Sourcing Co-Created Products: Should your Suppliers Collaborate?

DINAH COHEN-VERNIK
Department of Marketing
Jones Graduate School of Business
Rice University
6100 Main St, Houston, TX 77005
E-mail: dv6@rice.edu
Phone: 713-348-3729

OKSANA LOGINOVA
Department of Economics
University of Missouri
118 Professional Bldg, Columbia, MO 65211
E-mail: loginovao@missouri.edu
Phone: 573-882-0063

NILADRI B. SYAM
Department of Marketing
Trulaske College of Business
University of Missouri
431 Cornell Hall, Columbia, MO 65211
E-mail: syamn@missouri.edu
Phone: 573-882-9727

November 8, 2016

*Authors are listed alphabetically. Everyone contributed equally.
Abstract

We investigate the phenomenon of sourcing co-created products. Specifically, we study how a multi-product downstream firm should source from the upstream market, that is single-source versus multi-source, in a situation where the products are co-created with the suppliers. In business-to-business markets it is increasingly common for downstream firms to co-create the products with the help of their suppliers, and we contribute to the literature on sourcing strategies by incorporating product co-creation. We also model whether the downstream firm should establish a collaborative environment for its suppliers, and this is novel to the literature. Finally, we compare the sourcing strategies for co-created products with the case when the downstream firm makes a straight purchase.

We find that the downstream firm may be worse off when the upstream suppliers collaborate, unless the cross-effect of its and its suppliers’ investments is very large. Importantly, we derive this result for a completely general cost function. We have considered a very rich strategy space for the firm’s sourcing strategy, which includes, (a) co-creation or straight purchase, (b) single-sourcing or multi-sourcing, and (c) collaboration or no collaboration in the case of multi-source co-creation. We find that for an additively separable cost function, the downstream firm’s optimal strategy is multi-source co-creation without collaboration. An important economic force that our analysis has uncovered is that single-sourcing of co-created products completely destroys the downstream firm’s incentives to invest in co-creation. This result too has been derived for a completely general cost function, and thus we also contribute to the economics literature on holdup, which has not considered co-created products. This economic force is instrumental in ensuring that multi-sourcing dominates single-sourcing. Finally, we find that the incentives of the downstream firm to multi-source are stronger for co-created products than for straight purchase of standard products. Since the downstream firm internalizes some of the production in the case of co-created products, reducing its dependence on suppliers, one might speculate that the incentives to multi-source may be less for co-created products. Counter-intuitively, we show that the incentives to multi-source are even stronger, and are driven by endogenous investments of the firms.

Key words: co-creation, sourcing strategy, collaboration, holdup, game theory
1 Introduction

Firms in business-to-business (B2B) markets often develop innovative products with the help of their upstream suppliers. Such co-creation of products and services benefits both the upstream supplier and the downstream firm (Sawhney, Verona and Prandelli 2005; Jaworski and Kohli 2006; Kalaignanam and Varadarajan 2006; Moreau and Herd 2010; Moreau, Bonney and Herd 2011; Payne, Storbacka and Frow 2008). The fundamental concept of marketing is that the firm’s offering, and product design in particular, should focus on customers and their needs (Prahalad and Ramaswamy 2004; Jaworski and Kohli 2006), and the practice of product co-creation is consistent with this concept. Collaboration with the downstream firm enables the upstream supplier to develop products that truly reflect the voice-of-the-customer, since the downstream firm is closer to the end consumer ensuring that, compared to the upstream supplier, it has a deeper understanding of how products and processes are actually used (Griffin and Hauser 1993). In B2B contexts the lead-user literature, and the related user-design literature, have provided ample evidence of the advantages of user design for upstream suppliers who are able to benefit from user innovations to improve their offerings (Griffin and Hauser 1993; Randall, Terweisch and Ulrich 2007; von Hippel 1986; Thomke and von Hippel 2002; von Hippel and Katz 2002).

The downstream firm often co-creates with the upstream supplier so that both the firm and the supplier can influence the design of the final product and take advantage of the upstream supplier-led R&D innovations. For example, in automobile manufacturing the ‘German Model’ has OEMs (Original Equipment Manufacturers) work very closely with their upstream suppliers, and in fact, the suppliers undertake a large part of the R&D investments leading to innovations (Calzolari et al. 2015). Japanese automobile manufacturers also work on joint R&D with their suppliers, especially to reduce production costs, which later translates into lower prices and give the manufacturers a competitive edge. More generally, organizational learning has been identified as a key factor in obtaining sustainable competitive advantage, and inter-organizational knowledge sharing through co-creation, which often results in production efficiencies or product innovations, have been emphasized in the literature as a potent form of this kind of learning (Cohen and Levinthal 1990; Kogut and Zander 1992). Most papers on co-creation in marketing and in user-design literature have focused on the incentives of the upstream firm to collaborate with its customers on product design and R&D (Syam and Pazgal 2013; Redstrom 2006; Payne, Storbacka and Frow 2008). Our current work differs in that we adopt the perspective of the downstream firm in a B2B setting and investigate its incentives to co-create with upstream suppliers. Cohen-Vernik, Syam and Pazgal (2016) also investigate the incentives of downstream firms to co-create with upstream suppliers. However, in their setup competing downstream firms decide whether and how to co-create with a monopolistic upstream supplier, and therefore they do not consider multi-sourcing or single-sourcing in the presence of co-creation, which is a key focus of the current paper.

In this paper we study how a downstream firm, which is selling the products to the end consumers, should source those products from and co-create these product with multiple upstream suppliers. An added complexity occurs because the downstream firm sells multiple products. In this case the firm also has to decide if it will co-create different products with the same upstream supplier or with different ones. While the decision to involve one or more suppliers in the com-
pany’s sourcing strategy is important for non-co-created products, it is crucial for co-created products owing to the considerable investment that the downstream firm has to undertake. The literature on product sourcing has considered whether a downstream firm should source its products from one or multiple suppliers (Cohen and Young 2006; Mukherjee and Tsai 2013; Burke and Vakharia 2004; Burke, Carrillo and Vakharia 2007; Feng. 2012; Horowitz 1986; Shy and Stenbacka 2003), but has not considered co-creation in conjunction with multi-sourcing.

In actual practice, where products are sourced from suppliers, the suppliers and/or firms often undertake relationship-specific investments. In fact, the knowledge-sharing and collaborative R&D across vertical levels is very common in international trade, with buyers in developed countries sharing their research and development expertise with their suppliers in developing nations (Pack and Saggi 2001). See Spencer (2005) for a survey of the literature on outsourcing. We contribute to this literature by analyzing single versus multi-sourcing decisions when products are co-created by the supplier(s) and the downstream firm. This is new to the literature. Said differently, we contribute by incorporating endogenous R&D investment, both at the upstream and downstream levels, into the downstream firm’s decision to single-source or multi-source such co-created products. As an example, being a multi-product firm with various car models, Toyota needs to decide whether to work with a single supplier or with multiple suppliers to co-create its car models. This decision often depends on specific components. For instance, Toyota works with different suppliers in North America to design tires for each of its car models (Liker and Choi 2004). However, seats for both Camry sedan and Venza SUV are manufactured by Kentucky-based Johnson Controls with inputs from Toyota engineers. Thus, any downstream firm selling different products has to decide whether to co-create the various products with the same supplier or with different suppliers. We investigate this important tradeoff in this paper. The literature on ‘bi-sourcing’ analyzes the strategic implications of the downstream firm producing their own products as well as sourcing them from upstream suppliers (Du, Lu and Tao 2006; Beladi and Mukherjee 2012). This stream of literature also does not investigate endogenous collaborative R&D between firms at different vertical levels.

Auto manufacturer Toyota is frequently touted by upstream suppliers as the best car company to supply to, precisely because Toyota collaborates with them to improve the efficiency of their production systems. Indeed, Liker and Choi (2004) quote a senior executive of a Toyota supplier as stating that, “Toyota helped us dramatically to improve our production system...Toyota is our best customer”—Senior executive, supplier to Ford, GM, Chrysler, and Toyota, July 2001. Often these collaborations are aimed at improving efficiencies by reducing costs. For instance, in 1998 Toyota initiated “Construction of Cost Competitiveness for the 21st Century” (CCC21) program aimed at reducing costs of purchases from its suppliers by 30%. Such cost-reducing co-creation has been widely acknowledged as the cornerstone of Toyota’s highly rated and extensively studied lean manufacturing method, the Toyota Way. In this paper we model co-creation as cost-reducing R&D jointly performed by the upstream supplier(s) and downstream firm.

When a downstream firm works closely with different suppliers, another decision it has to make is how much collaboration among the suppliers to establish. For example, Toyota started its US suppliers’ association (Bluegrass Automotive Manufacturers Association, or BAMA) expressly to facilitate collaboration and knowledge sharing among its suppliers. As another example, the Heavy Duty Manufacturers Association (HDMA), which represents the interests of
suppliers to OEMs that sell heavy duty trucks like Mack etc., often holds Town Hall Meetings with the OEMs they sell to. Obviously, any given OEM can influence how much collaboration it wants its suppliers to have by being more or less involved in such suppliers’ association, even if, unlike Toyota, it has not directly formed its suppliers’ association. On the other hand, the Big Three US automakers are well known to foster rivalry, often unhealthy, among their suppliers as evident from this quote from a supplier: “Honda is a demanding customer, but it is loyal to us. [American] automakers have us work on drawings, ask other suppliers to bid on them, and give the job to the lowest bidder. Honda never does that.”– CEO, industrial fasteners supplier to Ford, GM, Chrysler, and Honda, April 2002 (Liker and Choi 2004). The phenomenon of upstream suppliers collaborating at the behest of the downstream firm is more general than just an automotive industry. Wiener and Saunders (2014) provide evidence that downstream clients in the IT industry often ‘force’ their upstream suppliers to cooperate. They state, “We define this concept as a situation where an external actor (the multi-sourcing client firm) creates and or- chestrates a market-like environment, in which a set of interdependent actors (the vendor firms) is required to compete and cooperate [our italics]” (pg 212-213). The authors further argue that this trend is increasing in the IT outsourcing industry. Such anecdotal evidence clearly suggests that downstream firms differ, both across industries and within an industry, in how much collaboration among their upstream suppliers they would like to have. Further, firms can facilitate such collaboration by various means. We analytically model the emerging phenomenon of supplier collaboration in the context of co-creation. To the best of our knowledge, our paper is the first to analyze the firm’s decision whether its suppliers should collaborate or not when the firm sources from multiple suppliers.

As already mentioned, many authors have investigated downstream firms’ strategic decisions to single-source or multi-source in the presence of multiple suppliers. Burke, Carrillo and Vakharia (2007) note that, almost 55% earned revenue of a manufacturing firm is spent on purchased materials. In the automobile industry, the suppliers to OEMs account for as much as 70% of the vehicle’s value. Thus, the procurement strategy of a downstream firm is of utmost importance. There could be many reasons for why a multi-product downstream firm may not want to entrust the supply of all products to the same supplier. For example, even in straight purchase without co-creation, a downstream firm may be better off with multiple suppliers because a single supplier may extract much of the firm’s surplus. When the downstream firm co-creates with the upstream suppliers it forms deeper links with its suppliers, often at considerable costs, and the importance of formulating a proper sourcing strategy is even greater compared to a situation where it does straight purchase from the upstream suppliers. Since it is always an option for the downstream firm to purchase from the upstream suppliers, we will also analyze single and multi-sourcing in the absence of co-creation, and this will serve as a benchmark. In this paper, we investigate whether the incentives to multi-source are weaker or stronger when the downstream firm co-creates with the upstream suppliers, in comparison with a situation where the downstream firm merely purchases from the upstream suppliers. The incorporation of multiple potential suppliers and the possibility of co-creation or straight purchase, with endogenous investment and pricing by both upstream suppliers and downstream firm, gives us a very rich strategy space to investigate the optimal sourcing strategy of the downstream firm.

To summarize, our research questions are: (1) What is the optimal sourcing strategy for a multi-product downstream firm that can choose to (a) either co-create with or purchase the prod-
ucts from its supplier(s), and (b) do so either from a single or multiple suppliers? (2) What are the optimal co-creation investments of the downstream firm and its upstream supplier(s), and how do they depend on whether the downstream firm does single or multi-sourcing? (3) If the downstream firm multi-sources the co-created products, will it prefer to establish a collaborative environment among its upstream suppliers? (4) Are the incentives to multi-source co-created products weaker or stronger compared to the downstream firm’s incentives to multi-source products simply purchased from the upstream market? Said differently, how does co-creation change the incentives to multi-source?

We analyze a model where a monopolistic downstream firm sells two horizontally differentiated products, for example a car manufacturer that sells both a mid-size sedan and a SUV. The downstream firm can choose to co-create its products with two different suppliers or with the same one. We conceptualize co-creation as the firms incurring bilateral or trilateral investments depending on whether the downstream firm co-creates with one or two upstream suppliers. Importantly, consistent with the co-creation literature which deals with collaboration between a firm and its customers, we too define co-creation as investments by firms at different hierarchical levels. Moreover, the co-creation in this paper is aimed at cost-reducing innovation (Levin and Reiss 1988; Sen and Tauman 2007). In this setup we endogenize the investment levels of the upstream supplier(s), the downstream firm, pricing by the suppliers and pricing by the downstream firm. Finally, since straight purchase is always an available strategy, we also analyze the incentives of the downstream firm to purchase both products from a single supplier or from multiple suppliers. In the case of straight purchase, the upstream suppliers can still invest in cost reduction technologies, but they do not benefit from any investment by the downstream firm. This provides a useful benchmark and allows us to compare the downstream firm’s sourcing strategies with and without co-creation. We now foreshadow our results.

We have four main results. First, we show that counter to intuition the downstream firm may be worse off when the upstream suppliers collaborate. We demonstrate this result for a very general cost function where investments at the upstream and downstream levels jointly decrease the marginal cost of production. The downstream firm is worse off when the cross-effect between the downstream and upstream investments (the second-order cross-partial derivative) is small. Intuitively, in a collaborative environment each supplier has an incentive to free-ride on the investment of the other, thus decreasing the ‘effective’ investment at the upstream level. Now when the cross-effect between the downstream and upstream investments is large, the downstream firm is motivated to decrease its investment because the upstream suppliers will pick up the slack through their own increased investment. The negative free-riding effect is muted. For low levels of the cross-effect, the negative free-riding effect dominates, and this adversely affects the downstream firm’s profit. It is noteworthy that if the cost function is additively separable, the cross-effect is zero. Thus, in the case of an additively separable cost function the downstream firm is always worse off when the upstream suppliers collaborate.

Second and interestingly, we find that when the downstream firm sources both products from a single supplier, it completely destroys the firm’s incentives to invest. With single-sourcing of co-created products, the downstream firm’s investment is zero. This result is related to the ‘holdup problem’ which arises when agents first make relationship-specific investments and then the prices are set (Grout 1984 and Tirole 1986). We demonstrate the holdup problem in the context of co-created products, and thus we contribute to the literature on holdup. Moreover, we
show that when the downstream firm multi-sources (co-creates its products with two suppliers), it chooses a positive level of investment. Thus, seen from the point of view of the downstream firm, multi-sourcing of co-created products softens the holdup problem. The reason why multi-sourcing leads to a higher level of investment by the downstream firm is that the presence of multiple suppliers implies that competition among them lowers the wholesale prices, leaving some surplus for the downstream firm. This provides incentives for the downstream firm to invest, whereas under single-sourcing the single upstream supplier extracts all the surplus from the downstream firm and the latter has no incentives to invest.

Third, we show that the downstream firm’s incentives to multi-source (versus single-source) are higher for co-created products than for non-co-created products. One might conjecture that, since the downstream firm internalizes some of the production when co-creating with the suppliers, it is not as dependent on them. Therefore, the downstream firm may face a smaller threat of procurement risk when dealing with a single source versus multiple sources in the presence of co-creation, as compared with a situation of straight purchase. On the contrary we find that the downstream firm has a stronger preference for multi-sourcing with co-created products than with non-co-created ones.

Fourth, we find that for any additively separable cost function, the optimal sourcing strategy for the downstream firm is multi-source co-creation without collaboration among the upstream suppliers. We have considered a very rich space for the firm’s sourcing strategy, which includes, (a) co-creation or straight purchase, (b) single-sourcing or multi-sourcing, and (c) collaboration or no collaboration among suppliers in the case of multi-source co-creation. Intuitively, co-creation allows the marginal cost of production to be very low because of cost-reducing investments by both upstream suppliers and the downstream firm, and this dominates straight purchase. Moreover, by not having the upstream suppliers collaborate the downstream firm also avoids the negative free-riding effect, and finally, by multi-sourcing the co-created products it mitigates the holdup problem, which would be severe with single-sourcing.

2 Model Setup

Consider a downstream firm that sells two horizontally differentiated products located at the opposite ends of the unit interval that represents the product space. Product 1 is located at 0 and product 2 is located at 1. The demand side consists of infinitely many consumers of total mass one. Each consumer has a most preferred product $x \in [0, 1]$, and hence derives the utility of $v - tx$ from consuming product 1 and $v - t(1 - x)$ from consuming product 2. Consumers as represented by $x$ are uniformly distributed over the unit interval.

There are many potential upstream suppliers. We analyze how the downstream firm co-creates the two products with them. The firm can choose to co-create with only one supplier (single-sourcing) or with two suppliers (multi-sourcing). The focus on co-creation with endogenous investments at both upstream and downstream levels distinguishes our work from the extant outsourcing literature.

One of our major contributions is that we investigate how co-creation changes the incentives of the downstream firm to either single or multi-source. Therefore, for purposes of comparison, we also analyze cases where there is outright purchase from upstream suppliers without any
co-creation. Another major contribution is to model whether the downstream firm will prefer to establish an environment for its suppliers to collaborate when it co-creates with both of them. We assume that when the downstream firm merely purchases the products from two suppliers, or when it co-creates one product from one supplier, but purchases the other product from another supplier, it will not establish a collaborative environment. This is consistent with the idea that collaboration between two suppliers is more important (and easier to implement) when both are working closely with the downstream firm. Hence, the possible strategies of the downstream firm are:

(A) Multi-Source Co-Creation: co-create product 1 with one supplier and product 2 with another supplier, possibly establishing an environment for the two suppliers to collaborate;

(B) Multi-Source Co-Creation and Purchase: co-create product 1 with one supplier and purchase product 2 from another supplier;

(C) Single-Source Co-Creation: co-create both products with one supplier;

(D) Multi-Source Purchase: purchase product 1 from one supplier and product 2 from another supplier;

(E) Single-Source Purchase: purchase both products from one supplier.

In this paper we study the situation where co-creation is aimed at enhancing the production efficiency. In such cost-reducing type of R&D the co-creating partners invest resources to reduce the production cost incurred by suppliers. The economics literature on R&D and innovation has dealt extensively with cost-reducing R&D, either by a firm alone or by a network of collaborating firms (Levin and Reiss 1988; Goyal and Moraga-Gonzalez 2001). Following these authors, we posit that when there is co-creation between the downstream firm and an upstream supplier, the R&D investment of the downstream firm, $e$, and the supplier’s own R&D investment, $I$, result in the supplier’s unit variable cost of $c(e, I)$. The cost function $c(e, I)$ has the following characteristics that are standard in the literature: $\partial c(e, I)/\partial e < 0$, $\partial c(e, I)/\partial I < 0$, $\partial^2 c(e, I)/\partial e^2 > 0$, $\partial^2 c(e, I)/\partial I^2 > 0$, and $\partial^2 c(e, I)/\partial e \partial I \geq 0$. These characteristics simply mean that co-creation investments lower the production costs, but at a decreasing rate. Such cost-reducing, rather than value-enhancing, co-creation is consistent with our assumption of horizontally differentiated products.

In the next five sections we analyze strategies A through E. The question of which strategy is the most profitable for the downstream firm gets answered in Section 8. The proofs of all lemmas and propositions are relegated to the Appendix.

3 Strategy A: Multi-Source Co-Creation

Under strategy A the downstream firm (F) co-creates product 1 with one supplier (S1) and product 2 with another supplier (S2). As already mentioned, in this situation, the downstream

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1This conceptualization of the results of joint R&D is identical to Levin and Reiss (1988), wherein the investment $x$ by a firm and $X$ by its linked collaborator result in a unit variable cost of $c(x, X)$ for the firm.
firm can choose whether to have its suppliers collaborate or not. We assume that this is a
dichotomous choice. Of course, we allow the suppliers to either accept or reject the downstream
firm’s exhortation to collaborate depending on their own incentives. In other words, there is
no forced collaboration, even though in some practical situations the downstream firm makes
forced collaboration a pre-condition (Wiener and Saunders 2014). Below is the timeline.

1. F decides whether to establish, at a fixed cost $\Gamma$, an environment in which S1 and S2 are
given an opportunity to collaborate.

2. If the collaborative environment is in place, S1 and S2 simultaneously choose whether to
collaborate or not. Only if both suppliers choose to collaborate, there will be collaboration
between them.

3. Simultaneously and independently, S1 and S2 choose their investment levels $I_1$ and $I_2$,
and F chooses its investment level $e$. In the absence of collaboration between suppliers
the costs of producing product 1 and product 2 are $c(e, I_1)$ and $c(e, I_2)$ respectively. When
there is collaboration between S1 and S2, the costs are $c(e, I_1 + \gamma I_2)$ and $c(e, I_2 + \gamma I_1)$,
respectively, where $\gamma \in (0, 1)$ represents the intensity of collaboration between two dif-
ferent manufacturers. We will refer to this stage as the product co-creation stage.

4. S1 and S2 simultaneously choose their wholesale prices $p_{1w}$ and $p_{2w}$. We will refer to this
stage as the wholesale pricing stage.

5. F chooses its retail prices $p_1$ and $p_2$. We will refer to this stage as the retail pricing stage.

6. Consumers make their purchasing decisions.

The first two stages capture the idea that establishing an overall collaborative environment
is a longer-term and more irreversible decision than specific investments and pricing.

3.1 Wholesale and Retail Pricing Stages

We start our analysis with the retail pricing stage, in which F, given $p_{1w}$ and $p_{2w}$, chooses $p_1$
and $p_2$. The results are summarized in Lemma 9 (in the Appendix). Proceeding with backward
induction, we analyze the wholesale pricing stage, in which S1 and S2, given $c_1$ and $c_2$, simulta-
neously choose $p_{1w}$ and $p_{2w}$. We record the obtained Nash equilibrium prices in Lemma 10 (also
relegated to the Appendix).

We use Lemma 9 and Lemma 10 to derive the intermediate (after the co-creation investments
have been made) profits of S1, S2, and F as functions of $c_1$ and $c_2$.

**Lemma 1** (Intermediate Profits under a Multi-Source Strategy). Suppose F sources the two
products from different suppliers. Then the intermediate profits of S1, S2, and F as functions of
$c_1$ and $c_2$ are

$$\pi_{ms}^{1}(c_1, c_2) = \frac{1}{48t}(8t - c_1 + c_2)(6t - 2c_1 + c_2),$$

---

We assume that $I_1$, $I_2$ and $e$ are observable but not verifiable, so cannot be specified in a contract.

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9
\[ \pi_{2}^{ms}(c_1, c_2) = \frac{1}{48t}(8t - c_2 + c_1)(6t - 2c_2 + c_1), \]

and

\[ \pi_{F}^{ms}(c_1, c_2) = v - 2t - \frac{3(c_1 + c_2)}{8} + \frac{(c_1 - c_2)^2}{128t}, \]

respectively.

The superscript ‘ms’ stands for ‘multi-source.’ Note that this lemma not only applies to strategy A, but also to the other two multi-source strategies, strategies B and D.

### 3.2 Product Co-Creation Stage

We now analyze the product co-creation stage, in which S1, S2, and F simultaneously choose co-creation investments \( I_1, I_2, \) and \( e \). Suppose there is collaboration between S1 and S2. S1, therefore, solves

\[
\max_{I_1} \pi_1^{ms}(c(e, I_1 + \gamma I_2), c(e, I_2 + \gamma I_1)) - I_1,
\]

S2 solves

\[
\max_{I_2} \pi_2^{ms}(c(e, I_1 + \gamma I_2), c(e, I_2 + \gamma I_1)) - I_2,
\]

and F solves

\[
\max_{e} \pi_{F}^{ms}(c(e, I_1 + \gamma I_2), c(e, I_2 + \gamma I_1)) - e.
\]

We take the first-order conditions to find the equilibrium values of \( I_1, I_2 \) and \( e \). The results are summarized in Lemma 3. The superscript ‘C’ stands for ‘collaboration.’

**Lemma 2** (Investments under Strategy A: Collaboration). Under strategy A, when S1 and S2 collaborate, they choose the investment levels \( I_1 = I_2 = I^C \), while F chooses the investment level \( e^C \). \( I^C \) and \( e^C \) are implicitly defined by

\[
\begin{cases}
-\frac{3}{4} \frac{\partial c(e^C, I^C + \gamma I^C)}{\partial e} = 1 \\
-\frac{1}{48t} (22t - 14\gamma t - (1 - \gamma)c(e^C, I^C + \gamma I^C)) \frac{\partial c(e^C, I^C + \gamma I^C)}{\partial I} = 1
\end{cases}
\]

(1)

Let \( \tilde{I}^C \equiv I^C + \gamma I^C \) denote the ‘effective’ investment of the upstream suppliers. We have:

\[
\begin{cases}
-\frac{3}{4} \frac{\partial c(e^C, \tilde{I}^C)}{\partial e} = 1 \\
-\frac{1}{48t} (22t - 14\gamma t - (1 - \gamma)c(e^C, \tilde{I}^C)) \frac{\partial c(e^C, \tilde{I}^C)}{\partial I} = 1
\end{cases}
\]

In Proposition 1 we examine the effect of collaboration parameter \( \gamma \) on co-creation investments \( \tilde{I}^C \) and \( e^C \). Interestingly, we find that higher levels of collaboration (higher \( \gamma \)) may be associated with lower levels of the effective investment \( \tilde{I}^C \).
Proposition 1 (Comparative Statics w.r.t. Collaboration Parameter $\gamma$). As $\gamma$ increases, $\tilde{I}^C$ decreases and $e^C$ increases if 

\[
\begin{align*}
\left(\gamma \frac{\partial c}{\partial I}\right)^2 + (22t - 14\gamma t - (1 - \gamma)c) \frac{\partial^2 c}{\partial e^2} \frac{\partial^2 c}{\partial e^2} & \geq \left(\gamma \frac{\partial c}{\partial e} \frac{\partial c}{\partial I} + (22t - 14\gamma t - (1 - \gamma)c) \frac{\partial^2 c}{\partial e \partial I}\right) \frac{\partial^2 c}{\partial e \partial I}.
\end{align*}
\]

(2)

$\tilde{I}^C$ increases and $e^C$ decreases if otherwise.

Observe that the inequality (2) is more likely to hold when the second-order cross partial derivative $\frac{\partial^2 c}{\partial e \partial I}$ is small relative to the second-order partial derivatives $\frac{\partial^2 c}{\partial e^2}$ and $\frac{\partial^2 c}{\partial I^2}$. There are three forces at work. First, there is a positive direct effect of $\gamma$ on $\tilde{I}^C$, as $\tilde{I}^C = I^C + \gamma I^C$. Second, there is a negative free-riding effect of $\gamma$ on $I^C$, and hence on $\tilde{I}^C$. Because there is collaboration among the suppliers, each supplier has an incentive to free-ride on the other, and this incentive to free-ride is stronger for higher levels of collaboration.

The suppliers operate in a competitive environment: any investment by a given supplier not only reduces its own marginal cost, but also the marginal cost of its rival, more so for higher values of $\gamma$. Again, each supplier is motivated to reduce its investment as $\gamma$ increases. Third, there are externalities among the suppliers and the downstream firm, owing to the assumption $\frac{\partial^2 c}{\partial e \partial I} > 0$. Positivity of the cross-partial derivative implies that the marginal effect of the cost-reducing investment by the upstream suppliers are higher the lower is $e^C$. Therefore, lower (higher) values of $e^C$ strengthen (weaken) the upstream suppliers’ incentives to invest.

When the externality effect (the cross-partial derivative $\frac{\partial^2 c}{\partial e \partial I}$) is large, the downstream firm is motivated to decrease its investment because the upstream suppliers will pick up the slack through their own increased investment. The externality effect then softens the negative free-riding effect. For large enough externality effect, this increase in investment by the upstream suppliers may reduce the negative free-riding effect enough so that the positive direct effect dominates (the second part of Proposition 1).

However, when the externality effect is small, it is not enough to reduce the free-riding effect at the upstream level sufficiently, and the downstream firm has to pick up the slack. The net result is that when $\gamma$ increases, the upstream suppliers reduce their investments because of the dominant free-riding effect (the first part of Proposition 1).

Note that if the cost function is additively separable, $c(e, I) = f(e) + g(I)$, the cross-partial derivative is zero. Therefore, the effective investment of the upstream suppliers decreases as $\gamma$ increases, which is undesirable for the downstream firm. This foreshadows the fact that for additively separable costs the downstream firm may not want to establish a collaborative environment among the upstream suppliers. We address this case in Section 3.3 and then later in Section 8.

Finally, we calculate $S_1$, $S_2$, and $F$’s equilibrium profits:

\[
\begin{align*}
\Pi_1^C & = \Pi_2^C = \pi^m_1(c(e^C, \tilde{I}^C), c(e^C, \tilde{I}^C)) - I^C = t - \frac{c(e^C, \tilde{I}^C)}{6} - I^C, \\
\Pi_F^C & = \pi^m_F(c(e^C, \tilde{I}^C), c(e^C, \tilde{I}^C)) - e^C - \Gamma = v - 2t - \frac{3c(e^C, \tilde{I}^C)}{4} - e^C - \Gamma.
\end{align*}
\]

(3)
Now suppose there is no collaboration between the suppliers. Setting $\gamma$ to zero in Lemma 2 yields:

**Lemma 3** (Investments under Strategy A: No Collaboration). Under strategy A, when $S1$ and $S2$ do not collaborate, they choose the investment levels $I_1 = I_2 = I^NC$, while $F$ chooses the investment level $e^{NC}$. $I^NC$ and $e^{NC}$ are implicitly defined by

$$ \begin{cases} 
-\frac{3}{4} \frac{\partial c(e^{NC}, I^{NC})}{\partial e} = 1 \\
-\frac{1}{48t} \frac{22t - c(e^{NC}, I^{NC})}{\partial I} = 1 
\end{cases} $$

The superscript ‘NC’ stands for ‘no collaboration.’ $S1$, $S2$, and $F$’s equilibrium profits are:

$$ \Pi_1^{NC} = \Pi_2^{NC} = \pi_{ms}^{NC}(c(e^{NC}, I^{NC}), c(e^{NC}, I^{NC})) - 6t - \frac{c(e^{NC}, I^{NC})}{6} - I^{NC}. $$

$$ \Pi_F^{NC} = \pi_F^{ms}(c(e^{NC}, I^{NC}), c(e^{NC}, I^{NC})) - e^{NC} = v - 2t - \frac{3c(e^{NC}, I^{NC})}{4} - e^{NC}. \quad (4) $$

### 3.3 Should the Downstream Firm Establish a Collaborative Environment?

Once offered an opportunity to collaborate, the two suppliers will collaborate if and only if $\Pi_1^C = \Pi_2^C \geq \Pi_1^{NC} = \Pi_2^{NC}$, or

$$ t - \frac{c(e^C, \tilde{I}^C)}{6} - I^C \geq t - \frac{c(e^{NC}, I^{NC})}{6} - I^{NC}, $$

$$ \frac{c(e^C, \tilde{I}^C)}{6} + I^C \leq \frac{c(e^{NC}, I^{NC})}{6} + I^{NC}. \quad (5) $$

Thus, the downstream firm should establish a collaborative environment if (5) holds and at the same time $\Pi_F^C \geq \Pi_F^{NC}$. The latter can be simplified to

$$ \frac{3c(e^C, \tilde{I}^C)}{4} + e^C + \Gamma \leq \frac{3c(e^{NC}, I^{NC})}{4} + e^{NC}. $$

For the special case of an additively separable cost function, we can make a sharp prediction about the downstream firm’s incentives to establish a collaborative environment for its suppliers. Suppose

$$ c(e, I) = f(e) + g(I), $$

with $f'(e) < 0$, $g'(e) < 0$, $f''(e) > 0$, and $g''(e) > 0$. Applying the results of Lemma 2 and Lemma 3 to $c(e, I) = f(e) + g(I)$ yields

$$ \begin{cases} 
-\frac{3}{4} f'(e^C) = 1 \\
-\frac{1}{48t} \frac{(22t - 14\gamma t - (1 - \gamma)(f(e^C) + g(\tilde{I}^C))g'(\tilde{I}^C))}{\partial I} = 1 
\end{cases} $$
and
\[
\begin{align*}
-\frac{3}{4} f'(e^{NC}) &= 1 \\
-\frac{41}{48} (22t - f(e^{NC}) - g(I^{NC}))g'(I^{NC}) &= 1
\end{align*}
\]

It immediately follows that \( e^C = e^{NC} \). Since the inequality (2) holds for an additively separable cost function, \( \bar{I}^C \) decreases in \( \gamma \) (Proposition 1), implying \( \bar{I}^C < I^{NC} \). Hence, \( c(e^C, \bar{I}^C) > c(e^{NC}, I^{NC}) \). Comparing

\[\Pi_F^C = v - 2t - \frac{3c(e^C, \bar{I}^C)}{4} - e^C - \Gamma \]

with

\[\Pi_{NC}^F = v - 2t - \frac{3c(e^{NC}, I^{NC})}{4} - e^{NC},\]

we see that the latter is higher. Thus, we conclude:

**Proposition 2** (No Collaboration among Suppliers for Additively Separable Costs).

*In the case of an additively separable cost function, the downstream firm obtains a higher profit under strategy A without collaboration than under strategy A with collaboration, even if the fixed cost of establishing a collaborative environment \( \Gamma \) is zero.*

Note that the result is very general and we have not assumed any specific functional form for the cost function to derive it. An additively separable cost function has zero cross-partial derivative: \( \partial^2 c/(\partial e \partial I) = 0 \), meaning there are no cross-effects between the downstream firm’s and the suppliers’ investments in co-creation. As shown in Proposition 1, for such cost functions the suppliers’ effective investment \( \bar{I}^C \) decreases as the intensity of collaboration \( \gamma \) increases, while the downstream firm’s investment remains the same (the externality effect is ‘turned off’). If so, the downstream firm is better off in a non-collaborative environment. The main point of Proposition 2 is to show that this intuition is valid even when all prices are endogenized. That is, strategy A without collaboration dominates the one with collaboration.

Later in Section 8 we analyze the entire game where the downstream firm chooses its optimal sourcing strategy from strategies A through E (given in Section 2). We show that additive separability does not conflict with the conditions under which the downstream firm prefers strategy A to all the others.

### 4 Strategy B: Multi-Source Co-Creation and Purchase

Under strategy B the downstream firm co-creates product 1 with one supplier (S1) and purchases product 2 from another supplier (S2). Below is the timeline.

1. Simultaneously and independently, S1 and S2 choose their investment levels \( I_1 \) and \( I_2 \), and F chooses its investment level \( e \). The costs of producing product 1 and product 2 are \( c(e, I_1) \) and \( c(0, I_2) \), respectively. Note that S2 does not benefit from the investment by F because it does not co-create with F.
2. S1 and S2 simultaneously choose their wholesale prices $p_1^w$ and $p_2^w$.

3. F chooses its retail prices $p_1$ and $p_2$.

Again, we start our analysis with the retail pricing stage (Lemma 9), then analyze the whole-
sale pricing stage (Lemma 10) and obtain the intermediate profits $\pi_{ms}^1(c_1, c_2)$, $\pi_{ms}^2(c_1, c_2)$ and $\pi_F^{ms}(c_1, c_2)$ (Lemma 1). We take one step back to analyze the first stage in which S1, S2, and F simultaneously choose $I_1, I_2$ and $e$. S1 solves

$$\max_{I_1} \pi_{ms}^1(e(I_1), c(0, I_2)) - I_1,$$

S2 solves

$$\max_{I_2} \pi_{ms}^2(e(I_1), c(0, I_2)) - I_2,$$

and F solves

$$\max_e \pi_F^{ms}(e(I_1), c(0, I_2)) - e.$$

The results are summarized in Lemma 4.

**Lemma 4** (Investments under Strategy B). **Under strategy B, S1 and S2 choose the investment levels** $I_1^\dagger$ and $I_2^\dagger$, while F chooses the investment level $e^\dagger$. $I_1^\dagger$, $I_2^\dagger$ and $e^\dagger$ are implicitly defined by

$$\left\{ \begin{array}{c}
- \left( \frac{3}{8} + \frac{c(0, I_2^\dagger) - c(e^\dagger, I_1^\dagger)}{64t} \right) \frac{\partial c(e^\dagger, I_1^\dagger)}{\partial e} = 1 \\
- \frac{1}{48t} \left( 22t - 4c(e^\dagger, I_1^\dagger) + 3c(0, I_2^\dagger) \right) \frac{\partial c(e^\dagger, I_1^\dagger)}{\partial I} = 1 \\
- \frac{1}{48t} \left( 22t - 4c(0, I_1^\dagger) + 3c(e^\dagger, I_1^\dagger) \right) \frac{\partial c(0, I_2^\dagger)}{\partial I} = 1
\end{array} \right.$$

F’s equilibrium profit is

$$\Pi_F^\dagger = \pi_F^{ms}(c(e^\dagger, I_1^\dagger), c(0, I_2^\dagger)) - e^\dagger = v - 2t - \frac{3c(e^\dagger, I_1^\dagger) + c(0, I_2^\dagger)}{8} + \frac{(c(0, I_2^\dagger) - c(e^\dagger, I_1^\dagger))^2}{128t} - e^\dagger.$$

The asymmetric nature of strategy B is to the downstream firm’s advantage, as seen from the term $(c(0, I_2^\dagger) - c(e^\dagger, I_1^\dagger))^2/(128t)$ in (6).

### 5 Strategy C: Single-Source Co-Creation

Under strategy C the downstream firm co-creates both products with one supplier (S). Below is the timeline.

1. Simultaneously and independently, S chooses its investment levels $I_1$ and $I_2$, and F chooses its investment level $e$. The costs of producing product 1 and product 2 are $c(e, I_1 + \delta I_2)$ and $c(e, I_2 + \delta I_1)$, respectively, where $\delta \in (0, 1)$ represents the knowledge spillover between two products produced by the same supplier. Further, we assume
that $\delta \geq \gamma$, which is consistent with the idea that knowledge spillover between two R&D processes is stronger when the products are developed by the same supplier than when they are developed by different but collaborating suppliers.

2. S chooses its wholesale prices $p_1^w$ and $p_2^w$.

3. F chooses its retail prices $p_1$ and $p_2$.

If $p_1^w$ and $p_2^w$ are too high, then F’s optimal choice of $p_1$ and $p_2$ will result in some consumers not buying at all. It is easy to show that in equilibrium S chooses $p_1^w$ and $p_2^w$ just low enough to avoid such an outcome. The detailed analysis of the retail and wholesale pricing stages under strategy C – a single-source strategy – appears in the Appendix as part of the proof of Lemma 5. The superscript ‘ss’ stands for ‘single-source.’

**Lemma 5** (Intermediate Profits under a Single-Source Strategy). Suppose F sources both products from the same supplier. Then the intermediate profits of S and F as functions of $c_1$ and $c_2$ are

$$
\pi_{SS}^S(c_1, c_2) = \frac{v}{2} - \frac{t}{2} - \frac{c_1 + c_2}{2} + \frac{(c_1 - c_2)^2}{16t}
$$

and

$$
\pi_{SS}^F(c_1, c_2) = \frac{t}{2} + \frac{(c_1 - c_2)^2}{32t},
$$

respectively.

It is left to determine the equilibrium levels of $I_1$, $I_2$ and $e$. F solves

$$
\max_e \pi_{SS}^F(c(e, I_1 + \delta I_2), c(e, I_2 + \delta I_1)) - e,
$$

or

$$
\max_e \frac{t}{2} + \frac{(c(e, I_1 + \delta I_2) - c(e, I_2 + \delta I_1))^2}{32t} - e.
$$

Note that the higher is the difference in the production costs, the higher is F’s profit. In the Appendix we show that the difference in the production costs in F’s profit function given above is decreasing in $e$. Therefore, the equilibrium level of investment by the downstream firm is zero.

**Proposition 3** (No Investment by the Downstream Firm under Single-Source Co-Creation). When the downstream firm co-creates both products with the same supplier, in equilibrium the firm will choose zero level of co-creation investment.

Proposition 3 is substantively important. It shows that the holdup problem, which refers to distortions in investment levels when relationship-specific investments are made prior to price setting, also exists in the world of co-created products. The reason for under-investment in the standard holdup problem is that an agent’s partner expropriates part of the return on its sunk investment, and anticipating this the agent lowers her investment below desirable levels. For the first formal proof of the under-investment hypothesis, please see Grout (1984) and Tirole (1986). As we have just seen, when the downstream firm co-creates with a single supplier it has
no incentives to invest. On the contrary, when F co-creates with both suppliers (strategies A and B) it chooses a positive level of investment. This is because the competition among the suppliers implies that they cannot extract all the downstream firm’s surplus, and therefore the downstream firm has incentives to invest in co-creation. Thus, with co-created products multi-sourcing is a means of softening the holdup problem for the downstream firm.

We will make a comparison of whether the downstream firm has stronger or weaker incentives to multi-source with co-created products, compared to non-co-created ones, after analyzing the benchmark cases of single and multi-sourcing with non-co-created products (strategies D and E, where we assume there is no investment by the downstream firm). The result stated in Proposition 3, that single-sourcing of co-created products completely destroys the downstream firm’s incentives to invest in co-creation, is an endogenous outcome of our analysis compared to the exogenously imposed lack of investment by the downstream firm in the case of non-co-created products.

To find S’s equilibrium investment levels, we solve

$$\max_{I_1, I_2} \pi^M \left( c(0, I_1 + \delta I_2), c(0, I_2 + \delta I_1) \right) - I_1 - I_2.$$

The results are summarized in the Lemma 6.

**Lemma 6** (Investments under Strategy C). Under strategy C, F chooses zero investment level, while S chooses the investment levels $I_1 = I_2 = I^*$, where $I^*$ is implicitly defined by

$$-\frac{1}{2}(1 + \delta) \frac{\partial c(0, I^* + \delta I^*)}{\partial I} = 1.$$

F’s equilibrium profit is

$$\Pi^*_F = \pi^M \left( c(0, I^* + \delta I^*), c(0, I^* + \delta I^*) \right) = \frac{t}{2}. \quad (7)$$

Clearly we can see that, for sufficiently high $v$, the downstream firm’s profit under strategy C is dominated by its profits under strategy A with or without collaboration (see (3) and (4)) and strategy B (6). Later in Section 8 we will compare the profits of the downstream firm under all strategies A through E.

### 6 Strategy D: Multi-Source Purchase

Under strategy D the downstream firm purchases product 1 from one supplier (S1) and product 2 from another supplier (S2). Below is the timeline.

1. S1 and S2 simultaneously choose their investment levels $I_1$ and $I_2$. The costs of producing product 1 and product 2 are $c(0, I_1)$ and $c(0, I_2)$, respectively.
2. S1 and S2 simultaneously choose their wholesale prices $p^w_1$ and $p^w_2$.
3. F chooses its retail prices $p_1$ and $p_2.$
Like strategies A and B, strategy D is a multi-source strategy; it leads to the same intermediate profit functions (Lemma 1). Proceeding with backward induction, we analyze the first stage in which S1 and S2 simultaneously choose $I_1$ and $I_2$. S1 solves
\[ \max_{I_1} \pi_{ms}^1(c(0, I_1), c(0, I_2)) - I_1 \]
and S2 solves
\[ \max_{I_2} \pi_{ms}^2(c(0, I_1), c(0, I_2)) - I_2. \]
We have:

**Lemma 7** (Investments under Strategy D). Under strategy D, S1 and S2 choose the investment levels $I_1 = I_2 = I^\dagger$, where $I^\dagger$ is implicitly defined by
\[ -\frac{1}{48t} (22t - c(0, I^\dagger)) \frac{\partial c(0, I^\dagger)}{\partial I} = 1. \]
F’s equilibrium profit is
\[ \Pi^\dagger_F = \pi_{ms}^F(c(0, I^\dagger), c(0, I^\dagger)) = v - 2t - \frac{3c(0, I^\dagger)}{4}. \] (8)

### 7 Strategy E: Single-Source Purchase

Under strategy E the downstream firm purchases both products from one supplier (S). Below is the timeline.

1. S chooses its investment levels $I_1$ and $I_2$. The costs of producing product 1 and product 2 are $c(0, I_1 + \delta I_2)$ and $c(0, I_2 + \delta I_1)$, respectively, with $\delta \in (\gamma, 1]$.

2. S chooses its wholesale prices $p_{1w}$ and $p_{2w}$.

3. F chooses its retail prices $p_1$ and $p_2$.

Because of the earlier result that under strategy C the downstream firm has no incentives to invest (Proposition 3), strategy E is payoff equivalent to strategy C. It is interesting to find that the downstream firm’s profit under single-source co-creation is the same as that under single-source purchase.

### 8 The Most Profitable Strategy for the Downstream Firm

In comparing strategies A through E, we adopt the viewpoint of the downstream firm. We gathered the firm’s profits (3), (4), (6), (7) and (8) in Table 1.
First note that for sufficiently high values of $v$ the single-source strategies C and E, with and without co-creation respectively, become inferior. In this section, we will consider the case of an additively separable cost function:
\[ c(e, I) = f(e) + g(I), \]
Table 1: Downstream Firm’s Profits under Strategies A–E

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Str. A w/ collaboration</td>
<td>$\Pi^C_F = v - 2t - \frac{3c(e^C, I^C + \gamma I^C)}{4} - e^C - \Gamma$, where $I^C$ and $e^C$ are defined in Lemma 2</td>
</tr>
<tr>
<td>Str. A w/o collaboration</td>
<td>$\Pi^{NC}_F = v - 2t - \frac{3c(e^{NC}, I^{NC})}{4} - e^{NC}$, where $I^{NC}$ and $e^{NC}$ are defined in Lemma 3</td>
</tr>
<tr>
<td>Strategy B</td>
<td>$\Pi^\dagger_F = v - 2t - \frac{3(c(e^\dagger, I^\dagger_1) + c(0, I^\dagger_2))}{8} + \frac{(c(0, I^\dagger_2) - c(e^\dagger, I^\dagger_1))^2}{128t} - e^\dagger$, where $I^\dagger_1$, $I^\dagger_2$, and $e^\dagger$ are defined in Lemma 4</td>
</tr>
<tr>
<td>Strategy D</td>
<td>$\Pi^\ddagger_F = v - 2t - \frac{3c(0, I^\ddagger)}{4}$, where $I^\ddagger$ is defined in Lemma 7</td>
</tr>
<tr>
<td>Strategies C and E</td>
<td>$\Pi^*_F = \frac{t}{2}$</td>
</tr>
</tbody>
</table>

with $f'(e) < 0$, $g'(e) < 0$, $f''(e) > 0$, and $g''(e) > 0$.

We already know from Proposition 2 that in this case strategy A without collaboration dominates the one with. How does strategy A without collaboration compare with the other two multi-source strategies, strategies B and D? To make sharper predictions of the most profitable strategy, in the following we assume a very commonly used cost function, keeping in mind that in our analysis, investments reduce the marginal costs of production

$$c(e, I) = \bar{c} - a\sqrt{e} - b\sqrt{I}.$$

We obtain the expressions for $e^{NC}$, $I^{NC}$ (investments under strategy A without collaboration), $e^\dagger$, $I^\dagger_1$, $I^\dagger_2$ (strategy B), and $I^\ddagger$ (strategy D). We collect all expressions in the following lemma.

Lemma 8. Suppose $c(e, I) = \bar{c} - a\sqrt{e} - b\sqrt{I}$. Then:

$$e^{NC} = \left(\frac{3a}{8}\right)^2,$$

$$I^{NC} = \left(\frac{22t - \bar{c} + a\sqrt{e^C}}{96t/b - b}\right)^2.$$
\[ e^† = \left( \frac{24t}{7ab/(96t/b - 7b) + 128t/a - a} \right)^2, \]
\[ I^†_1 = \frac{1}{4} \left( \frac{44t - 2\bar{c} + a\sqrt{e^†}}{96t/b - b} + \frac{7a\sqrt{e^†}}{96t/b - 7b} \right)^2, \]
\[ I^†_2 = \frac{1}{4} \left( \frac{44t - 2\bar{c} + a\sqrt{e^†}}{96t/b - b} - \frac{7a\sqrt{e^†}}{96t/b - 7b} \right)^2, \]
\[ I^‡ = \left( \frac{22t - \bar{c}}{96t/b - b} \right)^2. \]

Observe that \( I^{NC}_1 > I^†. \) It immediately follows that strategy A without collaboration dominates strategy D. Indeed, the downstream firm \( F \) can choose \( e = 0 \) under strategy A without collaboration and get a higher profit than under strategy D. Since \( e = e^{NC} \) is \( F \)'s best response to \( I_1 = I_2 = I^{NC} \), \( F \)'s equilibrium profit under strategy A without collaboration is even higher.

Next, consider the investments under strategy B. We see that \( I^†_1 > I^†_2 \), so \( c(e^†, I^†_1) < c(0, I^†_2) \). The asymmetry in the production costs is to the downstream firm’s advantage, as can be seen from the quadratic term in \( \pi_{ms}^F(c_1, c_2) \) (Lemma 1). The drawback of strategy B is that the downstream firm’s investment only lowers the cost of product 1, but not the cost of product 2. In our extensive numerical calculations we considered different values for parameters \( \bar{c}, a, b \) and \( t \) and found that strategy A without collaboration always yields a higher profit for the downstream firm than strategy B. Thus, we conclude:

**Proposition 4 (The Most Profitable Strategy).** In the case of an additively separable cost function of the form \( c(e, I) = \bar{c} - a\sqrt{e} - b\sqrt{I} \), the most profitable strategy for the downstream firm is strategy A without collaboration.

Note that because there are so many alternative sourcing strategies (strategies A through E) we have had to make many comparisons, yet we have assumed a specific functional form for the cost function only for the purposes of comparing strategy A with strategies B and D. For a very general additively separable cost function, we have shown that the downstream firm should not establish a collaborative environment under strategy A, and that strategy A dominates single-source strategies C and E. That the downstream firm should never employ strategies C and E, is true for any cost function.

How does co-creation affect the downstream firm’s incentives to multi-source (vs. single-source), compared to straight purchase? To answer this question, we compare the change in the downstream firm’s payoff when it switches from strategy C (single-source co-creation) to strategy A (multi-source co-creation) with that when it switches from strategy E (single-source purchase) to strategy D (multi-source purchase). Since

\[ \Pi_F^{NC} > \Pi_F^†, \]

we have

\[
\frac{\Pi_F^{NC}}{\Pi_F^C} - \frac{\Pi_F^*}{\Pi_F^D} > \frac{\Pi_F^*}{\Pi_F^C} - \frac{\Pi_F^*}{\Pi_F^E}.
\]
Proposition 5 (The Downstream Firm’s Incentives to Multi-Source). The downstream firm has stronger incentives to multi-source with co-created products than with non-co-created ones.

By incentives to multi-source we mean the increase in the downstream firm’s profit due to multi-sourcing compared to single-sourcing. The result in the above proposition seems counterintuitive at first. With non-co-created products the downstream firm often prefers multi-sourcing to single-sourcing since multi-sourcing reduces the dependence on a single supplier. One may think that since co-creation implies that the downstream firm internalizes some of the production, it is not as dependent on its suppliers. This would then imply that co-creation may reduce the motivation of the downstream firm to multi-source.

However, our analysis shows that the opposite is true: Co-creation increases the downstream firm’s incentives to multi-source. This is because in the world of co-created products multi-sourcing has two advantages over single-sourcing: lower input (wholesale) prices due to the competition at the upstream level and alleviation of the holdup problem (see the discussion after Proposition 3). In the world of non-co-created products, multi-sourcing has one advantage over single-sourcing, that of lower wholesale prices.

9 Conclusion

In this paper we investigate the phenomenon of sourcing co-created products. A multi-product downstream firm can source products from a single or from multiple suppliers. It can also choose whether to co-create the product with one or more suppliers or whether to just purchase the product from the upstream supplier market. Finally, if the downstream firm does source the co-created products from multiple suppliers, it could foster collaboration among the suppliers by, for example, playing an active role in suppliers’ associations. Thus, firms nowadays are confronted with a very complex decision when it comes to sourcing the products they sell to the end consumers. The literature on product sourcing has considered the issue of single-sourcing versus multi-sourcing, but only for non-co-created standard products. We contribute to this literature by investigating how a downstream firm should source products it co-creates with its suppliers. Another novel contribution is incorporating the decision of whether the downstream firm should establish a collaborative environment for its upstream suppliers. We add to the literature on product co-creation by considering how a firm should source such co-created products.

Co-creation of products has been steadily growing in practice, and in many B2B settings a large part of product and production related R&D is increasingly jointly carried out by upstream suppliers and downstream firms. For example, in the automobile industry OEMs work very closely with their suppliers to improve the design of cars and components thereof (Calzolari et. al. 2015). Parallel to the increase in co-creation as a business practice, especially in B2B markets, there is also an acceleration in the practice of outsourcing fueled by advances in procurement techniques. By some estimates, almost 55% of the earned revenue of a manufacturing firm is accounted for by purchased material (Burke, Carillo and Vakharia 2007). Thus, in studying a downstream firm’s sourcing strategies it is important to also understand how it would do sourcing when it co-creates with the upstream suppliers. Our research addresses this question.

In terms of managerial implications, we show that that the downstream firm may be worse off when the upstream suppliers collaborate, due to the fact that as the intensity of collaboration
increases the downstream firm’s investment could increase and the upstream suppliers’ investments could decrease. The only situation where the downstream firm would like to establish a collaborative environment for its upstream suppliers is when the cross-effect of its and its suppliers’ investments is very large. Thus, managers should very closely study how their and their supplier’s joint investments reduce costs before they decide whether or not to have their suppliers collaborate. This is a new insight from our analysis. We also show that single-sourcing of co-created products destroys the incentives to invest by the downstream firm, and indeed results in zero investment by it, so that such a strategy is dominated by multi-sourcing. The literature on holdup has shown that, in situations of straight purchase, investment levels are distorted when investments take place before price setting. We contribute to this literature by showing that this problem is pervasive, and even more intense, with co-created products as well. Importantly, we find that the downstream firm’s incentives to multi-source (versus single-source) are higher for co-created products compared to non-co-created products. One might speculate that because co-creation implies that the downstream firm internalizes some of the products, therefore with co-created products its incentives to multi-source would be less and the downstream firm would do well to have fewer but deeper relationships with upstream suppliers when they co-create. We show that such naive thinking ignores the endogenous investments which also change when the downstream firm single-sources or multi-sources.

We have considered co-creation as cost-reducing R&D jointly carried out by the downstream and upstream levels. Future work could consider demand-creating R&D and also competition at the downstream level.
References


Appendix

Lemma 9 (Retail Prices). Given $p_1^w$ and $p_2^w$, $F$ sets its retail prices to

$$p_1 = v - \frac{t}{2} + \frac{p_1^w - p_2^w}{4}$$

and

$$p_2 = v - \frac{t}{2} + \frac{p_2^w - p_1^w}{4}.$$  

Consumers with $x < \hat{x}$ purchase product 1, the rest purchase product 2, where

$$\hat{x} = \frac{1}{2} - \frac{p_1^w - p_2^w}{4t}.$$  

Proof. Suppose that $v$ is relatively high, so that in equilibrium all consumers buy (the market is fully covered). The ‘critical’ consumer is located at $\hat{x}$ that satisfies

$$v - t\hat{x} - p_1 = v - t(1 - \hat{x}) - p_2,$$

$$\hat{x} = \frac{t + p_2 - p_1}{2t}.$$  

Given $p_1^w$ and $p_2^w$, $F$ faces the following constrained optimization problem:

$$\max_{p_1, p_2} \hat{x}(p_1 - p_1^w) + (1 - \hat{x})(p_2 - p_2^w) = \frac{t + p_2 - p_1}{2t} (p_1 - p_1^w) + \frac{t + p_1 - p_2}{2t} (p_2 - p_2^w)$$

subject to

$$v - t\hat{x} - p_1 \geq 0.$$  

The constraint guarantees that the critical consumer buys the product; it holds with equality at the solution. Thus, we have

$$\max_{p_1, p_2} \frac{t + p_2 - p_1}{2t} (p_1 - p_1^w) + \frac{t + p_1 - p_2}{2t} (p_2 - p_2^w)$$

subject to

$$v - \frac{t + p_1 + p_2}{2} = 0.$$  

Substituting the constraint into the objective function yields

$$\max_{p_1} \frac{v - p_1}{t} (p_1 - p_1^w) + \frac{t - v + p_1}{t} (2v - t - p_1 - p_2^w),$$

F.O.C.  \( \frac{1}{t} (-p_1 + p_1^w + v - p_1 + 2v - t - p_1 - p_2^w - t + v - p_1) = 0, \)

or

$$-4p_1 + 4v - 2t + p_1^w - p_2^w = 0,$$

or

$$p_1 = v - \frac{t}{2} + \frac{p_1^w - p_2^w}{4}.$$  

25
hence

\[ p_2 = v - \frac{t}{2} + \frac{p_2^w - p_1^w}{4} \]

and

\[ \hat{x} = \frac{1}{2} - \frac{p_1^w - p_2^w}{4t} \].

\[ \square \]

Lemma 10 (Wholesale Prices). Given \( c_1 \) and \( c_2 \), S1 and S2 set their wholesale prices to

\[ p_1^w = \frac{6t + 2c_1 + c_2}{4} \]

and

\[ p_2^w = \frac{6t + 2c_2 + c_1}{4} \],

respectively.

Proof. S1 solves

\[ \max_{p_1^w} \hat{x}(p_1^w - c_1) = \left( \frac{1}{2} - \frac{p_1^w - p_2^w}{4t} \right) (p_1^w - c_1), \]

F.O.C. \[ \frac{1}{2} - \frac{p_1^w - p_2^w}{4t} - \frac{p_1^w - c_1}{4t} = 0. \]

Similarly, S2 solves

\[ \max_{p_2^w} (1 - \hat{x})p_2^w - c_2 = \left( \frac{1}{2} - \frac{p_2^w - p_1^w}{4t} \right) (p_2^w - c_2), \]

F.O.C. \[ \frac{1}{2} - \frac{p_2^w - p_1^w}{4t} - \frac{p_2^w - c_2}{4t} = 0. \]

We put the two first-order conditions into a system of equations and solve for \( p_1^w \) and \( p_2^w \):

\[ p_1^w = \frac{6t + 2c_1 + c_2}{4} \]

and

\[ p_2^w = \frac{6t + 2c_2 + c_1}{4} \].

\[ \square \]
Proof of Lemma 1

We use the results of Lemma 9 and Lemma 10 to obtain the intermediate profits of S1, S2, and F as functions of $c_1$ and $c_2$.

\[
\pi_{ms}^1(c_1, c_2) = \left( \frac{1}{2} - \frac{p_1^w - p_2^w}{4t} \right) (p_1^w - c_1) \\
= \left( \frac{1}{2} - \frac{c_1 - c_2}{16t} \right) \frac{6t - 2c_1 + c_2}{4} \\
= \frac{1}{48t} (8t - c_1 + c_2)(6t - 2c_1 + c_2).
\]

Similarly,

\[
\pi_{ms}^2(c_1, c_2) = \frac{1}{48t} (8t - c_2 + c_1)(6t - 2c_2 + c_1).
\]

Finally,

\[
\pi_{ms}^F(c_1, c_2) = \hat{x}(p_1^w - p_2^w) + (1 - \hat{x})(p_2^w - p_1^w) \\
= \left( \frac{1}{2} - \frac{p_1^w - p_2^w}{4t} \right) \left( \frac{v - t}{2} + \frac{p_1^w - p_2^w}{4} - p_1^w \right) \\
+ \left( \frac{1}{2} + \frac{p_1^w - p_2^w}{4t} \right) \left( \frac{v - t}{2} - \frac{p_1^w - p_2^w}{4} - p_2^w \right) \\
= v - \frac{t}{2} - \frac{p_1^w + p_2^w}{2} + \frac{(p_1^w - p_2^w)^2}{8t} = v - 2t - \frac{3(c_1 + c_2)}{8} + \frac{(c_1 - c_2)^2}{128t}.
\]

Proof of Lemma 2

S1 solves

\[
\max_{I_1} \frac{1}{48t} (8t - c(e, I_1 + \gamma I_2) + c(e, I_2 + \gamma I_1))(6t - 2c(e, I_1 + \gamma I_2) + c(e, I_2 + \gamma I_1)) - I_1,
\]

F.O.C. \[
\frac{1}{48t} \left( -\frac{\partial c(e, I_1 + \gamma I_2)}{\partial I} + \gamma \frac{\partial c(e, I_2 + \gamma I_1)}{\partial I} \right) (6t - 2c(e, I_1 + \gamma I_2) + c(e, I_2 + \gamma I_1)) \\
+ (8t - c(e, I_1 + \gamma I_2) + c(e, I_2 + \gamma I_1)) \left( -2 \frac{\partial c(e, I_1 + \gamma I_2)}{\partial I} + \gamma \frac{\partial c(e, I_2 + \gamma I_1)}{\partial I} \right) = 1.
\]

S2 solves

\[
\max_{I_2} \frac{1}{48t} (8t - c(e, I_2 + \gamma I_1) + c(e, I_1 + \gamma I_2))(6t - 2c(e, I_2 + \gamma I_1) + c(e, I_1 + \gamma I_2)) - I_2,
\]

F.O.C. \[
\frac{1}{48t} \left( -\frac{\partial c(e, I_2 + \gamma I_1)}{\partial I} + \gamma \frac{\partial c(e, I_1 + \gamma I_2)}{\partial I} \right) (6t - 2c(e, I_2 + \gamma I_1) + c(e, I_1 + \gamma I_2)) \\
+ (8t - c(e, I_2 + \gamma I_1) + c(e, I_1 + \gamma I_2)) \left( -2 \frac{\partial c(e, I_2 + \gamma I_1)}{\partial I} + \gamma \frac{\partial c(e, I_1 + \gamma I_2)}{\partial I} \right) = 1.
\]
F solves
\[
\max_e v - 2t - \frac{3(c(e, I_1 + \gamma I_2) + c(e, I_2 + \gamma I_1))}{8} + \frac{(c(e, I_1 + \gamma I_2) - c(e, I_2 + \gamma I_1))^2}{128t} - e,
\]

F.O.C.
\[
\begin{align*}
- \frac{3}{8} & \left( \frac{\partial c(e, I_1 + \gamma I_2)}{\partial e} + \frac{\partial c(e, I_2 + \gamma I_1)}{\partial e} \right) \\
+ & \frac{(c(e, I_1 + \gamma I_2) - c(e, I_2 + \gamma I_1))}{64t} \left( \frac{\partial c(e, I_1 + \gamma I_2)}{\partial e} - \frac{\partial c(e, I_2 + \gamma I_1)}{\partial e} \right) = 1.
\end{align*}
\]

We put the three first-order conditions into a system of equations to solve for the equilibrium values of $I_1$, $I_2$ and $e$. Obviously, in equilibrium M1 and M2 will choose the same level of investment; we denote it by $I^C$. Let $e^C$ denote the equilibrium level of F’s investment. $I^C$ and $e^C$ must satisfy
\[
\begin{cases}
- \frac{3}{4} \frac{\partial (e^C, I^C + \gamma I^C)}{\partial e} = 1 \\
- \frac{1}{48t} \left( (22t - 14\gamma t) - (1 - \gamma)(c(e^C, I^C + \gamma I^C)) \right) \frac{\partial (e^C, I^C + \gamma I^C)}{\partial I} = 1
\end{cases}
\]

Proof of Proposition 1

It follows from Lemma 3 that $\tilde{I}^C$ and $e^C$ are implicitly defined by
\[
\begin{cases}
- \frac{3}{4} \frac{\partial (e^C, \tilde{I}^C)}{\partial e} = 1 \\
- \frac{1}{48t} \left( (22t - 14\gamma t) - (1 - \gamma)(c(e^C, \tilde{I}^C)) \right) \frac{\partial (e^C, \tilde{I}^C)}{\partial I} = 1
\end{cases}
\]

Taking the total derivative with respect to $\gamma$ yields
\[
\begin{cases}
- \frac{3}{4} \left( \frac{\partial^2 c}{\partial e^2} \frac{de^C}{d\gamma} + \frac{\partial^2 c}{\partial e \partial I} \frac{d\tilde{I}^C}{d\gamma} \right) = 0 \\
- \frac{1}{48t} \left( -14t + c + \gamma \left( \frac{\partial c}{\partial e} \frac{de^C}{d\gamma} + \frac{\partial c}{\partial I} \frac{d\tilde{I}^C}{d\gamma} \right) \right) \frac{\partial c}{\partial I} \\
+ \left( (22t - 14\gamma t) - (1 - \gamma)(c(e^C, \tilde{I}^C)) \right) \left( \frac{\partial^2 c}{\partial e \partial I} \frac{de^C}{d\gamma} + \frac{\partial^2 c}{\partial I^2} \frac{d\tilde{I}^C}{d\gamma} \right) \right) = 0
\end{cases}
\]

This is a linear system of equations with respect to $\frac{de^C}{d\gamma}$ and $\frac{d\tilde{I}^C}{d\gamma}$:
Recall that $\frac{\partial c}{\partial e} < 0$, $\frac{\partial c}{\partial I} < 0$, $\frac{\partial^{2} c}{\partial e^{2}} > 0$, $\frac{\partial^{2} c}{\partial I^{2}} > 0$, and $\frac{\partial^{2} c}{\partial e \partial I} > 0$. We can rewrite our system of equations as

$$
\begin{align*}
C_1 \frac{deC}{d\gamma} + C_2 \frac{dIC}{d\gamma} &= 0 \\
C_3 \frac{deC}{d\gamma} + C_4 \frac{dIC}{d\gamma} &= -C_5
\end{align*}
$$

where

$$
C_1 = \frac{\partial^2 c}{\partial e^2}, \\
C_2 = \frac{\partial^2 c}{\partial e \partial I}, \\
C_3 = \gamma \frac{\partial c}{\partial e} \frac{\partial c}{\partial I} + (22t - 14\gamma t - (1 - \gamma)c) \frac{\partial^2 c}{\partial e \partial I}, \\
C_4 = \gamma \left(\frac{\partial c}{\partial I}\right)^2 + (22t - 14\gamma t - (1 - \gamma)c) \frac{\partial^2 c}{\partial I^2},
$$

and

$$
C_5 = -(14t - c) \frac{\partial c}{\partial I}
$$

are all positive. Solving the system of equations for $\frac{deC}{d\gamma}$ and $\frac{dIC}{d\gamma}$ yields

$$
\frac{deC}{d\gamma} = \frac{C_2C_5}{C_1C_4 - C_2C_3}
$$

and

$$
\frac{dIC}{d\gamma} = -\frac{C_1C_5}{C_1C_4 - C_2C_3}.
$$

Therefore, when $C_1C_4 - C_2C_3 > 0$, $\frac{deC}{d\gamma}$ is positive and $\frac{dIC}{d\gamma}$ is negative. Otherwise, $\frac{deC}{d\gamma}$ is negative and $\frac{dIC}{d\gamma}$ is positive.

**Proof of Lemma 4**

S1 solves

$$
\max_{I_1} \frac{1}{48t} (8t - c(e, I_1) + c(0, I_2))(6t - 2c(e, I_1) + c(0, I_2)) - I_1,
$$

F.O.C. $$
\frac{1}{48t} \left( - \frac{\partial c(e, I_1)}{\partial I} (6t - 2c(e, I_1) + c(0, I_2)) + (8t - c(e, I_1) + c(0, I_2)) \left( -2 \frac{\partial c(e, I_1)}{\partial I} \right) \right) = 1.
$$
S2 solves
\[
\max_{I_2} \frac{1}{48t} (8t - c(0, I_2) + c(e, I_1))(6t - 2c(0, I_2) + c(e, I_1)) - I_2,
\]
F.O.C. \[
\frac{1}{48t} \left( -\frac{\partial c(0, I_2)}{\partial I} (6t - 2c(0, I_2) + c(e, I_1)) + (8t - c(0, I_2) + c(e, I_1)) \left( -2 \frac{\partial c(0, I_2)}{\partial I} \right) \right) = 1.
\]

F solves
\[
\max_e v - 2t - \frac{3(c(e, I_1) + c(0, I_2))}{8} + \frac{(c(e, I_1) - c(0, I_2))^2}{128t} - e,
\]
F.O.C. \[
-3 \frac{\partial c(e, I_1)}{\partial e} + \frac{(c(e, I_1) - c(0, I_2))}{64t} \frac{\partial c(e, I_1)}{\partial e} = 1.
\]
Thus, the equilibrium values \( I_1^†, I_2^† \) and \( e^† \) must satisfy
\[
\begin{align*}
-\left( 3 + \frac{(c(0, I_2^†) - c(e^†, I_2^†))}{64t} \right) \frac{\partial c(e^†, I_1^†)}{\partial e} &= 1, \\
-\frac{1}{48t} (22t - 4c(e^†, I_2^†) + 3c(0, I_2^†)) \frac{\partial c(e^†, I_2^†)}{\partial I} &= 1, \\
-\frac{1}{48t} (22t - 4c(0, I_2^†) + 3c(e^†, I_1^†)) \frac{\partial c(0, I_2^†)}{\partial I} &= 1.
\end{align*}
\]

**Proof of Lemma 5**

When \( p_1^w \) and \( p_2^w \) are high, F solves
\[
\max_{p_1, p_2} \hat{x}_1 (p_1 - p_1^w) + (1 - \hat{x}_2) (p_2 - p_2^w),
\]
where
\[
v - p_1 - t \hat{x}_1 = 0 \quad \Rightarrow \quad \hat{x}_1 = \frac{v - p_1}{t}
\]
and
\[
v - p_2 - t (1 - \hat{x}_2) = 0 \quad \Rightarrow \quad 1 - \hat{x}_2 = \frac{v - p_2}{t}.
\]
Thus, we have
\[
\max_{p_1, p_2} \frac{v - p_1}{t} (p_1 - p_1^w) + \frac{v - p_2}{t} (p_2 - p_2^w).
\]
Differentiating with respect to \( p_1 \) and setting the derivative to zero yield
\[
p_1 = \frac{v + p_1^w}{2}.
\]
Differentiating with respect to \( p_2 \) and setting the derivative to zero yield
\[
p_2 = \frac{v + p_2^w}{2}.
\]
Hence,

$$\hat{x}_1 = \frac{v - p_1^w}{2t}$$

and

$$1 - \hat{x}_2 = \frac{v - p_2^w}{2t}.$$  

We want $\hat{x}_1$ and $1 - \hat{x}_2$ to sum up to one:

$$\frac{v - p_1^w}{2t} + \frac{v - p_2^w}{2t} = 1,$$

$$p_1^w + p_2^w = 2v - 2t.$$  

Next, given $c_1$ and $c_2$, S solves

$$\max_{p_1^w, p_2^w} \frac{v - p_1^w}{2t} (p_1^w - c_1) + \frac{v - p_2^w}{2t} (p_2^w - c_2)$$

subject to

$$p_1^w + p_2^w = 2v - 2t \Rightarrow p_2^w = 2v - 2t - p_1^w.$$  

Substituting the constraint into the objective function yields

$$\max_{p_1^w} \frac{v - p_1^w}{2t} (p_1^w - c_1) + \frac{2t - v + p_1^w}{2t} (2v - 2t - p_1^w - c_2),$$

F.O.C.

$$\frac{1}{2t} (-p_1^w + c_1 + v - p_1^w + 2v - 2t - p_1^w - c_2 - 2t + v - p_1^w) = 0,$$

$$-4p_1^w + 4v - 4t + c_1 - c_2 = 0,$$

$$p_1^w = v - t + \frac{c_1 - c_2}{4},$$

hence

$$p_2^w = v - t + \frac{c_2 - c_1}{4}.$$  

Finally, we derive S’s intermediate profit as a function of $c_1$ and $c_2$:

$$\pi_{\delta}^S(c_1, c_2) = \frac{v - p_1^w}{2t} (p_1^w - c_1) + \frac{v - p_2^w}{2t} (p_2^w - c_2)$$

$$= \left(1 - \frac{c_1 - c_2}{8t}\right) \left(v - t + \frac{c_1 - c_2}{4} - c_1\right) + \left(\frac{1}{2} + \frac{c_1 - c_2}{8t}\right) \left(v - t - \frac{c_1 - c_2}{4} - c_2\right)$$

$$= \frac{v - t}{2} - \frac{c_1 + c_2}{2} + \frac{(c_1 - c_2)^2}{16t}.$$  

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F’s intermediate profit as a function of $c_1$ and $c_2$ is:

\[
\pi^{ss}_{F}(c_1, c_2) = \frac{v - p_1}{t}(p_1 - p_1^w) + \frac{v - p_2}{t}(p_2 - p_2^w) \\
= \frac{(v - p_1^w)^2}{4t} + \frac{(v - p_2^w)^2}{4t} \\
= \frac{1}{4t} \left( \left( t - \frac{c_1 - c_2}{4} \right)^2 + \left( t + \frac{c_1 - c_2}{4} \right)^2 \right) \\
= \frac{1}{4t} \left( 2t^2 + 2 \left( \frac{c_1 - c_2}{4} \right)^2 \right) \\
= \frac{t}{2} + \left( \frac{c_1 - c_2}{4} \right)^2.
\]

**Proof of Proposition 3**

F solves

\[
\max_{e} \frac{1}{2} \left( \frac{t}{2} + \frac{(c(e, I_1 + \delta I_2) - c(e, I_2 + \delta I_1))^2}{32t} - e \right).
\]

The higher is the difference in the production costs, the higher is F’s profit. Suppose for definitiveness $I_2 > I_1$. Then

\[
|c(e, I_1 + \delta I_2) - c(e, I_2 + \delta I_1)| = c(e, I_1 + \delta I_2) - c(e, I_2 + \delta I_1)
\]

and

\[
\frac{\partial}{\partial e}(c(e, I_1 + \delta I_2) - c(e, I_2 + \delta I_1)) = -\int_{I_1}^{I_2} \frac{\partial^2 c(e, I)}{\partial e \partial I} \, dI < 0.
\]

It follows that zero investment level is F’s dominant strategy.

**Proof of Lemma 6**

S solves

\[
\max_{I_1, I_2} \frac{v}{2} - \frac{1}{2} \left( \frac{c(0, I_1 + \delta I_2) + c(0, I_2 + \delta I_1)}{2} + \frac{(c(0, I_1 + \delta I_2) - c(0, I_2 + \delta I_1))^2}{16t} \right) - I_1 - I_2,
\]

\[
\left\{ \begin{array}{l}
\frac{1}{2} \left( \frac{\partial c(0, I_1 + \delta I_2)}{\partial I} + \frac{\partial c(0, I_2 + \delta I_1)}{\partial I} \right) \\
+ \frac{\delta c(0, I_1 + \delta I_2) - c(0, I_2 + \delta I_1)}{8t} \left( \frac{\partial c(0, I_1 + \delta I_2)}{\partial I} - \frac{\partial c(0, I_2 + \delta I_1)}{\partial I} \right) = 1 \\
\frac{1}{2} \left( \frac{\delta c(0, I_1 + \delta I_2)}{\partial I} + \frac{\partial c(0, I_2 + \delta I_1)}{\partial I} \right) \\
+ \frac{\delta c(0, I_1 + \delta I_2) - c(0, I_2 + \delta I_1)}{8t} \left( \frac{\partial c(0, I_1 + \delta I_2)}{\partial I} - \frac{\partial c(0, I_2 + \delta I_1)}{\partial I} \right) = 1
\end{array} \right. 
\]
Depending on the parameters $t$ and $\delta$, as well as the functional from of the cost function, we can either get a symmetric solution, or two asymmetric ones. We are going to assume that $\delta$ is sufficiently close to one, so that the solution is symmetric: $I_1 = I_2 = I^*$. $I^*$ must satisfy

$$-\frac{1}{2}(1 + \delta) \frac{\partial c(0, I^* + \delta I^*)}{\partial I} = 1.$$  

**Proof of Lemma 7**

S1 solves

$$\max_{I_1} \frac{1}{48t} (8t - c(0, I_1) + c(0, I_2))(6t - 2c(0, I_1) + c(0, I_2)) - I_1,$$

F.O.C.  \[\frac{1}{48t} \left( -\frac{\partial c(0, I_1)}{\partial I} (6t - 2c(0, I_1) + c(0, I_2)) + (8t - c(0, I_1) + c(0, I_2)) \left( -2\frac{\partial c(0, I_1)}{\partial I} \right) \right) = 1.\]

S2 solves

$$\max_{I_2} \frac{1}{48t} (8t - c(0, I_2) + c(0, I_1))(6t - 2c(0, I_2) + c(0, I_1)) - I_2,$$

F.O.C.  \[\frac{1}{48t} \left( -\frac{\partial c(0, I_2)}{\partial I} (6t - 2c(0, I_2) + c(0, I_1)) + (8t - c(0, I_2) + c(0, I_1)) \left( -2\frac{\partial c(0, I_2)}{\partial I} \right) \right) = 1.\]

Obviously, in equilibrium S1 and S2 will choose the same level of investment; we denote it by $I^\dagger$. $I^\dagger$ must satisfy

$$-\frac{1}{48t} (22t - c(0, I^\dagger)) \frac{\partial c(0, I^\dagger)}{\partial I} = 1.$$  

**Proof of Lemma 8**

With $c(e, I) = \bar{c} - a\sqrt{e} - b\sqrt{I}$, the system of equations in Lemma 3 becomes

$$\begin{cases} \frac{3}{4} \frac{a}{2\sqrt{e^{NC}}} = 1 \\ \frac{1}{48t} (22t - \bar{c} - a\sqrt{e^{NC}} - b\sqrt{I^{NC}}) \frac{b}{2\sqrt{I^{NC}}} = 1 \\ \sqrt{e^{NC}} = \frac{3a}{8} \\ 22t - \bar{c} + a\sqrt{e^{NC}} + b\sqrt{I^{NC}} = \frac{96t}{b} \sqrt{I^{NC}} \end{cases}$$
so
\[ e^{NC} = \left( \frac{3a}{8} \right)^2 \]

and
\[ I^{NC} = \left( \frac{22t - \bar{c} + a\sqrt{e^{NC}}}{96t/b - b} \right)^2. \]

Setting \( e^{NC} \) to zero, we obtain
\[ I^\dagger = \left( \frac{22t - \bar{c}}{96t/b - b} \right)^2. \]

To find the investments under strategy B, we substitute \( c(e, I) = \bar{c} - a\sqrt{e} - b\sqrt{I} \) into the system of equations in Lemma 4:

\[
\begin{cases}
  \left( \frac{3}{8} + \frac{\bar{c} - a\sqrt{0} - b\sqrt{I}_2}{64t} - \left( \bar{c} - a\sqrt{e} - b\sqrt{I}_1 \right) \right) \frac{a}{2\sqrt{e}} = 1 \\
  \frac{1}{48t} \left( 22t - 4 \left( \bar{c} - a\sqrt{e} - b\sqrt{I}_1 \right) + 3 \left( \bar{c} - a\sqrt{0} - b\sqrt{I}_2 \right) \right) \frac{b}{2\sqrt{I}_1} = 1 \\
  \frac{1}{48t} \left( 22t - 4 \left( \bar{c} - a\sqrt{0} - b\sqrt{I}_2 \right) + 3 \left( \bar{c} - a\sqrt{e} - b\sqrt{I}_1 \right) \right) \frac{b}{2\sqrt{I}_2} = 1 \\
\end{cases}
\]

\[
\begin{cases}
  24t - b\sqrt{I}_2 + a\sqrt{e} + b\sqrt{I}_1 = \frac{128t}{a} \sqrt{e} \\
  22t - \bar{c} + 4a\sqrt{e} + 4b\sqrt{I}_1 - 3b\sqrt{I}_2 = \frac{96t}{b} \sqrt{I}_1 \\
  22t - \bar{c} + 4b\sqrt{I}_2 - 3a\sqrt{e} - 3b\sqrt{I}_1 = \frac{96t}{b} \sqrt{I}_2 \\
  24t + a\sqrt{e} - \frac{128t}{a} \sqrt{e} = b \left( \sqrt{I}_2 - \sqrt{I}_1 \right) \\
  7a\sqrt{e} = \left( \frac{96t}{b} \right) \left( \sqrt{I}_1 - \sqrt{I}_2 \right) \\
  44t - 2\bar{c} + a\sqrt{e} = \left( \frac{96t}{b} \right) \left( \sqrt{I}_1 + \sqrt{I}_2 \right) \\
\end{cases}
\]

\[
\begin{cases}
  24t + a\sqrt{e} - \frac{128t}{a} \sqrt{e} = \frac{7ab}{96t/b - 7b} \sqrt{e} \\
  \sqrt{I}_1 - \sqrt{I}_2 = \frac{7a\sqrt{e}}{96t/b - 7b} \\
  \sqrt{I}_1 + \sqrt{I}_2 = \frac{44t - 2\bar{c} + a\sqrt{e}}{96t/b - b}
\end{cases}
\]
\[
\begin{align*}
\sqrt{e^\dagger} &= \frac{24t}{7ab/(96t/b - 7b) + 128t/a - a} \\
\sqrt{I_1^\dagger} &= \frac{1}{2} \left( \frac{44t - 2\bar{c} + a\sqrt{e^\dagger}}{96t/b - b} + \frac{7a\sqrt{e^\dagger}}{96t/b - 7b} \right) \\
\sqrt{I_2^\dagger} &= \frac{1}{2} \left( \frac{44t - 2\bar{c} + a\sqrt{e^\dagger}}{96t/b - b} - \frac{7a\sqrt{e^\dagger}}{96t/b - 7b} \right)
\end{align*}
\]

or

\[
\begin{align*}
e^\dagger &= \left( \frac{24t}{7ab/(96t/b - 7b) + 128t/a - a} \right)^2 \\
I_1^\dagger &= \frac{1}{4} \left( \frac{44t - 2\bar{c} + a\sqrt{e^\dagger}}{96t/b - b} + \frac{7a\sqrt{e^\dagger}}{96t/b - 7b} \right)^2 \\
I_2^\dagger &= \frac{1}{4} \left( \frac{44t - 2\bar{c} + a\sqrt{e^\dagger}}{96t/b - b} - \frac{7a\sqrt{e^\dagger}}{96t/b - 7b} \right)^2
\end{align*}
\]