Forward Contracts, Market Structure, and the Welfare Effects of Mergers∗

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Abstract
We nest the multistage oligopoly model of Allaz and Vila (1993) within a richer industry structure to explore how the presence of a forward market affects the welfare consequences of mergers as well as the incentive to merge. Our key insight is that while the presence of a forward market is welfare enhancing, the incentive to sell in the forward market decreases with the level of concentration. As a consequence, firms have a greater incentive to merge and the reduction in consumer surplus due to mergers may be exacerbated. We explore the implications of these results for antitrust policy.

Keywords: forward contracting; hedging; mergers; antitrust policy
JEL classification: L13; L41; L44

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1 Introduction

Many imperfectly competitive industries are characterized by forward markets in which firms contract to supply output at a locked-in price upon the opening of a subsequent spot market. Among such industries are crude oil, tin, aluminum, copper, coffee, cocoa, and deregulated wholesale electricity. A long-standing result in the theoretical literature is that the presence of forward markets can result in greater output and lower prices (Anderson and Sundaresan (1984), Eldor and Zilcha (1990), Allaz and Vila (1993)). Little attention has been paid, however, to the role of competition in determining the magnitudes of these effects. One practical implication is that the theoretical literature provides scant guidance to antitrust authorities seeking to evaluate how to incorporate forward markets into merger analysis.

In the current study, we model an oligopolistic industry in which firms sell a homogeneous product and compete through their choices of quantities. Competition happens first in a contract market and later in a spot market. Following Allaz and Vila (1993), we model the level of forward contracting as a strategic variable of the firm. The forward market allows firms to make strategic forward sales that increase aggregate output in equilibrium. These forward sales discipline the exercise of market power in the spot market because firms are less sensitive to the price effect of increasing or decreasing output. Following Perry and Porter (1985), we model firms as having heterogeneous capital stocks that reflect their respective capacities. The model is rich enough to incorporate any number of firms as well as firms of varying sizes (i.e. capital stocks), thereby facilitating the analysis of any arbitrary change in structure. In this way, our analysis brings together two established theoretical literatures: one on the strategic incentive for forward contracting (e.g., Allaz and Vila (1993)), and the other on the effects of horizontal mergers with homogeneous products (e.g., Perry and Porter (1985); Farrell and Shapiro (1990)).

We develop three main sets of results. The first set of results establishes the relationship between market structure and the effect of forward contracting on welfare. We find that while the presence of a forward market always leads to weakly greater surplus relative to Cournot, the magnitude of this effect varies non-monotonically with competition. While surplus increases in the rates at which firms contract forward (their “hedge rates”), which are themselves larger when market structure is more conducive to competition, competition also leads to greater surplus in the absence of forward contracting. In the extreme cases of monopoly and perfect competition, respectively, forward contracting has no effect on welfare. It follows that the welfare-enhancing impact of a forward market is greatest at intermediate levels of competition.

The second set of results prove that forward contracts exacerbate the loss of consumer surplus caused by mergers if the market is sufficiently concentrated, but mitigate consumer surplus loss otherwise. This can be understood as the combination of two forces. As stated above, forward contracts discipline the exercise of market power. This would be sufficient to mitigate consumer surplus loss if hedge rates were to remain constant. However, firms have a greater
incentive to enter forward contracts if the industry is relatively competitive. It
follows that mergers reduce forward contracting and thereby soften an important
constraint on the exercise of market power. The latter effect dominates if the
market is sufficiently concentrated, such that the presence of forward market
amplifies consumer surplus loss.

The third set of results relates to the profitability of mergers. We prove that
the presence of a forward market makes all mergers privately profitable. To
motivate this result, we point out that mergers are not profitable in Cournot
models with constant marginal costs except in the case of merger to monopoly
(Salant, Switzer, and Reynolds (1983)). With marginal costs increasing due
to capital stocks, more mergers are profitable, but many still are not (Perry
and Porter (1985)). Forward markets create a prisoners’ dilemma for firms:
contracts are individually rational but collectively damaging. A merger reduces
the incentive for the merging firms and their competitors to contract forward,
which in turn causes price to increase by a greater amount than it would with
hedge rates fixed at the pre-merger level. The strength of this effect is sufficient
that even the smallest mergers are profitable.

One limitation of our model is that it does not incorporate that forward
contracts may reduce firms’ exposure to price volatility in the spot market.
However, Allaz (1992) shows that the risk-hedging and strategic motives can
coexist in equilibrium, with each contributing to an expansion of output relative
to the Cournot benchmark. The mechanisms that we identify extend to that
setting cleanly. Further, we anticipate that many of our results also would
extend to models in which forward contracts exist only to hedge risk; the basis
being that if mergers make the exercise of market power more profitable, but
for some limiting constraint, then they also introduce incentives to relax the
constraint. This principle applies well beyond models of forward contracting;
the dynamic price signaling game of Sweeting and Tao (2016) is one recent
example that shares a core intuition with our own research.

This study blends the literatures on horizontal mergers and strategic for-
ward contracting. In the former literature, Perry and Porter (1985) introduce
the concept of capital stocks to model mergers among Cournot competitors as
making the combined firm larger instead of merely reducing the number of firms.
McAfee and Williams (1992) solve for the equilibrium strategies under any ar-
bbitrary allocation of capital stocks. Farrell and Shapiro (1990) allow for fully
general cost functions which incorporate the possibility of merger-specific cost
efficiencies, and also develop the usefulness of examining “first-order” impact of
mergers. Jaffe and Wyle (2013) apply the first-order approach to study merger
effects under a general model of competition that nests conjectural variations,
Cournot, and Bertrand as special cases. The solution techniques that we employ
extend the methodologies developed in these articles. We also supplement our
theoretical results with Monte Carlo analyses, as in Miller, Remer, Ryan, and
Sheu (2017a) and Miller, Remer, Ryan, and Sheu (2017b).

The seminal article on strategic forward contracting is Allaz and Vila (1993).
The main result developed is that as the number of contracting stages increases
in a model of duopoly, total output approaches the perfectly competitive level.
The subsequent literature has gone in a number of directions. Hughes and Kao (1997) and Ferreira (2006) consider the importance of the assumption that contracts are observable to the market. Green (1999) extends the model to markets in which firms submit supply schedules. Ferreira (2003) explores equilibria of the game as number of contracting rounds approaches infinity. Mahenc and Salanie (2004) analyze the impact of forward contracting when firms compete via differentiated products Bertrand in the spot market. Liski and Montero (2006) consider the role of forward contracting in sustaining collusive outcomes. All of these studies suggest that the extent to which our results are applicable in real-world settings will depend on a number of features of the industry in question. Empirical evidence on the importance of forward contracting is presented in Wolak (2000), Bushnell (2007), Bushnell, Mansur, and Saravia (2008), Hortacsu and Puller (2008) and Brown and Eckert (2016).

Among the aforementioned studies, the closest to our research are Bushnell (2007) and Brown and Eckert (2016). Bushnell (2007) examines the welfare impact of a forward market for a symmetric $N$-firm oligopoly with a single round of forward contracting. The model is calibrated to a number of deregulated electricity markets in order to ascertain the impact of forward markets on prices and output. Mergers are not examined. Brown and Eckert (2016) allow firms to have heterogeneous capital stocks as we do, but the focus is primarily empirical and as a result, they do not address the same questions. Ours is the only study we are aware of that solves for the subgame perfect equilibrium of the game with an arbitrary number of contracting rounds and fully differentiated firms.

The paper proceeds as follows. Section 2 describes the model of multistage quantity competition and solves for equilibrium strategies using backward induction. Section 3 analyzes the welfare impact of forward contracting, showing that the welfare impact of a forward market is non-monotonic in concentration. Section 4 formally models the welfare impacts of mergers highlighting how the results differ from the baseline model of Cournot competition. Section 5 models the impact of a forward market on the incentives to collude. Section 6 addresses some policy implications of our results and Section 7 concludes with a discussion of the applicability of our results.

2 Model

2.1 Overview

We consider a modified Cournot model that features $T$ contracting stages. The model is a variant of Allaz and Vila (1993) but we allow for an arbitrary number of producers with heterogeneous production technologies as in McAfee and Williams (1992). Production occurs in the final stage, $t = 0$. In each of $t \in \{1, 2, \ldots, T\}$ periods prior to production, firms can contract at a set price to buy or sell output to be delivered at time $t = 0$. Denote each of these contracting stages as $T, \ldots, t, \ldots, 1$ such that stage $t$ occurs $t$ periods before production. At $t = 0$, production takes place, contracts are settled, and producers compete via
Cournot to sell any residual output in the spot market. The solution concept is Subgame Perfect Nash Equilibrium (“SPE”). Formally, let \( f_t^i \) denote the quantity contracted by producer \( i \in \{1, \ldots, N\} \) in stage \( t \), and let \( q_t^i = \sum_{\tau=t+1}^{T} f_{\tau}^i \) denote the producer’s forward position at the beginning of period \( t \). Forward contracts in stage \( t \) are agreed upon taking as given the forward price, \( P_t \), and the vector of forward positions, \( q_t = \{q_t^1, \ldots, q_t^N\} \), and with knowledge of the corresponding subgame equilibrium that follows. At \( t = 0 \), each producer sells \( q_0^i \) in the spot market taking into account the vector of forward positions \( q_0 = \{q_0^1, \ldots, q_0^N\} \) and given other producers’ output. This determines the producer’s output, \( q_i \), as the sum of its contracted and spot sales. Producers are “short” in the spot market if \( q_0^i > 0 \). Total output is the sum of all output and is defined \( Q = \sum_i q_i \). Buyers are passive entities and are represented by the linear inverse demand schedule \( P(Q) = a - bQ \), for \( a, b > 0 \).

Each producer \( i \) is characterized by its capital stock, \( k_i \), a proxy for its productive capacity. Total costs are \( C_i(q_i) = cq_i + dq_i^2 / 2k_i \), so that marginal costs, \( C_i'(q_i) = c + dq_i / k_i \), are increasing in output but decreasing in the capital stock. As a result, firms with greater capital stocks will have higher market shares owing to this cost advantage. We assume \( a > c \geq 0 \) to ensure that gains to trade exist. The parameter \( d \) is binary \( (d \in \{0, 1\}) \) and allows the model to nest constant marginal costs as a special case.

### 2.2 Spot market subgame

Solutions are obtained via backward induction: first considering the output decisions of producers in the spot market, given any vector of contracted quantities, and then considering the contract market. The spot price is determined by total output, \( Q(q_0) \), which is itself a function of the vector of forward positions, \( q_0 \). Producer \( i \) chooses its total output, \( q_i \) (the sum of forward and spot market quantities), taking as given \( q_0 \) as well as the vector of other producers’ output, \( q_{-i} \), to maximize the profit function,

\[
\pi^s_i(q_i; q_0^i, q_{-i}) = P(Q(q_0, q_{-i}))(q_i(q_0^i, q_{-i}) - q_0^i) - C_i(q_i(q_0^i, q_{-i})).
\]

Suppressing dependence on \( q_0 \) and \( q_{-i} \), the first-order condition implies that

\[
P(Q) + (q_i - q_0^i)P'(Q) = C_i'(q_i),
\]

If the producer holds a short position (i.e. \( q_0^i > 0 \)), then the inclusion of \( q_0^i \) in equation (1) says that, relative to Cournot, revenue is less sensitive to output since selling an additional unit has no effect on the price received from forward sales. This amounts to an outward shift in the firm’s marginal revenue function, holding fixed the output of other producers.\(^1\) If competing producers increase

\(^1\)Anderson and Sundaresan (1984) use this very argument to show that given a short forward position, a monopolist will necessarily increase output relative to Cournot. They rely on risk aversion to explain why a monopolist would hold a short position in the first place.
their output relative to Cournot due to their own forward positions, this will cause $i$’s marginal revenue function to shift back somewhat.

We derive closed-form expressions for equilibrium price and quantities by making use of the following terms:

$$\beta_i = \frac{bk_i}{bk_i + d}, \quad B = \sum_i \beta_i, \quad B_{-i} = \sum_{j \neq i} \beta_j, \quad F^0 = \sum_i \beta_i q^0_i, \quad F^0_{-i} = \sum_{j \neq i} \beta_j q^0_j$$

**Proposition 1** In the spot market subgame with vector of forward positions, $q^0$, there exists a unique Nash equilibrium in which price, total output and individual firms’ output are given by:

$$P(q^0) = c + \frac{a - c}{1 + B} - \frac{bF^0}{1 + B}$$

$$Q(q^0) = \left(\frac{a - c}{b}\right) \frac{B}{1 + B} + \frac{F^0}{1 + B}$$

$$q_i(q^0) = \left(\frac{a - c}{b}\right) \frac{\beta_i}{1 + B} + \frac{\beta_i}{1 + B} \left[(1 + B_{-i}) q^0_i - F^0_{-i}\right]$$

All proofs are in the Appendix. The above values have been expressed so as to illustrate the differences between the multi-stage model of competition considered here and a baseline model of Cournot competition without forward contracts in which $q^0_i = F^0_{-i} = F^0 = 0$. In Cournot, total output is increasing while price is decreasing in $B$. A larger value of $B$ corresponds to conditions typically associated with a more competitive industry: a larger number of firms, holding fixed capital stock per firm; greater capacity (i.e. capital stock) per firm, holding fixed the number of firms; and a more symmetric distribution of capacity among firms.

If $F^0$, a weighted average of producers’ forward positions, is positive (i.e. producers are short on net) then price is lower and total quantity is higher than under Cournot. This foreshadows the results obtained below. A given producer’s quantity may be higher or lower than the Cournot baseline, depending on how its forward position compares to that of other producers. One could imagine a producer would want to contract a large share of its productive capacity to grab a larger share of the overall market. However, since other producers are employing the same strategy, each must adjust its output to the contracted quantities of its rivals. We will be able to say more about which of these forces dominates after deriving the equilibrium in the contract market.

**2.3 Contract market**

Our formulation of the contract market follows Allaz and Vila (1993). In each contracting period, $\tau$, producers face a price, $P^\tau$, and decide how much to supply or demand. Speculators are passive entities and as such are not modeled.
Assume there are at least two such speculators whose roll consists of taking the opposite side of any long or short position if an arbitrage opportunity presents itself. With no uncertainty about the future, the spot price is known as are all prices and quantities in subsequent contracting rounds, conditional on equilibrium (pure) strategies. Perfect foresight along with competition among speculators rules out any price other than the resulting spot price. Therefore, we require \( P_\tau = \cdots = P_1 = P(Q(q^\tau)) \), where \( Q(q^\tau) \) is total output conditional on period-\( \tau \) forward positions, \( q^\tau \), taking as given the amount producers demand or supply at prices \( P^t, t \leq \tau \), and given equilibrium behavior in what follows. We refer to this as the “no arbitrage” condition.\(^2\) Finally, we assume no discounting of profits.\(^3\)

Consider then producer \( i \)'s decision of how much to supply (or demand) in the contract market. Taking as given \( q^\tau \) as well as the strategies of others, \( q_{-i} \), producer \( i \) chooses \( f^\tau_i \) to maximize its profit function,\(^4\)

\[
\pi_i(f^\tau_i; q^\tau, q_{-i}) = P\tau f^\tau_i + \sum_{t=1}^{\tau-1} P^t f^t_i + P(Q)(q_i - q^0_i) - C_i(q_i)
\]

The first line on the right-hand side says that the producer takes into account that transactions in the current period affect prices and quantities in subsequent contracting periods as well as in the spot market. The second line on the right-hand side follows from the no-arbitrage condition. This shows that when the producer believes that all subsequent forward prices will adjust to the rationally anticipated spot price, it need only be concerned with how its decision today affects the spot price.

The first-order condition implies that,

\[
P(Q) + (q_i - q^\tau_i) (1 + R^\tau_i) P'(Q) = C'_i(q_i), \tag{2}
\]

where \( R^\tau_i \equiv \sum_{j \neq i} \frac{\partial q_j}{\partial f^\tau_i} / \frac{\partial q_i}{\partial f^\tau_i} \) is the quantity response from all of producer \( i \)'s competitors to a marginal increase in \( i \)'s output, which in this instance is due to a marginal increase in \( f^\tau_i \). This term may be thought of as the conjectural variation in stage \( \tau \) about the combined output response to a unit change in producer \( i \)'s output, albeit one that is derived endogenously from equilibrium play. In a Cournot game with “Nash conjectures” (McAfee and Williams (1992)), this term is zero. But when competition spills across multiple periods as in the current setting, each producer recognizes that a marginal increase in its own short position, will reduce the amount competing firms produce. This creates an incentive for each firm to expand output beyond its Cournot level.

We derive \( R^\tau_i \) recursively, relying on equilibrium behavior.

\(^2\)The issue of commitment arises in that given a fixed number of contracting periods, a firm would always wish to to increase its contracting opportunities so as to disadvantage its rivals. Our results require that contracting frictions limit firms to a finite number of contracting periods.

\(^3\)Including a discount rate changes nothing. Cite source.

\(^4\)We suppress dependence on \( q^\tau \) and \( q_{-i} \) for readability.
Lemma 1 The conjectural variation in stage 1 with respect to producer $i$’s output as derived from Nash equilibrium behavior in the subgame beginning in stage 0 is,

$$R_i^1 = -\frac{B_{-i}}{1 + B_{-i}}.$$  

For any $\tau \geq 1$, define $\mu_i^\tau = \frac{\beta_i}{1 + R_i^\tau}$ and $M_{-i}^\tau = \sum_{j \neq i} \mu_j^\tau$. The conjectural variation in stage $\tau + 1$ with respect to producer $i$’s output as derived from SPE behavior in the subgame beginning in stage $\tau$ is,

$$R_i^{\tau+1} = -\frac{M_{-i}^\tau}{1 + M_{-i}^\tau}.$$  

We can use Lemma 1 to show how the firm’s problem is impacted by the presence of a forward market. It is evident that the marginal revenue curve facing firm $i$ in the contract market as expressed in equation (2) is flatter in own output than it would be under Cournot. Since $1 + R_i^\tau < 1$, a marginal increase in firm $i$’s contracted quantity does not reduce the price by as much as it would under Cournot because other firms respond by reducing their own output. Holding all other firms’ output fixed at their Cournot levels and assuming no forward position in period $\tau$ (i.e., $q_i^\tau = 0$), the inclusion of $1 + R_i^\tau$ in equation (2) pivots firm $i$’s marginal revenue curve up from the vertical axis, which suggests firm $i$ will increase output relative to Cournot. As we saw in the spot market subgame, incorporating a short position shifts the firm’s marginal revenue curve outward, thereby reinforcing this effect. However, if the same incentives facing firm $i$ lead other firms to increase their output relative to Cournot, firm $i$’s marginal revenue curve shifts down because quantities are strategic substitutes. This shift curbs firm $i$’s incentive to increase output relative to Cournot and may even decrease it if other firms increase their output by a large enough amount.

We can now derive the equilibrium of the full game. Let $M^\tau = \sum_i \mu_i^\tau$ for any $\tau \geq 1$ and for completeness of notation, let $R_i^0 = 0$ for all $i$ when $t = 0$.  

Proposition 2 There exists a unique SPE of the period $T$ stage game in which in each period, a producer anticipates producing $q_i$ and sells a strictly positive fraction of its uncommitted anticipated output so that $\sum_{t=\tau+1}^T f_i^t + q_\tau^i = q_i$. The equilibrium is characterized by a sequence of forward sales, $\{f_i^t\}_{i,t}$, a vector of outputs, $\{q_i\}_i$, total output, and price satisfying:

$$f_i^\tau = \frac{R_i^{\tau-1} - R_i^\tau}{1 + R_i^{\tau-1}} \left( q_i - \sum_{t=\tau+1}^{T} f_i^t \right)$$

$$q_i = \frac{a - c}{b} \frac{\mu_i^\tau}{1 + M^T}$$

$$Q = \frac{(a - c)}{b} \frac{M^T}{1 + M^T}$$

$$P = c + \frac{a - c}{1 + M^T}$$
Absent a contract market (i.e., \( R^T_i = 0 \) \( \forall i, t \)), \( \mu^T_i \) and \( M^T \) reduce to \( \beta_i \) and \( B \), respectively, so that the price and quantities in Proposition 2 collapse to their values in the Cournot game of McAfee and Williams (1992). We can assess the impact of a forward market more broadly by analyzing changes in equilibrium outcomes as \( T \) increases from zero as in Cournot to positive values. We have that,

**Corollary 1** For any \( T \in \{1, 2, \ldots \} \), price is (weakly) lower and total output is (weakly) higher in the SPE of the \((T+1)\)-stage game than in the \( T \)-stage game. Each inequality is strict outside of the monopoly case. An individual producer’s output can nevertheless be lower in the \((T+1)\)-stage game relative to the \( T \)-stage game if its capital stock is sufficiently small relative to that of its competitors.

Allaz and Vila (1993) provide a version of this result in the case of a symmetric two-firm oligopoly. When firms are symmetric, our model shows that all firms increase their output as \( T \) increases, as they do in Allaz and Vila (1993). Corollary 1 shows that this may no longer be the case when firms are asymmetric. This result suggests that the introduction of a forward market may increase concentration as measured by output, even as it improves welfare.

The impact of the forward market on output can be substantial. Consider the special case of constant marginal cost (\( d = 0 \)) and a single contracting stage (\( T = 1 \)). In this case, \( \beta_i = 1 \) so that \( h_i = \frac{N-1}{N} \), \( \mu^T_i = N \), and \( M^T = N^2 \). We calculate the increase in industry output due to the presence of a forward market by comparing the output in Proposition 2 to what the value would be if \( M^T = N^2 \) were replaced with \( B = N \). The presence of a forward market increases output by 140 percent when \( N = 2 \) and by nearly 600 percent when \( N = 6 \). These increases would be somewhat smaller if marginal costs were instead increasing but would be larger for \( T > 1 \).

3 Market Structure and Welfare

We now examine the role of market structure in evaluating the impact of a forward market on welfare. Whereas Allaz and Vila (1993) showed that welfare can span duopoly-Cournot to perfect competition levels as the number of contracting rounds increases, our focus is on how the welfare impact of a forward market is influenced by market structure. As such, we treat \( T \) as fixed, determined by the particulars of the industry.\(^5\)

3.1 Market structure and hedge rates

The welfare impact of a forward market is related to the fraction of each firm’s output that is contracted in the forward market, i.e. its “hedge rate.” The following result aids the understanding of this relationship.

\(^5\)Bushnell (2007) discusses the institutional details of forward sales within wholesale electricity.
Lemma 2 Given equilibrium strategies within the SPE of the \((T + 1)\)-stage game, the hedge rate can be expressed as 
\[ h_i = \frac{\partial q_i}{\partial q_i} = \frac{\partial |R_i^T|}{1 + \beta_i^T} \].

The result is fairly general in that it does not rely on the shape of the demand or cost functions. It follows from the fact that a firm, when deciding how much to supply on the contract market, takes into account that a marginal increase in supply will be met by a decrease in its competitors’ sales in subsequent periods. Thus, while a marginal increase in contracted supply on its own causes the price to decline, the corresponding decrease in competitors’ outputs partially offsets this. The optimum equates marginal revenue across each of \(T + 1\) stages much in the way that a third-degree price discriminating monopolist equates marginal revenue across customer segments.

A firm’s hedge rate depends at a first order on the amount of capital stock controlled by its competitors as well as the distribution of capital stock among them.\(^6\) Competitors with larger capital stocks produce more irrespective of hedging, so their response to firm \(i\)’s contracted quantity will be larger. At the same time, because larger firms make less efficient use of their capital stocks, firm \(i\)’s hedge rate is larger when the capital stocks of its competitors are more symmetrically distributed.\(^7\) The upshot is that the structural conditions which lead a firm to sell a larger fraction of its output in the contract market are the same conditions that lead to greater output in the baseline Cournot model.

As a further illustration, consider the perfectly symmetric case (i.e., \(\beta_i = \beta\) for all \(i\)). The (common) hedge rate when \(T = 1\) is,
\[ h^{(1)} = \frac{(N - 1) \beta}{1 + (N - 1) \beta}. \] (3)
That \(h^{(1)}\) is larger for larger values of \(N\) suggests that from a welfare perspective, a forward market is not a perfect substitute for a competitive industry structure because forward contracting is more prevalent when the industry is more competitive. This interpretation continues to hold for larger values of \(T\). To see this, we have from Lemmas 1 and 2 that the hedge rate when \(T = 2\) is,
\[ h^{(2)} = H(h^{(1)}) = \frac{(N - 1) \beta}{1 - \beta h^{(1)} + (N - 1) \beta}. \] (4)
Since \(H\) is monotonically increasing in \(h^{(1)}\) and larger for larger values of \(N\), it follows that iteration \(T - 1\), \(h^{(T)} = H^{T-1}(h^{(1)})\) (where the supercript \(T - 1\) reflects the number of iterations), is also larger for larger values of \(N\). Note that in the case of monopoly (\(N = 1\)), the hedge rate is zero for any \(T\) as forward contracting has no strategic impact.

\(^6\)In the game with \(T = 1\) contracting stages, a firm’s hedge rate, \(h_i = B_{-i}/(1 + B_{-i})\), depends only on the capital stocks of its competitors. But when \(T > 1\), the hedge rate depends on \(\mu^T_{-i}\), each of which depends on firm \(i\)’s capital stock through its influence on every other firm’s hedge rate. The effect of \(\beta_i\) on \(h_i\) is of a second-order magnitude, however.

\(^7\)Each \(\beta_i\) is concave in capital due to increasing marginal costs. Thus, firms with larger capital stocks produce less per unit of capital than smaller firms. Note that if marginal costs are constant (\(d = 0\)) then \(\beta_i = 1\) \(\forall i\).
3.2 Hedge rates and welfare

In Proposition 1, we saw that total output is increasing in $F^0$, a weighted-average of contracted output. Lemma 2 showed that a firm’s contracted output is increasing in its hedge rate, which itself is a function of market structure. In particular, when the market structure is more competitive—e.g., there are more firms or capital is distributed more symmetrically among a given number of firms—hedge rates are higher. This suggests that a forward market creates an additional channel through which market structure affects welfare.

To formalize this point, we first consider the industry-average Lerner Index, which summarizes the degree to which market output diverges from perfect competition and hence is useful as a proxy for consumer and total surplus (Shapiro (1989)). Let $s_i = q_i/Q$ denote firm $i$’s market share and let $\epsilon = -\left(\partial Q/\partial P\right)(P/Q)$ denote the price elasticity of demand.

**Lemma 3** Given a vector of hedge rates $h$, the Lerner Index derived from firms optimizing subject to $h$ equals

$$LI(h) \equiv \sum_i \left(\frac{P - C_i'}{P}\right) s_i = \sum_i \frac{s_i^2}{\epsilon} (1 - h_i)$$

Lemma 3 shows that each firm’s price-cost margin is a product of two terms, the typical Cournot term, $s_i^2/\epsilon$, and a term reflecting the importance of forward contracting, $(1 - h_i)$. The LI can be evaluated at the SPE hedge rates, but it also holds for an arbitrary vector of hedge rates, keeping in mind that $s_i$ and $\epsilon$ are themselves functions of the hedge rates. As hedge rates increase uniformly from zero to unity, price-cost margins and hence consumer and total surplus, span the Cournot outcome at one extreme and perfect competition at the other. Again holding $T$ fixed, the structural conditions that give rise to larger hedge rates are the same conditions that give rise to competitive outcomes in the absence of forward contracting.\(^8\)

3.3 Concentration and welfare

The results already established are sufficient to determine that forward markets have the greatest impact on outcomes in markets characterized by some intermediate level of competition/concentration. The $(T + 1)$-stage model is equivalent to the baseline model of Cournot competition in the monopoly case (Corollary 1), and both models converge to perfect competition as market shares approach zero (Lemma 3). Thus, if forward markets lower price and increase output (Corollary 1) then the magnitude of these effects must be maximized in markets with firms that have market shares bounded strictly by zero and unity.

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\(^8\)When $T$, the number of contracting rounds becomes large, hedge rates approach unity and outcomes become competitive even under industry structures that look very un-competitive. Allaz and Vila (1993) showed that, in a symmetric duopoly, as $T \to \infty$, output approaches the perfectly competitive level.
We present the result using both consumer surplus (CS) and total surplus (TS) as measures of welfare. These can be expressed as functions of demand elasticity ($\epsilon$) and the industry-average Lerner Index ($LI$):

\[
CS = \frac{a^2}{2b} \left( \frac{1}{1+\epsilon} \right)^2
\]
\[
TS = \frac{a(a-c)}{2b} \frac{1}{1+\epsilon} + \epsilon (CS) (LI),
\]

(5)

Let $\rho_{CS}$ denote the ratio of consumer surplus in the SPE of $(T + 1)$-stage model to consumer surplus in Cournot, holding constant all model parameters. Let $\rho_{TS}$ denote the analogous ratio with respect to total surplus.

Next, we define what it means for concentration to decrease from monopoly at one extreme to the limiting case of perfect competition. Assume that there is an infinite number of potential producers so that $N_k$ is the set of producers whose capital stock, $k_i$, is strictly positive under capital allocation $k$. We will then consider transfers of capital among a set of producers, $\tilde{N}$, that reduces the absolute difference in capital between every producer in $\tilde{N}$. Suppose that a transfer changes the capital allocation from $k$ to $k'$ where $k_i$ and $k'_i$ are elements of $k$ and $k'$, respectively. Following Waehrer and Perry (2003), an equilizing transfer is such that: (i) $|k_i - k_j| > |k'_i - k'_j|$ for every $i, j \in \tilde{N}$; (ii) $\sum_{i \in \tilde{N}} k_i = \sum_{i \in \tilde{N}} k'_i$; and (iii) $k_l = k'_l$ for all $l \notin \tilde{N}$. With respect to mergers, the pre-merger allocation of capital can be recovered from the post-merger allocation via an equilizing transfer.

We model perfect competition as the limiting case, as the number of firms with strictly positive capital stocks goes to infinity, of allocations in which all firms with strictly positive capital stocks are symmetric. A symmetric equalizing transfer is an equalizing transfer moving from $k$ to $k'$ in which: i) $k'_i = k'_j$ for every $i, j \in \tilde{N}$; and ii) $k'_i < k_i$ for every $i, j \in \tilde{N}$. Market structure approaches perfect competition from any arbitrary initial allocation of capital through a sequence of symmetric equalizing transfers.

**Proposition 3** If $k$ is the monopoly allocation, then any equilizing transfer from $k$ increases $\rho_{CS}$ and $\rho_{TS}$. For any allocation $k$ other than the monopoly allocation, there exists an allocation $\hat{k}(k)$, such that any symmetric equalizing transfer to $\hat{k}(k)$ causes $\rho_{CS}$ and $\rho_{TS}$ to decline.

Proposition 3 is one of our core results. The idea is that when markets are sufficiently concentrated, a small decrease in concentration increases welfare more in the presence of a forward market. A large enough decrease in concentration from an intermediate allocation can increase welfare relatively more in the absence of a forward market. In the remainder of this section, we use numerical techniques to illustrate how forward markets have the greatest impact on welfare with intermediate levels of competition/concentration.

We first compare the welfare statistics obtained with $T = 1$ rounds of forward contracting to those obtained in Cournot equilibrium ($T = 0$). To do so, we
create data on 9,500 “industries,” evenly split between \( N = 1, 2, \ldots, 20 \). For each industry, we calibrate the structural parameters of the model \((a, b, c, k)\) such that Cournot equilibrium exactly generates randomly-drawn market shares, an average margin, and normalizations on price and total output.\(^9\) We then obtain the welfare statistics that arise in Cournot equilibrium and with a single round of forward contracting.

Figure 1 summarizes the results. In each panel, the vertical axis provides the ratio of surplus with forward contracting to surplus with Cournot. The horizontal axes shows the Herfindahl-Hirschman index (“HHI”). The HHI is the sum of squared market shares, attaining a maximum of unity in the monopoly extreme and asymptotically approaching zero as the market approaches perfect competition. The HHI is an appealing statistic due to the theoretical link between HHI and welfare in the baseline Cournot model.\(^10\) Each dot represents a single industry, and the lines provide nonparametric fits of the data.

As shown, consumer surplus and total surplus are greater with forward contracting than with Cournot (because all dots exceed unity). Further, consistent with the corollary, the impact of a forward market is greatest at intermediate levels of competition.\(^11\) The gain in consumer surplus is maximized at an HHI

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\(^9\)We set \( P = Q = 100 \) and use a margin of 0.40.

\(^10\)Notice that when all \( h_i = 0, LI = HHI/\epsilon. \)

\(^11\)As there is not a one-to-one correspondence between HHI and consumer or total surplus,
around 0.30, which corresponds roughly to a symmetric three firm oligopoly. The gain in total surplus is maximized at an HHI around 0.40, between the symmetric triopoly and duopoly levels. The figure also shows that forward markets diminish producer surplus, particularly in unconcentrated markets.

It is also possible to compare the welfare statistics that arise with forward contracts to those obtained with perfect competition. This is especially tractable in the special case of symmetric firms and constant marginal costs \( (d = 0) \). The expressions in (5) can be presented as functions of the common hedge rate:

\[
CS(h^{(T)}) = \frac{(a - c)^2}{2} \left( \frac{N}{N + 1 - h^{(T)}} \right)^2
\]

\[
TS(h^{(T)}) = \frac{(a - c)^2}{2} \left( \frac{N}{N + 1 - h^{(T)}} - \frac{1}{2} \left( \frac{N}{N + 1 - h^{(T)}} \right)^2 \right)
\]

The analogous expressions with perfect competition are \( CS(1) = TS(1) = \frac{1}{2} (a - c)^2 \). Thus, the levels of consumer surplus and of total surplus with forward contracts, relative to perfect competition, are free of the demand and cost parameters and depend only on the number of firms and the hedge rate. This holds for any given hedge rate, including the SPE rates \( h^{(T)} \).

Figure 2 plots the ratios \( CS(h^{(T)})/CS(1) \) and \( W(h^{(T)})/W(1) \) for \( T = 0, \ldots, 3 \). Again, \( T = 0 \) corresponds to Cournot competition and \( h^{(0)} = 0 \). The horizontal axis in each panel is the number of firms \( (N = 1, \ldots, 10) \) which, under symmetry, is a sufficient statistic for concentration. As shown, consumer surplus and total surplus increase with \( N \) under Cournot equilibrium; in the limit as \( N \to \infty \) these welfare statistics approach the perfectly competitive level. Incorporating each round of contracting adds curvature to the relationship between surplus and the number of firms, such that surplus approaches the perfectly competitive level faster as \( N \) grows large. The “gap” between surplus with Cournot and surplus with forward contracts is largest for intermediate \( N \), again consistent with Proposition 3. Lastly, the figure is highly suggestive that forward markets amplify the impacts of market structure changes (e.g., mergers) on welfare in concentrated markets, but diminish impacts otherwise. We provide a more sophisticated analytical treatment of capital transfers in the next section.

### 4 Mergers

In this section, we analyze the welfare impacts of consolidation, which we treat as the transfer of capital stock from small to large firms. Mergers are inherently consolidating regardless of whether the larger or smaller firm is the acquirer since the merged firm’s capital stock will be larger than either of the merging parties’, leaving the capital stocks of other firms unchanged. Our interest extends beyond we view these results as illustrative. The advantage to using HHI to measure concentration is that it offers a complete ordering of any two capital allocations and hence allows us to plot the results. In the following section, we analyze a more theoretically-robust measure of concentration that does not offer a complete concentration-ordering of allocations.
mergers to partial acquisitions as many real-world applications involve the sale of individual plants. Even when evaluating full mergers, antitrust authorities must often consider whether and to what extent a partial divestiture might offset the anticompetitive harm from the merger.

Our results derive from an analytic “first-order” approach which we supplement in places with simulations. The analytic approach examines effects of small consolidating transfers, restricting attention to pairwise transfers of capital from any firm 2, say, to any firm 1, where $k_1 \geq k_2$.\footnote{Jaffe and Wyle (2013) and Farrell and Shapiro (1990) employ this approach, though they do not analyze how the merger changes firms’ conjectural variations as we do.} Holding fixed the total capital stock controlled by the two firms, a consolidation of capital among firms 1 and 2 is a transfer of some amount, $\Delta k$, such after the transfer, firm 1 has capital stock $k_1 + \Delta k$ while firm 2 has capital stock $k_2 - \Delta k$, leaving the total unchanged. Our analytical approach illuminates the mechanisms underlying our results while avoiding the integer problem inherent in the analysis of full mergers. Extrapolating to larger transfers such as full mergers involves integrating over these first-order effects. When first-order effects are insufficient to evaluate larger transfers or otherwise are aided by additional illustration, we provide simulations of full mergers. All of our results assume $T = 1$ round of forward contracting so we drop the superscripts when referring to terms like the conjectures that had been defined by stage. We close the section with a
discussion of how the results would change with additional contracting rounds.

4.1 Effects on consumer surplus

We begin by analyzing the effect of consolidation on consumer surplus. To the extent that antitrust agencies review mergers under a consumer surplus standard, our results should be directly applicable to antitrust policy. We can express consumer surplus within the SPE of the two-stage game as,

\[ CS = \int_0^Q (a - bt - P) \, dt = \frac{b}{2} Q^2. \]

It follows that any transfer of capital that reduces the equilibrium quantity reduces consumer surplus.

**Proposition 4** All consolidating transfers reduce consumer surplus within the SPE of the two-stage game. The loss of consumer surplus due to a consolidating transfer is mitigated if each firm’s hedge rate remains fixed at its pre-transfer value.

That consolidation leads to lower output should not be surprising as the result holds within the baseline model of Cournot competition. What it interesting is that the reduction in output is magnified when firms adjust their hedge rates in response to consolidation as they do in the SPE of the two-stage game. We can deconstruct the output effect into two components, a structural effect (SE), which measures the change in output holding each firm’s hedge rate fixed, and a hedging effect (HE), which measures the incremental change in output due to changes in how the new structure changes firms’ conjectural variations.

To see this, we have that the change in consumer surplus due to a consolidating transfer of capital is,

\[ \Delta CS = b \Delta Q = \frac{a - c}{(1 + M)^2} \sum_i \Delta \mu_i \]

where

\[ \Delta \mu_i = \begin{cases} \left( \frac{\mu_i}{\beta_i} \right)^2 \Delta \beta_i - \mu_i^2 \Delta R_i & \text{if } i = 1, 2 \\ -\mu_i^2 \Delta R_i & \text{if } i \neq 1, 2 \end{cases} \]

Collecting the \( \Delta \beta_i \) terms and the \( \Delta R_i \) terms, respectively, we can express the change in consumer surplus as, \( \Delta CS = SE^{CS} + HE^{CS} \), where,

\[ SE^{CS} = \frac{a - c}{(1 + M)^2} \left[ \left( \frac{\mu_1}{\beta_1} \right)^2 \Delta \beta_1 + \left( \frac{\mu_2}{\beta_2} \right)^2 \Delta \beta_2 \right] \]

\[ HE^{CS} = -\frac{a - c}{(1 + M)^2} \sum_i \mu_i^2 \Delta R_i \]

The structural effect is that a transfer of capital to firm 1 from the smaller firm 2 leads firm 1 to increase and firm 2 to decrease output. The net result is
a reduction in the combined output of the parties to the transaction. This is because of increasing marginal costs. As firm 1 increases its output in response to the added capital, its marginal cost rises which curbs firm 1’s incentive to expand output. Firm 2’s marginal cost declines as it lowers its output, which promotes greater output reduction. Third parties react to this net decline in the parties’ output by expanding output. The combined increase across all third parties is not enough to offset the decline in the parties’ output.

The change in structure due to a consolidating transfer changes what firms conjecture as to how their competitors will respond to a change in their contracted quantity. Third parties anticipate that the parties to the transaction will be less responsive to their contracted quantities on the basis that the parties are expected to produce less overall. This diminishes the strategic rationale for third parties to contract forward. Firm 1 anticipates that firm 2 will be less responsive to its contracted quantity, so its incentive to contract forward is reduced. Firm 2 anticipates that firm 1 will be more responsive to its contracted quantity, but the increase in firm 2’s contracted quantity does not offset the decrease in firm 1’s contracted quantity. It is clear that the net effect is a reduction in contracted quantity and a reduction in total output.

Next, we ask how the effect of a consolidating transfer on consumer surplus depends on concentration post- (or pre-) transaction. To measure concentration, we employ the transfer principle. According to Waehrer and Perry (2003), a capital allocation $k'$ is more concentrated than allocation $k$ by the transfer principle if and only if $k$ can be constructed from $k'$ by applying a finite series of equalizing transfers. It is evident that the consolidating transfers considered in Proposition 4 increase concentration under the transfer principle. The transfer principle is attractive here as it offers a clear link between concentration and welfare. Though the HHI is most commonly used to measure concentration in antitrust analysis, there are many ways to increase the HHI which have analytically ambiguous effects on the welfare impact of consolidation. As Waehrer and Perry (2003) show, concentration via the transfer principle is sufficient but not necessary for concentration via HHI.

**Proposition 5** Increasing concentration via the transfer principle increases $|\Delta CS|$.

Again, this result also holds within the baseline model, however, what the proof of this result reveals is that the hedging effect becomes more pronounced when concentration is greater. In this way, the structural and hedging effects are mutually reinforcing. It is natural to ask whether the effect of consolidation on consumer surplus is more pronounced within the two-stage game relative to the baseline Cournot game.

**Proposition 6** There exists a capital allocation $k$ such that $|\Delta CS|$ is larger within the SPE of the two-stage model than in Cournot. There exists a capital

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13 Pertaining to first-order effects, there is no distinction between pre- and post-transaction concentration.
14 Equalizing transfers were defined in Section 3.3.
allocation $k'$ that is less concentrated than $k$ under the transfer principle such that $|\Delta CS|$ is larger in Cournot than in the SPE of the two-stage model.

Proposition 6 says that the welfare effects of consolidating transfers within the two-stage model are greater than Cournot in markets that are sufficiently concentrated and small than Cournot in markets that are unconcentrated. The reason why the two-stage model doesn’t always lead to a greater reduction in consumer surplus is that consumer surplus depends on the pre-transaction hedge rate. Within the two-stage model, the effect of a change in structure on each firm’s output is proportional to its pre-transaction output. Recall that each firm’s output is larger within the two-stage model than the baseline, holding fixed the allocation of capital. Therefore, the structural effect is muted relative to Cournot, substantially so when the industry is fairly unconcentrated. Recall from Section 3.1 that hedge rates decline in concentration. It follows that as the capital stock becomes more concentrated, hedge rates decline and each firm’s output in the two-stage game converges to its output in the Cournot game. Proposition 6 establishes what amounts to a threshold level of concentration where the hedging effect exactly offsets the greater structural effect within Cournot.

Since Proposition 6 is a statement about first-order effects, it is ambiguous whether this result extends to large transactions including full mergers. It may for example be the case that allocation $k$ is sufficiently unconcentrated that an incremental transfer would reduce consumer surplus more under Cournot but a larger transfer would reduce consumer surplus more in the contracting model. We use simulations to get around this issue. Details of the simulations can be found in the Appendix.

Figure 3 examines how forward contracts affect consumer surplus losses due to mergers among the set of randomly drawn industries. Again, each industry represents a single parameterization obtained from randomly-drawn market shares and average margins. Within each industry we examine every pairwise merger. We calculate the ratio of consumer surplus loss with forward contracts to the corresponding loss without forward contracts (i.e., setting $h = 0$). This appears as the vertical axis on the graph. Values above unity represent mergers for which forward contracts amplify consumer surplus loss. The horizontal axis is the post-merger HHI. As shown, the relative consumer surplus loss with forward contracts increases with the post-merger HHI. Forward contracts tend to amplify loss for HHIs above 0.50, which is equivalent to symmetric duopoly.

4.2 Profitability

We now consider how the presence of a forward market affects the incentive to consolidate. Figure 4 plots the merging parties’ profits over the post-merger HHI for the two-stage model (Panel A) and Cournot (Panel B). It is evident that all mergers within the two-stage model are profitable whereas in the baseline model, many are not. In both models, merger profitability is increasing in HHI. This reflects two potential forces, the level of concentration pre-merger and
the size of the merger. Mergers of all sizes are more profitable in more highly concentrated industries, while large mergers may serve to significantly increase concentration within an industry.

This result helps to offer a more complete response to the “merger paradox.” Salant, Switzer, and Reynolds (1983) examined the incentive to merge within a symmetric model where firms compete on quantity which they produce at a constant marginal cost. They find that mergers are not profitable unless when combining at least 80 percent of firms. It is difficult to explain the prevalence of mergers in light of this result, hence the paradox. Deneckere and Davidson (1985) alter the assumption that firms compete on quantity and show that mergers are always profitable when firms offering differentiated products compete on price. But the assumption that products are differentiated may not be applicable in many settings such as the sale of commodities or wholesale electricity. Perry and Porter (1985) argue that the failure to explain the

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15Because prices are strategic complements, an increase in the merging firms’ prices is met by an increase in the prices of third-party goods, hence mergers are profitable. When firms instead compete on quantities, a decrease in the merging parties' quantities is met with an increase in third parties' quantities, so that mergers are only profitable if the third-party response is sufficiently muted.

16With undifferentiated products, Bertrand competition forces price to marginal cost, so that only mergers to monopoly are profitable.
profitability of mergers is actually a misconception since the mergers are not well-defined conceptually when firms can produce seemingly infinite quantities at a constant marginal cost. They propose a model of capital stocks, the same model we have adopted, and find that smaller mergers can indeed be profitable even when firms compete on quantities. Yet many mergers within their framework are unprofitable. Our simulations strongly suggest that supplementing Perry and Porter (1985) with a single round of forward contracting is sufficient for all mergers to be profitable.\footnote{One could also argue that there is no paradox at all even within the framework of Salant, Switzer, and Reynolds (1983). Suppose it is costly to simultaneously negotiate mergers among multiple firms, so that each merger must occur sequentially and involve no more than, say, two firms. In such a world, mergers that are unprofitable in the immediate term would be tolerated by forward looking firms if they lead in the long run to an industry structure where the terminal merger is profitable.}

5 Collusion

We turn now to the impact of forward markets on the sustainability of collusion. Liski and Montero (2006) examine the special of symmetric duopoly with constant marginal costs within an infinitely-repeated game in which every production period is preceded by a single round of forward contracting. We extend their analysis to incorporate an arbitrary number of symmetric firms and mul-
Multiple rounds of forward contracting. To do this, we place the model of Section 2 into an infinitely repeated game with the restrictions that $k_i = 1 \forall i$ and $d = 0$. Each production period is preceded by $T$ forward contracting rounds, and is accompanied by a spot market. We examine the conditions under which trigger strategies with Nash reversion constitute a SPE of the repeated game.

**Lemma 4** Any deviation occurs in the spot market.

The proof of Lemma 4 is a work in progress; however, it holds in the simpler setting of Liski and Montero (2006) and should generalize without difficulty. The critical discount rate that determines whether collusion is sustainable is

$$
\delta^{(T)}(N) = \frac{D - \frac{1}{N}\pi^{(0)}(1)}{D - \pi^{(T)}(N)}
$$

where $D$ is the profit realized in the period of deviation and $\pi^{(t)}(N)$ is the per-period Nash profit given $t$ rounds of forward contracting. Thus, for example, $\pi^{(0)}(1)$ is the monopoly profit and $\pi^{(T)}(N)$ is the reversion profit. Note that because the monopoly profit is invariant to the number of contracting rounds, $\pi^{(T)}(1) = \pi^{(0)}(1)$.

**Proposition 7** The critical discount rate, $\delta^{(T)}(N)$, decreases with the number of forward contracting rounds, $T$, for any fixed $N$.

The proof relies on the observation that $T$ affects $\delta^{(T)}(N)$ only through the reversion profit $\pi^{(T)}(N)$. Thus, the basic insight of Liski and Montero (2006) that forward markets can help sustain collusion by making punishment harsher holds in our more general setting.

Figure 5 plots the critical discount rates that arise with $T = 0, 1, 2$ in the special case of constant marginal costs and symmetry; if $T = 0$ then reversion profit is given by Cournot.\(^\text{18}\) As shown, the critical discount rate with $T = 0$ increases with $N$, the standard result that collusion is more difficult to sustain with more competitors. Adding a single round of forward contracting reduces the critical discount rate for each $N$ substantially, consistent with Proposition 7. Adding a second round has a smaller effect. Notice also that forward markets have more pronounced effects on the critical discount rate with small $N$.

### 6 Policy Implications

Our results have direct implications for antitrust policy. Antitrust authorities are tasked with upholding the relevant antitrust statutes and in doing so, challenging mergers and other such transactions that violate them. Though we have not modeled merger-specific efficiencies and other factors antitrust authorities may consider, our results point to situations where consolidating transactions

\(^\text{18}\)Our numerical calculations indicate that the critical discount rates are unaffected by the parameters $a$, $b$, and $c$ in this setting.
may be especially problematic and where considering the role of a forward market may change their determination. In theory, all of the consolidating transactions we have modeled are problematic as all reduce consumer surplus whether or not a forward market is present. In practice, however, one might expect consolidating transactions to be challenged only when the consumer surplus preserve in doing so exceeds the resource cost of litigation. In that sense, magnitudes matter.

An implication of Proposition 6 is that when evaluating a merger in an industry with a robust forward market, antitrust authorities may arrive at the incorrect conclusion if their analysis does not explicitly model the impact of a forward market. In particular, the more concentrated the market, the more likely it is than an analysis incorrectly based on static Cournot will fail to identify a merger that should be challenged. Figure 5 offers the same conclusion with respect to coordinated effects. This is a type-2 error in the parlance of statistical inference. If mergers in industries with robust forward markets have a higher type-2 error rate, then all else equal, these industries should see more consolidating transactions. This is because not only are highly concentrating transactions more likely to go unchallenged in industries with forward markets, but such transactions are more likely to be profitable in industries with forward markets.

There is also the possibility for type-1 errors when the industry is unconcentrated. However, since the loss of consumer surplus under the baseline Cournot model is likely to be quite small in these circumstances, such a merger is unlikely to be challenged.
7 Conclusions

We have analyzed consolidation in the presence of a forward market. Our results show that the welfare effects of consolidation are sensitive to the presence of a forward market in important ways. While our model presupposes the existence of a forward market, it is not hard to conceive of forward sales emerging organically. Whenever quantity is the strategic variable and whenever the terms of sale can be revealed to a firm’s competitors, a firm will have the strategic incentive to make sales in advance of production. To the extent that such transactions do occur, the applicability of our results may well extend beyond the commodities with established futures markets.

While our results should be relevant for policy makers in the merger review process, we believe an appropriate level of caution should be exercised. The model of capital stocks which we have employed throughout is limiting as it does not reflect firms’ actual marginal cost functions. In practice, consolidation may change the shape of firms’ marginal cost functions in ways that exacerbate or mitigate harm from mergers.

We have also assumed the strategic variable to be quantity. In wholesale electricity markets, spot prices are determined based on price-quantity schedules submitted by firms. In the supply-function equilibrium model of Klemperer and Meyer (1989), supply functions can be strategic substitutes or complements. Mahenc and Salanie (2004) study strategic complements in the context of differentiated Bertrand spot market competition and find that forward contracting increases spot market prices. We are aware of no studies that analyze the effect of mergers within this context. If consolidation lessens the incentive to contract in advance, then harm from consolidation is mitigated relative to our results.

Finally, we have assumed that all agents have perfect foresight so that the only motive for firms to sell in the contract market is to influence spot market competition. As we do not believe this to be the case in practice, our assumption of perfect foresight was made for the sake of tractability. Allaz (1992) and Hughes and Kao (1997) show that when foresight is imperfect and firms are risk averse, equilibrium hedge rates are higher than in the perfect-foresight case. How hedge rates change in response to a merger in this setting has not been explored to our knowledge. However, it is conceivable that our basic findings would still obtain. Consolidation, by increasing market power, increases the value to the merged firm of withholding output. To the extent that forward contracting even for the sake of hedging risk comes at the expense of exercising market power, mergers may well limit the incentive for firms to forward contract. We leave this issue and the other issues posed in this section to future research.
References


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Appendices

A Proofs

A.1 Proof of Proposition 1

Fixing the price at a candidate equilibrium value, $P$, and using the definition of $\beta_i$ given in the text, we can express equation (1) as,

$$q_i = \left( \frac{k_i}{bk_i + d} \right) (P - c) + \left( \frac{bk_i}{bk_i + d} \right) q_i^0$$

$$= \frac{\beta_i}{b} (P - c) + \beta q_i^0$$

Using the definitions of $B$ and $F^0$ from the text, we can express total quantity as,

$$Q = \sum_i q_i = \frac{B}{b} (P - c) + F^0$$

Substituting the identity $Q = (a - P)/b$ into the left-hand side of the above expression yields

$$\frac{a - P}{b} = \frac{B}{b} (P - c) + F^0$$

It is straightforward to solve the above for the equilibrium value of $P$, which we then plug into the above expressions for $Q$ and $q_i$ to obtain their equilibrium values.

A.2 Proof of Lemma 1

Consider $t = 1$. From the expression of $q_i$ in Proposition 1, we have that,

$$\frac{\partial q_i}{\partial f_{i1}} = \frac{\beta_i (1 + B_{-i})}{1 + B}$$

(A.1)

From the same expression of $q_i$, we also have that,

$$\frac{\partial q_i}{\partial f_{i1}} = -\frac{\beta_i \beta_j}{1 + B}$$

so that

$$\sum_{j \neq i} \frac{\partial q_j}{\partial f_{i1}} = -\frac{\beta_i B_{-i}}{1 + B}$$

(A.2)

Using (A.1) and (A.2), we have that,

$$R_i^1 = \sum_{j \neq i} \frac{\partial q_j}{\partial f_{i1}} \frac{\partial q_i}{\partial f_{i1}} = -\frac{B_{-i}}{1 + B_{-i}}$$
Now consider any \( t = \tau > 1 \). Fixing price at some candidate equilibrium, \( P \), and using the definition of \( \mu_i^\tau \) from the statement of the lemma, we can express equation (2) as,

\[
q_i = \mu_i^\tau \left( \frac{P - c}{b} \right) + \mu_i^\tau (1 + R_i^\tau) q_i^\tau
\]  

(A.3)

Define the following terms:

\[
F^\tau = \sum_i \mu_i^\tau (1 + R_i^\tau) q_i^\tau, \quad F_{-i}^\tau = \sum_{j \neq i} \mu_j^\tau (1 + R_j^\tau) q_j^\tau
\]

We can then express total output as,

\[
Q = \sum_i q_i = M^\tau \left( \frac{P - c}{b} \right) + F^\tau
\]  

(A.4)

Substituting \( Q = (a - P) / b \) into the above yields,

\[
\frac{a - P}{b} = M^\tau \left( \frac{P - c}{b} \right) + F^\tau
\]  

(A.5)

It is straightforward to solve the above expression for the equilibrium value of \( P \), which we then plug into (A.3) to obtain,

\[
q_i (q^\tau) = \left( \frac{a - c}{b} \right) \frac{\mu_i^\tau}{1 + M^\tau} + \frac{\mu_i^\tau}{1 + M^\tau} \left[ (1 + M_{-i}^\tau) (1 + R_i^\tau) q_i^\tau - F_{-i}^\tau \right]
\]  

(A.6)

Differentiating \( q_i (q^\tau) \) with respect to the firm’s own forward position yields,

\[
\frac{\partial q_i (q^\tau)}{\partial f_i^\tau} = \frac{\mu_i^\tau (1 + R_i^\tau)}{1 + M^\tau} (1 + M_{-i}^\tau)
\]  

(A.7)

Differentiating with respect to another firm’s position yields,

\[
\frac{\partial q_j (q^\tau)}{\partial f_i^\tau} = \frac{\mu_j^\tau (1 + R_j^\tau)}{1 + M^\tau} \mu^\tau
\]

so that,

\[
\sum_{j \neq i} \frac{\partial q_j (q^\tau)}{\partial f_i^\tau} = \frac{\mu_j^\tau (1 + R_j^\tau)}{1 + M^\tau} M_{-i}^\tau
\]  

(A.8)

Using (A.7) and (A.8), we have that,

\[
R_i^{\tau+1} = \sum_{j \neq i} \frac{\partial q_j}{\partial f_i^{\tau+1}} / \frac{\partial q_i}{\partial f_i^{\tau+1}} = - \frac{M_{-i}^\tau}{1 + M_{-i}^\tau}
\]
A.3 Proof of Proposition 2

Set $\tau = T$ in equation (A.5). By construction, $F^T = 0$ since $T$ is the first period in which forward contracts are bought or sold and $F^T$ has been defined as to reflect sales that occurred prior to period $T$. Solving for $P$, we have,

$$P = c + \frac{a-c}{1+MT} \quad (A.9)$$

Set $\tau = T$ in equation (A.4), where again, $F^T = 0$ by construction. Substituting in expression (A.9) for $P$ in equation (A.4), we have,

$$Q = \left(\frac{a-c}{b}\right)\frac{MT}{1+MT} \quad (A.10)$$

Finally, set $\tau = T$ in equation (A.6). Setting $q^T_i = F^T_i = 0$, we have,

$$q_i = \left(\frac{a-c}{b}\right)\frac{\mu^T_i}{1+MT} \quad (A.10)$$

We now proceed to characterize the firm’s forward sales. In equilibrium, it must be the case that for any period $\tau > 1$, $q_i (q^T) = q_i (q^{T-1})$. In other words period-$\tau$ behavior cannot cause firm $i$ to deviate from its strategy; if it did, then the strategy was not an equilibrium to begin with. Since the firm’s marginal cost in equation (2) is the same regardless of $\tau$, so too is its marginal revenue.

Equating marginal revenue between periods $T - 1$ and $T$, while using the fact that, $q^T_i = 0$ and $q^{T-1}_i = f^T_i$, we have,

$$(q_i - f^T_i) (1 + R^{T-1}_i) = q_i (1 + R^T_i)$$

It follows that the firm’s contracted quantity in period $T$ is,

$$f^T_i = \left(\frac{R^{T-1}_i - R^T_i}{1+R^{T-1}_i}\right) q_i,$$

where $q_i$ is the equilibrium value from equation (A.10). It’s uncommitted output at the beginning of period $T - 1$ is,

$$q_i - f^T_i = \frac{1+R^T_i}{1+R^{T-1}_i} \quad (A.11)$$

Continuing in this manner, we equate marginal revenue between periods $T - 1$ and $T - 2$, so that,

$$(q_i - f^T_i - f^{T-1}_i) (1 + R^{T-1}_i) = (q_i - f^T_i) (1 + R^T_i)$$

The firm’s contracted quantity in period $T - 1$ is,

$$f^{T-1}_i = \frac{R^{T-2}_i - R^{T-1}_i}{1+R^{T-2}_i} (q_i - f^T_i),$$
where \( q_i - f_i^T \) is the value from equation (A.11). Continuing in this manner, the expression for the firm’s forward quantities is true by induction.

### A.4 Proof of Corollary 1

Let \( Q^{(t)} \) denote total output in a game with \( t \) rounds of forward contracting. Further, let \( M^{(t)} = M^T \) when there are \( t \) rounds of forward contracting. To complete the notation, suppose that \( M^{(0)} = B \). We have that \( Q^{(t)} > Q^{(t-1)} \) if and only if \( M^{(t)} > M^{(t-1)} \).

We can construct any \( M^T \) recursively beginning with \( R_1^1 \) as given in Lemma 1. \( R_1^1 \) feeds into \( \mu_1^1 \), which feeds into \( M_1^1 \), which feeds into \( R_2^1 \) and so on.

**Claim 1** Outside the monopoly case, \( R_1^1 \in (-1, 0) \) and \( R_{t+1}^i < R_t^i \) regardless of \( T \).

**Proof.** Outside the monopoly case, each \( B_{-i} > 0 \). It is obvious that \( R_1^1 \in (-1, 0) \). Suppose by way of induction that \( R_t^i > R_{t-1}^i \) regardless of the number of contracting rounds in the game. If \( R_t^i > R_{t-1}^i \), then \( \mu_t^i > \mu_{t-1}^i \), which implies \( M_t^i > M_{t-1}^i \). This implies that,

\[
R_{t+1}^i = -\frac{M_{t-1}^i}{1 + M_t^i} < \frac{M_{t-1}^i}{1 + M_{t-1}^i} = R_t^i
\]

irrespective of \( T \). 

\( R_t^i < R_{T-1}^i \) implies that \( \mu_t^i > \mu_{t-1}^i \). From this we have that, \( M^T > M^{T-1} \), and it is evident that \( M_t^i = M^{(t)} \) regardless of the number of contracting rounds in the game. Since output is higher with more round of forward contracting, it is mechanically true that price is lower.

In the monopoly case, \( B_{-i} = 0 \) for the only producer \( i \) with strictly positive capital stock. It follows that \( R_1^1 = 0 \), which implies \( \mu_1^1 = \beta_i \), which implies \( M_1^1 = B \). Continuing in this manner, it is evident that for any \( t \), \( M_t^i = M^{t-1} \cdots = B \), so that total output and hence price are invariant to the number of contracting rounds.

By Proposition 2, an individual producer’s output is greater with \( T = 1 \) round of forward contracting if and only if,

\[
\frac{\mu_1^1}{1 + M^T} > \frac{\beta_i}{1 + B}
\]

After manipulating terms, this is equivalent to,

\[
\beta_i > \frac{1}{R_1^1} \left( \frac{1 + B_{-i}}{1 + M_1^1} - 1 \right)
\]

Since the right-hand side of the above expression is bounded above zero in all but the monopoly case, it follows that for \( \beta_i \) sufficiently close to zero, the condition fails.

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A.5 Proof of Lemma 2

It was established in the proof of Proposition 2 that a producer’s marginal revenue is equal across each period. Equating its period-$T$ marginal revenue with its period-0 marginal revenue, we have,

$$q_i (1 + R_i^T) = q_i - q_i^0$$

Rearranging terms, we have that,

$$\frac{q_i^0}{q_i} = |R_i^T|$$

A.6 Proof of Lemma 3

The solution to the producer’s problem in period $T$ is characterized by a modified version of equation (2) in which $\tau = T$ and $q_i^T = 0$ for all $i$. Rearranging terms, we have,

$$\frac{P - C_i'}{P} = -\frac{q_i}{P} P' (Q) (1 + R_i^T)$$

$$= -\frac{Q}{P} P' (Q) s_i (1 - h_i)$$

$$= \frac{s_i (1 - h_i)}{\epsilon}$$

where the second line uses the result of Lemma 2 that $h_i = R_i^T$ and uses the substitution, $q_i = s_i Q$. The third line uses the definition of demand elasticity, $\epsilon$. Pre-multiplying by $s_i$ then summing over all $i$ obtains the result.

A.7 Proof of Proposition 4

The proof proceeds in two parts, first showing that the structural effect is negative, then showing that the hedging effect is negative.

Lemma 5 \( SE^{CS} = \frac{a - c}{(1 + M)^2} \left[ \left( \frac{\mu_1}{\sigma_1} \right)^2 \Delta \beta_1 + \left( \frac{\mu_2}{\sigma_2} \right)^2 \Delta \beta_2 \right] \leq 0 \)

Proof. Using,

$$\Delta \beta_1 = bd \left( \frac{\beta_1}{bk_1} \right)^2 \Delta k$$

(A.12)

and,

$$\Delta \beta_2 = -bd \left( \frac{\beta_2}{bk_2} \right)^2 \Delta k = - \left( \frac{\beta_2}{bk_2} \right)^2 \left( \frac{\beta_1}{bk_1} \right)^{-2} \Delta \beta_1$$

(A.13)

\( SE^{CS} \) can be expressed as,
\[ SE^{CS} = \frac{a - c}{(1 + M^2)} \left[ \left( \frac{\mu_1}{b_{k1}} \right)^2 - \left( \frac{\mu_2}{b_{k2}} \right)^2 \left( \frac{\beta_2}{b_{k1}} \right)^2 \left( \frac{\beta_1}{b_{k1}} \right)^{-2} \right] \Delta \beta_1 \]

Since \( \Delta \beta_1 > 0 \), it is sufficient to show that the square-bracketed term is nonpositive. This reduces to,

\[ \left( \frac{\mu_1}{b_{k1}} \right)^2 - \left( \frac{\mu_2}{b_{k2}} \right)^2 \leq 0 \]

Using difference-of-squares (i.e. \( x^2 - y^2 = (x + y)(x - y) \)), it is sufficient that

\[ \frac{\mu_1}{b_{k1}} - \frac{\mu_2}{b_{k2}} \leq 0, \]

or equivalently,

\[ \mu_1 k_2 - \mu_2 k_1 \leq 0 \]

By construction, \( k_1 \geq k_2 \). We can define \( \delta \geq 0 \) such that \( k_2 \equiv k_1 - \delta \). The above inequality simplifies to,

\[ (\mu_1 - \mu_2) k_1 - \mu_1 \delta \leq 0 \tag{A.14} \]

Using the identity,

\[ \mu_i = \frac{\beta_i}{1 + \beta_i R_i} = \frac{\beta_i (1 + B - \beta_i)}{(1 + B) (1 - \beta_i) + \beta_i^2} \tag{A.15} \]

we have that,

\[ (\mu_1 - \mu_2) k_1 = \frac{(1 + B)(1 + B - \beta_1 - \beta_2)(\beta_1 - \beta_2) k_1}{[(1 + B) (1 - \beta_1) + \beta_1^2][(1 + B) (1 - \beta_2) + \beta_2^2]} \]

\[ = \frac{(1 + B)(1 + B - \beta_1 - \beta_2) \beta_1 (1 - \beta_2) \delta}{[(1 + B) (1 - \beta_1) + \beta_1^2][(1 + B) (1 - \beta_2) + \beta_2^2]} \]

If \( \delta = 0 \), then condition (A.14) holds trivially. If \( \delta > 0 \), condition (A.14) reduces to,

\[ - (1 + B) \beta_2 (1 - \beta_2) \leq (1 + B - \beta_1) \beta_2^2 \]

which is true by construction. \( \blacksquare \)

Lemma 6 \( HE^{CS} \equiv -\frac{a - c}{(1 + M)^2} \sum_i \mu_i^2 \Delta R_i \leq 0 \)

Proof. We have that,

\[ \Delta R_i = -\frac{\Delta B_{-i}}{(1 + B_{-i})^2}, \]

\[ \sum_i \mu_i^2 \Delta R_i \leq 0 \]
where,
\[
\Delta B_i = \begin{cases} 
\Delta \beta_2 & \text{if } i = 1 \\
\Delta \beta_1 & \text{if } i = 2 \\
\Delta \beta_1 + \Delta \beta_2 & \text{if } i > 2 
\end{cases}
\]

It follows that,
\[
HE^{CS} \left( \frac{a - c}{(1 + M)^2} \right)^{-1} = [\Delta \beta_1 + \Delta \beta_2] \sum_{j \neq 1, 2} \left( \frac{\mu_j}{1 + B_j} \right)^2 + \left( \frac{\mu_1}{1 + B_{-1}} \right)^2 \Delta \beta_2 + \left( \frac{\mu_2}{1 + B_{-2}} \right)^2 \Delta \beta_1
\]

It suffices to show that each of the square-bracketed terms are negative. From equations (A.12) and (A.13) we have that,
\[
\Delta \beta_1 + \Delta \beta_2 = \left[ 1 - \left( \frac{\beta_2}{bk_2} \right)^2 \left( \frac{\beta_1}{bk_1} \right)^{-2} \right] \Delta \beta_1 \\
\Delta \beta_1 = \left[ \left( \frac{\beta_1}{bk_1} \right)^2 - \left( \frac{\beta_2}{bk_2} \right)^2 \right] \left( \frac{\beta_1}{bk_1} \right)^{-2} \Delta \beta_1 \\
\Delta \beta_1 = \left[ \frac{\beta_1}{bk_1} - \frac{\beta_2}{bk_2} \right] \left( \frac{\beta_1}{bk_1} + \frac{\beta_2}{bk_2} \right) \left( \frac{\beta_1}{bk_1} \right)^{-2} \Delta \beta_1 \\
\Delta \beta_1 = \left[ \frac{\beta_2}{bk_1k_2} \left( \frac{\beta_1}{bk_1} + \frac{\beta_2}{bk_2} \right) \left( \frac{\beta_1}{bk_1} \right)^{-2} \Delta \beta_1 \leq 0
\]

This term is the structural effect in the baseline Cournot model. In the current setting, because the parties are expected to reduce output, they will be less responsive to forward sales of third parties.

Finally, we have that,
\[
\left( \frac{\mu_1}{1 + B_{-1}} \right)^2 \Delta \beta_2 + \left( \frac{\mu_2}{1 + B_{-2}} \right)^2 \Delta \beta_1 \\
= \left[ \left( \frac{\mu_2}{1 + B_{-2}} \right)^2 \left( \frac{\beta_2}{bk_2} \right)^{-2} - \left( \frac{\mu_1}{1 + B_{-1}} \right)^2 \left( \frac{\beta_1}{bk_1} \right)^{-2} \right] \left( \frac{\beta_2}{bk_2} \right)^2 \Delta \beta_1
\]

Due to difference-of-squares, it is sufficient that,
\[
\left( \frac{\mu_2}{1 + B_{-2}} \right) \left( \frac{\beta_2}{bk_2} \right)^{-1} - \left( \frac{\mu_1}{1 + B_{-1}} \right) \left( \frac{\beta_1}{bk_1} \right)^{-1} \leq 0
\]
Using equation (A.15), this is equivalent to,

\[
[(1 + B) (1 - \beta_1) + \beta_1^2] k_2 + [(1 + B) (1 - \beta_2) + \beta_2^2] k_1 \leq 0
\]

Using the identity, \( k_2 = k_1 - \delta \), this reduces to,

\[-(1 + B - \beta_1 - \beta_2) \beta_1 (1 - \beta_2) \delta - [(1 + B) (1 - \beta_1) + \beta_1^2] \delta \leq 0,
\]

which is true by construction.  

From Lemmas 5-6, we have that \( \Delta CS = SE^{CS} + HE^{CS} < 0 \), which establishes the first argument of the proposition. The second argument is that \( HE^{CS} < 0 \) which is shown by Lemma 6.

**A.8 Proof of Proposition 5**

We formally model two types of consolidating transfers: 1) transfers from firm 2 to firm 1; and 2) transfers among third parties. Once we show that \( \| \Delta CS \| \) is larger when the capital allocation is more concentrated by these two means, it is then obvious that the result continues to hold upon considering consolidating transfers from third parties to firm 1.

First we show that a consolidating transfer from firm 2 to firm 1 increases \( \| \Delta CS \| \). The proof of Proposition 4 shows that \( \Delta CS \) is proportional to \( \delta \equiv k_1 - k_2 \) such that an increase in \( \delta \) makes \( \Delta CS \) larger in absolute value.

Next, we show that a consolidating transfer among third parties increases \( \| \Delta CS \| \). From Lemma 7 (below), the structural effect becomes more pronounced as hedge rates decline toward zero. Increasing concentration among third parties has the effect of reducing hedge rates in the same way, which makes the structural effect more pronounced.

It remains to show that the conjectural effect becomes more pronounced as concentration increases among third parties. Let \( (CE^{CS})' \) denote the change in the conjectural effect due to an increase in concentration among third parties. We have that,

\[
(HE^{CS})' = -\frac{2 \cdot HE^{CS}}{1 + M} (M)' - \frac{a - c}{(1 + M)^2} \sum_i (\mu_i^2 \Delta R_i)' 
\]

We show that \( (M)' < 0 \) using the same arguments used to show \( \Delta CS < 0 \). There is a structural effect that reduces output across all the parties directly affected by the perturbation. There is also a hedging effect which reduces \( \mu_i \) for each of the non-affected parties. Since \( CE^{CS} < 0 \), it follows that,

\[-\frac{2 CE^{CS}}{1 + M} (M)' < 0.
\]

Using equation (A.15), it is easy to show that \( \mu_i^2 \Delta R_i \) is increasing in \( B_{-i} \). Since \( B_{-i} \) is decreasing in concentration, \(- (\mu_i^2 \Delta R_i)' < 0 \) for unaffected parties. For the affected parties, the argument is the same as used in the proof of Lemma 6 for why, on net, the parties to the transaction reduce their forward sales. That is, if we perturb the distribution of capital among a subset of third parties,
they will reduced their forward sales so that across the firms affected by the perturbation, $- \sum_i (\mu_i^2 \Delta R_i) < 0$.

### A.9 Proof of Proposition 6

The loss of consumer surplus in the baseline model due to a consolidating transfer is,

$$\Delta CS^0 = \frac{a - c}{(1 + B)^2} (\Delta \beta_1 + \Delta \beta_2)$$

**Lemma 7** $\Delta CS^0 \leq SE^{CS} \leq 0$. The first inequality is strict in all but the monopoly case. The second inequality is strict as long as $k_1 > k_2$.

**Proof.** It is sufficient to show that, $\left(\frac{\mu_1}{\beta_1}\right)^2 > \left(\frac{\mu_2}{\beta_2}\right)^2$. Using equation (A.15), this expression reduces to

$$(1 + B) (B - \beta_1 - \beta_2) + \beta_1 \beta_2 > 0 \quad (A.16)$$

which is true by construction. ■

The proof uses limits to show that as the industry approaches monopoly, the loss of surplus is greater under the two-stage model.

**Lemma 8** $\lim_{\kappa \to 1} SE^{CS} = \lim_{\kappa \to 1} \Delta CS^0 < 0$.

**Proof.** In the limit, $\beta_1 \to B$ and $\beta_2 \to 0$. From equation (A.15), we have that,

$$\lim_{\kappa \to 1} \left( \frac{\mu_1}{\beta_1} \right) = 1 \quad (1 + B) (1 - B) + B^2 = 1$$

and

$$\lim_{\kappa \to 1} \left( \frac{\mu_1}{\beta_1} \right) = \frac{1 + B}{1 + B} = 1.$$

Let $K$ denote industry capital stock. It follows that,

$$\lim_{\kappa \to 1} SE^{CS} = \lim_{\kappa \to 1} \Delta CS^0 = \lim_{\kappa \to 1} (\Delta \beta_1 + \Delta \beta_2) = \left( \frac{1}{(bK + d)^2} - \frac{1}{d^2} \right) bd\Delta k < 0$$

■

**Lemma 9** $\lim_{\kappa \to 1} HE^{CS} < 0$

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Proof. Using equation (A.15), we have that,
\[
\lim_{\kappa \to 1} HE^{CS} = -\frac{bB}{d} \Delta k < 0
\]

It follows that,
\[
\lim_{\kappa \to 1} \Delta CS = \lim_{\kappa \to 1} \Delta CS^0 + \lim_{\kappa \to 1} HE^{CS} < \lim_{\kappa \to 1} \Delta CS^0
\]

A.10 Proof of Lemma 4

To be written.

A.11 Proof of Proposition 7

It is apparent from equation (6) that \(\delta^{(T)}(N)\) decreases in \(T\) if and only if \(\pi^{(T)}(N)\) also decreases in \(T\). From Proposition 2 and the restrictions imposed in Section 5,
\[
\pi^{(T)}(N) = \frac{(a-c)^2}{N} \frac{M^{(T)}}{(1+M^{(T)})^2} \tag{A.17}
\]

\(M^{(0)} = N\) because \(\mu^0 = 1\). From Lemma 1, it can be derived that \(M^{(1)} = N^2\) in the symmetric model. It following immediately that \(\pi^{(0)}(N) > \pi^{(1)}(N)\). Next, consider any \(t = 2, 3, \ldots, T\). Again from Lemma 1,
\[
\mu^{(t)} = \frac{1}{1+R^{(t)}} = \frac{1}{1 - \frac{(N-1)\mu^{(t-1)}}{1+(N-1)\mu^{(t-1)}}} = 1 + (N-1)\mu^t > \mu^t
\]

It follows that \(M^{(t)} > M^{(t-1)}\) and \(\pi^{(t-1)}(N) > \pi^{(t)}(N)\), for \(t = 1, 2, \ldots, T\). QED.