Vertical Licensing, Input Pricing, and Entry

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Abstract

We explore the incentives of a vertically integrated incumbent firm to license the production technology of its core input to an external firm, transforming the licensee into its input supplier. We find that the incumbent opts for licensing even when licensing also transforms the licensee into one of its direct competitors in the final products market. In fact, the licensee’s entry into the final products market, although increases the competition and the cost that the licensor faces, it reinforces, instead of weakens, the licensing incentives. Furthermore, the licensee’s entry augments the positive welfare implications of vertical licensing.

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1 Introduction

Original brand manufacturers often license the production technology of their core inputs to external firms.\(^1\) Such a practice transforms the licensees into the licensors' input suppliers and potentially also into their direct competitors in the final products market. An illustration comes from the commercial aircraft market, where, in 2003, Boeing, the US aircraft manufacturer, licensed its wing production technology to Mitsubishi Heavy Industries, the Japanese engineering and electrical equipment producer. Boeing, as well as its main competitor, Airbus, had been already subcontracting the production of various aircraft components to external suppliers. However, never before a commercial aircraft manufacturer had subcontracted its wing production. Boeing’s decision was accepted with widespread criticism. First, because Boeing’s wing technology has been regarded as its crown jewel, and second, because the transfer of wing manufacturing effectively gave to Mitsubishi ‘total production competence’ with regard to commercial aircraft. Mitsubishi is now ready to launch its own passenger jet, which is due to enter service in 2018.\(^2\)

A similar illustration can be found in the electronic appliances market, where, in 1984, a Chinese firm, Haier, which at the time was operating as a producer of electrical equipment for original brand manufacturers, acquired the technology for producing high-quality refrigerators from a German original brand manufacturer, Liebherr. A year after the licensing of Liebherr’s technology, Haier introduced its own first four-star refrigerator in the market, evolving into an original brand manufacturer too. Haier soon became the leader refrigerator producer in China and, eventually, a major competitor in the global market.

The above illustrations give rise to a number of important questions regarding vertical licensing, such as: Does a firm wish to license its input production technology to an external firm when licensing can cause the entry of the licensee into its final product market? How the licensee’s entry affects the licensing incentives? What is the role of input pricing? What are the welfare implications of vertical licensing? In this paper, we address these questions.

To this end, we consider a framework in which two incumbent firms produce two compet-

\(^1\)According to empirical evidence, technology licensing is a common practice among firms (e.g., Caves et al., 1983, Nadiri, 1993, Anand and Khanna, 2000). Many well-known firms, such as IBM, Hitachi, Kodak, and Procter & Gamble, have licensed their technologies, earning millions of dollars in licensing revenue (e.g., A Market for Ideas, The Economist (October 20, 2005)).

\(^2\)Note that without the ability to manufacture wings, Mitsubishi could not become an independent player in the passenger jet market. For more information regarding this case, see e.g., Secret Wounds Of Globalism: Boeing Sells Its Technology – Cheap – To Japan, Forbes (January 5, 2014), and Boeing Outsourcing Gives Wing to Concerns, Chicago Tribune (December 21, 2003).
ing final goods using an input that they initially produce in-house. One of the incumbents considers licensing its input production technology to an external firm for a fixed licensing fee. When the licensing agreement is signed, the licensor sources the input from the licensee after bargaining over the terms of a two-part tariff contract. Moreover, when the licensing agreement is signed, the licensee can enter into the final goods market and compete with the two incumbents. We assume that competition in the final products market is in quantities and examine what happens when the licensee does not enter into the final goods market - the ‘no entry case’ - and when it enters - the ‘entry case’.

We find that independently of whether the licensee enters into the final goods market or not, the incumbent always opts for licensing. The key drivers of licensing, however, differ substantially among the entry and the no entry case. In the no entry case, licensing arises because of efficiency reasons: when the incumbent licenses its input technology, it becomes more efficient and, thus, enjoys a competitive advantage in the final goods market. The higher efficiency of the licensor is not due to a cost advantage of the licensee. It is due, instead, to input pricing. Specifically, in the no entry case, the licensee sets a wholesale price which is below the input’s marginal cost. We refer to this as the input pricing effect of licensing. The licensee does so because it can extract part of the resulting higher profits of the licensor through the fixed fee of the two-part tariff. In fact, due to the higher efficiency, the incumbent is willing to license its input production technology even for free when its bargaining power is sufficiently high.  

In the entry case, licensing results in an increase in the number of downstream competitors and, thus, in intensification of downstream competition. We refer to this as the competition intensity effect of licensing. The licensee in this case sets a wholesale price that exceeds the input’s marginal cost, decreasing the efficiency of the licensor. In the entry case too, licensing gives rise to two more effects: the market expansion effect and the business stealing effect. The market expansion effect refers to the fact that the entry of the licensee causes an increase of product variety, and thus, an expansion of the demand in the final products market. The business stealing effect refers instead to the fact that the licensee, by entering into the final products market, “steals” part of the sales and market share of the

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3 Licensing for free is observed in the market for open source software as well as in other markets. For instance, in January 2015, Toyota announced that it would make more than 5,600 patents on fuel-cell technologies available for use, free of royalty payments, to a wide array of companies in the transportation sector. For more on this, see e.g. Toyota Offers To License Hydrogen Fuel-Cell Patents, For Free, Green Car Reports (January 5, 2015), and Toyota To Share Hydrogen Fuel Cell Patents, Forbes (January 5, 2015).
licensor’s rival. Capturing the licensee’s profits, through the licensing fee, the licensor takes full advantage of both the market expansion effect and the business stealing effect, which work in favor of licensing and lead to its emergence in equilibrium.

Importantly, the entry of the licensee into the final goods market, although intensifies competition and reduces the licensor’s efficiency, strengthens, instead of weakens, the licensing incentives. Why is that? In the entry case, as mentioned above, the market expansion effect and the business stealing effect, which translate into a larger pie and a larger share of the pie for the licensor, respectively, are present. Moreover, in the entry case, the licensor and the licensee do not suffer severely from the competition intensity effect. This is so because they set the input price in such a way as to reduce the negative impact of the competition intensity effect. In other words, the licensee and the licensor use their vertical trading contract as an instrument that allows them to behave in a pro-collusive way. Clearly, this constitutes an additional motive for vertical licensing, which is absent in one-tier markets or in horizontal licensing.

Vertical licensing turns out to be desirable not only for the licensor, but also for the consumers and the economy as a whole. This holds both with and without the entry of the licensee in the final goods market. Intuitively, licensing results in lower final prices, and thus, in higher consumers’ surplus, either because it enhances the efficiency of the licensor (in the no entry case) or because it intensifies market competition (in the entry case). In fact, in the entry case, licensing is even more desirable because of the increased product variety and competition.

Examining various extensions of our main model, we demonstrate that the entry of the licensee into the final goods market reinforces the licensing incentives in other instances too, such as, when the rival incumbent also sources its input from the licensee or when competition in the final products market is in prices. In fact, we demonstrate that there are instances, in which the emergence of vertical licensing in equilibrium would be impossible without the entry of the licensee into the final goods market. This is so when the licensor is initially the only incumbent in the market or when the licensor and the licensee trade through a wholesale price contract. The licensee’s entry, though, can discourage licensing when it is through royalties.

Our work is related to the vast theoretical literature on technology licensing. This literature analyzes various aspects of licensing such as, the choice among royalties and licensing fees (e.g., Kamien and Tauman, 1986, Muto, 1993, Wang, 1998), the impact of
licensing on innovation (e.g., Gallini and Winter, 1985), the role of information asymmetries (e.g., Gallini and Wright, 1990, Beggs, 1992), and the choice among merger and licensing (e.g., Fauli-Oller and Sandonis, 2003). Most of the papers in this literature assume that the licensor and the licensee(s) are in different markets or in the same one-tier market. Exceptions include the papers by Mukherjee (2003), Arya and Mittendorf (2006), Mukherjee and Ray (2007), and Rey and Salant (2012), which, similar to us, examine licensing in a vertically related market. These papers, however, focus on settings in which the licensor and the licensee either operate in the same production stage of the vertical chain or, although they operate in different stages, they do not have a seller-buyer relationship.\(^4\) Moreover, even when they allow for entry in the market, triggered by licensing, they do not consider the entry of the licensee into the licensor’s final products market. Our paper complements the existing literature on technology licensing, focusing on the analysis of a situation, which, although it has been neglected by the literature, is present in a number of real world licensing cases (e.g., Boeing).

Our paper is also related to the literature on outsourcing. A number of papers within this literature (e.g., Pack and Saggi, 2001, Shy and Stenbacka, 2003, Sappington 2005, Arya et al., 2008a and 2008b, Lim and Tan, 2010) analyze a final product manufacturer’s ‘make-or-buy’ decision. That is, its choice among input production in-house and input sourcing from an external firm - outsourcing.\(^5\) Some of these papers assume that the input production is outsourced to an already existing vertically integrated rival. Others, instead, assume that it is outsourced to an independent upstream firm. In particular, Pack and Saggi (2001) and Goh (2005) examine a buyer’s incentives to outsource its technology to a supplier when technology diffusion can result in the entry of another firm downstream and upstream, respectively. Both papers find that entry, by driving the input price to marginal cost, can be beneficial for the incumbents engaged in technology transfer. In contrast to these papers, we consider the entry of the firm to which the input production is outsourced and we show that licensing is beneficial even when input price is greater than the marginal cost. Lim and Tan (2010), similarly to us, consider a setting in which the supplier becomes a direct competitor of the buyer after outsourcing. However, they focus on the role of the buyer’s rate of learning and brand equity, while we focus on the role of input pricing in the

\(^4\) An exception in this respect is the paper of Rey and Salant (2012) which considers vertical licensing.

\(^5\) The practice of outsourcing is obviously quite similar to the practice of licensing especially when licensing does not involve fees or royalties.
licensing incentives.

The remainder of the paper is organized as follows. In Section 2, we describe our main model. In Section 3, we examine the licensing incentives in both the no entry case and the entry case, and characterize the impact of entry on these incentives. In the following section, we evaluate the welfare implications of vertical licensing. In Section 5, we examine the robustness of our main findings when both of the incumbents source the input from the licensee. In Section 6, we discuss a number of other extensions of our main model. Finally, in Section 7, we conclude. All the proofs are included in Appendix B.

2 The Model

We consider a market consisting initially of two firms, firm 1 and firm 2. Each firm $i$, with $i = 1, 2$, produces a differentiated final good using, in a one-to-one proportion, a core input that it produces in-house at marginal cost $c > 0$.

Both firms hold a patent for their input production technologies. One of them, without loss of generality firm 1, considers licensing its input production technology to an external firm, firm $S$, for a fixed licensing fee, $F \geq 0$.\(^6\) When the licensing agreement is signed, the licensee (firm $S$) is in the position to produce the licensor’s (firm 1’s) input at marginal cost $c$.\(^7\)

The knowledge that firm $S$ acquires through licensing regarding the production of the final good’s core input could allow it to produce the final good too. Thus, licensing could also cause firm $S$’s entry into the final goods market. In what follows, we consider the ‘entry case’ in which firm $S$ enters into the final goods market and the ‘no entry’ case in which it does not enter.\(^8\) In both cases, we assume that when the licensing agreement is signed, firm 1 commits to sourcing the input from firm $S$. The input sourcing terms include the terms of a two-part tariff: a fixed fee, $T$, and a wholesale price per unit of input, $w$, that firm 1 pays to firm $S$. These terms are determined through Nash bargaining among firm $S$ and firm 1. The bargaining power of firm $S$ and firm 1 is given by $\beta$ and $1 - \beta$, respectively,

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\(^6\) As we demonstrate in Section 6, the licensor prefers using a fixed licensing fee to a variable royalty.

\(^7\) We abstract from assuming that firm $S$ is more efficient in input production than firm 1. Otherwise, the incentives for vertical licensing would be straightforward.

\(^8\) This could be so because, for instance, the cost of entry in the final goods market is prohibitively high and, thus, entry is blocked or because the ability of producing the core input does not suffice for the production of the final good too.
with $0 < \beta < 1$.\footnote{We should note that in the alternative scenario in which firm 1 can produce the input in-house in case of disagreement with firm $S$ during the negotiations, the equilibrium $w$ is the same as when firm 1 commits to sourcing the input from firm $S$. The only thing that changes then is $T$ which is lower since firm $S$ needs to compensate firm 1 for its outside option. As we will show later on, firm 1 manages to extract $T$ through the licensing fee. Thus, the change in $T$ would not affect our main conclusions.}

The (inverse) demand function for firm $i$’s final good is:

$$p_i(q_i, Q_{-i}) = a - q_i - \gamma Q_{-i}, \quad 0 < \gamma < 1, \quad a > c,$$

where $p_i$ and $q_i$ are the price and the quantity of firm $i$’s final good, respectively, and $Q_{-i}$ is the quantity of its rival(s)’ final good(s). In particular, $Q_{-i} = q_j$, with $i, j = 1, 2$ and $i \neq j$, in the no entry case, while $Q_{-i} = q_j + q_k$, with $i, j, k = 1, 2, S$ and $i \neq j \neq k$, in the entry case. The parameter $\gamma$ measures the degree of product differentiation; namely, the higher $\gamma$ is, the closer substitutes the final goods are.

The timing of moves is as follows. First, firm 1 decides whether to license its input technology to firm $S$. In case of licensing, it sets the licensing fee $F$ and, in turn, firm $S$ signs or not the licensing agreement. If the agreement is signed, in the following stage, firm 1 and firm $S$ negotiate over $(w, T)$. In the last stage, firm 1 and firm 2, as well as firm $S$ in the case of licensing and entry, choose their quantities simultaneously and separately.\footnote{The inclusion of bargaining over the licensing fee too would not affect qualitatively our main conclusions.}

We solve for the subgame perfect Nash equilibrium of this game.

We make the following assumption throughout in order to guarantee that all the firms face a non-negative marginal cost in all the cases under consideration:

**Assumption 1:** $a \leq 5c$

We start our analysis with the benchmark case in which there is no licensing. In the absence of licensing, firm 1 and firm 2 compete in the standard Cournot way. Specifically, each firm $i$ chooses its output in order to maximize its profits: $\pi_i(q_i, q_j) = (a - q_i - \gamma q_j)q_i - c q_i$, with $i, j = 1, 2$ and $i \neq j$. Solving the resulting system of first order conditions, we obtain the Cournot-Nash equilibrium quantities, $q_1^B$ and $q_2^B$, and the respective equilibrium profits, $\pi_1^B$ and $\pi_2^B$, included in Table 1 of Appendix A.

\footnote{We assume implicitly that firm 2 (besides firm 1 and firm $S$) observes the contract terms $(w, T)$ before choosing its own quantity. Still, our results would be exactly the same in the alternative scenario with partial observation of firms’ marginal costs. That is, in the scenario in which firm 2 does not observe the contract terms and, thus, the endogenous marginal cost of firm 1, while firms 1 and firm $S$ observe firm 2’s exogenous marginal cost given that firm 2 is vertically integrated. The analysis of this scenario is available from the authors upon request. In Section 6, we also examine what happens when firms compete in prices.}
3 Licensing Incentives

In this section, first, we analyze the licensing incentives with and without the entry of the licensee into the final goods market and, then, we examine how they are influenced by entry.

3.1 Licensing and No Entry

We start with the analysis of the case in which the licensing agreement has been signed and firm S has stayed out of the final goods market.

In the last stage, firm 2 faces the same maximization problem as in the benchmark case. Its competitor, firm 1, chooses \( q_1 \) in order to maximize its own (gross from \( T \) and \( F \)) profits:

\[
\pi_1(q_1, q_2, w) = (a - q_1 - \gamma q_2)q_1 - wq_1.
\]

Solving the system of the first order conditions, we derive the equilibrium quantities in terms of \( w \):

\[
q_1(w) = \frac{a(2 - \gamma) + \gamma c - 2w}{4 - \gamma^2} \quad \text{and} \quad q_2(w) = \frac{a(2 - \gamma) - 2c + \gamma w}{4 - \gamma^2}.
\] (1)

Obviously, a decrease in \( w \) results in higher output for firm 1 and lower output for firm 2.

In the following stage, firm S and firm 1 negotiate over \((w, T)\). In particular, they solve the following generalized Nash bargaining problem:

\[
\max_{w,T} \left[ \pi_S(w) + T \right]^\beta \left[ \pi_1(w) - T \right]^{1-\beta},
\] (2)

where \( \pi_S(w) = (w - c)q_1(w) \) are firm S’s profits and \( \pi_1(w) = \pi_1(q_1(w), q_2(w), w) \). Note that the disagreement payoffs of both firms are equal to zero since neither firm has an outside option. Maximizing (2) with respect to \( T \), we find:

\[
T = \beta \pi_1(w) - (1 - \beta)\pi_S(w).
\] (3)

Using (3), the gross (from \( F \)) profits of firm S and firm 1 can be rewritten as:

\[
\pi_S(w) + T = \beta(\pi_S(w) + \pi_1(w)) \quad \text{and} \quad \pi_1(w) - T = (1 - \beta)(\pi_S(w) + \pi_1(w)).
\] (4)

Substituting the above into (2), we obtain an expression which is proportional to the joint profits of firm S and firm 1. It follows that \( w \) is chosen to maximize these profits:

\[
\max_w \pi_S(w) + \pi_1(w) = (a - q_1(w) - \gamma q_2(w))q_1(w) - cq_1(w).
\] (5)
From the first order condition of (5), we obtain the equilibrium wholesale price and substituting it into (3), we also obtain the equilibrium fixed fee:

\[ w^{LN} = \frac{8c - 2(a + c)\gamma^2 + (a - c)\gamma^3}{4(2 - \gamma^2)} \quad \text{and} \quad T^{LN} = \frac{(a - c)^2(2 - \gamma)^2(2\beta + (1 - \beta)\gamma^2)}{8(2 - \gamma^2)^2}. \] (6)

One can easily check that \( w^{LN} < c \). That is, firm \( S \) subsidizes, through the wholesale price, the production of its customer, firm \( 1 \).\(^{12}\) As we saw above, by charging a lower wholesale price, firm \( S \) increases the aggressiveness of firm \( 1 \) in the final goods market and enhances its output at the expense of firm \( 2 \)’s output. Firm \( S \) has incentives to do so because it can use, in turn, the fixed fee \( T \) in order to capture part of the resulting higher firm \( 1 \)’s profits.\(^{13}\)

Clearly, the higher firm \( S \)’s bargaining power is, the larger is the share of firm \( 1 \)’s profits that it captures through \( T \).

The licensing fee is determined in the following way: firm \( 1 \) knows that firm \( S \) will reject the licensing agreement if and only if its profits without the agreement exceed its profits with the agreement. Since the former profits are equal to 0, it follows that firm \( 1 \) will optimally set: \( F^{LN} = \pi_S(w^{LN}) + T^{LN} \). Therefore, firm \( 1 \)’s net equilibrium profits in the licensing and no entry case are: \( \pi_1^{LN} = \pi_1(w^{LN}) - T^{LN} + F^{LN} = \pi_1(w^{LN}) + \pi_S(w^{LN}) \).

Obviously, firm \( 1 \) captures not only the profits from its own sales in the final goods market but also firm \( S \)’s profits from the input sales, i.e., it captures all of its joint profits with firm \( S \). Comparing firm \( 1 \)’s equilibrium profits in the licensing and no entry case \( \pi_1^{LN} \) (included in Table 1 of Appendix A) with its profits under no licensing \( \pi_1^B \), we reach the following conclusion.

**Proposition 1** When firm \( S \) does not enter into the final goods market, firm \( 1 \) licenses its input technology.

As Proposition 1 informs us, firm \( 1 \) always licenses its input technology to firm \( S \) when the latter does not enter into the final goods market. It is important to note that this holds not only when firm \( 1 \) charges a positive fixed fee for the licensing agreement, but also when it offers the licensing agreement for free \( (F = 0) \), as long as its bargaining power is sufficiently high. This is stated formally in the following Corollary.

\(^{12}\) Assumption 1 guarantees that \( w^{LN} > 0 \).

\(^{13}\) A similar rationale can be found in the delegation literature (e.g., Vickers, 1985, Fershtman and Judd, 1987, Sklivas, 1987).
Corollary 1 When firm $S$ does not enter into the final goods market and licensing is for free ($F = 0$), firm 1 licenses its input technology if and only if $\beta$ is sufficiently low.

Why does firm 1 have incentives to provide its input production technology to firm $S$ even for free? Recall that when firm 1 produces the input in-house, it faces marginal cost $c$. When, instead, it licenses its input technology, it faces a lower marginal cost $w^{LN} < c$. We refer to the cost reduction that firm 1 enjoys through the input price as the input pricing effect of licensing. It follows that due to the input pricing effect, licensing results in an increase in firm 1’s efficiency; hence, in a cost advantage for firm 1 in the final goods market. A straightforward implication of this is that the gross from $T^{LN}$ profits of firm 1 are larger under licensing. When firm $S$’s bargaining power is not too large, firm $S$ obtains only a small share of these profits through $T^{LN}$. As a consequence, firm 1 is willing, then, to license its input technology even for free. Clearly, when a licensing fee is applied and firm 1 fully captures the profits that arise from its own sales in the final goods market, licensing incentives are always present independently of the bargaining power distribution.\footnote{As we demonstrate in Section 6, when firm 1 bargains with firm $S$ over the licensing fee too, it always has incentives for licensing.}

The above result is in accordance with a result from the literature on vertical separation, according to which vertical separation and, thus, external input sourcing can be preferred to vertical integration for strategic reasons. In particular, a number of papers within this literature (e.g., Vickers, 1985, Jansen, 2003) demonstrate that in settings with downstream quantity competition (strategic substitutability), a vertically separated upstream firm sets the wholesale price below the marginal cost in order to increase its downstream customer’s efficiency and profits and extract the latter through the fixed fee. This mechanism obviously coincides with the one that we have identified above. The main difference is that in our setting, licensing allows the downstream, and not the upstream, vertically separated firm to extract the profits.

3.2 Licensing and Entry

We turn now to the examination of the case in which the licensing agreement has been signed and firm $S$ has entered into the final goods market.

In the last stage of the game, firms 1, 2, and $S$ choose their outputs in order to maximize
their respective profits:

\[
\pi_1(q_1, q_2, q_S, w) = (a - q_1 - \gamma q_2 - \gamma q_S)q_1 - wq_1; \tag{7}
\]

\[
\pi_2(q_1, q_2, q_S, w) = (a - q_2 - \gamma q_1 - \gamma q_S)q_2 - c q_2; \tag{8}
\]

\[
\pi_S(q_1, q_2, q_S, w) = (a - q_S - \gamma q_1 - \gamma q_2)q_S - c(q_1 + q_S) + wq_1. \tag{9}
\]

Solving the system of the first order conditions, we find:

\[
q_1(w) = \frac{a(2 - \gamma) + 2cw - (2 + \gamma)w}{2(2 - \gamma)(1 + \gamma)}; \tag{10}
\]

\[
q_2(w) = q_S(w) = \frac{2(a - c) - \gamma(a - w)}{4 + 2\gamma - 2\gamma^2}. \tag{11}
\]

In the second stage, firm S and firm 1 solve the following maximization problem:

\[
\max_{w, T} [\pi_S(w) - d_S + T]^\beta [\pi_1(w) - T]^{1-\beta}, \tag{12}
\]

where \(\pi_1(w)\) and \(\pi_S(w)\) are found after substituting (10) and (11) into (7) and (9), respectively. It is important to note that while the disagreement payoff of firm 1 continues to be null, the same does not longer hold for firm S’s disagreement payoff. This is so because firm S now has an outside option in its bargaining with firm 1: in case of disagreement, firm S can still have profits from its own sales in the final goods market. In particular, its disagreement payoff is given by \(d_S = (a - q_S^D - \gamma q_2 B)q_S^D - c q_S^D\), where \(q_S^D = q_1^B\).

Maximizing (12) with respect to \(T\), we obtain:

\[
T = \beta \pi_1(w) - (1 - \beta) [\pi_S(w) - d_S]. \tag{13}
\]

Using the above expression, we find:

\[
\pi_S(w) - d_S + T = \beta [\pi_S(w) + \pi_1(w) - d_S]; \tag{14}
\]

\[
\pi_1(w) - T = (1 - \beta) [\pi_S(w) + \pi_1(w) - d_S]. \tag{15}
\]

Substituting (14) and (15) into (12), we note that the latter reduces to an expression proportional to the joint profits of firm S and firm 1 minus firm S’s disagreement payoff.
The wholesale price that maximizes this expression is:

\[
w^{LE} = \frac{a(2 - \gamma)(1 - \gamma)\gamma + c(4 + \gamma(2 - \gamma - 3\gamma^2))}{2(2 + 2\gamma - 2\gamma^2 - \gamma^3)}.
\]  

(16)

It can be confirmed that \(w^{LE} > c\). In other words, in contrast to the no entry case, when firm \(S\) enters into the final goods market, the wholesale price exceeds the input’s marginal cost; there is an inverse input pricing effect in place. Why is that? Initially, one might think that this result is driven by firm \(S\)’s incentive to "raise rival’s cost" in order to increase its own market share in the final goods market. However, this is not so here. As we saw above, the wholesale price is chosen in order to maximize the joint profits of firm \(S\) and firm 1 and not the profits of firm \(S\) alone. Essentially, the two firms manage to behave as a multi-product firm through the setting of the wholesale price. But why their joint profits are higher when the wholesale price exceeds the marginal cost? Firm \(S\)’s entry into the final goods market intensifies downstream competition. We refer to this as the competition intensity effect of licensing. The two firms alleviate the negative impact of the competition intensity effect on their joint profits through the setting of a higher wholesale price that leads them in turn into producing a lower joint output. In other words, the two firms set the wholesale price in a pro-collusive way.

It remains to determine the presence or absence of licensing incentives in the first stage of the game. We can obtain the equilibrium fixed fee \(T^{LE}\) after substituting (16) into (13). In turn, substituting the resulting \(T^{LE}\) and \(w^{LE}\) into (14) and (15), we can also obtain the gross from the licensing fee equilibrium profits of firm 1 and firm \(S\). For the same reasons as the ones explained in subsection 3.1, firm 1 extracts, through the licensing fee, the profits of firm \(S\): \(F^{LE} = \pi_S(w^{LE}) + T^{LE}\). Therefore, firm 1’s net equilibrium profits (included in Table 2 of Appendix A) in the licensing and entry case are: \(\pi_1^{LE} = \pi_1(w^{LE}) + \pi_S(w^{LE})\). Comparing \(\pi_1^{LE}\) with \(\pi_1^B\), we find that firm 1 has incentives to license its input technology even when licensing reinforces competition.

**Proposition 2** When firm \(S\) enters into the final goods market, firm 1 licenses its input technology.

The intuition is as follows. As mentioned above, licensing, when it is accompanied by entry, gives rise to the competition intensity effect and the inverse input pricing effect. These effects clearly have a negative impact on the profits that firm 1 obtains from its own sales
in the final goods market. Thus, if licensing was for free, firm 1 would never opt for it in the entry case. Besides though the competition intensity effect and the inverse input pricing effect, when licensing triggers entry, it also brings about a business stealing effect and a market expansion effect. The former refers to the fact that firm 2’s output falls as the number of firms in the market increases. In other words, when firm S enters into the final goods market, it “steals” part of the sales and market share of firm 1’s rival. The market expansion effect refers instead to the fact that the entry of firm S corresponds to an increase in the number of differentiated final products and, thus, to an increase in product variety that in turn expands the demand in the final goods market.\(^{15}\) Both of these effects augment the joint profits of firm S and firm 1 that the licensor enjoys, making licensing profitable.

An important observation is that the incentives for vertical licensing can be stronger than the incentives for horizontal licensing. To see this, let us briefly consider horizontal licensing. In particular, consider a setting in which firm 1 and firm 2 operate in an one-tier market and firm 1 licenses its production technology to an independent firm which enters into this one-tier market. In such a case, as Mankiw and Whinston (1986) demonstrate, entry gives rise again to the competition intensity effect, the business stealing effect and the market expansion effect. It does not, however, give rise to the inverse input pricing effect. Thus, it does not allow the licensor and the licensee to alleviate the negative impact of the competition intensity effect and behave in a pro-collusive way. It follows from this that input pricing - the vertical contract - constitutes an instrument that can allow the firms involved in vertical licensing to behave in a pro-collusive way, strengthening the licensing incentives.

\(^{15}\)The market expansion effect appears to be present in the licensing case between Boeing and Mitsubishi. The latter is about to introduce the Mitsubishi Regional Jet (MRJ) - a new and differentiated product in the market for short-haul passenger aircrafts known as “regional jets”. In particular, the MRJ will come in versions seating roughly 70-90 passengers, while Boeing and Airbus’ aircrafts seat over 100 passengers. Demand for regional jets has picked up after the MRJ made its maiden flight in November 2015. In fact, Mitsubishi already has 243 orders and 204 options for these jets. According to an article on the press (*Mitsubishi Aims for the Sky After Jet Takes Off*, The Wall Street Journal, November 11, 2015) “... estimates that carriers will order 4,360 regional jets through 2034 and it is predicted that Mitsubishi will capture 27% of that market.” For more on this see e.g., *Can Mitsubishi Heavy Industries’ MRJ Regional Jet Lift It To A 'Buy'?*, Forbes (October 20, 2014), *Japanese Planemaker in Talks for Major Deal*, Financial Tribune (July 13, 2016).
3.3 The Impact of Entry

Having explored the incentives for licensing both with and without entry, we are now able to characterize the impact that entry has on them.

Proposition 3 The entry of firm S into the final goods market reinforces firm 1’s licensing incentives.

Interestingly, the entry of the licensee into direct competition with the licensor has a positive instead of a negative impact on the licensing incentives. Why is that? The entry of the licensee increases the intensity of competition faced by the licensor in the final goods market. This - the competition intensity effect - is clearly a negative effect for the licensor. At the same time though, the entry allows firm 1 to steal away market share from its rival - firm 2 - as well as to enjoy a larger market size. These effects - the business stealing effect and the market expansion effect - augment firm 1’s profits and along with the entry’s inverse input pricing effect, which mitigates the negative impact of the competition intensity effect, render entry desirable.

Entry does not only alter the licensing incentives, as Proposition 4 states below, it also alters the impact that product differentiation has on them.

Proposition 4 An increase in product differentiation has a negative impact on the licensing incentives when firm S does not enter into the final goods market and a positive one when it enters.

In the no entry case, the higher is product differentiation, the higher is the equilibrium wholesale price and, thus, the smaller is the subsidy that firm S offers to firm 1. In other words, the weaker is the competition in the final goods market, the lower are the incentives of firm S to enhance the competitive position of its customer by decreasing the latter’s variable cost and, thus, the smaller is firm 1’s efficiency enhancement. An implication of this that when licensing is not accompanied by entry, it is more likely to be observed in markets with less differentiated products.

In the entry case, instead, a decrease in product differentiation has two negative implications for the licensor: it enhances the competition intensity effect and it weakens the market expansion effect. In fact, in the entry case, the relationship between the equilibrium wholesale price and product differentiation is U-shaped: $\partial w^{LE} / \partial \gamma < 0$ if and only if
\( \gamma \geq 0.43792 \). Intuitively, an increase in the wholesale price decreases the negative impact of the competition intensity effect. When, however, the competition in the final goods market is already too fierce, a (further) increase in the wholesale price is avoided because it can result in the market foreclosure of firm 1.\(^{16}\) As a consequence, when licensing is accompanied by entry, licensing incentives are stronger in markets with more differentiated products.\(^{17}\)

4 Welfare Implications of Vertical Licensing

We have already seen that vertical licensing is desirable from firm 1’s viewpoint. Next, we examine whether vertical licensing is also desirable from a welfare viewpoint.

**Proposition 5** Vertical licensing both with and without the entry of firm S into the final goods market always has a positive impact on consumers’ surplus and on total welfare. Its impact is larger with entry than without entry.

Vertical licensing is beneficial both for the consumers and for the economy as a whole. Intuitively, the positive impact of licensing on consumers’ surplus is due to the increase in firm 1’s efficiency in the no entry case and to the increase in product variety and competition intensity in the entry case. In turn, the positive impact of licensing on total welfare is driven by its positive impact on consumers’ surplus and on the licensor’s profits (Propositions 1 and 2) that outweigh its negative impact on firm 2’s profits. In fact, vertical licensing is even more desirable from a welfare viewpoint when it triggers entry into the final goods market than when it does not. Intuitively, with entry, even though firm 1 is less efficient, market competition is fiercer and the market is larger.

Recently, the EU and the US both revised their rules for the assessment of technology licensing agreements under respectively the EU competition law and the US antitrust law. The new regulations continue to reflect the view that licensing, by facilitating the diffusion of technology, is in most cases pro-competitive. Based on this view, a block exemption applies to all the licensing agreements between firms that have limited market shares.\(^{18}\)

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\(^{16}\)For a review of the market foreclosure issues that arise in the presence of vertical integration see e.g., Rey and Tirole (2007).

\(^{17}\)In the extreme case in which the final goods tend to be homogeneous, firm 1 is indifferent between entry and no entry. The market expansion effect is absent then and the competition intensity effect along with the business stealing effect cancel out with the input pricing effect.

\(^{18}\)According to the new EU regulation, Regulation N. 316/2014, in the case of licensing agreements between non-competitors, the block exemption applies when the individual market share of each party does not exceed
Our analysis puts forward an additional and novel channel through which licensing can be pro-competitive even in cases in which the firms involved in licensing have large market shares. In particular, it demonstrates that vertical licensing, by triggering entry into multiple stages of a market, it can generate more intense product market competition, and thus, it can further increase welfare.

5 Common Input Supplier

We have assumed throughout our analysis that firm 2 produces its input in-house both with and without licensing. We alter this assumption now and assume that when licensing takes place, firm 2 also sources the input from firm S.

A consequence of the above modification is that the bargaining game that takes place over the contract terms under licensing differs from the respective one in our main model. In particular, firm S now bargains with two, instead of one, firms. In modeling the resulting multilateral bargaining game, we invoke the Nash equilibrium of simultaneous generalized Nash bargaining games, in which the bargaining power of firm S and each firm i, with i = 1, 2, continues to be β and 1 − β, respectively. A key assumption that underlies this modeling approach is that firm S bargains with each firm i simultaneously and separately. In order to avoid the multiple equilibria that can arise in such a setting due to the multiplicity of the beliefs that the downstream firms can form when they receive out-of-equilibrium offers, we impose pairwise proofness on the equilibrium contracts (e.g., Horn and Wolinsky, 1988, O’Brien and Shaffer, 1992, Milliou and Petrakis, 2007, Alipranti et al., 2014). That is, we require that a contract between firm S and firm i is immune to a bilateral deviation of firm S with firm j. Moreover, in order to ensure that all the firms face a non-negative marginal cost in all the cases under consideration, we assume that a < 3c.

In the no entry case, since firm S bargains with two firms, it now has an outside option: the profits that it would make when one of the downstream firms acts as monopolist facing

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30%. Respectively, according to the US Antitrust Guidelines for the Licensing of Intellectual Property (updated on August 12, 2016 by the US Department of Justice and the Federal Trade Commission), the US Agencies will not challenge an agreement if the licensor and the licensee collectively account for no more than 20% of each relevant market significantly affected by the licensing restraint.

19 In Section 6, we confirm and reinforce this point by showing that licensing can be welfare-enhancing even when the licensor is initially a monopolist and/or when it has a cost advantage relative to the other incumbent.

the equilibrium contract terms. The resulting equilibrium wholesale prices, with \( i = 1, 2 \), are:\textsuperscript{21}

\[
w_i^{CN} = c - \frac{\gamma^2(a - c)}{2(2 - \gamma^2)} < w^{LN}
\]

We observe that \( w_i^{CN} < c \). That is, similarly to the main model, in the no entry case, firm \( S \) subsidizes the final good producers via the wholesale prices. In fact, it subsidizes the downstream production even more now. The reason for the subsidization differs from the one in the main model. The subsidization here is due to the "commitment problem" faced by the upstream monopolist, firm \( S \). That is, it is due to the fact that when firm \( S \) trades with firm \( i \), it cannot commit that it will not offer better trading terms to firm \( j \).\textsuperscript{22} An important implication of this is that the gross from the licensing fee profits of firm \( S \) are negative when \( \beta < \overline{\beta}(\gamma) \equiv \frac{\gamma^3}{4 - 2\gamma - 2\gamma^2 + \gamma}, \) with \( \frac{\partial \overline{\beta}(\gamma)}{\partial \gamma} > 0, \overline{\beta}(1) = 1 \) and \( \overline{\beta}(0) = 0 \), i.e., when either products are too close substitutes or when products are sufficiently close substitutes and firm \( S \)'s bargaining power is sufficiently low. This is so because, when products are close substitutes and, thus, downstream competition is fierce, firm \( S \)'s incentives to behave opportunistically are strong; hence, its"commitment problem" is severe. Given this, when \( \beta < \overline{\beta}(\gamma) \), firm 1 has to offer a negative licensing fee to firm \( S \); firm 1 has to pay for the licensing agreement.

Respectively, in the entry case, firm \( S \)'s outside option also differs from the one in our main model. Its outside option now includes not only its profits from its own sales in the final goods market, but also its profits from its input sales, at the equilibrium wholesale price, to firm \( i \), with \( i = 1, 2 \). This results in the following equilibrium wholesale prices:

\[
w_i^{CE} = \frac{a(2 - \gamma)\gamma + c(2 + \gamma)^2}{4 + 6\gamma} > w^{LE}
\]

As in the main model, we observe that \( w_i^{CE} > c \). But, firm 1, firm \( S \) and firm 2 behave now as a multi-product monopolist, in the setting of the wholesale price, while in our main model only firm 1 and firm \( S \) could behave as a multi-product firm. A consequence of this is that the equilibrium wholesale price is now higher, \( w_i^{CE} > w^{LE} \).

**Proposition 6** When both firm 1 and firm 2 source the input from firm \( S \) under licensing,

\( (i) \) in the no entry case, firm 1 licenses its input technology if and only if \( \gamma < 0.891638 \)

\textsuperscript{21} The detailed equilibrium analysis is included in Appendix A.

\textsuperscript{22} For more on this, see e.g., Milliou and Petrakis (2007), Aliprantis et al. (2014).
and \( \beta > \beta_1(\gamma) \), with \( \frac{\partial \beta_1}{\partial \gamma} > 0 \), \( \beta_1(0.891638) = 1 \) and \( \beta_1(0) = 0 \), and \( \beta_1(\gamma) > \beta(\gamma) \) for all values of \( \gamma \).

(ii) in the entry case, firm 1 always licenses its input technology.

(iii) entry always reinforces firm 1’s licensing incentives.

(iv) licensing always has a positive impact on consumers’ surplus and on total welfare and its positive impact is higher with entry than without entry if and only if products are sufficiently differentiated.

As Proposition 6 informs us, firm 1 opts for licensing even when it shares its input supplier with its rival. This holds unless licensing is not accompanied by entry and the the final products are too close substitutes and/or the licensee does not have a lot of bargaining power. This is so because then firm 1 has to pay firm S for the licensing agreement.

Proposition 6 also informs us that the impact of the entry on the licensing incentives and the welfare implications of licensing are again positive. However, now, licensing with entry is more beneficial, relatively to licensing without entry, for the consumers and the whole economy if and only if product differentiation is sufficiently high. This is so, because in the entry case, when product differentiation is low, the market expansion effect is weak and does not outweigh the lower efficiency of firm 1 and firm 2, which is due to the higher wholesale prices.

**Corollary 2** (i) In the no entry case, firm 1’s licensing incentives are stronger when both firm 1 and firm 2 source the input from firm S under licensing than when only firm 1 sources the input from firm S if and only if \( \gamma < 0.881239 \) and \( \beta > \beta_2(\gamma) \), with \( \frac{\partial \beta_2}{\partial \gamma} > 0 \), \( \beta_2(0.881239) = 1 \) and \( \beta_2(0) = 0 \).

(ii) In the entry case, firm 1’s licensing incentives are stronger when both firm 1 and firm 2 source the input from firm S under licensing than when only firm 1 sources the input from firm S.

The use of the licensee as input supplier by the licensor’s rival does not discourage licensing. In fact, it tends to encourage licensing unless the final products are too close substitutes and/or firm S’s bargaining power is not too high, and thus, firm 1 has to pay firm S for the licensing agreement. It is interesting to stress that in the entry case, the stronger licensing incentives when the licensee acts as a common supplier, arise from the fact that the inverse input pricing effect is stronger: all the firms in the market, and not
just firm 1 and firm $S$ manage to behave as a multi-product monopolist. A consequence of this is that in the common supplier case, the competition intensity effect is weaker and the business stealing effect is stronger. Even though the licensing incentives can be stronger when both firms 1 and 2 source the input from firm $S$ than when only firm 1 sources the input from firm $S$, the positive welfare implications of licensing are smaller in the former case. This is because the inverse input pricing effect does not only lower efficiency, but it also weakens the market expansion effect.

6 Extensions - Discussion

In this section, we discuss briefly a number of further extensions of our main model to extract some additional insights.

(i) Wholesale Price Contract

We consider here what would happen if input trading occurred through a wholesale price contract that includes only $w$. For simplification reasons, we assume that firm $S$ makes a take-it-or-leave-it offer to firm 1 regarding $w$.  

Under licensing, due to the lack of the fixed fee in the wholesale price contract, firm $S$ is not in the position to extract part of firm 1’s profits and double marginalization is present both with and without entry, $\tilde{w}^{LN} > c$ and $\tilde{w}^{LE} > c$. Therefore, under trading through a wholesale price, there is an inverse input pricing effect both in the no entry case and in the entry case.

**Proposition 7** When vertical trading takes place through a wholesale price contract, firm 1 licenses its input technology if and only if firm $S$ enters into the final goods market.

When firms trade through a wholesale price contract, licensing does not arise in equilibrium when it is not accompanied by entry. Intuitively, the licensor is less efficient with licensing due to the inverse input pricing effect. Its lower efficiency translates into lower profits with licensing in the no entry case. The licensing incentives are restored when entry occurs mainly because of the market expansion effect which is in place under a wholesale price contract too.

In fact, the market expansion effect results also in a greater consumer and total welfare with licensing and entry than without licensing.

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23 As we discuss later on, the inclusion of bargaining would not affect our main results.

24 If we included bargaining over $w$, the equilibrium wholesale prices would be lower than $\tilde{w}^{LN}$ and $\tilde{w}^{LE}$. In fact, they would be decreasing with the bargaining power of firm 1. Clearly, this means, that the licensing
On the basis of the above, we can draw two conclusions. First, the contract type used in input trading can be crucial for the licensing incentives: when the licensee does not enter into the final goods market, licensing arises in equilibrium when firms trade through a two-part tariff contract, but not when they trade through a wholesale price contract. And second, when input trading takes place through a wholesale price contract, the licensee’s entry not only does not discourage vertical licensing, but, in fact, it constitutes its driving force.

(ii) Ex-ante Monopoly

We have assumed throughout our analysis that there are initially two vertically integrated incumbents in the market. One might wonder whether this assumption is innocuous. In order to examine this, we consider here the alternative case in which only firm 1 is initially in the market - firm 1 is an ex-ante monopolist. Our main conclusion is included in the following Proposition.

**Proposition 8** When firm 1 is initially a monopolist in the market, it has incentives to license its input technology if firm S enters into the final goods market. Otherwise, it is indifferent between licensing and no licensing.

In the no entry case now, the licensee, in contrast to our main analysis, does not subsidize the production of firm 1. This occurs because firm 1 does not face any competition, and thus, firm S has no reason to increase its aggressiveness. An implication of this is that firm 1 and firm S operate as a vertically integrated firm; hence, they operate as in the benchmark case of no licensing. This means that firm 1 is now indifferent between licensing without entry and no licensing.

In the entry case, there are two important differences compared to our main analysis. The first is that the competition intensity effect is stronger because firm S’s entry into the final goods market transforms the latter from a monopoly to a duopoly. The second is that the business stealing effect is absent since firm 2 does not exist in the final goods market. In light of these, one would expect that firm 1 does not opt for licensing now. This is not so though; firm 1 opts for licensing even when licensing causes the loss of its monopoly status. The presence of licensing incentives, which is more striking here than in our main analysis,
is driven exclusively by the market expansion effect and, thus, by the positive impact that entry has on the size of the final goods market.

(iii) Cost Asymmetry
One might wonder what would happen if the two incumbents had asymmetric costs, and in particular, if the licensor had a cost advantage relative to the other incumbent. In order to examine this, we assume now that the marginal costs that firm 1 and firm 2 face initially are $c$ and $c_2$, respectively, with $c < c_2$. Under this assumption, we find that the results of our main model are qualitatively robust. Moreover, we find that the licensing incentives, both under entry and no entry, are stronger when the licensor enjoys a cost-advantage than when it does not and the positive impact of licensing on consumer and total welfare is larger under entry. This occurs because when the licensor enjoys a cost advantage, then both the input pricing effect and the market expansion effect are stronger, while the competition intensity effect is weaker.

(iv) Bargaining over the Licensing Fee
We consider here what would happen if the licensor bargained with the licensee not only over the input’s trading terms but also over the licensing fee. To do so, we assume that firm 1 and firm $S$ bargain in a Nash bargaining fashion in the first stage of the game over $F$ and that they bargaining powers are the same as the ones in their negotiations over the input’s terms of trade.

When the bargaining power of firm $S$ is positive, firm 1 is not in the position to extract all of firm $S$’s profits through $F$. Still, even in this case, firm 1 always has incentives to license its input technology both with and without entry. This holds because during the negotiations over $F$, firm $S$ has to compensate firm 1 for the profits that the latter would make without licensing - otherwise, firm 1 would not offer a licensing agreement to firm $S$ and the latter would make zero profits. Because of this, in the extreme case in which firm 1 has no bargaining power ($\beta = 1$), it is indifferent among licensing and no licensing. In all other cases, it prefers licensing to no licensing and the higher is its bargaining power the more it prefers licensing.

The allocation of the licensing fee does not affect consumer and total welfare. Therefore, the conclusions of our main analysis regarding the welfare implications of licensing remain unchanged in the presence of bargaining over $F$.

(v) Licensing through Royalty
One might wonder what would happen if licensing took place though a per-unit of output royalty, $r \geq 0$, instead of through a fixed licensing fee.

Clearly, when a royalty is used, firm 1 is not able to extract all the profits of firm $S$. Moreover, firm $S$’s marginal cost is then $r + c$ instead of $c$. The increase in the input supplier’s marginal cost can translate into worse input sourcing terms for firm 1 and, thus, into lower firm 1’s profits from its own sales in the final goods market. Not surprisingly thus, in the no entry case, firm 1 always optimally sets $r^N = 0$. Given this, does it have incentives to license its technology? The answer to this question is already provided in our main analysis. More specifically, recall that when $F = 0$, firm 1 opts for licensing if and only if its bargaining power is sufficiently high and the licensee does not enter into the final goods market.

In the entry case, when the royalty is imposed only on firm 1’s output, then again $r^E = 0$. But when it is imposed either only on firm $S$’s output or on both firm $S$’s and firm 1’s output, $r^E > 0$. When the latter occurs, then, in contrast to what happens when licensing takes place through a fixed fee, the entry of the licensee into the final goods market discourages licensing incentives - licensing does not arise in equilibrium. This is so mainly for two reasons. First, because the market expansion effect is weaker when $r^E > 0$. And second, because since now firm 1 does not manage to extract firm $S$’s profits, it is not in the position to take full advantage of the market expansion effect.

It follows from the above that the form of the licensing contract, whether it is a fixed licensing fee or a variable royalty, can affect the licensing incentives. It also follows that when licensing is through a royalty, entry can discourage instead of encourage licensing. This is in sharp contrast with our main conclusion. Still, we should mention that firm 1 is better off under licensing through a fixed licensing fee than under licensing through a royalty. Therefore, firm 1 would choose to license its technology through a fixed licensing fee and not through a royalty. Actually, firm 1’s choice would be aligned with the interest of consumers and the economy as a whole since the positive impact of licensing on consumer and total welfare is even larger when licensing takes place through a fixed licensing fee. Stated differently, not only from the licensor’s viewpoint but also from a welfare viewpoint, vertical licensing is preferable when it occurs through a fixed licensing fee.

(vi) Downstream Price Competition

Next, we discuss what happens when firms compete in prices in the final products market.
instead of in quantities.

As it is well known from the literature, prices, in contrast to quantities, are strategic complements. Because of this, under downstream price competition, in the no entry case, firm $S$ does not subsidize firm 1’s final production via the wholesale price; it does not wish its downstream partner to behave aggressively in the final market competition - it charges a wholesale price that exceeds its marginal cost. Still, similarly to our main model, firm 1 always licenses its input technology. In fact, it licenses its technology even for free if the bargaining power of firm $S$ is low enough. Why is that? As the literature on vertical separation (e.g., Bonanno and Vickers, 1988, Lin, 1988, Gal-Or, 1990, Cyrenne, 1994) has demonstrated, vertical separation (vertical licensing in our case) dampens downstream competition, leading to higher downstream profits that the upstream firm (the downstream firm in our case) extracts through the fixed fee.

In the entry case now, firm 1 has incentives to license its input technology, but not always. It has incentives to do so if and only if the final products are not too close substitutes ($\gamma < 0.9793$). Its licensing incentives are driven now by the market expansion effect alone; the business stealing effect is absent under price competition. Competition in the market is fiercer under price competition than under quantity competition. Hence, the competition intensity effect of licensing is stronger in the former case and dominates its market expansion effect when products tend to be homogeneous since then downstream competition is already quite fierce. Clearly, this implies that in markets in which firms’ products are quite similar, vertical licensing, when it is accompanied by entry, is more likely to be observed when firms compete in quantities than when they compete in prices.

It follows that the impact of entry on the licensing incentives is positive under downstream price competition too, but only as long as the final products are not too close substitutes. Licensing continues to be welfare-enhancing under downstream price competition but only when it triggers entry. In the no entry case, licensing is now welfare detrimental due to the decrease that it causes (through the input price) in the licensor’s efficiency.

7 Concluding Remarks

We have examined the incentives of a vertically integrated incumbent to license its input technology to an external firm. We have done so in a setting in which after the signing of the licensing agreement, the licensor sources the input from the licensee and the licensee
can also enter into the final goods market and compete with the licensor.

We have shown that licensing emerges in equilibrium not only when the licensee does not enter into direct competition with the licensor, but also when it enters. In fact, we have shown that in the latter case, although market competition is more intense, the licensing incentives are stronger. Intuitively, in the absence of entry, vertical licensing is motivated by the low input price at which the licensor sources the input from the licensee. The low input price translates into higher efficiency and, in turn, into larger market share and profits for the licensor.

When, instead, the licensee enters into the final goods market, first, the licensor is less efficient - it pays an input price that exceeds the input’s marginal cost - and second, it competes with more firms than in the no licensing case. Still, the licensor opts for licensing because the entry of the licensee results in the expansion of the final goods market and in business stealing from the rival incumbent firm. In fact, the entry of the licensee encourages instead of discourages the licensing incentives.

Our welfare analysis has revealed, first, that vertical licensing is always beneficial both for the consumers and for the economy as a whole, and second, that it is even more beneficial in the entry case. Importantly, it has indicated that vertical licensing can enhance welfare even more than horizontal licensing, by triggering entry into more than one stages of a market, and thus, by generating more intense product market competition in multiple production stages.

Extending our main analysis in various directions, we have shown that in many instances the emergence of vertical licensing in equilibrium would be impossible without the entry of the licensee into the final goods market. This holds, for instance, when the licensor and the licensee trade via a wholesale price contract or when the licensor is initially the only incumbent in the market. Moreover, the entry of the licensee reinforces the licensing incentives when the rival firm also sources its input from the licensee as well as when downstream competition is in prices. In contrast, the entry of the licensee seems to discourage licensing when licensing takes place through royalties.

Summing up, we have provided an explanation for the commonly observed practice of vertical licensing in markets where licensing can transform the licensee not only into an input supplier but also into a direct competitor of the licensor in the final products market. Our explanation lies on strategic considerations and not on exogenously assumed efficiencies of the licensee. Clearly, if we had assumed that the licensee is either more efficient in input
production than the licensor or that the input production is characterized by economies of scale, then vertical licensing and the positive impact of entry on the licensing incentives would have been much less surprising than in our setting.

Still, we should mention that our analysis is just a first step in the direction of understanding licensing in vertically related markets. In future work, we plan to extend our analysis by endogenizing firms’ investments in the improvement of the input production technology and examining how they could affect and be affected by licensing.

8 Appendix A

Table 1: Equilibrium Values in the Benchmark Case and in the Licensing with No Entry Case

<table>
<thead>
<tr>
<th>Case</th>
<th>( q_1^B = q_2^B = \frac{a-c}{2+\gamma} )</th>
<th>( \pi_1^B = \pi_2^B = \frac{(a-c)^2}{(2+\gamma)^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1^{LN} = \frac{(a-c)(2-\gamma)}{2(2-\gamma^2)} )</td>
<td>( q_2^{LN} = \frac{(a-c)(4-\gamma(2+\gamma))}{4(2-\gamma^2)} )</td>
<td></td>
</tr>
<tr>
<td>( \pi_1^{LN} - T^{LN} = \frac{(1-\beta)(a-c)^2(2-\gamma)^2}{8(2-\gamma^2)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_2^{LN} = \frac{(a-c)(4-\gamma(2+\gamma))^2}{16(2-\gamma^2)^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{LN} = \frac{\beta(a-c)^2(2-\gamma)^2}{8(2-\gamma^2)} )</td>
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<td></td>
</tr>
<tr>
<td>( \pi_1^{LN} = \frac{(a-c)^2(2-\gamma)^2}{8(2-\gamma^2)} )</td>
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</table>

Table 2: Equilibrium Values in the Licensing with Entry Case

<table>
<thead>
<tr>
<th>Case</th>
<th>( q_1^{LE} = \frac{(a-c)(4-2\gamma-\gamma^2)}{4(2+2\gamma-2\gamma^2-\gamma)} )</th>
<th>( q_2^{LE} = q_3^{LE} = \frac{(a-c)(4-3\gamma^2)}{4(2+2\gamma-2\gamma^2-\gamma)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_1^{LE} - T^{LE} = \frac{(1-\beta)(a-c)^2(4-2\gamma-\gamma^2)^2}{8(2+\gamma)^2(2+2\gamma-2\gamma^2-\gamma)} )</td>
<td></td>
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</tr>
<tr>
<td>( \pi_2^{LE} = \frac{(a-c)^2(4-3\gamma^2)^2}{16(2+2\gamma-2\gamma^2-\gamma)^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F^{LE} = \frac{(a-c)^2(16(1+\gamma)-8\gamma^2(2+\gamma)+\beta(4-2\gamma-\gamma^2)^2)}{8(2+\gamma)^2(2+2\gamma-2\gamma^2-\gamma)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_1^{LE} = \frac{(a-c)^2(8-8\gamma+\gamma^2)}{8(2+2\gamma-2\gamma^2-\gamma)} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Equilibrium Values in the Licensing with Entry and Wholesale Price Contract Case
$$\hat{q}_{1}^{LE} = \frac{(a-c)(4-2\gamma-\gamma^2)}{2(8+8\gamma-3\gamma^2-2\gamma^3)}$$

$$\hat{q}_{2}^{LE} = \hat{q}_{S}^{LE} = \frac{(a-c)(2-\gamma)(4+3\gamma^2)}{2(8+8\gamma-3\gamma^2-2\gamma^3)}$$

$$\pi_{1}(\hat{q}_{1}^{LE}) = \frac{(a-c)^2(4-2\gamma-\gamma^2)^2}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2}$$

$$\hat{p}_{2}^{LE} = \frac{(a-c)^2(2-\gamma)^2(4+3\gamma^2)^2}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2}$$

$$\hat{p}_{1}^{LE} = \frac{(a-c)^2(6-\gamma)(2-\gamma)}{4(8+8\gamma-3\gamma^2-2\gamma^3)^2}$$

$$\hat{p}_{1}^{LE} = \frac{(a-c)^2(56+8\gamma+48\gamma^2-6\gamma^3-7\gamma^4+\gamma^5)}{2(8+8\gamma-3\gamma^2-2\gamma^3)^2}$$

### 8.1 Common Input Supplier

(a) No Entry

After each firm $i$ chooses its quantity in order to maximize its profits: $\pi_i(q_i, q_j, w_i) = (a - q_i - \gamma q_j)q_i - w_i q_i$ with $i, j = 1, 2$ and $i \neq j$, we obtain the equilibrium quantities for given levels of input prices:

$$q_i(w_i, w_j) = \frac{a(2-\gamma) - 2w_i + \gamma w_j}{4 - \gamma^2}. \quad (17)$$

Letting $w_j^{CN}$ denote the equilibrium outcome of the negotiations of firm $S$ and firm $j$, $w_i$ is chosen to maximize the generalized Nash bargaining product:

$$\max_{w_i, T_i} [\pi_S(w_i, w_j^{CN}) + T_i + T_j - d_{S_i}(w_j^{CN}, T_j^{CN})]^{\beta} [\pi_i(w_i, w_j^{CN}) - T_i]^{1-\beta}, \quad (18)$$

where $\pi_S(w_i, w_j^{CN}) = (w_i - c)q_i(w_i, w_j^{CN}) + (w_j^{CN} - c)q_j(w_i, w_j^{CN})$ and $\pi_i(w_i, w_j^{CN}) = \pi_i(q_i(w_i, w_j^{CN}), q_j(w_i, w_j^{CN}), w_i, w_j^{CN})$. Note that firm $S$’s profits arise from sales to two instead of one final good producer. We notice here that firm $S$ has an outside option, which means that its disagreement payoff is no longer null. In particular, if an agreement between firm $S$ and firm $i$ is not reached, then firm $S$’s disagreement payoff is given by $d_{S_i}(w_j^{CN}, T_j^{CN}) = (w_j^{CN} - c)q_j^{mon}(w_j^{CN}) + T_j^{CN}$, where $q_j^{mon}(w_j^{CN}) = (\alpha - w_j^{CN})/2$ is the quantity expected to be produced by the monopolist final good producer which faces an input price $w_j^{CN}$. In other words, in case of disagreement with one of the final good producers, firm $S$ is expected to receive from the remaining firm in the final goods market the equilibrium fixed fee $T_j^{CN}$ plus the revenues from input sales at the equilibrium wholesale price $w_j^{CN}$.

From the first order conditions of (18) we obtain the equilibrium wholesale price and
the respective net equilibrium profits:

\[ w_i^{CN} = w_j^{CN} = w_j^{CN} = c - \frac{\gamma^2(a - c)}{2(2 - \gamma^2)}; \quad (19) \]

\[ \pi_i^{CN} = \frac{(a - c)^2(2 - \gamma)((1 + \beta)(4 - 2\gamma - 2\gamma^2) - \gamma^3(1 - \beta))}{8(2 - \gamma^2)^2}; \quad (20) \]

\[ \pi_j^{CN} = \frac{(1 - \beta)(a - c)^2(2 - \gamma)^2}{8(2 - \gamma^2)}. \quad (21) \]

(b) Entry

After each firm \( i \) chooses its quantity in order to maximize its profits: \( \pi_i(q_i, q_j, q_S, w_i, w_j) = (a - q_i - \gamma q_j - \gamma q_S)q_i - w_i q_i \), with \( i, j = 1, 2 \) and \( i \neq j \), while firm \( S \) chooses its quantity in order to maximize its profits given by: \( \pi_S(q_i, q_j, q_S, w_i, w_j) = (a - q_S - \gamma q_i - \gamma q_j)q_S - c(q_i + q_j + q_S) + w_i q_i + w_j q_j \), we obtain the equilibrium quantities for given levels of input prices:

\[ q_i(w_i, w_j) = \frac{a(2 - \gamma) - (2 + \gamma)w_i + \gamma(c + w_j)}{2(2 - \gamma)(1 + \gamma)}; \quad (22) \]

\[ q_S(w_i, w_j) = \frac{a(2 - \gamma) - (2 + \gamma)w_i + \gamma(c + w_j)}{2(2 - \gamma)(1 + \gamma)}; \quad (23) \]

Letting \( w_j^{CE} \) denote the equilibrium outcome of the negotiations of firm \( S \) and firm \( j \), \( w_i \) is chosen to maximize the generalized Nash bargaining product:

\[ \max_{w_i, T_j} \left[ \pi_S(w_i, w_j^{CE}) + T_i + T_j - d_{S_2}(w_j^{CE}, T_j^{CE}) \right]^\beta \left[ \pi_i(w_i, w_j^{CE}) - T_i \right]^{1-\beta}, \quad (24) \]

where \( \pi_S(w_i, w_j^{CE}) \) and \( \pi_i(w_i, w_j^{CE}) \) are found after substituting \( q_S(w_i, w_j) \) and \( q_i(w_i, w_j) \) into \( \pi_S(q_i, q_j, q_S, w_i, w_j) \) and \( \pi_i(q_i, q_j, q_S, w_i, w_j) \). In the entry case firm \( S \) has an outside option that differs from the one of the no entry case. In particular, if an agreement between firm \( S \) and firm \( i \) is not reached, then firm \( S \)'s disagreement payoff is given by \( d_{S_2}(w_j^{CE}, T_j^{CE}) = (a - q_S^D - \gamma q_j^D(w_j^{CE}) - c) q_S^D + (w_j^{CE} - c) q_j^D(w_j^{CE}) + T_j^{CE} \), where \( q_i^D = q_i^D = \frac{a - c}{2\gamma} \) and \( q_j^D(w_j^{CE}) = \frac{a - w_j^{CE}}{2\gamma} \) are the quantities expected to be produced, respectively, by firm \( S \) and firm \( j \) which faces an input price \( w_j^{CE} \). In other words, in case of disagreement with firm \( i \), firm \( S \) can still have profits from its own sales in the final goods market, as well as it receives the equilibrium fixed fee \( T_j^{CE} \) plus the revenues from input sales at the equilibrium wholesale price \( w_j^{CE} \) from the rival remaining firm in the final goods market.
From the first order conditions of (24) we obtain the equilibrium wholesale price and the respective net equilibrium profits:

\[ w^{CE} = w_i^{CE} = w_j^{CE} = \frac{a(2 - \gamma)\gamma + c(2 + \gamma)^2}{4 + 6\gamma}; \]  

(25)

\[ \pi_1^{CE} = \frac{(a - c)^2A}{8(2 - \gamma^2)^2} \text{ and } \pi_2^{CE} = \frac{(1 - \beta)(a - c)^2(16(1 - \gamma - \gamma^2) - 8\gamma^3 + 7\gamma^4 + \gamma^5)}{4(2 + \gamma)^2(2 + 3\gamma)^2} \]  

(26)

where \( A = 32 + 16\beta + 80\gamma + 16\beta\gamma + 64\gamma^2 - 16\beta\gamma^2 + 8\gamma^3 - 8\beta\gamma^3 - 10\gamma^4 + 7\beta\gamma^4 - \gamma^5 + \beta\gamma^5 > 0. \)

9 Appendix B

Proof of Proposition 1: Calculating the difference in the profits of firm 1 in the case of licensing and no entry and in the benchmark case, we find:

\[ \pi_1^{LN} - \pi_1^B = \frac{(a - c)^2\gamma^4}{8(2 + \gamma)^2(2 - \gamma^2)} > 0. \]

Thus, firm 1 always has incentives to license its input technology under no entry.

Proof of Corollary 1: Calculating the difference in the profits of firm 1 in the case of licensing and no entry without receiving the licensing fee, and in the benchmark case, we find that \( \pi_1(q_1^{LN}) - T^{LN} - \pi_1^B = \frac{(a - c)^2(16\beta - 8\beta\gamma^2 - \gamma^4 + \beta\gamma^4)}{8(2 + \gamma)^2(-2 + \gamma^2)} > 0 \) if and only if \( 0 < \beta < \frac{\gamma^4}{16 - 8\gamma^2 + \gamma^4}. \)

Thus, under no entry firm 1 has incentives to license its input technology for free when its bargaining power is sufficiently high.

Proof of Proposition 2: Calculating the difference in the profits of firm 1 in the case of licensing and entry and in the benchmark case, we find:

\[ \pi_1^{LE} - \pi_1^B = \frac{(a - c)^2(4 - 2\gamma - \gamma^2)^2}{8(2 + \gamma)^2(2 + 2\gamma - 2\gamma^2 - \gamma^3)} > 0. \]

Therefore, firm 1 always has incentives to license its input technology under entry.

Proof of Proposition 3: Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:

\[ \pi_1^{LE} - \pi_1^{LN} = \frac{(a - c)^2(1 - \gamma)(8 - 8\gamma + 2\gamma^3 - \gamma^4)}{8(2 - \gamma^2)(2 + 2\gamma - 2\gamma^2 - \gamma^3)} > 0. \]

It follows that firm 1 has stronger incentives to license its input technology with entry than
without entry.

**Proof of Proposition 4:** We differentiate \((\pi^L_N - \pi^B_1)\) and \((\pi^L_E - \pi^B_1)\) in terms of \(\gamma\), respectively:

\[
\frac{\partial (\pi^L_N - \pi^B_1)}{\partial \gamma} = \frac{(a-c)^2 \gamma^3 (4 + \gamma - \gamma^2)}{(2 + \gamma)^3 (-2 + \gamma^2)^2} > 0
\]

\[
\frac{\partial (\pi^L_E - \pi^B_1)}{\partial \gamma} = \frac{(a-c)^2 (-4 + 2\gamma + \gamma^2)(48 + 16\gamma - 44\gamma^2 - 14\gamma^3 + 4\gamma^4 + \gamma^5)}{8(2 + \gamma)^3 (2 + 2\gamma - 2\gamma^2 - \gamma^3)^2} < 0
\]

It follows that an increase in product differentiation has a negative impact on the licensing incentives under no entry and a positive one under entry.

**Proof of Proposition 5:** In the benchmark case, the consumers’ surplus is:

\[
CS^B = aq_1^B + aq_2^B - \frac{1}{2} [(q_1^B)^2 + (q_2^B)^2 + 2\gamma q_1^B q_2^B] - p_1 q_1^B - p_2 q_2^B = \frac{(a-c)^2(1 + \gamma)}{(2 + \gamma)^2}.
\]

In the case of licensing with no entry, the consumers’ surplus is:

\[
CS^{LN} = aq_1^{LN} + aq_2^{LN} - \frac{1}{2} [(q_1^{LN})^2 + (q_2^{LN})^2 + 2\gamma q_1^{LN} q_2^{LN}] - p_1 q_1^{LN} - p_2 q_2^{LN} = \frac{(a-c)^2 32 (1 - \gamma^2) + \gamma^3 (4 + 5\gamma)}{32(2 - \gamma^2)^2}.
\]

In the case of licensing with entry, the consumers’ surplus is:

\[
CS^{LE} = aq_1^{LE} + aq_2^{LE} + aq_S^{LE} - \frac{1}{2} [(q_1^{LE})^2 + (q_2^{LE})^2 + (q_S^{LE})^2 + \\
2\gamma q_1^{LE} q_2^{LE} + 2\gamma q_1^{LE} q_S^{LE} + 2\gamma q_2^{LE} q_S^{LE}] - p_1 q_1^{LE} - p_2 q_2^{LE} - p_S q_S^{LE} = \\
\frac{(a-c)^2 48 + 80\gamma - 84\gamma^2 - 108\gamma^3 + 43\gamma^4 + 30\gamma^5}{32(2 + 2\gamma - 2\gamma^2 - \gamma^3)}.
\]

Calculating the following differences: \(CS^{LN} - CS^B = \frac{(a-c)^2 \gamma^2 (32 + 16\gamma - 28\gamma^2 - 8\gamma^3 + 5\gamma^4)}{32(2 + \gamma)^2 (2 - \gamma^2)^2}\) and \(CS^{LE} - CS^B = \frac{(a-c)^2 (4 - 2\gamma - \gamma^2)(16 + 40\gamma - 34\gamma^3 - 7\gamma^4 + 2\gamma^5)}{32(2 + \gamma)^2 (2 - \gamma^2)^2 (2 + 2\gamma - 2\gamma^2 - \gamma^3)^2}\), we find that they are always positive.

Total welfare is defined as the sum of consumers’ and producers’ surplus, namely:

\(W^k = CS^k + \pi^k_1 + \pi^k_2 + \pi^k_S\), where \(k = B, LN\) and \(LE\). Calculating the following differences:

\(W^{LN} - W^B = \frac{(a-c)^2 (32 - 16\gamma - 20\gamma^2 + 8\gamma^3 + 3\gamma^4)}{32(2 + \gamma)^2 (2 - \gamma^2)^2}\) and \(W^{LE} - W^B = \frac{(a-c)^2 (4 - 2\gamma - \gamma^2)(12 + 12\gamma - 11\gamma^2 - 6\gamma^3)}{32(2 + \gamma)^2 (2 - \gamma^2)^2 (2 + 2\gamma - 2\gamma^2 - \gamma^3)^2}\),

we find that they are always positive. Thus, both the consumers’ surplus and total welfare are greater under licensing than under no licensing.

Moreover, calculating the following differences, \(CS^{LE} - CS^{LN} = \frac{(a-c)^2 (1 - \gamma) B}{32(2 + \gamma)^2 (2 + 2\gamma - 2\gamma^2 - \gamma^3)^2}\), where \(B = 64 + 128\gamma - 144\gamma^2 - 272\gamma^3 + 104\gamma^4 + 200\gamma^5 - 20\gamma^6 - 60\gamma^7 - \gamma^8 + 5\gamma^9 > 0\), and
\[ W^{LE} - W^{LN} = \frac{(a-c)^2(1-\gamma)\Gamma}{32(2-\gamma)^2(2+2\gamma-2\gamma^2-\gamma^3)^2}, \]
where \( \Gamma = 192 - 496\gamma^2 + 80\gamma^3 + 376\gamma^4 - 40\gamma^5 - 124\gamma^6 - 4\gamma^7 + 17\gamma^8 + 3\gamma^9 > 0, \)
we find that they are always positive and, therefore, both consumers’ surplus and total welfare are greater with entry than without entry.

**Proof of Proposition 6:**

(i): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing without entry and in the benchmark case, we find:

\[
\pi_1^{CN} - \pi_1^B = \frac{(a-c)^2(32\beta - 32\beta \gamma^2 - 16\gamma^3 - 6\gamma^4 + 10\beta \gamma^4 - 4\gamma^5 - \gamma^6 + \beta \gamma^6)}{8(2 + \gamma)^2(-2 + \gamma^2)^2} > 0,
\]

if and only if \( \gamma < 0.891638 \) and \( \beta > \beta_1(\gamma) = \frac{-16\gamma^3 - 6\gamma^4 + 4\gamma^5 + \gamma^6}{-32 + 32\gamma^2 - 10\gamma^3 + 10\gamma^4}, \) with \( \frac{\theta_1}{\theta_7} > 0, \) \( \beta_1(0.891638) = 1 \) and \( \beta_1(0) = 0, \) and \( \beta_1(\gamma) > \overline{\beta}(\gamma) \) for all values of \( \gamma. \) Namely, firm 1 does not have always incentives for licensing without entry.

(ii): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing with entry and in the benchmark case, we find:

\[
\pi_1^{CE} - \pi_1^B = \frac{(a-c)^2\Delta}{4(2 + \gamma)^2(2 + 3\gamma)^2} > 0,
\]

where \( \Delta = 16 + 16\beta + 32\gamma + 16\beta \gamma + 28\gamma^2 - 16\beta \gamma^2 + 8\gamma^3 - 8\beta \gamma^3 - 10\gamma^4 + 7\beta \gamma^4 - \gamma^5 + \beta \gamma^5 > 0. \)
Therefore, firm 1 always has incentives to license its input technology under entry.

(iii): In the Common Input Supplier case, calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:

\[
\pi_1^{CE} - \pi_1^{CN} = \frac{(a-c)^2E}{8(2 + \gamma)^2(2 + 3\gamma)^2(-2 + \gamma^2)^2} > 0,
\]

where \( E = 128 + 256\gamma - 256\beta \gamma + 96\gamma^2 - 416\beta \gamma^2 - 128\gamma^3 + 192\beta \gamma^3 - 56\gamma^4 + 464\beta \gamma^4 + 192\gamma^5 - 16\beta \gamma^5 + 138\gamma^6 - 174\beta \gamma^6 - 24\gamma^7 - 12\beta \gamma^7 - 29\gamma^8 + 23\beta \gamma^8 - 2\gamma^9 + 2\beta \gamma^9 > 0. \) It follows that firm 1 has stronger incentives to license its input technology with entry than without entry.

(iv): In the Common Input Supplier case, in the case of licensing without entry and in the case of licensing with entry, the consumers’ surplus are given by:

\[
CS^{CN} = \frac{(a-c)^2(2 - \gamma)^2(1 + \gamma)}{4(-2 + \gamma^2)^2} \quad \text{and} \quad CS^{CE} = \frac{(a-c)^2(12 + \gamma(28 + 9\gamma))}{8(2 + 3\gamma)^2}.
\]

Calculating the differences in consumers’ surplus between licensing and no licensing,
namely, \( CS^{CN} - CS^B = \frac{(a-c)^2\gamma^2(1+\gamma)(8-3\gamma^2)}{4(2+\gamma)^2(-2+\gamma)^2} \) and \( CS^{CE} - CS^B = \frac{(a-c)^2(16+32\gamma-8\gamma^2-8\gamma^3+9\gamma^4)}{8(2+\gamma)^2(2+3\gamma)^2} \), we find that they are always positive.

Total welfare is defined as the sum of consumers and producers’ surplus, namely:
\[ W^k = CS^k + \pi^k_1 + \pi^k_2 + \pi^k_S, \]
where \( k = B, CN \) and \( CE \). Calculating the following differences, \( W^{CN} - W^B = \frac{(a-c)^2(8-5\gamma^2-\gamma^3)}{4(2+\gamma)^2(-2+\gamma)^2} \) and \( W^{CE} - W^B = \frac{(a-c)^2(48+32\gamma-56\gamma^2-8\gamma^3+3\gamma^4)}{8(2+\gamma)^2(2+3\gamma)^2} \), we find that they are always positive. Thus, both the consumers’ surplus and total welfare are greater under licensing than under no licensing.

Calculating the following differences, we find \( CS^{CE} - CS^{CN} = \frac{(a-c)^2Z}{8(2+3\gamma)^2(-2+\gamma)^2} > 0 \)
where \( Z = 16 + 16\gamma - 60\gamma^2 - 48\gamma^3 + 6\gamma^4 + 10\gamma^5 + 9\gamma^6 > 0 \) if and only if \( \gamma < 0.54958 \) and \( W^{CE} - W^{CN} = \frac{(a-c)^2H}{8(2+3\gamma)^2(-2+\gamma)^2} > 0 \) where \( H = 48 - 16\gamma - 116\gamma^2 + 32\gamma^3 + 42\gamma^4 - 2\gamma^5 + 3\gamma^6 > 0 \) if and only if \( \gamma < 0.714414 \). Thus, both consumers’ surplus and total welfare are higher with entry than without entry if and only if products are sufficiently differentiated.

**Proof of Corollary 2:**

(i): Calculating the difference in the profits of firm 1 in the case of licensing and no entry when firm 1 and 2 source the input from firm \( S \) with the case when only firm 1 sources the input from firm \( S \), we find that, \( \pi^{CN}_1 - \pi^{LN}_1 = \frac{(a-c)^2(2-\gamma)\Theta}{8(-2+\gamma)^2} > 0 \) where \( \Theta = 4\beta - 2\beta\gamma - 2\beta\gamma^2 - 2\beta^3 + \beta\gamma^3 > 0 \) if and only if \( \gamma < 0.881239 \) and \( \beta > \beta_2(\gamma) = \frac{2\gamma^3}{4-2\gamma-2\gamma^2+\gamma} \), with \( \frac{\partial\beta_2}{\partial\gamma} > 0 \), \( \beta_2(0.881239) = 1 \) and \( \beta_2(0) = 0 \). Thus, under no entry firm 1 has stronger licensing incentives when both firm 1 and firm 2 source the input from firm \( S \) than when only firm 1 sources the input from firm \( S \) if the products are not close substitutes or when products are sufficiently close substitutes and firm \( S \)’s bargaining power is sufficiently high.

(ii): Calculating the difference in the profits of firm 1 in the case of licensing and entry when firm 1 and 2 source the input from firm \( S \) with the case when only firm 1 sources the input from firm \( S \), we find that, \( \pi^{CE}_1 - \pi^{LE}_1 = \frac{(a-c)^2I}{8(2+\gamma)^2(2+3\gamma)^2(-2+\gamma)^2} > 0 \) where \( I = 64\beta + 64\gamma + 128\beta\gamma + 240\gamma^2 - 64\beta\gamma^2 + 160\gamma^3 - 192\beta\gamma^3 - 200\gamma^4 + 28\beta\gamma^4 - 180\gamma^5 + 96\beta\gamma^5 + 11\gamma^6 + 8\beta\gamma^6 + 24\gamma^7 - 18\beta\gamma^7 + 2\gamma^8 - 2\beta\gamma^8 > 0 \). Thus, under entry firm 1 always has stronger licensing incentives when both firm 1 and firm 2 source the input from firm \( S \) than when only firm 1 sources the input from firm \( S \).

**Proof of Proposition 7:** Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the case of licensing without entry, we find:
\[ \pi^{CE}_1 - \pi^{LN}_1 = \frac{(a-c)^2K}{8(2+\gamma)^2(8 + 8\gamma - 3\gamma^2 + 2\gamma^3)^2} > 0, \]
where \( K = 512 + 256\gamma - 448\gamma^2 - 32\gamma^3 + 170\gamma^4 - 32\gamma^5 - 35\gamma^6 + 8\gamma^7 + 4\gamma^8 > 0 \). Therefore, firm 1 always has stronger incentives to license its input technology with entry than without entry.

**Proof of Proposition 8:** Calculating the difference in the profits of firm 1 in the case of licensing with entry and in the benchmark case, when firm 1 is initially a monopolist, we find:

\[
\frac{\pi_{1}^{LE} - \pi_{1}^{B}}{\pi_{1}^{B}} = \frac{(a - c)(1 - \gamma)^2}{4 - 3\gamma^2} > 0.
\]

Therefore, firm 1 has always incentives to license its input technology under entry.

Calculating the difference in the profits of firm 1 in the case of licensing without entry and in the benchmark case, we observe that \( \frac{\pi_{1}^{LN} - \pi_{1}^{B}}{\pi_{1}^{B}} = 0 \), namely, firm 1 is indifferent between licensing without entry and no licensing.

10 References


63-80.


