Searching for a bargain

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VERY PRELIMINARY

Abstract

We consider a model of consumer search with differentiated products where firms may make a lower offer to consumers that initially walk away. The probability of receiving such an offer is exogenous. In equilibrium, consumers with a high match value buy directly, while those with an intermediate match value gamble on receiving a lower price. Surprisingly, we find that equilibrium posted prices are lower than in a benchmark when prices cannot be negotiated. Consumers are better off, while firms are worse off. Total welfare decreases, as there is more search.

1 Introduction

Although a common occurrence in non-Western countries, bargaining in stores is rarely observed in Western countries. This difference may be put aside as a cultural difference. However, public commentators regularly refer to evidence that bargaining off the posted price is in fact possible in stores such as jewellers, shoe shops, travel agents, furniture stores and electronics stores (see e.g. The Guardian, 2008, Time, 2013, The New York Times, 2013). One explanation as to why haggling in stores is still a rarely seen sight seems to be that consumers are largely unaware that there is even a chance they might succeed.

An academic piece of evidence of the possibility of bargaining in retail stores is provided by Shelegia and Sherman (2014). In their experimental study, they consider the retailers’ characteristics associated with the option to bargain. In this experiment, they sent out research assistants to pose as consumers in over 300 stores throughout the city.

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of Vienna, Austria with the simple instruction to ask for a discount. A discount was offered approximately 40% of the time on products ranging from a backpack to a bottle of perfume to a surveillance camera for babies. Conditional on getting a discount, the average size of the discount was noted to be approximately 25 EUR, and the average discount percentage almost 10%. The experiment clearly shows that retail stores do not necessarily commit to their prices.

In this paper, we model markets in which firms are unable to fully commit to their posted prices. Consumers engage in sequential search and firms sell differentiated products which closely relates the model to that of Anderson and Renault (1999). Firms post prices, that consumers can only observe if they visit. If consumers walk away from a firm, then with some exogenous probability, the firm can make a second, lower price offer. Hence, consumers with a high match value will buy right away while those with a lower match value may reject in the hope of receiving a better offer. Intuitively, one may expect that having the option to lower its price will only imply that firms post higher prices. We show, however, that that is not the case. In equilibrium, consumers are better off, while firms are worse off. Welfare decreases, as consumers search more.

In the past, researchers have in the context of bargaining in consumer markets mainly considered the choice between committing to posted prices and allowing for bargaining without posted prices (e.g., Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, Camera and Delacroix, 2004). More recently, a small but growing literature considers bargaining off posted prices which is where this paper clearly fits in.

Still, the setup of these papers largely differs. In part this can be attributed to the fact that there is no one clear way as to how the process of bargaining should be modelled. In earlier research, authors include consumer bargaining costs (Desai and Purohit, 2004, King and Patras, 2014); seller bargaining costs (King and Patras, 2014); relative bargaining skill between buyer and seller (Desai and Purohit, 2004); two types of consumers, price-takers and hagglers (Desai and Purohit, 2004, Gill and Thanassoulis, 2009, Gill and Thanassoulis, 2016); a(n) (exogenous) probability of successful bargaining (King and Patras, 2014, Gill and Thanassoulis, 2016); consumers who non-sequentially search for second quotes (Gill and Thanassoulis, 2009). In our model, consumers can credibly convey their rejection of the posted price and some do so strategically in the
hope of obtaining a second offer.

The focus of these papers also differs from ours. Desai and Purohit (2004) ask the question whether retailers can gain a strategic advantage by permitting bargaining. They find that, depending on the parameters and in particular the proportion of haggling consumers, retailers find themselves in a prisoners’ dilemma either where they end up with a fixed price policy where symmetric haggling policies would have earned higher profits, or vice versa. Different from Desai and Purohit (2004), we consider in this paper a non-binary bargaining policy. That is, firms deal with a frequency of how often they engage in bargaining with consumers rather than whether to permit it or not.

Gill and Thanassoulis (2009) study an oligopoly competing in Cournot-fashion to sell a homogeneous good. Consumers have individual valuations for the good. Moreover, the group of consumers is split into two groups: price takers and bargainers. Bargainers attempt to extract multiple second quotes from a number of firms while price takers submit to the Cournot market list price. The authors find that an increase in the proportion of bargainers increases both the list price and the lowest price offered to bargainers. The part of the model that concerns itself with the bargaining consumers closely relates to Burdett and Judd’s (1983) model of non-sequential search. Our framework, on the other hand, is largely based on Anderson and Renault’s (1999) model of sequential search. Furthermore, Gill and Thanassoulis (2009) are interested in the effects of an increase in the proportion of bargainers in the market on prices, while we focus on effects on search and welfare when all consumers have a chance of getting a second quote.

King and Patras (2014), in their model, consider a monopolist who can either choose to allow bargaining or commit not to bargain. Furthermore, the choice to bargain, if allowed, is made by the buyer endogenously. Efficient bargaining is assumed. The posted price is shown to be pushed above the monopoly price when the monopolist allows for bargaining. Under mild conditions on the costs of bargaining for the consumer and the seller, the monopolist’s expected profits are higher when bargaining is allowed. Modelling a monopoly rather than a competitive setting allows the authors to include an endogenous choice of bargaining. In contrast, our interest also lies in the potential of bargaining in a competitive setting.

Gill and Thanassoulis (2016), similar to the bargaining process we study, include a
probability of successful bargaining. The process differs in that consumers do not have to abandon the list price first. When the two firms, each on one end of the Hotelling line (Hotelling, 1929), compete, first in list prices, then in discount prices, the authors find that a misallocation of goods causes welfare to increasingly decline with an increasing proportion of bargainers relative to price takers. In addition, the authors find that a higher proportion of bargainers leads to both higher list prices as well as higher discount prices. An important difference with our paper is that we consider consumers who each time they visit a firm weigh the option in front of them with an uncertain outside option, while Gill and Thanassoulis (2016) consider completely informed consumers.

The remainder of the paper is organized as follows. In Section 2, we set out the model. In Section 3, we derive results for the benchmark model where bargaining is excluded. We then analyze the full model in Section 4. In Section 5, we derive comparative statics. Section 6 considers an endogenous probability of obtaining a second offer. Robustness is discussed in Section 7. Finally, Section 8 concludes.

2 The model

The framework in which we operate is largely based on Anderson and Renault (1999). There are infinitely many\(^1\) single-product firms whose marginal production costs equal zero. Products are horizontally differentiated and sold to a unit mass of consumers. A consumer incurs search costs \(s\) when a firm is visited. Search is sequential. Consumer \(j\) buying a product at firm \(i\) at price \(p_i\) obtains utility

\[ u_{ij} = v + \varepsilon_{ij} - p_i. \]

\(v\) is the baseline valuation for any consumer buying any firm’s product. We assume \(v\) is sufficiently high, such that the market is always fully covered in equilibrium. \(\varepsilon_{ij}\) is consumer \(j\)’s match value for product \(i\). This is private information. It is common knowledge that \(\varepsilon_{ij}\) is independently and identically distributed across consumers and firms with distribution function \(F(\varepsilon)\). To derive closed-form solutions, we assume match values are uniformly distributed on \([0, 1]\) interval. The robustness of the results to this simplifying assumption will be discussed in Section 7.1.

\(^1\)We refer to Section 7.2 for a discussion on the assumption of infinitely many firms in this model.
A consumer that visits a firm and rejects its price will receive a better offer with exogenous probability \( \gamma \). The better offer is denoted \( p_b \) and will be derived endogenously. Both the consumers and the firms are aware of this probability \( \gamma \). Consumers that do not receive or that reject \( p_b \) will visit the next firm, incurring search costs \( s \).

We thus rule out that a consumer that fails to receive a better offer will buy at the higher price \( p \) anyway. For example, there may be psychological costs associated with the shame of buying at the posted price after walking away but not obtaining a second offer. Without this assumption, consumers would always reject \( p \). Also, we crucially assume that firms do not know in advance whether they will offer a second, lower price to a particular consumer. For example, this may be left at the discretion of its employees, that decide on the basis of factors uncorrelated with a consumer’s match value.

The timing of the model is thus as follows. First, nature sets \( \gamma \). Second, firms set posted prices \( p \), unobservable to consumers. Third, the consumer randomly visits a firm and observes its posted price and the match value it provides. Fourth, the consumer either accepts or rejects this price. If she rejects, then with probability \( \gamma \), the firm makes a second offer that may again be accepted or rejected by the consumer. If she rejects the second offer, or if the firm fails to make one, the consumer may choose to incur search costs \( s \) to visit the next firm where this process repeats itself, etcetera, ad infinitum.

In the remainder, we will first analyze a benchmark model without second offers. We then analyze the full model as described above, and an extension in which \( \gamma \) is set endogenously. We will omit the consumer index \( j \) for ease of exposition.

### 3 Benchmark: no second offers

We first consider the benchmark case where \( \gamma = 0 \), so there are no second offers. We are thus in the regular Anderson and Renault (1999) framework, but with infinitely many firms. Yet, we will derive the equilibrium in a slightly different manner that will prove helpful in deriving the equilibrium of the full model.

Consider a symmetric equilibrium where all firms charge \( p^* \). Denote a consumer’s expected equilibrium utility from participating in this game as \( v + \Delta \). At her first visit, this consumer observes match value \( \varepsilon_i \) and equilibrium price \( p^* \). With infinitely many firms, continuing search yields expected utility \( v + \Delta - s \). Hence, she is indifferent between
searching and not when \( \varepsilon_i = \hat{\varepsilon} \) with

\[
\hat{\varepsilon} \equiv p + \Delta - s.
\] (1)

This implies that \( \Delta \) should satisfy

\[
\Delta = \int_0^{\hat{\varepsilon}} (\Delta - s) dF(\varepsilon) + \int_{\hat{\varepsilon}}^{1} (\varepsilon - p^*) dF(\varepsilon).
\] (2)

This can be seen as follows. By construction, \( \Delta \) is this consumer’s equilibrium expected utility from participating. If she does, by visiting the first firm, she will continue search if she finds a match value smaller than \( \hat{\varepsilon} \). If so, her continuation utility is \( \Delta - s \). With \( \varepsilon_i > \hat{\varepsilon} \), she stops searching and obtains \( \varepsilon_i - p^* \).

For a uniform distribution, we can solve (2) to find

\[
\Delta = \frac{1 + \hat{\varepsilon}}{2} - p^* - \frac{\hat{\varepsilon}}{1 - \hat{\varepsilon}} s.
\] (3)

Substituting (3) into (1),

\[
\hat{\varepsilon} = 1 - \sqrt{2s}.
\] (4)

Suppose firm \( i \) defects from the tentative equilibrium \( p^* \) by setting price \( p_i \). Defining \( \delta \equiv p_i - p \), the joint probability that firm \( i \) is visited as the \( k \)th (of \( N \) total) firm and sells to this consumer is given by

\[
\lim_{N \to \infty} \frac{1}{N} F(\hat{\varepsilon})^k (1 - F(\hat{\varepsilon} + \delta)),
\]

as this is true if she did not buy at any earlier firm but does buy at \( i \). Firm \( i \)'s expected profits are thus given by

\[
\Pi_i(p_i) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} F(\hat{\varepsilon})^k (1 - F(\hat{\varepsilon} + \delta)) p_i = \lim_{N \to \infty} \frac{1}{N} \left[ 1 - F(\hat{\varepsilon} + \delta) \right] p_i.
\]

Taking the first-order condition, imposing symmetry and using (4) yields equilibrium prices and industry profits

\[
p^{bench} = \Pi^{bench} = \sqrt{2s}.
\]

4 Introducing second offers

We now consider the full model described in Section 2. We first solve for a firm’s optimal bargaining price for given \( \gamma \) and \( p \). We then derive optimal consumer behavior for given
\( \varepsilon_i, \gamma, p \) and inferred bargaining price \( p_b \). Finally, we infer the posted price given consumer responses. We look for a symmetric pure strategy equilibrium in posted prices.

Suppose again that all firms charge \( p^* \). We look for an equilibrium where a consumer accepts the posted price if her match value is high enough. Rejecting a price implies that she will forgo the current match value with probability \( 1 - \gamma \). This is not worthwhile if \( \varepsilon_i \) is larger than some cut-off value \( \bar{\varepsilon} \).

Again denote the consumer’s equilibrium utility from participating in this game as \( \Delta \). Consider a consumer that rejects the price \( p_i \) of firm \( i \). From this, the firm can infer that \( \varepsilon_i < \bar{\varepsilon} \). If the firm now offers a bargaining price \( p_b \), the consumer will accept if her utility from doing so, \( \varepsilon_i - p_b \), exceeds her expected utility from continuing search, \( \Delta - s \). For the firm, the probability that the consumer will accept the second offer equals

\[
Pr(\varepsilon_i > p_b + \Delta - s | \varepsilon_i < \bar{\varepsilon}) = (\bar{\varepsilon} - p_b - \Delta + s) / \bar{\varepsilon}.
\]

Firm \( i \)’s expected profit from this consumer is thus

\[
p_b(\bar{\varepsilon} - p_b - \Delta + s) / \bar{\varepsilon}.
\]

Taking the first-order condition yields

\[
p_b = (\bar{\varepsilon} - \Delta + s) / 2.
\] (5)

The consumer accepts \( p_b \) if \( \varepsilon_i > \bar{\varepsilon} \), with

\[
\bar{\varepsilon} \equiv (\bar{\varepsilon} + \Delta - s) / 2.
\] (6)

Now move back to the stage where the consumer decides whether to accept or reject the posted price. If she accepts, she obtains \( \varepsilon_i - p \). If she rejects, she obtains \( \gamma(\varepsilon_i - p_b) + (1 - \gamma)(\Delta - s) \). Hence, she is indifferent when \( \varepsilon_i = \bar{\varepsilon} \). Using (5), this yields

\[
\bar{\varepsilon} = \frac{2p}{2 - \gamma} + \Delta - s.
\] (7)

For the equilibrium value of \( \Delta \), this implies

\[
\Delta = \int_0^{\bar{\varepsilon}} (\Delta - s) dF(\varepsilon) + \gamma \int_{\varepsilon}^{\bar{\varepsilon}} (\varepsilon - p_b) dF(\varepsilon)
+ (1 - \gamma) \int_{\varepsilon}^{\bar{\varepsilon}} (\Delta - s) dF(\varepsilon) + \int_1^{\bar{\varepsilon}} (\varepsilon - p) dF(\varepsilon).
\] (8)
Using (5)–(8), this yields
\[
\Delta = 1 + s - p - \sqrt{2s - \frac{\gamma(1 - \gamma)p^2}{(2 - \gamma)^2}}. \tag{9}
\]
Hence
\[
p_b = \frac{p}{2 - \gamma} \tag{10}
\]
\[
\bar{\epsilon} = 1 + \frac{\gamma p}{2 - \gamma} - \sqrt{2s - \frac{\gamma(1 - \gamma)p^2}{(2 - \gamma)^2}}, \tag{11}
\]
\[
\tilde{\epsilon} = 1 - \frac{(1 - \gamma)p}{2 - \gamma} - \sqrt{2s - \frac{\gamma(1 - \gamma)p^2}{(2 - \gamma)^2}}. \tag{12}
\]

Suppose firm \(i\) defects from the tentative equilibrium \(p^*\) by setting price \(p_i\). Given that \(i\) would be visited \(k\)-th, the probability the consumer arrives at firm \(i\) is
\[
[(1 - \gamma)(F(\bar{\epsilon}) - F(\tilde{\epsilon})) + F(\tilde{\epsilon})]^{k-1} = [(1 - \gamma)F(\bar{\epsilon}) + \gamma F(\tilde{\epsilon})]^{k-1}.
\]
Since \(\Delta\), the expected utility from playing the game, is unchanged by firm \(i\)'s defection, the reservation values at firm \(i\) simply become
\[
\bar{\epsilon}_i = \frac{2p_i}{2 - \gamma} + \Delta - s, \tag{13}
\]
\[
\tilde{\epsilon}_i = (\bar{\epsilon}_i + \Delta - s)/2 = \frac{p_i}{2 - \gamma} + \Delta - s, \tag{14}
\]
and clearly
\[
p_{b,i} = \frac{p_i}{2 - \gamma}. \tag{15}
\]
Firm \(i\)'s profit function is now given by
\[
\Pi_i(p_i) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} [(1 - \gamma)(F(\bar{\epsilon}_i) - \gamma F(\tilde{\epsilon}_i))]^k \times [(1 - F(\bar{\epsilon}_i))p_i + \gamma (F(\bar{\epsilon}_i) - F(\tilde{\epsilon}_i))p_{b,i}].
\]
Using (9)–(15), taking the first-order condition, and imposing symmetry, we are able to derive equilibrium price and, subsequently, industry profit
\[
p_{sec} = \sqrt{\frac{2(2 - \gamma)^4s}{(4 - 3\gamma)(4 - 3\gamma^2)}}, \tag{16}
\]
\[
\Pi_{sec} = \sqrt{\frac{2(2 - \gamma)^4(4 - 3\gamma)s}{(4 - 2\gamma - \gamma^2)^2(4 - 3\gamma^2)}}. \tag{17}
\]
5 Comparative statics

Most surprisingly, we find

Theorem 1. Posted prices are lower when firms have the option to make second offers: 

\[ p^{sec} < p^{bench} \forall \gamma \in (0, 1). \]

As a result, industry profits are lower: 

\[ \Pi^{sec} < \Pi^{bench} \forall \gamma \in (0, 1). \]

Moreover, equilibrium prices and profits converge to those in the benchmark if \( \gamma \) converges to either 0 or 1.

\[ \square \]

Proof. See Appendix.

One would expect that the option firms have to make a second offer if consumers reject the posted price, would lead to firms charging a posted price that is higher than it otherwise would be. That however, is not the case here. With the exogenous probability of a second offer, firms cannot commit to their posted price. Consumers’ outside option improves upon visiting a firm as it does not with certainty include search costs. This gives consumers more bargaining power, forcing firms to lower their prices. The higher the probability of a second offer, the stronger this effect, and the lower the posted price. Yet, the posted price is increasing in \( \gamma \) for \( \gamma > 2/3 \). A substantial share of consumers buying from a firm now buys at the discounted price, hence it becomes profitable to increase the posted price. In the extreme, when \( \gamma \to 1 \), all consumers buy at the discounted price, so that the posted price converges to the equilibrium price without bargaining. With \( \gamma = 0 \), we are back in the benchmark case.

Expected consumer surplus can now be computed as the sum of baseline valuation \( v \) and \( \Delta \), respective for the benchmark and full model case:

\[ CS^{bench} = v + \Delta^{bench} \]
\[ = v + 1 + s - 2\sqrt{2s}, \]

and

\[ CS^{sec} = v + \Delta^{sec} \]
\[ = v + 1 + s - \left( \sqrt{\frac{4 - 3\gamma}{4 - 3\gamma^2}} \right) \times 2\sqrt{2s}. \]
Furthermore, we can compute expected total welfare for the two models which is the sum of consumer and industry profit. For the benchmark, it follows that

\[ TW^{\text{bench}} = v + 1 + s - \sqrt{2s}, \]

and in the model with second offers

\[ TW^{\text{sec}} = v + 1 + s - \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2}} \times \sqrt{2s}. \]

Before we compare these welfare measures, it is useful to first compute the sources of possible differences. Apart from prices, these sources include expected search costs and match values. Note that prices do not influence total welfare as a difference in prices is merely a shift between consumer and firm. Denote \( \Lambda \) as total expected search costs, we find for the benchmark case

\[ \Lambda^{\text{bench}} = \lim_{N \to \infty} \sum_{k=0}^{N} F(\hat{\varepsilon})^k(1 - F(\hat{\varepsilon}))ks = \sqrt{s/2} - s, \]

while for the full model, we find

\[
\begin{align*}
\Lambda^{\text{sec}} &= \lim_{N \to \infty} \sum_{k=0}^{N} [(1 - \gamma)F(\bar{\varepsilon}) + \gamma F(\bar{\varepsilon})]^k[1 - F(\bar{\varepsilon}) + \gamma(F(\bar{\varepsilon}) - F(\hat{\varepsilon}))]ks \\
&= \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2}} \times \sqrt{s/2} - s.
\end{align*}
\]

The match value a consumer can expect to obtain ex ante in the benchmark is

\[ MV^{\text{bench}} = \lim_{N \to \infty} \sum_{k=0}^{N} F(\hat{\varepsilon})^k(1 - F(\hat{\varepsilon}))E(\varepsilon|\varepsilon > \hat{\varepsilon}) = 1 - \sqrt{s/2}, \]

and in the full model

\[
\begin{align*}
MV^{\text{sec}} &= \lim_{N \to \infty} \sum_{k=0}^{N} [(1 - \gamma)F(\bar{\varepsilon}) + \gamma F(\bar{\varepsilon})]^k[(1 - F(\bar{\varepsilon}))E(\varepsilon|\varepsilon > \bar{\varepsilon}) + \gamma(F(\bar{\varepsilon}) - F(\varepsilon))E(\varepsilon|\varepsilon < \varepsilon < \bar{\varepsilon})] \\
&= 1 - \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2}} \times \sqrt{s/2}.
\end{align*}
\]

We can then show:

**Theorem 2.** There is more search, and lower match values, hence lower welfare, when firms have the option to make a second offer: \( \Lambda^{\text{sec}} > \Lambda^{\text{bench}}; MV^{\text{sec}} < MV^{\text{bench}}; TW^{\text{sec}} < TW^{\text{bench}} \forall \gamma \in (0, 1) \). Yet, consumer surplus is then higher: \( CS^{\text{sec}} > CS^{\text{bench}} \forall \gamma \in (0, 1) \).
Proof. See Appendix.

Hence, for consumers the adverse effect of searching more and lower match values is more than outweighed by the beneficial effect of paying lower prices. The intuition behind these results is as follows. Since, as we explained earlier, rejecting the posted price has become cheaper, consumers can be more critical. This leads to more search and hence higher expected search costs. More search usually leads to higher match values, but because accepting lower match values may come with lower prices, this does not hold up here. With prices being transfers between consumers and producers, lower matching and more search leads to lower welfare overall.

Finally, we will shortly comment on the effects of changes in search costs $s$. As $s$ increases, consumers become less critical as search becomes a less attractive option. In turn, average prices paid increase, while average match values (of the goods bought) decrease. Thus, producers gain while consumers lose. Overall, welfare decreases as $s$ increases.

6 Extension: endogenous $\gamma$

In this section, we consider $\gamma$ as a decision variable in the model. One possible interpretation for an exogenous $\gamma$ is that firms only sometimes pick up on consumers rejecting the posted price. We can then interpret an endogenous parameter as a way for firms to pick up on as many rejecting consumers as they want.

The timing of the game as played by a single consumer is as follows. First, firms set $\gamma$. Second, firms post prices, unobservable to consumers. Third, the consumer randomly visits a firm and observes its posted price and the match value it provides. Fourth, the consumer either accepts or rejects this price. If she rejects, then with probability $\gamma$, the firm makes a second offer that may again be accepted or rejected by the consumer. If she rejects the second offer, or if the firm fails to make one, the consumer may choose to incur search costs $s$ to visit the next firm where this process repeats itself, etcetera, ad infinitum.

Suppose all firms but firm $i$ set $p^*$ and $\gamma^*$. Instead, firm $i$ defects to $\gamma_i$ and thus also
reconsiders his price $p_i(\gamma_i)$. Firm $i$'s profit function is now given by

$$
\Pi_i(p_i, \gamma_i) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} [(1 - \gamma) F(\bar{\varepsilon}) + \gamma F(\bar{\varepsilon})]^k \times [(1 - F(\bar{\varepsilon}_i))p_i + \gamma_i(F(\bar{\varepsilon}_i) - F(\bar{\varepsilon}_i))p_{b,i}],
$$

where it is easily shown that, similar to (13), (14), (15), we now have

$$
\bar{\varepsilon}_i = \frac{2p_i}{2 - \gamma_i} + \Delta - s, \quad (19)
$$

$$
\bar{\varepsilon}_i = (\bar{\varepsilon}_i + \Delta - s)/2 = \frac{p_i}{2 - \gamma_i} + \Delta - s, \quad (20)
$$

$$
p_{b,i} = \frac{p_i}{2 - \gamma_i}, \quad (21)
$$

and $\Delta$ still as in (9).

Since we solve by backward induction, given the timing of the game, we first take the first-order condition with respect to $p_i$. Using that all other firms set the equilibrium price as in (16), we find

$$
p_i(\gamma_i) = \frac{(2 - \gamma_i)^2}{4 - 3\gamma_i} \sqrt{\frac{2(4 - 3\gamma)s}{4 - 3\gamma^2}}.
$$

Plugging this expression back into the profit function, one finds

$$
\Pi_i(\gamma_i) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N} [(1 - \gamma) F(\bar{\varepsilon}) + \gamma F(\bar{\varepsilon})]^k \times \left[(1 - \Delta + s)\sqrt{\frac{2(4 - 3\gamma)s}{4 - 3\gamma^2}} - \frac{2(4 - 3\gamma)s}{4 - 3\gamma^2}\right] \frac{(2 - \gamma_i)^2}{4 - 3\gamma_i},
$$

where the second equality follows when we plug in $\Delta$. Taking the first-order condition with respect to $\gamma_i$ and then imposing symmetry such that $\gamma_i = \gamma$ yields the condition

$$
\frac{(2 - \gamma)(2 - 3\gamma)}{(4 - 3\gamma)^2} = 0.
$$

The condition is solved only by $\gamma = 2/3$. However, at this point profits are minimized. Hence, the two boundary cases (which are effectively equal to each other) are profit maximizing and therefore indicate the equilibria:

$$
\gamma^* = 0; \quad \gamma^* = 1.
$$
This result clearly reveals that firms also individually are better off committing to their prices in this model regardless of what other firms choose to do. One theory is that the driving factor behind this result is that firms cannot differentiate between consumers that reject in the hope of a better offer and consumers that reject because their valuation for the product is too low. This makes the lack of commitment to prices here a tool not strong enough for firms to gain strategic advantage.

7 Robustness

In this section, we discuss robustness of the results to some key assumptions in our models.

7.1 Uniformly distributed match values

In the analyses, we have throughout assumed consumers’ match values to be uniformly distributed between zero and one. This allowed us to derive closed-form solutions in our model. The question arises which, if any, results still hold when a different distribution is assumed. Unfortunately, closed-form solutions could not be obtained when employing a general distribution. Claims about robustness of the results to this assumption can therefore not be defended.

7.2 Infinitely many firms

We have assumed in our analyses an infinite number of competing firms. Now consider the case of a limited number, say \( N \), firms. When arriving at the \( N \)-th firm, when the consumer rejects the posted price, he either obtains a second offer or he gets nothing, since we assume that no recall is possible. At the \((N - 1)\)-th firm, rejecting the posted price means either a second offer or another chance at playing the game. The point is that, when a consumer is at the \( N \)-th firm, he is more likely to take the posted price than when he is at the first firm as the risk of getting nothing is further away at that point. In other words, the outside option decreases as the consumer has visited more firms. The less firms there are, the lower the outside option becomes already at the first firm. Of course, the firm does not know how many firms the consumer has visited before him, which makes for a complicated analysis. Nonetheless, we conjecture that the less firms
there are, the higher prices will be, and the sum of profits over all the firms per consumer would increase at the expense of consumer surplus.

We are inclined to say that for the benchmark model where there are no second offers, the mechanism is very similar and the comparatives statics results derived in Section 5 sill hold. However, there is one important difference between the two models which may complicate those results. If a consumer arrives at the last firm without a chance of a second offer, he will always buy and so all consumers still buy in this model. With a chance of a second offer, some consumers with low match values may find it worthwhile to ‘gamble’ for a better offer still which may also lead them to end up without the product. Hence, at this point, the baseline valuation $v$ becomes of some import. However, when $v$ is high, no consumer would likely take the gamble. When $v$ is low, some consumers ending up not buying the product has less of an impact on consumer welfare.

Unfortunately, even for the case of a duopoly, the analysis gets rather complicated, if not impossible to find closed-form solutions. With our thoughts written here, however, we conjecture our main results regarding the relation between the model with and the model without second offers still hold.

### 7.3 One second offer

Why would, by the same logic of making a second offer, a firm not (sometimes) make a third or a fourth offer? This process would resemble a sort of one-sided bargaining procedure where at each refusal of the consumer, the firm learns more about his hidden match value. It seems, however, there is no reason to think this would end better for firms than it did when there is only a chance of a second offer. Therefore, we conjecture our results are robust to such an extension.

### 8 Conclusion

In this paper, we studied the effects of bargaining on consumer prices. We modelled bargaining by giving firms the option to offer a lower price if consumers reject its posted price. Surprisingly, we find that the possibility of bargaining lowers posted prices. With the prospect of a better deal, consumers with an intermediate valuation have a better outside option. Consumers thus gain bargaining power that forces firms to lower their
prices.

One might argue that a critical assumption that drives this result is the firms’ inability to differ between those leaving consumers that hope to get a bargain and those that simply value their product too little, though it remains for future research to actually show. Consumers not only benefit from their gained power through lower prices, they also obtain the luxury to be more critical which leads to a better allocation of goods in the sense that, on average, consumers value the product they end up with more. Although consumers pay on average more search costs, surplus is shown to be higher. Total welfare, on the other hand, is lower as the better allocation of goods does not outweigh the additional search undertaken for it. Finally, we have also shown in this paper that firms are, independent of the choice of other firms, individually better off to commit to their prices.

Appendix

Proof Theorem 1

We compare equilibrium prices

\[ p^{\text{bench}} = \sqrt{2s}, \]
\[ p^{\text{sec}} = \sqrt{\frac{(2 - \gamma)^4}{(4 - 3\gamma)(4 - 3\gamma^2)}} \times \sqrt{2s}. \]

Let \( g_1 : [0, 1] \rightarrow \mathbb{R} \) be defined by

\[ g_1(x) = \sqrt{\frac{(2 - x)^4}{(4 - 3x)(4 - 3x^2)}}, \]

such that

\[ p^{\text{sec}} = g_1(\gamma) \times p^{\text{bench}}. \]

To show:

\[ g_1(x) \leq 1 \quad \forall x \in [0, 1]; \quad g_1(x) < 1 \quad \forall x \in (0, 1). \] (22)

These inequalities hold if and only if the function \( h_1 : [0, 1] \rightarrow \mathbb{R} \) defined by

\[ h_1(x) = (4 - 3x)(4 - 3x^2) - (2 - x)^4 \]
satisfies the properties

\[ h_1(x) \geq 0 \quad \forall x \in [0, 1]; \quad h_1(x) > 0 \quad \forall x \in (0, 1). \tag{23} \]

The function \( h_1 \) is a quartic function (fourth-degree polynomial) and therefore continuous. It can be rewritten as

\[ h_1(x) = 20x - 36x^2 + 17x^3 - x^4. \]

On its domain, we find two roots at \( x = 0 \) and \( x = 1 \). Furthermore, one can easily see that the derivative evaluated at zero

\[ h_1'(0) = 20 > 0, \]

which implies that for any value of \( x \) strictly between zero and one, \( h_1(x) > 0 \). We have now shown (23) holds, implying (22).

For profits

\[ \Pi_{bench} = \sqrt{2}s, \]
\[ \Pi_{sec} = \sqrt{\frac{(2 - \gamma)^4(4 - 3\gamma)}{(4\gamma^2 - 2\gamma)^2(4 - 3\gamma^2)}} \times \sqrt{2}s. \]

Let \( g_2 : [0, 1] \to \mathbb{R} \) be defined by

\[ g_2(x) = \sqrt{\frac{(2 - x)^4(4 - 3x)}{(4 - 2x - x^2)^2(4 - 3x^2)}}, \]

such that

\[ \Pi_{sec} = g_2(\gamma) \times \Pi_{bench}. \]

To show:

\[ g_2(x) \leq 1 \quad \forall x \in [0, 1]; \quad g_2(x) < 1 \quad \forall x \in (0, 1). \tag{24} \]

These inequalities hold if and only if the function \( h_2 : [0, 1] \to \mathbb{R} \) defined by

\[ h_2(x) = (4 - 2x - x^2)^2(4 - 3x^2) - (2 - x)^4(4 - 3x) \]

satisfies the properties

\[ h_2(x) \geq 0 \quad \forall x \in [0, 1]; \quad h_2(x) > 0 \quad \forall x \in (0, 1). \tag{25} \]
The function $h_2$ is an sixth-degree polynomial and therefore continuous. It can be rewritten as

$$h_2(x) = 112x - 256x^2 + 168x^3 - 12x^4 - 9x^5 - 3x^6.$$ 

On its domain, we find two roots at $x = 0$ and $x = 1$. Furthermore, one can easily see that the derivative evaluated at zero

$$h_2'(0) = 112 > 0,$$

which implies that for any value of $x$ strictly between zero and one, $h_2(x) > 0$. We have now shown (25) holds, implying (24).

**Proof Theorem 2**

We compare equilibrium total expected search costs

$$\Lambda^{bench} = \sqrt{s/2} - s,$$

$$\Lambda^{sec} = \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2}} \times \sqrt{s/2} - s.$$

Let $g_3: [0, 1] \to \mathbb{R}$ be defined by

$$g_3(x) = \sqrt{\frac{(4 - 3x)(4 - 3x^2)}{(4 - 2x - x^2)^2}},$$

such that

$$\Lambda^{sec} - \Lambda^{bench} = (g_3(\gamma) - 1)\sqrt{s/2}.$$

Hence, to show:

$$g_3(x) \geq 1 \quad \forall x \in [0, 1]; \quad g_3(x) > 1 \quad \forall x \in (0, 1). \quad (26)$$

These inequalities hold if and only if the function $h_3: [0, 1] \to \mathbb{R}$ defined by

$$h_3(x) = (4 - 2x - x^2)^2 - (4 - 3x)(4 - 3x^2)$$

satisfies the properties

$$h_3(x) \leq 0 \quad \forall x \in [0, 1]; \quad h_3(x) < 0 \quad \forall x \in (0, 1). \quad (27)$$
The function $h_3$ is a fourth-degree polynomial and therefore continuous. It can be rewritten as

$$h_3(x) = -4x + 8x^2 - 5x^3 + x^4.$$ 

On its domain, we find two roots at $x = 0$ and $x = 1$. Furthermore, one can easily see that the derivative evaluated at zero

$$h'_3(0) = -4 < 0,$$

which implies that for any value of $x$ strictly between zero and one, $h_3(x) < 0$. We have now shown (27) holds, implying (26).

For the expected match value, we found

$$MV^{\text{bench}} = 1 - \sqrt{s/2},$$

$$MV^{\text{sec}} = 1 - \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2} \times \sqrt{s/2}}.$$ 

Hence,

$$MV^{\text{sec}} - MV^{\text{bench}} = (1 - g_3(\gamma)) \sqrt{s/2}.$$ 

Since

$$g_3(x) > 1 \quad \forall x \in (0, 1),$$

indeed

$$MV^{\text{sec}} < MV^{\text{bench}}.$$ 

For consumer surplus

$$CS^{\text{bench}} = v + 1 + s - 2\sqrt{2s},$$

$$CS^{\text{sec}} = v + 1 + s - \left(\sqrt{\frac{4 - 3\gamma}{4 - 3\gamma^2}}\right) \times 2\sqrt{2s}.$$ 

Now,

$$CS^{\text{sec}} - CS^{\text{bench}} = \left(1 - \sqrt{\frac{4 - 3\gamma}{4 - 3\gamma^2}}\right) 2\sqrt{2s}.$$ 

The inequality trivially follows as

$$\frac{4 - 3\gamma}{4 - 3\gamma^2} < 1 \quad \forall \gamma \in (0, 1)$$

$$\implies CS^{\text{sec}} > CS^{\text{bench}} \quad \forall \gamma \in (0, 1).$$
For total welfare

\[ TW^{\text{bench}} = v + 1 + s - \sqrt{2s}, \]

\[ TW^{\text{sec}} = v + 1 + s - \sqrt{\frac{(4 - 3\gamma)(4 - 3\gamma^2)}{(4 - 2\gamma - \gamma^2)^2}} \times \sqrt{2s}. \]

So we now have

\[ TW^{\text{sec}} - TW^{\text{bench}} = (1 - g_3(\gamma)) \sqrt{2s}. \]

As we have already shown

\[ g_3(x) > 1 \quad \forall x \in (0, 1), \]

from which we trivially conclude

\[ TW^{\text{sec}} > TW^{\text{bench}} \quad \forall \gamma \in (0, 1). \]
References


