Abstract

Why do manufacturers oppose “loss leading” of their products by retailers and why do their protests receive support from consumer interest groups and policymakers? We provide a solution to this puzzle when consumers are salient thinkers, showing how in this case loss leading shifts profits from manufacturers of high-quality (branded) products to retailers and how, when the extent of one-stop shopping is sufficiently large and retail competition sufficiently intense, this crowds out high quality. We identify conditions for when a prohibition of loss leading enhances consumer and overall welfare and for when such a policy backfires.

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Introduction

The initial quote is not unusual as milk is an important “loss leader” in many countries. In Germany, Wal-Mart’s attempt to gain market share through heavy discounts has led to a landmark decision of the highest court in 2002 and subsequently to a change of the national competition law: it now bans loss leading explicitly in the food retailing industry. But why do manufacturers oppose deep discounts, which after all may considerably expand demand for their product? And why are their demands supported by policymakers and consumer interest groups, even though consumers should benefit from lower prices? Our paper provides answers to these questions. Manufacturers’ fears of lower profits and consumer interest groups’ concerns about a reduction in quality both prove to be unfounded in our baseline model with fully rational consumers, but they receive support when consumers are salient thinkers. Still, even in the latter case we show how the prohibition of below-cost pricing can backfire.

In our model, brand manufacturers negotiate with retailers, which may alternatively stock a lower-quality (or possibly private label/store brand) product instead. We are 

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This quote is not unusual in any respect. While it dates back to 2011, searching this very same online newspaper generates many more headlines of heavy discounting of dairy products in the UK, such as: 

“Farmers’ fury at 89p supermarket milk: Blockade threat as price of four pints is slashed” (MailOnline for the UK's Daily Mail, 14 October 2014, while the first quote is from the 15 January 2011 edition). Notably, the latter article states: “Farmers are threatening protests after Iceland cut the price of a four-pint milk carton from £1 to 89p. The budget store is using milk as a loss leader – selling below cost price to lure in customers – with the result it is even undercutting discount chains Aldi and Lidl [which are hard discounters of German origin that increasingly expand globally, the authors].” Depending on the national cuisine, also other staple goods are common loss leaders. For instance, in Spain the government repeatedly undertakes efforts to curb discounts on olive oil (e.g., “Deal Could Stop Use of Olive Oil as Loss Leader”, OliveOilTimes, 4 March, 2013). These efforts are helped by Spain’s legal prohibition of loss leading, of which recently the German discounter Lidl fell foul when selling wine at a discount (“Lidl rapped for selling wines at a loss”, TheDrinksBusiness, 16 October 2014). Wine is also an example of a loss leader in a category with more branded goods.

We refer to loss leaders as (retail) products that are sold at low prices, and in particular below cost, to attract consumers who generate additional sales from more profitable products or services.

This goes back to a decision of the German competition authority in 2000 to prohibit below-cost selling by Wal-Mart, as well as Germany’s two leading discounters Aldi and Lidl. Wal-Mart challenged this decision and lost on appeal. Wal-Mart exited the German market in 2006.

To our knowledge, European countries where concerns have led to a full or partial (e.g., sector specific) prohibition of loss leading include, next to Germany, Belgium, France, and Ireland. While U.S. federal law does not forbid below-cost selling or loss leading per se, several states have enacted below-cost selling laws. California goes even further and rules in its Business and Professions Code Section 17044 that “[i]t is unlawful for any person engaged in business within this State to sell or use any article or product as a ‘loss leader’ as defined in Section 17090 of this chapter.”
primarily interested in the case where the provision of the higher-cost but higher-quality product is more efficient. Retailers in turn compete for final consumers who are one-stop shoppers and who make their respective shopping decision based on a limited number of items. The number of other products in their basket thus captures the extent of one-stop shopping, while competition on the selected (potentially loss-leading) items occurs through promotional discounts (as in Varian 1980). After presenting our baseline analysis, we follow Bordalo et al. (2013) and stipulate that consumers may put too much weight on the attribute of a product, that is price or quality, along which the product differs more, in relative terms, compared to the market average. When consumers are salient thinkers in this sense, manufacturers have indeed reason to resist deep discounts in their product category, just as in our introductory example: When their product category is used so as to attract one-stop shoppers, the prevailing lower price level in this product category makes it more attractive for retailers to offer cheaper substitutes of lower quality. Even when such substitution is not yet observed, we find that the fact that so-called “known-value items” (KVIs)\(^5\) serve as loss leaders reduces rather than enhances brand manufacturers’ bargaining position vis-à-vis retailers. Once substitution takes place, also policymakers’ and consumer groups’ concerns about quality are no longer unfounded. However, a ban of below-cost pricing may backfire, as it can increase firm profits at the expense of consumers while making it at the same time more, instead of less, likely that low quality is provided. It backfires when it is overlooked that such a ban constrains retailers’ pricing for both low-quality and high-quality products. We derive precise conditions for when such a prohibition increases welfare and for when not.

Various policy reports, such as OECD (2007), have stressed that loss leading for promoted products is an essential part of competition in the retailing industry. We do not dispute this, but our analysis also draws attention to potential detrimental effects. Chen and Rey (2012) show how when retailers are asymmetric and a lack of competition allows for such price discrimination, loss leading can be used to screen consumers according to their shopping cost. In our model, potentially deep discounts on one product arise as (multi-product) retailers make a positive margin on other products that consumers purchase on their shopping trip and for which consumers do not (ex-ante) compare prices.\(^6\)

\(^5\)This terminology is frequently used in marketing (cf. http://www.mckinsey.com/industries/retail/our-insights/pricing-in-retail-setting-strategy) and stresses precisely the point that consumers compare stores based on these, rather than all, items.

\(^6\)This broadly follows Lal and Matutes (1994), albeit we do not endogenize advertising in our model. There is also a small literature that analyzes the possible incidence of below-cost pricing when consumers are equally informed about all prices, e.g., due to differences in demand elasticities (see Bliss (1988),
and welfare losses from loss leading arise particularly when there is intense competition between retailers.\textsuperscript{7} Notably in terms of consumer welfare, there is also an immediate tension when a ban on below-cost pricing dampens retail competition.\textsuperscript{8} In our model, a ban is unambiguously negative when consumers make an unbiased choice, while we derive conditions for when with salient thinkers, through shoring up the provision of high quality, it increases efficiency.

In terms of positive implications, our results may provide new insights into the rapid spread of private labels in many countries, as we show how this may directly relate to a change in shopping behavior with a shift towards one-stop shopping.\textsuperscript{9} Again, consumers’ tendency towards salient thinking provides here the link. In the fields of psychology, marketing, and behavioral economics, researchers have long documented consumers’ tendency to pay more attention to particularly salient characteristics. Important papers in this vein include Huber et al. (1982), Thaler (1985), Simonson (1989), and Tversky and Simonson (1993).\textsuperscript{10} A formalization of salience, on which we rely on, has recently been provided by Bordalo et al. (2013). As we discuss in detail below, what drives our results is the specification that salience is determined by how different a product’s attribute is relative to the market average. The motivation for our contribution is different from Bordalo et al. (2014), which considers a model of undifferentiated, duopolistic competition. Our focus

\textsuperscript{7}Note that we abstract from theories of harm that focus on other, potentially smaller and less competitive retailers. Absent an outright prohibition by law, a prosecution of loss leading along such lines would typically require to find predatory behavior. For a detailed treatment of predatory behavior see Bolton et al. (2000). Chen and Rey (2012) discuss in some detail relevant case material and findings from recent sector inquiries.

\textsuperscript{8}Allain and Chambolle (2005) and Rey and Vergé (2010) show how also intrabrand, next to interbrand competition, can thereby be dampened, as below-cost pricing regulations may allow manufacturers to impose price floors.

\textsuperscript{9}The market share of private labels in European food retailing has risen significantly, with now more than 40\% in some countries, such as the UK. Frequently, but not exclusively, private labels are positioned at the lower end of the quality and price range. See European Commission (2011) for an overview across Europe. Bergès-Sennou et al. (2004) provide an earlier guide to the academic literature, stressing different roles for the introduction and positioning of private labels. Especially in Europe, concerns about a potentially inefficient proliferation of private labels, combined with an exercise of buyer power, have recently resurfaced in sector inquiries and have led to various policy reports (e.g., European Commission 2014).

\textsuperscript{10}For example, Huber et al. (1982) show that the choice among two alternatives can crucially be affected if a third, dominated alternative is added (the so-called “attraction effect”). Similarly, Simonson (1989) demonstrates that adding an alternative that is particularly good on one dimension, but bad on another (e.g., a product with very high quality, but also a very high price) may tilt consumers’ choice among the initially available alternatives (“compromise effect”). Overall, the literature stresses the importance of the choice context for the weights consumers put on different product attributes. Wedell (1991) provides a psychological analysis of the associated “weight-shift” of attributes. We do however not claim that the chosen framework is fully consistent with these earlier theories and their implications.
is instead both on the vertical relation between retailers and manufacturers as well as on the interaction of one-stop shopping and the degree of retail competition. In fact, we will find a gradual reduction in brand manufacturers' profits precisely when the extent of loss leading increases but retailers still stock the high-quality product. Inderst and Obradovits (2016) also do not consider the vertical dimension in their model with shrouded charges and salient thinking. In fact, more generally the literature has largely neglected the consideration of vertical relationships in models of promotions, as in ours (cf. however Janssen and Shelegia (2015), where qualities are however homogeneous and search is sequential). Also in a model in the spirit of Varian (1980), Armstrong and Chen (2009) analyze pricing and product quality choice when some consumers are assumed to be inattentive to product quality. As we endogenize consumers’ potentially biased attention to different product attributes, salience affects the market outcome only under particular circumstances, notably when retail competition is intense and the extent of one-stop shopping large.

The remainder of this article is organized as follows. Section 2 reviews our main results. Section 3 sets out the model for the baseline case, which is analyzed in Section 4. Section 5 contains the main analysis with salient thinkers when there is no policy intervention and Section 6 introduces a ban on below-cost pricing. Section 7 concludes. All proofs are relegated to the appendix. We also collect additional material in an online appendix.

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11Still, shrouded charges increase firms’ margins and thereby reduce headline prices, similar to an increase in the extent of one-stop shopping. While there is also no analysis of a ban of below-cost pricing, the two papers differ also in various aspects of the considered game, notably as in the present paper we consider the simultaneous choice of products and prices, reflecting retailers’ ability to frequently change their offering (or, what often amounts to the same, the positioning of products in their shops).

12Promotions (sales) are a defining feature of modern retailing competition and account for a large share of the observed price variation in retailing. Among the numerous empirical studies documenting the vast prevalence of retail promotions are Volpe (2013), Nakamura and Steinsson (2008), Berck et al. (2008), Hosken and Reiffen (2004), Pesendorfer (2002), and Villas-Boas (1995).

13There is also no vertical dimension in Inderst and Obradovits (2015), which extends Narasimhan’s (1988) influential work with asymmetric firms to analyze how salience affects promotion intensity and thereby also product choice.

14Other relevant models in the developing field of behavioral industrial organization include Ellison and Ellson (2009), Spiegler and Eliaz (2011), Hefti (2012), and de Clippel et al. (2013). In all of these papers, consumers’ attention is limited to a subset of alternative choices. Although this “consideration set” can be manipulated by firms’ strategic decisions (such as advertising), the evaluation of options within a given consideration set is exogenous and does not depend on the attributes of available options. In Koszegi and Szeidl (2013), consumers’ utility weighting of product attributes increases in the range of the respective attributes across all choices in consumers’ consideration set. Their analysis focuses on (distortions in) consumers’ intertemporal choice. Consumers’ inattention to certain product characteristics may also be driven by rational consumers’ efficient allocation of attention. Such models of “rational inattention” include Matějka and McKay (2012), Persson (2013), and Gabaix (2014).
2 Guide to Results

In this section we review our main results, both for the benchmark case with rational consumers and for the case with salient thinkers.

**Benchmark Case.** Our benchmark model with rational consumers considers a market populated by shoppers, who compare the offers of several retailers, and non-shoppers, who only frequent their “own” retailer. The fraction of shoppers provides a convenient measure of the degree of competition, while we know from Varian (1980) that such models give rise to promotional strategies (through mixed strategies). Shoppers compare retailers only on the basis of a “known value item” (KVI) (while holding rational beliefs about retailers’ offering of all other products). As the extent of one-stop shopping increases, retailers’ prices for the KVI will drop below cost, which is when we describe the respective products as “loss leaders”. The focus of our analysis is on whether and how this affects quality choice and manufacturer profits. Guided by the policy concerns described in the Introduction, we focus on the case where a more efficient high-quality variant is provided by some brand manufacturer, while lower-quality variants are provided competitively (or are produced as private labels).

With rational consumers, we find that while one-stop shopping can lead to loss leading on the KVI, so that its price may drop even drastically compared to that of other, non-promoted products, this has no effect on either manufacturer profits or the choice of quality (Proposition 1).\(^{15}\) Notably, there is no difference between retailers’ choice of quality for the loss-leading product and all other, non-promoted products. In the benchmark case, there is thus no scope for a ban of below-cost pricing to increase efficiency. Instead, when it constrains retailers’ pricing decision, such a policy reduces competition to the detriment of consumers. We show how notably manufacturers of the promoted, formerly loss-leading item strictly benefit from this as their brands essentially become gatekeepers when retailers want to access shoppers: Retailers that instead offer lower-value variants for the KVI, to which the prohibition of loss leading equally applies, can then no longer successfully compete for shoppers. While brand manufacturers of loss leaders would thus lobby for a ban of below-cost pricing, in the benchmark case with rational consumers their

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\(^{15}\) More precisely, this holds in the case where all consumers have the same reservation value, so that total demand is inelastic (up to the point where it is “choked off” completely). When total demand is elastic as consumers differ in their reservation value for shopping, we show that the profits of the brand manufacturer of the loss leader even increase as the extent of one-stop shopping increases and the price at which retailers sell its product thus decreases.
interests are orthogonal to those of consumers (Proposition 2).

**Salient thinkers.** Our main interest in this paper lies with the analysis of the case where consumers are salient thinkers. Salient thinkers put too much weight on the attribute along which a particular offering differs most relative to other offers in the market. While we also characterize the respective pricing equilibria, with and without a prohibition of below-cost pricing, our main interest lies again with manufacturer profits and quality. Intuitively, salient thinking does not affect retailers’ choice of products that are not promoted and potentially used as loss leaders. But salient thinking and the extent of one-stop shopping now have jointly an effect on the provision of the loss leader.

To see this, consider a candidate equilibrium where the more efficient high quality is chosen also for the loss leader. Salient thinking makes the deviation of a retailer that replaces the respective brand manufacturer’s product by a lower-quality variant more profitable if the corresponding price cut is sufficient to make price salient, so that the quality difference matters less in the eyes of salient-thinking consumers. The extent of one-stop shopping is crucial, as making the lower price, rather than the lower quality, salient is achieved with a smaller price cut when prices for the loss leader at other retailers are already low. This follows directly from the definition of salience that we apply, following Bordalo et al. (2013), which relies on the principle of “diminishing sensitivity”: What matters for which attribute becomes salient is the relative difference compared to other offers, so that notably the same absolute price decrease looms relatively larger when the overall price level is lower. There are now two consequences from this observation. First, when the non-salient attribute is discounted sufficiently in consumers’ decision-making and when the extent of one-stop shopping is sufficiently large, so that the promoted price is sufficiently low, there no longer exists an equilibrium where all retailers offer the more efficient high-quality variant of the KVI. Second, even when still only high quality is provided, brand manufacturers’ profits may decrease when their product is promoted more aggressively. While in the latter case a retailer still finds it profitable to supply also the high-quality variant of the loss leader, its next-best option of supplying a lower-quality variant becomes more attractive when all other retailers promote this product more aggressively (and when consumers discount non-salient items sufficiently). These implications of salient thinking and one-stop shopping (Propositions 3 and 4) provide our starting point for the final policy analysis.

In stark contrast to the case with rational consumers, when consumers are salient
thinkers, the prohibition of below-cost pricing can, at the same time, increase overall efficiency as well as both consumer welfare and brand manufacturers’ profits (Proposition 6). While we derive precise conditions for when this is the case, we also show how such a policy can backfire and even reduce overall efficiency by making it less, instead of more, likely that the more efficient high-quality product is chosen for the promoted KVI (Proposition 7). The key to understand this is the recognition that the policy applies to the provision of both the low-quality product and the high-quality product, and we characterize when it constrains one more than the other.

Overall, while we can thus indeed provide a rationale for why brand manufacturers, consumer interest groups, and policy makers may all see benefits in prohibiting loss leading, even though this may reduce retail competition, and why, in addition, this may not be in retailers’ interest, our analysis also warns that such a prohibition can backfire if it is applied in a too simplistic manner.

3 The Benchmark Model with Rational Consumers

We model a market where consumers, albeit they potentially purchase a basket of products, focus their attention both on a particular product as well as on particular (“salient”) attributes. In the baseline model, we first provide a benchmark by abstracting from the role of salient thinking. Retailers compete for final consumers, who are one-stop shoppers and thus purchase all products at a single retailer. Manufacturers with different qualities and in different product categories compete for shelf space. We are interested primarily in retailers’ and consumers’ product selection (and its implication for welfare) as well as in manufacturers’ profits, and the respective implications of a policy of prohibiting loss leading.

Products are sold through \( n = 1, \ldots, N \) retailers. Given the richness of our model, we next impose symmetry along various dimensions. We suppose that each retailer stocks \( I \geq 1 \) products. When we thus compare \( I = 1 \) with \( I > 1 \) (e.g., \( I = 2 \)), consumers engage in one-stop shopping only in the latter case. It is convenient to also denote the respective sets of retailers and products by \( N \) and \( I \), respectively. The price of product \( i \) at retailer \( n \) is \( p_{ni} \), and we suppose that the respective quality can be described by a real-valued variable \( q_{ni} \).

Each product \( i \) can be supplied at different qualities. We again impose symmetry,
which, as we will see, serves primarily the purpose of simplifying the exposition.\footnote{Precisely, we will subsequently see that, for the purpose of our analysis, we can summarize the provision of all products \( i > 1 \) into a single variable (capturing retailers’ equilibrium profit margins from these products).} We thus suppose that a product can be supplied in two qualities \( q_H > q_L \) with respective constant marginal costs of production \( c_H > c_L \).\footnote{We abstract from retailers’ own (handling) costs, which however can be included without affecting results.} As we want to investigate whether claims regarding the negative implications of loss leading are justified (cf. the Introduction), we focus on the case where high quality is more efficient: Denoting \( \Delta_q = q_H - q_L \) and \( \Delta_c = c_H - c_L \), we thus suppose that \( \Delta_q > \Delta_c \).\footnote{We have however fully solved also the case where \( \Delta_q < \Delta_c \), for which we indeed find that there is no scope for policy intervention. Results can be obtained from the authors upon request.}

Again motivated by our introductory remarks, we focus on the profits of high-quality (brand) manufacturers and thus stipulate that for any retailer \( n \) and category \( i \), the respective high-quality product can be produced by a single such brand manufacturer, while instead the low-quality variant either is produced by two or more undifferentiated suppliers, or represents a private label of the respective retailer. The obvious implication of these specifications is that the low-quality variant will always be procured by retailers at cost. For concreteness only, we presently abstract from private labels and stipulate that low quality is, for each retailer \( n \) and each product category \( i \), offered by at least two undifferentiated manufacturers. With respect to the supply of high-quality (branded) products, we can without loss of generality focus on so-called “two-part tariff contracts”, which stipulate a fixed fee \( T_{ni} \) together with a constant (per-unit) wholesale price \( w_{ni} \) that the retailer has to pay. We specify below how supply contracts are determined.

We now turn to consumers. Suppose first that \( I = 1 \), so that there is no one-stop shopping. Here, our model fully follows Varian (1980). We stipulate that a fraction \( (1 - \lambda)/N \) of consumers can only shop at their (local) retailer \( n \) (for each \( n \in N \)), so that a total fraction \( 1 - \lambda \) of consumers does not compare offers. In contrast, the remaining fraction \( \lambda \) of consumers, called “shoppers”, is free to choose any retailer, so that \( \lambda \) also captures the intensity of competition.

Suppose next that there is one-stop shopping as \( I > 1 \). We now stipulate that only the offer of product \( i = 1 \) is observed before a consumer enters the respective shop. Shoppers thus observe offers \( (q_{ni}^1, p_{ni}^1) \) across retailers, while non-shoppers observe the respective offer only for their (local) retailer. No consumer observes offers for products \( i > 1 \) before they enter a shop, though they hold (rational) expectations \( (\hat{q}_{ni}^i, \hat{p}_{ni}^i) \). Once in a shop, a consumer
then decides which subset of products \( I' \subseteq I \) to buy, thereby realizing the respective utility \( \sum_{i \in I'} (q_n^i - p_n^i) \), where we normalize consumers’ reservation value to zero and assume that consumers demand at most one unit in each product category.\(^{19}\)

As we discussed in the Introduction, product \( i = 1 \) represents a “known value item” (KVI). For our purpose it is inconsequential whether consumers’ limited ex-ante knowledge of offers for products \( i > 1 \) is due to limited attention or memory or whether it follows from limits to advertising space.\(^{20}\) While we find that products in category \( i = 1 \) will not always be offered below cost (notably not when \( I = 1 \)), for convenience only we will still refer to them as loss leaders.

**Market Game.** The following description of the market game fully summarizes our model.

\( t = 1: \) *Manufacturer competition:* At each retailer \( n \) and for each product category \( i \), the single brand manufacturer with high quality \( q_H \) and costs \( c_H \) and at least two non-brand manufacturers with quality \( q_L \) and costs \( c_L \) compete by simultaneously offering contracts to their retailer.

\( t = 2: \) *Retailer choice:* Retailers simultaneously choose, first, which product to stock for any of the \( I \) categories, and, second, which prices \( p_n^i \) to set.

\( t = 3: \) *Consumer choice:* Of the fraction \( 1 - \lambda \) of non-shopping consumers, each randomly frequents any of the \( N \) retailers. The fraction \( \lambda \) of shopping consumers decides which of the \( N \) retailers to visit, depending on, first, observed prices and quantities \( (q_n^1, p_n^1) \) for product \( i = 1 \) and, second, when there is one-stop shopping as \( I > 1 \), on the anticipated choices \( (\hat{q}_n^i, \hat{p}_n^i) \) for products \( i > 1 \). Once in a shop, any consumer purchases a product \( i \) if and only if \( q_n^i - p_n^i \geq 0 \).

\(^{19}\)Our analysis is thus heavily simplified by the specification that a consumer visits at most one shop, so that he indeed buys any of the \( I \) products either at the same shop or not at all. However, this outcome would also arise endogenously when we specified some arbitrary small costs \( \varepsilon > 0 \) of visiting (other) shops. Monopoly prices for products for which prices are not observed (ex-ante) without incurring these costs (cf. below) would then be an implication of the well-known “Diamond paradox” in markets with search costs.

\(^{20}\)As discussed in the Introduction, KVIs (or loss leaders) are often staple products that are frequently purchased by most customers. One way to endogenize the choice of the loss-leading category would thus be to stipulate that, first, only \( i = 1 \) is indeed purchased by all consumers, while all other products \( i > 1 \) are purchased by a (possibly considerably) smaller subset of consumers and, second, consumers can at most memorize one offer.
4 Benchmark Analysis

Part of the subsequent analysis for the baseline model follows well-established results, which is why we can be short, though all remaining gaps are filled in the proofs in the appendix.

So as to avoid double-marginalization, in equilibrium products are provided at a marginal wholesale price that is equal to marginal cost.\textsuperscript{21} For what follows, to streamline also the exhibition in the proofs, we confine ourselves to the characterization of equilibria with this feature, noting however that this must uniquely hold when retailers’ prices are not constrained by policy.\textsuperscript{22} Depending on the chosen quality $q^i_n$, the retailer’s marginal cost of offering product $i$ is thus either $c_L$ or $c_H$. Given competition for the provision of the low-quality product, the respective offers will not contain a positive fixed part: Low-quality products are thus offered by manufacturers at cost.\textsuperscript{23} In contrast, the offer of the respective high-quality manufacturer at retailer $n$ may contain a fixed part $T^i_n \geq 0$. When this offer is accepted, $T_n$ is thus the profit of the high-quality manufacturer. By optimality, the specification of $T_n$ will leave the respective retailer just indifferent between acceptance and rejection.

We now proceed as follows. We first consider the case where there is no one-stop shopping as $I = 1$, in which case our analysis can closely follow that of Varian (1980). As we show subsequently, the inclusion of products $i > 1$ will then be relatively immediate, so that we choose to collect our formal results once we have introduced one-stop shopping.

\textsuperscript{21}Note that we consider a situation in which there are different brand and non-brand manufacturers at each retailer. This may be realistic for some products that retail chains procure centrally and then distribute themselves across their different outlets. Formally, this allows us to abstract from having to choose between observable contracts, under which manufacturers that serve different retailers could dampen intrabrand competition, and non-observable contracts, where a manufacturer’s own opportunism problem may make this impossible. One case where such an opportunism problem does not arise is where, in case of a single brand manufacturer that serves all retailers, the supply of products is modeled by a so-called “Nash-in-Nash” framework with simultaneous bilateral negotiations between each retailer and different representatives (agents) of the manufacturer. Such an approach is frequently used in the literature, and we can easily extend our results in this way. As is well known, when instead a single manufacturer makes simultaneous offers to all retailers, then the specification of out-of-equilibrium beliefs (in case of non-observable offers) becomes crucial. We have not extended our analysis in this direction.

\textsuperscript{22}In the subsequently analyzed case of a prohibition of below-cost pricing, this is no longer uniquely the case when the constraint on retail prices essentially turns each retailer into a monopolist (with inelastic residual demand).

\textsuperscript{23}We focus on non-dominated strategies.
4.1 The Case without One-Stop Shopping ($I = 1$)

Given $I = 1$, we presently drop the superscript $i(= 1)$. Compared to Varian’s (1980) seminal analysis, our model is in fact only made slightly more complicated by the addition of vertical contracting and endogenous product (quality) choice.

To see this, note first that given $\Delta_q > \Delta_c$, all retailers choose high quality in equilibrium. Suppose to the contrary that, in an equilibrium at $t = 2$, some retailer $n$ would instead choose $q_n = q_L$ and some price $p_n$. In this case the retailer and the high-quality manufacturer could however jointly realize strictly higher profits by offering instead $q_H$ at a price $p_n + \Delta_q$, so that this leaves consumers indifferent (and thus does not affect expected demand), while the margin would increase by $\Delta_q - \Delta_c > 0$. All retailers must thus offer high quality.

Even though all retailers are henceforth expected to choose the same qualities and are therefore symmetric, it is well known that for $N > 2$ there are multiple pricing equilibria, albeit profits are uniquely pinned down (cf. Varian (1980) and notably Baye et al. (1992)). Ignoring fixed fees $T_n$, with symmetric qualities each retailer makes exactly the (gross) profits that it would realize when choosing the highest feasible price, which is $p_n = q_H$, and thereby attracting only its fraction of non-shoppers, $(1 - \lambda)/N$. Hence, we have for each retailer profits (gross of the fixed fee paid to the brand manufacturer) of

$$\pi = (q_H - c_H)(1 - \lambda)/N.$$ Intuitively, all profits that could be realized with shoppers are fully competed away in equilibrium (cf. below for the characterization of the (mixed strategy) pricing equilibrium).

Suppose now a retailer would deviate and choose the low-quality variant. Then, we can likewise show that its (deviating) profit becomes

$$\pi_d = (q_L - c_L)(1 - \lambda)/N.$$ By optimality for the brand manufacturer, the fixed fee $T_n$ extracts exactly the difference between $\pi$ and $\pi_d$, so that $T_n = (\Delta_q - \Delta_c)(1 - \lambda)/N$. In words, the brand manufacturer extracts the incremental surplus that a retailer realizes with the (non-contested) fraction of non-shoppers. We have thus uniquely characterized the equilibrium provision of products as well as profits. We next turn to the characterization of equilibrium prices.

It is well known from the seminal work of Varian (1980) that the considered demand system does not afford an equilibrium where all retailers choose pure strategies.\footnote{Intuitively, as low quality is inferior given $\Delta_q > \Delta_c$, we can show that the deviating retailer finds it optimal to target only its respective fraction of non-shoppers. This can also be seen from inspection of the gray line in Figure 1 below, which represents the profit of a deviating low-quality retailer as a function of the respective price (and for the case without salience).}

We

\footnote{When $p_n = p > c$ was the (symmetric) deterministic equilibrium price, a retailer that does not yet...}
denote price strategies by the CDF $F_n(p_n)$ with support $p_n \in P_n$, as well as lower and upper boundaries $\underline{p}_n$ and $\overline{p}_n$, respectively. In line with the literature, we sometimes refer to choices $p_n < \overline{p}_n$ as promotions. The lower boundary $\underline{p}_n$ thus denotes the deepest promotion of retailer $n$.

It is also known that a unique equilibrium exists when $N = 2$, in which the two retailers choose symmetric strategies and where the support is a convex set $P_n = [\underline{p}_n, \overline{p}_n]$. But it is further known that when $N > 2$, there exist multiple pricing equilibria. This multiplicity is however inconsequential for our present analysis, which is so for two reasons. First, recall that we already uniquely pinned down equilibrium profits, regardless of the ensuing pricing equilibrium. Second, we know that in any pricing equilibrium, the lowest promotion that is chosen with positive probability is the same, and that the respective value $\underline{p}$ solves the following equation:

$$\left[\frac{1}{N} - \frac{\lambda}{1 - \lambda N} + \lambda\right] (p - c_H) = \frac{1 - \lambda}{N} (q_H - c_H).$$

(1)

Intuitively, a retailer is then indifferent between, on the one hand, choosing $p_n = \underline{p}$ and attracting all shoppers, so that its demand is $(1 - \lambda)/N + \lambda$, and, on the other hand, choosing $p_n = q_H$ and attracting only the fraction $(1 - \lambda)/N$ of consumers. This indifference condition (1) can be solved for

$$\underline{p} = c_H + (q_H - c_H) \frac{1 - \lambda}{1 - \lambda + \lambda N}. \tag{2}$$

As this always strictly exceeds $c_H$, without one-stop shopping there is clearly no scope for below-cost pricing in equilibrium.

To be more specific, we will later focus on symmetric pricing equilibria, so that all retailers with the same quality choose the same pricing strategy. Presently, with $I = 1$, the symmetric pricing strategy $F_n(p) = F(p)$ is immediately obtained from the following indifference requirement: For a retailer to be indifferent for all $p \in [\underline{p}, q_H]$, all other $N - 1$ retailers must choose prices according to $F(p)$ so that indeed

$$\left[\frac{1}{N} - \frac{\lambda}{1 - \lambda N} + \lambda[1 - F(p)]^{N-1}\right] (p - c_H) = \pi,$$

attract all shoppers would find it strictly profitable to marginally lower its price and thereby attract all shoppers. Also $p_n = c$ cannot constitute a pricing equilibrium, as each firm would have an incentive to increase its price and serve its locked-in consumers at a positive margin.

26For instance, some retailers may then choose the highest price $p_n = q_n$, leaving it to other (at least two) retailers to compete for shoppers.

27More formally, for a given equilibrium we have $\underline{p} = \inf_{n \in N, p_n \in P_n} p_n$. 

13
where we used that shoppers are attracted with probability \([1 - F(p)]^{N-1}\). Substituting equilibrium retailer profits (gross of fixed fees paid to manufacturers), \(\pi = (q_H - c_H)(1 - \lambda)/N\), we have

\[
F(p) = 1 - \frac{1 - \lambda}{\lambda N} \left[ \frac{q_H - c_H}{p - c_H} - 1 \right].
\]

Note that \(F(p)\) shifts downwards (in the sense of first-order stochastic dominance) when there are more shoppers in the market (higher \(\lambda\)), similar to the reduction in the deepest promotion \(p\) derived in (2). We postpone a formal statement of all the preceding observations (regarding qualities, profits, and prices) until we have considered also the case with \(I > 1\) products (and thus one-stop shopping).

### 4.2 One-stop Shopping \((I > 1)\)

Recall that even shoppers do not observe the offers of products \(i > 1\) before entering a shop, but only that of the potential loss leader \(i = 1\). Optimally, all retailers thus set prices \(p^i_n = q^i_n\) for \(i > 1\). As consumers thus rationally anticipate that \(\tilde{p}^i_n = q^i_n\) for \(i > 1\), they anticipate to realize no surplus on these products. Furthermore, given \(\Delta_q > \Delta_c\), it is immediate that in equilibrium, the high-quality product must be stocked for products \(i > 1\). Finally, it is then also intuitive that the respective brand manufacturer can extract again the incremental surplus realized with non-shoppers, so that \(T^i_n = (\Delta_q - \Delta_c)(1 - \lambda)/N\) for all \(i > 1\). This mirrors the results for the provision of the single product without one-stop shopping.

Turning to \(i = 1\), we note first that, by the argument in the preceding section, also the potential loss leader must be provided at high quality in an equilibrium. We turn now to pricing for \(i = 1\). When retailer \(n\) attracts a shopper with the offer of \(p^1_n\), it realizes not only the margin \(p^1_n - c_H\) on \(i = 1\), but also the additional margin \((I - 1)(q_H - c_H)\) on all other products that the consumer will put in his basket. This intensifies price competition. We express this insight first in terms of the lowest price that retailers will offer (which again will be the same in any symmetric or asymmetric pricing equilibrium). To derive this deepest promotion, note that with general \(I \geq 1\) the indifference condition (1) becomes

\[
\left[ \frac{1 - \lambda}{N} + \lambda \right] \left[ (\bar{p} - c_H) + (I - 1)(q_H - c_H) \right] = I \frac{1 - \lambda}{N} (q_H - c_H),
\]

where the right-hand side captures a retailer’s gross profits when it only attracts non-shoppers and extracts from these the total margin \(I(q_H - c_H)\) and where the left-hand side captures gross profits when, at the lowest price \(\bar{p}\), all shoppers of mass \(\lambda\) are attracted in
addition. Note that we drop the superscript \( i = 1 \) wherever this does not cause confusion. Solving (4) we now have

\[
p = c_H + (q_H - c_H) \left[ 1 - I \left( \frac{\lambda N}{1 - \lambda + \lambda N} \right) \right].
\] (5)

Clearly, the lowest promotion \( p \) is strictly decreasing in the scope of products \( I \) that consumers purchase during their one-stop shopping trip. For \( I > 1 \) pricing now involves loss leading (with positive probability) when \( p < c_H \), which transforms to

\[
I > \frac{1 - \lambda + \lambda N}{\lambda N}.
\] (6)

For instance, when we have \( I = 2 \), the fraction of shoppers must be sufficiently large so that \( \lambda > 1/(N + 1) \). Hence, even with only two competing retailers (\( N = 2 \), \( \lambda > 1/3 \) is sufficient to induce loss leading (and this condition gets relaxed as \( N \) increases). Note however that such numerical examples must be considered with care, as they are clearly driven by the imposed symmetry across products. In fact, when we instead denote by \( v \) the total margin that retailers earn with all other \( (i > 1) \) products, \( v = (I - 1)(q_H - c_H) \), the respective condition becomes\(^{28}\)

\[
v > \frac{1 - \lambda}{\lambda N} (q_H - c_H).
\]

For what follows, it is convenient to restrict consideration to parameter values for which \( p > 0 \). Clearly, negative prices would seem unreasonable, as they should attract (professional) arbitrageurs that would buy only the loss leader.\(^{29}\) Note however that our subsequent characterization of (constrained) pricing under a prohibition of loss leading will also apply to the case where we invoke this (non-negative price) constraint explicitly.

Finally, also the characterization of a symmetric pricing equilibrium immediately extends to the case of one-stop shopping, once we recall that \( (I - 1)(q_H - c_H) \) represents the additional margin earned with each attracted consumer. From this we obtain, with a slight change to (3),

\[
F(p) = 1 - \sqrt[\lambda N]{\frac{1 - \lambda}{\lambda N} \left[ \frac{I(q_H - c_H)}{p - c_H + (I - 1)(q_H - c_H) - 1} \right]}. \] (7)

We now summarize the characterization both for the case without one-stop shopping \( (I = 1) \) and for that with one-stop shopping \( (I > 1) \):

\(^{28}\)Precisely, we can write \( p = c_H + (q_H - c_H) \left( \frac{1 - \lambda}{1 - \lambda + \lambda N} - v \left( \frac{\lambda N}{1 - \lambda + \lambda N} \right) \right) \).

\(^{29}\)This may not yet be the case when \( 0 < p < c_H \), as retailers typically restrict the volume that each individual buyer may purchase.
Proposition 1  In the benchmark case with rational consumers, we have the following unique characterization of equilibrium product choice and profits for all products $I \geq 1$:

i) **Quality:** Both for the (potential) loss leader $i = 1$ and for all other products $i > 1$, high quality is chosen.

ii) **Prices:** Products $i > 1$ are always offered at prices equal to consumers’ willingness to pay, $p_n^i = q_H$. Instead, the price for $i = 1$ depends on the extent of one-stop shopping ($I$) as follows: The lowest price at which it is offered in equilibrium, $p$ as given by (5), is strictly decreasing in $I$ and it falls below the cost with which the high-quality product is procured when $I$ is sufficiently large so that (6) holds.

iii) **Profits:** At each retailer $n$ and for all product categories $i$, i.e., again independent of whether $i = 1$ or $i > 1$, the respective high-quality manufacturer realizes the same profit $(\Delta_q - \Delta_c)(1 - \lambda)/N$.

**Proof.** See Appendix A.

Proposition 1 serves as a benchmark case. Only the price of the potential loss leader $i = 1$ is responsive to an increase in the extent of one-stop shopping, but not the price of all other products and also not the quality of products in any category. The profit of a brand manufacturer in any given category $i$ also does not depend on $I$, irrespective of whether the price at which the product offered remains unchanged (that is, for $i > 1$) or whether it strictly decreases (that is, for $i = 1$).\(^{30}\)

For future reference we now report explicitly the realized values for (consumer) surplus and individual firm profits. In the presently analyzed setting, where only high quality is offered in equilibrium, this proves to be straightforward. Clearly, total realized surplus is given by $W = I(q_H - c_H)$, reflecting the facts that there is a total mass one of consumers and that the provision of products is symmetric ($q_i = q_H$ for all $i$). This surplus is shared as follows. Each brand manufacturer realizes the same profit $\Pi^M_i = \Pi^M = (\Delta_q - \Delta_c)(1 - \lambda)/N$ and each retailer the profit $\Pi^R = I(q_L - c_L)(1 - \lambda)/N$. Aggregating over all retailers and manufacturers, total firm profits thus add up to $N(\Pi^R + I\Pi^M) = I(1 - \lambda)(q_H - c_H)$. The difference to total surplus is equal to consumer rent: $CS = I\lambda(q_H - c_H)$. Thus, consumers

\(^{30}\)While the respective result is expressed in terms of the lower boundary $p$, for a given equilibrium, notably that with symmetric prices, an increase in $I$ shifts the distribution of prices for $i = 1$ downwards in the sense of First-Order Stochastic Dominance. Note that while we use differences in the extent of one-stop shopping ($I$) in order to generate situations where there is in fact loss leading—as an increase in $I$ intensifies competition for the “known-value item” (KVI) $i = 1$—we do not conduct a comparative analysis of profits and welfare in $I$. For this we would have to determine where and under what conditions consumers bought a particular subset of products “before” these were included into their one-stop shopping trip. Instead, our main comparative analysis will be in terms of policy implications for a given value of $I$.\)
and firms share the total surplus according to the fraction $\lambda$ (going to consumers) and $1 - \lambda$ (going to firms).\(^{31}\)

For the benchmark case with rational consumers, results are thus not consistent with the claims of brand manufacturers that their profits decline when their product category is a loss leader or when loss leading increases, which is the case when the extent of one-stop shopping increases but also when there is more competition (higher $\lambda$).\(^{32}\) And also the fears of consumer advocates and policymakers regarding a negative impact of loss leading on quality prove to be unfounded. Both results will however be strikingly different in case consumers are salient thinkers.

At this point it is instructive to briefly consider an extension of the benchmark model that allows for elastic demand. Such an extension, through introducing heterogeneity in consumers’ reservation values for shopping, is presented in Online Appendix B. As we show there, in this case indeed the opposite prediction to the aforementioned claim of manufacturers is obtained, which we report as follows.\(^{33}\)

**Observation.** When total demand (for shopping) is elastic as consumers have heterogeneous reservation values, in the benchmark model a brand manufacturer makes strictly higher profits when the extent of one-stop shopping $I$ increases, as this expands total demand and thus the incremental profit that its higher quality generates. This holds notably for the loss leader $i = 1$ even though the respective retail price decreases and lies below costs for high $I$.

### 4.3 Prohibition of Below-Cost Pricing

In the benchmark case, there is no scope for policy intervention to improve aggregate welfare. But when a prohibition of loss leading dampens price competition, this can reduce consumer welfare. We derive this formally in what follows, which also sets the stage for the markedly different implications when consumers are salient thinkers. We also derive implications for firm profits and the pricing equilibrium, which are of separate interest.

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\(^{31}\)The only effect that the extent of one-stop shopping $I$ thus has on surplus and profits is that it “scales up” the respective values when more or less products are bought and consumed. Note that in what follows we will always be interested in a comparative analysis (e.g., regarding a change in policy) for a given value of $I$, rather than in a comparative analysis in $I$. For the latter one would clearly have to specify how consumers have purchased the different sets of products before these were combined into their one-stop shopping trip, which is outside the scope of our analysis.

\(^{32}\)The latter is immediate from inspection of $p$ and $\Pi_M$ and thus not reported separately.

\(^{33}\)Given its additional complexity, we must however leave a similar extension of our subsequent model with salient consumers to future research, which is why we fully relegate the formalization and proof of the Observation to Online Appendix B.
Consider thus a ban on below-cost pricing, so that \( p^i_n \geq c^i_n \).\(^{34}\) Clearly, this affects at most the price for the potential loss leader and only so when there is one-stop shopping with \( I > 1 \).\(^ {35}\) More precisely, equilibrium prices are clearly only affected when otherwise \( p < c_H \), which is the case when (6) holds. While the prohibition then constrains promotions, we can show that, at first, it does not negatively affect consumer welfare. The proof of Proposition 2 provides details on retailers’ (symmetric) pricing in this regime: They then choose with strictly positive probability the lowest possible price for the loss leader, \( c_H \), and otherwise mix over a range \([p_r, q_H]\), where the lower boundary \( p_r \) lies strictly above \( c_H \). Hence, retailers choose a “quasi-bimodal” distribution of prices.\(^ {36}\) In this regime, which applies when

\[
I := \frac{1 - \lambda + \lambda N}{\lambda N} < I < \frac{1}{\lambda} =: \overline{I},
\]

firm profits and aggregate consumer welfare still remain unaffected by the prohibition of loss leading.\(^ {37}\) Expected prices increase however when

\[
I > \overline{I}.
\]

When the extent of one-stop shopping is sufficiently large so that (9) holds, retailers choose the lowest possible price \( p_n = c_H \) with probability one. Thus, when now the extent of one-stop shopping further increases (higher \( I \)), firms’ profits are shielded from an increase in competition for the loss-leading category: In this case, the prohibition of below-cost pricing strictly increases firm profits to the detriment of consumers.

\(^{34}\)We are aware that vertical contracts could, in principle, be used to partially circumvent restrictions on below-cost pricing, which is a feature that does not arise in other contributions in the literature as they do not explicitly consider retailers (even though this is precisely the area to which a possible prohibition is typically applied). For instance, if we were to consider only two-part tariff contracts, a prohibition that targeted only the marginal wholesale price could induce firms to reduce this price component and simultaneously increase the fixed part (e.g., through a higher slotting allowance). Other policy regimes could, for instance, compare average wholesale and retail prices, which would generate additional complications in a setting as ours, with (random) promotions. We must leave a detailed comparison of various policy approaches to future research.

\(^{35}\)Our focus is on retail prices, profits, and welfare. Even with a prohibition of below-cost pricing, intuitively it is still optimal for manufacturers to set the marginal wholesale price equal to cost, \( w_n = c \). However, this is no longer uniquely optimal when the prohibition effectively constrains competition (with \( p_n = w_n = c \)) so that for each retailer (residual) demand becomes inelastic.

\(^{36}\)As noted above, while we do not incorporate such a feature in our analysis, even absent the prohibition of below-cost pricing, retailers may face a lower boundary on the price of any particular good, as choosing a price below this boundary would trigger purchases from reselling intermediaries that would not purchase any other products. Our characterization clearly also applies then, and we obtain here more generally a “quasi-bimodal” distribution of prices.

\(^{37}\)However, the prohibition of below-cost pricing erodes the difference in consumer welfare between shoppers and non-shoppers. In an extended model, where \( \lambda \) was made endogenous, this would then reduce consumers’ incentives to shop, which would in turn lead to higher expected prices already in this regime. Endogenizing the fraction of shoppers may provide an interesting extension for future work.
The extent of this transfer of rents from consumers to firms that the prohibition of loss leading induces when (9) holds can be derived easily as follows. Note first that with unrestricted competition, firms’ joint profits are equal to the total rent that is realized with non-shoppers, \((1 - \lambda)I(q_H - c_H)\). When due to a prohibition of below-cost pricing it holds that \(p_n = c_H\), firms jointly extract from both non-shoppers and shoppers the total rent that they realize with all products but \(i = 1\) (which is priced at costs), \((I - 1)(q_H - c_H)\). The difference between \((I - 1)(q_H - c_H)\) and \((1 - \lambda)I(q_H - c_H)\) thus constitutes the respective rent transfer, i.e., the difference in consumer surplus \(CS\), which equals \((q_H - c_H)(\lambda I - 1)\) and is indeed strictly positive if and only if (9) holds.

Before we collect results, we finally observe how this increase in firm profits is shared between manufacturers and retailers. We find that the increase in firm profits from a prohibition of below-cost pricing is fully appropriated by manufacturers. This is again easy to see once we note that when a retailer deviates to stocking a low-quality version of \(i = 1\), then the prohibition of loss leading clearly also applies to this (lower-quality) product. (In fact, this will be the main channel through which a prohibition positively affects welfare with salient thinkers in our subsequent analysis.) But when (9) holds and all competing retailers thus choose \(p_n' = c_H\), a deviating retailer’s offer of \(p_n = c_L\) for a low-quality product will not attract any shoppers, given that \(\Delta_q > \Delta_c\). In this sense, a prohibition of below-cost pricing essentially turns the producer of the high-quality version of \(i = 1\) into a “gatekeeper” with respect to shoppers. A retailer that stocks the low-quality version instead can only attract non-shoppers, which thus yields the same outside option (deviation) profit for the retailer as in the case without policy interference. Retailers’ profits are thus not affected by the imposition of a ban on below-cost pricing.\(^{38}\)

There is now a final interesting twist in the determination of firm profits that affects how the windfall gains from policy interference, \((q_H - c_H)(\lambda I - 1)\), are shared between manufacturers. We always find an equilibrium where all benefits accrue to the manufacturers of loss leaders. But there are also other equilibria (that is, for the contractual game in \(t = 1\)), where some of these gains are shared with high-quality providers of products \(i > 1\). This follows intuitively from the complementary nature of all high-quality products: While the provider of product \(i = 1\) acts like a “gatekeeper” to shoppers, the retailer must

\(^{38}\)Note that we could clearly not derive such a nuanced difference between the implications for retailers’ and manufacturers’ profits when we abstracted from vertical contracting. As the impact on notably small manufacturers’ profits is however at the heart of policy concerns, this additional layer of complexity in our analysis is thus indeed satisfied. In fact, this will be even more so the case with salient thinking, as then there can be a strict conflict of interest between retailers and manufacturers with regards to the introduction of such a ban.
stock high quality in all categories to realize the maximum surplus. While with a prohibition of below-cost pricing there can thus exist multiple equilibria for how (in \( t = 1 \)) profits are shared (through the joint determination of fixed fees \( T_i^0 \)), we can show that, first, the provider of product \( i = 1 \) always extracts the highest share and that, second, there exists also an equilibrium where it extracts all of the transfer of consumer rent that is induced by the policy. As this is not of separate interest for our analysis, we streamline the exposition by stating the respective results only in the proof of the following Proposition.

**Proposition 2** In the benchmark case with rational consumers, prohibiting below-cost pricing has no impact on the provision of (high-quality) products and thus on aggregate efficiency. While it affects retailers’ pricing when otherwise the lowest promotion would fall below costs, \( \underline{p} < c_H \) as (6) holds, it only starts to affect aggregate firm profits and thus also aggregate consumer welfare when the extent of one-stop shopping is sufficiently large, so that also \( I > T \). Then, all retailers deterministically set the price of the former loss leader \( i = 1 \) equal to cost, \( p_n = c_H \). This leads to a strictly positive increase \((q_H - c_H)(\lambda I - 1)\) in firm profits, at the detriment of consumers, which is entirely appropriated by brand manufacturers.

**Proof.** See Appendix A.

That brand manufacturers, and notably those of loss leaders, would lobby for the prohibition of below-cost pricing is thus not surprising. But in the benchmark case with rational consumers, brand manufacturers benefit from such prohibition if and only if consumers are worse off, and there are no overall efficiency gains. Recall moreover that brand manufacturers, even those of loss leaders, are not negatively affected by below-cost pricing (Proposition 1), even when their products are promoted at prices below costs \( (\underline{p} < c_H) \). But when there is a prohibition of below-cost pricing, this reduces retail competition, as such competition effectively only takes place for the loss leader, and we have shown that notably brand manufacturers of such loss leaders are then in a position to appropriate the additional windfall profits.

5 **Loss Leading when Consumers are Salient Thinkers**

In what follows, we extend the analysis to the case with salient thinkers, starting again with a situation where below-cost pricing is allowed. In a first step, we introduce the concept of salient thinking. There, it is again instructive to first focus on the case without
one-stop shopping, $I = 1$. Subsequently, we analyze the equilibrium outcome, focusing again on the choice of quality as well as on manufacturer profits, for which finally the implications of a ban of below-cost pricing are analyzed.

As discussed in the Introduction, the main theme of our analysis is that consumers’ attention is limited and selective. We now extend this insight to consumers’ comparison of the different product attributes, price and quality. Note first that such a comparison clearly applies only to the loss-leading product $i = 1$, for which consumers observe and compare prices and qualities when they decide which retailer to visit. For this reason we suppress now the index $i = 1$, though given the additional complexity, we will also do so subsequently (under one-stop shopping) whenever this does not cause confusion. To streamline the further exposition, we will also suppress in what follows the strategic behavior of all manufacturers of products in the (non loss-leading) categories $i > 1$. This is done by specifying that retailers can always procure the respective high- and low-quality products at cost.\(^{39}\)

### 5.1 Salient Thinking

A consumer who shops around compares all alternatives $n \in N$ that are not strictly dominated, including by his outside option of value zero. Denote this set by $N_+$, which may be empty, in which case the consumer chooses his outside option.\(^{40}\) Clearly, the interesting case, where salience will play a role, is that where $N_+$ is neither empty nor a singleton. This consideration set will now be important for the determination of which attribute is more salient for a given offer, and this will ultimately matter as non-salient attributes will be “discounted” when a consumer makes his choice. Both stages are now formally described.

For this define the averages $P = \frac{1}{|N_+|} \sum_{n \in N_+} p_n$ and $Q = \frac{1}{|N_+|} \sum_{n \in N_+} q_n$. These averages describe, within the considered choice context, the respective “reference good”. Suppose

\(^{39}\)A full analysis of the model where all products are supplied strategically can be obtained upon request. While this leads to additional complexity for describing the equilibrium outcome, the only point where it makes a slight difference is when a prohibition of below-cost pricing constrains competition sufficiently. In that case, high-quality manufacturers of products $i > 1$ are (also) able to realize strictly higher profits than without such a policy.

\(^{40}\)Precisely, using already that $p_n = q_n$ for all $i > 1$, $N_+$ consists of all $n \in N$ such that $q_n - p_n \geq 0$ and that there does not exist some $n' \in N$ with both $q_{n'} \geq q_n$ and $p_{n'} \leq p_n$ (one strictly). See Bordalo et al. (2013) for a discussion and motivation of the removal of strictly dominated alternatives (“editing”). We find such “editing” particularly convincing in the present context, where, without it, a consumer would essentially keep in his consideration set offers of exactly the same product but with different prices. We extend such “editing” to a comparison with the outside option.
that the retailer’s price is below the average price, \( p_n < P \), and also quality is below the average quality, \( q_n < Q \). Then, we say that price is salient when \( \frac{p_n}{P} < \frac{q_n}{Q} \) and that quality is salient when the converse holds strictly. That is, the lower price, but not the lower quality, is salient when price is relatively lower (that is, in percentage terms), compared to the average of all considered offers, than quality, again compared to the average. Suppose next that the retailer’s price and quality are both higher than the respective average, as \( p_n > P \) and \( q_n > Q \). Then price is salient when now \( \frac{p_n}{P} > \frac{q_n}{Q} \), while when the converse holds strictly, quality is salient.

The preceding specification borrows from Bordalo et al. (2013), as discussed in the Introduction. They motivate this on the basis of results from cognitive psychology, whereby stimuli are perceived with diminishing sensitivity.\(^{41}\) As we point out below, this feature is key for our results to hold. What now heavily simplifies our analysis is the observation that for all offers, the same attribute is salient, and that the respective condition simplifies to a pairwise comparison: When \( p_L \) is the lowest price for a low-quality product and \( p_H > p_L \) that for a high-quality product, price is salient for all offers when\(^{42}\)

\[
\frac{p_L}{p_H} < \frac{q_L}{q_H}
\] (10)

and quality when the converse of (10) holds strictly.

We come now to the implications of salience, where we again rely on the specification in Bordalo et al. (2013). In any pairwise comparison between different offers, for each offer the respective non-salient attribute is “discounted” by some factor \( \delta \in [0, 1] \), compared to what a rational consumer would do. Then, when quality is salient, the best high-quality offer will be strictly preferred when \( q_H - \delta p_H > q_L - \delta p_L \) (i.e., \( \delta(p_H - p_L) < q_H - q_L \)), while when price is salient, it is only preferred when \( \delta q_H - p_H > \delta q_L - p_L \) (i.e., \( p_H - p_L < \delta(q_H - q_L) \)).\(^{43}\) Clearly, the two conditions coincide when \( \delta = 1 \), while as \( \delta \) becomes smaller, the wedge between the two conditions becomes larger. To complete the specification, we suppose that

\(^{41}\)The respective simple ratio property is then obtained from a set of additional axioms. For these as well as additional motivation, we refer to their seminal work.

\(^{42}\)To see this, suppose that \( L \geq 1 \) retailers charge the lowest price \( p_L \) for a low-quality product, whereas \( H \geq 1 \) retailers charge the lowest price \( p_H \) for a high-quality product, so that the average price is thus \( P = \frac{Lp_L + Hp_H}{L + H} \) and the average quality \( Q = \frac{Lq_L + Hq_H}{L + H} \). Then, for a low-quality offer price is salient if \( \frac{p_L}{P} < \frac{q_L}{Q} \), which indeed transforms to (10), and likewise the condition \( \frac{p_H}{P} < \frac{q_H}{Q} \) for a high-quality offer also transforms to (10). The same transformations apply when quality is salient. Incidentally, when there is no binding constraint that prohibits loss leading, given the nature of the subsequently derived equilibrium, we will have with probability one that \( L = 1 \) and \( H = 1 \), provided that indeed \( |N_+| > 1 \). This will be different only when policy constrains pricing (as then with positive probability, prices of two or more retailers will be set at the lowest possible bound).

\(^{43}\)Note here that we already use that a consumer rationally expects zero surplus from all products \( i > 1 \).
when a consumer is indifferent, he randomizes with equal probability over all respective offers, though this symmetric tie-breaking rule is without consequences for our results.

Note that the preceding description was guided by the application to shoppers’ choice. It however also applies to non-shoppers, albeit then the description is trivial as \( N_+ \) contains a single element: The respective consumer prefers to visit “his” retailer, rather than taking up his outside option, when \( q_n \geq p_n \). To summarize, while non-shoppers always consider only a single product, for shoppers the respective consideration set is larger, which is why salient thinking may matter and distort their choice. In the following analysis, we will however not be concerned with the direct implications of salient thinking on consumers’ (and notably shoppers’) true surplus. We must leave such an analysis to future work as our focus is primarily on the implications of a ban on below-cost pricing, rather than on a more detailed analysis of consumers’ behavior and consumer welfare in the unconstrained equilibrium.

### 5.2 Analysis: High-Quality Equilibrium

We now ask first whether and when there still exists an equilibrium where only high-quality products are offered and we are then interested in how salient thinking, combined with one-stop shopping, affects manufacturer profits. Clearly, with \( q_n = q_H \) for all \( n \in N \), salient thinking does not matter on equilibrium. Still, as we show, salience affects the profitability of a retailer’s strategy to deviate and stock the low-quality product. This affects both manufacturer profits and the condition for when the high-quality equilibrium exists.

Consider thus a deviating retailer \( n \) choosing \( q_n = q_L \) and some price that we denote by \( p_n = p_L \), which shoppers thus compare with the high-quality offer at the lowest price \( p_H = \min_{n' \neq n} p_{n'} \). To make price salient, \( p_L \) must satisfy (10), which we can rewrite as follows: With \( \Delta p = p_H - p_L \), it must hold that \( \frac{\Delta p}{p_H} > \frac{\Delta q}{q_H} \), i.e., the respective price decrease must be larger in percentage terms compared to the difference in qualities. When \( p_H \) is lower, so as to achieve this \( p_L \) needs to undercut \( p_H \) by less in absolute terms (\( \Delta p \)). This is precisely how both competition and one-stop shopping affect the profitability of a low-quality deviation under salience: Under one-stop shopping it is more likely that the lowest price of a rival (\( p_H \)) is lower, and the same applies when competition is more intense. By the preceding argument, this allows the deviating retailer, who offers a low-quality product, to make price salient with a smaller absolute discount (\( \Delta p \)) than what would be necessary when \( p_H \) was larger. And when a deviating low-quality retailer makes
price salient, this attenuates the quality difference (precisely, by the factor \(\delta\)). This is the mechanism by which salient thinking can increase the profits of a retailer that deviates to offering low quality, which in turn lowers the profits that a brand manufacturer can extract from the retailer—albeit, by the preceding arguments, only when salient thinkers discount non-salient attributes sufficiently and only when loss leading ensures that prices are sufficiently low. What is more, when these forces become sufficiently strong, a brand manufacturer may no longer be able to make the provision of high quality strictly optimal for any given retailer.

**Proposition 3** Suppose consumers discount non-salient attributes. Then all retailers still provide the high-quality variant of the potential loss leader \(i = 1\) if and only if

\[
\Delta_c \leq \max\{\delta \Delta_q, \frac{\Delta_q}{q_H}p\}, \tag{11}
\]

in which case the price equilibrium is unaffected. However, even when (11) holds, a brand manufacturer of \(i = 1\) realizes strictly lower profits (for \(\delta < 1\)) if

\[
I > \frac{q_H(\Delta_q - \Delta_c)}{(q_H - c_H)\Delta_q}. \tag{12}
\]

Instead, salient thinking has no implications for the choice of qualities and prices in categories \(i > 1\).

**Proof.** See Appendix A.

Thus, when (11) holds, still all retailers offer the high-quality variant in all categories and also prices are unchanged compared to the case without salient thinking. However, when the extent of one-stop shopping is large, so that (12) holds, brand manufacturers of the loss leader earn strictly less than when consumers are rational thinkers (and thus also strictly less than manufacturers of all other products \(i > 1\)). Being a potential loss leader then indeed strictly hurts the respective brand manufacturer. We now discuss first condition (11), noting that we subsequently fully analyze also the case where this condition does not hold. We then discuss condition (12).

**Discussion of condition (11).** The two parts in condition (11) formalize the discussion preceding Proposition 3. First, the stronger condition \(\delta \Delta_q \geq \Delta_c\) (rather than only \(\Delta_q \geq \Delta_c\)) ensures that consumers’ *perceived* difference between high and low quality exceeds the respective cost difference even if price is salient, such that only high quality will
be provided in equilibrium. Second, when this is not the case such that $\delta \Delta_q < \Delta_c$, a retailer’s profitability of deviating to low quality (with a corresponding optimal deviation price) depends on the prevailing price level, precisely on the already previously used lower boundary of the support (and thus the deepest promotion), $p$.\footnote{Recall also that the same lower boundary applies irrespective of whether for $N > 2$ we choose a symmetric or an asymmetric pricing equilibrium when $q_n = q_H$.} We now provide a more formal (and thus more precise) discussion of the second part of condition (11).

We show in the proof of Proposition 3 that when $\delta \Delta_q \geq \Delta_c$ does not hold (though not necessarily otherwise, as we discuss subsequently), a deviating retailer that offers a low quality finds it optimal to lower its price sufficiently so as to capture the shoppers with probability one. (Of course, the retailer could do the same also with high quality by choosing $p = p$, but this does not increase its profit.) The highest price for which the deviating retailer can achieve this is given by $p_L = p_{q_H}^{q_L}$.\footnote{It should be noted that $p_L = p_{q_H}^{q_L}$ indeed makes price salient with probability one as in an equilibrium where all (other) retailers offer high quality all prices will be above $p$ with probability one (i.e., there is no mass point at $p$).} The second part of (11) (for existence) is now obtained by comparing the respective (deviating) margin for $i = 1$, $p_L - c_L$, with that from still offering high quality (and also attracting all shoppers), $p - c_H$, where the former is not strictly higher only as long as

$$\frac{p q_L}{q_H} - c_L \leq p - c_H.$$  

This transforms to $\Delta_c \leq \frac{\Delta_q}{q_H} p$, which is the second part of (11). For what follows, it is now useful to express this condition also in terms of the extent of one-stop shopping: $I \leq \bar{I}$ with

$$\bar{I} := \frac{q_H(\Delta_q - \Delta_c)}{\Delta_q(q_H - c_H)} \left( \frac{1 - \lambda + \lambda N}{\lambda N} \right).$$

**Discussion of condition (12).** When in addition (12) holds, so that the extent of one-stop shopping is sufficiently large, salient thinking reduces the profits of high-quality manufacturers even when still only high-quality products are supplied. As we discussed above, the presence of salient thinkers then erodes the relative value of stocking a high-quality loss leader, that is, compared to the retailer’s alternative to deviate and offer a low-quality variant at a sufficiently discounted price. We now provide additional intuition for condition (12) by a more formal argument.

When there are no salient thinkers, a deviating retailer’s profit is maximized when it chooses $p_L = q_L$ and thus only attracts non-shoppers (while, by construction, a high-quality
retailer would be indifferent between pricing at the highest price \( q_H \) and strictly lower). This follows intuitively as the low-quality product is less efficient, so that when retailers with a high-quality loss leader are indifferent between such a strategy and attracting shoppers (with positive probability), the deviating retailer must strictly prefer the former.

A first insight is now that, at least for sufficiently small \( \varepsilon > 0 \) and corresponding prices \( q_L - \varepsilon \), a deviating retailer’s profits are strictly higher with salient thinkers than without (though clearly not always higher than when (still) setting \( \varepsilon = 0 \), as we argue subsequently). To show this, we need an auxiliary result, which we will use also in what follows. To derive this, take some price \( p_L \) for low quality and compare it to the minimum price \( p_H \) of all high-quality offers of \( i = 1 \). Comparing thus \((q_L, p_L)\) to \((q_H, p_H)\), we ask now when the condition that price is salient is also sufficient for that a shopper indeed prefers the low-quality offer: \( p_H - p_L > \delta \Delta q \). From substitution of the two conditions we obtain that this is the case if and only if

\[
p_L > \delta q_L. \tag{13}
\]

That is, when (13) holds, we know that making price salient is sufficient to ensure that the (deviating) offer is indeed preferred by shoppers. When we now consider a price \( p_L = q_L - \varepsilon \), condition (13) is clearly satisfied for all sufficiently small \( \varepsilon < (1 - \delta)q_L \). In such a sufficiently small neighborhood of \( q_L \), we thus know that the likelihood with which the deviating retailer attracts shoppers, \( \Pr(p_H \geq p_L q_H q_L) \), is indeed strictly higher than when there are no salient thinkers (i.e., compared to the respective probability \( \Pr(p_H \geq p_L + \Delta q) \)).

While we have thus shown that with salient thinkers a deviating (low-quality) retailer benefits more from a slightly lower price \( q_L - \varepsilon \) than without salient thinkers, such a lower price may still be less profitable than setting \( p_L = q_L \) (and thus only attracting non-shoppers, in which case salience does not matter for the deviating retailer’s profit).

Condition (12) is now derived precisely from the requirement that such a marginally lower price, so as to attract shoppers with positive probability, is indeed more profitable than setting \( p_L = q_L \).\footnote{More precisely, we thus ask when \( \frac{p_L}{p_H} < \frac{q_L}{q_H} \) (which transforms to \( p_H > p_L \frac{q_H}{q_L} \)) implies \( p_H - p_L > \delta \Delta q \) (and thus \( p_H > p_L + \delta \Delta q \)), which is clearly the case if \( p_L \frac{q_H}{q_L} > p_L + \delta \Delta q \). This can finally be transformed to obtain (13).}

Intuitively, attracting shoppers becomes more profitable when the

\[
\frac{p_L}{p_H} < \frac{q_L}{q_H} \quad \text{or} \quad p_H - p_L > \delta \Delta q,
\]

which clearly holds for all sufficiently small \( \varepsilon < (1 - \delta)q_L \). However, while we have thus shown that a slightly lower price, say \( p_L = q_L - \varepsilon \), is indeed more profitable than \( p_L = q_L \), this is not necessarily the case for all low-quality offers, as \( p_L = q_L - \varepsilon \) may still be less profitable than \( p_L = q_L \).}

\footnote{Note that this observation holds for all values \( \delta < 1 \), albeit we can already observe from (13) that the range of lower prices \( p_L \) for which price salience implies that shoppers indeed prefer low quality is clearly larger when the non-salient attribute is discounted more (lower \( \delta \)).}

\footnote{As we observed that \( \delta \) does not affect the profitability of setting a \textit{marginally} smaller price, this is also why (12) is independent of \( \delta \).}
additional margin that is realized from one-stop shopping becomes thereby larger, which is how we expressed condition (12).49

**Profits of the loss-leading brand manufacturer.** For our subsequent comparative analysis it is now instructive to explicitly characterize the profits of the loss-leading brand manufacturer when (11) and (12) hold jointly. We observed that when (12) holds, then a retailer’s deviating profit is strictly higher than in the case without salient thinkers. Recall that there we obtained for all brand manufacturers, including $i = 1$, profits of $\Pi_i^M = (\Delta_q - \Delta_c)(1 - \lambda)/N$. These are now strictly lower for $i = 1$ precisely as it becomes more attractive for the retailer to deviate to the lower-quality version. To obtain an explicit characterization, we have two cases to distinguish. In the first case, it is optimal for a deviating retailer to lower $p_L$ sufficiently so that it captures shoppers with probability one. The respective offer that makes price salient with probability one, $p_L = p^{\delta q_L}_{q_H}$, then indeed attracts shoppers for sure when the respective price $p_L$ satisfies (13). Instead, if this is not the case, it is more profitable for the retailer to lower the price only up to $p_L = \delta q_L$, implying that the retailer’s lower price will not be salient with probability one and that it also does not attract shoppers for sure.

The two cases are depicted in Figure 1, which shows how a deviating retailer’s profits change in the chosen price $p_L$. The black solid curve depicts the situation that we described first, where for sufficiently low $\delta$ ($\delta \leq p/q_H$), the deviating profit is maximized by reducing the price sufficiently so that price becomes salient with probability one. Instead, there is an interior optimum for larger values of $\delta$ (here, $\delta = 0.85$ is depicted), where shoppers are attracted with a probability that lies strictly between zero and one. This is showcased by the dashed profit line, which is notably different for $p_L < \delta q_L$. The benchmark case with rational consumers ($\delta = 1$) is depicted in gray. We note that all considered profits are gross of fixed fees, including the equilibrium profit $\pi = I(q_H - c_H)\frac{1 - \lambda}{N}$. Clearly, in all cases that are considered in Figure 1 this is higher than the maximum deviation profit, as otherwise there would not be an equilibrium where all retailers offer high quality also for $i = 1$.

49 Admittedly, this is only half of the truth, as when $I$ increases, also the distribution of equilibrium (high-quality) prices changes, precisely in the sense of a “downwards” (First-Order Stochastic Dominance) shift. Thereby, for given $p_L$, also $\Pr(p_H \geq p_L \frac{q_H}{q_L})$ becomes smaller, which exerts a countervailing force, making a price reduction to $p_L = q_L - \varepsilon$ less attractive. That the “margin effect”, as discussed in the main text, is however stronger, is not incidental. This is because, following an increase in $I$, the equilibrium price distribution adjusts precisely so that high-quality firms remain indifferent between attracting shoppers and not attracting shoppers. But with salient thinkers, the impact of a price reduction $\varepsilon > 0$ is larger for the deviating low-quality retailer.
Figure 1: Example of a deviating low-quality retailer’s expected profit as a function of its deviation price \( p_L \leq q_L \). The parameters used are \( q_H = 1, c_H = 0.75, q_L = 0.5, c_L = 0.4, N = 3, I = 2, \lambda = 0.25 \) (such that \( p = 0.75 = c_H \)).

The subsequent characterization is derived in the proof of Proposition 3. It will be used below for a more detailed comparative analysis of profits and it also summarizes formally the preceding discussion of retailers’ best deviation.

**Corollary 1** Suppose (11) and (12) hold, so that a high-quality equilibrium exists and a loss-leading brand manufacturer of product \( i = 1 \) makes strictly lower profits with salient thinkers than without salient thinkers. In this case the respective profits are given by

\[
(1 - \lambda)(\Delta_q - \Delta_c)/N - \lambda \left[ c_H \frac{q_L}{q_H} - c_L + (I - 1)(q_H - c_H)\frac{\Delta_q}{q_H} \right]
\]

when \( \delta \leq \frac{p}{q_H} \) and by

\[
(1 - \lambda)(\Delta_q - \Delta_c)/N - (1 - \lambda)(1 - \delta) \frac{q_L c_H - q_H c_L + (I - 1)(q_H - c_H)\Delta_q}{\delta q_H - c_H + (I - 1)(q_H - c_H)} / N
\]

when \( \delta > \frac{p}{q_H} \).

### 5.3 Provision of High and Low Quality

When (11) holds, salient thinking is only relevant off-equilibrium, as it affects a retailer’s outside option when it chooses low quality and lowers its price. When however (11) no
longer applies, we find that both high and low quality is offered in equilibrium, which would not be the case without salient thinkers. Now, however, while we already explained why a high-quality equilibrium does not exist when (11) no longer holds, it is also immediate that it can not be an equilibrium that all retailers choose low quality for the loss-leading product \( i = 1 \). We know that as then profits from shoppers are fully competed away, each retailer would earn strictly less than when offering instead high quality and setting \( p_n = q_H \) for \( i = 1 \).50

Denote now the likelihood that, in a symmetric equilibrium, a retailer chooses high quality by \( \alpha \). Depending on its choice, a retailer then uses different pricing strategies, which we denote by \( F_L \) and \( F_H \). We relegate a characterization of the respective pricing strategies to the proof, focusing instead again on equilibrium qualities and profits.

**Proposition 4** Suppose consumers discount non-salient attributes and that now (11) does not hold. While for all products, \( i > 1 \), salient thinking still has no implication for the choice of qualities and prices, now each retailer offers the high quality of the (potential) loss-leading product \( i = 1 \) only with probability

\[
\alpha^* = \sqrt{\frac{N - 1}{\lambda N} \left[ \frac{\Delta_q - \Delta_c}{(q_L c_H - q_H c_L) + (I - 1)\Delta_q (q_H - c_H)} \right]} < 1
\]

(14)

and the low quality with probability \( 1 - \alpha^* \). The corresponding pricing strategies for \( i = 1 \), \( F_L \) and \( F_H \), are such that price is always salient if both qualities coexist, and all shoppers surely choose the low-quality product unless all retailers stock the high-quality product. A brand manufacturer of the loss leader now makes zero profits, regardless of whether a retailer chooses high or low quality in this category.

**Proof.** See Appendix A.

When (11) no longer holds, the difference between the profits of brand manufacturers of the loss leader \( i = 1 \) and the profits of the producers of non loss leaders \( i > 1 \) thus becomes particularly stark: The former realize zero profits, either as a retailer no longer chooses high quality in this category or as it still does so, but is now indifferent between choosing low and high quality. This implication for the profits of loss leader manufacturers is intuitive: Given retailers’ indifference between stocking high or low quality for the loss leader, they are not prepared to pay a premium for high quality for product \( i = 1 \) (and only such a

50Note here as well that in a candidate equilibrium where retailer \( n \) is supposed to choose low quality for \( i = 1 \), the respective high-quality manufacturer would offer its product at cost \( (T_n = 0) \).
premium would leave the respective manufacturer with strictly positive profits). We next combine the insights from Propositions 3 and 4 in a comparative analysis. Subsequently, we analyze the implications of a ban of loss leading.

5.4 Comparative Analysis

Note first that we have $\alpha^* = 1$ when (11) holds (i.e., only the high quality is provided). Recall also that, as established in the proof of Proposition 4, in equilibrium price is salient whenever the low-quality product is offered by any retailer and that in this case all shoppers purchase low quality.

**Proposition 5** Suppose consumers are salient thinkers. The extent of salient thinking ($\delta$) as well as that of one-stop shopping ($I$) have no implication for the provision of products $i > 1$. This is different for the loss leader $i = 1$: First, the respective brand manufacturer’s profits are lower when non-salient attributes are discounted more (lower $\delta$) and when the extent of one-stop shopping is higher (higher $I$). Second, when (11) no longer holds, also the likelihood that any retailer inefficiently provides low quality, $1 - \alpha^*$, as well as the likelihood with which any consumer then purchases low quality, which is $1 - \alpha^*$ for non-shoppers and $1 - (\alpha^*)^N$ for shoppers, are strictly higher when the extent of one-stop shopping increases.

**Proof.** See Appendix A.

The preceding results speak right to the concerns of manufacturers, consumer interest groups, and policymakers, as discussed in the Introduction. Brand manufacturers of $i = 1$ are now rightly concerned when under one-stop shopping their product becomes a loss leader, with the minimum promoted price $p$ strictly decreasing in $I$. They are worse off than high-quality manufacturers (of non loss-leaders) whose products are not promoted, and they are worse off compared to the benchmark case where consumers are not overtly attentive to salient attributes. Policy makers are now equally rightly concerned about the possibility that loss leading induced by increasing one-stop shopping may lead to inefficiently lower product quality.
6 Prohibiting Below-Cost Pricing when Consumers are Salient Thinkers

As discussed in the Introduction, there are two distinct policy rationales that support calls for prohibiting below-cost pricing. First, there are concerns that “excessively low” prices for loss-leading products will lead to inefficiently low quality. Second, policymakers seem also to be concerned with the distribution of profits between retailers and manufacturers. With respect to the latter consideration it should be noted that the protection of smaller and potentially dependent manufacturers is an objective that is enshrined explicitly in the competition law of various European jurisdictions. The possibility that such “excessively low” prices, as the outcome of retailers’ competition for one-stop shoppers, would be to the detriment of the respective manufacturers has thus not only a political dimension, but also a legal dimension in some countries. In our benchmark case without salient thinkers, we showed however that a prohibition of loss leading did not increase welfare and that it only positively affected manufacturer profits at the detriment of consumers (leaving retailers’ profits unaffected). Consumer advocates and those supporting (smaller) manufacturers should thus not be seen on the same side.

Given the documented inefficiency of the outcome with salient thinkers, provided that one-stop shopping indeed leads to a sufficient decrease in the price of the loss leader, this generates the potential for beneficial policy intervention. In fact, the preceding description of why salient thinking increases a retailer’s incentives to deviate from the provision of high quality for the loss leader (and why this then leads to a reduction in the respective brand manufacturer’s profits) seems to provide an immediate intuition for why a prohibition of loss leading can now be beneficial. What such a simple argument would however neglect is that the prohibition applies also to the (price of the) provision of high quality. As we show next, this makes the implications of a ban on below-cost pricing much more nuanced when there is salient thinking. In particular, we also derive conditions for when a prohibition backfires and reduces welfare.

To streamline the subsequent exposition, we focus on a situation where the degree of salient thinking $\delta$ is sufficiently strong ($\delta \Delta_q < \Delta_c$) such that the inefficient low-quality product will indeed be provided with positive probability if the degree of one-stop shopping is high. Obviously, when policy aims first and foremost at an increase in overall efficiency, a prohibition of below-cost pricing should only be considered in this case.\(^{51}\)

\(^{51}\)However, we can also show that if this is not the case, as with $\delta \Delta_q \geq \Delta_c$ we have $\alpha^* = 1$ for any
6.1 The Case where a Ban of Below-Cost Pricing Is Beneficial

We show that a ban has the desired positive impact if

\[ \frac{q_H}{q_L} > \frac{c_H}{c_L}. \]  

(15)

The case where this is seen most immediately is that where retailers’ incentives to cut prices are sufficiently strong so that when the ban is imposed, the price of product \( i = 1 \) will be equal to cost with probability one. Consider now a (candidate) high-quality equilibrium, where thus \( p_n = c_H \) holds for all retailers for the promoted product \( i = 1 \). When (15) holds, this immediately implies that a deviating retailer who would instead stock the low-quality variant could not attract shoppers, such that this cannot be profitable.\(^{52}\) This ensures an equilibrium product provision of \( \alpha_{reg}^* = 1 \), i.e., the efficient outcome. We can show that this insight fully extends to all other cases, where retailers’ incentives to cut prices are not yet sufficiently strong to ensure that \( p_n = c_H \) holds with probability one. (Recall at this stage that for intermediate values of \( I \), retailers follow a “quasi-bimodal” strategy of choosing with strictly positive probability either \( p_n = c_H \) or strictly higher prices.)

The preceding argument also implies that, as the ban of below-cost pricing constrains a deviating (low-quality) retailer more, it reduces the value of retailers’ outside option vis-à-vis brand manufacturers of \( i = 1 \), so that the latter can now extract higher profits. Interestingly, this may already hold for values of \( I \) where the ban of below-cost pricing does not yet impact equilibrium prices, given that it may still already affect a retailer’s best deviation profit. To see this more directly, consider the case depicted in Figure 1 above, where a deviating retailer would optimally choose \( p_L = \frac{p_{qL}}{q_H} \) for \( \delta \) sufficiently low. But this price is no longer feasible when there is a ban of below-cost pricing given that \( p_{qL} < c_L \) (as depicted in the figure, where \( c_L = 0.4 \)), though the lowest high-quality price is still feasible when \( p \geq c_H \) (which holds with equality in the given numerical example). These two conditions can be jointly satisfied if and only if (15) holds.

When \( I \) is sufficiently high so that \( p_n = c_H \), we already know that this reduces price competition sufficiently so that rent is transferred from consumers to firms. When (15) holds, this is again fully appropriated by brand manufacturers.\(^{53}\) We return to consumer degree of one-stop shopping \( I \), the policy will never decrease efficiency.

\(^{52}\)Recall that without salient thinkers, the analogous condition was precisely that the high-quality product was more efficient, \( \Delta_q > \Delta_c \).

\(^{53}\)Recall that in order to refine the set of (profit-sharing) equilibria (at \( t = 0 \)), we have stipulated that manufacturers of \( i = 1 \) move first with their offer to retailers. With this specification, we show in the subsequent proof that profits of manufacturers for all categories \( i > 1 \) remain unchanged when a ban of below-cost pricing is imposed.
surplus after stating now the implications for overall efficiency and brand manufacturers’ profits.

**Proposition 6** Suppose consumers are salient thinkers and that condition (15) holds. Then a prohibition of below-cost pricing has always a non-negative effect on efficiency and the profits of brand manufacturers of the loss-leading product $i = 1$. Precisely, such a policy now ensures that regardless of the extent of one-stop shopping ($I$) and the extent of salient thinking ($\delta$), only the more efficient high-quality product is offered in equilibrium also for the loss leader ($\alpha^{*}_{\text{reg}} = 1$).

**Proof.** See Appendix A.

Recall that without salient thinkers, under a ban of below-cost pricing, brand manufacturers’ profits could only increase to the detriment of consumers. With salient thinkers, however, there are now two key differences. The first difference is that such a ban can increase overall efficiency, so that firm profits can be higher even without a transfer of consumer rent. The second difference is that now there is the possibility of a transfer of profits from retailers to the brand manufacturers of the loss leaders. We now shed more light on how, because of these two differences, both brand manufacturers as well as consumers can benefit from a ban of below-cost pricing.

For this, we first return to the case without a ban. There, we can show that joint firm profits, that is of both retailers and manufacturers, are always equal to $I(q_H - c_H)(1 - \lambda)$, i.e., the total surplus that could be realized with non-shoppers. Note that this holds also when from $\alpha < 1$ the low-quality product is stocked with positive probability. The resulting inefficiency is thus fully borne by consumers. To spell this out more formally, note that when retailers choose the high-quality product for $i = 1$ only with probability $\alpha$, total welfare is

$$W = I(q_H - c_H) - [(1 - \lambda)(1 - \alpha) + \lambda(1 - \alpha^N)] (\Delta_q - \Delta_c).$$

This takes into account that non-shoppers purchase a high-quality loss leader with probability $\alpha$ and shoppers only with probability $\alpha^N$, given that the lowest-priced low-quality product always attracts the shoppers if at least one such product is offered in equilibrium. Subtracting from this total firm profits $I(q_H - c_H)(1 - \lambda)$, true consumer surplus in the unregulated case is thus

$$CS = I(q_H - c_H)\lambda - [(1 - \lambda)(1 - \alpha) + \lambda(1 - \alpha^N)] (\Delta_q - \Delta_c).$$
When there is a ban on below-cost pricing, where we know that only high quality is provided, we need to distinguish between two cases. We know that for \( I \leq \bar{I} \), a ban does not result in a transfer of rents from consumers to firms, so that then \( CS_{\text{reg}} = I(q_H - c_H)\lambda \), while we also know that for \( I > \bar{I} \), consumer rent is strictly lower, as it then equals only the total surplus from the loss leader: \( CS_{\text{reg}} = q_H - c_H \).

Now when \( \tilde{I} < \bar{I} \), there exists an interval of values \( I \in (\tilde{I}, \bar{I}] \) so that the ban of below-cost pricing leads surely to strictly higher consumer surplus (given that \( \delta \Delta_q < \Delta_c \) such that \( \alpha < 1 \)).\(^{54}\) Then, the ban prevents a reduction in overall efficiency that would otherwise be fully borne by consumers, while there is no dampening of competition as still \( I \leq \bar{I} \). In this region, also manufacturer profits in the loss-leading category strictly increase, as without a ban they would not obtain positive profits. Thus in the considered case (and clearly, from continuity of profits and consumer welfare, still when \( I \) is somewhat higher), interests of brand manufacturers of the loss leader and of consumers are aligned: Both strictly benefit from a ban of below-cost pricing. However, when \( I \) is sufficiently large, so that without the ban the competitive price of the “known-value item” (KVI) \( i = 1 \) is sufficiently low, for consumers the negative effect of a higher price dominates and interests are no longer aligned with those of the loss-leading manufacturer. A ban is thus only strictly beneficial for consumers when the extent of one-stop shopping takes on intermediate values, while it is neutral for low values of \( I \) and makes consumers strictly worse off for high \( I \). Figure 2 illustrates the preceding discussion. For any given extent of one-stop shopping \( I \), it compares the unconstrained outcome with that under a ban on below-cost pricing. We see that \( \alpha < \alpha_{\text{reg}} = 1 \) when \( I > \bar{I} \) and we see that with respect to consumer surplus there are three different cases: When \( I \leq \tilde{I} \) the ban has no impact on consumer surplus. Then there is an intermediate range where the ban increases consumer surplus, \( CS_{\text{reg}} > CS \), while for sufficiently high \( I \) the ban leads to strictly lower consumer surplus with \( CS_{\text{reg}} < CS \).

**Corollary 2** Suppose consumers are salient thinkers and that condition (15) holds. Then a sufficient condition for that a ban on below-cost pricing also strictly increases consumer surplus is that \( \tilde{I} < I < \bar{I} \) (given that \( \delta \Delta_q < \Delta_c \)), while when the extent of one-stop shopping \( I \) is sufficiently large, consumers are strictly worse off.

\(^{54}\)Rewriting \( \tilde{I} < \bar{I} = 1/\lambda \), we can see that the condition is more likely to hold when, holding now \( I \) fixed, competition is otherwise not too strong (low \( \lambda \)). This is intuitive as otherwise the (expected) competitive price for \( i = 1 \) would be so low that the ban leads to lower consumer surplus already for low \( I \). Observe also that \( N \) affects positively the incentives for a retailer to choose low quality so as to attract (salient-thinking) shoppers, as shoppers then become relatively more important (in size) compared to the retailer’s share of non-shoppers \( (1/N) \). Taken together, the condition is thus more likely to hold when \( \lambda \) is low and \( N \) is high.
Figure 2: Depiction of the equilibrium product-choice-probability $\alpha$ and consumers’ expected surplus, both under a ban of below-cost pricing and without, for the case where $q_H > q_L$. The parameters used are $q_H = 1$, $c_H = 0.75$, $q_L = 0.5$, $c_L = 0.4$, $N = 3$, $\lambda = 0.25$, $\delta < \Delta c / \Delta q$. 
When consumers are salient thinkers, our analysis thus provides some support for the claims of brand manufacturers, policy makers and consumer advocates that below-cost pricing reduces (consumer) welfare, and that a ban would therefore be justified. But we also show that these claims indeed only receive some support: Such a policy may instead also backfire. Even when condition (15) holds, as we presently stipulate, consumers are actually worse off precisely when unconstrained competition would lead to particularly low prices for the loss leader. And we show next that when condition (15) does not hold, even the preceding welfare results are overturned (albeit, as we show as well, brand manufacturers of loss-leading products would then not want to lobby for such a ban).

6.2 The Case where Policy Backfires

We show that in terms of overall efficiency, a ban of below-cost pricing backfires precisely when the converse of condition (15) holds. Again, the intuition is seen most immediately when we take the case where the ban sufficiently constrains pricing so that, in a candidate high-quality equilibrium, we would have \( p_n = c_H \) for \( i = 1 \) at all retailers. Suppose now that a retailer deviates and offers a low-quality version of product \( i = 1 \) at price \( c_L \), for which its lower price becomes salient due to \( \frac{c_H}{q_L} < \frac{c_L}{q_L} \) (and also provides a higher perceived utility due to \( \delta \Delta_q < \Delta_c \)). Hence, as this deviation expands the retailer’s demand for products \( i > 1 \) while keeping its (zero) margin on product \( i = 1 \) unchanged, this clearly constitutes a profitable deviation.55 The pitfall of a ban of below-cost pricing is thus that it also constrains retailers’ pricing with a high-quality product, and not only with a low-quality product.56 We now have that \( \alpha_{\text{reg}}^* < \alpha^* \) can hold strictly and that even \( \alpha_{\text{reg}}^* = 0 \) may hold, i.e., that all retailers offer the low quality for sure when there is a ban of below-cost pricing, while this was never the case without a ban. From our preceding discussion this implies that consumers are always worse off, as they both have to bear the costs of this increased inefficiency and may suffer from higher prices when the ban sufficiently constrains competition.

Figure 3 depicts the case where condition (15) no longer holds. Again, we compare, for any given \( I \), efficiency and consumer surplus with and without a ban of below-cost pricing. We see that precisely when the extent of one-stop shopping is sufficiently large with \( I > \hat{I} \), the ban has the described negative implication for efficiency as then \( \alpha_{\text{reg}}^* < \alpha^* \).

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55 Albeit not the most profitable one, as we can easily show.
56 By the same token, when all retailers offer low quality for \( i = 1 \) at a price \( p_n = c_L \), then a deviating retailer that offers high quality could not make quality salient, as the retailer could not reduce its price sufficiently.
Figure 3: Depiction of the equilibrium product-choice-probability $\alpha$ and consumers' expected surplus, both under a ban of below-cost pricing and without, for the case where $q_H < q_L$. The parameters used are $q_H = 1$, $c_H = 0.65$, $q_L = 0.5$, $c_L = 0.3$, $N = 2$, $\lambda = 0.25$, $\delta < \Delta_c/\Delta_q$.

And we see that from then onwards also consumers surplus is strictly smaller with the ban, $CS_{reg} < CS$.

Finally, we now find that a prohibition of below-cost selling never harms retailers, while they may even strictly benefit if the extent of one-stop shopping is large (at the same time, we can show that the ban keeps loss-leading manufacturers’ profits unaffected). Precisely, retailers benefit strictly in the case where the ban constrains pricing sufficiently so that $p_n = c_L$ (now for the low-quality equilibrium that arises when $I \geq T$, cf. Figure 3). The intuition for this is as follows. Recall that we stipulated that the low-quality version (in each product category $i \in I$) was competitively provided, implying that, in contrast to the preceding cases, in the considered equilibrium there is no longer a “gatekeeper” for the “known-value item” $i = 1$ that controls a retailer’s access to shoppers. As retailers are shielded from further competition for the provision of product $i = 1$ due to the ban of below-cost pricing when $I \geq T$, they can appropriate the ensuing transfer of rents from consumers.

We collect our findings in the subsequent proposition. As its proof is somewhat tedious, we relegate it to the online appendix. There, we also derive in more detail when the
respective assertions hold indeed strictly.

**Proposition 7** Suppose consumers are salient thinkers and that the converse of condition (15) holds strictly (and $\delta \Delta_q < \Delta_c$). Then a prohibition of below-cost pricing always has a negative effect on efficiency and consumer surplus, while it keeps manufacturers’ profits unchanged.

**Proof.** See Online Appendix B.

Hence, when condition (15) no longer applies, the impact of a ban of below-cost pricing is in conflict with two out of the three aforementioned policy goals: While it does not affect the profits of the manufacturers of loss leaders, it leads to lower welfare and lower consumer surplus. While we do not advocate for a “sophisticated” policy response that would be contingent on condition (15), which may be hard to establish, we still have the following clear-cut recommendation: If policymakers’ main priority lies in maximizing overall efficiency or in supporting the profits of the loss-leading manufacturers, in our model they should impose such a ban when this is advocated for by the affected manufacturers, albeit this risks hurting consumers precisely when the loss-leading price would otherwise be particularly low. In contrast, when manufacturers of loss leaders remain silent regarding a ban of below-cost pricing, there is no scope for beneficial policy intervention.57

### 7 Conclusion

Our analysis in this paper is motivated by the following two observations. First, one-stop shopping leads consumers to base their choice of retailers only on a comparison of a selected number of products (“known-value items”). These are consequently the products on which price competition is fiercest, leading potentially to loss leading. Second, consumers’ attention to different attributes of a product, notably price and quality, may not be “fixed”, but may depend instead on market circumstances, precisely on whether a particular offer is “saliently different” along the respective attribute, compared to the other offers in the market. Our analysis captures these two features by combining a model of one-stop stopping and limited information about prices, set into a model of sales (Varian 1980), with recent developments in behavioral economics (precisely, the formalization of salience in Bordalo et al. (2013)).

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57 Note at this point that we stipulated an overall inelastic demand, as is widely done in the literature on “promotions”. This excludes a deadweight loss when prices rise. We must leave it to future research to extend, in particular, the analysis with salient thinkers in this direction.
We derive novel normative and positive implications from this model, showing in particular the difference that the presence of salient thinkers makes, compared to a baseline case where consumers have rational attention. Our first implications concern manufacturers’ profits. The role of salient thinkers is crucial, as an increase in the extent of one-stop shopping will only then negatively affect manufacturer profits. Thus, when consumers are salient thinkers, but not so otherwise, we can indeed support manufacturers’ concerns when retailers use the respective product category for loss leading. The second set of implications that we derive concerns the choice of product quality. We show how high-quality products may be crowded out inefficiently when these are used for promotions and when competition is fierce or the extent of one-stop shopping is large.

As discussed in the Introduction, these implications directly relate to the ongoing policy debate about possible detrimental effects of retailers’ deep discounting in loss-leading product categories. We thus explicitly consider a policy of prohibiting below-cost pricing. This only affects product choice when consumers are salient thinkers, but not so otherwise. And we identify the precise circumstances when this intervention increases or decreases overall efficiency. That brand manufacturers, rather than retailers, lobby for a prohibition of below-cost pricing is in our model a necessary, albeit in no way a sufficient condition for that such a policy enhances efficiency or even consumer welfare. But we also find that interests of policy makers, consumer advocates and brand manufacturers may indeed be aligned when consumers are salient thinkers.

8 References


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9 Appendix A: Proofs

Proof of Proposition 1. To streamline the exposition of the proof, we first recognize that, by the arguments in the main text, each retailer will stock high quality at categories \(i > 1\) and that the corresponding prices equal the respective valuation \(q_H\). We next extend this result also to \(i = 1\). We argue to a contradiction and suppose \(q_n = q_L\) (suppressing for now the notation for \(i = 1\)). We next write expected demand at each retailer as a function of consumers’ (perceived) net surplus \(s_n\), which is \(s_n = q_L - p_n\), given the anticipated monopolistic pricing at all products \(i > 1\). Given the anticipated strategies at all other retailers \(n' \neq n\), in an equilibrium retailer \(n\) faces some expected demand \(X_n(s_n)\) (which, at this point, we need not derive explicitly). Take now some equilibrium price \(p_n\) (i.e., a price for \(i = 1\) in the respective support of retailer \(n\)) and expected demand \(X_n(q_L - p_n)\). Consider a deviation to \(q_n = q_H\) and the choice of a price \(\hat{p}_n = p_n + \Delta q\), which thus realizes the same expected demand. In case the high-quality manufacturer offered his product at the wholesale price \(w_n = c_H\), the retailer’s increase in profit (gross of \(T_n\)) would then be at least

\[
\kappa = \frac{1 - \lambda}{N} (\Delta q - \Delta c) > 0.
\]

By setting \(T_n = \kappa/2\) the respective high-quality manufacturer can thus ensure that its (deviating) offer is accepted for sure and that it generates strictly positive profits, which results in a contradiction to the claim that retailer \(n\) offers low quality for product \(i = 1\).

We next turn to wholesale contracts, noting first that we restrict consideration to supporting an equilibrium where marginal wholesale prices equal marginal costs. Take first \(i = 1\) and note that, at retailer \(n\), the respective price \(p_n^1\), where we now make the dependency on \(i = 1\) explicit, then maximizes

\[
\left[ (p_n^1 - w_n^1) + (I - 1)(q_H - c_H) \right] X_n(q_H - p_n^1),
\]

so that joint profits of retailer \(n\) and the respective high-quality manufacturer are clearly maximized when \(w_n^1 = c_H\). This extends now also to the providers of products \(i > 1\) as follows. Then, the respective objective function of the retailer equals

\[
\left[ (p_n^1 - c_H) + (I - 2)(q_H - c_H) + (q_H - w_n^1) \right] X_n(q_H - p_n^1),
\]

which indeed equals that in (16).

Next, given marginal wholesale prices equal to marginal manufacturer costs, the determination of fixed fees follows from the argument in the main text as follows. From Baye et
al. (1992) we know retailers’ unique on-equilibrium gross profits $\pi$, as stated in the main text. We show next that when a retailer rejects the offer of the high-quality manufacturer of any category $i$, the deviation profit, gross of the fixed fees $T_n^i$ for all other manufacturers $j$, is obtained by attracting only the respective locked-in fraction of consumers. We show this first when $i = 1$. To show this for any price equilibrium (given multiplicity when $N > 2$), consider generally any two levels of net utility that a retailer may offer to consumers, $s' < s''$. We show that when offering $s'$ is weakly preferred for a retailer that (on-equilibrium) chooses $q_n^1 = q_H$, offering the lower net utility is strictly preferred when the retailer deviates to $q_n = q_L$. Formally: Making use of the expression $X_n(s)$ for expected demand, as well as prices $p'_H = q_H - s'$ and $p''_H = q_H - s''$ with high quality and prices $p'_L = q_L - s'$ and $p''_L = q_L - s''$ with low quality, we claim that

$$
[(q_H - s' - c_H) + (I - 1)(q_H - c_H)] X_n(s') \geq [(q_H - s'' - c_H) + (I - 1)(q_H - c_H)] X_n(s'')
$$

implies

$$
[(q_L - s' - c_L) + (I - 1)(q_H - c_H)] X_n(s') > [(q_L - s'' - c_L) + (I - 1)(q_H - c_H)] X_n(s''),
$$

which indeed holds from $\Delta_q > \Delta_c$.\(^{58}\) As we know that offering zero net utility ($p_n = q_H$) yields the equilibrium profits, offering zero net utility (now $p_n = q_L$) must then indeed be uniquely optimal when deviating to $q_N = q_L$. From the respective expressions $\pi$ and $\pi_d$, we then obtain from a retailer’s indifference, which must hold by optimality for the manufacturer, that

$$
T_n^1 = \pi - \pi_d = (\Delta_q - \Delta_c) \frac{1 - \lambda}{N}.
$$

We can now apply this argument also to all categories $i > 1$, after noting the equivalence of the respective expressions as used already when we compared (16) with (17). \textbf{Q.E.D.}

**Proof of Proposition 2.** As in the proof of Proposition 1, it is again immediate that high quality is provided at all categories $i > 1$. Take now $i = 1$, where the respective price may now be constrained. Recall that, without such a constraint, the argument in Proposition

\(^{58}\)More precisely, when we impose equality on the first condition, we can write this as

$$
[(q_L - s' - c_L) + [(q_H - c_H) - (q_L - c_L)] + (I - 1)(q_H - c_H)] X_n(s') =
[(q_L - s'' - c_L) + [(q_H - c_H) - (q_L - c_L)] + (I - 1)(q_H - c_H)] X_n(s''),
$$

i.e.,

$$
[(q_L - s' - c_L) + (I - 1)(q_H - c_H)] X_n(s') =
[(q_L - s'' - c_L) + (I - 1)(q_H - c_H)] X_n(s'') + (\Delta_q - \Delta_c) [X_n(s'') - X_n(s')].
$$

From this, our second inequality follows as $\Delta_q > \Delta_c$ and as clearly $X_n(s'') > X_n(s')$. 

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44
relied on arguing to a contradiction, as a retailer and a high-quality manufacturer could then profitably deviate by providing consumers with the same net utility through a high-quality product and pocketing the difference $\Delta_q - \Delta_c$. This argument now still fully applies after noting that when some retailer $n$ can offer consumers the net utility $s_n = q_L - p_n$ with a low-quality product while this still satisfies the policy constraint, then from $\Delta_q > \Delta_c$ the constraint is also satisfied after deviating to high quality and adjusting the price upwards by $\Delta_q$. Next, again as in Proposition 1, it is straightforward that marginal wholesale prices equal to marginal costs constitute an equilibrium outcome.\(^{59}\)

With these preliminary results at hand, we turn now to a characterization of a symmetric pricing equilibrium. This is then followed by a determination of manufacturer profits (that is, fixed fees). It now proves useful to use the notation $v = (I - 1)(q_H - c_H)$ and to also express thresholds in terms of $v$, rather than $I$, albeit there clearly exists a one-to-one mapping between those two. More precisely, we have that $v = \frac{1 - \lambda}{\lambda N} (q_H - c_H)$ and $\bar{v} = \frac{1 - \lambda}{\lambda} (q_H - c_H)$.

**Lemma 1** Take the baseline model without salient thinkers and suppose that below-cost pricing is prohibited. Then for $v \leq \underline{v}$, the prohibition does not bind and thus does not affect the set of pricing equilibria. For $v > \underline{v}$, we have the following characterization:

i) When $\underline{v} < v < \bar{v}$, then there exists a (for $N = 2$ unique) symmetric pricing equilibrium in which, for the price of the product $i = 1$, retailers sample $c_H$ with probability $\beta^* \in (0, 1)$, whereas with the remaining probability, they sample prices continuously from the CDF

$$F_r(p) := 1 - \frac{N^{-1}}{1 - \beta^*} \sqrt{\frac{1 - \lambda}{\lambda N} \left( \frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}$$  

with support $[p_r, q_H]$, where $\beta^*$ is defined implicitly by the unique solution to

$$\frac{1 - (1 - \beta)^N}{\beta} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

and

$$p_r := c_H - v + \frac{q_H - c_H + v}{1 + \frac{\lambda N}{1 - \lambda} (1 - \beta^*)^{N-1}} \in (c_H, q_H).$$

ii) When $v \geq \bar{v}$, all retailers set for $i = 1$ the minimum price $p_n = c_H$.

\(^{59}\)Recall that we do not claim uniqueness, which would precisely not hold when the policy sufficiently constrains competition so that a retailer’s residual demand becomes inelastic.
**Proof of Lemma 1.** Recall that \( p < c_H \) holds if and only if \( v > \bar{v} \). Since in any pricing equilibrium with high-quality products, no matter whether symmetric or asymmetric, no firm samples prices below \( p \), the set of pricing equilibria is clearly not affected if \( v \leq \bar{v} \).

Suppose next that \( v \geq \bar{v} \). If all retailers choose \( p_n = c_H \), each makes a profit of \( \frac{v}{N} \). Note that here and in what follows, the considered profits are gross of the fixed payment to the manufacturer. Deviations downward are impossible due to the price floor and the best upward deviation is to \( q_H \), which realizes \( (q_H - c_H + v) \frac{1-\lambda}{N} \) and does not exceed \( \frac{v}{N} \) if and only if \( v \geq \bar{v} \).

To show uniqueness of this equilibrium, assume the contrary, such that at least one retailer’s (possibly mixed) pricing strategy does not put all probability on \( c_H \). Denote the largest upper support of retailers’ pricing strategy by \( \hat{p} \in (c_H, q_H] \). Clearly, it cannot be part of an equilibrium that multiple retailers have a mass point at \( \hat{p} \), which implies that there must be at least one retailer that never attracts the shoppers when sampling \( \hat{p} \), thereby realizing profits

\[
(\hat{p} - c_H + v) \frac{1-\lambda}{N} \leq (q_H - c_H + v) \frac{1-\lambda}{N}.
\]

Given this inequality, by deviating to \( c_H \) this retailer could for \( v > \bar{v} \) strictly increase its profit to at least \( \frac{v}{N} \) (as at worst, all other retailers charge \( c_H \) for sure).

Suppose finally that \( v \in (\bar{v}, \bar{v}) \). It is straightforward to verify that \( p_n = c_H \) and \( p_n \in [\underline{p}, q_H] \) all yield the same profit of \( \frac{1-\lambda}{N}(q_H - c_H + v) \). Precisely, for \( \underline{p} \) note that the expected profit is \( (p_n - c_H + v) \left[ \frac{1-\lambda}{N} + \lambda(1-\beta)^{N-1} \right] \), as shoppers can only be attracted if none of the \( N-1 \) rivals samples \( c_H \). And with \( p \in (\underline{p}, q_H) \), the expected profit is \( (p - c_H + v) \left\{ \frac{1-\lambda}{N} + \lambda[(1-\beta)(1-F_r(p))]^{N-1} \right\} \), as shoppers can only be attracted if all rivals sample a price above \( p \). Provided that \( \beta \in (0, 1) \), which will be verified below, \( F_r(p) \) in (18) is strictly increasing in \( p \), with \( F_r(\underline{p}) = 0 \) and \( F_r(q_H) = 1 \). Finally, if a retailer samples \( c_H \), its expected profit can be written as

\[
\pi_i(c_H) = (c_H - c_H + v) \left[ \frac{1-\lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1-\beta)^{N-1-j} \frac{\lambda}{j+1} \right],
\]

as it has to share the shoppers with \( j \in \{0, ..., N-1\} \) rivals, which happens with probability \( \binom{N-1}{j} \beta^j (1-\beta)^{N-1-j} \), respectively. Using that

\[
\sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1-\beta)^{N-1-j} \frac{1}{j+1} = \frac{1}{\beta N} \left[ 1 - (1-\beta)^N \right],
\]
which follows from the binomial theorem, this uniquely pins down $\beta = \beta^*$. (Note that the left-hand side of (19) is strictly decreasing in $\beta$ for $\beta \in (0, 1)$, as it can be rewritten as $f(\beta) = \sum_{k=0}^{N-1} (1 - \beta)^k$.) Note finally that no retailer can profitably deviate to $p_n \in (c_H, p_r)$ as, by construction, a strictly higher profit is realized with $p_r$ instead. **Q.E.D. (Lemma 1)**

We finally characterize the equilibrium fixed transfers $T_i^i$, where we focus on equilibria in which all transfers and thus profits are symmetric for manufacturers of products $i > 1$. To abbreviate the exposition, for now we simply refer to the respective transfers by $T^i$ and to that for $i = 1$ by $T^1$. Note first that obviously, compared to the case where below-cost pricing was not prohibited, the equilibrium outcome differs only when retailer competition is sufficiently constrained so that total industry profits are strictly higher (and thus given by $(I-1)(q_H - c_H)$ across all retailers). It is next straightforward from optimality that in equilibrium $(T^1, T^i)$ are jointly determined so that when a retailer was to reject any of the respective contracts, he would then (weakly) prefer to reject also all other contracts, thereby realizing profits of $I \frac{1-\lambda}{N} (q_L - c_L)$ (where it is now more convenient not to substitute $v$). With this in mind, all equilibrium pairs $(T^1, T^i)$ must satisfy first the indifference requirement that

$$\frac{I-1}{N} (q_H - c_H) - T^1 - (I-1)T^i = I \frac{1-\lambda}{N} (q_L - c_L).$$

In addition, we have the following two conditions for manufacturers’ minimum profits. For $i > 1$ this is given by $\frac{1-\lambda}{N} (\Delta_q - \Delta_c)$, as otherwise a manufacturer could marginally raise its transfer, so that a retailer would reject all other contracts but the one of the deviating manufacturer. The same minimum profits apply also to $i = 1$, but here profits must also lie above an additional boundary. This follows from the “gatekeeping” role of $i = 1$, as when a retailer only accepts the contract of $i = 1$, then the maximum joint profits are given as the maximum of either $\frac{1-\lambda}{N} [(q_H - c_H) + (I-1)(q_L - c_L)]$ or $\frac{1}{N} (I-1)(q_L - c_L)$. Deducting retailers’ rejection profits of $I \frac{1-\lambda}{N} (q_L - c_L)$, we have the following minimum profits of the respective brand manufacturers:

$$T^i = \frac{1-\lambda}{N} (\Delta_q - \Delta_c),$$

$$T^1 = \max \left\{ \frac{1-\lambda}{N} (\Delta_q - \Delta_c), \frac{\lambda I-1}{N} (q_L - c_L) \right\}.$$

To complete the specification, note that for $i = 1$ we then have from application of $T^i$,
together with (21), the upper boundary

$$T^1 = \frac{1 - \lambda}{N} (\Delta_q - \Delta_e) + \frac{\lambda - 1}{N} (q_H - c_H).$$

Overall, we can support as part of an equilibrium all combinations $$(T^1, T^i)$$ where $$T^1 \in [T^1, T^i]$$ and where $$T^i$$ then satisfies (21). Q.E.D.

**Proof of Proposition 3.**

Roadmap. We start by proving that condition (11) is necessary for a high-quality equilibrium to exist (Lemma 2). We proceed to show that condition (11) is also sufficient for a high-quality equilibrium to exist. This is done in three steps. (A) In Lemma 3, we first characterize a retailer’s optimal deviation price when it deviates to stocking low quality; (B) in Lemma 4, the corresponding maximal deviation profit is pinned down; and (C) in Lemma 5, this maximal deviation profit is compared to retailers’ gross profits in a high-quality equilibrium, and we establish that deviating is not profitable if condition (11) holds. The final step of our proof is given by Lemma 6, where we use retailers’ maximal deviation profits (as characterized in Lemma 4) to solve for the equilibrium profits of brand manufacturers $$i = 1$$, given that condition (11) is satisfied. The statement about manufacturer profits follows immediately from this.

Before proceeding with the proof, recall that for brevity’s sake we presently restrict consideration to the strategic provision of products in category $$i = 1$$. Also, from the arguments for the case with rational consumers it is again immediate that there exists an equilibrium where the product is offered at a marginal wholesale price equal to cost. Observe finally that we again abbreviate expressions in the proof by denoting

$$v = (I - 1)(q_H - c_H).$$

**Lemma 2** A high-quality equilibrium can only exist if condition (11) holds.

**Proof.** We show that when condition (11) does not hold, then a retailer $$n$$ could profitably deviate and offer instead a low-quality product. For this we first derive some characteristics that apply to any (candidate) high-quality equilibrium. The first is that

$$\bar{\pi} = \frac{1 - \lambda}{N} (q_H - c_H + v)$$

(22)

provides an upper boundary for each retailer’s gross profits. This is obtained by setting $$T_n = 0$$ and using that in any pricing equilibrium with $$q_n = q_H$$ and symmetric retailer costs $$c_H$$, all retailers must realize gross profits of $$\bar{\pi}$$. A formal proof for this can be found
in Baye et al. (1992), Lemmas 7 and 8, as stated within their proof of Theorem 1. We next derive, across all equilibria, the lowest price that any retailer may charge. As the retailer can ensure the profit (22), gross of any $T_n$, by charging $p_n = q_H$, we can obtain this price $p$ from the requirement

$$(p - c_H + v) \left( \frac{1 - \lambda}{N} + \lambda \right) = \tilde{\pi}.$$  

The respective minimum price is thus $p$, as defined in the main text.

Consider now a deviation of retailer $n$ to stocking $q_n = q_L$. This deviating retailer can attract all shoppers with probability one by pricing at the minimum of $p \frac{q_L}{q_H}$ (which guarantees that price is salient) and $p - \delta \Delta_q$ (which guarantees that the retailer attracts the shoppers, provided that price is salient). If the minimum is given by the latter condition ($p > \delta q_H$), the deviating retailer’s profit with this price satisfies

$$(p - \delta \Delta_q - c_L + v) \left( \frac{1 - \lambda}{N} + \lambda \right) > \tilde{\pi} \quad \text{if} \quad \Delta_c > \Delta^q \frac{q_H}{p}. \tag{23}$$

If instead $p \leq \delta q_H$, the deviating retailer’s profit satisfies

$$(p - \delta \Delta_q - c_L + v) \left( \frac{1 - \lambda}{N} + \lambda \right) > \tilde{\pi} \quad \text{if} \quad \Delta_c > \delta \Delta_q. \tag{24}$$

Hence, no matter whether $p > \delta q_H$ or $p \leq \delta q_H$, if it holds that

$$\Delta_c > \max\{\delta \Delta_q, \Delta^q \frac{q_H}{p}\}, \tag{25}$$

a deviation to choosing $q_n = q_L$ is strictly profitable. This proves that condition (11) (the converse of condition (25)) is indeed necessary. Q.E.D. (Lemma 2).

**Lemma 3** Define $\delta := \frac{p}{q_H}$ and consider a (candidate) equilibrium in which all retailers choose the high-quality product and a symmetric pricing strategy. Then, the optimal deviation price $p^*_{\text{dev}}$ of a retailer offering a low-quality product is

$$p^*_{\text{dev}} = \begin{cases} q_L & \text{if } \frac{q_H}{q_L} > \frac{c_H - v}{c_L - v} \\ \{p_{\text{dev}} | p_{\text{dev}} \in [p \frac{q_L}{q_H}, q_L]\} & \text{if } \delta \leq \delta \quad \text{and} \quad \frac{q_H}{q_L} = \frac{c_H - v}{c_L - v} \\ \{p_{\text{dev}} | p_{\text{dev}} \in [\delta q_L, q_L]\} & \text{if } \delta > \delta \quad \text{and} \quad \frac{q_H}{q_L} = \frac{c_H - v}{c_L - v} \\ \{p_{\text{dev}} | p_{\text{dev}} \in [p - \delta(q_H - q_L), \delta q_L]\} & \text{if } \delta > \delta \quad \text{and} \quad \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \quad \text{and} \quad \delta > \frac{c_H - c_L}{q_H - q_L} \\ \{p_{\text{dev}} | p_{\text{dev}} \in [p - \delta(q_H - q_L), \delta q_L]\} & \text{if } \delta > \delta \quad \text{and} \quad \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \quad \text{and} \quad \delta = \frac{c_H - c_L}{q_H - q_L} \\ \{p_{\text{dev}} | p_{\text{dev}} \in [p - \delta(q_H - q_L), \delta q_L]\} & \text{if } \delta > \delta \quad \text{and} \quad \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \quad \text{and} \quad \delta < \frac{c_H - c_L}{q_H - q_L}. \end{cases}$$
Proof. Denote the (random) minimum price of all \( N - 1 \) rivals by \( \tilde{p}_{\text{min}} \), which is distributed according to

\[
F_{\text{min}}(p_{\text{min}}) = 1 - \frac{1 - \lambda}{\lambda N} \left( \frac{q_H - c_H + v}{p_{\text{min}} - c_H + v} - 1 \right).
\]

We now consider two possibilities. In the first case, the deviating retailer’s price \( p_{\text{dev}} \) is such that quality is salient with \( \frac{q_H}{q_L} > \frac{\tilde{p}_{\text{min}}}{p_{\text{dev}}} \). Note that then the retailer can only attract the shoppers when \( q_L - \delta p_{\text{dev}} \geq q_H - \delta \tilde{p}_{\text{min}} \). But this condition, together with that of “quality salience”, can only hold jointly if both \( \tilde{p}_{\text{min}} < p_{\text{dev}} \frac{q_H}{q_L} \) and \( \tilde{p}_{\text{min}} \geq \frac{q_H - q_L}{\delta} + p_{\text{dev}} \), which requires that

\[
p_{\text{dev}} \frac{q_H}{q_L} > \frac{q_H - q_L}{\delta} + p_{\text{dev}}.
\]

Solving this for \( p_{\text{dev}} \), this condition is clearly incompatible with \( p_{\text{dev}} \leq q_L \), which is required to ensure that the offer is accepted by any consumer. It thus follows that the deviating retailer can only attract the shoppers with its low-quality product if price is salient. Precisely, then two conditions must hold jointly: The “salience constraint”

\[
\tilde{p}_{\text{min}} \geq p_{\text{dev}} \frac{q_H}{q_L} \tag{26}
\]

and the “competition constraint”

\[
\tilde{p}_{\text{min}} \geq p_{\text{dev}} + \delta(q_H - q_L). \tag{27}
\]

The salience constraint binds if \( p_{\text{dev}} \geq \delta q_L \) and the competition constraint binds if \( p_{\text{dev}} \leq \delta q_L \).

Clearly, the competition constraint (27) is irrelevant if the deviating retailer can already guarantee to win all shoppers for a price larger than \( \delta q_L \). This is the case if the price which deterministically wins all shoppers under a binding salience constraint (26), \( p_{\text{dev}} = p_{\text{dev}} \frac{q_H}{q_L} \), exceeds \( \delta q_L \). Solving the requirement yields \( \delta \leq \frac{\tilde{p}_{\text{min}}}{q_H} < 1 \). Consider this case first (Case A) and then the complementary case (Case B).

**Case A:** \( \delta \leq \frac{\tilde{p}_{\text{min}}}{q_H} \). For all (relevant) values of \( p_{\text{dev}} \geq \delta q_L \), the retailer’s expected profit is then given by

\[
\pi(p_{\text{dev}}) = (p_{\text{dev}} - c_L + v) \left( \frac{1 - \lambda}{N} + \lambda \Pr \left( \tilde{p}_{\text{min}} \geq p_{\text{dev}} \frac{q_H}{q_L} \right) \right) \\
= (p_{\text{dev}} - c_L + v) \left( \frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{p_{\text{dev}} \frac{q_L}{q_H} - c_H + v} \\
\propto \frac{p_{\text{dev}} - c_L + v}{p_{\text{dev}} \frac{q_H}{q_L} - c_H + v}.
\]
Taking the derivative with respect to \( p_{\text{dev}} \), its sign is equal to the sign of \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} \). Hence, for \( \delta \leq \delta' \), there are three cases. If \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} > 0 \), the retailer’s optimal deviation price is \( q_L \). If \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} < 0 \), its optimal deviation price is \( p_{\text{dev}}^{\delta q_L} \). And finally, if it holds (non-generically) that \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} = 0 \), the deviating retailer is indifferent between choosing any price in the interval \( [p_{\text{dev}}^{\delta q_L}, q_L] \).

**Case B:** \( \delta > \delta' \). Recall that in this case it is not possible for the deviating retailer to attract all shoppers for sure while the salience constraint (26) binds. Hence, pricing at or below \( \delta q_L \) may become optimal. Before turning to this possibility, note that for prices \( p_{\text{dev}} \geq \delta q_L \) the findings from Case A immediately carry over, so that the deviating retailer’s expected profit behaves monotonically in \( p_{\text{dev}} \). Repeating now the same exercise for \( p_{\text{dev}} < \delta q_L \), the deviating retailer’s expected profit can be written as

\[
\pi(p_{\text{dev}}) = (p_{\text{dev}} - c_L + v) \left( \frac{1 - \lambda}{N} + \lambda \Pr \{ \tilde{p}_{\min} \geq p_{\text{dev}} + \delta(q_H - q_L) \} \right)
= (p_{\text{dev}} - c_L + v) \left( \frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{p_{\text{dev}} + \delta(q_H - q_L) - c_H + v}
\propto \frac{p_{\text{dev}} - c_L + v}{p_{\text{dev}} + \delta(q_H - q_L) - c_H + v}.
\]

Taking the derivative with respect to \( p_{\text{dev}} \), its sign is equal to the sign of \( \delta(q_H - q_L) - (c_H - c_L) \). Combining this with the observation from Case A that the deviating retailer’s expected profit for prices above \( \delta q_L \) is increasing (decreasing) if and only if \( \frac{\mu}{q_L} > \frac{c_H - v}{c_L - v} \), we have the following cases:

i) if \( \delta(q_H - q_L) - (c_H - c_L) > 0 \) and \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} > 0 \), the optimal deviation price is \( q_L \).

ii) if \( \delta(q_H - q_L) - (c_H - c_L) < 0 \) and \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} < 0 \), the optimal deviation price is any price in the interval \( [\delta q_L, q_L] \).

iii) and if \( \delta(q_H - q_L) - (c_H - c_L) < 0 \) and \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} < 0 \), the optimal deviation price is \( \delta(q_H - q_L) \).

Note next that for \( \delta > \delta' \), there are no other cases (i.e., cases where \( \delta(q_H - q_L) - (c_H - c_L) \leq 0 \) and \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} \geq 0 \)). To see this, it suffices to show the converse, namely that for \( \delta > \delta' \), \( \frac{\mu}{q_L} - \frac{c_H - v}{c_L - v} \geq 0 \) implies \( \delta(q_H - q_L) - (c_H - c_L) > 0 \). For a proof, observe that from

\[ ^{60} \text{A prime indicates non-generic parameter combinations.} \]
and if it holds that

\[ \pi \text{ does not exceed its (candidate equilibrium) profit of }  \tilde{\pi} \]

Clearly, a high-quality equilibrium exists if a retailer's maximal deviation profit

\[ \text{A high-quality equilibrium exists if condition (11) holds.} \]

\textbf{Lemma 5} Consider a (candidate) equilibrium in which all retailers choose the high-quality product and a symmetric pricing strategy. Then, the optimal deviation profit \( \pi^*_{\text{dev}} \) of a retailer offering a low-quality product is

\[
\pi^*_{\text{dev}} = \begin{cases} 
(q_L - c_L + v) \frac{1-\lambda}{N} & \text{if } \frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v} \\
(p_{\frac{q_H}{q_L}} - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta \leq \delta_0 \\
\frac{\delta q_L - c_L + v}{\delta q_H - c_L + v} (q_H - c_H + v) \left( \frac{1-\lambda}{N} + \lambda \right) & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \delta_0 \text{ and } \delta \geq \frac{\Delta_q}{\Delta_c} \\
(p - \delta(q_H - q_L) - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \delta_0 \text{ and } \delta < \frac{\Delta_q}{\Delta_c}.
\end{cases}
\]

\textbf{Lemma 5} A high-quality equilibrium exists if condition (11) holds.

\textbf{Proof.} Clearly, a high-quality equilibrium exists if a retailer's maximal deviation profit does not exceed its (candidate equilibrium) profit of \( \bar{\pi} = (q_H - c_H + v) \frac{1-\lambda}{N} \). We now consider the different possible cases, as pinned down in Lemma 4 above. First, since \( (q_L-c_L+v) \frac{1-\lambda}{N} < \bar{\pi} \) due to \( \Delta_q > \Delta_c \), a high-quality equilibrium exists whenever \( \frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v} \).

Second, if it holds that \( \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \) and \( \delta \leq \delta_0 \), a high-quality equilibrium exists if and only if

\[
\left( \frac{q_L}{q_H} - c_L + v \right) \left( \frac{1-\lambda}{N} + \lambda \right) \leq \bar{\pi}.
\]

(28)
Using the fact that $\tilde{\pi}$ can also be written as $(p - c_H + v)\left(\frac{1}{N} + \lambda\right)$, (28) becomes

$$\frac{p}{q_H} = \tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}.$$  

Hence, for $\frac{q_H}{q_L} < \frac{c_H - v}{c_L - v}$ and $\delta \leq \tilde{\delta}$, a high-quality equilibrium exists if and only if $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$. Third, if $\frac{q_H}{q_L} < \frac{c_H - v}{c_L - v}$ and $\delta > \tilde{\delta}$ and $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$, a high-quality equilibrium exists if and only if

$$\frac{\delta q_L - c_L + v}{\delta q_H - c_H + v}(q_H - c_H + v)\left(\frac{1 - \lambda}{N} + \lambda\right) \leq \tilde{\pi}.$$  

A straightforward comparison reveals that this is the case if and only if $\delta \geq \frac{\Delta_c}{\Delta_q}$, which is true by assumption. And fourth, if $\frac{q_H}{q_L} < \frac{c_H - v}{c_L - v}$ and $\delta > \delta$ and $\tilde{\delta} < \frac{\Delta_c}{\Delta_q}$, a high-quality equilibrium can not exist. To see this, observe that in this case we can write

$$\pi^*_\text{dev} = (p - \delta(q_H - q_L) - c_L + v)\left(\frac{1}{N} + \lambda\right) = \tilde{\pi} + (\Delta_c - \delta\Delta_q)\left(\frac{1 - \lambda}{N} + \lambda\right) > \tilde{\pi},$$  

where the last inequality comes from $\delta < \frac{\Delta_c}{\Delta_q}$. To sum up all four cases, a high-quality equilibrium thus exists if either $\frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v}$, or $\delta \leq \tilde{\delta}$ and $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$, or $\delta > \tilde{\delta}$ and $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$. These conditions are now finally simplified.

Note for this that condition $\frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v}$ is redundant as it implies $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$ (but not vice versa).\(^{61}\) Focusing on the two remaining conditions (a) $\delta \leq \tilde{\delta}$ and $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$ and (b) $\delta > \tilde{\delta}$ and $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$, the respective parameter space can be separated into two regimes. First, if it holds that $\tilde{\delta} \geq \frac{\Delta_c}{\Delta_q}$, the considered high-quality equilibrium exists for all values of $\delta$, as either (a) or (b) must be satisfied. Second, if it holds that $\tilde{\delta} < \frac{\Delta_c}{\Delta_q}$, the candidate equilibrium only exists if $\delta \geq \frac{\Delta_c}{\Delta_q}$, as (a) can not be satisfied and $\delta \geq \frac{\Delta_c}{\Delta_q}$ becomes the binding condition in (b). This concludes the proof that condition (11) is also sufficient for a high-quality equilibrium to exist. Q.E.D. (Lemma 5).

The following characterization of manufacturer profits (for $i = 1$) follows immediately from Lemma 4 and noting that by optimality retailers are made indifferent between acceptance and rejection.

\(^{61}\)One possibility to show this is by isolating $\frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v}$ for $c_L$, which gives $c_L \geq v + \frac{q_H}{q_L}(c_H - v)$. Inserting this into $\frac{\Delta_c}{\Delta_q}$ gives an upper bound for the latter. One can then prove via direct manipulation that even at this upper bound, it must hold that $\frac{\Delta_c}{\Delta_q} \leq \tilde{\delta}$.
Lemma 6  Given that condition (11) holds, each brand manufacturer \( i = 1 \) makes a profit of

\[
\Pi^M = \begin{cases} 
\frac{1-\lambda}{N} (\Delta_q - \Delta_c) & \text{if } \frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v} \text{ and } \delta \leq \frac{p}{q_H} \\
\frac{1-\lambda}{N} (\Delta_q - \Delta_c) - \lambda \left[ (c_H - v) \frac{q_L}{q_H} - (c_L - v) \right] & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta \leq \frac{p}{q_H} \text{ and } \Delta_c \leq \delta \Delta_q \\
\frac{1-\lambda}{N} (\Delta_q - \Delta_c) - \frac{1-\lambda}{N} \left[ (1-\delta) \frac{q_L (c_H - v) - q_H (c_L - v)}{\delta q_H - (c_H - v)} \right] & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \frac{p}{q_H} \text{ and } \Delta_c \leq \delta \Delta_q.
\end{cases}
\]

Note that an immediate implication is that the profits of brand manufacturers \( i = 1 \) are reduced, relative to the benchmark case with rational consumers, if and only if \( \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \). Inserting \( v \), this condition is easily transformed to condition (12), as stated in the proposition.

This concludes the proof of Proposition 3. Q.E.D.

Proof of Corollary 1.  See Lemma 6 in the proof of Proposition 3 above. The respective expressions in the corollary are obtained immediately by inserting \( v \). Q.E.D.

Proof of Proposition 4.  We first characterize the symmetric (candidate) equilibrium and then show that retailers have indeed no incentive to deviate. Note that we abbreviate again \( v = (I - 1)(q_H - c_H) \). We claim that under the given parameter constraints, a mixed-strategy equilibrium exists in which each retailer stocks \( q_H (q_L) \) with probability \( \alpha^* (1 - \alpha^*) \), where for convenience we restate

\[
\alpha^* = \frac{1-\lambda}{\lambda N} \sqrt{\frac{\Delta_q - \Delta_c}{\frac{q_L (c_H - v) - q_H (c_L - v)}{\delta q_H - (c_H - v)}}}.
\]

Conditional on stocking \( q_H \), retailers draws prices from the CDF

\[
F_H(p) := 1 - \frac{1}{\alpha^*} \sqrt{\frac{1-\lambda}{\lambda N} \left( \frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}
\]

with support

\[
[p_H, \bar{p}_H] = \frac{\Delta_c}{\Delta_q} \left[ q_H, q_H \right].
\]
Conditional on stocking $q_L$, retailers draw prices from the CDF

$$F_L(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_L + v} - 1\right)} - \alpha^*}{1 - \alpha^*}$$

with support

$$[p_L, p_H] = [c_L - v + (q_H - c_H + v) - \lambda N \frac{\Delta_c}{\Delta_q} q_L].$$

Note first that the defined supports are such that $p_H = q_L$ and $\delta q_L - \frac{\Delta_c}{\Delta_q} q_L < \delta q_H - \frac{\Delta_c}{\Delta_q} q_H$, where the latter follows from the proposition’s parameter constraint $\Delta_c > \Delta q$. This implies that price is always salient if high- and low-quality products coexist in the market, and that price salience ensures that shoppers receive a higher perceived utility from buying the low-quality product, no matter which prices are drawn. Hence, for any price $p$ in the support of $F_H(.)$, a high-quality firm’s probability of attracting shoppers is

$$(\alpha^* [1 - F_H(p)])^N,$$  \hspace{1cm} (29)

while for any price $p$ in the support of $F_L(.)$, a low-quality firm’s probability of attracting shoppers is

$$(\alpha^* + (1 - \alpha^*) [1 - F_L(p)])^N.$$  \hspace{1cm} (30)

With this we can confirm that retailers are indeed indifferent between choosing low or high quality (where $T_n = 0$) and choosing a price in the respective support $[p_L, p_H]$ and $[p_L, p_H]$. Precisely, using (29) we have

$$\pi_H(p) = (p - c_H + v) \left\{ \frac{1}{N} + \lambda [\alpha^*(1 - F_H(p))]^{N-1} \right\} = \frac{1 - \lambda}{N} (q_H - c_H + v) = \Pi_R,$$

while using (30) we have

$$\pi_L(p) = (p - c_L + v) \left\{ \frac{1}{N} + \lambda [\alpha^* + (1 - \alpha^*)(1 - F_L(p))]^{N-1} \right\} = \Pi_R.$$

Below we show that there is also no profitable deviation to prices $p \notin [p_H, p_H]$ and $p \notin [p_L, p_L]$. Before doing so, we show that the distribution functions are well-behaved. For this note first that $\alpha^* \in (0, 1)$, i.e., that

$$\frac{1 - \lambda}{\lambda N} \left[ \frac{\Delta_q - \Delta_c}{q_H (c_H - v) - (c_L - v)} \right] \in (0, 1).$$

That the expression is indeed positive follows from $\frac{\Delta_q}{q_H} (c_H - v) - (c_L - v) > 0$, which is obtained from using $\Delta_c > \frac{\Delta q}{q_H} p$ and $p > c_H - v$. To see that the expression is smaller
than 1, we obtain from substituting for $p$ that this is equivalent to $\Delta_c > \frac{\Delta_q}{q_H} p$, again as required by the proposition. Both $F_H(.)$ and $F_L(.)$ are clearly (strictly) increasing over the respective supports and substitution of $\alpha^*$ reveals that they are also well-behaved at the boundaries.

Consider now deviations to prices outside the respective supports. If some retailer $n$ chooses $q_n = q_H$ but deviates to a price $p_{dev} < p_H$, its offer is clearly preferred to any other offer of a high-quality product and it is also preferred to the lowest-price offer of a low-quality product if, for the respective minimum $\tilde{p}_{min}$, it holds that $\tilde{p}_{min} > p_{dev} \frac{q_L}{q_H}$, so that quality becomes salient, or $\tilde{p}_{min} > p_{dev} - \delta \Delta q$, so that the deviating offer is preferred even if price is salient. It thus follows for the expected deviation profits that

$$\pi_H(p_{dev}) = (p_{dev} - c_H + v) \cdot \left\{ \frac{1 - \lambda}{N} + \lambda \left[ \alpha^* + (1 - \alpha^*) \left[ 1 - F_L \left( \min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta q \right\} \right) \right] \right\}^{N-1},$$

which from inserting $F_L(.)$ transforms to

$$\pi_H(p_{dev}) = \frac{1 - \lambda}{N} (q_H - c_H + v) \left( \frac{p_{dev} - c_H + v}{\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta q \right\} - c_L + v} \right).$$

The sign of the function’s derivative is equal to the sign of $(c_H - v) \frac{q_H}{q_H} - (c_L - v)$ if $p_{dev} \frac{q_L}{q_H} \leq p_{dev} - \delta \Delta q$ or the sign of $\Delta c - \delta \Delta q$ if $p_{dev} \frac{q_L}{q_H} > p_{dev} - \delta \Delta q$, respectively. The former is strictly positive due to $\Delta_c > \frac{\Delta_q}{q_H} p$, as we have already shown, while the latter is strictly positive as $\Delta_c > \delta \Delta q$. Thus, we have shown that $\pi_H(p_{dev}) < \Pi^R$ if $p_{dev} < p_L$ for a high-quality firm.

Consider finally deviations by low-quality firms, where we need to consider deviations $p_{dev} > p_L$. Clearly, by construction the profit can only exceed $\Pi^R$ if it still attracts shoppers, for which it is in turn necessary that all other retailers choose high quality and that price remains salient. Thus, even when we only consider these constraints, the deviation profit is bounded by

$$\pi_L(p_{dev}) = (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[ \alpha^* \left( 1 - F_H \left( \frac{q_H}{q_L} \right) \right) \right] \right\}^{N-1},$$

which from inserting $F_H(.)$ transforms to

$$\pi_L(p_{dev}) = \frac{1 - \lambda}{N} (q_H - c_H + v) \left( \frac{p_{dev} - c_L + v}{p_{dev} \frac{q_L}{q_H} - c_H + v} \right).$$
The sign of the derivative is determined by $-\left[ (c_H - v) \frac{\Delta u}{q_H} - (c_L - v) \right]$, which is strictly negative due to $\Delta c > \Delta q q_H$ (see above). Thus, we have also shown that $\pi_L(p_{dev}) < \Pi^R$ if $p_{dev} > p_L$ for a low-quality firm. Q.E.D.

**Proof of Proposition 5.** The claim regarding the comparative statics of manufacturer profits of product $i = 1$ follows straightforwardly from inspection of these profits, as stated in Corollary 1. It is moreover immediate to see that $\alpha^*$ is strictly decreasing in $I$, such that both consumers’ probability of purchasing high quality, as well as retailers’ probability of stocking high quality, strictly decrease in the extent of one-stop shopping. Q.E.D.

**Proof of Proposition 6.**

Roadmap. We start by proving that the regulation ensures existence of a high-quality equilibrium, given that $\frac{\Delta u}{c_H} > \frac{\Delta u}{c_L}$ (Lemma 7). We proceed to show that under the same condition, the regulation weakly increases the profits of manufacturers $i = 1$, and weakly decreases the profits of retailers (Lemma 8). We abbreviate again $v = (I - 1)(q_H - c_H)$ (with $v = \frac{1-\lambda}{\lambda N} (q_H - c_H)$ and $\bar{v} = \frac{1-\lambda}{\lambda} (q_H - c_H)$) and use as well that there exists an equilibrium where the wholesale price (for $i = 1$) equals the respective marginal cost of production.

**Lemma 7** If below-cost pricing is prohibited and $\frac{\Delta u}{c_H} > \frac{\Delta u}{c_L}$, a high-quality equilibrium always exists.

Proof. We assume that the (regulated) high-quality equilibrium of Lemma 1 is played, and then prove that deviating to low quality, combined with an optimal deviation price, does not pay. Note that since a deviating low-quality firm’s offer can never attract the shoppers if quality is salient, it is sufficient to check that no retailer has an incentive to deviate if it only needs to make price salient in order to attract the shoppers. In order to show this, we have to consider three cases: (i) $v \geq \bar{v}$, (ii) $v \in (\underline{v}, \bar{v})$, and (iii) $v \leq \underline{v}$.

(i) If $v \geq \bar{v}$, any high-quality equilibrium is characterized by all firms charging $c_H$ deterministically, giving rise to an equilibrium profit of $\frac{v}{N}$. A deviating retailer can only attract the shoppers with its deviation price $p_{dev}$ if $\frac{ch}{p_{dev}} > \frac{q_H}{q_L}$ (in order to make price salient). This requires $p_{dev} < \frac{q_H}{q_L} c_H$, which is however no longer feasible as $\frac{q_H}{q_L} c_H < c_L$ (as follows from $\frac{q_H}{q_L} > \frac{c_H}{c_L}$). A deviating retailer can therefore only target non-shoppers, which yields less than $\frac{v}{N}$ due to $v \geq \bar{v}$.

(ii) If $v \in (\underline{v}, \bar{v})$, symmetric high-quality equilibria are characterized by retailers sampling $c_H$ with positive probability $\beta^* \in (0, 1)$, while they draw prices continuously from...
a compact interval \([p_r, q_H]\) \((p_r > c_H)\) with remaining probability. Now it is possible for a deviating retailer to attract shoppers with positive probability. More precisely, a deviating retailer’s expected profit when charging a deviation price \(p_{dev} \in \left[\frac{q_H}{q_L} p_r, q_L\right]\) (provided that this is not below costs of \(c_L\)) is given by

\[
\pi_{dev}(p_{dev}) = (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[ (1 - \beta^*) \left( 1 - F_r \left( \frac{q_H}{q_L} p_{dev} \right) \right) \right] \right\}^{N-1},
\]

as the shoppers can only be attracted (price can only be made salient) if all rivals price above \(\frac{q_H}{q_L} p_{dev}\), which happens with probability \(\left[ (1 - \beta^*) (1 - F_r(\frac{q_H}{q_L} p_{dev})) \right]^{N-1} \). Inserting \(F_r(.)\) from Lemma 1, the above profit function simplifies to

\[
\pi_{dev}(p_{dev}) = \left( p_{dev} - c_L + v \right) \frac{1 - \lambda}{N} \left( \frac{q_H - c_H + v}{\frac{q_H}{q_L} p_{dev} - c_H + v} \right),
\]

which is strictly increasing (decreasing) in \(p_{dev}\) if \(v < \frac{q_H c_L - c_H q_L}{q_H - q_L} (v > \frac{q_H c_L - c_H q_L}{q_H - q_L})\). If it is strictly increasing in \(p_{dev}\), the optimal deviation price is \(q_L\) for a maximal deviation profit of \(\frac{1 - \lambda}{N} (q_L - c_L + v)\), which is less than the candidate equilibrium profit. If it is strictly decreasing in \(p_{dev}\), the optimal deviation price is \(\frac{q_H}{q_L} p_r\) and yields an expected profit of

\[
\left( \frac{q_L}{q_H} p_r - c_L + v \right) \frac{1 - \lambda}{N} \left( \frac{q_H - c_H + v}{p_r - c_H + v} \right).
\]

Comparing this with the candidate equilibrium, deviating would only pay if \(p_r < \frac{q_H (c_H - c_L)}{q_H - q_L}\). However, this cannot be the case, as \(p_r > c_H\), but \(\frac{q_H (c_H - c_L)}{q_H - q_L} < c_H\) for \(\frac{q_H}{q_L} > \frac{c_H}{c_L}\).

(iii) If \(v \leq \tilde{v}\), the regulation does not affect on-equilibrium pricing. Furthermore, from Proposition 3 we know that if any deviation was possible, the high-quality equilibrium would exist if \(\Delta_v \leq \tilde{\Delta}_v\), which is equivalent to

\[
v \leq \tilde{v} := c_H + \frac{(1 - \lambda) q_H - \Delta_v q_H (1 - \lambda + \lambda N)}{\lambda N}.
\]

But since \(v < \tilde{v}\) for \(\frac{q_H}{q_L} > \frac{c_H}{c_L}\), this must always be satisfied in the considered case. \textbf{Q.E.D.} (Lemma 7)

\textbf{Lemma 8} If below-cost pricing is prohibited and \(\frac{q_H}{c_H} > \frac{q_L}{c_L}\), the profits of manufacturers \(i = 1\) weakly increase, while those of retailers weakly decrease.
Proof. We again consider three different subcases: \( v \leq \underline{v} \), \( v \geq \bar{v} \), and \( v \in (\underline{v}, \bar{v}) \). If \( v \leq \underline{v} \), a high-quality equilibrium exists both under the regulation and without it (as a high-quality equilibrium always exists if either \( \Delta_q \geq \Delta_c \) or \( v < \bar{v} \), and the latter is implied by \( v \leq \underline{v} \) and \( \frac{\mu}{q_L} > \frac{c_H}{c_L} \)). Moreover, the regulation does not affect retailers’ equilibrium pricing and gross profits. But it may reduce their scope for deviations, as it is possible that \( p_{\text{dev}} \leq c_L \) even though \( \frac{\mu}{q_L} > \frac{c_H}{c_L} \) or \( \Delta_q \geq \Delta_c \). Hence, under the considered parameters, retailers’ equilibrium profits weakly decrease under the regulation for \( v \leq \underline{v} \), while manufacturers’ equilibrium profits weakly increase.

If \( v > \bar{v} \), under the regulation all retailers choose \( p_n = c_H \) for a gross profit of \( \frac{v}{N} \). Because they find it impossible to attract shoppers when deviating (as pricing below \( \min \{ \frac{\mu}{q_L} c_H, c_H - \Delta_q \} \) is prohibited if either \( \frac{\mu}{q_L} > \frac{c_H}{c_L} \) or \( \Delta_q \geq \Delta_c \)), their optimal deviation price is \( q_L \) for a maximal deviation profit of \( \frac{v}{N} (q_L - c_L + v) \). Hence, retailer (manufacturer) profits weakly decrease (increase) in the specified parameter region. Note that if it moreover holds that \( v \geq \bar{v} \) and \( \Delta_q \geq \Delta_c \), manufacturer (retailer) profits strictly increase (decrease), as no high-quality equilibrium would exist without the regulation, while manufacturers can appropriate a strictly positive profit of \( \frac{v}{N} - \frac{1 - \lambda}{N} (q_L - c_L + v) \) with the regulation (and retailer profits strictly decrease from \( \frac{v}{N} - \frac{1 - \lambda}{N} (q_H - c_H + v) \) to \( \frac{v}{N} - \frac{1 - \lambda}{N} (q_L - c_L + v) \)).

Finally, if \( v \in (\underline{v}, \bar{v}) \), retailers choose \( c_H \) with positive probability and sample prices up to \( q_H \) with remaining probability. Again, due to \( \frac{\mu}{q_L} > \frac{c_H}{c_L} \) or \( \Delta_q \geq \Delta_c \), it is impossible for deviating retailers to choose a price that captures all shoppers deterministically. If a deviating retailer chooses a (feasible) deviation price \( p_{\text{dev}} \), it makes an expected profit of

\[
\pi_{\text{dev}}(p_{\text{dev}}) = \left( p_{\text{dev}} - c_L + v \right) \cdot \left\{ \frac{1}{N} + \lambda \left( 1 - \beta^* \right) \left( 1 - F_r \left( \max \left\{ \frac{q_H}{q_L} p_{\text{dev}}, p_{\text{dev}} + \Delta_q \right\} \right) \right) \right\}^{N-1} \\
= \left( p_{\text{dev}} - c_L + v \right) \left( 1 - \frac{\lambda}{N} \right) \left( \frac{\max \left\{ \frac{q_H}{q_L} p_{\text{dev}}, p_{\text{dev}} + \Delta_q \right\} - c_H + v}{\max \left\{ \frac{q_H}{q_L} p_{\text{dev}}, p_{\text{dev}} + \Delta_q \right\} - c_H + v} \right),
\]

where the maximum operator follows from the fact that shoppers can only be attracted if both price is made salient and a higher perceived utility under salient prices is offered, and the second line follows from inserting \( F_r(.) \) (see Lemma 1) and simplifying. If we contrast this with the deviation profit without the regulation (cf. the proof of Lemma 3

\footnote{Strictly speaking, a high-quality equilibrium would still exist for \( v = \bar{v} \), but with manufacturers making zero profits.}
above), we note that the two functions are identical. Hence, under the pricing regulation, a retailer’s optimal deviation price (if it could freely choose any price) remains unaffected, but it may not be allowed to set such a price because of the prohibition of below-cost pricing, such that its deviation profit decreases. Hence, also for \( v \in (v, \bar{v}) \), retailers’ profits weakly decrease, while manufacturers’ profits weakly increase (and strictly so if \( v \geq \bar{v} \) and \( \delta \Delta_q < \Delta_c \), as without the regulation, no high-quality equilibrium would exist\(^{63}\)). Q.E.D. (Lemma 8)

This concludes the proof of Proposition 6. Q.E.D.

\(^{63}\)Apart from the borderline case where \( v = \bar{v} \), in which manufacturers make zero profits.
10 Online Appendix B: Omitted Material and Proofs

10.1 Omitted Proof for the Case where the Regulation Backfires (Proposition 7)

We first prove the following claim, which describes the different regimes for equilibrium product choice under the pricing regulation and \( \frac{q_H}{c_H} < \frac{q_L}{c_L} \). Once more, we abbreviate \( v = (I - 1)(q_H - c_H) \), with \( \underline{v} = \frac{1-\lambda}{\lambda N}(q_H - c_H) \) and \( \bar{v} = \frac{1-\lambda}{\lambda}(q_H - c_H) \).

Claim. If consumers are salient thinkers and \( \frac{q_H}{c_H} < \frac{q_L}{c_L} \), a prohibition of below-cost pricing has the following consequences for retailers’ product choice:

(I) If \( \delta \Delta_q \geq \Delta_c \) or \( v \leq \bar{v} := c_H + \frac{(1-\lambda)q_H - \Delta_c}{\lambda N} q_H (1-\lambda + \lambda N) \lambda < \underline{v} \) (or both), retailers always stock the high-quality product.

(II) If \( \delta \Delta_q < \Delta_c \) and \( \bar{v} < v \leq \bar{v} \), where \( \bar{v} \in (\underline{v}, \bar{v}) \) is defined implicitly by the unique solution to

\[
v \left(1 - \frac{\alpha^*(v)}{1 - \alpha^*(v)}\right) = \frac{1-\lambda}{\lambda}(q_H - c_H),
\]

retailers stock the high-quality product with probability \( \alpha^*(v) \in (0, 1) \) (as defined in Proposition 4), while they stock the low-quality product with complementary probability. Moreover, \( \alpha^*(v) \) is strictly decreasing in \( v \).

(III) If \( \delta \Delta_q < \Delta_c \) and \( \bar{v} < v < \bar{v} \), retailers stock the high-quality product with probability \( \tilde{\alpha}(v) \in (0, \alpha^*(v)) \), where \( \tilde{\alpha}(v) \) is defined implicitly by the unique solution to

\[
\frac{1-\alpha^N}{1-\alpha} = \frac{1-\lambda}{\lambda v}(q_H - c_H),
\]

while they stock the low-quality product with complementary probability. With \( \tilde{\alpha}(\bar{v}) = \alpha^*(\bar{v}) \) and \( \tilde{\alpha}(\bar{v}) = 0 \), it holds that \( \tilde{\alpha}(v) < \alpha^*(v) \) for all \( v \in (\bar{v}, \bar{v}) \).

(IV) Finally, if \( \delta \Delta_q < \Delta_c \) and \( v \geq \bar{v} \), retailers always stock the low-quality product.

The claim is proven by a series of lemmas.

Lemma 9 If \( \frac{q_H}{c_H} < \frac{q_L}{c_L} \) and below-cost pricing is prohibited, a high-quality equilibrium, as characterized by Lemma 1, exists if and only if \( \delta \Delta_q \geq \Delta_c \) or \( v \leq \bar{v} \).
Proof of Lemma 9. We first prove existence of a high-quality equilibrium if either \( \delta \Delta_q \geq \Delta_c \) or \( v \leq \bar{v} \). Start with the former case, such that \( v \) can take an arbitrary value. If \( v \geq \bar{v} \), we know from Lemma 1 that in a high-quality equilibrium, all retailers must charge \( c_H \) deterministically, for an equilibrium profit of \( \frac{v}{N} \). From \( \delta \Delta_q \geq \Delta_c \), it follows immediately that deviations which attract the shoppers are impossible, since doing so requires pricing below \( c_H - \delta \Delta_q \) (in order for the competition constraint to be satisfied), which falls short of \( c_L \) and is therefore prohibited. Hence, the optimal deviation price is \( q_L \), which yields lower profits due to \( v \geq \bar{v} \). If \( v \leq \bar{v} \), price setting in a high-quality equilibrium is not constrained. Then, at worst, the optimal deviation price is not constrained either. But even if this is true, we know from Proposition 3 that a high-quality equilibrium exists for \( \delta \Delta_q \geq \Delta_c \).

Consider finally the intermediate case in which \( v \in (\bar{v}, \bar{v}) \). Lemma 1 characterizes a hypothetical high-quality equilibrium in this case. A retailer’s expected deviation profit when charging a (feasible) deviation price \( p_{dev} \) is given by

\[
\pi_{dev}(p_{dev}) = (p_{dev} - c_L + v) \cdot \left( \frac{1 - \lambda}{N} + \lambda \left[ 1 - \beta^* \right] \right) \left( 1 - F_r \left( \max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta \Delta_q \right\} \right) \right)^{N-1} \]

\[
= (p_{dev} - c_L + v) \left( \frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{\max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta \Delta_q \right\} - c_H + v},
\]

where the maximum operator ensures that both the salience constraint and competition constraint are satisfied, and the second line follows from inserting \( F_r(.) \) and simplifying. Using this, it is not hard to show that, for the given parameter range, the optimal deviation price is interior and equals \( \delta q_L \). Hence, if this price is even feasible in face of the regulation, the optimal deviation profit is given by

\[
(\delta q_L - c_L + v) \left( \frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{\delta q_H - c_H + v}.
\]

It is then straightforward to show that this optimal deviation profit falls short of the candidate equilibrium’s profit of \( \frac{1 - \lambda}{N} (q_H - c_H + v) \) as \( \delta \Delta_q \geq \Delta_c \).

We check next possible deviations when \( v \leq \bar{v} \). Note then that since \( \bar{v} < v \) for \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \), on-equilibrium pricing is not affected under the regulation. Hence, at worst the regulation also doesn’t restrict the set of optimal deviations, as was true in the baseline model with salience. From Proposition 3 it thus follows that a high-quality equilibrium exists if \( \Delta_c \leq \frac{\Delta}{q_H} p \), which is equivalent to \( v \leq \bar{v} \).
We will proceed to show that no high-quality equilibrium can exist in the complementary case where both \( \delta \Delta q < \Delta c \) and \( v > \tilde{v} \). If \( v \in (\tilde{v}, \tilde{v}] \), the equilibrium pricing of a high-quality equilibrium would still be unaffected, as high-quality firms’ lowest price \( p \) would satisfy \( p \geq c_H \). Then there are two cases. If \( \delta \leq \frac{p}{\tilde{q}H} \), a deviating retailer can attract the shoppers deterministically by pricing at \( \tilde{q}H \) (as the salience constraint is binding for low \( \delta \)). This price is feasible because \( \tilde{q}H > c_L \) due to \( p \geq c_H \) and \( \frac{\tilde{q}H}{\tilde{q}L} < \frac{c_L}{c_H} \). It is then easy to show that the resulting deviation profit \( \pi_{\text{dev}} = (\tilde{q}H - c_H + v) \left( \frac{1-\lambda}{N} + \lambda \right) \) exceeds the profit \( (q_H - c_H + v) \left( \frac{1-\lambda}{N} + \lambda \right) \) in a hypothetical high-quality equilibrium if \( \Delta c > \frac{\Delta q}{\tilde{q}H} \), i.e., if \( v > \tilde{v} \). Second, if it instead holds that \( \delta > \frac{p}{\tilde{q}H} \), a deviating retailer can attract the shoppers deterministically by pricing at \( p - \delta \Delta q \) (as the competition constraint is binding for high \( \delta \)). This price is feasible because \( p - \delta \Delta q > c_L \) due to \( p \geq c_H \) and \( \delta \Delta q < \Delta c \). The corresponding deviation profit of \( \pi_{\text{dev}} = (p - \delta \Delta q - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) \) again exceeds the hypothetical equilibrium profit in a high-quality equilibrium, provided that \( \delta \Delta q < \Delta c \).

We now show that no high-quality equilibrium can exist for \( v \in (\tilde{v}, \tilde{v}] \). This is because, although the regulation becomes binding and retailers’ pricing in a hypothetical high-quality equilibrium becomes restricted to prices at or above \( c_H \), retailers’ expected gross profit stays at \( (q_H - c_H + v) \frac{1-\lambda}{N} \) (compare with Lemma 1). Hence, similar to the case where \( v \in (\tilde{v}, \tilde{v}] \) discussed before, if \( \delta \leq \frac{\tilde{q}H}{\tilde{q}L} \), a deviating retailer can attract all shoppers by pricing at \( c_H \frac{\tilde{q}L}{\tilde{q}H} - \epsilon \), which is permissible as \( c_H \frac{\tilde{q}L}{\tilde{q}H} > c_L \). It is then easy to show that the resulting deviation profit of \( (c_H \frac{\tilde{q}L}{\tilde{q}H} - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) \) strictly exceeds \( (q_H - c_H + v) \frac{1-\lambda}{N} \) for \( v > \tilde{v} \). If it holds in contrast that \( \delta > \frac{\tilde{q}H}{\tilde{q}L} \), a deviating retailer can attract all shoppers by pricing at \( c_H - \delta \Delta q - \epsilon \), which is feasible because \( c_H - \delta \Delta q > c_L \). The corresponding deviation profit of \( (c_H - \delta \Delta q - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) \) also strictly exceeds \( (q_H - c_H + v) \frac{1-\lambda}{N} \), as follows from \( \delta \Delta q < \Delta c \) and \( v > \tilde{v} \).

Observe finally that a high-quality equilibrium also cannot exist for \( v \geq \tilde{v} \) and \( \delta \Delta q < \Delta c \). In this case, the high-quality retailers would charge \( c_H \) deterministically for a profit of \( \frac{\tilde{q}H}{\tilde{N}} \) (see Lemma 1). If \( \delta \leq \frac{\tilde{q}L}{\tilde{q}H} \), deviating to low quality with a deviation price of \( \frac{\tilde{q}L}{\tilde{q}H} c_H - \epsilon > c_L \) would attract all shoppers and give a deviation profit of \( (\frac{\tilde{q}L}{\tilde{q}H} c_H - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) \), which clearly exceeds \( \frac{\tilde{q}H}{\tilde{N}} \). If instead \( \delta > \frac{\tilde{q}L}{\tilde{q}H} \), deviating to low quality with a deviation price of \( c_H - \delta \Delta q - \epsilon > c_L \) would result in a deviation profit of \( (c_H - \delta \Delta q - c_L + v) \left( \frac{1-\lambda}{N} + \lambda \right) \), which is also larger than \( \frac{\tilde{q}H}{\tilde{N}} \) because \( \delta \Delta q < \Delta c \). Q.E.D.

The following sequence of lemmas characterizes the symmetric equilibrium in product
choice and pricing under the regulation if \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \), \( \delta \Delta_q < \Delta_c \), and \( v > \tilde{v} \) such that no high-quality equilibrium exists.

**Lemma 10** If \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \), \( \delta \Delta_q < \Delta_c \), and \( v \in (\tilde{v}, v_0] \), the equilibrium is still characterized by Proposition 4.

**Proof of Lemma 10.** From the proof of Proposition 4, we know that in this candidate equilibrium, the lowest price a low-quality firm samples is \( p_L = c_L - v + (q_H - c_H + v) \frac{1 - \lambda}{1 - \lambda + \lambda N} \), while the lowest price a high-quality firm samples is \( p_H = \frac{\Delta c}{\Delta_q} q_H \). Observe first that \( p_H \) exceeds \( c_H \) for every \( v \), so the regulation clearly does not bind for high-quality firms. And it also does not bind for low-quality firms, provided that \( v \leq \tilde{v} \). Hence, as the equilibrium pricing is not affected for \( v \in (\tilde{v}, v_0] \), which was part of an equilibrium without the regulation (see the proof of Proposition 4), the corresponding strategy-combination still constitutes an equilibrium with the regulation. Q.E.D.

**Lemma 11** If \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \), \( \delta \Delta_q < \Delta_c \), and \( v \geq \bar{v} \), all retailers choose the low-quality product and set \( p_n = c_L \).

**Proof of Lemma 11.** Note that each retailer makes a profit of \( \frac{v}{N} \) in this candidate equilibrium. As further undercutting is impossible, the best possible deviation while keeping low-quality is to price at \( q_L \) for a profit of \( (q_L - c_L + v) \frac{1 - \lambda}{N} \), which falls short of \( \frac{v}{N} \) for \( v \geq \bar{v} \). Deviating to high-quality while rendering quality salient is impossible, as the highest such price, \( p_{dev} = \frac{q_H}{q_L} c_L \) is below \( c_H \). With a price of \( q_H \) (in order to fully exploit its loyal consumers) the expected profit \( (q_H - c_H + v) \frac{1 - \lambda}{N} \) does not exceed \( \frac{v}{N} \) if \( v \geq \bar{v} \). The other possibly optimal deviation is to charge \( c_L + \delta \Delta_q \), which is the highest price that attracts shoppers although price, rather than quality, is salient. However, this price is below \( c_H \) if \( \delta \Delta_q < \Delta_c \). Q.E.D.

The following technical lemma is needed for a characterization of the remaining case where \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \), \( \delta \Delta_q < \Delta_c \), and \( v \in (v, \bar{v}) \).

**Lemma 12** There exists a unique \( \tilde{v} \in (v, \bar{v}) \) such that

\[
v \left( \frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H),
\]

where

\[
\alpha^*(v) = \sqrt{N - \frac{\Delta_q - \Delta_c}{\lambda N} \left[ \frac{q_H}{q_H} (c_H - v) - (c_L - v) \right]},
\]

as defined in Proposition 4.
**Proof of Lemma 12.** Note first that the RHS of equation (31) is independent of $v$. Hence, it is sufficient to prove that (1) $v \left( \frac{1-\alpha^N(v)}{1-\alpha(v)} \right) < \frac{1-\lambda}{\lambda} (q_H - c_H)$, (2) $\overline{v} \left( \frac{1-\alpha^N(\overline{v})}{1-\alpha(\overline{v})} \right) > \frac{1-\lambda}{\lambda} (q_H - c_H)$, and (3) $v \left( \frac{1-\alpha^N(v)}{1-\alpha(v)} \right)$ is strictly increasing in $v$ over the relevant range. For (1), note that since $\tilde{v} < v$, $\alpha^*(\tilde{v}) = 1$, $\alpha^*(v)$ is strictly decreasing in $v$, and $\frac{1-\alpha^N}{1-\alpha}$ is strictly increasing in $\alpha$, the LHS for $v = \overline{v}$ must fall short of $v \left( \lim_{\alpha \to 1} \frac{1-\alpha^N}{1-\alpha} \right) = vN$, which is the RHS of equation (31). For (2), note that $\frac{1-\alpha^N}{1-\alpha}$ strictly exceeds 1 for all $\alpha \in (0, 1)$. Hence, the LHS of equation (31) for $v = \overline{v}$ must exceed $\overline{v}$, which is the RHS of the equation.

For (3), we first make use of the implicit definition of $\alpha^*(v)$, which is given by

$$\pi_H(p_H) = \left( \frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) \left( \frac{1-\lambda}{N} + \lambda \alpha^{N-1} \right) \equiv \frac{1-\lambda}{N} (q_H - c_H + v).$$

Implicit differentiation establishes that

$$\frac{d\alpha^*(v)}{dv} = -\frac{\alpha^*(v)}{\left( \frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) (N-1)}.$$

As $v \left( \frac{1-\alpha^N(v)}{1-\alpha(v)} \right)$ can be rewritten as $v \sum_{j=0}^{N-1} \alpha^*(v)^j$, the derivative of the latter with respect to $v$ is then given by

$$\sum_{j=0}^{N-1} \alpha^*(v)^j + v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \frac{d\alpha^*(v)}{dv} = \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \left( \frac{\alpha^*(v)}{\Delta_c q_H - c_H + v} (N-1) \right)$$

$$> \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^{j} \left( \frac{1}{v(N-1)} \right)$$

$$> \sum_{j=0}^{N-1} \alpha^*(v)^j - (N-1) \alpha^*(v)^j \left( \frac{1}{N-1} \right) = 0,$$

where the third line follows from $\frac{\Delta_c}{\Delta_q} q_H < \frac{\Delta_c}{\Delta_q}$. This finalizes the proof of Lemma 12. Q.E.D.

**Lemma 13** If $\frac{\Delta_c}{\Delta_q} q_H < \frac{\Delta_c}{\Delta_q}$, $\delta \Delta_q < \Delta_c$, and $v \in (v, \tilde{v})$, the following constitutes an equilibrium. With probability $\alpha^* \in (0, 1)$ as defined in Proposition 4, retailers choose the high-quality product and sample prices according to the CDF $F_H(p)$, again as defined in Proposition 4. With probability $1 - \alpha^*$, retailers choose the low-quality product. If they do so, they choose $p_n = c_L$ with probability $\tilde{\beta}(v) \in (0, 1)$, where $\tilde{\beta}(v)$ is defined implicitly by

$$v \left( \frac{1 - \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^N}{(1 - \alpha^*(v)) \tilde{\beta}} \right) = \frac{1-\lambda}{\lambda} (q_H - c_H),$$

(32)
with \( \hat{\beta}(v) = 0 \), \( \tilde{\beta}(\hat{v}) = 1 \), and \( \tilde{\beta}'(v) > 0 \). With the remaining probability \( 1 - \tilde{\beta}(v) \), low-quality retailers sample prices continuously from a CDF \( F_{L,r}(p) \) with support \([p_{L,r}, \Delta_q q_L] \), where

\[
p_{L,r} := c_L - v + \frac{(q_H - c_H + v) \frac{1-\lambda}{N}}{\frac{1-\lambda}{N} + \lambda \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta}(v) \right]^{N-1}} \in (c_L, \Delta_q q_L),
\]

and

\[
F_{L,r}(p) := 1 - \frac{\sqrt{\frac{1-\lambda}{N} \left( \frac{q_H - c_H + v}{p - c_L + v} - 1 \right) - \alpha^*(v)}}{(1 - \alpha^*(v)) \left( 1 - \tilde{\beta}(v) \right)}.
\]

**Proof of Lemma 13.** Because the lower support bound of high-quality firms (which is given by \( \frac{\Delta_c}{\Delta_q} q_H \) and thus strictly exceeds \( c_H \), as follows from \( \frac{q_H}{q_L} < \frac{c_H}{c_L} \)) and the upper support bound of low-quality firms are the same as in the mixed-product equilibrium without the regulation (see the proof of Proposition 4), price will always be salient and high-quality firms cannot serve shoppers if both low-quality and high-quality products are introduced to the market. Hence, from the proof of Proposition 4, high-quality firms are still indifferent between sampling any price in their specified support, and make an expected profit of \((q_H - c_H + v) \frac{1-\lambda}{N}\). A low-quality firm’s expected profit when sampling \( p_L = \frac{\Delta_c}{\Delta_q} q_L \) is \((\frac{\Delta_c}{\Delta_q} q_L - c_L + v) \frac{1-\lambda}{N} + \lambda (\alpha^*)^{N-1}\), as it can only attract the shoppers if all of its rivals stock \( q_H \). By construction of \( \alpha^* \), this also yields an expected profit of \((q_H - c_H + v) \frac{1-\lambda}{N}\). If a low-quality firm samples \( p_{L,r} \), its expected profit is given by

\[
\pi_L(p_{L,r}) = (p_{L,r} - c_L + v) \left\{ \frac{1-\lambda}{N} + \lambda \left[ \alpha^* + (1 - \alpha^*) \left( 1 - \tilde{\beta} \right) \right] \right\}^{N-1},
\]

as it can only attract the shoppers if all rivals either stock \( q_H \), or stock \( q_L \), but do not sample \( c_L \). Setting this equal to \((q_H - c_H + v) \frac{1-\lambda}{N}\), we find \( p_{L,r} \). If a low-quality firm samples an arbitrary price \( p \) in its support, its expected profit is given by

\[
(p - c_L + v) \left\{ \frac{1-\lambda}{N} + \lambda \left[ \alpha^* + (1 - \alpha^*) \left( 1 - \tilde{\beta} \right) \right] \left( 1 - F_{L,r}(p) \right) \right\}^{N-1},
\]

as it can only attract shoppers if all of its rivals either choose \( q_H \), or choose \( q_L \) but do not charge a lower price than \( p \). Setting this equal to \((q_H - c_H + v) \frac{1-\lambda}{N}\), we find the CDF \( F_{L,r}(p) \) reported in the lemma.
If a low-quality firm samples $c_L$, its expected profit is

$$\pi_L(c_L) = (c_L - c_L + v) \left\{ \frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \left[ \left( 1 - \alpha^* \right) \tilde{\beta} \right]^{j} \left[ 1 - \left( 1 - \alpha^* \right) \tilde{\beta} \right]^{N-1-j} \frac{\lambda}{j+1} \right\}$$

$$= v \left\{ \frac{1 - \lambda}{N} + \lambda \left[ \frac{1 - \left( 1 - \alpha^* \right) \tilde{\beta}}{N(1 - \alpha^*) \tilde{\beta}} \right] \right\},$$

as it has to share the shoppers with $j \in \{0, \ldots, N-1\}$ rivals which also sample $c_L$ with probability $\binom{N-1}{j} \left[ \left( 1 - \alpha^* \right) \tilde{\beta} \right]^{j} \left[ 1 - \left( 1 - \alpha^* \right) \tilde{\beta} \right]^{N-1-j}$, and where the second line is obtained by proper manipulation and making use of the binomial theorem. Setting this equal to $(q_H - c_H + v) \frac{1 - \lambda}{N}$ and simplifying, we find the implicit definition of $\tilde{\beta}(v)$ as provided in the lemma.

Taken together, each price in retailers’ support thus indeed yields the same expected profit. Further, low-quality retailers have no profitable deviation price as pricing below $c_L$ is prohibited, pricing between $c_L$ and $p_{L,r}$ is strictly dominated by pricing at $p_{L,r}$, and pricing above their upper support bound $\frac{N}{N} q_L$ was already shown to be inferior in the proof of Proposition 4, where the high-quality product is stocked with the same probability and high-quality firms use the same strategy as in the present candidate equilibrium. Deviating high-quality firms can never guarantee to attract all shoppers, as they would have to price below $c_L \frac{q_H}{q_L}$, which falls short of $c_H$. If a high-quality firm deviates to $p_{\text{dev}} \in \left[ p_{L,r} \frac{q_H}{q_L}, \frac{N}{N} q_H \right)$, its expected profit is given by

$$\pi_H(p_{\text{dev}}) = (p_{\text{dev}} - c_H + v) \cdot \left\{ \frac{1 - \lambda}{N} + \lambda \left[ \alpha^* + (1 - \alpha^*) \left( 1 - \tilde{\beta} \right) \left( 1 - F_{L,r} \left( \min \left\{ p_{\text{dev}} \frac{q_L}{q_H}, p_{\text{dev}} - \delta \Delta q \right\} \right) \right) \right]^{N-1} \right\}$$

$$= (p_{\text{dev}} - c_H + v) \left( \frac{q_H - c_H + v}{\min \left\{ p_{\text{dev}} \frac{q_L}{q_H}, p_{\text{dev}} - \delta \Delta q \right\} - c_L + v} \right),$$

where the second line follows from inserting $F_{L,r}(\cdot)$, as found in the lemma, and simplifying. (Note that the term $\min \left\{ p_{\text{dev}} \frac{q_L}{q_H}, p_{\text{dev}} - \delta \Delta q \right\}$ appears because a deviating high-quality firm can win the shoppers if either quality is salient, or price is salient but its offer still provides a higher perceived utility). The derivative of $\pi_H(p_{\text{dev}})$ with respect to $p_{\text{dev}}$ is strictly positive for all $p_{\text{dev}}$, which implies that also firms with $q_H$ have no profitable deviation.

In the proof of Proposition 4, it has already been established that $\alpha^*$ and $F_H(p)$ are well-behaved. Furthermore, provided that $\tilde{\beta}(v) \in [0, 1)$, $F_{L,r}(p)$ is also well-behaved, as it
is strictly increasing in \( p \), with \( F_{L,r}(p_{L,r}) = 0 \) and \( F_{L,r} \left( \frac{\Delta_q}{q_H}, q_L \right) = 1 \). It remains to show that \( \tilde{\beta}(v) \) and \( p_{L,r} \) are well-behaved, where for convenience we restate the implicit definition of \( \tilde{\beta}(v) \):

\[
v \left( \frac{1 - \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^N}{(1 - \alpha^*(v)) \tilde{\beta}} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H).
\]

Note that for \( v = v \), the above equation can only be satisfied if \( (1 - \alpha^*) \tilde{\beta} = 0 \), which, as \( \alpha^* \in (0, 1) \) for \( v > \tilde{v} \), implies that \( \tilde{\beta}(v) = 0 \). Setting \( \tilde{\beta} = 1 \), this becomes

\[
v \left( \frac{1 - \alpha^*(v)^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H),
\]

where the unique solution is given by \( \tilde{v} \in (v, \overline{v}) \) (see Lemma 12 above). We will now prove that \( \tilde{\beta}(v) \) is strictly increasing in \( v \). To see this, note first that

\[
v \left( \frac{1 - \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^N}{(1 - \alpha^*(v)) \tilde{\beta}} \right) = v \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j \tilde{\beta} \left( \frac{\alpha^*(v)}{dv} \right).
\]

Plugging this into the above implicit definition of \( \tilde{\beta}(v) \) and differentiating implicitly, we find that \( \tilde{\beta}(v) \) is strictly increasing in \( v \) if

\[
\sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left( \frac{\alpha^*(v)}{dv} \right) > 0.
\]

To see that this is indeed the case, note that

\[
\sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left( \frac{\alpha^*(v)}{dv} \right)
= \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left( -\frac{\alpha^*(v)}{q_H - c_H + v(N - 1)} \right)
> \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left( -\frac{\alpha^*(v)}{v(N - 1)} \right)
> \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j - \sum_{j=0}^{N-1} (N - 1) \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j \tilde{\beta} \left( \frac{\alpha^*(v)}{N - 1} \right)
= \sum_{j=0}^{N-1} \left[ 1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j \left( 1 - \alpha^*(v) \tilde{\beta} \right) \geq 0.
\]
where the second line follows from implicitly differentiating the definition of $\alpha^*(v)$, and the third line follows from $\frac{uH}{ql} < \frac{cH}{cL}$. Hence, $\bar{\beta}(v)$ is strictly increasing in $v$, with $\bar{\beta}(v) = 0$ and $\bar{\beta}(\bar{v}) = 1$. From this it follows directly that $p_{L,r}(\bar{v}) = c_L$, whereas using the definition of $\alpha^*(v)$ yields $p_{L,r}(\bar{v}) = \frac{\Delta c}{\Delta q} q_L$. A proof that $p_{L,r}(v)$ is strictly increasing in $v$ is omitted for brevity. Q.E.D.

**Lemma 14** If $\frac{uH}{ql} < \frac{cH}{cL}$, $\delta \Delta q < \Delta_c$, and $v \in (\bar{v}, \bar{v})$, the following constitutes an equilibrium. Retailers choose the high-quality product with probability $\bar{\alpha}(v)$, where $\bar{\alpha}(v)$ is the unique solution to

$$\frac{1 - \alpha_N}{1 - \alpha} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

with $\bar{\alpha}(\bar{v}) = \alpha^*(\bar{v})$, $\bar{\alpha}(\bar{v}) = 0$, and $\bar{\alpha}'(v) < 0$. High-quality retailers sample prices continuously from a CDF $F_{H,r}(p)$ with support $[p_{L,H}, q_H]$, where

$$p_{H,r} := c_H - v + \frac{(q_H - c_H + v)\frac{1 - \lambda}{N}}{1 - \lambda + \lambda \bar{\alpha} N - 1} \in \left(\frac{\Delta c}{\Delta q} q_H, q_H\right)$$

and

$$F_{H,r}(p) := 1 - N \frac{1 - \lambda}{\lambda v} \frac{1}{N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1\right).$$

With the remaining probability $1 - \bar{\alpha}(v)$, retailers choose $q_L$ and $p_n = c_L$.

**Proof of Lemma 14.** As $p_{H,r} > \frac{\Delta c}{\Delta q} q_H > c_L \frac{uH}{ql}$, price is always salient if both high-quality and low-quality products are in the market. Then, all shoppers prefer a low-quality firm’s offer, as $\delta q_H - p_{H,r} < \delta q_L - c_L$ due to $p_{H,r} > c_H$ and $\delta \Delta q < \Delta_c$. If a high-quality firm samples an arbitrary price $p$ in its support, its expected profit is given by

$$(p - c_H + v) \left\{ \frac{1 - \lambda}{N} + \lambda \bar{\alpha} (1 - F_{H,r}(p)) \right\}^{N-1},$$

as it can only attract the shopper if all of its rivals stock $q_H$ and sample a higher price than $p$. Setting this equal to $(q_H - c_H + v)\frac{1 - \lambda}{N}$, we find the CDF $F_{H,r}(p)$ and $p_{L,\bar{v}}$. Clearly, provided that $\bar{\alpha}(v) > 0$, $F_{H,r}(p)$ is well-behaved, as it is strictly increasing in $p$, with $F_{H,r}(p_{L,H}) = 0$ and $F_{H,r}(q_H) = 1$.

If a retailer chooses $q_L$ and $p_n = c_L$, its expected profit is given by

$$\pi_L(c_L) = (c_L - c_L + v) \left[ \frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} (1 - \bar{\alpha})^j \bar{\alpha}^{N-1-j} \frac{\lambda}{j+1} \right]$$

$$= v \left[ \frac{1 - \lambda}{N} + \lambda \frac{1 - \bar{\alpha}^N}{(1 - \bar{\alpha}) N} \right],$$

$$69$$
as it has to share the shoppers with \( j \in \{0, \ldots, N-1\} \) rivals, which happens with probability \((N-1) (1-\tilde{\alpha})^j \tilde{\alpha}^{N-1-j}\), and where the second line again follows from the binomial theorem. Setting this equal to \((q_H - c_H + v)\frac{1 - \lambda}{N}\) yields the implicit definition of \(\tilde{\alpha}(v)\) in the lemma. Note that the LHS of this is strictly increasing in \(\alpha\), which implies that \(\tilde{\alpha}(v)\) must be strictly decreasing in \(v\). It is also easy to check that \(\tilde{\alpha}(\bar{v}) = 0\). Furthermore, comparing the above equation with the definition of \(\hat{\alpha}^*(v)\), which is the unique \(v\) that satisfies

\[
\frac{1 - \alpha^*(\hat{v})^N}{1 - \alpha^*(\hat{v})} = \frac{1 - \lambda}{\lambda \hat{v}} (q_H - c_H),
\]

it is apparent that \(\tilde{\alpha}(\hat{v}) = \alpha^*(\hat{v})\). Using the latter two results, from the definition of \(p_{H,r}\) it immediately follows that \(p_{H,r}(\bar{v}) = q_H\), whereas also using the definition of \(\alpha^*(v)\) yields \(p_{H,r}(\hat{v}) = \frac{\Delta}{\Delta_q} q_H\). It can likewise be established that \(p_{H,r}(v)\) is strictly increasing in \(v\).

It remains to show that no firm can have a profitable deviation. Note first that due to the regulation, it is impossible for deviating high-quality firms to render quality salient if any rival stocks \(q_L\). Hence, the best deviation a high-quality retailer can make is to charge the highest price for which its offer wins although price, rather than quality, is salient. But this price, \(p_{dev} = c_L + \delta \Delta_q\), is prohibited due to \(\delta \Delta_q < \Delta_c\). If a low-quality firm chooses \(p_{dev} > c_L\), its expected profit is at best (that is, for \(\delta = 0\)) given by

\[
\pi_L(p_{dev}) = (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[ \tilde{\alpha} \left( 1 - F_{H,r}\left( p_{dev} \frac{q_H}{q_L} \right) \right) \right]^{N-1} \right\}
\]

\[
= (p_{dev} - c_L + v) \frac{1 - \lambda}{N} \left( \frac{q_H - c_H + v}{p_{dev} \frac{q_H}{q_L} - c_H + v} \right),
\]

where the second line follows from inserting \(F_{H,r}(\cdot)\) and simplifying. From this, it is easy to show that \(\pi_L(p_{dev})\) is strictly decreasing in \(p_{dev}\), from which it follows that low-quality firms’ optimal deviation price is \(p_{H,r} \frac{q_L}{q_H} > c_L\) for a maximal deviation profit of

\[
\left( p_{H,r} \frac{q_L}{q_H} - c_L + v \right) \frac{1 - \lambda}{N} \left( \frac{q_H - c_H + v}{p_{H,r}(\hat{v}) - c_H + v} \right).
\]

This could only exceed the candidate equilibrium’s profit \((q_H - c_H + v)\frac{1 - \lambda}{N}\) if \(p_{H,r} < \frac{\Delta}{\Delta_q} q_H\), which is not satisfied. Hence, also low-quality firms do not have a profitable deviation. Q.E.D.

For the comparison of \(\tilde{\alpha}(v)\) and \(\alpha^*(v)\), we finally establish the following.

Lemma 15 \(\tilde{\alpha}'(v) < \alpha'^*(v)\) for all \(v \in [\hat{v}, \bar{v}]\).
Proof of Lemma 15. We first note that the implicit definition of \( \tilde{\alpha}(v) \) can be rewritten as

\[
\sum_{j=0}^{N-1} \alpha^j = \frac{1 - \lambda}{\lambda v} (q_H - c_H).
\]

Implicitly differentiating this, we obtain that

\[
|\tilde{\alpha}'(v)| = \frac{1 - \lambda}{\lambda v^2} (q_H - c_H) \left( \sum_{j=0}^{N-1} \frac{1}{j! \tilde{\alpha}(v)^j} \right) > \frac{1}{v(N-1)},
\]

where the second line follows from using \( \frac{1 - \lambda}{\lambda v} (q_H - c_H) = \sum_{j=0}^{N-1} \tilde{\alpha}(v)^j \) by the above definition. We have next that

\[
|\alpha^*(v)| = \frac{\alpha^*(v)}{(\frac{\Delta c}{\Delta q} q_H - c_H + v) (N-1)} < \frac{\alpha^*(v)}{v(N-1)},
\]

where the expression for \( \alpha^*(v) \) has already been established in the proof of Lemma 12, and the inequality follows from \( \frac{\Delta c}{\Delta q} q_H < \frac{\Delta c}{\Delta q} c_L \). Comparing the lower bound of \( |\tilde{\alpha}'(v)| \) with the upper bound of \( |\alpha^*(v)| \), it is clear that \( \tilde{\alpha}(v) \) must have a larger absolute slope than \( \alpha^*(v) \) if \( \alpha^*(v) \leq 1 \), which is indeed the case for all \( v \in [\hat{v}, \bar{v}] \). Q.E.D.

Taken together, Lemmas 9 to 15 prove our claim regarding equilibrium product choice under the regulation and \( \frac{q_H}{c_H} < \frac{q_L}{c_L} \). Q.E.D.

Hence, it is evident that the below-cost pricing regulation strictly decreases efficiency (by reducing the equilibrium probability \( \alpha \) of retailers choosing high quality for category \( i = 1 \)) if and only if \( \delta \Delta q < \Delta c \) and \( v > \hat{v} \). Otherwise, efficiency remains unchanged. We next turn to the claim regarding the profits of manufacturers \( i = 1 \) and retailers.

Lemma 16 If below-cost pricing is prohibited, \( \frac{q_H}{c_H} < \frac{q_L}{c_L} \), and \( \delta \Delta q < \Delta c \), the profits of manufacturers \( i = 1 \) remain unchanged, while those of retailers weakly increase.

Proof. Without the regulation, from Proposition 3 and \( \delta \Delta q < \Delta c \), it follows that a high-quality equilibrium exists if and only if \( \Delta c \leq \frac{\Delta q}{q_H} q_H \), which is equivalent to \( v \leq \hat{v} \). Similarly, we have established above that also under the regulation, a high-quality equilibrium exists
if and only if $v \leq \tilde{v}$. If no high-quality equilibrium exists (and therefore, retailers either mix between stocking the high and low-quality product, or deterministically stock the low-quality product), manufacturers cannot extract a positive profit, as retailers are at best indifferent between choosing high quality or low quality. Hence, both under the regulation and without it, manufacturers make zero profits for $v \geq \tilde{v}$.

If $v < \tilde{v}$ such that a high-quality equilibrium exists (and manufacturers’ profits are strictly positive), manufacturer profits do not depend on the regulation. To see this, note first that for $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, it holds that $\tilde{v} < v$, which implies that whenever a high-quality equilibrium exists, retailers’ on-equilibrium pricing is not affected by the pricing regulation (since $p > c_H$). But also the maximum deviation profit of a retailer is not affected, since deviating retailers can guarantee to attract all shoppers by pricing at $\min\{p_{2/3}, p - \delta \Delta_q\} > \min\{c_H \frac{q_L}{q_H}, c_H - \delta \Delta_q\}$, which does not fall short of $c_L$ and therefore is not prohibited due to $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $\delta \Delta_q < \Delta_c$. Hence, for $v \leq \tilde{v}$, the regulation does not affect retailers’ on-equilibrium gross profits and their deviation profits. Note finally that without the regulation, for any $v > \tilde{v}$, retailers make an expected profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$. Under the regulation, they also make an expected profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$ for $v \in (\tilde{v}, \bar{v}]$, but $\frac{v}{N} > \frac{1-\lambda}{N}(q_H - c_H + v)$ for $v > \bar{v}$. Hence, retailers’ expected profits strictly increase for $v > \bar{v}$, but are unaffected for any $v \leq \bar{v}$. Q.E.D.
10.2 Elastic Demand in the Baseline Model (Observation 1)

We extend the baseline model with rational consumers to allow for a continuous elastic demand for shopping. For this we suppose that each consumer has an independently drawn reservation value for shopping: $\theta \geq 0$ with CDF $G(\theta)$. Thus, using already that consumers’ rationally anticipated net surplus from all products $i > 1$ is zero, a consumer benefits from visiting retailer $n$ only when, with respect to product $i = 1$, it holds that $q_n - p_n \geq \theta$. For a given realization of qualities and prices, the demand $D_n$ for retailer $n$ is thus now composed as follows. Given consumer surplus at retailer $n$, $s_n = q_n - p_n$, denote the respective maximum across all retailers by $s^{\text{max}} = \max_{n' \in N}(q_{n'} - p_{n'})$ and by $N^{\text{max}}$ the number of retailers for which $s_n = s^{\text{max}}$. Then demand is given by

$$D_n = G(s_n) \left(1 - \frac{\lambda}{N}\right) \quad \text{if } s_n < s^{\text{max}}$$

and by

$$D_n = G(s_n) \left[1 - \frac{\lambda}{N} + \frac{\lambda}{N^{\text{max}}}\right] \quad \text{if } s_n = s^{\text{max}},$$

where we assume that shoppers randomize with equal probability when indifferent, albeit this specific tie-breaking rule is inconsequential for the subsequent analysis.

As this is a straightforward extension from the case where consumers all have the same reservation value of shopping (of zero), we already make use of the observations that, first, for all products the high quality is stocked and, second, there is an equilibrium where marginal wholesale prices equal marginal costs. Observe next that a retailer’s expected (gross) profit with any non-shopping local consumer is

$$\pi = (p - c_H) + (I - 1)(q_H - c_H))G(q_H - p).$$

We suppose for convenience that $g/G$ is weakly increasing in its argument, so that the respective (“monopoly”) price $p^m = \arg \max_p \pi$ is uniquely determined and results in (“monopoly”) profits of $\pi^m$. Thus, when a retailer only attracts non-shoppers, then the maximum profit is $\frac{1 - \lambda}{N} \pi^m$ (gross of any fixed fee paid to the manufacturer).

We next extend our characterization of a symmetric pricing equilibrium. There, all retailers will randomize according to some continuous CDF $F(p)$ over the support $p \in [\underline{p}, p^m]$. A retailer choosing $p = \underline{p}$ can be certain to sell to all shoppers whose reservation value is sufficiently low, while with $p = p^m$ the retailer sells only to its local non-shoppers (with sufficiently low reservation value). To make the retailer indifferent between these...
two choices, we know already that \( p \) must satisfy
\[
\left( \frac{1 - \lambda}{N} + \lambda \right) (p - c_H + (I - 1)(q_H - c_H))G(q_H - p) = \frac{1 - \lambda}{N} \pi^m, \tag{36}
\]
while the distribution is finally obtained from the requirement to make each retailer indifferent also with respect to all \( p \in (p, p^m) \):
\[
F(p) = 1 - \frac{N - 1}{\lambda N} \sqrt{\frac{\pi^m}{(p - c_H + (I - 1)(q_H - c_H))G(q_H - p) - 1}}. \tag{37}
\]
This extends the characterization of the pricing equilibrium.

We turn finally to a characterization of manufacturer profits, which again are uniquely determined (that is, across all pricing equilibria). We focus on \( i = 1 \) with (suppressing the subscript \( i = 1 \)) respective profits \( \Pi^M \). From the respective indifference condition for each retailer, we now have for each manufacturer
\[
\Pi^M = \frac{1 - \lambda}{N} (\pi^m_H - \pi^m_L),
\]
where we extended the notation in (35) as follows: \( \pi^m_H \) denotes the maximum profit when the retailer stocks \( q_H \) at cost \( c_H \) and \( \pi^m_L \) is the respective maximum profit when the retailer instead stocks \( q_L \) at cost \( c_L \). Using uniqueness of the respective prices \( p^m_H \) an \( p^m_L \) and appealing to the envelope theorem, we have that
\[
\frac{d\Pi^M}{dI} = (q_H - c_H) \frac{1 - \lambda}{N} [G(q_H - p^m_H) - G(q_L - p^m_L)].
\]
For Observation 1 to hold, i.e., that a brand manufacturer is now strictly better off when \( I \) increases, it thus remains to prove that
\[
p^m_H - p^m_L < \Delta_q. \tag{38}
\]
It is now convenient to denote more generally \( p^m(q, c) \) as the "monopoly price" when, at \( i = 1 \), quality \( q \) is stocked at cost \( c \). With this we rewrite
\[
p^m_L = \arg \max_p [(p - c_L + (I - 1)(q_H - c_H))G(q_L - p)]
= \arg \max_p ((p - \Delta_q) - c_L + (I - 1)(q_H - c_H))G(q_L - (p - \Delta_q)) - \Delta_q
= p^m(q_H, \Delta_q + c_L) - \Delta_q.
\]
Hence, the requirement (38) transforms to
\[
p^m_H = p^m(q_H, c_H) < p^m(q_H, \Delta_q + c_L),
\]
which follows as, when this is interior for a non-degenerate \( G(\cdot), \partial p^m(q, c)/\partial c > 0 \) and as \( \Delta_q + c_L > c_H \) from \( \Delta_q > \Delta_c \). Q.E.D.