A theory of recommended price dispersion

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January 3, 2017

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Abstract

This paper contributes to the theory of recommended retail prices. We consider a model with two firms that compete by setting prices in a homogeneous product market. In this market, firms first set a recommended retail price that serves as an upper bound on their actual retail price. The key element is that a fraction of consumers is partially informed, such that these consumers make their purchase decision solely based on recommended prices. We show that if the partially informed consumers use a simple heuristic, recommended retail prices lead to lower retail prices on average. This effect is weakened if consumers have rational expectations.

JEL-codes: C72, L13

Keywords: recommended retail prices, search, partially informed consumers

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1 Introduction

This paper contributes to the theory of recommended retail prices. It is inspired by the practice of gasoline majors in the Netherlands to set “advisory prices”; recommended prices that serve as an upper bound on the prices that may actually be charged by local gasoline stations. Still, the model also applies to many other instances of such pricing practices. [...] Our model builds on Varian’s (1980) celebrated model of sales to explain these phenomena.

We consider a model with two firms that compete by setting prices in a homogeneous product market. They play a two-stage game. In the first stage firms simultaneously and independently set recommended retail prices. In the second stage, after having observed each other’s recommended retail prices, they simultaneously and independently set their actual prices. Recommended prices serve as an upper bound on the actual prices firms can set.

Throughout, we assume that there are three types of consumers. Some consumers are completely uninformed, pick a firm at random, and buy there. Other consumers are completely informed and go to the cheapest firm. Note that these two types of consumers are identical to those in the Varian (1980) model. Yet, we also introduce a third group of partially informed consumers that can only observe the recommended prices. There are many reasons as to why that may be the case. Our main argument is that recommended prices are more readily observable, and easier to check, than actual prices at individual outlets. In our model, these partially informed consumers give recommended prices some bite.

Regarding the behavior of partially informed consumers, we look at two possibilities. First, we assume that they behave according to a simple heuristic and visit the firm with the lowest recommended price. However, it turns out that this heuristic may lead to the wrong decision; often firms with a lower recommended price charge higher actual prices on average. We also look at the case in which partially informed consumers are rational, in the sense that they visit the firm with the lowest expected actual price, given the two recommended prices that they observe.
In equilibrium, we find that firms always use mixed strategies, both in the first as well as in the second stage. The equilibrium of the full game is hard to find, and involves solving a delay-differential equation. The possibility to set recommended prices is effectively a prisoners’ dilemma; both firms have an individual incentive to set such prices, yet, when both do, expected profits for both firms turn out to be lower.

Our game-theoretical model is based on our observations from the Dutch retail gasoline market, where a number of large players compete. These firms operate numerous outlets, which all charge different prices. Still, these firms work with a list price or recommended retail price that is widely publicized, for examples on websites such as http://www.nu.nl/brandstof. Consumers that fully rely on these recommended retail prices know that if they visit a gasoline retail outlet, they will never pay a price that is higher than the recommended retail price of that particular firm. Yet, in most cases, they will end up paying a lower price.

The empirical study on the Dutch gasoline market by Faber and Janssen (2008) suggests that this may facilitate collusion. We provide a different interpretation. We extend the canonical model by Varian (1980) with consumers that are not informed about all retail prices, but can observe recommended prices. Recommended retail prices then serve as an upper bound on the retail prices of a firm. We study the effect this has on competition. Our paper thus provides an alternative rationale on the use of recommended retail prices.

There is a substantial theoretical literature on recommend retail prices, also known as manufacturer suggested retail prices. Most of this work focuses on vertical relations in the industry. For instance, Buehler and Gärtner (2013) argues that manufacturers are better informed about demand uncertainty and use recommended prices to convey this information to retailers. Lubensky (2013) agrees with this explanation, but it is consumers (not retailers) who are informed about aggregate market conditions. Finally, Puppe and

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1In the real world, recommended prices are often set by a manufacturer and apply to its retailers. Yet, our model still makes sense if a manufacturer and retailer sign a contract that maximizes their joint profits (as is common in the literature on vertical restraints, see e.g. Whinston, 1989).
Rosenkranz (2011) note that recommended prices are potentially used by manufacturers to shift the profit from downstream to upstream firms.

Our explanation fits more in the behavioral industrial organization literature, where firms try to exploit boundedly-rational consumers. Heidhues and Kószegi (2014) present an interesting, behavioral rationale for recommended prices. They note that firms may set a high price for an extended period, which establishes a high reference price. In turn, this boosts demand during a short period of sales. On average, under some assumptions, profits can be higher than if firms were to use a strategy with a constant price level. We could interpret the reference price as the recommended retail price and, during the sales period, firms give a discount on the recommended price. However, this is not consistent with the empirical observations mentioned above, since, in the Dutch retail gasoline market, the discounts are persistent over time.

The paper is organized as follows. In Section 2, we introduce a two-stage game where firms decide on the recommended price in stage 1 and the actual retail price in stage 2. Section 3 presents the solution of the second stage of the game when consumers use a simple search heuristic. In Section 4, we show that in equilibrium, the firms will always use recommended prices, and solve for the first-stage equilibrium strategy for a subset of the parameter space. Finally, Section 5 explores the implications when consumers are fully rational.

2 The game

We consider a market with two firms: $i = 1, 2$. There is no cost of production and a unit mass of consumers with unit demand and a common willingness-to-pay that is normalized to 1. In stage 1, firms simultaneously and unilaterally set recommended prices or list prices $P_i$. In the second stage, after having observed all recommended prices, they decide on an actual price $p_i$. We impose that recommended prices serve as an upper bound on actual prices, so $p_i \leq P_i$. 
There are three types of consumers. A fraction $\lambda$ is fully informed, and can observe all prices. These consumers buy from the firm with the lowest price. A fraction $\mu$ is partially informed. These consumers can observe recommended prices, try to make an inference on that about actual prices, and then decide where to buy. For now, we assume that partially informed consumers use a simple rule of thumb and go to the firm with the lowest recommended price. As we shall see, this is not always the rational thing to do.\footnote{In case the firm with the lowest recommended price has a higher expected price, some of the partially informed consumers should instead buy from the firm with the highest recommended price such that both firms charge the same price in expectation. In Section 5, we analyze a variation of the model with fully rational partially informed consumers.} The remaining fraction $1 - \mu - \lambda$ of consumers is completely uninformed. They pick a firm at random. Throughout the paper, we assume that $\lambda > 0$, $\mu > 0$ and $\lambda + \mu < 1$. Note therefore that our model is a variation of Varian (1980): to his model, we add partially informed consumers and recommended prices. This game is also close to Obradovits (2014).

3 Solving stage 2

We solve by backward induction and first consider stage 2 of the game. Given the recommended prices $P_1$ and $P_2$ that are set in stage 1, what actual prices will firms charge? Denote the firm that has set the (weakly) lowest recommended price in stage 1 as firm $L$, and its recommended price as $P_L$. Similarly, the firm with the highest recommended price is denoted by $H$, and its price as $P_H$. Define the ratio of recommended prices as $R$, i.e.

$$R \equiv \frac{P_H}{P_L}.$$  

By construction, $R \geq 1$. There are two cases to consider: the firms have set identical recommended prices such that $R = 1$, or they have set different recommended prices such that $R > 1$. 

\footnote{In case the firm with the lowest recommended price has a higher expected price, some of the partially informed consumers should instead buy from the firm with the highest recommended price such that both firms charge the same price in expectation. In Section 5, we analyze a variation of the model with fully rational partially informed consumers.}
3.1 Identical recommended prices

In this case, solving the model is a standard exercise that follows Varian (1980). First, note that in this case, a fraction $\lambda$ of consumers goes to the cheapest firm, whereas the remaining fraction $1 - \lambda$ picks a firm at random. It is easy to see that an equilibrium in pure strategies does not exist. We thus look for a symmetric equilibrium in mixed strategies, in which both firms set a price that is a draw from a probability distribution with cumulative density function $F(p)$ on some interval $[\underline{p}, \bar{p}]$. Consider firm 1. Its expected profits when charging some price $p \in [\underline{p}, \bar{p}]$ equal

$$\pi(p) = \left(\frac{1}{2} (1 - \lambda) + \lambda (1 - F(p))\right) p. \tag{1}$$

This is because a firm will always sell to its share of uninformed consumers $\frac{1}{2} (1 - \lambda)$, and serve the informed consumers if its price is lower than that of its competitor, which happens with probability $1 - F(p)$. With the usual arguments, in equilibrium we necessarily have $\bar{p} = P_L$, so by construction all prices in the support of $F$ should yield profits of $\frac{1}{2} (1 - \lambda) P_L$. From (1), this implies

$$F(p) = \frac{1 + \lambda}{2\lambda} - \frac{(1 - \lambda) P_L}{2\lambda p},$$

with support $\left[\frac{1 - \lambda}{1 + \lambda} P_L, P_L\right]$.

3.2 Different recommended prices

In this case, all informed consumers go to the cheapest firm, uninformed consumers pick a firm at random, while all partially informed consumers go to firm $L$, which has the lower recommended price. We thus have a homogeneous product duopoly with price setting where, first, the two firms differ in the number of ‘loyal’ customers they face and, second, may differ in the upper bound they face on their price. In that sense, our analysis is a combination of that in Narasimhan (1988), that looks at different numbers of loyal customers, and Obradovits (2014), that considers different upper bounds on prices in the second stage of his model.
For ease of exposition, denote by \( \alpha_L \) the captured base of firm \( L \), and by \( \alpha_H \) that of firm \( H \). Thus, \( \alpha_L \) is the share of total consumers that definitely buys from firm \( L \), while \( \alpha_H \) is the share of total consumers that definitely buys from firm \( H \). As we assume presently that the partially informed consumers all go to the firm with the lower recommended price, we have

\[
\alpha_L = \frac{1 - \lambda - \mu}{2} + \mu = \frac{1 - \lambda + \mu}{2}; \quad \alpha_H = \frac{1 - \lambda - \mu}{2}.
\]

Obviously \( 0 < \alpha_H < \alpha_L < 1 \) and \( \alpha_L + \alpha_H = 1 - \lambda < 1 \).

Based on the literature, our conjecture is that the (asymmetric) Nash equilibrium is of the following form: with probability \( \sigma_i \), firm \( i \) sets a price equal to its recommended price \( P_i \), while with probability \( 1 - \sigma_i \), it draws a price from some distribution \( F_i \) with support \([\nu, P_i] \).

Consider firm \( H \) first. By setting its price equal to its recommended price, it can guarantee a profit of \( \alpha_H P_H \). Now suppose it slightly undercuts the recommended price of the other firm, \( P_L \). It will then definitely sell to its captured consumers at a profit of \( \alpha_H P_L \). Moreover, with probability \( \sigma_L \), its price is lower than that of firm \( L \), and it attracts all non-captured consumers. Thus, its expected profits when slightly undercutting \( P_L \) equal \( \alpha_H P_L + \sigma_L (1 - \alpha_L - \alpha_H) P_L \). In a mixed strategy equilibrium, firm \( H \) should be indifferent between these two options, so we need

\[
\alpha_H P_L + \sigma_L (1 - \alpha_L - \alpha_H) P_L = \alpha_H P_H,
\]

which pins down \( \sigma_L \);

\[
\sigma_L = \frac{\alpha_H (R - 1)}{1 - \alpha_L - \alpha_H}.
\]

For now, we assume that this is well-defined in the sense that \( \sigma_L \in (0, 1) \).

Suppose next that firm \( H \) sets a price \( p \) in the interval \([\nu, P_L] \). It will then definitely sell to its captured consumers at a profit of \( \alpha_H p \). With probability \( \sigma_L \), firm \( L \) sets \( P_L \), and firm \( H \) attracts all non-captured consumers. Finally, with probability \( 1 - \sigma_L \), firm \( L \) draws its price from \( F_L \). In that case, the conditional probability that firm \( H \) attracts all non-captured consumers is \( 1 - F_L(p) \). Hence, firm \( H \)'s profits from setting such a price are
\[ \alpha_H p + [\sigma_L + (1 - \sigma_L)(1 - F_L(p))](1 - \alpha_L - \alpha_H)p. \]  
Equilibrium requires that this equals \( \alpha_H P_H \), the profits firm \( H \) obtains from charging \( P_H \). Hence, we require

\[ \alpha_H p + [\sigma_L + (1 - \sigma_L)(1 - F_L(p))](1 - \alpha_L - \alpha_H)p = \alpha_H P_H; \]

This pins down \( F_L \):

\[ F_L(p) = \frac{1 - \alpha_L - \alpha_H P_H/p}{1 - \alpha_L - \alpha_H R}. \]

To find the lower bound \( \nu \), we impose \( F_L(\nu) = 0 \) to find

\[ \nu = \frac{\alpha_H P_H}{1 - \alpha_L}. \]

Now consider firm \( L \). If it sets its price equal to the lower bound \( \nu \), it will attract all \( 1 - \alpha_H \) consumers that are not captured by firm \( H \). Its profits of doing so equal \((1 - \alpha_H)\nu\). Suppose that it charges its recommended price \( P_L \). With probability \( \sigma_H \) it then attracts all non-captured consumers. Otherwise, it only attracts its captured consumers. Expected profits then equal \( \sigma_H(1 - \alpha_H)P_L + (1 - \sigma_H)\alpha_L P_L \). Again, equilibrium requires that firm \( L \) is indifferent between these two prices, i.e. we need

\[ (1 - \alpha_H) \cdot \frac{\alpha_H P_H}{1 - \alpha_L} = \sigma_H(1 - \alpha_H)P_L + (1 - \sigma_H)\alpha_L P_L. \]

This pins down \( \sigma_H \):

\[ \sigma_H = \frac{(1 - \alpha_H)\alpha_H R - (1 - \alpha_L)\alpha_L}{(1 - \alpha_L)(1 - \alpha_L - \alpha_H)}. \]

For now, we assume that this is well-defined in the sense that \( \sigma_H \in (0, 1) \). Note that \( \sigma_L = \sigma_H \) if \( \alpha_L = \alpha_H \) (cf. Obradovits, 2014).

Suppose finally that firm \( L \) sets a price \( p \) in the interval \([\nu, P_L]\). It will then definitely sell to its captured consumers at a profit of \( \alpha_L p \). With probability \( \sigma_H \), firm \( H \) sets \( P_H \), and firm \( L \) attracts all non-captured consumers. Lastly, with probability \( 1 - \sigma_H \), firm \( H \) draws its price from \( F_H \). In that case, the conditional probability that firm \( L \) attracts all non-captured consumers is \( 1 - F_H(p) \). Hence firm \( L \)'s profits from setting such a price are

\[ [\sigma_H + (1 - \sigma_H)(1 - F_H(p))](1 - \alpha_L - \alpha_H)p + \alpha_L p. \]

Equilibrium requires that these profits
equal those from setting $\nu$. Hence, we require

$$[\sigma_H + (1 - \sigma_H)(1 - F_H(p))](1 - \alpha_L - \alpha_H)p + \alpha_L p = \frac{(1 - \alpha_H)\alpha HP_H}{1 - \alpha_L},$$

After some straightforward algebra, we see that $F_H(p) = F_L(p) \equiv F(p)$.

As noted, for this to be an equilibrium, we need $\sigma_L \in [0, 1]$ and $\sigma_H \in [0, 1]$. For $\sigma_L \geq 0$, we need $P_L \leq P_H$, which is satisfied by assumption. The conditions to have $\sigma_L \leq 1$ and $\sigma_H \leq 1$ both collapse to $R \leq (1 - \alpha_L)/\alpha_H$. For $\sigma_H \geq 0$, we need $R \geq (1 - \alpha_L)\alpha_L/(1 - \alpha_H)\alpha_H$. Combining inequalities, we thus need

$$R_0 \equiv \frac{(1 - \alpha_L)\alpha_L}{(1 - \alpha_H)\alpha_H} \leq R \leq \frac{1 - \alpha_L}{\alpha_H} \equiv R_1.$$ 

Note that $(1 - \alpha_L)\alpha_L > (1 - \alpha_H)\alpha_H$ and hence that $R_0 > 1$.\(^3\) It also implies

$$\frac{R_1}{R_0} = \frac{1 - \alpha_H}{\alpha_L} > 1,$$

hence the interval $(R_0, R_1)$ is well-defined. We thus observe that this type of equilibrium requires that the ratio of recommended prices is neither too small nor too large.

### 3.2.1 Large recommended price ratio

Suppose now that $R > R_1$. In that case, (2) cannot be satisfied. In other words, in that case firm $H$’s profits from charging $P_L$ are always lower than its profits from charging $P_H$. Hence $p_H = P_H$. Firm $L$ then simply charges the best reply to $P_H$, which is $p_L = P_L$. All consumers that are not captured by firm $H$ will then buy at firm $L$, so its profits are $(1 - \alpha_H)P_L$, while the profits of firm $H$ equal $\alpha_H P_H$.

### 3.2.2 Small recommended price ratio

Suppose finally that $R \leq R_0$. Then we may venture that $\sigma_H = 0$, such that the firm with the highest recommended price will always undercut $P_L$. However, firm $L$ will still charge

\(^3\)To see that $(1 - \alpha_L)\alpha_L > (1 - \alpha_H)\alpha_H$, observe that this is equivalent to showing that $\alpha_L - \alpha_H > \alpha_L^2 - \alpha_H^2 = (\alpha_L + \alpha_H)(\alpha_L - \alpha_H)$. Dividing both sides by $\alpha_L - \alpha_H > 0$ yields the condition $\alpha_L + \alpha_H < 1$, which is true by assumption.
the recommended price with positive probability \( \sigma_L \). Moreover, we assume that both firms’
strategies have the same support \([\nu, P_L]\).

Observe that firm \( L \) can guarantee a payoff of \( \alpha_L P_L \) by charging the recommended price.
Firm \( H \)’s strategy must keep \( L \) indifferent when it charges \( p \in [\nu, P_L] \):

\[
\alpha_L P_L = \alpha_L p + (1 - \alpha_L - \alpha_H)p(1 - F_H(p)), \tag{3}
\]

which is true at \( p = P_L \) since \( F_H(P_L) = 1 \). Note that \( F_H(\nu) = 0 \) implies \( \nu = \frac{\alpha_L P_L}{1 - \alpha_H} \).

Similarly, to keep firm \( H \) indifferent when it charges \( p \in [\nu, P_L] \), we obtain the requirement

\[
\text{constant} = \alpha_H p + (1 - \alpha_L - \alpha_H)p(\sigma_L + (1 - \sigma_L)(1 - F_L(p))). \tag{4}
\]

To determine the constant, i.e. the expected profit level of firm \( H \), note that at \( p = \nu \), firm
\( H \) deterministically attracts all consumers that are not captured by firm \( L \), hence

\[
\text{constant} = (1 - \alpha_L)\nu.
\]

Inserting \( \nu \), the expected profit level of firm \( H \) is thus given by \( \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} P_L \). Using this expected profit level and \( F_L(P_L) = 1 \), we can determine \( \sigma_L \):

\[
\frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} P_L = \alpha_H P_L + (1 - \alpha_L - \alpha_H)P_L\sigma_L,
\]

which implies that

\[
\sigma_L = \frac{\alpha_L - \alpha_H}{1 - \alpha_H}.
\]

Then we can derive the expressions for \( F_L \) and \( F_H \) from (4) and (3), respectively. Remark
that the constant in (4) is equal to the expected profit level of firm \( H \). It turns out that

\[
F_L(p) = F_H(p) = 1 - \frac{\alpha_L(P_L/p - 1)}{1 - \alpha_L - \alpha_H}.
\]

We now summarize the relevant information that will be used for stage 1 of the game in
each of the different cases.

**Proposition 1.** The (expected) equilibrium profits of stage 2 of the game are given by
- Case A: $R < R_0$. The expected profit of firm $L$ is $\alpha_L P_L$. The expected profit of firm $H$ is $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L$.

- Case B: $R \in [R_0, R_1]$. The expected profit of firm $L$ is $\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L} P_H$. The expected profit of firm $H$ is $\alpha_H P_H$.

- Case C: $R > R_1$. The expected profit of firm $L$ is $(1-\alpha_H) P_L$. The expected profit of firm $H$ is $\alpha_H P_H$.

### 3.3 Properties of the equilibrium in stage 2

The results we derived above already allow us to pin down some interesting implications concerning the frequency and depth of discounts that firms give vis-à-vis their recommended price.

**Result 1.** The minimum discount that firm $H$ offers is $P_H - P_L$.

The firm with the highest recommended price will never set a small discount, but always undercuts the lowest recommended price.

**Result 2.** In case $B$, firm $H$ is more likely to offer a discount: $\sigma_H < \sigma_L$. In case $A$, firm $H$ always offers a discount.

Since the firm with the highest recommended price is more likely to offer a discount, the partially informed consumer may benefit from buying from the firm with the highest recommended price. Figure 1 shows that the partially informed consumers’ heuristic indeed fails for a range of $P_L$. In particular, this happens near the point where case $B$ shifts to case $A$. In fact, we can show that this is true in general.

**Proposition 2.** There exists a unique $R^* \in (R_0, R_1)$ such that for all $R < R^*$ ($R > R^*$), the expected price of firm $H$ is lower (higher) than the expected price of firm $L$.

**Proof.** Observe first that if $R > R_1$ ($P_L < P_H/R_1$), we are in case $C$ where both firms set a price equal to their recommended price and the expected price of firm $H$ is higher than the expected price of firm $L$. Note next that if $R < R_0$ ($P_L > P_H/R_0$), we are in case $A$
where the only difference between firm $L$ and firm $H$ is that firm $L$ sets its price equal to its recommended price with a strictly positive probability. Hence the expected price of firm $H$ is lower than the expected price of firm $L$. To complete the proof, we now establish that for case $B$ where $R \in [R_0, R_1]$ ($P_L \in (P_H/R_1, P_H/R_0)$), firm $L$’s expected price strictly increases in $P_L$, while firm $H$’s price strictly decreases in $P_L$. This implies that there must be a unique value of $P_L \in (P_H/R_1, P_H/R_0)$ such that the firms’ expected prices are equal. To see this, note that after some straightforward calculation, we find for case $B$ that

$$E_{p_L} = \frac{\alpha_H}{1 - \alpha_H - \alpha_L} \left[ P_H - P_L + P_H \log \left( \frac{1 - \alpha_L}{\alpha_H} \frac{P_L}{P_H} \right) \right],$$

while

$$E_{p_H} = \frac{P_H}{(1 - \alpha_H)(1 - \alpha_L)} \left[ \alpha_H(1 - \alpha_H)P_H/P_L - \alpha_L(1 - \alpha_L) + (1 - \alpha_H)\alpha_H \log \left( \frac{1 - \alpha_L}{\alpha_H} \frac{P_L}{P_H} \right) \right].$$

From this, we easily obtain

$$\frac{dE_{p_L}}{dP_L} = \frac{\alpha_H}{1 - \alpha_H - \alpha_L} (P_H/P_L - 1) > 0,$$

while

$$\frac{dE_{p_H}}{dP_L} = -\frac{P_H}{P_L} \left[ \frac{\alpha_H(1 - \alpha_H)}{(1 - \alpha_L)(1 - \alpha_H - \alpha_L)} \right] (P_H/P_L - 1) < 0.$$

\section{Solving stage 1}

First, it is straightforward to establish the following:

\textbf{Theorem 1.} \textit{There exists no symmetric pure-strategy equilibrium.}

\textbf{Proof.} Suppose that in equilibrium $P_L = P_H = P^*$. Equilibrium profits would then equal $(1 - \lambda) P^*/2$. Now suppose firm $L$ defected to setting some slightly lower recommended price $\hat{P}_L < P^*$. It would then find itself in case $A$ above, making profits $(1 - \lambda + \mu) \hat{P}_L/2$. Hence this defection is profitable (for $\hat{P}_L$ sufficiently close to $P^*$) and the original situation is not an equilibrium. \hfill \blacksquare

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Corollary 1. In equilibrium, recommended prices will be used.

Proof. An equilibrium without recommended prices is equivalent to \( P_L = P_H = 1 \), which would constitute a (non-existent) symmetric pure-strategy equilibrium. ■

The challenge is now to derive the (symmetric) mixed-strategy equilibrium.\(^4\) Let the support of the equilibrium recommended price distribution \( G(\cdot) \) be denoted by \([P, \bar{P}]\). We first establish some preliminary results.

Lemma 1. \( G(\cdot) \) is atomless.

Proof. Suppose to the contrary that \( G(\cdot) \) had an atom at some recommended price \( P^* \). Then, both firms would choose \( P^* \) with strictly positive probability \( \beta > 0 \), giving rise to profits of \( \frac{1-\lambda}{2} P^* \) in that case. Consequently, by an identical argument as in the proof of Theorem 1 above, either firm could profitably transfer its probability mass at \( P^* \) to some marginally lower recommended price \( \hat{P} < P^* \) (since this would lead to strictly higher profits with probability \( \beta \)). Hence, there can be no atoms in equilibrium. ■

Lemma 2. There exists no equilibrium in which \( \frac{P}{\bar{P}} \leq R_0 \), such that only case A would be relevant for every pricing subgame.

Proof. Suppose to the contrary that this was the case. Then, the firm with the lower recommended price would always make an expected profit of \( \pi_L = \alpha_L P_L \), while the firm with the higher recommended price would always make an expected profit of \( \pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L < \pi_L \).

Recalling that firms’ candidate equilibrium CDF must be atomless (see Lemma 1 above), a firm setting \( P \in [P, \bar{P}] \) as recommended price would thus make an expected profit of

\[
\Pi_i(P) = \int_P^\bar{P} \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} PG'(P) dP + [1-G(P)]\alpha_L P.
\]

\(^4\)Existence of such an equilibrium follows from Dasgupta and Maskin (1986). However, this is no guarantee for a well-behaved equilibrium distribution, e.g. Ewerhart (2015) provides an example of a game whose mixed-strategy equilibrium has countably-infinite many mass points.
Since every recommended price in each firm’s support must yield the same expected profit, it follows that
\[
\Pi'_i(P) = \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} PG'(P) + [1 - G(P)]\alpha_L - \alpha_L PG'(P) = 0.
\]
The general solution to the above first order differential equation is given by
\[
G(P) = 1 + k \cdot P^{-\frac{1 - \alpha_H}{\alpha_L - \alpha_H}},
\]
where \(k\) is obtained from the terminal condition \(G(\overline{P}) = 1\). But the latter implies \(k = 0\), such that the candidate CDF becomes \(G(P) = 1\). Clearly, this is incompatible with the considered equilibrium.

**Lemma 3.** \(\overline{P} = 1\).

**Proof.** Suppose to the contrary that \(\overline{P} < 1\). Then, if firm \(i\) deviates and sets some recommended price \(P_i \in (\overline{P}, 1]\), it makes an expected subgame profit of either \(\alpha_H P_i\) (if the other firm chooses its recommended price \(P_j \leq P_i/R_0\) such that either case \(B\) or case \(C\) applies) or \((1 - \alpha_L)\alpha_L P_i / R_0\) (if \(P_j \in (P_i/R_0, \overline{P}]\) such that case \(A\) applies). Hence, we can write firm \(i\)’s overall expected profit at \(P_i\) as
\[
\pi_i(P_i) = G(P_i/R_0)\alpha_H P_i + \int_{P_i/R_0}^{\overline{P}} \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} PG'(P)dP.
\]
Taking the derivative with respect to \(P_i\) yields
\[
\pi'_i(P_i) = G(P_i/R_0)\alpha_H + G'(P_i/R_0)\alpha_H P_i/R_0 - 1/R_0 \left[ \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} \right] G'(P_i/R_0) P_i/R_0
\]
\[= G(P_i/R_0)\alpha_H.\]
Hence, \(\lim_{\epsilon \downarrow 0} \pi'_i(\overline{P} + \epsilon) = G(\overline{P}/R_0)\alpha_H > 0\), where the last inequality follows from Lemma 2. But then, since setting a recommended price marginally higher than \(\overline{P}\) would pay, and since this would be feasible due to \(\overline{P} < 1\), the latter cannot be part of an equilibrium. ■

From Lemma 3, we thus know that the support of the equilibrium distribution \(G(\cdot)\) is \([\underline{P}, 1]\), with \(\underline{P}R_0 < 1\) due to Lemma 2. Now, recall that if a firm sets a recommended price
$P \in [P, 1]$, its expected profit depends on whether it has the lowest recommended price and whether case $A$, $B$ or $C$ is relevant. So, we have to take a weighted average over these possible cases and ensure that firms’ expected profit does not vary with $P$, such that

$$
\text{constant} = G(P/R_0)\alpha_H P + \frac{(1 - \alpha_L)\alpha_L}{\alpha_H} \int_{\max\{P/R_0, L\}}^P sG'(s)ds + [G(PR_0) - G(P)]\alpha_L P
+ \frac{(1 - \alpha_H)\alpha_H}{\alpha_L} \int_{\min\{PR_1, 1\}}^1 sG'(s)ds + [1 - G(PR_1)](1 - \alpha_H)P. \tag{5}
$$

We thus have to solve a delay-differential equation (since $G$ is evaluated at $R_0/P, P, R_0 P$ and $R_1 P$). The procedure detailed below works under the assumptions that $R_1 P \geq 1$ (such that case $C$ will never occur in any pricing subgame) and $P \geq 1/R_0^2$.

The first step is to partition the support: $I_1 = [P, 1/R_0], I_2 = [1/R_0, PR_0)$ and $I_3 = [PR_0, 1]$, which by assumption are non-empty intervals. Denote the part of the distribution function in partition $i = 1, 2, 3$ by $G_i$. Obviously $G_1(P) = 0$, $G_1(1/R_0) = G_2(1/R_0)$, $G_2(PR_0) = G_3(PR_0)$, and $G_3(1) = 1$.

Note next that for recommended prices $P \in I_1$, we have $PR_0 \in I_3$ and $P/R_0 < P$. Hence (5) reduces to

$$
\text{constant} = \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} \int_{P}^1 sG'_1(s)ds + [G_3(PR_0) - G_1(P)]\alpha_L P + \frac{(1 - \alpha_H)\alpha_H}{1 - \alpha_L} \int_{R_0 P}^1 sG'_3(s)ds. \tag{6}
$$

Taking the derivative with respect to $P$ and simplifying eventually yields

$$
G_1(P) + \left(\frac{\alpha_L - \alpha_H}{1 - \alpha_H}\right) PG'_1(P) - G_3(PR_0) = 0.
$$

For recommended prices $P \in I_2$, we have $PR_0 > 1$ and $P/R_0 < P$. Therefore (5) reduces to

$$
\text{constant} = \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} \int_{P}^{1/R_0} sG'(s)ds + \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} \int_{1/R_0}^P sG'(s)ds + (1 - G_2(P))\alpha_L P.
$$

Taking the derivative with respect to $P$ yields

$$
1 - G_2(P) - \left(\frac{\alpha_L - \alpha_H}{1 - \alpha_H}\right) PG'_2(P) = 0.
$$
which implies that $G_2$ has the form

$$G_2(P) = 1 - B_0 P^{-1/k},$$

(7)

where

$$k = \frac{\alpha_L - \alpha_H}{1 - \alpha_H} \in (0, 1),$$

(8)

and $B_0$ is a coefficient to be determined.

For recommended prices $P \in \mathcal{I}_3$, we have $PR_0 > 1$ and $P/R_0 \in \mathcal{I}_1$. In this case, (5) reduces to

$$\text{constant} = G_1(P/R_0) \alpha_H P + \frac{(1 - \alpha_L)\alpha_L}{1 - \alpha_H} \left[ \int_{P/R_0}^{R_0} sG_1'(s) ds + \int_{P/R_0}^{R_0} sG_2'(s) ds + \int_{P/R_0}^{P} sG_3'(s) ds \right] + [1 - G_3(P)] \alpha_L P.$$  

(9)

Differentiating with respect to $P$ and simplifying we eventually obtain

$$1 - G_3(P) - k PG_3'(P) + G_1(P/R_0) \frac{\alpha_H}{\alpha_L} = 0.$$  

(9)

We now employ a trick to solve for $G_1$ and $G_3$: We first introduce the new variable $z \equiv P/R_0$, which we substitute in (9) to get

$$G_1(z) = \frac{\alpha_L}{\alpha_H} \left[ k R_0 z G_3'(R_0 z) - (1 - G_3(R_0 z)) \right].$$

(10)

After taking the derivative with respect to $z$ and simplifying, we obtain

$$G_1'(z) = \frac{\alpha_L}{\alpha_H} \left[ R_0 G_3'(R_0 z) (k + 1) + k R_0^2 z G_3''(R_0 z) \right].$$

We then plug these expressions for $G_1$ and $G_1'$ into (6), which, after simplification, yields a second-order differential equation for $G_3$ of the following form:

$$1 - G_3(P) \left( \frac{\alpha_L - \alpha_H}{\alpha_L} \right) - k(2 + k) PG_3'(P) - k^2 P^2 G_3''(P) = 0,$$

(11)

We can now conjecture that $G_3(P)$ has the following functional form:

$$G_3(P) = a + b_1 P^{c_1} + b_2 P^{c_2},$$

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such that $PG'_3(P) = b_1 c_1 P^{c_1} + b_2 c_2 P^{c_2}$ and $P^2G''_3(P) = b_1 c_1 (c_1 - 1) P^{c_1} + b_2 c_2 (c_2 - 1) P^{c_2}$.

Substituting these expressions and comparing coefficients, we find that
\begin{equation}
    a = \frac{\alpha_L}{\alpha_L - \alpha_H},
\end{equation}
\begin{equation}
    c_{1,2} = -\frac{1}{k} \left( 1 \pm \sqrt{\frac{\alpha_H}{\alpha_L}} \right),
\end{equation}
while $b_1$ and $b_2$ are still unspecified.

Note that both $c_1$ and $c_2$ are defined by the solution to an identical quadratic equation.\(^5\) However, $c_1$ and $c_2$ are still uniquely determined (up to permutation), as otherwise (if the same root was taken twice), $G_3(P)$ would collapse to the form $G_3(P) = a + bP^c$, which, with just one free parameter $b$, would be insufficient to give a general solution to a second-order differential equation. For concreteness, let
\begin{equation}
    c_1 = -\frac{1}{k} \left( 1 - \sqrt{\frac{\alpha_H}{\alpha_L}} \right),
\end{equation}
\begin{equation}
    c_2 = -\frac{1}{k} \left( 1 + \sqrt{\frac{\alpha_H}{\alpha_L}} \right).
\end{equation}
Hence we have that
\begin{equation}
    G_3(P) = \frac{\alpha_L}{\alpha_L - \alpha_H} + b_1 P^{-\frac{1}{c_1}} \left( 1 - \sqrt{\frac{\alpha_H}{\alpha_L}} \right) + b_2 P^{-\frac{1}{c_2}} \left( 1 + \sqrt{\frac{\alpha_H}{\alpha_L}} \right),
\end{equation}
where $b_1$ and $b_2$ are yet to be determined.

The requirement $G_3(1) = 1$ then immediately pins down the equilibrium relationship between $b_1$ and $b_2$. Using the above equation, we find that
\begin{equation}
    b_2 = b_2(b_1) = 1 - a - b_1.
\end{equation}

Analogously to above, we next introduce the variable $q \equiv PR_0$, which we substitute in (6) to get
\begin{equation}
    G_3(q) = G_1(q/R_0) + k(q/R_0)G'_1(q/R_0).
\end{equation}

\(^5\)Precisely, $k^2 c_i^2 + 2kc_i + \frac{\alpha_L - \alpha_H}{\alpha_L} = 0$ for $i = 1, 2$. 

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After taking the derivative with respect to $q$, we obtain

$$G_3'(q) = G_1'(q/R_0)/R_0 + kG_1'(q/R_0)/R_0 + k(q/R_0^2)G_1''(q/R_0).$$

We once more plug these expressions for $G_3$ and $G_3'$ into the relevant equation (9), which, after simplification, yields a second-order differential equation for $G_1$ of the following form:

$$1 - G_1(P) \left( \frac{\alpha L - \alpha H}{\alpha L} \right) - k(2 + k)PG_1'(P) - k^2P^2G_1''(P) = 0. \quad (18)$$

Observe that this differential equation perfectly coincides with the one obtained for $G_3(P)$ above. Hence, an analogous argument reveals that $G_1(P)$ must have the following form:

$$G_1(P) = a + \beta_1 P^{c_1} + \beta_2 P^{c_2},$$

where $a$, $c_1$ and $c_2$ have already been specified above.

Indeed, using equations (15) and (10), we find that, conditional on $b_1$, it holds that

$$G_1(P) = a + \left[ b_1 R_0^{c_1} \sqrt{\frac{\alpha L}{\alpha H}} \right] P^{c_1} + \left[ -b_2(b_1) R_0^{c_2} \sqrt{\frac{\alpha L}{\alpha H}} \right] P^{c_2},$$

which pins down $\beta_1$ and $\beta_2$ as functions of $b_1$:

$$\beta_1(b_1) = b_1 R_0^{c_1} \sqrt{\frac{\alpha L}{\alpha H}}, \quad (19)$$

$$\beta_2(b_1) = -b_2(b_1) R_0^{c_2} \sqrt{\frac{\alpha L}{\alpha H}}. \quad (20)$$

Note next that $G_1(1/R_0)^{1} \overset{\gamma}{=} G_2(1/R_0)$. From this, we can pin down $B_0$ as a function of $b_1$,

$$B_0(b_1) = \frac{1 - a - \beta_1(b_1) R_0^{-c_1} - \beta_2(b_1) R_0^{-c_2}}{R_0^{1/k}}.$$

Inserting $\beta_1(b_1)$, $\beta_2(b_1)$ and using that $b_1 + b_2(b_1) = 1 - a$ then leads to

$$B_0(b_1) = \frac{(1 - a) \left( 1 + \sqrt{\frac{\alpha L}{\alpha H}} \right) - 2b_1 \sqrt{\frac{\alpha L}{\alpha H}}}{R_0^{1/k}}. \quad (21)$$
Using moreover that \( G_2(P R_0) = G_3(P R_0) \), we obtain the following equation for \( b_1 \), conditional on \( P \):

\[
1 - B_0(b_1) \cdot (P R_0)^{-1/k} = a + b_1(P R_0)^{c_1} + (1 - a - b_1)(P R_0)^{c_2}.
\]

Note that this equation is linear in \( b_1 \), hence we can directly solve for \( b_1 \), given \( P \):

\[
b_1(P) = \frac{(1 - a)[1 - (P R_0)^{c_2}]}{(P R_0)^{c_1} - (P R_0)^{c_2} - e(P R_0)^{-1/k}}, \tag{22}
\]

where

\[
d = (1 - a) \left( 1 + \sqrt{\frac{\alpha_L}{\alpha_H}} \right) R_0^{-1/k}, \tag{23}
\]

\[
e = 2 \left( \sqrt{\frac{\alpha_L}{\alpha_H}} \right) R_0^{-1/k}. \tag{24}
\]

In turn, one can immediately calculate \( b_2(b_1) \), \( \beta_1(b_1) \), \( \beta_2(b_1) \) and \( B_0(b_1) \) as functions of \( b_1(P) \). Therefore, any given \( P \) fully characterizes our three equilibrium CDFs \( G_1(P) \), \( G_2(P) \) and \( G_3(P) \).

The final step to solve for equilibrium is the consistency requirement that

\[
G_1(P; a, \beta_1(b_1(P)), \beta_2(b_1(P))) = 0. \tag{25}
\]

Unfortunately, it turns out that this equation is highly complex, such that it generally

\[\text{does not permit a closed-form solution. However, after solving numerically for } P, \text{ we can}\]

immediately determine the full equilibrium CDF by making use of the above equations.

The respective candidate equilibrium then indeed constitutes an equilibrium if our initial

\[\text{assumptions are satisfied: } P \in [1/R_0^2, 1/R_0) \text{ and } R_1 P \geq 1.}\]

\[\text{Thus, to solve for the equilibrium in the first round, the following algorithm is used:}\]

1. Determine \( R_0 \), \( k \), \( a \), \( c_1 \), \( c_2 \), \( d \) and \( e \) by inserting the parameters \( \alpha_L \) and \( \alpha_H \).
2. Numerically solve the consistency requirement (25) in order to obtain \( P \).
3. If \( P \in [1/R_0^2, 1/R_0) \) and \( R_1 P \geq 1 \), proceed with step 4. Otherwise, the conjectured equilibrium fails to exist for the considered parameter combination.
4. Use equation (22) to solve for \( b_1(P) \).
5. Set \( b_2 = 1 - a - b_1 \) and use equations (19), (20) and (21) to solve for \( \beta_1 \), \( \beta_2 \) and \( B_0 \). The equilibrium CDFs \( G_1 \), \( G_2 \) and \( G_3 \) are then fully pinned down.
In the remainder of this section, we will demonstrate the equilibrium effects of recommended prices on consumer welfare. For the example parameter values $\lambda = \frac{1}{3}$ and $\mu = \frac{1}{8}$ (which correspond to $\alpha_L = 19/48$ and $\alpha_H = 13/48$), Figure 2 shows the equilibrium distribution of recommended prices.

Observe that the median recommended price is approximately 0.81, which is already quite close to the median retail price of approximately 0.67 if firms could abstain from recommended prices (which is equivalent to $P_H = P_L = 1$). This shows that firms’ self-imposed upper bounds on their retail prices may induce lower retail prices. Nevertheless, consumer welfare might still go down if lower price caps (in the form of recommended prices) would lead to significantly lower discounts on retail prices. Simulations show that this is not the case: with recommended prices, the median retail price drops to approximately 0.56. Hence, consumers benefit from the existence of recommended retail prices. The intuition behind this result is as follows. In the first stage of the game, there is fierce competition for the partially informed consumers, while in the second stage, there is again competition for the fully informed consumers. Since firms would like to commit not to use recommended prices, this is essentially a prisoner’s dilemma.

5 Rational consumers

So far, we assumed that partially informed consumers use a simple heuristic: they visit the firm with the lowest recommended price. Yet, as we saw, this is not always the rational thing to do: in the equilibrium of the second stage, it may very well happen that the firm with the highest recommended price has the lowest expected price. In this section, we therefore impose that partially informed consumers are fully rational.

Suppose that firms set recommended prices $P_L$ and $P_H$ such that $P_L < P_H$. In this case, we denote the expected price of firm $L$ in the second stage if a fraction $\theta$ of partially informed consumers visits firm $L$ as $E(p_L; P_L, P_H, \theta)$. Similarly, the expected price of firm $H$ if a fraction $\theta$ of partially informed consumers visits firm $L$ is denoted $E(p_H; P_H, P_L, 1 - \theta)$. So
Figure 1: Expected price of firm $L$ (black line) and firm $H$ (red line) as a function of the recommended price of firm $L$ ($P_H = 1$, $\lambda = \frac{1}{3}$, $\mu = \frac{1}{8}$, thin lines indicate case boundaries).

Figure 2: Equilibrium distribution of recommended prices ($\lambda = \frac{1}{3}$, $\mu = \frac{1}{8}$).
far we have assumed that $\theta = 1$. But if $E(p_L; P_H, P_L, 1) > E(p_H; P_H, P_L, 0)$, that cannot be part of an equilibrium with rational consumers. In that case, we need partially informed consumers to randomize, and we need them to visit firm $L$ with probability $\tilde{\theta}$ such that $E(p_L; P_L, P_H, \tilde{\theta}) = E(p_H; P_H, P_L, 1-\tilde{\theta})$. Note that $E(p_L; P_L, P_H, \tilde{\theta})$ and $E(p_H; P_H, P_L, 1-\tilde{\theta})$ can simply be found by solving our baseline model with $\alpha_L = \left(1 - \lambda + \tilde{\theta}\mu\right)/2$ and $\alpha_H = \left(1 - \lambda + (1 - \tilde{\theta})\mu\right)/2$.

Hence, our algorithm to deal with fully rational consumers is as follows:

1. Solve the model under the assumption that partially informed consumers always go to the firm with the lowest recommended price, as we did in the previous section.

2. For recommended prices $(P_L, P_H)$ such that $E(p_L; P_L, P_H, 1) > E(p_H; P_H, P_L, 0)$, look for $\tilde{\theta}$ such that $E(p_L; P_L, P_H, \tilde{\theta}) = E(p_H; P_H, P_L, 1-\tilde{\theta})$. As $E(p_L; P_L, P_H, \theta)$ is strictly increasing in $\theta$ and $E(p_H; P_H, P_L, 1-\theta)$ is strictly decreasing in $\theta$, such a $\theta$ necessarily exists.

Figure 3 gives a detail of Figure 1, but now also includes the scenario of fully rational partially informed consumers. Interestingly, if consumers are rational, expected prices may be higher. This can be understood as follows. Note first that if all consumers go to the firm with the lowest recommended price, then competition for these consumers is relatively fierce. Having rational consumers, however, necessarily implies that not all consumers go to the firm with the lowest recommended price, provided that consumers’ heuristic would fail if they did. Hence, competition for these consumers becomes less fierce, and in equilibrium, the expected actual prices may be higher as a result.

We finally turn to the question under which circumstances recommended prices will be used when the partially informed consumers are fully rational. The following proposition gives a surprisingly simple answer.

**Proposition 3.** Suppose that the partially informed consumers are fully rational. In this case, if $\lambda \geq 1/3$, then for every permissible $\mu \in (0, 1 - \lambda)$, there exists an equilibrium in which no recommended prices are used. [Open question: Can we show that no other (at least
Figure 3: Expected price of firm $L$ (black line) and firm $H$ (red line) as a function of the recommended price of firm $L$ under the heuristic, and if consumers are rational (blue line). Note that the left-hand panel is a detail of Figure 1. The right-hand panel gives the required value of $\tilde{\theta}$. 
symmetric) equilibria exist?] If instead \( \lambda < \frac{1}{3} \), then for every permissible \( \mu \in (0, 1 - \lambda) \), recommended prices will be used in any equilibrium.

Proof. We start with the case \( \lambda \geq \frac{1}{3} \). We thus need to show that \( P_1 = P_2 = 1 \), with corresponding expected profits of \( \frac{1-\lambda}{2} \) for each firm, does not give rise to a profitable (first period) deviation. Suppose first that firm \( i \in \{1, 2\} \) deviates to some \( P_i \leq 1/R_1 \) (such that \( P_j/P_i = 1/P_i \geq R_1 \)). From case \( C \), we know that its expected profit is then given by \( (1 - \alpha_H)P_i \), which is thus maximized for the boundary \( P_i = 1/R_1 \), i.e. the maximal deviation profit in case \( C \) is given by

\[
(1 - \alpha_H)/R_1 = \frac{\alpha_H(1 - \alpha_H)}{1 - \alpha_L} = \frac{(1 - \lambda - \mu)(1 + \lambda + \mu)}{2(1 - \lambda - \mu)}.
\]

A straightforward manipulation then shows that this does not exceed the candidate equilibrium profits of \( \frac{1-\lambda}{2} \) if and only if \( \lambda \geq \frac{1-\mu}{3} \). Clearly, this is satisfied for \( \lambda \geq 1/3 \), as assumed.

Suppose next that firm \( i \) deviates to some \( P_i \in (1/R_1, 1/R_0) \) (such that \( P_j/P_i \in (R_0, R_1) \)) for which the expected price of firm \( H \) still strictly exceeds the expected price of firm \( L \) (from Proposition 2, we know that this is the case when \( P_i \) lies sufficiently close above \( 1/R_1 \)). From case \( B \), we can again infer that the deviating firm’s expected profit is then given by \( \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L} \) (independent of \( P_i \)). But from the above argument, where we worked with the same (maximal) deviation profit for case \( C \), we already know that such a deviation is not profitable.

We still have to check whether a deviation to an even higher recommended price may pay. For such prices, in an equilibrium with perfectly rational partially informed consumers, only a fraction \( \tilde{\theta} \) of these consumers goes to firm \( L \), and \( \tilde{\theta} \in (1/2, 1) \) is determined endogenously such that firms’ expected prices are equalized. However, the structure of the ensuing pricing equilibrium is still the same as in case \( B \), just firms’ shares of loyal consumers are adjusted, with \( \tilde{\alpha}_L = \frac{1-\lambda-\mu}{2} + \tilde{\theta} \mu \) and \( \tilde{\alpha}_H = \frac{1-\lambda-\mu}{2} + (1 - \tilde{\theta}) \mu \). Hence, we still know that the deviating
firm’s (with the lower recommended price) expected profit is given by

$$
\frac{(1 - \tilde{\alpha}_H)\tilde{\alpha}_H}{1 - \tilde{\alpha}_L} = \left( \frac{1 - \frac{\lambda + \mu}{2} - \tilde{\theta} \mu}{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu} \right) \left( \frac{1 + \frac{\lambda - \mu}{2} + \tilde{\theta} \mu}{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu} \right).
$$

One can now easily establish that the derivative of the deviating firm’s expected profit with respect to \( \tilde{\theta} \) equals

$$
\mu \left[ 1 - \frac{\lambda(1 + \lambda)}{\left( \frac{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu}{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu} \right)^2} \right].
$$

(26)

The statement is now proven if we can show that this is (weakly) negative for \( \lambda \geq 1/3 \), which implies that a deviating firm’s expected profit is maximal for the lowest possible \( \tilde{\theta} = 1/2 \), such that it is indeed optimal to set \( P_i = 1 \) (this is because for all \( P_i < 1 \), it holds that \( \tilde{\theta} > 1/2 \)). We thus want to show that

$$
1 - \frac{\lambda(1 + \lambda)}{\left( \frac{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu}{1 + \frac{\lambda + \mu}{2} - \tilde{\theta} \mu} \right)^2} = 1 - \frac{\lambda(1 + \lambda)}{\left( \frac{1 + \lambda}{2} - \mu(\tilde{\theta} - 1/2) \right)^2} \leq 0.
$$

Clearly, since \( \tilde{\theta} \geq 1/2 \), the inequality is hardest to satisfy for \( \mu = 0 \). Hence, a sufficient condition is that \( 1 - \frac{\lambda(1 + \lambda)}{\left( \frac{1 + \lambda}{2} \right)^2} \leq 0 \), which is equivalent to \( \lambda \geq 1/3 \), as assumed.

We now turn to the complementary case where \( \lambda < 1/3 \). We want to establish that for this case, a deviation from \( P_1 = P_2 = 1 \) always pays, such that recommended prices will be used in any equilibrium. Clearly, a sufficient condition for this is that a marginal deviation downward is always profitable. For any given \( \tilde{\theta} \in (1/2, 1) \), we have already pinned down the derivative of a deviating firm’s expected profit with respect to \( \tilde{\theta} \) (see expression (26) above). Evaluated at \( \tilde{\theta} = 1/2 \) (which is the relevant value for a marginal deviation from the candidate equilibrium), this becomes

$$
\mu \left[ 1 - \frac{\lambda(1 + \lambda)}{\left( \frac{1 + \lambda}{2} \right)^2} \right] = \mu \left( 1 - \frac{4\lambda}{1 + \lambda} \right).
$$

Note that this is strictly positive for \( \lambda < 1/3 \), which means that a marginal deviation of \( P_i \) downward in order to increase \( \tilde{\theta} \) always pays (for \( \lambda < 1/3 \)). This completes the proof. ■
[Intuition...]
References


