Sticks or carrots or both? *

Frances Z. Xu Lee

Abstract

To punish an agent, the principal sometimes has to incur costs. We show under what conditions the principal optimally chooses to only use rewards, or to only use costly punishment, or to use a combination of both in the contract offered to the agent. Even though agent’s limited liability alone causes too little effort to be implemented, the principal may motivate too much effort when costly punishment is an available instrument. An agent’s payoff may decrease when the effort becomes more productive in creating a good outcome.

1. Introduction

Punishments (sticks) are widely used. The court puts a criminal into jail. A parent puts a child into time-outs. A coach forces a student to run 10 laps. A teacher puts a student into detention. A manager sends a worker to work on the most unpleasant task. These punishments are typically costly to the principal because it is costly to overpower the resistance of the agent to enforce the punishment and sometimes the principal internalizes some of the suffering of the agent, e.g. when the principal is a parent. This paper incorporates costly punishment into a principal-agent model with agent moral hazard and limited liability.\(^1\)

In practice, some schemes exclusively use rewards (carrots), some exclusively use punishments (sticks) and some mix both of them. Potty training a toddler is usually a reward scheme full of jelly beans and stickers. Lack of a crime is not rewarded but a crime is punished with

\(^*\)Loyola University Chicago. francesxu312@gmail.com. The author is grateful for comments from Jacques Cremer and Wing Suen.

\(^1\)Costly punishment will not be used if there is no limited liability: the principal can make the agent pay the principal which is a form of punishment that directly benefits the principal and the principal can always achieve the first best.
community service, jail time or even death penalty. The law and economics literature, to be reviewed in more detail, has compared the reward (carrots) with the punishment (sticks). It has however viewed reward and punishment as mutually exclusive choices. Instead, we show that a scheme that involves both might be optimal: committing to punish the bad outcome as well as rewarding the good outcome. Teachers at school uses both rewards and punishments: medals for good performance and detentions for bad performance. The US government gives conditional foreign aids while threatens economic sanctions that are also costly to the US economy.\(^2\)

Our paper endogenizes the extents of both the reward and the punishment, optimally chosen by a principal in a contract offered to the agent when the agent has a moral hazard problem.

When effort causes the chance of the good outcome being sufficiently high relative to the chance of the bad outcome, using punishment to motivate effort is cheaper than using reward for the principal. This point has been made in different settings. Dari-Mattiacci and Geest (2009) shows that, in a multiple-period setting where each period is constrained by the same per period capacity of a reward or a punishment which only affects the payoffs of the party with moral hazard, a punishment can motivate more effort over time than a reward. In their non-principal-agent model, sticks are always preferable to carrots.\(^3\) Wittman (1984) incorporates an “administrative cost” for a social planner in executing either reward or punishment. He argues that if a punishment successfully implement the efficient outcome then no administrative cost is incurred.

We show that this intuition of the superiority of the punishment over reward is not the complete story when one incorporates the outside option of the agent.\(^4\) When the outside option is very good, the principal is forced to only use reward for the agent. When the outside option is sufficient low, the choice just boils down to which of the reward or punishment is cheaper in expectation to motivate effort. When the outside option is in an intermediate range, rewarding the good outcome while punishing the bad outcome is optimal because it takes advantage of the lower execution cost of punishment as a threat to motivate effort while keeping the agent from quitting with the prospect of the reward. The interaction of the

\(^2\)U.S. has passed a bill that allows Cuba to buy goods from US by credit, rather than cash, which is a reward for Cube’s modernization.

\(^3\)They also argue that an exogenous level of punishment can cause too much effort which can be mitigated by allowing several persons to share the punishment, while we show that an endogenously chosen level of punishment may lead to too much effort because punishment is very cheap to the principal if the good outcome is very likely.

\(^4\)Even a citizen of a country has the option to quit: to immigrate or to revolt.
agent’s incentive compatibility constraint and individual rationality constraint creates interesting comparative statics. As it becomes more likely to attain the good outcome under effort, the principal may optimally move from a scheme with both punishment and reward to a scheme that only rewards. This is because the more relaxed incentive compatibility constraint allows the principal to optimally reduce both the punishment and the reward. However, the reward cannot be reduced to nothing given a moderately good outside option of the agent.

When an agent’s effort becomes more productive, one would expect that the agent’s payoff in equilibrium to be better. This however is not always the case. When the effort is not very productive, the principal optimally chooses not to use punishment because of too high a chance of having to execute it. The agent’s payoff is above the outside option because only reward is meted out. However, as the effort becomes more productive, the principal may optimally shift to using both reward and punishment and keeping the agent at the outside option. If one interprets our principal as a parent and our agent as a child, our paper is related to studies on parenting strategies. Burton, Phipps and Curtis (2002) documented empirical evidences and argued that the parenting strategy (the amount of praise or scolding as a function of the child’s behavior) should be modelled as being chosen according to the characteristics of the child. For an outside option that is reasonable for the parenting context, our model implies that the middle ability agents (children) are pushed to their IR constraints. Lower ability children are placed under pure reward schemes which always give them payoffs above their outside options. Top ones are under pure punishment schemes but they are so capable that they can often avoid the punishment by avoiding bad outcomes and they also enjoy a payoff above their outside options. The middle ones are parented with both rewards and punishment but they are the most unhappy ones.

The literature on punishment in experiments or psychology tend to only look at the differential effects of punishment and rewards on the recipients without taking into account the implementers’ incentives. For example, Fryer et. Al. (2012) shows that loss aversion makes a reduction in bonus to be more effective than an increase in bonus in changing public school teacher’s performances.

Section 2 lays out the basic model. Section 3 presents the benchmark where punishment is not available to be part of the scheme. Section 4 derives the least costly way to implement and examines the principal’s decision over whether or not to implement effort. It also shows how the intensity and the direction of the rewards and punishments change with exogenous variables. Section 5 considers the case where the principal internalizes some of the agent’s

5 Sometimes the parents lay out the rules ahead of time as if offering a contract. Sometimes, the contract is implicit through repeated actions.
payoff.

2. The Basic Model

An outcome is either good or bad: \( \{o_1, o_2\} \) with \( o_2 \) being the good outcome.\(^6\) An agent can exert a hidden effort to improve the probability of the good outcome. The effort costs \( c \) to the agent and increases the chance of a good outcome by \( \theta \) (the productivity of effort). Without the effort, the chance of the good outcome is \( p \). Assume that \( \theta < 1 - p \). The bad outcome’s value to the principal is \( v \), and the good outcome’s value to the principal to be \( v_+ + v \). All parameters are common knowledge.

The timing is the following: (1) Principal offers a scheme that maps the outcomes of a task to reward or punishment levels. (2) The agent chooses whether or not to exert effort. (3) Outcome realizes and punishment or reward is executed.

A reward means that the principal chooses an action that gives the agent a positive payoff. The reward here is a direct monetary transfer. That is, to give agent a payoff \( x > 0 \), the cost to the principal is \( x \). A punishment means that the principal takes an action that gives the agent a negative payoff \( x \) with \( x < 0 \). Because of limited liability, a monetary transfer from the agent to the principal is not possible. Punishing the agent is costly to the principal. The cost to the principal is assumed to be \( \lambda(-x) > 0 \) with \( \lambda > 0 \). This is a direct cost.\(^7\) Let the principal choose a reward or punishment contingent on each outcome: \( x_i \in (-\infty, \infty) \) for \( i \in \{1, 2\} \) with \( x_1 \) being the agent’s payoff (before effort costs) under outcome 1 and \( x_2 \) being the agent’s payoff (before effort costs) under outcome 2.

What is new here is that we allow the principal to have a tool: a punishment that, if executed, destroys the payoffs of both the principal and the agent. For example, a parent can put a child into a time-out facing the corner of the walls and the parent chooses how many minutes this time-out is. The child’s dis-utility increases in the number of minutes linearly. During the whole length of the time-out, the parent has to give sufficient monitoring to make sure that the child is caught immediately if he or she tries to get away from the corner of the walls. The cost to the parent is also increasing linearly in the number of minutes of time-out. The slope for the parent depends on the opportunity cost of the time of the parent as well as how much this parent internalizes the child’s dis-utility.

Principal’s payoff is the value from the outcome minus any reward/punishment cost. The

\(^6\)This can be generalized to \( N \) many outcomes.

\(^7\)In Section 5, we will examine the case where there is indirect cost coming from principal internalizing the agent’s payoff.
agent, if accepting the scheme from the principal, gets the reward/punishment payoff minus any effort cost. That is, when not paying effort and not getting any reward or punishment, the agent’s payoff after accepting the scheme is normalized to be 0. The outside option of the agent is \( t \). This is the payoff the agent gets if it turns down the offer from the principal. We allow \( t \) to be negative. Think of a child. Quitting from a parent results in a payoff that is worse than if he or she stays with a parent with no reward or punishment. The principal’s outside option is 0. Let \( w = v + pv \) denote the expected value when the agent does not exert effort. Assume that \( w > t \) so the relationship is worthwhile even in the absence of effort. All players are risk neutral and maximize expected payoffs. We consider Subgame Perfect Equilibrium.

3. Benchmark: limited liability without costly punishment

We first consider what happens if a punishment is not available for the principal to use and the limited liability is the only constraint. The principal’s problem of implementing effort in the least costly way is to decide on a level of reward \( x_2 \) for the good outcome:

\[
\min_{x_2} \quad (p + \theta)x_2 \\
\text{s.t.} \quad \theta x_2 \geq c \quad \text{(IC)} \\
(p + \theta)x_2 - c \geq t \quad \text{(IR)} \\
x_2 \geq 0
\]

It is immediate that the optimal reward level is

\[
x_2 = \max\left\{ \frac{c + t}{p + \theta}, \frac{c}{\theta} \right\}
\]

When \( t \) is big (\( t > \frac{c}{\theta} \)), the IR constraint is binding and when \( t \) is small (\( t < \frac{c}{\theta} \)), the IC constraint is binding.

The principal has also the option of not motivating effort while keeping the agent in the relationship. Its payoff depends on the outside option because of limited liability. When the outside option of the agent is better than the situation of getting no reward and paying no effort (\( t > 0 \)), one of the least costly ways to keep agent in the relationship while giving up on effort is to pay the agent \( t \) regardless of outcome. That is, the cost of keeping the agent in the relationship is \( t \). If IR is binding when implementing effort (\( t > \frac{c}{\theta} \)), the cost of implementing effort is exactly \( c + t \). That is, motivating the effect will cost the principal an additional \( c \), so the principal’s decision on whether to implement is always efficient, i.e., based on the sign of
\( \theta v - c \). When IC is binding, the cost of implementing effort is above \( c + t \), so there is too little incentive to implement effort.

When the outside option of the agent is negative relative to the situation of getting no reward and paying no effort \( (t < 0) \), the principal cannot take advantage of the bad outside option because of limited liability. The principal’s payoff of not motivating effort is then \( w \). A negative outside option implies that IC is binding, so the cost of implementing effort is \( (p + \theta) \frac{c}{\theta} > c \). Therefore, there is too little incentive to implement effort.

It is a standard result that limited liability leads to too little incentive to implement effort in general.

**Lemma 1.** When punishment cannot be used in the scheme, there is too little incentive to implement effort if and only if \( t < \frac{p}{\theta} c \). The effort level is always efficient otherwise.

**Proof.** The cost to the principal of implementing effort is \( \max\{c + t, c + \frac{p}{\theta} c\} \). The cost to the principal of not implementing effort is \( \max\{t, 0\} \). There are three cases. Case 1. \( t \geq \frac{p}{\theta} c \), then the former is \( c + t \) while the latter is \( c \). The difference is \( c \), which means the decision of whether or not to implement effort is efficient. Case 2. \( 0 \leq t < \frac{p}{\theta} c \). The former is \( c + \frac{p}{\theta} c \) while the latter is \( t \). The difference is larger than \( c \), which means there is too little incentive to implement effort. Case 3. \( t < 0 \). The former is \( c + \frac{p}{\theta} c \) while the latter is \( 0 \). The difference is larger than \( c \), which means there is too little incentive to implement effort. \( \square \)

### 4. Main analysis

**The Least costly way to implement effort**

The principal’s problem of implementing effort in the least costly way is:

\[
\min_{x_1, x_2} \begin{cases} 
(1 - p - \theta) x_1 + (p + \theta) x_2 & \text{if } x_1 \geq 0, x_2 \geq 0 \\
(1 - p - \theta) \lambda(-x_1) + (p + \theta) x_2 & \text{if } x_1 \leq 0, x_2 \geq 0 \\
(1 - p - \theta) \lambda(-x_1) + (p + \theta) \lambda(-x_2) & \text{if } x_1 \leq 0, x_2 \leq 0 
\end{cases}
\]

s.t. \( \theta(x_2 - x_1) \geq c \) \hspace{1cm} \text{(IC)}

\( (1 - p - \theta) x_1 + (p + \theta) x_2 - c \geq t \) \hspace{1cm} \text{(IR)}

The case of \( x_1 \geq 0 \) and \( x_2 \leq 0 \) (rewarding bad outcome and punishing good outcome) is omitted because it will clearly violate the IC constraint. We will use the term “pure reward” for a scheme where a positive reward is given for the good outcome and no punishment is
By construction, it satisfies (IR) for any \( x \). Suppose the least costly scheme has \( x_1 = 0 \) and \( x_2 = x \). Therefore, (IR) has to bind and also a scheme with \( x_1 = 0 \) and \( x_2 = x \) must also be satisfied with \( x \).

By construction, it also satisfies (IR). Since the original contract satisfies (IC), \( x_1 = 0 \) and \( x_2 = x \) and \( \epsilon > 0 \). Suppose (IR) is not binding. Consider another scheme with \( x_1 = (1 - p - \theta)x_1 + (p + \theta)x_2 \). Since (IR) is satisfied with \( x_1 \) and \( x_2 \), (IR) must also be satisfied with \( x_1' \) and \( x_2' \) by construction for any \( \epsilon > 0 \). Since (IC) is satisfied with \( x_1, x_2, x_2 - x_1 > \frac{\epsilon}{p} \). We have \( x_2 - x_1' > x_2 - x_1 + \frac{1}{1-p}(x_2 - x_1) - \epsilon > \frac{\epsilon}{p} + \frac{1}{1-p}(x_2 - x_1) - \epsilon \). Therefore, for any \( \epsilon \) in \( (0, \frac{1}{1-p}(x_2 - x_1)) \), the (IC) holds for \( x_1', x_2' \). Because the new scheme \( x_1', x_2' \) involves lower punishment for both outcomes than scheme \( x_1, x_2 \), the principal’s cost is lower under \( x_1', x_2' \).

Suppose the least costly scheme has \( x_1 > 0 \) and \( x_2 > 0 \). Suppose (IR) is not binding. Consider another scheme with \( x_1' = 0 \) and \( x_2' = \frac{1}{1-p}(x_2 - x_1) - \epsilon > 0 \). By construction, it satisfies (IR) for \( \epsilon \) small enough. Since the original contract satisfies (IC), \( x_2 > x_1 + \frac{\epsilon}{p} \). This implies \( x_2' - x_1' > \frac{x_1}{p} + \frac{\epsilon}{p} > \frac{\epsilon}{p} \) if \( \epsilon \) is small enough, which satisfies (IC). Then this new scheme \( x_1', x_2' \) reduces the cost to implement effort, which is a contradiction. Therefore, (IR) has to bind.

Consider another scheme with \( x_1' = 0 \) and \( x_2' = \frac{1}{1-p}(x_2 - x_1) + \frac{\epsilon}{p} > 0 \). By construction, it also satisfies (IR). Since the original contract satisfies (IC), \( x_2 > x_1 + \frac{\epsilon}{p} \). This implies \( x_2' - x_1' > \frac{x_1}{p} + \frac{\epsilon}{p} > \frac{\epsilon}{p} \), so the new scheme strictly relaxes (IC). Therefore, \( x_1', x_2' \) implement effort and results in the same cost to the principal.

We now simplify the principal’s problem in minimizing the cost of implementing the effort by restricting to the domain of \( x_1 \leq 0 \) and \( x_2 \geq 0 \), keeping in mind that Lemma 2 implies that if a pure reward scheme is optimal with (IR) binding then a double reward scheme is also optimal.

**Lemma 2.** A scheme with \( x_1 < 0 \) and \( x_2 < 0 \) (double punishment) can never be the least costly scheme. If a scheme with \( x_1 > 0 \) and \( x_2 > 0 \) (double reward) is the least costly scheme, then IR must bind and also a scheme with \( x_1' = 0 \) and \( x_2' > 0 \) (pure reward) will also be the least costly scheme.

**Proof.** Suppose the least costly scheme has \( x_1 < 0 \) and \( x_2 < 0 \). Consider another scheme with \( x_1' = (1 - p - \theta)x_1 + (p + \theta)x_2 \) and \( x_2' = 0 \). Since (IR) is satisfied with \( x_1, x_2 \), (IR) must also be satisfied with \( x_1' \) and \( x_2' \) by construction for any \( \epsilon > 0 \). Since (IC) is satisfied with \( x_1, x_2, x_2 - x_1 > \frac{\epsilon}{p} \), we have \( x_2' - x_1' = x_2 - x_1 + \frac{1}{1-p}(x_2 - x_1) - \epsilon > \frac{\epsilon}{p} + \frac{1}{1-p}(x_2 - x_1) - \epsilon \). Therefore, for any \( \epsilon \) in \( (0, \frac{1}{1-p}(x_2 - x_1)) \), the (IC) holds for \( x_1', x_2' \). Because the new scheme \( x_1', x_2' \) involves lower punishment for both outcomes than scheme \( x_1, x_2 \), the principal’s cost is lower under \( x_1', x_2' \).

Suppose the least costly scheme has \( x_1 > 0 \) and \( x_2 > 0 \). Suppose (IR) is not binding. Consider another scheme with \( x_1' = 0 \) and \( x_2' = (1 - p - \theta)x_1 + (p + \theta)x_2 \). Since (IR) is satisfied with \( x_1, x_2 \), (IR) must also be satisfied with \( x_1' \) and \( x_2' \) by construction for any \( \epsilon > 0 \). Since (IC) is satisfied with \( x_1, x_2, x_2 - x_1 > \frac{\epsilon}{p} \), we have \( x_2' - x_1' = x_2 - x_1 + \frac{1}{1-p}(x_2 - x_1) - \epsilon > \frac{\epsilon}{p} + \frac{1}{1-p}(x_2 - x_1) - \epsilon \). Therefore, for any \( \epsilon \) in \( (0, \frac{1}{1-p}(x_2 - x_1)) \), the (IC) holds for \( x_1', x_2' \). Because the new scheme \( x_1', x_2' \) involves lower punishment for both outcomes than scheme \( x_1, x_2 \), the principal’s cost is lower under \( x_1', x_2' \).

We now simplify the principal’s problem in minimizing the cost of implementing the effort by restricting to the domain of \( x_1 \leq 0 \) and \( x_2 \geq 0 \), keeping in mind that Lemma 2 implies that if a pure reward scheme is optimal with (IR) binding then a double reward scheme is also optimal.
The principal’s problem of implementing effort in the least costly way is:

\[
\min_{x_1, x_2} \quad (1 - p - \theta)\lambda(-x_1) + (p + \theta)x_2
\]

\[
s.t. \quad \theta(x_2 - x_1) \geq c
\]

\[
(1 - p - \theta)x_1 + (p + \theta)x_2 - c \geq t
\]

\[
x_1 \leq 0 \quad \text{and} \quad x_2 \geq 0
\]

Both punishment and reward contribute to the IC constraint. The marginal cost of punishment to the principal is \((1 - p - \theta)\lambda\), while the marginal cost of reward to the principal is \((p + \theta)\). When \(\theta\) is high, the chance of the punishment being executed is low while the chance of the reward being executed is high. Therefore, when \(\theta\) is high it is more economical for the principal to use punishment to satisfy the agent’s IC constraint. The cutoff value for \(\theta\) is \(\frac{\lambda}{1+\lambda} - p\).

When \(\theta < \frac{\lambda}{1+\lambda} - p\), the principal has a preference for using reward to fulfill IC. When the outside option is sufficiently good, the pure reward is set to meet IR, with IC being lax. When the option option is worse, the pure reward is set to meet IC, with IR being lax.

When \(1 - p > \theta > \frac{\lambda}{1+\lambda} - p\), the principal has a preference for using punishment to fulfill IC. However, when the outside option \(t\) is sufficiently attractive, the principal has to reply on a reward to satisfy agent’s IR constraint. The optimal contract is pure reward to meet the binding IR and IC will be left lax. When the outside option is in an intermediate range, despite the principal’s preference for punishment to meet IC, she has to use some reward to meet IR. Both IR and IC are binding and the contract is a mixed scheme. When the outside option is sufficiently unattractive, the principal has no fear of the agent quitting the relationship. She uses pure punishment to meet the binding IC with IR being lax. Proposition 1 summarizes the optimal scheme of implementing effort.

**Proposition 1.** Let \(\bar{t}(\theta) = \frac{p}{\theta}c\) and \(\underline{t}(\theta) = -\frac{1-p}{\theta}c\). The least costly contract to implement effort when Principal has to bear a linear direct cost of punishment are the following.

1. \(\theta < \frac{\lambda}{1+\lambda} - p\). The scheme is pure reward. (1) If \(t > \bar{t}(\theta)\), then \(x_1 = 0\) and \(x_2 = \frac{t+c}{p+\theta}\), IR is binding and IC is lax. The cost of implementing the effort is \(c + t\). (2) If \(t < \bar{t}(\theta)\), then \(x_1 = 0\) and \(x_2 = \frac{c}{\theta}\), IR is lax and IC is binding. The cost of implementing the effort is \(c + \frac{p}{\theta}c\).

2. \(\theta > \frac{\lambda}{1+\lambda} - p\). The form of the scheme depends on \(t\). (1) If \(t > \bar{t}(\theta)\), then \(x_1 = 0\) and \(x_2 = \frac{t+c}{p+\theta}\) (pure reward). IR is binding and IC is lax. The cost of implementing the effort is \(c + t\). (2) If \(t \in (\underline{t}(\theta), \bar{t}(\theta)]\), then \(x_1 = -\left(\frac{p}{\theta}c - t\right)\) and \(x_2 = t + \frac{1-p}{\theta}c\) (reward and punishment). Both IR and IC are binding. The cost of implementing the effort is \((p + \theta)(t + \frac{1-p}{\theta}c) + (1 - p - \theta)(\frac{p}{\theta}c - t)\lambda\).
(3) If $t \leq \bar{t}(\theta)$, $x_1 = -\frac{c}{\theta}$ and $x_2 = 0$ (pure punishment). IR is lax and IC is binding. The cost of implementing the effort is $(1 - p - \theta)\frac{c}{\theta}$. 

When the pure reward is optimal, there is also contracts that reward both outcomes that are equally optimal. This is because IC is lax, one can move a little bit of reward to the bad outcome, keeping the expected total reward the same and keeping IC still satisfied. In all other cases, the above optimal contracts are the only optimal ones for implementing effort. Figure 1 summarizes the least costly way to implement effort for the principal.

![Figure 1. Principal’s least costly way to implement effort.](image)

**Principal’s incentive to motivate effort**

The principal can choose not to implement effort. Its payoff depends on the outside option because of limited liability. Let $K_0$ denote the principal’s minimized costs when not motivating effort but still having the agent in the relationship (i.e. accepting the scheme offered). When the outside option of the agent is positive relative to the situation of getting no reward or punishment and paying no effort, one of the least costly ways to keep agent in the relationship is to pay the agent $t$ regardless of outcome. When the outside option of the agent is negative relative to the situation of getting no reward or punishment and paying no effort,
the principal cannot take advantage of the bad outside option because of limited liability.

\[
K_0 = \begin{cases} 
    t & \text{if } t > 0 \\
    0 & \text{if } t \leq 0 
\end{cases}
\]

Put in another way: there is a basic cost of keeping the agent in the relationship if \( t > 0 \) at \( t \). Otherwise, there is no cost in keeping the agent in the relationship.

Let \( K_1 \) denote the principal’s minimized cost of implementing effort from Proposition 1. Assuming breaking the indifference in favor of motivating effort, the principal chooses to implement effort if and only if \( \theta v \geq K_1 - K_0 \). The efficient decision is to implement effort if and only if \( \theta v \geq c \). Therefore, we define the following terms. We says that the principal has “too much incentive to implement effort” if \( K_1 - K_0 < c \) and “too little incentive to implement effort” if \( K_1 - K_0 > c \). When \( K_1 - K_0 < c \), we have \( \frac{K_1 - K_0}{\theta} < \frac{c}{\theta} \), so there exists a parameter range \( v \in (\frac{c}{\theta}, \frac{K_1 - K_0}{\theta}) \) such that the efficient decision is to not implement effort, but the equilibrium outcome is to implement effort, and for all other possibilities of \( v > 0 \), the efficient and equilibrium decisions coincide. When \( K_1 - K_0 > c \), the opposite is true: we have \( \frac{K_1 - K_0}{\theta} > \frac{c}{\theta} \), so there exists a parameter range \( v \in (\frac{c}{\theta}, \frac{K_1 - K_0}{\theta}) \) such that the efficient decision is to implement effort, but the equilibrium outcome is to not implement effort, and for all other possibilities of \( v > 0 \), the efficient and equilibrium decisions coincide.

Case 1. \( t > \bar{t}(\theta) > 0 \). The cost of implementing effort is \( K_1 = c + t \), while the cost of not implementing effort but keeping the agent in the relationship is \( K_0 = t \). So the outcome is always efficient. Here, limited liability is not binding. Since only rewards are used, which are transfers from the principal to the agent, and the agent’s IR constraint is binding, the principal fully internalizes the agent’s payoff.

Case 2. \( t \leq \bar{t}(\theta) \) and \( \theta < \frac{\lambda}{1 + \lambda} - p \). There are two subcases. (1) \( t > 0 \), so \( K_0 = t \). Since IR is lax when implementing effort, the cost of implementing effort is greater than \( c + t \), so there is less incentive to implement effort. More specifically, \( K_1 - K_0 - c = (\frac{c}{\theta} t - t) > 0 \) implies \( K_1 - K_0 > c \). (2) \( t < 0 \). so \( K_0 = 0 \). The cost of implementing effort is \( K_1 = c + \frac{c}{\theta} t \), so \( K_1 - K_0 = c + \frac{c}{\theta} t > c \). Therefore, there is also too little incentive for effort. Intuitively, a slack IR and limited liability lead to under-supply of effort, which is a standard result.

Case 3. \( t \leq \bar{t}(\theta) \) and \( \theta > \frac{\lambda}{1 + \lambda} - p \). This is when punishment is used. Since the principal chooses to use punishment, it reduces the cost of implementing effort relative to the no-punishment benchmark.
Proposition 2. Under the following non-empty parameter cases, the efficient outcome is no effort but the equilibrium outcome is agent exerting effort:

(1) \( t > \hat{t}(\theta) \), \( t \in (t(\theta), \bar{t}(\theta)] \) and \( t > 0 \), so \( K_0 = t \).

\[
K_1 = (p + \theta)(t + \frac{1 - p}{\theta} c) + (1 - p - \theta)(\frac{p}{\theta} c - t)\lambda
\]

Note that \( K_1 - K_0 \) is decreasing in \( \theta \). When \( \theta = 1 - p \), \( K_1 - K_0 = c \). Since \( \theta < 1 - p \), \( K_1 - K_0 > c \). That is, too little incentive for effort.

(2) \( t \in (t(\theta), \bar{t}(\theta)] \) and \( t < 0 \), so \( K_0 = 0 \).

\[
K_1 = (p + \theta)(t + \frac{1 - p}{\theta} c) + (1 - p - \theta)(\frac{p}{\theta} c - t)\lambda
\]

Note that \( K_1 - K_0 \) is decreasing in \( \theta \). When \( \theta = 1 - p \), \( K_1 - K_0 = c + t < c \). Therefore, there exists a cutoff for \( \theta \), denoted by \( \tilde{\theta}(t) \in (\frac{\lambda}{1 + \lambda} - p, 1 - p) \) such that the incentive to implement is too strong above it and too little below it.

(3) \( t \leq \hat{t}(\theta) \) and \( \theta > \frac{\lambda}{1 + \lambda} - p \). This implies \( t < 0 \), so \( K_0 = 0 \). Therefore, \( K_1 - K_0 = (1 - p - \theta)\frac{c}{\theta} \lambda \). Note that \( K_1 - K_0 \) is decreasing in \( \theta \). When \( \theta = \frac{\lambda}{1 + \lambda} - p \), we have \( K_1 - K_0 = c + \frac{pc}{\theta} > c \). When \( \theta = 1 - p \), \( K_1 - K_0 = 0 < c \). Therefore, there is a cutoff for \( \theta \), \( \tilde{\theta} = \frac{\lambda}{1 + \lambda}(1 - p) \), such that the incentive to implement is too strong above it and too little below it. Proposition 2 highlights the case of over supplying the effort.

The interesting part is that effort may be oversupplied as in contrast to Lemma 1. This happens when the outside option is bad so that IC is binding and when the productivity of effort is high. When effort is implemented, the chance of having to carry out a punishment is so low that implementing effort has very low expected cost to the principal. Think of the extreme case when \( \theta = 1 - p \). The effort can completely eliminate the chance of a bad outcome and thus any cost of punishment. The principal will always want to implement effort even if \( \theta v < c \).

Comparative statics with respect to \( \theta \)
As the productivity of the effort increases ($\theta$ increases), the least costly way of implementing effort involves a decreasing the intensity of incentives: the magnitude of reward and punishment (if any) are decreasing. This is because a higher $\theta$ makes both IR and IC constraints easier to satisfy and both reward and punishment are costly to the principal. Moreover, when $\theta$ reaches the critical value of $\frac{\lambda}{1+\lambda} - p$, there is a discrete drop in the reward and a discrete increase in punishment, reflecting that it becomes cost effective to replace reward with punishment to motivate effort.

Assume that the value of the good outcome $v$ is sufficiently large: $v > \frac{\lambda}{1+\lambda}c/(\frac{\lambda}{1+\lambda} - p)$ such that it is worthwhile to motivate effort even when punishment is not available. The change in the rewards and punishment in equilibrium is shown by Figure 2, fixing the outside option $t$ to be in a negative range: $t \in (-\frac{1-p}{\frac{\lambda}{1+\lambda} - p}, -c)$:

![Figure 2](image_url)

**Figure 2.** How reward and punishment levels change with $\theta$ in equilibrium for moderately negative outside option.

This range of $t$ gives the richest changes in the reward and punishment when $\theta$ changes. For example, if $t$ is more negative, the equilibrium contract will move through no incentive, pure reward and pure punishment, without a range for both reward and punishment. When $t > 0$, the figure will be similar, except that punishment’s magnitude drops to zero while reward’s magnitude is still positive. The corresponding payoffs of the agent is shown in Figure
3. Since $t < 0$. The principal wants to motivate effort by paying reward for the good outcome if and only if $\theta v > c + \frac{p}{\lambda} c$. Therefore, the first cutoff $\theta_1$ is the solution to $\theta v = c + \frac{p}{\lambda} c$. The cutoff $\theta_2$ is where the principal is indifferent between satisfying IC with the reward and the punishment. That is, $\theta_2 = \frac{\lambda}{1+\lambda} - p$. When $v$ is sufficiently positive, $\theta_1 < \theta_2$. The cutoff $\theta_3$ is where the IR is just binding if IC is entirely satisfied using punishment. Therefore, $\theta_3 = \frac{1-p}{c} c$.

When $\theta \leq \theta_1$, there is no motivation for effort, the agent’s payoff stays at 0. When $\theta \in (\theta_1, \theta_2]$, only rewards is used so the agent’s payoff is positive, but declining as reward drops. When $\theta \in (\theta_2, \theta_3]$, both reward and punishment are used and the agent’s payoff is kept at the outside option. When $\theta \in (\theta_3, 1-p]$, only punishment is used so the agent’s payoff is negative, but increasing as punishment drops.

**Figure 3.** How agent’s payoff changes with $\theta$ in equilibrium for moderately negative outside option.

As $\theta$ increases from below threshold $\theta_2$ to above threshold $\theta_2$, the agent’s payoff discretely drops. Note that the threshold $\theta_2 = \frac{\lambda}{1+\lambda} - p$ depends on $p$. Therefore, an increase in $p$ can also cause the agent’s payoff to discretely drop from $\frac{pc}{\theta}$ to $t$.

**Proposition 3.** There exists $\theta' > \theta''$ such that the agent’s equilibrium payoff is higher under $\theta''$ (lower productivity of effort) than under $\theta'$ (higher productivity of effort). There exists $p' > p''$ such that the agent’s equilibrium payoff is higher under $p''$ (lower prospect without effort) than under $p'$ (higher prospect without effort).
This explains that punishment is rarely used on very young children, but more commonly used as one grows older, presumably with better ability to create a good outcome both with and without effort.

5. Cost of punishment is indirect

In this section, we assume that there is no direct cost in executing the punishment, but the principal internalizes the agent’s payoff to some extent. The principal’s payoff is the payoff in Section 3 plus $\delta U_A$ where $U_A$ is the agent’s payoff with $\delta \in (0, 1)$.

This implies that any punishment $x_1 < 0$ suffered by the agent upon a bad outcome imposes a cost of $\delta(-x_1)$ on the principal as well and the principal’s cost of rewarding the agent is reduced by $\delta x_2$ as well. Moreover, when implementing the effort, the principal also bears a cost equal to $\delta c$ by internalizing the agent’s effort cost. Therefore, the principal’s problem of implementing effort in the least costly way is:

$$
\min_{x_1, x_2} \quad (1 - p - \theta)(-x_1)(\lambda + \delta) + (p + \theta)x_2(1 - \delta) + \delta c \\
\text{s.t.} \quad \theta(x_2 - x_1) \geq c \quad \text{(IC)} \\
(1 - p - \theta)x_1 + (p + \theta)x_2 - c \geq t \quad \text{(IR)} \\
x_1 \leq 0 \quad \text{and} \quad x_2 \geq 0
$$

Note that this is equivalent to:

$$
\min_{x_1, x_2} \quad (1 - p - \theta)\frac{\lambda + \delta}{1 - \delta}(-x_1) + (p + \theta)x_2 \\
\text{s.t.} \quad \theta(x_2 - x_1) \geq c \quad \text{(IC)} \\
(1 - p - \theta)x_1 + (p + \theta)x_2 - c \geq t \quad \text{(IR)} \\
x_1 \leq 0 \quad \text{and} \quad x_2 \geq 0
$$

Setting $\hat{\lambda} = \frac{\lambda + \delta}{1 - \delta}$, this problem becomes essentially the same as the one when the cost of punishment is a direct cost. Therefore, we have the analogy to Proposition 1:

**Proposition 4.** Let $\bar{\ell}(\theta) = \frac{\theta}{\theta}$ and $\bar{t}(\theta) = -\frac{1 - p}{\theta}c$. The least costly contract to implement effort when the Principal internalizes $\delta$ proportion of the agent’s payoff are the following.

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8The case of full internalization $\delta = 1$ is not interesting because it implies that reward is free to the principal and the principal is happy to pay the agent an infinite amount for a good outcome.
1. $\theta < \frac{\lambda + \delta}{1 + \lambda} - p$. The scheme is pure reward. (1) If $t > \bar{t}(\theta)$, then $x_1 = 0$ and $x_2 = \frac{t + \delta}{p + \theta}$, IR is binding and IC is lax. The cost of implementing the effort is $(1 + \delta)c + (1 - \delta)t$. (2) If $t < \bar{t}(\theta)$, then $x_1 = 0$ and $x_2 = \frac{\delta}{\theta}$, IR is lax and IC is binding. The cost of implementing the effort is $(1 + \delta)c + (1 - \delta\frac{p}{\theta})c$.

2. $\theta > \frac{\lambda + \delta}{1 + \lambda} - p$. The form of the scheme depends on $t$. (1) If $t > \bar{t}(\theta)$, then $x_1 = 0$ and $x_2 = \frac{t + \delta}{p + \theta}$ (pure reward). IR is binding and IC is lax. The cost of implementing the effort is $(1 + \delta)c + (1 - \delta)t$. (2) If $t \in (\bar{t}(\theta), \bar{t}(\theta))$, then $x_1 = -\frac{\theta}{\delta}(c - t)$ and $x_2 = t + \frac{1 - \delta}{\theta}c$ (reward and punishment). Both IR and IC are binding. The cost of implementing the effort is $(1 - p - \theta)((\frac{\theta}{\delta} c - t)(\lambda + \delta) + (p + \theta)(t + \frac{1 - \delta}{\theta}c)(1 - \delta) + \delta c$. (3) If $t \leq \bar{t}(\theta)$, $x_1 = -\frac{\theta}{\delta}$ and $x_2 = 0$ (pure punishment). IR is lax and IC is binding. The cost of implementing the effort is $(1 - p - \theta)\frac{\theta}{\delta}(\lambda + \delta) + \delta c$.

6. Conclusion

In this paper, we investigate the optimal choice of carrots or sticks considering the principal’s costs of using both. Sticks are often costly to the principal because of the resistance of the agent to the punishment or because the principal internalizes some of the agent’s payoff. We derive insights on when to promise only carrots for a good outcome, when to promise only sticks for a bad outcome and when to promise both, taking into account the outside option of the agent.

References


