price leads to

\[
(\tilde{p} - c)D'(\tilde{p}) + D(\tilde{p}) - \varphi'(\tilde{p}) = 0.
\]

Equation \((3.6)\) shows that the optimal retail price \( \tilde{p} \) ends up being above the monopoly price \( p^m \). This extra distortion reflects the familiar “double marginalization” that was already stressed in the seminal work of Spengler (1950).

The optimal effort satisfies the same rule as before, up to the fact that the base profit is now expressed at price \( \tilde{p} \). Let \( \tilde{\pi} = (\tilde{p} - c)D(\tilde{p}) - \varphi(\tilde{p}) \) denote the manufacturer’s profit in that scenario. The optimal promotional effort satisfies:

\[
(3.7) \quad \psi'(\tilde{e}) = \theta \tilde{\pi}.
\]

Obviously, effort is downward distorted in that scenario; i.e., \( \tilde{e} < e^m \). The double marginalization dissipates profit along the supply chain and thus reduces the benefit of promotional services.

We further impose a condition meaning that the retail margin in that double-marginalization scenario is not enough to cover the maximal feasible effort.

Assumption 1.

\[
(1 + \theta)\varphi(\tilde{p}) < \psi(1).
\]

4. Downstream Moral Hazard

This section analyzes the set of incentive constraints that characterize the vertical relationship between manufacturers and retailers. We still assume that fixed fees are not
available to extract the retailer’s downstream profit. Being offered a wholesale contract 
\((w, z)\), the retailer optimally chooses the retail price and the promotional effort, i.e., the 
pair \((p, e)\), so as to solve the following problem:

\[
\max_{(p,e)} (p - w) D(p) (1 + \theta e) + \theta e z D(p) - \psi'(e).
\]

The pair \((p, e)\) are viewed as moral hazard variables whose choice is induced by the 
wholesale contract \((w, z)\). Provided that the relationship between those variables is one-
to-one, optimizing over wholesale contracts is akin to optimizing over the retailer’s choice 
variables.

Turning now to the optimal choice of retail price and effort induced by the manufac-
turer, and assuming concavity of the above maximand, we derive the following first-order 
conditions with respect to \(p\) and \(e\) respectively as:

\[
\begin{align*}
(4.1) & \quad \frac{1}{1 + \theta e} \frac{p - w}{p} + \frac{\theta e}{1 + \theta e} \frac{p - w + z}{p} = -\frac{D(p)}{p D'(p)}, \\
(4.2) & \quad \theta (p - w + z) D(p) = \psi'(e).
\end{align*}
\]

These conditions have an intuitive meaning. Equation (4.1), shows that the inverse 
elasticity of demand should be equal to an average of the retail price cost-margins with 
and without backwards rebates. If the promotional effort is significant \((e \text{ large})\), this 
average is very close to the margin taking into account the backwards rebate \(z\) only. 
When effort is less significant, this backward rebate plays much less role.

The second condition, Equation (4.2) tells us that the retailer’s effort comes from 
equating the marginally disutility of effort with the downstream profit that accrues to 
the retailer in case demand has been boosted. This condition showcases the role of the 
backwards rebates in boosting effort.

Instead of viewing the choice of \((p,e)\) in terms of the contract \((w, z)\), we may as well 
“invert” the system of Equations (4.1) and (4.2) and view the contracting variables \((w, z)\) 
in terms of the pair \((p, e)\) that the upstream manufacturer may want to implement. So 
doing yields

\[
\begin{align*}
(4.3) & \quad p - w = -\frac{D(p)}{D'(p)} (1 + \theta e) - \frac{e \psi'(e)}{D(p)}, \\
(4.4) & \quad z = (1 + \theta e) \frac{\psi'(e)}{\theta D(p)} + (1 + \theta e) \frac{D(p)}{D'(p)},
\end{align*}
\]

The first of these conditions (4.3) is particularly important. Later, it will allows us to 
find out restrictions on implementable pairs \((p, e)\) that ensure a positive margin \(p - w\).

Taking stock of (4.3) and (4.4), we may express, still in terms of the pair \((p, e)\) to be
implemented, the manufacturer’s and the retailer’s profit respectively as

\[(4.5) \quad V(p, e) = (p - c)D(p)(1 + \theta e) - \psi(e) - U(p, e),\]

and

\[(4.6) \quad U(p, e) = (1 + \theta e)\varphi(p) - \psi(e).\]

These expressions showcase that the manufacturer appropriates the whole profit of the vertical structure minus what is left to the retailer. In turn, and as we discussed earlier on, the retailer’s downstream profit net of the disutility of effort \(4.6\) must remain non-negative for any implementable pair \((p, e)\).

Observe that the retailer has always the option of choosing \(p = w\) and \(e = 0\); an option which ensures at least zero net profit. Any acceptable wholesale contract \((w, z)\) must ensure that the retailer at least breaks even. Hence, \(U(p, e)\) must remain non-negative under all circumstances below.

5. “Head-to-Head” Competition between Upstream Manufacturers

We consider now the case where upstream manufacturers compete “head-to-head” for the retailer’s services. More precisely, manufacturers compete à la Bertrand in offering (exclusive) contracts \((w, z)\). Those contracts maximize the retailer’s net profit subject to the moral hazard incentive constraints \((4.3)\) and \((4.4)\) that determine the retail price and the promotional effort in terms of \((w, z)\). If contracts were not maximizing the retailer’s net profit, a manufacturer could make a better deal with the retailer and get a positive profit. Moral hazard incentive constraints follow again from the manufacturers’ lack of control of retail prices and promotional effort. Competition is thus akin to shifting all bargaining power towards the retailer. Competition between upstream manufacturers thus drives their profit to zero:

\[(5.1) \quad ((w - c)(1 + \theta e) - \theta ez)D(p) = 0.\]

Formally, an equilibrium contract must now solve:

\[\min_{(w, z, p, e)} \max_{(p)} (p - w)D(p)(1 + \theta e) + \theta ezD(p) - \psi(e)\]

subject to \((4.3), (4.4),\) and \((5.1)\).

It is straightforward to see that \((w^* = c, z^* = 0)\) is the solution to the above problem.

**Proposition 1.** “Head-to-head” competition between upstream manufacturers implements the vertically integrated outcome:

- \(p^* = p^m\) and \(e^* = e^m\).
• The retailer captures all profits of the vertical structure.

A wholesale contract that sells at marginal cost, \( w^* = c \), and entails no rebate, \( z^* = 0 \), makes the retailer a residual claimant for the maximization of the overall profit of the vertical structure. There is no need to use backwards rebates to induce effort, the incremental monopoly profit obtained by the retailer from boosting demand are enough to do so.

Observe that there are actually other equilibrium pairs of wholesale price and rebate. Indeed, any pair \((w, z)\) such that (5.1) is binding provides the retailer with the incentives to perform effort \( e^m \) and to charge final consumers a price \( p^m \). An implication is thus that producers which may be able to command a positive wholesale margin \((w > c)\) have to concede in turn sufficiently high a retrospective payment \( z > 0 \) so that their break-even constraint binds.

Importantly, this outcome is such that
\[
 p^m = p^* > w^* = c.
\]

In other words, the retailer imposes the monopoly margin \( p^m - c \) on consumers; the retail price is always above cost. Imposing a ban on resale below cost would have no bite in this environment. RBC laws have only an impact when upstream manufacturers collude and collectively behave as a monopolist as we will see next.

**Proposition 2.** Resale-below-cost laws have no bite when upstream manufacturers compete “head-to-head” for the retailer’s services.

**Remark.** The model of competition that we considered above, namely “head-to-head” competition in contracts, may appear somewhat extreme. A bit of product differentiation between the manufacturers’ goods would certainly change the analysis of the model but we are very confident that its main thrust would carry over. At equilibrium, retail prices would remain above marginal cost provided that goods are not too differentiated and RBC laws would again have no bite when manufacturers compete. Section 8 below analyzes how product differentiation impacts on collusive strategies and pricing practices.

### 6. Upstream Collusion in Unregulated Environments

We now suppose that there are several upstream manufacturers which are all identical, i.e., they all have the same marginal cost and sell the same homogenous good with demand \( D(p) \). Those manufacturers can now collude to exert market power and provide cheaper incentives to their retailer. Collusion between those manufacturers is akin to having a single upstream monopolistic manufacturer keeping all bargaining power when negotiating with the retailer. Then, we may assume without loss of generality that those collusive profits are redistributed equally among the collusive manufacturers.\(^6\)

\(^6\) Assuming perfect collusion certainly yields an upper bound on the benefits of collusion and, more implicit collusion may certainly attenuate the impact of the manufacturers’ market power without changing the direction of our results.
The optimal pair \((p^c, e^c)\) that would be induced by the collusive manufacturers must solve:

\[
(P^c) : \max_{(p,e)} V(p,e) \text{ subject to }
\]

\[
(6.1) \quad U(p,e) = (1 + \theta e)\varphi(p) - \psi(e) \geq 0.
\]

**Proposition 3.** Suppose that Assumption 1 holds and that upstream manufacturers collude.

- The optimal retail price and promotional effort are respectively given by

\[
(6.2) \quad (p^c - c)D'(p^c) + D(p^c) - (1 - \lambda^c)\varphi'(p^c) = 0,
\]

and

\[
(6.3) \quad \theta [(p^c - c)D(p^c) - (1 - \lambda^c)\varphi(p^c)] = \lambda^c \psi'(e^c)
\]

where \(\lambda^c\) is the non-negative Lagrange multiplier of the retailer’s break-even condition (6.1)

- The retailer makes zero profit, i.e., \(\lambda^c > 0\).

Remind that Assumption 1 implies that the retail margin in the “double-marginalization” scenario does not suffice to cover the cost of the maximal feasible effort, i.e., \(e = 1\). Then, the manufacturer must reduce the retailer’s effort to ensure that the retail profit from sales (including instances where backwards rebates are offered and instances where they are not) covers its disutility. Importantly, the retailer’s break-even constraint (6.1) still holds as an equality at the optimal contract even if fixed fees are not available to extract downstream profit. The retailer is indifferent between selling at \(p = w\) while exerting no effort and choosing the pair \((p^c, e^c)\) that maximizes the collusive manufacturers profit. The optimal contract moves along the retailer’s break-even condition towards a point that maximizes the profit of the collusive manufacturers.

Even if fixed fees are not available to extract the retailer’s profit, collusive manufacturers still can use the wholesale price to do so while keeping the retrospective rebate to induce the promotional effort. More precisely, using Equation (4.3) and the binding participation constraint (6.1), we obtain the following expression of the retailer’s base profit

\[
(6.4) \quad (p^c - w^c)D(p^c) = -R(e^c) < 0.
\]

This expression echoes our earlier formula (3.4) found in scenarios where fixed fees are available. The optimal wholesale contract is such that the base profit compensates for the non-negative retailer’s liability rent that must be given up by the manufacturer to induce effort \(e\) in a moral hazard setting. In other words, the condition \(e^c > 0\) implies below-cost pricing:

\[
p^c < w^c.
\]

With such optimal wholesale contract, the retailer cannot recover the loss on the base profit, \((p^c - w^c)D(p^c) < 0\) if he exerts less than the requested effort \(e^c\). A negative price-cost margin acts as an incentive device to exert enough promotional effort so as to
recover those losses with the profit made when demand is high thanks to the retailer’s effort.

We now turn to further comparative statics.

**Proposition 4.** With collusive upstream manufacturers, the retail price is shifted above the optimal price for the vertically integrated structure but remains below the “double-marginalization” price:

\[ c < p^m < p^c < \tilde{p}. \]

This proposition is remarkable. With two-part tariffs, collusive manufacturers could make their retailer a residual claimant for the choice of effort and retail price and thereby achieve the profit \( \pi^m \) of the vertically integrated structure. The monopoly price would then be charged to final consumers. Without the possibility of using a fixed fee, collusive manufacturers must still incentivize the retailer for his promotional effort and extract his profit. This extraction role is best accomplished by using the base profit \((p - w)D(p)\) which plays the role of a fixed fee. Upstream managers choose a negative value for this base profit, i.e., implement a downstream retail price below the wholesale price. Then, retailers can only break even if they are rewarded with backwards rebates following a boost in demand.

Of course, the base profit is a poor redistributive instrument to shift profit from retailers to manufacturers. Using that instrument entails some allocative distortions and the retail price is shifted towards the “double-marginalization” price \( \tilde{p} \). This upward shift of the retail price is intuitive. Indeed, Equation (6.4) shows that the base profit must compensate for the liability rent that accrues to the retailers under moral hazard. This moral hazard constraint is akin to a limit in how surplus can be freely allocated between parties. It is thus not surprising that the retail price comes closer to what was found in the benchmark setting of Section 3.2 where there was no way to allocate surplus except through the wholesale price.

**Remark.** It is important to understand that, in our setting, the retail price is set below cost in order to solve a moral hazard problem and discipline the retailer. This incentive argument does not rely on other often found reasons for below-cost pricing, like predation, vertical opportunism (Allain and Chambolle, 2011), multiproduct pricing (Bliss, 1988), or screening of different kinds of consumers according to their shopping costs (Chen and Rey, 2012 and 2016).

### 7. Upstream Collusion Under the Shadow of Resale-Below-Cost Laws

In this section, we assume that, by regulation, retail prices cannot be set below cost. The following non-negativity constraint must thus always hold:

\[ p - w \geq 0. \]  

(7.1)

We then investigate the consequence of such a constraint on retail prices and profits.
**The Role of the Retailer’s Liability Rent.** Because the retail price margin satisfies the moral hazard incentive constraint (4.3), the non-negativity constraint (7.1) amounts in fact to imposing:

\[(7.2)\quad U(p, e) = (1 + \theta e)\varphi(p) - \psi(e) \geq R(e).\]

In other words, imposing a ban on below-cost pricing is akin to imposing that the base profit \((p - w)D(p)\) that accrues to the retailers can no longer be used to extract the retailer’s overall profit. The retailer can no longer run a loss if he fails in boosting demand through his promotional effort. The only channel to reward effort is through setting a backwards margin which is large enough. The contractual constraint imposed by regulation refers to an argument which is by now familiar from the moral hazard literature. When payments to the retailer cannot be negative, collusive upstream manufacturers must give up a positive liability rent \(R(e)\) if they want to induce a positive level of effort \(e\) from the retailer. This quantity is found on the right-hand side (7.2), and it makes this constraint clearly harder to satisfy than the break-even condition (6.1) that prevails absent regulation.

Because \(\varphi(p)\) is non-increasing in \(p\), constraint (7.2) is relaxed by choosing a retail price below its value absent regulation. At the same time, constraint (7.2) is also relaxed by reducing effort so as to reduce the retailer’s liability rent. Imposing a ban on below-cost pricing is thus akin to replace the cost of effort \(\psi(e)\) by a more costly “virtual” disutility of effort \(e\psi'(e) = \psi(e) + R(e)\). Reducing effort becomes thus an attractive channel for relaxing constraint (7.2). Under regulation, collusive manufacturers tend to reduce effort more than absent such regulation.

Formally, we may now write the manufacturer’s problem as follows:

\[
(P^g) : \max_{(p,e)} (p - c)D(p)(1 + \theta e) - \psi(e) - U(p, e)
\]

subject to (7.2).

For the time being, we will content with stating the optimal policy under a resale-below-cost law. We can now establish the following results.

**Proposition 5.** Suppose that Assumption 1 holds, that upstream manufacturers collude but that there is a ban on below-cost pricing.

- **The optimal retail price and promotional effort are respectively given by:**

\[(7.3)\quad (p^g - c)D'(p^g) + D(p^g) - (1 - \lambda^g)\varphi'(p^g) = 0,
\]

and

\[(7.4)\quad \theta \left[ (p^g - c)D(p^g) - (1 - \lambda^g)\varphi(p^g) \right] = \lambda^g [\psi'(e^g) + e^g\psi''(e^g)]
\]

where \(\lambda^g\) the non-negative Lagrange multiplier for constraint (7.2).

- **The retail price is equal to the wholesale price:**

\[p^g = w.\]

---

7See Laffont and Martimort (2002, chapter 4).
• The retailer’s profit is always non-negative, i.e., \( \lambda^g > 0 \):

\[
U(p^g, e^g) = R(e^g) > 0.
\]

The optimality conditions (7.3) and (7.4) are pretty similar to (6.2) and (6.3) found in unregulated environments. The only change comes from the fact that with a ban on below-cost pricing, everything happens as if the disutility of effort \( \psi(e) \) was now replaced by a “virtual” disutility of effort \( e^g \psi'(e^g) = \psi(e) + R(e) \) that accounts for the extra limited liability rent of effort left to the retailer when the retail price cannot be set below the wholesale price. In particular, it follows from (7.4) that:

\[
(7.5) \quad \theta \left[ (p^g - c)D(p^g) - (1 - \lambda^g)\varphi(p^g) \right] > \lambda^g \psi'(e^g).
\]

Had the Lagrange multipliers been unchanged when moving to a regulated environment, i.e., \( \lambda^g = \lambda^c \), the previous inequality shows that effort is distorted downward. At the same time, the retail price should be reduced towards the monopoly level to increase the right-hand side of (7.5). This points at the direct effects of a ban on below-cost pricing. Of course, indirect effects may also come from a change in the values of the Lagrange multipliers \( \lambda^c \) and \( \lambda^g \) as one moves from one institutional environment to the other. A priori, replacing the break-even condition (6.1) by the more stringent condition (7.2) calls for increasing the value of the Lagrange multiplier, and we may expect \( \lambda^c \leq \lambda^g \). Those indirect effects may further modify equilibrium prices and efforts.

Further Exploring the Impact of a Ban on Below-Cost Pricing. The comparison between, on the one hand, \( (p^c, e^c) \), and, on the other hand, \( (p^g, e^g) \) is thus made difficult because of the endogeneity of the Lagrange multipliers \( \lambda^c \) and \( \lambda^g \). These comparative statics can nevertheless be further explored in some specific environments. To this end, consider the following linear-quadratic example:

\[
(7.6) \quad D(p) = 1 - p \quad \text{and} \quad R(e) = \psi(e) = \frac{e^2}{2}.
\]

Next proposition demonstrates that the direct effects stressed above dominate the indirect ones.

Proposition 6. Suppose that demand is linear and the disutility of effort is quadratic. Imposing a ban on below-cost pricing

• decreases effort

\[ e^c > e^g; \]

• increases the retail price

\[ p^c < p^g; \]

• relaxes the shadow cost of the feasibility conditions

\[ 0 < \lambda^g < \lambda^c < 1. \]

Given that a linear demand and a quadratic disutility of effort may be viewed as first-order approximations of more complex specifications of preferences and technologies,
Proposition 6 suggests that there might be a strong presumption that the retail price increases following a ban of below-cost sales. Far from promoting competition in the hypothetical scenario where below-cost pricing would be used for predatory purposes, such a law may harm consumers and reduce overall welfare.

**Evaluating Remedies.** Our analysis gives a clear idea of what should be the remedies paid to the retailer once upstream collusion has been denounced by an Antitrust Authority. Indeed, absent collusion, the retailer appropriates the whole profit of the vertically integrated structure while, with collusion under a *RBC* law, it only appropriates the corresponding liability rent.

Formally, we obtain the following expression of the remedies $\Delta$ to be paid:

$$\Delta \equiv (1 + \theta e^m)\pi^m - \psi(e^m) - R(e^g)$$

Retailer’s profit with upstream competition Liability rent under collusion and *RBC* law

Using (3.2), this expression can be rewritten as:

$$\Delta \equiv \frac{\pi^m}{\text{Monopoly profit}} + \frac{R(e^m) - R(e^g)}{\text{Difference in liability rents with and without collusion}}.$$

Remedies are thus the sum of two terms. The first term is the monopoly level of the based profit that accrues to the retailer if, with competition, he was not undertaking any effort. The second term is the incremental value of his liability rent as collusion is banned and effort is increased from $e^g$ to $e^m$.

To give a maybe more practical way of computing these remedies, we again consider the linear-quadratic specification (7.6). There, we can really compute:

$$\pi^m = \frac{(1 - c)^2}{4}, \quad D(p^m) = \frac{1 - c}{2}, \quad e^m = \frac{\theta(1 - c)^2}{4}.$$

The market outcome under collusion and a *RBC* law is more complex to derive. Details can be found within the proof of Proposition 6 in the Appendix but, for the sake of the comparison, we obtain:

$$p^g = c + \frac{(3 - 2\lambda^g)(1 - c)}{2(2 - \lambda^g)}, \quad D(p^g) = \frac{1 - c}{2(2 - \lambda^g)}, \quad e^g = \frac{\theta(1 - c)^2}{8(2 - \lambda^g)\lambda^g}$$

where $\lambda^g$ solves

$$1 = \frac{\theta^2(1 - c)^2}{16} \left[ \frac{2 - 3\lambda^g}{(\lambda^g)^2(2 - \lambda^g)} \right].$$

What is striking with this linear-quadratic specification is that remedies can be expressed only in terms of values of the demand function before and after collusion has been denounced, namely $D(p^g)$ and $D(p^m)$. Indeed, tedious computations show that the percentage of after-collision profits that should be paid in terms of remedies is worth:

$$\frac{\Delta}{\pi^m} \equiv 1 + \frac{(\lambda^g)^2(2 - \lambda^g)}{2 - 3\lambda^g} \left( 1 - \frac{1}{4(2 - \lambda^g)^2(\lambda^g)^2} \right).$$
8. Differentiated Products

We now suppose that there are two potential upstream manufacturers who sell differentiated goods. We denote by \( D_i(p_i, p_{-i}) \) the demand for good \( i \) \((i = 1, 2)\) when the retail price is \( p_i \) and the price of the substitute if \( p_{-i} \). To simplify the analysis, we also assume symmetry across products, that is, goods are produced at the same marginal cost \( c \) and demands are symmetric:

\[
D_1(p_1, p_2) = D_2(p_2, p_1).
\]

Accordingly, we will sometimes omit indices to denote \( D_i(p_i, p_{-i}) = D(p_i, p_{-i}) \). Goods are substitutes, i.e., for all nonnegative price vectors \((p_1, p_2)\), we have:

\[
0 > -\frac{\partial D}{\partial p_2}(p_1, p_2) > \frac{\partial D}{\partial p_1}(p_1, p_2).
\]

That is, the price effect of a marginal change of price \( p_i \) on own demand \( D_i(\cdot) \) dominates the cross-effect on demand \( D_{-i}(\cdot) \) for the good sold by the other manufacturer.

In this multiproduct context, we also assume that the successes in boosting demand for either good are independent and that the retailer’s overall disutility from exerting an effort pair \((e_1, e_2)\) is additive and writes as \( \psi(e_1) + \psi(e_2) \).

**Upstream Collusion.** Suppose that the two manufacturers collude when designing the wholesale prices \((w_1, z_1)\) and \((w_2, z_2)\) they offer to their common retailer. Given a multidimensional wholesale contract \((w, z)\), the retailer chooses a vector of retail prices \( p = (p_1, p_2) \) and a vector of promotional efforts \( e = (e_1, e_2) \) that now solve:

\[
\max_{(p, e)} \sum_{i=1}^{2} \left[ (p_i - w_i) D_i(p_i, p_{-i}) + \theta e_i (p_i - w_i + z_i) D_i(p_i, p_{-i}) - \psi(e_i) \right].
\]

Assuming concavity of the objectives, the first-order conditions for this problem describes the retailer’s incentive compatibility constraints. Those constraints can be written as, for \( i = 1, 2 \),

\begin{align*}
(8.1) \quad & (1 + \theta e_i) \left[ (p_i - w_i) \frac{\partial D_i}{\partial p_i}(p_i, p_{-i}) + D_i(p_i, p_{-i}) \right] + \theta e_i z_i \frac{\partial D_i}{\partial p_i}(p_i, p_{-i}) \\
& + (1 + \theta e_{-i})(p_{-i} - w_{-i}) \frac{\partial D_{-i}}{\partial p_i}(p_{-i}, p_i) + \theta e_{-i} z_{-i} \frac{\partial D_{-i}}{\partial p_i}(p_{-i}, p_i) = 0,
\end{align*}

and

\begin{align*}
(8.2) \quad & \theta (p_i - w_i + z_i) D_i(p_i, p_{-i}) = \psi'(e_i).
\end{align*}

We are looking for a symmetric wholesale contract \((w, z)\) that implements a symmetric
pair \((p, e)\) such that
\[
\frac{1}{1 + \theta e} (p - w) + \frac{\theta e}{1 + \theta e} (p - w + z) = -\frac{D(p, p)}{\partial D/\partial p_1(p, p) + \partial D/\partial p_2(p, p)},
\]
(8.3)
\[
\theta(p - w + z) D(p, p) = \psi'(e).
\]
(8.4)

From now on, the analysis borrows much from what was done in Section 6 and most details will be omitted. All previous results carry over once one has noticed that \(D(p, p)\) stands now for the single-product demand \(D(p)\) that was used in the single-product scenario. With this observation being made, we are led to define the retailer’s overall profit from sales as
\[
\tilde{\varphi}(p) = -\frac{D^2(p, p)}{\partial D/\partial p_1(p, p) + \partial D/\partial p_2(p, p)} > 0.
\]

Mimicking our earlier findings in the case of a single-product monopolist, we shall now also assume that \(\tilde{\varphi}'(p) \leq 0\). Assumption 1 is now replaced by

**Assumption 2.**
\[
(1 + \theta) \tilde{\varphi}(\tilde{p}) < \psi(1)
\]
where
\[
(\tilde{p} - c) \left( \frac{\partial D}{\partial p_1}(\tilde{p}, \tilde{p}) + \frac{\partial D}{\partial p_2}(\tilde{p}, \tilde{p}) \right) + D(\tilde{p}, \tilde{p}) - \tilde{\varphi}'(\tilde{p}) = 0.
\]

When collusion takes place in an unregulated environment, the retailer’s break-even constraint in a symmetric setting where efforts and retail prices are identical on each product line can be written as:
\[
U(p, e) = 2 [\tilde{\varphi}(p)(1 + \theta e) - \psi(e)] \geq 0.
\]
(8.5)

We can immediately get:

**Proposition 7.** Suppose that Assumption 2 holds and that upstream manufacturers collude.

- The optimal symmetric retail price and promotional effort are respectively given by
\[
(p^e - c) \left( \frac{\partial D}{\partial p_1}(p^e, p^e) + \frac{\partial D}{\partial p_2}(p^e, p^e) \right) + D(p^e, p^e) - (1 - \lambda^e) \tilde{\varphi}'(p^e) = 0.
\]
(8.6)

and
\[
\theta [(p^e - c) D(p^e, p^e) - (1 - \lambda^e) \tilde{\varphi}(p^e)] = \lambda^e \psi'(e^e).
\]
(8.7)

where \(\lambda^e\) is the non-negative Lagrange multiplier of the retailer’s break-even condition (8.5).

- The retailer makes zero profit, i.e., \(\lambda^e > 0\).

These findings are pretty similar with those found in the case where manufacturers sell an homogenous good. The novel insight of Proposition 7 is that collusive manufacturers
have to jointly raise retail prices for both goods to avoid that the demand for one be substituted away by demand for the other had the price of only one good been raised.

When collusion instead takes place under the shadow of RBC laws, the condition on non-negative margins, i.e., \( p \geq w \), in a symmetric setting where efforts and retail prices are identical on each product line becomes:

\[
U(p, e) = 2[(1 + \theta e)\tilde{\phi}(p) - \psi(e)] \geq 2R(e). \tag{8.8}
\]

**PROPOSITION 8.** Suppose that Assumption 2 holds, that upstream manufacturers collude but that there is a ban on below-cost pricing.

- The optimal symmetric retail price and promotional effort are respectively given by

\[
(p^g - c) \left( \frac{\partial D}{\partial p_1}(p^g, p^g) + \frac{\partial D}{\partial p_2}(p^g, p^g) \right) + D(p^g, p^g) - (1 - \lambda^g)\tilde{\phi}'(p^g) = 0. \tag{8.9}
\]

and

\[
\theta [(p^g - c)D(p^g, p^g) - (1 - \lambda^g)\tilde{\phi}(p^g)] = \lambda^g(\psi'(e^g) + e^g\psi''(e^g)). \tag{8.10}
\]

where \( \lambda^g \) is the non-negative Lagrange multiplier for the non-negativity constraint (8.8).

- The retail price is equal to the wholesale price, i.e., \( \lambda^g > 0 \):

\[
p^g = w. \]

- The retailer’s profit is always non-negative:

\[
U(p^g, e^g) = 2R(e^g) > 0.
\]

**LINEAR-QUADRATIC SPECIFICATION WITH MULTIPLE PRODUCTS.** To illustrate, consider the multiproduct version of our previous linear-quadratic example, with

\[
D(p_1, p_2) = \frac{2 + \gamma}{2(1 + \gamma)} \left( 1 - p_1 - \frac{\gamma}{2}(p_1 - p_2) \right) \tag{8.11}
\]

where \( \gamma \in [0, +\infty) \) relates to the degree of differentiation between downstream products. Notice that \( \gamma = 0 \) when goods are not related and \( \gamma = +\infty \) when they are substitutes.\(^8\)

Repeating the analysis performed in the case of a single product, we may obtain expressions for the multipliers \( \lambda^c \) and \( \lambda^g \) respectively associated to the constraints (8.5)

\(^8\)This demand function is chosen to be consistent with the demand chosen in the case of a monoproduct retailer. More specifically, we use Shubik and Levitan (1980)’s linear demand system where demands can be consistently derived from the optimization problem of a representative consumer with (gross) utility given by \( q_0 + \sum_{i=1,2} q_i - \frac{\alpha}{2} \left( \sum_{i=1,2} q_i \right)^2 - \frac{\alpha}{1+\gamma} \left( \sum_{i=1,2} q_i^2 - \frac{1}{2} \left( \sum_{i=1,2} q_i \right)^2 \right) \), where \( q_0 \) is the numéraire and \( \alpha = \frac{1}{1+\gamma} \) is chosen so that the demand for one product when this is the only available product is \( D(p) = 1 - p \). This allows to perform a meaningful comparison between the case where one product only is available and the case where several products are available.
and (8.8) that prevail in the absence and in the presence of a RBC law respectively. These expressions allow us to obtain the following comparative statices (where we make explicit the dependence of equilibrium values on the differentiation parameter \( \gamma \))

**PROPOSITION 9.** Suppose that there is collusion between upstream manufacturers selling differentiated product (with collusion being regulated or not by a RBC law).

- \( p^c(\gamma) \) and \( p^g(\gamma) \) are both increasing with \( p^c(\gamma) < p^g(\gamma) \).
- \( \lambda^c(\gamma) \) and \( \lambda^g(\gamma) \) are both decreasing with \( \lambda^c(\gamma) > \lambda^g(\gamma) \).

As collusive manufacturers sell goods which are closer substitutes, leaving to their common retailer enough profits from sales to satisfy the feasibility conditions (either the zero-profit condition (8.5) or the positive-margin condition (8.8)) becomes more difficult. As in the single-product scenario, there are again two consequences of hardening those feasibility conditions. The first *direct effect* is that profits from sales should be increased while effort must be reduced. Increasing the profits from sales certainly means that backwards rebates should be raised, and this especially true that below-cost pricing is banned. On the other hand, the shadow costs of these feasibility conditions should increase when those constraints are hardened by diminishing differentiation. Yet, as in the single-product scenario, the linear-quadratic specification is such that this *indirect effect* is dominated. The equilibrium values of the multipliers \( \lambda^c(\gamma) \) and \( \lambda^g(\gamma) \) diminishes as goods are less differentiated. This requires higher upward price distortions and retail prices come closer to those observed with double marginalization.

### 8.1. Collusive Manufacturers under Resale-Below-Cost Laws

The logic here replicates that seen in the case of a single manufacturer. Again, extracting the retailer’s profit requires to set retail prices below the wholesale price. Extending our previous findings, we get

\[
 p^c - w^c = -\frac{R^c(\epsilon^c)}{D(p^c, p^c)} < 0.
\]

Still relying on the symmetry of our setup, we may derive the optimal (symmetric) retail price and effort when a RBC law applies.

**PROPOSITION 10.** Suppose upstream manufacturers collude. The optimal symmetric retail price and effort \((p^g, e^g)\) solve

\[
 (p^g - c) \left( \frac{\partial D}{\partial p_1}(p^g, p^g) + \frac{\partial D}{\partial p_2}(p^g, p^g) \right) + D(p^g, p^g) = (1 - \lambda^g)\tilde{\phi}'(p^g), \\
 \theta \left( (p^g - c)D(p^g, p^g)(1 - \lambda^g)\tilde{\phi}'(p^g) \right) = \lambda^g (\psi'(e^g) + R^c(e^g)),
\]

where \( \lambda^g \in (0, 1) \) is the Lagrange multiplier of the binding constraint \( p - w \geq 0 \), which writes as

\[
 (1 + \theta e^g)\tilde{\phi}(p^g) = \psi(e^g) + R(e^g).
\]
In the multiproduct linear-quadratic setup, we derive, all else equal, the following value of the multiplier $\lambda^g$

$$
1 = \frac{\theta^2[1 - (1 - \gamma)c]^2}{16(1 - \gamma)} \frac{1 - 3\lambda^g}{(\lambda^g)^2(2 - \lambda^g)}.
$$

By an argument that replicates that of the previous section, the comparison between $\lambda^c$ and $\lambda^g$ in this collusive environment gives us $\lambda^g < \lambda^c$.

This condition again pushes the retail price up when a RBC law is imposed. The reasoning is similar to that in the case of a mono-output upstream manufacturer and it won’t be repeated here.
9. References


Appendix

Proof of Proposition 1. Inserting the values of $(w, z)$ obtained from (5.1) into the maximand $(P^*)$ amounts to rewrite $(P^*)$ as:

$$(P^*) : \max_{(p,e)} (p-c)D(p)(1+\theta e) - \psi(e)$$

subject to (4.3) and (4.4).

The solution is given by $p^* = p^m$ and $e^* = e^m$. Inserting into (4.3) and (4.4) gives $w^* = c$ and $z^* = 0$, which obviously satisfy (5.1).

Proof of Proposition 2. Immediate.

Proof of Proposition 3. Expressing both the maximand in $(P^c)$ as written in the text and the constraints in terms of $(p,e)$, this optimization problem becomes:

$$(P^c) : \max_{(p,e)} [(p-c)D(p) - \varphi(p)] (1+\theta e)$$

subject to (6.1).

Let denote by $\lambda^c \geq 0$ the Lagrange multiplier of the retailer’s break-even condition (6.1). We may write the corresponding Lagrangean as

$$[(p-c)D(p) - \varphi(p)] (1+\theta e) + \lambda^c [\varphi(p)(1+\theta e) - \psi(e)].$$

Assuming concavity of this Lagrangean in $(p,e)$ and optimizing with respect to $p$ and $e$ respectively yields the following first-order Karush-Khùn-Tucker conditions (6.2) and (6.3).

We now have to prove that $\lambda^c > 0$. Suppose the contrary, i.e., $\lambda^c = 0$. Then, we would have $p^c \equiv \tilde{p}$ and $e^c \equiv 1$. Unfortunately, those values of the retailer’s control variables do not satisfy the break-even condition (6.1) when Assumption 1 holds. In other words, the retailer’s break-even condition (6.1) is binding.

Proof of Proposition 4. From (3.6), it follows that necessarily $\lambda > 0$. From (6.2) it also follows that

$$(p^c - c)D'(p^c) + D(p^c) - \varphi'(p^c) = \lambda^c \varphi'(p^c).$$

Notice now that $\varphi'(p^c) < 0$ from the assumptions made on the demand $D(\cdot)$. Hence, we obtain

$$(p^c - c)D'(p^c) + D(p^c) - \varphi'(p^c) > 0.$$  

Assuming the quasi-concavity of the manufacturer’s objective $(p-c)D(p) - \varphi(p)$, it follows that $p^c < \tilde{p}$.

Observe now that the vertically integrated outcome entails

$$\varphi(p^m) = \pi^m = \frac{\psi'(e^m)}{\theta}.$$

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Thus,

\[(A.1) \quad (1 + \theta e^m)\pi^m - \psi(e^m) = \frac{\psi'(e^m)}{\theta} + R(e^m) > 0.\]

Observe that the value of the Lagrange multiplier \(\lambda^c\) is obtained by inserting into Equation (A.1) the values of \(p \) and \(e \) obtained from (6.2) and (6.3). For \(\lambda^c = 1\), these values are \(p^m \) and \(e^m \) respectively. Condition (A.1) then tells us that the participation constraint (6.1) is strictly satisfied for these values, a contradiction with the fact that \(\lambda^c > 0\). We deduce from this that \(\lambda^c < 1\) necessarily.

Inserting into (6.2), we get

\[(p^c - c)D'(p^c) + D(p^c) = -(1 - \lambda^c)\varphi'(p^c) < 0.\]

Therefore, \(p^c > p^m\). This concludes the proof.

\[\square\]

**Proof of Proposition 5.** We may express \((P^g)\) in a more compact form as:

\[(P^g) : \max_{(p,e)} [(p - c)D(p) - \varphi(p)] (1 + \theta e)\]

subject to

\[(A.2) \quad (1 + \theta e)\varphi(p) - e\psi'(e) \geq 0,\]

Let denote by \(\lambda^g\) the Lagrange multiplier for constraint (A.2). We may write the corresponding Lagrangean as

\[[(p - c)D(p) - \varphi(p)] (1 + \theta e) + \lambda^g [\varphi(p)(1 + \theta e) - e\psi'(e)].\]

Assuming concavity of this Lagrangean in \((p, e)\) and optimizing with respect to \(p\) and \(e\) respectively yields the following first-order Karush-Khun-Tucker conditions (7.3) ad (7.4).

We now prove that (A.2) is binding. To prove this, suppose the contrary, i.e., \(\lambda^g = 0\). Then, we would have \(p^g \equiv \bar{p}\) and \(e^g \equiv 1\). Unfortunately, those values of the retailer’s control variables do not satisfy the break-even condition (7.2) and hence the harder constraint (7.2) when Assumption 1 holds. In other words, the non-negativity condition (7.2) is binding.

The last item follows from observing that

\[(1 + \theta e^g)\varphi(p^g) = e^g\psi'(e^g) > \psi(e^g)\]

when \(e^g > 0\).

\[\square\]

**Proof of Proposition 6.** Let denote by \(P(\lambda)\) the solution to the following equation:

\[(A.3) \quad (P(\lambda) - c) D'(P(\lambda)) + D(P(\lambda)) - (1 - \lambda)\varphi'(P(\lambda)) = 0,\]

This expression gives us the retail price as a function of the Lagrange multiplier \(\lambda\) under both scenarios, i.e., whether there is ban on below-cost pricing or not.
For the linear-quadratic specifications, we have \( \varphi(p) = (1 - p)^2 \) and \( \varphi'(p) = -2(1 - p) \). We immediately find:

\[
(A.4) \quad P(\lambda) - c = \frac{(3 - 2\lambda)(1 - c)}{2(2 - \lambda)} \quad \Leftrightarrow \quad 1 - P(\lambda) = \frac{1 - c}{2(2 - \lambda)}.
\]

**Upstream Collusion without regulation.** Let denote by \( E^c(\lambda) \) the retailer’s effort as a function of the Lagrange multiplier \( \lambda \). It is now solution to the following equation:

\[
(A.5) \quad \theta \left[ (P(\lambda) - c) D(P(\lambda)) - (1 - \lambda) \varphi(P(\lambda)) \right] = \lambda \psi'(E^c(\lambda)).
\]

The value of the Lagrange multiplier \( \lambda^c \) is then obtained as a solution to the retailer’s binding break-even constraint that can be written as:

\[
(A.6) \quad (1 + \theta E^c(\lambda)) \varphi(P(\lambda)) = \psi(E^c(\lambda)).
\]

Inserting into (A.5) yields

\[
E^c(\lambda) = \frac{\theta(1 - c)^2}{4(2 - \lambda)\lambda}.
\]

The value of \( \lambda^c \) follows from inserting the expressions of \( P(\lambda) \) and \( E^c(\lambda) \) so obtained into (A.6):

\[
(A.7) \quad 1 = \frac{\theta^2(1 - c)^2}{8} \left[ \frac{2 - 3\lambda^c}{(\lambda^c)^2(2 - \lambda^c)} \right].
\]

The right-hand side of (A.7) is decreasing over \([0, 1]\) with a unique solution \( \lambda^c \in (0, \frac{2}{3}) \).

**Upstream Collusion with a RBC Law.** For a given value of \( \lambda \), the retailer’s effort \( E^g(\lambda) \) now solves:

\[
(A.8) \quad \theta \left[ (P(\lambda) - c) D(P(\lambda)) - (1 - \lambda) \varphi(P(\lambda)) \right] = \lambda \left[ \psi'(E^g(\lambda)) + E^g(\lambda) \psi''(E^g(\lambda)) \right].
\]

The corresponding Lagrange multiplier \( \lambda^g \) now solves:

\[
(A.9) \quad (1 + \theta E^g(\lambda)) \varphi(P(\lambda^g)) = E^g(\lambda) \psi'(E^g(\lambda)).
\]

With the linear-quadratic specification, we obtain:

\[
(A.10) \quad E^g(\lambda) = \frac{\theta(1 - c)^2}{8(2 - \lambda)\lambda} = \frac{E^c(\lambda)}{2}.
\]

Inserting into (A.9), we obtain the following expression for \( \lambda^g \)

\[
(A.11) \quad 1 = \frac{\theta^2(1 - c)^2}{16} \left[ \frac{2 - 3\lambda^g}{(\lambda^g)^2(2 - \lambda^g)} \right].
\]

Again, the right-hand side of (A.11) is decreasing over \([0, 1]\) with a unique solution \( \lambda^g \in (0, \frac{2}{3}) \).

Comparing (A.7) and (A.11) immediately yields \( \lambda^g < \lambda^c \). Moreover, we have:

\[
e^g = E^g(\lambda^g) = \frac{E^c(\lambda^g)}{2} < \frac{E^c(\lambda^c)}{2} < E^c(\lambda^c) = e^c
\]

since \( E^c(\lambda) \) is decreasing in \( \lambda \). Hence, effort is lower under a RBC law.

Finally, inserting \( \lambda^g < \lambda^c \) into the expressions of the retail prices (A.4) ends the proof. \(\square\)
Proof of Proposition 7. Similar to that of Proposition 4 and is thus omitted.

Proof of Proposition 8. Similar to that of Proposition 5 and is thus omitted.

Proof of Proposition 9. Mimicking the analysis found in the proof of Proposition 6 with the expression of linear demands given in (8.11) but omitting details, we obtain the following expressions of the multipliers \( \lambda^c(\gamma) \) and \( \lambda^g(\gamma) \):

\[
1 = \frac{\theta^2(1 - c)^2}{16} \left( 2 + \frac{\gamma}{1 + \gamma (2 - \lambda^c(\gamma))(\lambda^c(\gamma))^2} \right) 2 - 3\lambda^c(\gamma),
\]

\[
1 = \frac{\theta^2(1 - c)^2}{32} \left( 2 + \frac{\gamma}{1 + \gamma (2 - \lambda^g(\gamma))(\lambda^g(\gamma))^2} \right) 2 - 3\lambda^g(\gamma).
\]

These formulas are similar to the ones found when there is a single mono-output manufacturer, which are actually obtained by setting \( \gamma = 0 \) above.

Notice first that, whatever the degree of product differentiation \( \gamma \), we always obtain that \( \lambda^c(\gamma) < \lambda^g(\gamma) \).

Because \( \frac{2 + \gamma}{1 + \gamma} \) is decreasing in \( \gamma \) and \( \frac{2 - 3\lambda}{(2 - \lambda)\lambda^2} \) is decreasing in \( \lambda \), these Lagrange multipliers are both decreasing functions of \( \gamma \).

Inserting these findings into (A.4) ends the proof.