Mergers on Networks

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Abstract

I take a closer look at the type of mergers where each firm owns multiple shops across a country. The current approach of the European Commission is based on the analysis of shops’ catchment areas, where each area is studied in isolation. I show how to extend the Commission’s approach to take overlaps in catchment areas into account. In effect, I compute the general equilibrium whereas the Commission considers partial equilibria. I run a simulation to compare price increases under both approaches and find that the ranking of price increases coincides in less than 60% of cases. Furthermore, there are cases where general equilibrium analysis predicts completely opposite results from those of partial equilibrium analysis done by the Commission. In light of these differences a revision of the European Commission practice is recommended.

Key Words: mergers, networks, spatial competition, consumer demand.

JEL Classification: D43, D85, L13, L14, L40.

1 Introduction

In the recent past the European Commission investigated a number of merger cases where the merging parties owned multiple shops across a country.1 A typical case looks as follows. The shops can be supermarkets, pharmacies, gas stations, swimming pools, cinemas, etc. Due to transportation

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1See cases M.1221, M.1684, M.4686, M.6506, M.6822.
costs, the competition between the shops is local in scope. The number of shops can go into hundreds or thousands and it is infeasible for the merging parties or the Commission to analyze them all on a case by case basis. Initial screening for potentially problematic areas is necessary. In many cases the Commission has adopted the following approach.

Given the type of shops under investigation, the maximum travel time is defined so that it is sensible to assume that consumers are willing to travel that amount of time to reach a competing shop. Then, for each shop of the merging parties a local market is defined with the center at that shop and capturing all competing shops that fall within the maximum travel time. For each of the so-defined local markets a concentration analysis is performed. Normally, the European Commission follows the general guidelines with some industry-specific modifications, e.g. local markets with the HHI below 2 000 and the HHI increase below 250 are considered non-problematic.\(^2\) All local markets that are not automatically cleared will require more thorough investigation.

In effect, the Commission studies what impact a merger can have on prices using partial equilibrium analysis. However, local markets overlap and if the corresponding connections are taken into account, i.e. if we look at the general equilibrium, the effect on prices can potentially be different. Just how important is this possible difference between partial and general equilibrium analyses and just how large can it be?

Firstly, some mergers are cleared subject to divestments. In such cases the initial screening procedure might also be used to decide on which divestments are necessary. If the general equilibrium analysis delivers a different set of problematic areas in comparison with the partial equilibrium analysis, then that means the firms might be divesting wrong shops (from the society’s point of view). Secondly, the outcome of the initial screening procedure might decide the outcome of the merger case. We can speculate that in the worst case scenario a merger might be cleared unconditionally using the partial equilibrium analysis whereas it should be blocked based on the general equilibrium analysis, or vice versa.

In this paper I combine the insights of the literature on differentiated Bertrand competition and the literature on network games to setup a model that, on one hand, is in line with the European Commission’s practice of defining local markets and, on the other hand, allows for the general equilibrium analysis. I then present an example where the general and partial equilibrium approaches deliver opposite results. In light of this example, there is scope for improvement within the current antitrust practices when it comes to mergers of firms with multiple points of sale.

In general, competition economists are aware that general equilibrium effects can be important. See, e.g. the *chain of substitution* argument in Bishop and Walker (2010, p. 145). The argument goes as follows. Suppose the geography is such that shops A and B compete with each other, shops B and C compete with each other, but not shops A and C directly (see Fig. 1). The chain of substitution argument says that even if shops A and B belong to the merging parties, there might not be a substantial increase in prices after the merger, because the prices in shop B will be constrained by shop C, and the prices in shop A will be constrained by shop B. Whereas partial equilibrium analysis identifies a monopoly market centered around shop A.\(^3\) In this paper I undertake a more formal analysis of this critique and show with a specific example what the chain of substitution argument might imply for merger analysis.

I define consumer preferences following Dixit (1979), Singh and Vives (1984), and Häckner (2000). Conceptually, Dixit’s specification implies that consumers diversify their purchases across all neighbouring shops. Given a suitable time horizon, this feature is realistic for some of the examples given earlier: supermarkets, gas stations. Another possible modelling approach is to use discreet choice models. Arguably, discreet choice models are more suitable for, e.g., swimming pools. However, these models—of which the logit model is the simplest example—are likely to deliver multiple equilibria when complex networks are considered. And multiplicity of equilibria is a hurdle for normative applications of the model.

When the Commission defines local markets, they define which shops exhibit competitive pressure on the current shop. I will assume that consumers are concentrated at shop locations. Then saying that shop A exhibits competitive pressure on shop B is equivalent to saying that consumers that are located at B can shop not only at B but also at A. In this way, I can take the local markets as defined by the Commission as a starting point and from there draw the corresponding network of shops and consumers.

Placing consumers at network nodes results in a model that is easily tractable even for thousands of shops. Following Hotelling, one can also place consumers at the edges, see Heijnen and Soetevent (2014). These models can give new theoretical insights but they are hardly tractable for

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\(^3\)The European Commission has accepted the chain of substitution argument in its most general form—by considering a global market instead of a set of local markets when local markets’ overlaps are large enough—in cases M.1221 and M.1684.
empirical work with many locations.

Generally speaking, I analyze a linear-quadratic network game, with firms competing in prices. This type of games has been well studied, see e.g., Ballester et al. (2006), Ballester and Calvó-Armengol (2010). These authors, as well as Bramoullé et al. (2014), also remark that a linear-quadratic network game can be rationalized with linear-quadratic utility functions. I follow up on their remarks with a formal specification suitable for studying price competition. Bloch and Quéré (2013) also explore linear-quadratic games where several locations can be owned by the same firm, but they give a behavioral foundation to their model due to a different focus of their paper.

Once the network game is set up, it is straightforward to compute general equilibrium prices before and after the merger. The next step is to compare the resulting increase in prices with the European Commission’s approach. I do not adhere to the Commission’s guidelines literally, meaning that I do not compute HHI changes. Instead—for the ease of comparison—I compute price changes for each local market defined as a neighbourhood subgraph of the shop in question. In this way I can compute price increases using partial equilibria and thus neglect chain of substitution effects.

I compare both approaches by computing the rank correlation of the respective price increases. I analyze all possible shop partitions into firms and all possible mergers on all graphs of size 6. In some case the rank correlation equals 1 implying that the largest price increases are identified for the same markets irrespective of the chosen approach. In many other cases the rank correlation is negative. I present an example with 6 shops, where the price increase among the 3 shops of the merging parties is completely opposite between the two approaches. The local market that is expected to have the largest price increase when using partial equilibrium analysis is expected to have the smallest price increase when using general equilibrium analysis, and vice versa. (The result also holds if I use HHI increases instead of local price increases when emulating Commission’s analysis.)

My analysis implies that there are potential merger cases where the European Commission’s approach leads to the wrong conclusions. How likely that is to happen in practice is a different question. A simulation based on an actual past case of the Commission might help to answer that question but the required data is normally not publicly available.

The rest of the paper is organized as follows. In Section 2 and 3 I setup and solve a general model of mergers on networks. In Section 4 I consider an example with a merger over a graph of 6 shops. A simulation over all possible graphs of size 6 is presented in Section 5. Section 6 briefly concludes.
2 Model Setup

There are \( M \) firms and \( N \) shops. The ownership structure is given by matrix \( f \) such that \( f_{ij} = 1 \) whenever firm \( i \) owns shop \( j \) and \( f_{ij} = 0 \) otherwise. I impose \( \sum_i f_{ij} = 1 \), i.e. only one firm can own a given shop. Then

\[
s_i = \{ j | f_{ij} = 1 \}
\]

(1)

gives the set of all the shops owned by firm \( i \) and

\[
v(j) = \sum_i i f_{ij}
\]

(2)

gives the firm that owns shop \( j \).

Every shop \( i \) sets its own price \( p_i \). In practice, firms can sometimes have uniform price policies so that each shop belonging to the same firm sets the same price. However, product quality (e.g., product variety in grocery stores) and service quality (e.g., opening hours, queue waiting times) may still vary between the shops. The quality effectively constitutes the negative of a price: higher quality is more costly for the firms to produce and it is beneficial to consumers. Therefore the assumption that prices can vary across shops owned by the same firm is realistic. The fact that the European Commission analyzes local markets in such merger cases instead of a single national market further corroborates this assumption.

There is a unit mass of consumers located around each shop.\(^4\) Consumers can shop at their own location as well as at the neighbouring shops. Formally, let \( g \) be an adjacency matrix such that \( g_{ij} = 1 \) if consumers from location \( j \) can shop at location \( i \) and \( g_{ij} = 0 \) otherwise. By construction, \( g_{ii} = 1 \). An equivalent interpretation is that whenever \( g_{ij} = 1 \) shop \( i \) puts competitive pressure on shop \( j \).

I allow \( g \) to be asymmetric, i.e. a network. If shop 1 is a specialized grocery store and shop 2 is a supermarket, then 2 puts competitive pressure on 1 whereas 1 puts less or even no competitive pressure on 2. The solution in this section is given for the general case, but for the example in Section 4 it suffices to consider a symmetric \( g \), i.e. a graph.

Let

\[
N_i^+ = \{ j | g_{ji} = 1 \}
\]

(3)

denote the in-neighbourhood of \( i \). \( N_i^+ \) is a set of all the shops that put competitive pressure on shop \( i \), plus shop \( i \) itself.

Following H"ackner (2000),\(^5\) a representative consumer located at \( i \) has

\(^4\)The model can be readily extended to heterogeneous demand, in which case it can be calibrated to fit an actual allocation of market shares of different shops.

\(^5\)There is a typo in that paper on p. 234. In the utility definition it should be either \( \gamma \sum_{i \neq j} q_i q_j \) or \( 2 \gamma \sum_{i>j} q_i q_j \) as otherwise the formula is not consistent with Singh and Vives (1984).
utility $U_i$ defined as follows:

$$U_i = \alpha \sum_{j \in N_i^+} q_{ij} - \frac{1}{2} \left( \sum_{j \in N_i^+} q_{ij}^2 + \gamma \sum_{k,j \in N_i^+ \atop k \neq j} q_{jk}q_{ik} \right) + q_0 =$$

$$\alpha \sum_{j} g_{ji}q_{ij} - \frac{1}{2} \left( \sum_{j} g_{ji}q_{ij}^2 + \gamma \sum_{k,j \in N_i^+ \atop k \neq j} g_{ji}g_{kj}q_{ij}q_{ik} \right) + q_0,$$  \hspace{1cm} (4)

where $q_{ij}$ is the amount of goods that consumer located at $i$ buys at location $j$, $q_0$ is the outside good, $\alpha > 0$, and $0 \leq \gamma < 1$. With this $\gamma$ range the goods offered at different shops are gross substitutes. The associated budget constraint is given by

$$\sum_{j \in N_i^+} p_j q_{ij} + q_0 \leq I.$$  \hspace{1cm} (5)

Define

$$q_j = \sum_i q_{ij}$$  \hspace{1cm} (6)

to be the total amount of goods purchased at shop $j$. Then the profits of firm $i$, which equal the sum of the profits of its shops, are given by

$$\pi_i = \sum_{j \in s_i} q_j (p_j - c) = \sum_j f_{ij}q_j (p_j - c),$$  \hspace{1cm} (7)

where $c$ denotes marginal costs.

The timing of the model is as follows. In period one the firms simultaneously set the prices in their shops. In period two consumers shop so as to maximize their utility given the prices. I focus on the Nash equilibrium of this game.

Games where players have incomplete information about the network have received attention in the literature. For example, Galeotti et al. (2010) assume that each player knows his node’s degree but not the degree of his neighbours. Where does the current model stand when we consider the information requirements for the players? Do we assume that the firms know their competitors, the competitors of their competitors, etc.? While assuming such knowledge might be unrealistic, this assumption is not required. For the considered Nash equilibrium to be a practical solution concept it suffices to assume that the firms know the residual demand on their products given the prices of their competitors. This is a milder assumption that is satisfied if, for example, firms regularly conduct marketing research to reveal the demand they face.
3 General Equilibrium

Denote with $d_i^+ = |N_i^+| = \sum_j g_{ji}$ the in-degree of shop $i$. From (4) and (5) it follows that the consumer utility is maximized when

$$q_{ij} = g_{ji} \left( \beta_i + \delta_i \sum_k g_{ki}p_k - \eta p_j \right),$$  \hspace{1cm} (8)

where

$$\beta_i = \frac{\alpha}{1 + \gamma(d_i^+ - 1)},$$ \hspace{1cm} (9)

$$\delta_i = \frac{\gamma}{\alpha(1 - \gamma)} \beta_i,$$ \hspace{1cm} (10)

$$\eta = \frac{1}{1 - \gamma}.$$ \hspace{1cm} (11)

Summing up over $i$ we obtain

$$q_j = \sum_i g_{ji} \beta_i + \sum_{i,k} g_{ji}g_{ki}\delta_i p_k - \eta p_j \sum_i g_{ji}.$$ \hspace{1cm} (12)

Substituting this expression for $q_j$ into (7) and differentiating $\pi_r$ with respect to $p_t$ (whenever $f_{rt} = 1$) gives

$$\frac{\partial \pi_r}{\partial p_t} = a_{rt} + \sum_j b_{rtj}p_j - h_t p_t,$$ \hspace{1cm} (13)

where

$$a_{rt} = -c \sum_{i,j} f_{rj}g_{ji}g_{ti} \delta_i + \eta c \sum_i g_{ti} + \sum_i g_{ti} \beta_i,$$ \hspace{1cm} (14)

$$b_{rtj} = (1 + f_{rj}) \sum_i g_{ji}g_{ti}\delta_i,$$ \hspace{1cm} (15)

$$h_t = 2\eta \sum_i g_{ti}.$$ \hspace{1cm} (16)

Putting all the first order conditions together and solving gives the prices in the general equilibrium:

$$p = X^{-1}y,$$ \hspace{1cm} (17)

where $X_{ij} = b_{v(i)j} - \mathbf{1}_{i=j}h_i$, $y_i = -a_{v(i)i}$, and $\mathbf{1}_A = 1$ if $A$ is true and $\mathbf{1}_A = 0$ otherwise.

It is straightforward to derive sufficient second order conditions for the profit maximization problem. Whenever shops $t$ and $k$ belong to firm $r$ ($f_{rt} = 1$, $f_{rk} = 1$) we have

$$\frac{\partial \pi_r}{\partial p_t \partial p_k} = b_{rtk} - \mathbf{1}_{t=k}h_t.$$ \hspace{1cm} (18)
For each firm $r$ I require the Hessian matrix $\left\{ \frac{\partial \pi_r}{\partial p_t \partial p_k} \right\}$ to be negative semidefinite. Then $\pi_r$ is concave for each $r$ and (17) gives profit maximizing equilibrium prices.

A block-diagonal matrix is negative semidefinite if and only if all the block submatrices are negative semidefinite. Further, renumbering both rows and columns in the same way does not change whether a matrix is negative semidefinite or not. Therefore the Hessian matrices for all firms are negative semidefinite if and only if matrix $\Omega$ is negative semidefinite, where

$$\Omega_{ij} = b_{v(i)j} f_{v(i)j} - 1_{i=j} h_i.$$ (19)

4 Example

Consider the example depicted in Fig. 2. There are 4 firms denoted A through D. Between them they have 6 shops denoted 1 through 6. Firms C and D are planning a merger. What would be the expected price increase across all the shops?

In this example we have

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}. $$ (20)

Let $f^*$ denote the ownership matrix before the merger and $f^{**}$—after the
merger. Then

\[
f^* = \begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 
\end{pmatrix}
\]

(21)

\[
f^{**} = \begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 
\end{pmatrix}
\]

(22)

Finally, for this example take \( \alpha = 1.0, \gamma = 0.9, c = 0 \).

Then from (17) for each shop \( i \) we can compute prices before and after the merger, denote them respectively \( p^*_i \) and \( p^{**}_i \). These are the prices in the general equilibrium. However, the current approach of the European Commission is to consider partial equilibria. We can emulate that approach by considering each shop together with its closest neighbours as a separate graph. For example, the graph centered around shop 5 includes shops 5, 6, and 3. For each such subgraph \( j \) let \( \tilde{p}^*_ji \) and \( \tilde{p}^{**}ji \) denote the prices of shop \( i \) before and after the merger.

Fig. 3, left side, plots \( \tilde{p}^{**}ji / \tilde{p}^*_ji \) against \( p^{**}i / p^*_i \) for \( i \in \{3,5,6\} \), i.e. for the shops of the merging parties. We can see that the expected increases in prices are completely opposite depending on the chosen approach. If we use the partial equilibrium approach, then the local market around shop 5 becomes a monopoly and a four-fold price increase is expected. Whereas if we use the general equilibrium approach, the price rise at 5 is the smallest among the shops of the merging parties because the price at 5 is limited by the price at 6, which in turn continues to be under competitive pressure from the three neighbouring shops. And the situation is just the opposite when we look at 3. According to the partial equilibrium approach there is no expected price increase, because the local market around 3 does not change after the merger. However, according to the general equilibrium approach
this area is the most problematic, because the only remaining competitor in the area, shop 4, is now surrounded by the shops of the merging parties that are all raising prices.

In practice, the Commission often will not compute price increases as such but rather approximate them with HHI increases. Fig. 3, right side, plots HHI increases computed on the respective neighbourhood subgraphs against $p_i^{**}/p_i^*$. The outcome is the same: the partial equilibrium approach yields results that are completely at odds with the results from the general equilibrium.

5 Simulation

How common is the problem depicted in the example? If the discrepancy between partial and general equilibria manifests itself in a small number of instances, then it is only of a theoretical interest. However, if the problem is sufficiently common, then there is a non-negligible chance that wrong problematic areas are given attention in the type of mergers we consider, that wrong divestments are chosen, or even that wrong clearing or blocking decisions are made.

Ex-post analysis of past merger cases would be most illuminating but as a first step I undertake the following simulation. I consider all possible graphs of 6 nodes, all possible partitions of shops into firms for each of these graphs, and all possible mergers on top of that. The graphs represent travelling times between the shops. If a graph is planar, then it is always possible to come up with geography that rationalizes that graph. It is not always possible to do so for non-planar graphs. Therefore I drop non-planar graphs. Further, I drop cases where there is only one firm (no merger possible) and a number of cases with corner solutions.

To compare all the resulting cases, we need to assign weights to them. Saying that all cases are equally likely is one possibility but I opt for a less ad-hoc approach and make the following probability assumptions about the building blocks of the model.

Let $\rho$ denote the probability that there is an edge between any two nodes of a graph, and let $P(g)$ denote the probability that a given graph is characterized by an adjacency matrix $g$. Then

$$\ln P(g) = \sum_{i>j} g_{ij} \ln \rho + \sum_{i>j} (1 - g_{ij}) \ln(1 - \rho).$$

(23)

I further assume there are $n$ potential firms ($n = 6$ in this simulation) and that each shop $i$ has an equal probability to belong to any of these potential firms. Then all possible ownership matrices are equally likely and the probability of a given matrix $f$ is $P(f) = \frac{1}{n^n}$. Finally, I assume that any merger is equally likely as well.
Then, for each specific case, i.e. for each specific graph, ownership matrix, and merger, we have a well-defined probability. Analogously with the earlier analysis, for each such case I compute the general equilibrium, the partial equilibria, the resulting increases in prices under both approaches (only for the shops of the merging parties), and finally I rank the increases in prices and compute the rank correlation between general and partial equilibria approaches. The cumulative distribution function of the resulting rank correlation is depicted in Fig. 4.

General and partial equilibria approaches give the same ranking of price increases with probability of less than 60%. In the remaining cases the ranking diverges between the two approaches. Moreover, with probability of about 5%–10% the rank correlation is negative and in about 2% of cases it equals $-1$, i.e. general and partial equilibria approaches yield completely opposite results in those cases.

6 Concluding Remarks

The European Commission plays a fundamental role in ensuring a fair, competitive, and united European market. The decisions of the Commission have direct impact on business profits and on consumer welfare. In this paper I show that the approach the European Commission uses for mergers of firms that have multiple points of sale can yield completely incorrect predictions. Namely, the European Commission undertakes partial equilibrium analysis which in some cases gives results that are at odds with the results of the general equilibrium. Without considering a real merger case it is dif-

![Figure 4: CDFs of Rank Correlation of Price Increases](image-url)
difficult to judge whether the problem I identify can manifest itself in practice. However, given the theoretical possibility that the results can change substantially, given the profits and consumer welfare at stake, and given that the general equilibrium analysis can be performed relatively easily on the basis of already defined local markets, there is no reason not to advice a change of practice to the European Commission.

References


