Competition and welfare consequences of information platforms

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Abstract
Exogenous shocks in the informational level of a subset of consumers may affect the market structure, equilibrium and welfare. Information platforms (e.g. Yelp, Airbnb) provide information about experience goods, such as restaurants and lodging. Their existence may cause an heterogeneity in the level of information available to potential consumers, depending on their access to the platform. This study fosters our understanding of how information platforms impacted competition, profits and welfare. Using a spokes model of horizontal competition, I show that platforms may enhance welfare by increasing the value of realised transactions (better matching) and by increasing the size of the market. However, they may also enable producers to extract a larger share of surplus and variety may decrease. Therefore, consumers are always ex ante better off with the platform, however some of them will be worse off ex post, for their preferred variety has disappeared.

JEL Classification: D02, D21, D43, D61, D83, L11, L13, L15

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1 Introduction

Information platforms (also known as information aggregators) publish opinions and facts, allowing both objective and subjective information on a product to be easily accessible to potential customers. Tourist and restaurant guides (e.g. Lonely Planet and Michelin guide) were arguably the forerunners of current information aggregators, providing objective facts (such as location and contact information) as well as a subjective review. However, the technology in the past did not allow for constant updates; besides, having only one review was a limitation. Online information platforms appeared in the 2000s and have enjoyed a rapid expansion ever since. Information platforms improved upon guides by allowing consumers to post their reviews, thus ensuring an always up-to-date monitoring of the firm, more variety in reviews and a lower risk of bribery. Furthermore, online aggregators are often enhanced with features that bring down search costs to (almost) zero.

Despite their heterogeneity, aggregators all operate in markets with asymmetric information regarding experience goods. The lack of information alters both consumers’ expected utility and total welfare by inducing mismatch costs. By mismatch costs I refer to the welfare loss that occurs in three different cases: i) poor match of consumers with products (i.e. alternative transactions would produce a larger surplus), ii) welfare decreasing transactions (i.e. the ex-post consumer’s valuation of the good is lower than the costs), and iii) failure to generate surplus-enhancing transactions (i.e. a consumer refrains from consumption, while at least one welfare-enhancing transaction is feasible). Mismatch costs may occur with either vertically or horizontally differentiated goods. I restrain my attention to the latter case.

Producers may fail to be credible if they try to release information on product characteristics. Furthermore, a producer would not provide the important service of gathering together the information about all the competitors, which is crucial to abate the searching cost. Examples of information platforms are: Airbnb, Booking and RatesToGo in the lodging sector or ClubKviar, OpenTable, TheFork and Urbanspoon in the restaurant industry. Foursquare, TripAdvisor and Yelp are active in several sectors.

For example, TheFork features an average of 70+ reviews per restaurant.

This includes, for example, filters to refine queries, or to ensure that only available products are displayed, and personalised suggestions based on the customer’s consumption history.

This includes membership policy, source of information and offered complementary services. For example, everyone can leave a review on TripAdvisor and Yelp. Only the own staff and certified consumers can post on ClubKviar, so as to reduce the risk of fraudulent reviews. ClubKviar offers to its members a 30% discount, a search engine allows to refine searches by location, day, price, atmosphere, etc.

The case of vertically differentiated goods is analysed in (Akerlof, 1970)’s market for lemons: quality goods are not traded even in the presence of demand for them, for consumers are afraid of buying a low-quality good. Common solutions include insurances and warranties. Insurance is not effective in the case of horizontally differentiated goods, as the perception of quality is subjective.

Firms’ reputation and word-of-mouth may help solve this issue, if repeat purchases are likely. Their effectiveness is limited, however, in markets where most of the consumers tend to be one-time visitors, such as holiday lodging, exclusive restaurants, or museums.
versely, successful information platforms can effectively inform customers and help in the matching process. Their independency and their reputation vouch for them, and regrouping all the information is necessary to make search costs negligible. By reducing mismatch costs, aggregators affect both the quantity and quality (i.e. the generated surplus) of transactions, thereby impacting social welfare from the demand side. In addition, they may play a role in fostering competition among firms, thus altering both the market equilibrium and welfare from the supply side. Most importantly, because only a subset of the consumers makes use of the aggregators, an asymmetry in the structure of the market appears, with informed and uninformed consumers.

This work provides a theoretical framework aimed at understanding the impact of review platforms on pricing, consumers’ behaviour and welfare. More generally, it can be considered as a study of the impact of an exogenous informational shock that affects a subset of consumers, in a horizontally differentiated market. I propose a stylised model loosely inspired by *ClubKviar* (restaurant) and *HomeAway* (lodging).7

I assume that goods differ in variety but not in quality: i.e. firms compete horizontally. This reflects the recent tendency of platforms to specialise on a specific quality level, similarly to what happens with restaurant guides (think, for example, of the Michelin guide): the reputation of the aggregator becomes the guarantee of the vertical positioning (*ClubKviar*, for example, specialises in high quality restaurants, but its brand *KviarCasual* covers the segment slightly below. The same happens with *Luxury, HomeAway* and *Toprural* for lodging.) Horizontal differentiation can be interpreted as the combination of several components, amongst which the style (types of regional cuisine, for restaurants; architectural styles for lodging) and the atmosphere (e.g. modern and trendy, romantic, old style). Agents’ information is crucial: I assume that consumers have no ex-ante information, hence they cannot tell goods apart. Absent the platform, they decide whether or not to consume based on expectations. If they decide to consume, they choose the seller randomly. By contrast, the platform credibly reveals relevant pieces of information.8 Both the total lack of ex-ante information and the full information setting provided by the platform are extreme, but harmless, assumptions.

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7Stewart Masters, CEO of *ClubKviar*, shared with me – off the record – several insights about the functioning of *ClubKviar*. The model that I propose tries to be as general as possible, and therefore it should not be considered as a description of either *ClubKviar* or *HomeAway*, but rather as a general tool to be used in understanding the main driving forces in the market. *ClubKviar* belongs to GrupoMercantis, which also owns an aggregator operating in the leisure sector (*Kviarcity*), and two group-buying services (*Triavip* for triathlon products, and *Destinity* for travels and tourism). *HomeAway* is a vacation rental marketplace and currently a subsidiary of Expedia. *HomeAway* owns several brands, including VRBO, VacationRentals, Luxury, Toprural, all operating in the vacation rental marketplace, but covering different market segments. Both *ClubKviar* and *HomeAway* represent instructive examples of online platforms that sellers can use to attract more consumers, and that consumers can use to reduce the mismatch cost.

8A restaurant platform, for instance, provides detailed information about location, type of cuisine, menu, price, atmosphere, and several other characteristics. Filters allow customers to narrow down the search.
The driving force of the model comes from the differential information available through the platform, and not from its level.

I compare the equilibrium without and with an information platform. In the latter case, sellers endogenously decide whether to be indexed on the aggregator or not. In the current version of the model, consumers’ decision to use the platform is exogenous, however, a micro-founded extended version should be soon available, in which agents’ decision to use the platform is endogenous. I compute the change in welfare that follows the appearance of an information platform. I focus on the impact of the first aggregator entering a market, while the analysis of the role of competition in the platforms market is left for future analysis. I consider the short-, medium- and long-term equilibria in terms of number of firms and welfare. I use the spokes model of horizontal differentiation, which is an extension of the Hotelling model, and also a special case of Hart (1985). The general properties of the spokes model are studied and discussed in Chen and Riordan (2007). Figure 1 illustrates this model graphically. The spokes model has been used extensively in the most recent literature, in a broad set of different environments (Caminal and Claici, 2007; Caminal and Granero, 2012; Germano and Meier, 2013; Mantovani and Ruiz-Aliseda, Forthcoming; Reggiani, 2014).

Figure 1: Spokes model with 7 spokes ($\bar{N} = 7$) and 4 firms ($N = 4$)

Whereas the Hotelling model has the form of a single line, the spokes model takes the shape of a star: it could be seen as the union of several Hotelling lines of equal length. Consumers are uniformly distributed over all the spokes. Transport costs are a function of the distance that a consumer travels to reach a product/firm. The usual interpretation applies, that location represents tastes and variety, therefore distance and transport costs represent the consumer’s disutility from consuming a good of a variety other than the preferred one. The intersection of all the spokes is denoted centre. Each spoke (denoted $l_i$) has an origin, which is the point on the segment opposite to the centre. On each and every spoke there can be at most one firm, located at the origin of the spoke. Firms’ location is exogenous.\(^9\) As discussed in Chen and Riordan (2007), the spokes model is characterised by the fact that – contrary to the circular

\(^9\)See Reggiani (2014) for the study of the endogenous location of firms in a spokes model.
model à la Salop – entry and exit of firms do not imply an unrealistic relocation of firms. This is a particularly convenient and realistic feature when it comes to aggregators. Indeed, in the presence of the platform, an online firm is simultaneously facing two different sets of competitors (on- and off-line). The Salop model would conceptually fail to adapt to this case, as the automatic reposition along the circle depending on the number of competitors would mean that the firm’s very same product has different characteristics depending on how it is purchased.\textsuperscript{10} Another feature that makes the spokes model more general and realistic is that it does not require the market to be fully covered in order to feature some competition amongst firms. Hence, it is possible to model in a meaningful way the case in which firms have some market power, without being a monopolist, and still allow for the entry of a platform to have an expansionary effect on the market.

The standard spokes model has a technical limitation when there are no firms located on some spokes and transport costs are low enough for firms to be willing to serve customers located on spokes other than their own. All the agents distributed on empty spokes form a mass of consumers willing to switch from one seller to another at any marginal change in prices. This generates a discontinuity in the demand functions that inhibits the existence of a pure strategy equilibrium. Chen and Riordan (2007) discuss this problem, the possible ways to deal with it and the disadvantages of each. The weakest condition to avoid this issue is to assume that consumers attach a positive valuation to consumption only to a finite number of products. Following Chen and Riordan (2007), I assume that each consumer positively values two products.\textsuperscript{11} Notice that this assumption is redundant in the setting of section 2.1, but it is necessary for the remaining ones.

I assume that agents’ valuations are large enough to ensure that some transactions are \textit{ex-ante} profitable even when agents are uninformed about the firms’ characteristics (location). When the platform is available, agents choose whether to use it or not. If they do so, users learn the location of all the firms indexed therein. However, it is a fact that only a share of the population uses information aggregators. This may be due to some agents’ lack of technological skills, to learning costs of using the platform, or to the fact that using aggregators may also be time-consuming. In this preliminary version of the model, for the sake of tractability, I assume that a share of the population – denoted \textit{surfers} – bears no cost of using the aggregator, while the remaining – denoted \textit{walkers} – has a prohibitive cost. Consequently, \textit{surfers} always make use of the aggregator, while \textit{walkers} never do so and choose the restaurant randomly out of a list (e.g. yellow pages). An extension with consumers

\textsuperscript{10}Within this context, it would mean that a given restaurant would propose, for example, an old-fashion atmosphere to off-line consumers, and a trendy one to those who booked on-line.

\textsuperscript{11}An alternative solution to deal with the existence of an equilibrium problem would be to assume that agents have different valuations for each product.
endogenously choosing whether or not to use the platform will be available soon.

Firms trade off the benefits of being online (i.e. the possibility to expand their customers base) with its costs (i.e. the aggregator’s fee), and they choose to be listed on the aggregator or not. I denote the former \( e \)-sellers and the latter \( c \)-sellers, where \( e \) stands for “electronic” while \( c \) stands for “conventional”. \( C \)-restaurants are not listed on the aggregator’s website, hence they can only serve \textit{walkers}. \( E \)-restaurants are available both online and offline, hence they can serve both \textit{surfers} and \textit{walkers}.

All sellers pay an entry fee (e.g. a license). Furthermore, \( e \)-sellers pay a fee to the aggregator to be listed online. I assume the fees to be fixed and to be paid once every period.\(^{12}\) This allows us to distinguish between the short, medium and long run. In the short run, firms decide whether to be listed online or not, taking the others’ decisions as fixed. In the medium run, the number of \( e \)-sellers adjusts and ensures that the profit is the same for both types of firms. In the long run, free entry ensures that the total number of competing firms (both online and offline) adjusts to restore the zero-profit condition. I begin with a long run equilibrium with no aggregator. I study the short- and medium-run effects of the aggregator’s entry, and finally I compare the long run equilibrium without and with the aggregator.

This model provides a rationale for the existence of a separating equilibrium, in which only some firms and some consumers decide to use the aggregator (i.e. \( e \)-sellers and \textit{surfers}). The model could be extended to allow for a different product valuation for \textit{walkers} and \textit{surfers}.\(^{13}\) In that case, the aggregator may serve another important role, which is to allow the firm to distinguish between two types of consumers – \textit{walkers} and \textit{surfers} – with different willingness to pay. Hence, the aggregator becomes an instrument through which it is possible for the restaurant to price discriminate.\(^{14}\) Price discrimination is not considered in the baseline model, in order to disentangle the welfare effects of the aggregator from the standard welfare effects of price discrimination. Subsection 1.1 briefly discusses the literature on the welfare impact of 3rd price discrimination, while Appendix A.2 discusses the extension of the baseline model in this direction.

Uncertainty in the absence of the aggregator reduces agents’ \textit{ex-ante} willingness to pay, which in turn decreases the rent that firms can extract from each consumer and may also lead

\(^{12}\)This is a simplifying assumption. Aggregators use very different fee schemes, ranging from a per-year fee (\textit{HomeAway, Tripadvisor, VRBO}) to a per-reservation fee (\textit{TheFork}) or a percentage of the final price paid by the consumer (\textit{Airbnb, Wimdu}).

\(^{13}\)Surfers are characterised by a low cost of using the aggregator. Whether we interpret it as a low opportunity cost of time or as a tech-friendly profile, one may expect \textit{surfers} to also have a lower willingness to pay for the product.

\(^{14}\)If both groups have the same valuation for the product, there is no room for price discrimination. As long as valuation is correlated with one’s status, \( e \)-restaurants can use the aggregator as a device to recognise the types of consumers and may use that information when choosing prices.
to a partially uncovered market. It is interesting to notice that, because of the incomplete
information, the share of market that is uncovered (if any) consists of the consumers located
closest to the firm. This means that, in a situation in which sellers propose very extreme and
possibly unconventional products (e.g. avant-garde restaurants), active consumers are among
those with the least extravagant tastes, while those who like unorthodox products will refrain
from consuming. The intuition is that the uninformed average consumer knows that all sellers
offer a product that is neither very close nor too far from what they like. In contrast, the
eccentric consumer knows that few sellers may offer their preferred product, whereas most
of them offer something that is extremely far from it. As a consequence, the best strategy
for the eccentric and uninformed consumer is not to consume, because the probability of
randomly selecting a good match is too low. The aggregator solves the information problem
and reduces the average transport costs mainly for agents with unconventional tastes (that
is, those who are farther away from the centre of the star), but it may strengthen competition
amongst firms. Hence, transactions generate more surplus, but firms are not necessarily able
to profit from that. I conclude that platforms have an impact on i) profits (positive in the
short run, negative in the medium run), ii) variety (the long-run equilibrium number of firms
decreases), and iii) both the share of consumers served and their surplus (positive the former,
uncertain the latter). Firms face a Bertrand supertrap since each of them, individually, has
an incentive to be listed on the aggregator; however this results in a decrease in medium-
run profits for all of them. I show that total welfare increases in the long run, and this
should be interpreted as a combination of three different factors: the aggregator reduces the
inefficiencies due to asymmetric information, it expands the market that is covered, and finally
it increases competition amongst firms. Consumers are \textit{ex-ante} better off, however, because
variety decreases, this cannot be considered a Pareto improvement, for some consumers are
\textit{ex-post} worse off.

1.1 Related literature

Besides the aforementioned literature on the spokes model, this model is interconnected with
several strands of the literature. Review aggregators have been used in several and hetero-
genous empirical studies, either for their interest \textit{per se} or because they offer an extremely
rich database that can be used for different purposes. An example of the latter is Davis
et al. (2015) who use information from Yelp to study how spatial and social frictions affect
consumption. Ghose et al. (2012) show that consumers base their purchase on reviews. How-
ever, DellaVigna and Pollet (2007, 2009) show that consumers tend to disregard several pieces
of information, Pope (2009) shows evidence of rank-heuristic behaviours.\textsuperscript{15} Much attention has been devoted to the impact on profits of review-based rankings. Aggregators rank firms based on the consumers' rates. Anderson and Magruder (2012) and Luca (2011) estimate the increase in profits due to higher rank for restaurants rated on Yelp. These results suggest a short-run business-stealing effect for vertically differentiated firms. The effect is larger for restaurants for which few sources of information are available outside Yelp. Their results are consistent with other studies on online reviews (e.g. Chevalier and Mayzlin, 2006, considering the impact of book reviews on their sales), as well as on the impact of reviews by professional critics (Reinstein and Snyder, 2005; Hilger et al., 2011) and of offline word of mouth (Duflo and Saez, 2002; Sorensen, 2006; Moretti, 2011). However, the results on the impact of reviews on profit suggest that vertical differentiation within a single aggregator is unlikely to be long-lasting. In equilibrium, we should expect vertical convergence within aggregators.\textsuperscript{16} Both the theoretical (Vial and Zurita, 2013) and empirical (Cabral and Hortacsu, 2010) literature indeed suggest that - in the long run - poorly ranked firms either disappear or converge to their competitors’ quality. This supports my choice of focusing on horizontal competition within an aggregator.\textsuperscript{17}

Clearly, the effectiveness of online review aggregators depends on their credibility. Credibility is undermined by the risk of fake reviews.\textsuperscript{18} Some aggregators (e.g. ClubKviar) increased the cost of posting fake reviews by allowing only certified consumers to post them. Degan (2006); Mayzlin (2006); Luca and Zervas (Forthcoming); Mayzlin et al. (2013) study the phenomenon of manipulated reviews. Dai et al. (2012) propose an improved econometric methodology to rank options based on reviews and minimise the impact of possibly manipulated reviews.

Online aggregators can be considered as a further evolution within the internet economics and management. The economic literature has thoroughly analysed strengths and weaknesses of the different online selling and advertising methods, and their impact on market competition and welfare. Alba et al. (1997) and Bakos (1997) are amongst the first to discuss electronic sales. Jin and Kato (2007) discuss possible advantages and disadvantages of online selling. Anderberg and Andersson (2003); Arabshahi (2010); Dholakia (2010); Byers et al. (2011); Jing and Xie (2011) and Chen and Zhang (Forthcoming), consider group buying and the Groupon

\textsuperscript{15}Agents engaging in rank-heuristic behaviours tend to focus on the rank, although more precise data are available. In particular, when the rank is associated to a continuous measure, such as a grade, they prefer disproportionately the better ranked option, even when the difference in grade is negligible.

\textsuperscript{16}Anecdotal evidence suggests that aggregators specialise on different price ranges.

\textsuperscript{17}Vertical differentiation is instead more likely to arise when different aggregators compete. However, this such analysis is beyond the scope of this work.

\textsuperscript{18}Tripadvisor was fined as much as 500k euros by the Italian antitrust authority in 2014, for it failed to adopt controls to prevent false reviews. The British authority had previously forced Tripadvisor to stop advertising that their reviews are accurate.
experience. Edelman et al. (2011) examine the use of group-buying as a device to introduce price discrimination and as an advertising device. See Liang et al. (2014) and the references therein for an overview of the most recent literature on group-buying. Biyalogorsky et al. (2001) focus on referral-reward programs, Shaffer and Zhang (2002) on one-to-one promotion, and Xie and Shugan (2001) on advance selling. Aggregators may also play a role that is similar to advertisement: Rysman (2004); Busse and Rysman (2005) consider advertising on Yellow Pages, while Evans (2009) focuses on online advertising. In my model, possible advertising effects are not taken into account.

Information aggregators could be seen as an electronically enhanced Yellow Pages service or more generally a search engine. Both aggregators and search engines can list firms and provide some basic and objective characteristics. In the practice, search engines tend to differ from aggregators in a few respects, although it would be technologically feasible for a search engine to integrate the features of aggregators. First and most importantly, search engines do not allow users to leave reviews, which is the main channel of transmission of third-party information. Secondly, although both are financed by firms (aggregators set a fee, while search engines rely on firms’ advertisement), the aggregator is meant to be neutral, and its reputation is based on that. In other words, aggregators treat all the listed firms equally. When the aggregator ranks firms, it makes it based on the measures decided by the user (price, location, availability, other users’ ranking, etc.). However, search engines charge firms in exchange for visibility, hence search outcomes are a weighted compromise between users interest for the best possible match and firms interest to be listed on top. Eliaz and Spiegler (2011) study the search engines trade-off and shows that they have an incentive to degrade the quality of results compared to the minimising search cost outcome. Bar-Isaac et al. (2012) study how search engines affect the firms’ incentives to design products for niches (markets with few enthusiastic consumers), as opposed to be mass-oriented, trying to attract many consumers, although with low willingness to pay.

The aggregator allows restaurants to tell *surfers* and *walkers* apart. As previously discussed, should status be correlated with valuation, restaurants would use this information to price discriminate. The idea of selective discounting first appeared in Varian (1980) and Narasimhan (1984, 1988). In the baseline model, I deliberately avoid any price discrimination effect, to be able to focus on the impact of the aggregator on welfare through the reduction in mismatch costs. However, in appendix A.2 I briefly discuss the role of selective pricing in this framework. Price discrimination may be welfare-improving as it allows to serve weaker markets which would otherwise be cut out (Schmalensee, 1981; Varian, 1985; Stigler, 1987). When 3rd price discrimination doesn’t affect the equilibrium quantity exchanged, nor the identity of the consumers, then it decreases welfare when the demand function is linear, while it may
increase it otherwise (Aguirre et al., 2010; Aguirre, 2011; Chen and Schwartz, Forthcoming).

In the model, gathering information is prohibitively costly without the aggregator, and there is no search costs when using the aggregator. This seems reasonable when available information provided by the firm itself is not credible and hence agents do not engage in searching activity. This setting could be relaxed, by assuming that agents can invest in searching when the aggregator is not available, in which case it would seem reasonable to assume that search costs without the aggregator are larger. The baseline model can be interpreted as the limit case of costs being either infinite or zero. The literature on search costs has used different models depending on whether goods are homogeneous or not. For the former case, see Janssen et al. (2011) and the literature therein. The literature with differentiated products was pioneered by Wolinsky (1986); Anderson and Renault (1999). Moraga-González and Petrikaitė (2013) show some interesting welfare results with relevant policy implications when firms merge in markets with search costs. They show that mergers reduce search costs because the merged firms will gather products: a demand-side efficiency is identified, that may justify mergers. In my model, search costs are prohibitive when offline (hence, never realised in equilibrium) and null otherwise. Not allowing for offline searching implies increasing the value for consumers of being online. Therefore, assuming that searching offline is not viable implies an over-estimated reduction of mismatch costs and an underestimated reduction of search costs. Should the model be generalised to the case in which agents always incur positive and not-prohibitive search costs, the welfare enhancing effect of the aggregator could be seen as the sum of the reductions in both search and transport costs.

My work is close in spirit with Salop and Stiglitz (1977). The authors consider a society with two groups of agents, one that may gather information about a product and another whose cost of search is prohibitive and remains uninformed. The crucial difference between our works is the type of product differentiation. In Salop and Stiglitz (1977), there is one homogeneous product, possibly sold at different prices. Searching serves the purpose of reducing the purchase price, but the product consumed remains unchanged. Lack on information explains price diversity in equilibrium. An increase in the share of informed consumers strengthens competition and decreases prices. All agents prefer to buy from the cheapest store, but they may buy elsewhere to avoid the search cost. Search costs generate inefficiency and welfare loss: under unitary demand and fully covered market, a lower price is simply implying a smaller transfer from consumers to producers. In my model, on the opposite, the equilibrium price is the same everywhere, while each agent values the good differently. Becoming informed increases welfare, for the match becomes more accurate (transport costs decrease). Agents information is not responsible per se of a decrease in price, since better matches call for more consumers’ surplus to extract, and I will show that it is precisely due
to the presence of uniformed consumers that prices cannot increase and that firms are not able to extract surplus from informed consumers. da Graça and Masson (2013) points out, in a framework similar to the one that I analyse, that information may be bad for consumers surplus. The idea behind their result is that information on the true value of a good leads to a change in the willingness to pay, the composition of the demand, and hence the price that a monopolist would charge. Therefore, previously undervalued goods would be sold at a higher price, hurting consumers. This mechanism doesn’t appear in my work because online prices cannot increase as long as firms are also serving uninformed buyers whose willingness to pay remained unaltered. da Graça and Masson (2013) focus on the effect on consumers surplus: it should be noticed that part of the welfare change corresponds to a shift between consumers and firms, while part of it comes from a change in total consumption. In my model, information never leads to an increase in price (informed agents can always act as uninformed) and therefore the model always predicts (weak) increases in consumption and decreases in prices, and furthermore changes in consumption are not necessarily due to changes in prices.

Online review aggregators represent an example of two-sided market. In this model I abstract from this problem mainly by focusing on the case of one aggregator (at most) and of consumers with a totally inelastic demand for the use of the aggregator, however appendix A.1 briefly overviews the two-sided markets literature and uses the insights from it to discuss the robustness of this model’s results.

The remainder of the paper is organised as follows: section 2 introduces the baseline model: subsections 2.1 and 2.2 consider the case when the aggregator is not available and when the aggregator is available, respectively. Section 3 studies the market equilibrium, while section 4 studies the welfare consequences of aggregators. Section 5 concludes. Appendix A includes some extensions of the baseline model. All the proofs are relegated to appendix B.

2 The model

Consider a spokes model, as described in Figure 1, with $\bar{N} \geq 4$ spokes populated by a uniformly distributed population of mass one, where $\bar{N}$ is exogenously determined. Spokes have a length $\frac{1}{2}$. The intersection point is named centre, while the termination of a spoke opposite to the centre is named origin.

Each restaurant is randomly located at the origin of an empty spoke; there can be at most one restaurant at each spoke. $\bar{N}$ represents the total number of restaurants, with $N \in [2, \bar{N}]$. Restaurants can be of two types: the conventional ones ($c$-restaurants) can only be booked offline; the electronic ones ($e$-restaurants) can be booked both offline and online: $n \leq N$.

\footnote{Notice that when $N = 2$ the spokes model reduces to the standard Hotelling model.}
denotes the number of e-restaurants. Both $n$ and $N$ are endogenous. Denoting spokes by $l_i$, with $i \in (1, N)$, and ordering them by type of restaurant, spokes $l_1$ to $l_n$ have an e-restaurant, spokes $l_{n+1}$ to $l_N$ have a c-restaurant, and finally spokes $l_{N+1}$ to $l_\bar{N}$ are empty.

Each restaurant must pay a yearly fixed fee $f$ to be active. Furthermore, e-restaurants pay an extra fee $f^e$ to be available online. For the sake of simplicity, variable production costs are normalised to 0. The aggregator is a non-strategic agent that acts as a device that restaurants use (upon payment of the fixed and exogenous fee $f^e$) to provide credible information (i.e. their location) to potential consumers and that consumers can use at no cost, in order to obtain the information and to book restaurants. Consumers are of two types: walkers have no access to the aggregator, while surfers have. The share of walkers in society is $\alpha$.

Following the literature on spokes, each consumer has two preferred spokes: the one where (s)he is located and another – randomly assigned – one.\footnote{This assumption, as discussed in the introduction, is needed to avoid mass points and discontinuity in demand functions and hence to guarantee the existence of a symmetric pure strategy equilibrium in the standard spokes model (see Chen and Riordan, 2007). It should be noticed that this assumption is redundant in section 2.1, however, it guarantees the existence of a unique, pure strategy equilibrium in section 2.2 and after.} The assumption means that consumers assign a positive value $v$ to meals from restaurants located on their two preferred spokes. Instead, their valuation for any other meal is 0. In other words, a consumer located on spoke $l_j$ has a valuation $v > 0$ for a meal from either restaurant $j$ or $k$, where $k \neq j$ represents the other (randomly assign) restaurant. The same consumer has valuation $v = 0$ for any meal other than $j$ or $k$. For an agent located on spoke $l_j$, $x_j \in [0, 1/2]$ measures an agent’s distance from the origin of the spoke $l_j$, while $x_m \in [1/2, 1]$ is the distance from the origin of any spoke other than $l_j$, with $x_j + x_m = 1$. It is now possible to define by $(x_m; l_j; l_k)$ the consumer located on spoke $l_j$ at location $x_m$ and with valuation $v > 0$ for meals served at $l_j$ and $l_k$. The distance between a consumer and a restaurant represents the difference between the consumer’s preference and the product characteristics. Consumers incur in a mismatch cost equal to the distance between their location and the restaurant’s. Hence the mismatch cost for consumer $(x_m; l_j; l_k)$ when buying from restaurant $r$ is

\[
M_r(x_m; l_j; l_k) = \begin{cases} 
(1 - x_m) & \text{if } r = j \\
x_m & \text{if } r \neq j,
\end{cases}
\]

(1)

where $(1 - x_m)$ and $x_m$ are the distances in each case.

2.1 Without aggregator

I use the afore described setting to study the equilibrium when the aggregator is not available and, therefore, consumers ignore the characteristics of the restaurants. In such a case, consumers know their own valuation $v$, their type $(x_m; l_j; l_k)$, however, they don’t know the
location of restaurants, and hence the possible mismatch cost: meals are an experience good and, therefore, consumers ignore how much they will like a restaurant until they consumed the meal. This means that consumers can randomly choose any restaurant \( i \in \{1, \ldots, N\} \) (for example, out of a list of them) and consume a meal there, but the realisation of the mismatch cost occurs when the consumption decision is irreversible. Hence, the consumption decision is made based on the expected utility of consumption.

**Lemma 1.** The expected mismatch cost for an agent consuming a meal in a randomly selected restaurant is:

\[
EM = \left( \frac{N - 2}{N} - x_m + \frac{1}{N} \right).
\]

(2)

The expected valuation of a consumer is \( 2v/\bar{N} \), therefore, the expected utility of an agent is positive if:

\[
x_m \leq \frac{\bar{N}}{N - 2} \left( \frac{2v}{N} - p - \frac{1}{N} \right).
\]

(3)

Denoting by \( \tilde{x}_m \) the agent indifferent between buying and restraining from it, the symmetric, pure strategy interior equilibrium implies that

\[
\tilde{x}_m = \frac{1}{N - 2} \left( v + \frac{N - 4}{4} \right),
\]

(4)

\[
p = \frac{v}{N} - \frac{1}{4}.
\]

(5)

Each firm covers a demand \( D = \frac{(4v - N)}{2(N - 2)N} \), and profits are \( \pi = \frac{(4v - N)^2}{8(N - 2)^2N} \). To ensure an interior equilibrium, \( \tilde{x}_m \in [1/2, 1] \) is required. This implies that \( v \in \left[ \frac{1}{4} \left( N, (3\bar{N} - 4) \right) \right] \).

The equilibrium price (5) is the monopoly price. Indeed, according to the so called Diamond’s paradox (Diamond, 1970), the profit-maximising price is the monopolist’s one when consumers exerts no search effort and consider that firms ex-ante are all equals. It is interesting to notice (equation 2) that expected mismatch costs are increasing in the number of spokes \( \bar{N} \). This comes from the fact that it increases the probability of being located in a spoke where there is no restaurant. This also means that the location \( \tilde{x}_m \) of the indifferent consumer is decreasing in \( \bar{N} \). Instead, expected mismatch costs are not affected by \( N \). This occurs because the two opposed forces cancel out: on the one side an increase in \( N \) reduces the probability of being located in a spoke where there is no restaurant, but on the other, conditional on being at a spoke with a restaurant, it equally increases the probability of consuming a meal in a restaurant on a different spoke from the one where the consumer is located, given that the consumption decision is made before knowing the location of the restaurant.

From Lemma 1, when the aggregator is not available agents are willing to purchase only as long as they are located sufficiently close to the centre (i.e. consumers within the dashed
circle in figure 2). This is a quite surprising result, as it says that – absent information – the share of market which is less likely to be covered is the one closest to firms’ location. This is a consequence of the fact that the expected mismatch cost is increasing in $x_m$: agents have a large ($\frac{N-1}{N}$) probability of consuming a product located on a spoke other that theirs, hence expected mismatch costs are lower for agents located closer to the centre. The inefficiency becomes clear, the likelihood to consume is inversely related to the ex-post mismatch cost.

2.2 With aggregator

This subsection considers the full information case. An aggregator lists all restaurants and provide all their characteristics: that is, all of them are $e$-restaurants, hence $n = N$. All consumers benefit make use of the aggregator, i.e. all consumers are surfers, hence $\alpha = 0$.

The aggregator allows surfers to discover the characteristics (i.e. the location) of a restaurant before consumption, which means that consumers know ex-ante and for each restaurant both the mismatch cost and the valuation of the meal in case of consumption. This benchmark case corresponds to the original setting in Chen and Riordan (2007).
Figure 3 depicts a case with four $e$-restaurants and no $c$-restaurants. Lemma 2 defines the unique, symmetric, pure strategy equilibrium in the market.

**Lemma 2.** The demand faced by one restaurant is

\[
D_r = \begin{cases} 
\frac{2}{N}(v - p_r) & \text{if } v - p_r \leq \frac{1}{2} \\
\frac{2}{N} \left( \frac{1}{N-1} \right) \sum_{s \neq r; s \leq n} \frac{p_s - p_r + \frac{1}{2}}{2} + \left( \bar{N} - n \right) (v - p_r) & \text{if } \frac{1}{2} < v - p_r \leq 1 \\
\frac{2}{N} \left( \frac{1}{N-1} \right) \sum_{s \neq r; s \leq n} \frac{p_s - p_r + \frac{1}{2}}{2} + \left( \bar{N} - n \right) & \text{if } v - p_r > 1.
\end{cases}
\]

The unique symmetric, Nash equilibrium in pure strategies implies that

\[
p^a = \begin{cases} 
v - \frac{1}{2} & \text{if } 1 < v < \frac{4N-n-3}{2(2N-n-1)} \\
\frac{4N-n-3}{2(2N-n-1)} (n-1) + 2(\bar{N}-n)v & \text{if } \frac{4N-n-3}{2(2N-n-1)} \leq v < 2 \\
v - 1 & \text{if } 2 \leq v \leq \frac{2N-2}{n-1} \\
\frac{2N-2}{n-1} & \text{if } \frac{2N-2}{n-1} < v.
\end{cases}
\]

Figure 4 represents the optimal price schedules $p$ - when the aggregator is not available, as defined in Eq. (5) - and $p^a$ - when the aggregator operates in the market, Eq. (7). Notice that the intersection point of $p$ with the horizontal axis occurs at $v = \bar{N}/4$. Finally, notice that $p$ and $p^a$ intersect when $v = \frac{N(8N-3n-5)}{4(n-1)}$ and $p^a$ is in its last (horizontal) segment, while $p^a < p$ whenever $v > \frac{N(8N-3n-5)}{4(n-1)}$. A higher product valuation should entail higher prices. This is true in the setting without the aggregator, and hence with no information about the restaurants' location. Under full information (i.e. with the aggregator), however, competition becomes fiercer along consumers' valuation and hence the two (countervailing) effects result in a weakly increasing price schedule.

\(^{21}\)In figure 4, the intersection is arbitrary drawn as if $\bar{N} = 8$, hence at $v = 2$. 

---

**Figure 4:** Price schedule. Without aggregator ($p$, Eq. 5) and with the aggregator ($p^a$, Eq. 7)
As depicted in figure 4, absent the aggregator and depending on the parameter values, there may be no market when the valuation of the good is low. Instead, when consumers are informed, some transactions always occur in equilibrium. Information allows to expand the market, and hence it is likely to enhance welfare. However, the more relevant and common case is when \( v \geq (\bar{N})/4 \) and hence a market for the product exists even before the introduction of the aggregator.

3 Market equilibrium

In this section, I study the equilibrium when an aggregator is available. I suppose that a share \( \alpha \) of the population are walkers, that is, they never use the aggregator, remain uninformed, and therefore base their consumption decision on expectations and learn their valuation and mismatch cost only after consuming, as in subsection 2.1. The \( 1 - \alpha \) surfers always recover the information available. The value \( \alpha \) is exogenous.\(^{22}\)

By definition, c-restaurants are not listed online, hence they do not disclose information, while e-restaurants are listed online. Restaurants endogenously decide whether to be listed online or not: \( n \) represents the number of e-restaurants, \( N - n \) is the number of c-restaurants. Restaurants pay \( f^e \) to be listed online. Walkers randomly buy at any of the \( N \) restaurants, hence they may consume at a c- or e-restaurant. Surfers use the information available to maximise their utility: they may find a suitable match among the e-restaurants, or decide to act as a walker whenever this is a better strategy.

Therefore, e-restaurants may serve both groups of consumers: surfers and the share of walkers that randomly decide to purchase there, while c-restaurants instead only serve walkers. However, with probability \( \epsilon = \left( \frac{N-n-1}{N-1} \right) \left( \frac{N-n}{N} \right) \) none of the two favourite restaurants of a surfer is online. In such case, surfers are free to act like walkers and randomly select a restaurant. I assume that, alike walkers, surfers buying offline will randomly purchase from any available restaurant.\(^{23}\)

\(^{22}\)As discussed in the introduction, assuming that a fixed share of the population makes use of the aggregator can be interpreted as the extreme case in which walkers have a prohibitive cost of using the aggregator – this could be due to limited IT skills or high opportunity cost of time – while surfers have no cost of usage. The appendix discusses the consequences of this assumption.

\(^{23}\)This assumption, made for tractability reasons, implies that surfers cannot randomise within the subset of offline restaurants, excluding e-restaurants. This assumption is realistic when it is not possible to directly match the online firm with the offline one. This occurs, for example, when the aggregator doesn’t provide some crucial contact information before the transaction occurs: a notable case would be the one of Airbnb, where the host’s phone and address are not revealed. Another case in which the matching is not possible is when the database of firms is large and users focus on the results of an advanced search: when no firms fit the introduced criteria, the user learn that no restaurants in the database are a good fit, but for the user it would be too costly to check which firms belong to the database. By relaxing this assumption, any surfer acting alike walkers has still an informational advantage (the surfer is able to exclude from the set any e-restaurant, which would be a poor match) hence, the model that I propose underestimates welfare benefits, by over-estimating
The pricing strategy of e-restaurants is bounded by the legal environment, the type of technology and the agreement signed with the aggregator. Sub-sections 3.1 and 3.2 consider two plausible alternative situations. In the former, e-restaurants can treat surfers and walkers differently, i.e. they can charge $p^s$ to surfers and $p^w$ to walkers. In the latter, e-restaurants are bounded to propose one single price $p^e$ to all of their consumers. This may occur for legal reasons, as part of an agreement with the aggregator, or whenever the seller cannot distinguish the two types of buyers as, for example, when reservations are not made directly through the aggregator’s platform.

3.1 E-restaurants charge different prices to surfers and walkers

When e-restaurants can charge different prices to surfers and walkers, the online and offline markets can be treated separately. E-restaurants charge a price $p^s$ to surfers who purchase through the aggregator. Both e- and c-restaurants face the same problem when choosing the price strategy for walkers. I denote $p^w$ the price that both types of restaurant charge to walkers.

Amongst walkers, the agent that is indifferent between purchasing and restraining from it is

$$\hat{x}_d^w = \frac{\bar{N}}{(\bar{N}-2)} \left( \frac{2v}{\bar{N}} - \frac{1}{\bar{N}} - p^w \right).$$

(8)

The maximisation problems of e- and c-restaurants are respectively:

$$\max_{p^w, p^s} \frac{2(\alpha + (1-\alpha)\epsilon)}{N} \left( \hat{x}_d^w - \frac{1}{2} \right) p^w + (1-\alpha)D_r p^s$$

(9)

and

$$\max_{p^w} \frac{2(\alpha + (1-\alpha)\epsilon)}{N} \left( \hat{x}_d^w - \frac{1}{2} \right) p^w,$$

(10)

where $D_r$ is defined by equation (6). In both maximisation problems, the first term is the profit from the offline market.

Noticing the separability between $p^s$ and $p^w$, it is immediate to conclude that both prices coincide with those obtained in sections 2.1 and 2.2, for the only difference here with respect to the previous sections is due to a resize of the marker.

Lemma 3 summarises the result of the maximisation.

**Lemma 3.** The offline price is $p^w = \frac{v}{\bar{N}} - \frac{1}{4}$. The online price is

$$p^s = \begin{cases} 
\frac{v - \frac{1}{2}}{(n-1)+2(\bar{N}-n)v} & \text{if } 1 < v < \frac{4\bar{N}-n-3}{2(2\bar{N}-n-1)} \\
\frac{4\bar{N}-n-3}{2(2\bar{N}-n-1)} v & \text{if } \frac{4\bar{N}-n-3}{2(2\bar{N}-n-1)} \leq v < 2 \\
\frac{v - 1}{2\bar{N}-n-1} & \text{if } 2 \leq v \leq \frac{2\bar{N}-2}{n-1} \\
\frac{2\bar{N}-2}{n-1} & \text{if } \frac{2\bar{N}-2}{n-1} < v.
\end{cases}$$

(11)

the mismatch cost for such consumers.
The two prices are equal \( p^w = p^s \) for \( \tilde{v} = \frac{\tilde{N}(8\tilde{N}-3n-5)}{4(n-1)} \). The crossing occurs when \( p^s \) is in its last region \( (v > \frac{2\tilde{N}-2}{n-1}) \). The offline price is lower \( (p^w < p^s) \) if and only if \( v < \tilde{v} \).

From lemma 3 we learn that e-restaurants are willing to charge a lower price to surfers than the regular one for walkers, whenever the good’s valuation is sufficiently large. Such a result if compatible with the observed evidence that the restaurants that are listed on some aggregators (e.g. ClubKviar) propose a substantial discount to users that book through the web-site. However, the opposite is also true: for low valuation goods, firms would prefer to charge surfers more than walkers. This can occur only when e-restaurants can identify surfers and avoid any arbitrage, as assumed in this subsection. Subsection 3.3 considers the case in which surfer can pay the walkers’ price when this is lower.

Figure 5: Consumers served: walkers in the left chart and surfers in the right chart

Figure 5 shows the consumers that are served. The left chart refers to walkers: within the circle \( (x \leq \tilde{x}_d^w) \), all walkers decide to consume (red, continuous lines), while those located outside it restrain from consuming (black, dotted lines). The right chart refers to surfers: the green, continuous lines indicate locations where surfers consume: all surfers located on any spoke with an e-restaurant consume, as well as any surfer within the circle \( (x \leq \tilde{x}_d^s) \) and located on a spoke without e-restaurant.\(^{24}\)

**Lemma 4.** C-restaurants sell at the same price as before the aggregator’s appearance. However, the size of the population that they serve decreases from 1 to \( \alpha + \epsilon(1-\alpha) \). Therefore, their gross profit shrinks to

\[
\pi_d^c = (\alpha + \epsilon(1-\alpha)) \frac{(4v - \tilde{N})^2}{8 (\tilde{N} - 2) N \tilde{N}}. \tag{12}
\]

\(^{24}\)The dimension of the two circles was arbitrarily set: depending on the parameters values, it may be that \( \tilde{x}_d^w > \tilde{x}_d^s \).
The e-restaurant’s gross profit is \( \pi_e = (\alpha + \epsilon(1 - \alpha)) \frac{(4v - \bar{N})^2}{8(\bar{N} - 2)\bar{N}} + (1 - \alpha)D_r p^e \), with

\[
D_r p^e = \begin{cases} 
\frac{1}{N} \left(1 - \frac{1}{2}\right) & \text{if } 1 < v < \frac{4\bar{N} - n - 3}{2(2\bar{N} - n - 1)} \\
\frac{(n-1)+2(\bar{N}-n)v^2(2\bar{N}-n-1)}{N(\bar{N} - 1)} & \text{if } \frac{4\bar{N} - n - 3}{2(2\bar{N} - n - 1)} \leq v < 2 \\
\frac{(2\bar{N}-n-1)^2}{(n-1)N(\bar{N} - 1)} & \text{if } 2 \leq v \leq \frac{2\bar{N} - 2}{n-1} \\
\frac{(\bar{N}-n)^2}{(n-1)N(\bar{N} - 1)} & \text{if } \frac{2\bar{N} - 2}{n-1} < v.
\end{cases}
\] (13)

It is immediate to notice that c-restaurants’ gross profit is less than what any restaurant earns when the aggregator is not available, since \( \alpha + \epsilon(1 - \alpha) < 1 \), and that e-restaurants’ gross profit is larger than the one of c-restaurants’ (\( \pi_e \geq \pi_c \)): comparing net profits, e-restaurants are more profitable whenever \( f^e < (1 - \alpha)D_r p^e \).

3.2 E-restaurants charge a same price to all their consumers

When e-restaurants cannot charge different prices to surfers and walkers, they choose one single price that they charge to all their consumers. C-restaurants may charge a different price. Through this subsection, I consider the case in which e-restaurants cannot sell at different prices to surfers and walkers. In this case, e-restaurants charge a price \( p^e \) to all their consumers, while c-restaurants will set a price \( p^c \). Walkers face now an uncertainty about the price that they will pay, since they ignore which type of restaurant they will attend. The location of the indifferent walker, which now depends on the expected price, is now defined by:

\[
\tilde{x}_u = \frac{\bar{N}}{(\bar{N} - 2)} \left( \frac{2v}{\bar{N}} - \frac{1}{N} - \frac{n}{ntp^e} - \frac{N - n}{nt} \right),
\] (14)

where the subscript \( u \) stands for “uncertainty”, for walkers cannot anticipate the price that they will be charge, because it depends on the type of restaurant where they consume. This means that an increase in \( p^e \) negatively affects the demand for e-restaurants and, therefore, that the offline equilibrium price changes with respect to the previous case. The maximisation problem for the e- and c-restaurants is respectively:

\[
\max_{p^e} \left( \frac{2(\alpha + (1 - \alpha)\epsilon)}{N} \left( \tilde{x}_u - \frac{1}{2} \right) + (1 - \alpha)D_r \right) p^e
\] (15)

and

\[
\max_{p^e} \frac{2(\alpha + (1 - \alpha)\epsilon)}{N} \left( \tilde{x}_u - \frac{1}{2} \right) p^e,
\] (16)

From the first order conditions, we obtain that

\[
p^e = \frac{N}{n} \left( \frac{v}{N} - \frac{1}{4} - \frac{n}{2N} p^e \right),
\] (17)
Proposition 1. When e-restaurants must charge the same price to both walkers and surfers, the equilibrium is defined as follows: c-restaurants charge \( p^c = \max p^c, 0 \), where \( p^c \) is defined by equation (19). E-restaurants charge \( p^{c^*} = \max\{p^c, 0\} \) when \( p^{c^*} > 0 \) and charge instead \( p^e \) when [TBA].

Prices \( p^c \) and \( p^e \) are defined by equations (18) and [TBA]. For \( v \) sufficiently low \( \left( v < \frac{(2N-n-1)/(n-1)-\Psi_1}{\Psi_5} \right) \), the price \( p^c \) is less then or equal to \( p^e \), which is the price charged to surfers in subsection 3.1 (when e-restaurants are able to set two different prices). Both \( p^e \) and \( p^c \) are increasing in \( v \): \( \partial p^e/\partial v \geq 0 \) and \( \partial p^c/\partial v \geq 0 \).

Picture 6 depicts \( p^e \) and \( p^c \) together, under the assumption that \( N < 3n \). The optimal price charged to walkers follows a similar behaviour as the one charged to surfers: they share the same intervals that define the pieces of the equations, however slopes and intercepts depend on the parameter values and so does the relative position of \( p^e \) and \( p^c \). Hence, it is not possible to draw the two in one single graph.

\[
p^e = \begin{cases} 
\Psi_1^e v - \Psi_1^t t & \text{in Region } R^e 1 \\
v - \frac{t}{2} & \text{in Region } R^e 2 \\
\Psi_3^e v + \Psi_3^t t & \text{in Region } R^e 3 \\
v - t & \text{in Region } R^e 4 \\
\Psi_5^e v + \Psi_5^t t & \text{in Region } R^e 5, 
\end{cases} 
\]

and

\[
p^c = \begin{cases} 
\hat{\Psi}_1^c v - \hat{\Psi}_1^c t & \text{in Region } R^c 1 \\
\left( \frac{4N^2-nN^2}{2N(N-n)} \right) v - \frac{t}{4} & \text{in Region } R^c 2 \\
\hat{\Psi}_3^c v - \hat{\Psi}_3^c t & \text{in Region } R^c 3 \\
\left( \frac{2N-nN}{2N(N-n)} \right) v - \left( \frac{N-n}{N-n} \right) \frac{t}{4} & \text{in Region } R^c 4 \\
\hat{\Psi}_5^c v - \hat{\Psi}_5^c t & \text{in Region } R^c 5, 
\end{cases} 
\]

where the equations for \( \Psi_j^i \), \( \hat{\Psi}_j^i \) and \( R^e j \) are defined in the proof of proposition 1.

3.3 E-restaurants can offer discounts to surfers

In this subsection, I consider the case in which the seller can propose two different prices but cannot tell the two types of buyers apart. Therefore, e-restaurants cannot charge surfers more than walkers, for the surfers’ incentive compatibility constraint is binding.

3.4 OLD E-restaurants charge a same price to all their consumers

When the restriction \( p^e_s \leq p^e_w \) is binding, e-restaurants charge a same price \( p^e \) to both surfers and walkers. The maximisation problems becomes then:
Figure 6: Price schedule. Charging different prices ($p^s$, Eq. 11) and charging one price ($p^e$, Eq. 18)

\[
\max_{p^e} \left( \frac{2\alpha}{N} \left( \bar{x}_w - \frac{1}{2} \right) + (1 - \alpha)D_r \right) p^e
\]

and

\[
\max_{p^e} \left( \frac{2\alpha}{N} \left( \bar{x}_w - \frac{1}{2} \right) + \frac{2(1 - \alpha)\epsilon}{N - n} \left( \bar{x}_s - \frac{1}{2} \right) \right) p^e,
\]

with $\bar{x}_w$ and $\bar{x}_s$ defined by equation (??) and (14), clearly replacing $p^e_w = p^e$.

The F.O.Cs are:

\[
\alpha \left( \frac{t}{2} - \left( \frac{2\epsilon}{N} - \frac{t}{N} - \frac{2\alpha}{N} p^e - \frac{N-n}{N} p^e \right) \frac{N}{(N-2)} \right) = \frac{1 - \alpha}{N} \left\{ \begin{array}{ll} v - p^e & \text{if } v - p^e \leq \frac{t}{2} \\ \frac{(n-1)\epsilon}{N} + (N-n)(v-2p^e) & \text{if } \frac{t}{2} < v - p^e \leq t \\ \frac{(n-1)\epsilon}{N} + (N-n)t & \text{if } v - p^e > t. \end{array} \right.
\]

and

\[
\alpha \left( \frac{t}{2} - \left( \frac{2\epsilon}{N} - \frac{t}{N} - \frac{n}{N} p^e - \frac{2N-n}{N} p^e \right) \frac{N}{(N-2)} \right) = \frac{1 - \alpha}N \left( \frac{N - n}{N - n - 2} \right) \left( \frac{2v - t - 2(N-n)p^e}{(N-n-2)} - \frac{t}{2} \right),
\]

This can be rewritten as

\[
\phi_1 v - \phi_2 t - \phi_3 p^e = \phi_4 p^e
\]

\[
\Psi_1 v - \Psi_2 t - \Psi_3 p^e = \Psi_4 p^e,
\]

where:
$$
\phi_1 = \frac{2\alpha}{N(N-2)} + \begin{cases} 
\frac{1-n}{N(N-n)} & \text{if } v - p^c \leq \frac{t}{2} \\
\frac{n(1-n)}{N(N-1)} & \text{if } \frac{t}{2} < v - p^c \leq t \\
0 & \text{if } v - p^c > t,
\end{cases}
$$

$$
\phi_2 = \frac{\alpha\bar{N}}{N(N-2)} - \begin{cases} 
0 & \text{if } v - p^e \leq \frac{t}{2} \\
\frac{(1-n)(1-(n-1))}{2N(N-1)} & \text{if } \frac{t}{2} < v - p^e \leq t \\
\frac{2\alpha\bar{N}}{N^2(N-2)} + \frac{(1-n)(4N-3n-1)}{2N(N-1)} & \text{if } v - p^e > t,
\end{cases}
$$

$$
\phi_3 = \frac{2\alpha\bar{N} + 1}{N^2(N-2)} + \frac{(1-n)(4N-3n-1)}{2N(N-1)} \frac{\bar{N}n}{2N(N-1)}
$$

$$
\phi_4 = \frac{\alpha\bar{N}(N-n)}{N^2(N-2)}.
$$

Notice that $\phi_i > 0$ for $i = 1, 3, 4$, for any $\alpha \in (0, 1)$. The sign of $\phi_2$ depends on $\alpha$: it exist a value $\bar{\alpha} \in [0, 1]$ such that $\phi_2 > 0$ if $\alpha > \bar{\alpha}$.

**Lemma 5.** The equilibrium prices, using equations (24) and (25), are:

$$
p^c = \frac{\phi_1 \Psi_4 - \phi_4 \Psi_1 + (\Psi_2 \phi_4 - \phi_2 \Psi_4)}{\phi_3 \Psi_4 - \phi_4 \Psi_3} t
$$

$$
p^c = \frac{\phi_1 \Psi_1 - \phi_1 \Psi_3 + (\phi_2 \Psi_3 - \phi_3 \Psi_2)}{\phi_3 \Psi_4 - \phi_4 \Psi_3} t.
$$

**Proposition 2.** Both prices are increasing in $v \left(\frac{\partial p^c}{\partial v} > 0, \frac{\partial p^e}{\partial v} > 0\right)$. The optimal price that c-restaurants charges is always decreasing in transport costs $\left(\frac{\partial p^c}{\partial t} < 0\right)$, while in the case of e-restaurants the effect of transport costs depends on the share $\alpha$ of walkers: it exists a threshold $\bar{\alpha}$ such that $\frac{\partial p^c}{\partial t} < 0$ for $\alpha > \bar{\alpha}$ and $\frac{\partial p^c}{\partial t} > 0$ for $\alpha < \bar{\alpha}$.

Proposition 2 can be easily interpreted. It is natural that an increase in the good valuation, hence in buyers’ willingness to pay, leads to an increase in prices. Under no information (hence, in the case of walkers), an increase in transport costs has only a negative impact, by decreasing agents’ willingness to pay. However, in the case of surfers, we observe two countervailing effects: on the one side, agents’ net valuation for the good decreases, but transport costs also have an impact on competition, by reducing the number of buyers for which firms compete. When the number of walkers is sufficiently large, the first effect prevails and an increase in $t$ is reflected by a decrease in $p^c$. On the opposite, when e-restaurants serve mainly surfers, the competition effect prevails and an increase in transport costs can lead to an increase in prices.
I restrain my focus on the case of \( v \geq (\bar{N} t)/4 \), that is, when the market exists even in the absence of the aggregator.

One last point to notice is that with probability \( \epsilon = \left( \frac{\bar{N} - n - 1}{N - 1} \right) \left( \frac{\bar{N} - n}{N} \right) \) neither of the (two) favourite restaurants of a surfer is listed online. Alike walkers, they may decide to randomly select one of the available restaurants or restrain from purchasing.

**Lemma 6.** When the aggregator is available, it is still optimal for \( c \)-restaurants to behave the same as without it and charge walkers a price

\[
p^e = \frac{(4v - \bar{N} t)}{4N}.
\]

However, the size of the population that they serve decreases from 1 to \( \alpha + \epsilon(1 - \alpha) \). Therefore, their profit is

\[
\pi^c = (\alpha + \epsilon(1 - \alpha)) \frac{(4v - \bar{N} t)^2}{8(N - 2) NN^t}.
\]

E-restaurants serve walkers at the same price as \( c \)-restaurants, while the online price is

\[
p^e = \begin{cases} \frac{4v - \bar{N} t}{4N} & \text{if } \frac{N}{4} \leq \frac{v}{t} < \frac{N(8N - 3n - 5)}{4(n - 1)} \\ \frac{2N - n - 1}{n - 1} t & \text{if } \frac{N(8N - 3n - 5)}{4(n - 1)} < \frac{v}{t}. \end{cases}
\]

Profits come from selling both offline and online. The gross profit of online firms is

\[
\pi^e = \pi^c + (1 - \epsilon)(1 - \alpha)p^e D_r
\]

, where:

\[
p^e D_r = \begin{cases} \frac{S + n - 2}{4(N - 1)} + \frac{(\bar{N} - n)}{N t} \frac{(4v - \bar{N} t)}{2N^2} & \text{if } \frac{v}{t} \in \left( \frac{N}{4}, \frac{3}{N - 1} \right] \quad \text{(Region A)} \\ \frac{(4v - \bar{N} t)(2N - n - 1)}{4N^2(N - 1)} & \text{if } \frac{v}{t} \in \left( \frac{N}{4}, \frac{3}{N - 1}, \frac{8N - 3n - 5}{n - 1} \right] \quad \text{(Region B)} \\ \frac{2N - n - 1}{n - 1} \frac{t}{N(N - 1)(n - 1)} & \text{if } \frac{N(8N - 3n - 5)}{4(n - 1)} < \frac{v}{t} \quad \text{(Region C)}. \end{cases}
\]

Online restaurants must pay a fee \( f^e \) to the aggregator. Hence, their net profit is \( \pi^e - f^e \).

Lemma 6 provides interesting insights on the role of competition in this model. Restricting our attention to the case in which the good valuation is large enough for a market to exist even if the aggregator is not available, the fact that surfers can always purchase offline puts an upperbound to prices which limits restaurants’ profits. If they could, \( e \)-restaurants would charge larger prices to surfers than to walkers any time that \( \frac{v}{t} < \frac{N(8N - 3n - 5)}{4(n - 1)} \). However, surfers can always decide to buy offline from a restaurant discovered online and hence \( e \)-restaurants are constrained to charge \( p^e \leq p^e \) to surfers. Therefore, in regions A and B, \( e \)-restaurants charge everyone the same. In region C, competition online is strong enough to play a role in the determination of prices. As a matter of facts, it pushes online prices below the offline ones.
3.5 Number of firms

The decision to be online is endogenous: restaurants compare their profit offline $\pi^c$ with the profit online, net of the aggregator fee, $\pi^e - f^e$. The number of restaurants active in each market depends on profitability. To study it, let first formally define the timing considered.

The initial condition is the long run equilibrium previous to the entry of the aggregator. In such a setting, the equilibrium condition implies that the number $N_0$ of active firms is such that profit is equal to the licence cost: $\pi = f$. Following the entry of the aggregator, in the short run firms decide whether to be listed on the aggregator or not. Their decision is based on the comparison of profits of $e$- and $c$-restaurants (respectively, $\pi^e$ and $\pi^c$). The number $n$ of $e$-restaurants increases as long as it is profitable. The medium run equilibrium is reached for $n = n_0$, that is when the net of fees profits equate: $\pi^e - f^e = \pi^c$. The long run equilibrium occurs when the total number of active firms is $N = N_1$, where $N_1$ is the number of active firms such that profits on both markets equal the license cost: $\pi^e - f^e = \pi^c = f$.

The initial number of active restaurants $N_0$, such that $\pi = f$ is

$$N_0 = \frac{(4v - \bar{N} t)^2}{8(N-2)\bar{N}t}.$$  

Replacing $N_0$ in equation (33), it is possible to obtain the short and medium run equilibrium profit for $c$-restaurants:

$$\pi^c = (\alpha + \epsilon (1 - \alpha)) f. \quad (37)$$

$E$-restaurants serve both surfers and walkers. From equation (35), it is possible to notice that the number $N$ of $c$-restaurants does not affect profits obtained from serving surfers.

In the medium run, the number of $e$-restaurants adjusts, hence $n_0$ is such that $\pi^e - f^e = \pi^c$. Noticing that $\pi^e = (\pi^e + (1 - \epsilon)(1 - \alpha)p^e D_r)$, it is immediate to conclude that $n_0$ is implicitly defined by:

$$(1 - \epsilon)(1 - \alpha)p^e D_r = f^e, \quad (38)$$

where $(1 - \epsilon) = \frac{n(2N-n-1)}{N(N-1)}$ and $p^e D_r$ is defined by equation (36). Depending on the region and on the value of the parameters (in particular the value of $v$ and $t$), corner solutions may arise, with therefore $n_0 = 0$ (no restaurant is listed online, and the equilibrium is defined by
section 2.1) or $n_0 = N$ (all the restaurants are listed online, and the equilibrium is defined by section 2.2).

**Proposition 3.** In the long run, entry of the aggregator decreases the number of active firms, hence a reduction in variety is observed. The total number of active firms is

$$N_1 = (\alpha + \epsilon(1 - \alpha)) \frac{(4\nu - \tilde{N}t)^2}{8(N - 2)} \tilde{N}t = (\alpha + \epsilon(1 - \alpha)) N_0. \quad (39)$$

Proposition 3 is discussed in section 4, together with proposition 4.

## 4 Welfare analysis

This section uses the results from the previous one to determine the welfare effects of the entry of the aggregator.

**Proposition 4.** In the short run, entry of the aggregator enhances online firms’ profits. In the medium run, firms’ entry in the online market reduces everyone’s profits and firms are locked in a Bertrand supertrap. Indeed, for all firms medium run profits are lower than before the aggregator’s entry.

Figure 8 depicts the evolution of firms’ profits in the different equilibria.

![Figure 8: Comparison of profits in the short, medium and long run.](image)

Proposition 4 shows that e-restaurants benefit from the presence of the aggregator in the short run, but their medium run profit is lower than if the aggregator hadn’t entered the market. In other words, restaurants face a sort of prisoner dilemma: they use the aggregator to increase their profit in the short run, which is possible for the combined effect of both
an increase in the total size of the market and a business stealing mechanism from c- to e-restaurants. However, being listed online is detrimental to medium run profits, for competition increases: for low valuations of the good, online e-restaurants are bound to maintain prices low to compete with c-restaurants, and when the valuation is high it’s the competition amongst e-restaurants that keeps prices down. Competition in the walkers market remains constant, but the size of the market shrinks, for now surfers buy online. Ex-post, all firms are hurt by the presence of the aggregator, but there is nothing that they can do and – given the existence of the aggregator – their optimal strategy may still be to be listed online.

The decrease in profits has a direct consequence on variety. Proposition 3, through equation (39), shows that the entry of the aggregator implies a reduction of firms in the long run equilibrium. Firms exit the market, and this could have two consequences on welfare: on the one side, less firms could imply larger transport costs. On the other hand, active firms are responsible of the deadweight loss corresponding to sunk entry costs, hence its reduction would be beneficial for welfare. The standard result in horizontal competition is that there is excess of variety (that is, too many firms enter the market, taking into account the trade-off between transport and sunk costs), hence a reduction in variety is usually welfare enhancing. In this case this result is even stronger. As a matter of fact, in the offline market, price and equilibrium transport costs do not depend on the number of firms, as explained in subsection 2.1 (for the different forces cancel out). Hence, any reduction in the number of firms has the unique effect of reducing sunk costs. Concerning the online market only, the total number of active firms is irrelevant for the equilibrium, and what matters is the number \( n \) of online firms. Hence, any reduction in the number of firms is irrelevant at any internal solution with \( N \geq n \).\(^{25}\) This implies that, from a welfare perspective, the optimal balance between sunk and transport costs implies \( N = n \), that is, all firms are listed online.

**Proposition 5.** Entry of the aggregator strictly enhances consumers’ surplus in the medium and long run. In the long run, the entry of the aggregator always enhances total welfare. The long run total welfare increase due to the entry of the aggregator is:

\[
\Delta W = \pi^a + \frac{(1 - \alpha)n}{N} \left( S_j^e - S_j^c \right) + \frac{\bar{N} - n}{N - 1} (S_k^e - S_k^c) > 0.
\]  

(40)

where \( \pi^a \) is the aggregator’s profit, \( S_j^e = \frac{4(v - p^e) - t}{s} \), \( S_j^c = \frac{\bar{x}_m(\frac{2v}{N} - p^c - \frac{t}{2})}{2} \).

\[ S_k^e = \begin{cases} 
\frac{(v - p^e - t/2)^2}{8} & \text{if } \gamma < \frac{3\bar{N}}{4(N-1)} \\
\frac{4(v - p^e - t/2)^2 - 3t}{8} & \text{if } \gamma \geq \frac{3\bar{N}}{4(N-1)}.
\end{cases} \]

\(^{25}\)Whether the equilibrium conditions \( n_0 \) and \( N_1 \) satisfy the condition for an internal solution \( N \geq n \) depends on the parameter values \( f \) and \( f^e \). Whenever this is not the case, by a fixed point argument it is possible to show that the equilibrium would be \( N = n \in (N_1, n_0) \).

\(^{26}\)Notice that \( \frac{3\bar{N}}{4(N-1)} < \frac{N(sN-3n-5)}{4(n-1)} \), hence in the first segment \( p^e = p^c \).
The term \((S_j^c - S^c)\) represents the increase in surplus of a surfer consuming from the spoke where (s)he is located, whereas \((S_k^c - S^c)\) is the increase in surplus of a surfer consuming from a spoke different from where (s)he is located.

A revealed preference argument is sufficient to guarantee that consumers’ surplus can only be weakly larger with the aggregator. Indeed, consumers’ surplus without aggregator is null, and the option of purchasing offline remains always available to consumers. Therefore, nobody would “become a surfer” unless it is weakly profitable.

The increase in consumers welfare comes, at least partially, from the increase in available information that reduces inefficient mismatch costs. Indeed, informed consumers are able to consume the most valuable product, that is the one that maximises the difference between value and transport costs.\(^{27}\) Such increase in the value of transactions may have an expansionary effect: if \(\frac{v}{4} < \frac{3N-4}{4}\) (regions A and part of B), the market is partially uncovered without the aggregator. As previously noticed, consumers located closer to the origin of spokes (and hence to firms) are not served in the incomplete information setting – although they are those with the lowest transportation cost – because they show the highest expected transportation cost. In the complete information framework, they are the first to be served and those who most benefit from the transaction. Therefore, we obtain an increase in welfare due to an expansion of the market. Notice that in regions A and B the price is the same with and without the aggregator, which implies that this effect is a consequence of the increase in information and it is not related to the possible change in competition among firms, which only affects the equilibrium price in region C. A further, positive, effect may arise because online prices are lower than offline in region C. Firms, due to increased competition, extract a lower share of surplus.

The total long run welfare is equal to the long run consumers surplus by the zero-profit condition. Therefore, it is immediate to conclude that the increase in total surplus is equal to the increase in the consumers’ one.

\(^{27}\) Surfers learn the firms’ location before consuming and therefore they are able to avoid unnecessary transport costs and to consume products that they do not value, that is, the aggregator solves the information issue and reduces mismatch costs.
5 Final remarks

I use the spokes model of horizontal competition to analyse the impact of the entry of an online review aggregator, such as ClubKviar or Opentable, on both the market equilibrium and welfare. I show that aggregators are welfare-enhancing.

The increase in welfare comes through three channels. Firstly, aggregators expand the market when it is not initially covered, by allowing for some additional consumers to be reached. This is possible because consumers can learn the location of the restaurant via the aggregator and thus every efficient transaction takes place. Secondly, the lack of information absent the aggregator is also responsible for mismatch costs in the form of inefficient transactions taking place (a consumer may attend a restaurant which is not her/his best choice and even agree on transactions that generate a negative surplus). Entry of the aggregator guarantees that all, and only, surplus-enhancing transactions take place. Finally, when the consumers’ valuation of the good is large enough, entry of the aggregator makes competition fiercer and this may lead to a decrease in prices. To some extent, we can view the aggregator as a device boosting some of the positive effects of the internet, such as the reduction in search costs and the increase in the likelihood of a match occurring, as documented in Rapson and Schiraldi (2013) for the second-hand car market in California.

Profits tend to increase in the short run, partly because of a larger share of the market being served, and partly as a business-stealing effect (from offline to online firms). Entry of additional online firms pushes both online and offline medium-run profits down, however, to a level that is lower than before the aggregator’s entry. Therefore, firms face a prisoner’s dilemma (or Bertrand supertrap) situation, in which it is optimal for firms to resort to the aggregator as a deviation from the previous equilibrium, yet they would be better off if they did not.

In the long run, the zero profit condition induces a reduction in the active number of firms in the market, hence variety decreases. This could affect consumers’ surplus if they have a strong taste for variety. However, any such decrease in welfare is compensated by the dynamics described above. Furthermore, as it is standard in the horizontal competition literature, the total number of firms tends to exceed the optimum when we account for both consumers’ transport costs and firms’ sunk entry costs. Taking all different forces into consideration, the total welfare effect is always positive.

The model provides several testable predictions, including the number of active firms shrinking when an aggregator enters a market; profits increasing in the short run for the firms listed online first, and long run profits remaining constant.
Appendix A  Discussions and Extensions

A.1 Networks and two-sided markets

A two-sided market (2SM) is a platform with two distinct user groups, both interested in the size of the other. Typical examples go from credit cards to yellow pages. In the latter example, advertisers are interested in the number of consumers, while users care about the number of firms listed. The optimal strategy for a two-sided platform (e.g. the credit card or the yellow pages company) depends on the network effects, because it may be profitable to be aggressive on one side of the market, in order to become attractive on the other and the platform may shift surplus extraction from one side to the other.

Aggregators are a two-sided market: consumers care about the number of e-restaurants that are listed, and restaurants care about the number of surfers using the platform. Shall one study competition amongst different aggregators, it would be crucial to take the 2SM aspects into account. However, in this work I focus on the impact of having one aggregator in the market in the most parsimonious way. The combination of three following modelling choices allowed to disregard the 2SM specificities:

- there is only one aggregator,
- surfers have no cost of using the aggregator, hence all of them always use it,
- restaurants pay a fixed (and exogenous) fee to be listed online.

Having only one aggregator, which charges an exogenous fee, I avoid any consideration on the optimal pricing strategy of the platform. The assumption on surfers’ connection cost ensures that restaurants are willing to be listed online, and the zero cost for surfers ensures that they are willing to use the aggregator even when the number of listed restaurants is small. Furthermore, having only one aggregator, users don’t have to choose which platform to use.

Clearly, these are simplifying assumptions, that may be relaxed in future analysis, but they allow to disentangle the effect of the aggregator on the market from any other factor. Although this model departs from the issues treated within this branch of literature, the general insights hold, especially when moving to the case with more than one aggregator or if the number of users (surfers) is endogenous. For a review of the literature on two-sided markets, see Roson (2005); Rysman (2009); Filistrucchi et al. (2013, 2014).

In the aggregator framework, if each of them specialises on one sub-market (e.g. each aggregator could focus on a vertically different type of restaurant), my results still hold. When the number of aggregators grows, network externalities will play a crucial role and the 2SM
literature may shed light on the possible behaviour of each aggregator. Armstrong (2006) discusses the effect of platform competition, distinguishing between the case of single and multi-homing (that is, if users can use both platforms or if the two are mutually exclusive). Results and especially, the maximum number of aggregators that can survive in a market, would depend crucially on whether we assume that consumers and restaurants can use more than one platform or not. In both cases, competition amongst aggregators is likely to have important consequences on welfare. Section 4 concludes that the presence of an aggregator is beneficial, and the mechanism goes through the fact that it reduces transport costs and improves the quality of matches by spreading information at a negligible cost. When several aggregators compete, this welfare-enhancing property of the aggregator is weakened. Indeed, either the users’ cost increases (if surfers have to compare the information on several platforms, the process becomes less efficient), or the quality of information decreases (if surfers only use one aggregator, which lists only a fraction of the e-restaurants).

A.1.1 Endogenous use of the aggregator

The model requires that an exogenous fraction $\alpha$ of consumers never uses the aggregator, while the remaining always does. The implicit assumption is that the cost of using the aggregator is prohibitive for walkers and null for surfers. Relaxing this assumption may have consequences on the results of the model.

Notice that, if surfers bare a cost of using the aggregator, this reduces their surplus and hence welfare decreases. However, by revealed preference, any consumer choosing to use the aggregator will always find its use welfare enhancing. Therefore, the change on welfare would only be quantitative but not qualitative.

Endogenising the decision to become a surfer may complicate the equilibrium substantially. As in any two-sided market, the number of surfers is crucial to determine e-restaurants’ profits and hence their number, which itself affects the number of surfers. Suppose that consumers bare a cost $c$ of using the aggregator, and that $c$ is heterogeneously and randomly distributed following a continuous c.d.f. $\Phi$. Given the own location and the equilibrium online price, an agent is indifferent between being a surfer or a walker when the expected surplus online is equal to the usage cost. Surfers are then characterised by the equation

$$\frac{n}{N}(v - p^e - (1 - x_m)t) + \frac{n(N-n)}{N(N-1)}(v - p^e - x_mt) - c \geq \frac{2n}{N} - p^e - t \left( \frac{N-2}{N}x_m + \frac{1}{N} \right).$$

The equilibrium share of surfers will depend on the distribution $\Phi$.

Intuitively, $\alpha$ should be increasing in the number of e-restaurants, and decreasing in prices. The number of restaurants should increase with the number of consumers and decrease with the price. If an equilibrium exists, it is reasonable to expect that it will be qualitatively similar to the one in the baseline model. However, the existence of an equilibrium is not ensured,
because combining the heterogeneity in transport costs with the one to access the aggregator may produce a mass of indifferent consumers as well as some discontinuities and non-linearities in the demand function. The main drawback, besides the reduced mathematical tractability, is that both the existence conditions and the results depend dramatically on the shape of $\Phi$.

### A.2 Price discrimination

When consumers self-select into two groups, i.e. *surfers* and *walkers*, restaurants can tell them apart and therefore they may decide to use this information to practice price discrimination. In the baseline model, this does not occur because the valuation of the good is the same for both groups, hence restaurants cannot increase profits through discrimination.

In this model, *surfers* can always buy offline, therefore, the online price can at most be as large as the offline one, and in some cases it is optimal for the online price to be lower. This limits the options for an e-restaurant to increase profits through price discrimination. Indeed, even if e-restaurants knew that *surfers* willingness to pay is larger, they could not take advantage of it through a price discrimination mechanism. The exception would be when restaurants find it optimal not to serve *walkers* and charge the larger price to *surfers*.

However, if *surfers* are willing to pay less than the others,\(^{28}\) then aggregators may have a further expansionary effect on the market. Suppose, as depicted in figure 9, that the valuation $v^w$ of *walkers* is larger than the one of *surfers* ($v^s$). Then, the offline price may be (depending on the relative size of the two groups) such that *surfers* are excluded from the market. Once the aggregator starts operating, e-restaurants can charge a lower price to *surfers* and start serving both types of consumers. While the intuition of market expansion is easy to understand and general (it occurs as long as the optimal price for *walkers* is above the piecewise price schedule for *surfers*), providing a formal solution for the model with different valuation proves a delicate task. Indeed, the solution depends on the assumption on the difference in valuation, and hence on the type of intersection (if any) between the two price schedules. Clearly, welfare increases whenever the aggregator allows to serve a larger subset of the population.

\(^{28}\)Supposing that *surfers* are willing to pay less seems reasonable. Surfers have no cost of using the aggregator, hence they are likely to have better IT knowledge and a lower cost of time. Both characteristics could be correlated with age (negatively), and low income and willingness to pay (positively).
Appendix B  Proofs

Proof of Lemma 1. The expected mismatch cost for an agent consuming a meal in a randomly selected restaurant is:

\[ EM = \Pr(j > N)x_m + \Pr(j \leq N) (\Pr(r = j)(1 - x_m) + \Pr(r \neq j)x_m) \]
\[ = \frac{\bar{N} - N}{N} x_m + \frac{N}{N} \left( \frac{1}{N}(1 - x_m) + \frac{N - 1}{N} x_m \right) \]
\[ = \left( \frac{\bar{N} - 2}{N} x_m + \frac{1}{N} \right). \] (41)

The expected valuation of a consumer is \(2v/\bar{N}\), where \(2/\bar{N}\) is the probability that agent \((x_m; l_j; l_k)\) randomly consumes either \(j\) or \(k\). Therefore, the expected utility of an agent is positive if:

\[ 0 \leq \frac{2v}{\bar{N}} - p - EM \]
\[ 0 \leq \frac{2v}{\bar{N}} - p - \left( \frac{\bar{N} - 2}{N} x_m + \frac{1}{N} \right) \]
\[ x_m \leq \frac{\bar{N}}{\bar{N} - 2} \left( \frac{2v}{\bar{N}} - p - \frac{1}{N} \right). \] (42)

From equation (42) follows that \(p = \frac{2v}{N} - \left( \frac{\bar{N} - 2}{N} \bar{x}_m + \frac{1}{N} \right)\). Notice that, since consumers randomly choose where to consume, each firm faces the same demand \(D = \frac{1}{N} \frac{2}{\bar{N}} \bar{N} (\bar{x}_m - \frac{1}{2}) = \frac{2}{N} (\bar{x}_m - \frac{1}{2})\).

A restaurant’s profit function is therefore \(\pi = pD - f = \left( \frac{2v}{N} - \left( \frac{\bar{N} - 2}{N} \bar{x}_m + \frac{1}{N} \right) \right) \frac{2}{N} (\bar{x}_m - \frac{1}{2}) - f\). Maximising profit with respect to \(\bar{x}_m\), the first order condition implies that \(\bar{x}_m = \frac{1}{N - 2} \left( v + \frac{\bar{N} - 4}{4} \right)\). The equilibrium price and the other results in the lemma directly follow. \(\square\)

Proof of Lemma 2. Consumers \((x_m; l_j; l_k)\) can be of three types: type-0 consumers are those with both \(j > n\) and \(k > n\), i.e. no restaurant with positive valuation is available;
type-I consumers are those with either \( j < n \) or \( k < n \), i.e. only one restaurant with positive valuation is available; type-II consumers are those with both \( j < n \) and \( k < n \), i.e. both restaurants with positive valuation are available. Clearly, type-0 consumers are not active on the market, and can be disregarded.

For any \( j \neq k \), it is never rational for firm \( k \) to have \( |p_k - p_j| > 1 \), for then all consumers would always find it more convenient to purchase from firm \( j \). Therefore, I focus on the case \( |p_k - p_j| < 1 \). To construct the demand faced by firm \( r \), I consider type-I and type-II consumers separately. By construction, \( x_m \) is the distance of a consumer from the origin of any spoke \( m \) other than the own. With a little abuse of notation, define \( x_r \) as the distance of a consumer from the origin of spoke \( r \). Then,

\[
x_r = \begin{cases} 
1 - x_m & \text{if } r = j \\
x_m & \text{if } r = k 
\end{cases}
\]  

(43)

Because type-I consumers positively value only one available product, firm \( r \) is a monopolist for the consumers interested in its product. Consumer \((x_m; l_j; l_k)\) is willing to buy from firm \( r \) if \( v - p_r - x_r \geq 0 \), which is equivalent to say \( x_r \leq v - p_r \). Therefore, the demand that firm \( r \) faces is

\[
D^I_r = \begin{cases} 
\frac{2(\bar{N} - n)}{N(N - 1)} (v - p_r) & \text{if } v - p_r \leq 1 \\
\frac{2(\bar{N} - n)}{N(N - 1)} & \text{if } v - p_r > 1,
\end{cases}
\]  

(44)

where \( \frac{2}{N} \) represents the density of the distribution of consumers, while \( \frac{(\bar{N} - n)}{(N - 1)} \) is the probability \( Pr(j = r \land k > n \lor j > n \land k = r) \) (i.e. the probability that the agent is of type-I, with \( r \) as the unique valuable product). The type-I market is covered if and only if \( v - p_r \geq 1 \).

In the case of type-II consumers, firm \( r \) competes with each firm \( s \neq r \) (with \( s \leq n \)) for both consumers \((x_m; l_r; l_s)\) and \((x_m; l_s; l_r)\). Such consumers prefer to purchase from firm \( r \) as long as \( x_r \leq \frac{p_s - p_r + 1}{2} \). Hence, the demand faced by firm \( r \) is

\[
D^II_r = \frac{2}{N(N - 1)} \sum_{s \neq r; s \leq n} \frac{p_s - p_r + 1}{2}.
\]  

(45)

The demand faced by firm \( r \) is obtained by summing (44) and (45), as long as the equilibrium price \( p \) is such that \( v - p > \frac{1}{2} \), otherwise firms are never serving anyone that is not located on their own spoke. In such case, each firm is a monopolist on its spoke, it faces a demand \( D_r = \frac{2}{N} \bar{x}_m \) with \( \bar{x}_m = v - p \). The unique profit maximising equilibrium under such circumstances is \( p = \frac{v}{2} \).
Hence, the total demand is

\[
D_r = \begin{cases} \frac{2}{N}(v - p_r) & \text{if } v - p_r \leq \frac{1}{2} \\ \frac{1}{N} \left( \sum_{s \neq r; s \leq n} \frac{p_s - p_r + 1}{2} + (N - n)(v - p_r) \right) & \text{if } \frac{1}{2} < v - p_r \leq 1 \\ \frac{2}{N} \left( \sum_{s \neq r; s \leq n} \frac{p_s - p_r + 1}{2} + (N - n) \right) & \text{if } v - p_r > 1. \end{cases}
\]

(46)

It directly follows, from the maximisation problem, that the optimal price schedule is

\[
p^a = \begin{cases} v & \text{if } v \leq 1 \\ v - \frac{1}{2} & \text{if } 1 < v < \frac{4N - n - 3}{2(2N - n - 1)} \\ \frac{(n-1) + 2(N-n)v}{4N - 3n - 1} & \text{if } \frac{4N - n - 3}{2(2N - n - 1)} \leq v < 2 \\ v - 1 & \text{if } 2 \leq v < \frac{2N - 2}{n - 1} \\ \frac{2N - n - 1}{n - 1} & \text{if } \frac{2N - 2}{n - 1} < v. \end{cases}
\]

(47)

However, under the assumptions that \( \bar{N} \geq 4 \) and \( v \geq \frac{N}{4} \), we can disregard the first interval.

**Proof of Lemma 3.** This lemma relies mainly on the results in the previous lemmas. As for the crossing of \( p^a \) and \( p^w \), it is straightforward to check that in all regions but the last one, the crossing of the two functions occurs outside (to the left) of the region’s interval. Concerning the region defined by \( \frac{2\bar{N} - 2}{n - 1} < v \), it is immediate to obtain that \( \frac{2\bar{N} - 1}{n - 1} = \frac{v}{N} - \frac{1}{4} \) for \( v = \frac{8\bar{N} - 3n - 5}{4(n-1)} \bar{N} \). In order to check that the crossing occurs within the region, we need to solve the inequality \( \frac{8\bar{N} - 3n - 5}{4(n-1)} \bar{N} > \frac{2\bar{N} - 2}{n - 1} \).

This simplifies into \( (8\bar{N} - 3n - 5)\bar{N} > 8\bar{N} - 8 \), assuming that \( n > 1 \), which must be the case, otherwise the optimal price \( p^a \) would be negative.

Because \( (8\bar{N} - 3n - 5)\bar{N} > (8\bar{N} - 3\bar{N} - 5)\bar{N} \), a sufficient condition for the inequality to hold is to have that \( (8\bar{N} - 3\bar{N} - 5)\bar{N} > 8\bar{N} - 8 \), which simplifies into \( 5(\bar{N} - 1)\bar{N} > 8(\bar{N} - 1) \), hence \( 5\bar{N} > 8 \), which is always verified.

**Appendix C  Old proofs**
Proof of Proposition 1. The maximisation problem for the $e$- and $c$-restaurants is respectively:

$$\max_{p^e} \left( \frac{2(\alpha + (1 - \alpha)e)}{N} \left( \bar{x}_u - \frac{1}{2} \right) + (1 - \alpha)D_r \right) p^e$$

(48)

and

$$\max_{p^e} \frac{2(\alpha + (1 - \alpha)e)}{N} \left( \bar{x}_u - \frac{1}{2} \right) p^e,$$

(49)

The FOCs are:

$$\frac{2(\alpha + (1 - \alpha)e)}{N} \left( \bar{x}_u - \frac{1}{2} - p^e \frac{\bar{N}}{N-2} \frac{n}{N} \right) + (1 - \alpha) \left( D_r + p^e \frac{\partial D_r}{\partial p^e} \right) = 0$$

(50)

and

$$\left( \bar{x}_u - \frac{1}{2} \right) + p^e \frac{\bar{N}}{N-2} \frac{N-n}{Nt} = 0,$$

(51)

The two simplify into:

$$-\frac{(\alpha + (1 - \alpha)e)\bar{N}}{N(N-2)} \left( \frac{2v}{N} - \frac{t}{2} - \frac{2n}{N} p^e - \frac{N-n}{N} p^e \right) = 2(1 - \alpha) \left( \frac{v - 2p^e}{N} \right) - \frac{(n-1)\frac{v}{2} - (N-n)(v-2p^e)}{(N-1)}$$

if $v - p^e \leq \frac{t}{2}$

$$= \frac{\frac{v}{2} - (N-n)(v-2p^e)}{(N-1)}$$

if $\frac{t}{2} < v - p^e \leq t$

$$= \frac{v - p^e}{N}$$

(52)

and

$$\frac{N-n}{N} p^e = \frac{n}{N} - \frac{t}{4} - \frac{n}{2N} p^e$$

(53)

Combining the two:

$$-\frac{(\alpha + (1 - \alpha)e)(v - \frac{t}{4} - \frac{3n}{2N} p^e)}{N(N-2)} = \frac{1}{N^2} \left( \frac{v - 2p^e}{N} \right) - \frac{(n-1)\frac{v}{2} - (N-n)(v-2p^e)}{(N-1)}$$

if $v - p^e \leq \frac{t}{2}$

$$= \frac{(n-1)\frac{v}{2} - (N-n)t}{(N-1)}$$

if $\frac{t}{2} < v - p^e \leq t$

$$= \frac{v - p^e}{N}$$

(54)

$$p^e = 2N \left\{ \begin{array}{ll}
\frac{(\alpha + (1 - \alpha)e)N^2(\frac{N}{2} - \frac{1}{4}) + (1 - \alpha)N(\bar{N}-2)v}{4(1-\alpha)N^2(N-2)+3n(\alpha+(1-\alpha)e)N^2} & \text{if } v - p^e \leq \frac{t}{2} \\
\frac{(\alpha + (1 - \alpha)e)(N-1)N^2(\frac{N}{2} - \frac{1}{4}) + N(\bar{N}-2)(1-\alpha)((\bar{N}-n)v + (n-1)\bar{t})}{(N-1)(4N-3n-1)N^2(N-2)(1-\alpha)+3n(\alpha+(1-\alpha)e)\bar{N}} & \text{if } \frac{t}{2} < v - p^e \leq t \\
\frac{(\alpha + (1 - \alpha)e)N^2(\frac{N}{2} - \frac{1}{4}) + 2N-n-1N(\bar{N}-2)(1-\alpha)\bar{t}}{3n(\alpha+(1-\alpha)e)N(N-2)(N-2)(1-\alpha)} & \text{if } v - p^e > t.
\end{array} \right.$$  

(55)

Solving the equation, and computing the intervals, we obtain

$$p^e = \begin{cases} 
\Psi_1^e v - \Psi_1^e t & \text{in Region } R^e1 \\
\frac{v}{2} & \text{in Region } R^e2 \\
\Psi_2^e v + \Psi_3^e t & \text{in Region } R^e3 \\
v - t & \text{in Region } R^e4 \\
\Psi_5^e v + \Psi_5^e t & \text{in Region } R^e5
\end{cases}$$

(56)
where

\[ \Psi_1' = 2N \frac{(N(N-2) - (1 - \alpha) + \alpha + (1 - \alpha) \alpha N)}{(N-2) - (1 - \alpha) + \alpha + (1 - \alpha) \alpha N} \]

\[ \Psi_1 = \frac{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)}{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} \]

\[ \Psi_3' = 2N \frac{(N(N-1)N^2 + (N-3)N^2(N-4)(2)(1-a))}{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} \]

\[ \Psi_3 = \frac{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)}{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} \]

\[ \Psi_5' = 2N \frac{(N(N-1)N^2 + (n-1)N^2(N-2)(1-\alpha))}{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} \]

\[ \Psi_5 = \frac{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)}{3N(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} \]

and regions are defined as follows.

**Region R¹**: \( \Psi_1' < \frac{1}{2} \left( \frac{3N(N-2)N^2 + 4N^2(N-2)(1-\alpha)}{3N(N-2)N^2 + 4N^2(N-2)(1-\alpha)} \right) \)

**Region R²**: \( \Psi_1' < \frac{1}{2} \left( \frac{3N(N-2)N^2 + 4N^2(N-2)(1-\alpha)}{3N(N-2)N^2 + 4N^2(N-2)(1-\alpha)} \right) \)

**Region R³**: \( \Psi_3' < \frac{1}{2} \left( \frac{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)}{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)} \right) \)

**Region R⁴**: \( \Psi_5' < \frac{1}{2} \left( \frac{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)}{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)} \right) \)

**Region R⁵**: \( \Psi_5' < \frac{1}{2} \left( \frac{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)}{3N(N-1)N^2 + 4N^2(N-2)(1-\alpha)} \right) \)

In order to study the properties of \( p' \), I study first the characteristics of some specific points.

The intersection of \( p' \) with the vertical ax occurs at \( -\Psi_1' \), which is always above \( -\frac{1}{2} \) under the assumption of \( N < 3n: \)

\[ \frac{3n(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)}{3n(\alpha+1-\alpha)\alpha N^2 + 4N^2(N-2)(1-\alpha)} < \frac{1}{2} \frac{N(N-3)N^2}{4N^2} \frac{N^2(N-2)}{N(N-2)} < \frac{1}{2} \]

At point A, we move from region \( R¹ \) to region \( R² \); at point B from region \( R² \) to region
$R^c 3$; at point C from region $R^c 3$ to region $R^c 4$; at point D from region $R^c 4$ to region $R^c 5$; at point E, $p^e$ equals $p^s$.

Concerning point A, notice that it corresponds to a value of $\frac{\alpha}{1}$ which is less than 1, whenever $\Psi_i < \frac{1}{2} + \Psi_1$.

This simplifies to $3n\bar{N} + (\bar{N} - 4) N > 0$, which is always true.

Concerning point B, notice that it corresponds to a value of $\frac{\alpha}{1}$ which is less than 1, whenever $\Psi_3 < \frac{1}{2} - \Psi_3$.

This simplifies to $N^2(1 - \alpha) \left[ n - 1 - (4\bar{N} - 3n - 1) (\bar{N} - 2) \right] (\bar{N} - 2) < (\alpha + (1 - \alpha)\epsilon) \left[ 3n \left( \bar{N} - 1 \right) \bar{N} + N \left( \bar{N} - 3 \right) \right]$. The left hand side of this expression is negative whenever $\bar{N} > \frac{3n}{8} (3 + \sqrt{9(3+n)^2 - 16(5n+3)})$. This is always verified, since

$$\frac{3n}{8} !(3 + \sqrt{9(3+n)^2 - 16(5n+3)}) < 3.$$

Concerning point C, notice that it corresponds to a value of $\frac{\alpha}{1}$ which is less than 2, whenever $2\Psi_3 < 1 - \Psi_3$.

This simplifies to $2n \left( \bar{N} - 3 \right) \bar{N} + 4 \left( n\bar{N}^2 - N \right) + (\bar{N} - 4) N > 0$. This is always true, for the assumption $N < 3n$ ensures that $\bar{N}^2 > \frac{N}{n}$.

Concerning point D, notice that it corresponds to a value of $\frac{\alpha}{1}$ which is smaller than 2, whenever $\frac{1 + \Psi_3}{1 - \Psi_3} < 2$.

This simplifies to $0 < (9n\bar{N}^2 - 35N) \bar{N} + 3n \left( \bar{N}^2 - 3n \right) \bar{N} + 12n \left( \bar{N} - 3 \right) \bar{N}^2 + 21n\bar{N} + 32N + 3nN\bar{N}$. By the assumption that $N < 3n$, one can see that $(9n\bar{N}^2 - 35N) > 0$, all the other terms are also clearly positive.

Concerning point E, notice that it corresponds to a value of $\frac{\alpha}{1}$ which is larger than $\frac{2(\bar{N} - 1)}{n - 1}$, whenever $\frac{2(\bar{N} - n - 1)}{n - 1} - \Psi_1 \geq \frac{2(\bar{N} - 1)}{n - 1}$.

This simplifies to $2N^2(1 - \alpha) (n - 1) (2\bar{N} - n - 1) (\bar{N} - 2)^2 > [9N\bar{N} + 18n\bar{N}^2 + 6n^2\bar{N}^2 - 12n\bar{N}^3 - 6n^2\bar{N} - (1 - \alpha)\epsilon] (\bar{N} - 1) \bar{N}$.

While the left hand side is always positive, we can show that the right hand side is negative by showing that $9N\bar{N} + 18n\bar{N}^2 + 6n^2\bar{N}^2 - 12n\bar{N}^3 - 6n\bar{N} < 0$. A sufficient condition for that is $\bar{N} > \frac{3 + \sqrt{25}}{2}$, which is always true. This ensures a crossing between $p^e$ and $p^s$ in the region in which $p^s$ is flat.

\[\Box\]
Proof of Proposition 2. From equation (7) we notice that, for \( v/t > 2(\bar{N} - 1)/(n - 1) \), the price \( p^e_\ell \) is constant with respect to the valuation, while lemma ?? states that \( \partial p^e_\ell / \partial t > 0 \), therefore, it always exist a value of \( v \) such that \( p^e_\ell > p^e_0 \).

To ensure that a lower bound \( v \) exists, such that \( p^e_\ell < p^e_0 \) for any \( 0 < v < \bar{v} \), it is sufficient to notice that: i) from equation (7), \( p^e_\ell > 0 \) for any \( v > 0 \); ii) \( p^e_\ell \leq 0 \) for \( 0 < v \leq \bar{v} \). 

Remark i) is immediate. To show that \( p^e_\ell \leq 0 \) notice that: i) from equation (7), \( p^e_\ell \). To obtain the price online, we must then compare prices in equation (7) with the offline price. Regardless of the concavity with respect to \( \alpha \), if \( \alpha \) is defined as the horizontal ax varies and the number of crossings between \( p^e_\ell \) and \( p^e_\ell \) (once or more) depends on the value of \( \bar{N} \).

Proof of Proposition 2. From the (linear) expression of the equilibrium prices \( p^e \) and \( p^c \), it is immediate to conclude that the sign of the derivatives is the same as the coefficient of the parameters. Therefore, \( \partial p^c / \partial t = (\phi_3 / \phi_4 - \Psi_2 / \Psi_4) / (\phi_3 / \phi_4 - \Psi_2 / \Psi_4) \), \( \partial p^e / \partial t = (\Psi_2 / \Psi_4 - \phi_2 / \phi_3) / (\Psi_2 / \Psi_4 - \phi_2 / \phi_3) \).

The sign of the expressions is unequivocal for all but \( \partial p^c / \partial t \). In order to prove that the sign of the derivative is negative for \( \alpha > \bar{\alpha} \) and positive for \( \alpha < \bar{\alpha} \), it is sufficient to notice the following:

1. \( \lim_{\alpha \to 0} \left( \Psi_2 / \Psi_4 - \phi_2 / \phi_4 \right) / \left( \phi_3 / \phi_4 - \Psi_3 / \Psi_4 \right) > 0 \);
2. \( \lim_{\alpha \to 1} \left( \Psi_2 / \Psi_4 - \phi_2 / \phi_4 \right) / \left( \phi_3 / \phi_4 - \Psi_3 / \Psi_4 \right) < 0 \);
3. The expression \( \left( \Psi_2 / \Psi_4 - \phi_2 / \phi_4 \right) / \left( \phi_3 / \phi_4 - \Psi_3 / \Psi_4 \right) \) is a function of degree 2 of \( \alpha \).

Regardless of the concavity with respect to \( \alpha \), or the sign of its derivative, the function can only cross the axis once in the interval \( \alpha \in [0, 1] \), hence the sign of the derivative will be always positive before the crossing, and negative after it. The value \( \alpha = \bar{\alpha} \) is defined as the one at which the derivative takes value 0.

Proof of Lemma 6. As previously discussed, prices online can be at most as large as offline price. To obtain the price online, we must then compare prices in equation (7) with the offline price.
\[ \frac{\partial p^*}{\partial \alpha} \]

Figure 10: Sign of the derivative: \( \frac{\partial p^*}{\partial \alpha} \) can only cross once.

\( \frac{v}{\tau} \leq 1 \): Solving \( \frac{v}{\tau} > \frac{4v-Nt}{4N} \), it is immediate to obtain condition \( v \left( \frac{N-2}{2N} \right) > -\frac{\tau}{4}, \) which is always verified.

\( 1 < \frac{v}{\tau} < \frac{4N-n-3}{2(2N-n-1)} \): Solving \( v - \frac{\tau}{2} > \frac{4v-Nt}{4N} \), the condition obtained is \( v > \frac{N+1}{N-1} t \), which is always verified for \( 1 < \frac{v}{\tau} \) and \( N > 2 \).

\( 2 \leq \frac{v}{\tau} < \frac{4N-n-3}{2(2N-n-1)} \): Solving \( \frac{(n-1)t+2(N-n)v}{4N-3n-1} < \frac{4v-Nt}{4N} \), the condition obtained is

\( (4\bar{N} - n - 5) \frac{\bar{N}}{4} < (4\bar{N} - 3n - 1 - 2\bar{N}^2 + 2n\bar{N}) \frac{v}{\bar{N}}. \)

This must be solved separately, depending on the sign of the right-hand side.

If \((4\bar{N} - 3n - 1 - 2\bar{N}^2 + 2n\bar{N}) < 0\), then \( \frac{(n-1)t+2(N-n)v}{4N-3n-1} < \frac{4v-Nt}{4N} \) if and only if \( \frac{v}{\bar{N}} < \frac{(4N-n-5)}{(4N-3n-1-2\bar{N}^2+2n\bar{N})} \frac{\bar{N}}{4} \); but the RHS is negative, hence the condition is never verified for \( \frac{v}{\bar{N}} > \frac{4N-n-3}{2(2N-n-1)} > 0 \).

Moving to the case of \((4\bar{N} - 3n - 1 - 2\bar{N}^2 + 2n\bar{N}) > 0\), the condition boils down to \( \frac{v}{\bar{N}} > \frac{(4\bar{N}+n-5)}{(4N-3n-1-2\bar{N}^2+2n\bar{N})} \frac{\bar{N}}{4} \). For this condition to be verified withing the interval of existence, one needs \( \frac{(4\bar{N}+n-5)}{(4N-3n-1-2\bar{N}^2+2n\bar{N})} \frac{\bar{N}}{4} < 2 \), which is equivalent to say \((5\bar{N} - 8) (3n - 4\bar{N} + 1) > 0\), and hence \( n > \frac{4\bar{N}-1}{3} \). However, \( \frac{4\bar{N}-1}{3} > \bar{N} \), and it cannot be that \( n > \bar{N} \). Hence, the interval is empty and always \( \frac{(n-1)t+2(N-n)v}{4N-3n-1} > \frac{4v-Nt}{4N} \).

\( 2 \leq \frac{v}{\tau} \leq \frac{2N-n-2}{n-1} \): Solving \( v - t > \frac{4v-Nt}{4N} \), the condition obtained is \( v > \frac{3}{4} \frac{\bar{N}}{N-1} t \), which is always verified for \( 2 \leq \frac{v}{\tau} \leq \frac{2N-n-2}{n-1} \) and \( \bar{N} > 2 \).

Finally, \( p^*D_r \) is simply obtained by multiplying \( p^* \) by \( D_r \) respectively, Eq. (34) and Eq.
This would give:

\[
p^e D_r = \begin{cases} 
\frac{4 \bar{N}v - 4v + \bar{N}t}{2N^2} + (N-n)v \frac{(4v-Nt)}{2N^2} & \text{if } \frac{v}{t} < \frac{1}{4} \bar{N}^{-1} \quad \text{(Region D)} \\
\left(\frac{N+n-2}{4(N-1)} + \frac{(N-n)v}{Nt}\right) \frac{(4v-Nt)}{2N^2} & \text{if } \frac{v}{t} \in \left[\frac{\bar{N}}{4(N-1)}, 1\right] \quad \text{(Region A')} \\
\frac{(2N-n-1)\bar{N}t}{N(N-1)(n-1)} & \text{if } \frac{\bar{N}v}{Nt} > \left(\frac{3}{N-1} \left(8N-3n-5\right)\right) \quad \text{(Region B)} \\
\frac{N(8N-3n-5)}{4(n-1)} & \text{if } \frac{N(8N-3n-5)}{4(n-1)} < \frac{v}{t} \quad \text{(Region C)},
\end{cases}
\]

However, the condition \(v \geq (\bar{N}t)/4\) implies a higher lower bound for Region A', while Region D is outside the parameter's space.

**Proof of proposition 3.** In the long run, the number of firms active on the market adjusts, and \(\pi^e - f^e = \pi^c = f\). From equation 33, \(\pi^c = (\alpha + \epsilon(1-\alpha)) \frac{(4v-Nt)^2}{8(N-2)Nt}\), hence \(N_1 = (\alpha + \epsilon(1-\alpha)) \frac{(4v-Nt)^2}{8(N-2)Nt}\).

**Proof of proposition 4.** By construction the medium run profit is such that \(\pi^e - f^e = \pi^e\). From equation (37) we have that \(\pi^e = (\alpha + \epsilon(1-\alpha)) f\), hence we obtain that in the medium run \(\pi^e - f^e = \pi^c = \alpha f\).

This is clearly lower than the firms' profits before the entry of the aggregator, which was \(\pi = f\). Hence, all firms' medium run profits are lower than their profit in the no-aggregator equilibrium.

**Proof of proposition 5.** First notice that the zero-profit condition guarantees that long run welfare is null both before and after the entry of the aggregator. Hence, the change in long run welfare is given by the aggregator's profit \(\pi^a\) (the sum of the paid fees, net of any cost to deliver the service) and by the change in consumers' surplus.

Furthermore, the surplus of agents who purchase offline is the same both before and after the introduction of the aggregator, hence the variation in consumers' surplus is produced by those consumers who purchase online when the aggregator is available.

Let's consider first the case of a spoke with an \(e\)-restaurants located there and then the opposite case. The surplus of a surfer located at \(x_j\) when consuming from firm \(l_j\) is \(v - p^e - x_j t\). Notice that for the considered range of value for \(v/t\), \(e\)-restaurants always serve all surfers located on their own spoke. Denote by \(S_j^c\) the surplus of all the surfers located on such spoke.

Then we immediately obtain \(S_j^c = \frac{4(e-p^e) - t}{8}\), by computing the area of the trapezoid depicted in figure 11.

To obtain the change in surplus, we need to subtract the surplus \(S_j^c\) that would have been generated by the active consumers in the absence of the aggregator. \(S_j^c = \frac{x_n(\frac{Nt}{2} - p^e - \frac{t}{2})}{2}\) is given
by the area of the triangle in figure 12. It is immediate to remark that $v - p^e - \frac{t}{2} > \frac{2v}{N} - p^e - \frac{t}{2}$, because $p^e \leq p^c$. Therefore, $S^e < S^j$.

Let’s now consider $S^e$, that is, the surplus of surfers located on an empty spoke, whose second preferred restaurant is online. To do so, I distinguish the case of fully covered market ($\frac{v}{t} \geq \frac{3N}{4(N-1)}$) depicted in picture 13 a), and partially covered market ($\frac{v}{t} < \frac{3N}{4(N-1)}$) depicted
Surplus is obtained by computing the two areas: when the market is fully covered, \( S^e_k = \frac{4(v-p^e)-2t}{8} \), whereas \( S^e_k = \frac{(v-p^e-t/2)^2}{24} \) if the market is not fully covered.

The surplus when the aggregator is not available, as before, is \( S^c = \hat{x}_m(\frac{2v}{N}-p^e-\frac{t}{2}) \), and it is depicted in figure 14. From the picture it is immediate to notice that again \( S^e_k > S^c \). \(^{29}\)

![Figure 14: Willingness to pay - surfers on a spoke without an e-restaurant.](image)

To finally compute the total variation in surplus, I compute the weighted sum of \( S^e_j \) and \( S^e_k \), accounting for the probability of each spoke to have an e-restaurant located there. Then, \( \Delta W = \pi^a + \frac{(1-n)n}{N} \left( \left( S^e_j - S^c \right) + \frac{N-n}{N-1} \left( S^e_k - S^c \right) \right) > 0. \)

\(^{29}\)The position of \( \frac{2v}{N} - p^e - \frac{t}{2} \) in figure 14 has been arbitrarily located below \( v - p^e - t \), although the opposite may also be true. However, this is irrelevant for the proof.
References


Edelman, B., Jaffe, S., Kominers, S. D., 2011. To groupon or not to groupon: The profitability of deep discounts.


