Delegated Certification*

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Abstract

In many markets, regulators rely on expert certifiers to provide buyers with reliable information about products. Should regulators set rules requiring certifiers to acquire information, or should the decision whether or not to acquire information be left in the hands of certifiers? We show that if candidates to be certified are able to disclose information about their types, then inflexible rules imposed on certifiers could worsen certification, by crowding out information disclosure from candidates. Our results also shed light on the pervasiveness of delegation in certification, by showing that even a regulator with the expertise needed to acquire information could benefit from delegating that task to a certifier. We discuss those findings in connection with credit rating.

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1 Introduction

In a broad range of markets, regulators delegate certification to private firms. For example, the main goal of the Securities and Exchange Commission (SEC) is to assure that all investors have adequate information about the financial products they trade\footnote{From the SEC website: “The Securities and Exchange Commission is concerned primarily with promoting the disclosure of important market-related information. [...] The Office of Credit Ratings assists the Commission in executing its responsibility [...] through the oversight of credit rating agencies.” The first half of the quote can be found on page \url{www.sec.gov/about/whatwedo.shtml}, the second part of the quote on page \url{www.sec.gov/ocr}.} This function is carried out, in part, by private companies with specific expertise, such as credit rating agencies, under the regulatory supervision of the SEC\footnote{While for some time credit rating agencies were granted considerable freedom, the Dodd-Frank Act has encouraged the SEC to step up significantly its supervision of Credit Rating Agencies, including requiring a disclosure of credit rating methodologies, see \url{www.sec.gov/spotlight/dodd-frank/creditratingagencies.shtml}.} Additional examples are discussed later in this introduction.

The present paper addresses a number of fundamental policy issues: should regulators set rules requiring certifiers to acquire information about products sold, or should some room to manoeuvre be left to certifiers? A key trade-off emerges, from the regulators’ viewpoint: stringent rules can raise the amount of information acquired by certifiers, but could crowd out information voluntarily disclosed by certified entities. Credit rating agencies, for instance, base their ratings both on information received by issuers and on information acquired elsewhere: e.g. “In issuing and maintaining its ratings, Fitch relies on factual information it receives from issuers and underwriters and from other sources Fitch believes to be credible.”\footnote{Source: \url{https://www.fitchratings.com/site/definitions} See also Cohn, Strobl and Rajan (2016).} How then should credit rating agencies be regulated?

We develop a tractable analytical framework to explore the central trade-off described in the previous paragraph. We show that inflexible rules imposed on certifiers could make certification \textit{worse}. Our results also shed light on the pervasiveness of delegation in certification, by showing that even a regulator with the expertise needed to acquire information could benefit from delegating certification.

Our stylized model has the following features. Each candidate to be certified is either good or bad. A regulator seeks to identify which of the candidates belong to the good type, and delegates the task of certifying the candidates to a certifier. Our setting combines information acquisition by the certifier, and information disclosure from the candidates. Specifically, the certifier forms prior beliefs based on the applicant pool, and possesses two more sources
of information: (a) a costly test revealing a signal correlated with the type of the candidate tested, and (b) hard information voluntarily disclosed by the candidates themselves. We model disclosure as Bayesian persuasion ([Kamenica and Gentzkow 2011]): candidates commit to (possibly noisy) disclosure rules before learning their type.

We compare two regulatory frameworks, based on the certification quality they induce. In the mandatory framework, the regulator requires that the certifier runs the test on each candidate in the pool. By contrast, in the discretionary framework, the certifier chooses whether or not to run the test, and makes this decision contingent on information disclosed by the candidates. Our main findings are as follows. First, no regulatory framework is always optimal: rules improve certification quality when the applicant pool is good, but worsen certification quality when the pool is bad (Result 1). Second, on average (i.e. before knowing the applicant pool), rules are optimal if and only if signals from the test are either very accurate or very inaccurate (Result 2).

While both results provide novel insights, Result 2 is perhaps the most relevant for policy, since regulations typically cannot be changed overnight and, therefore, ought to be evaluated according to their average performance. Results 1 and 2 are based on the following observations. As running the test involves a cost, in the discretionary framework the certifier performs the test only when uncertainty about a candidate’s type is sufficiently large. This has two implications, from the candidates’ perspective. First, when the certifier is pessimistic enough, she may reject candidates without even running the test in the first place. This creates incentives for the candidates to disclose information in order to encourage the certifier to conduct the test, and thereby mitigate the risk of being rejected. Second, the discretionary framework in effect gives each candidate the option to avoid being subjected to the test: all the candidate needs to do is to disclose information inducing sufficiently optimistic beliefs about his type so as to discourage the certifier from subjecting him to the test.

The encouragement effect of the discretionary framework unambiguously improves the aggregate amount of information available to the certifier. The impact of the discouragement effect on this aggregate amount of information is however more ambiguous, since candidates

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4Models of hard information disclosure typically exhibit ‘unraveling’, whereby candidates disclose all private information. In practice, unraveling seems not to occur in certification (see e.g. [Dranove and Jin 2010] for a survey of the empirical literature connected to certification). The approach proposed by Bayesian persuasion is one way to reconcile theory with empirics. [Best and Quigley 2016] microfound Bayesian persuasion through reputational incentives.

5The quality of certification is 1 if the certifier makes the correct certification decision (certify a good candidate or reject a bad one), and 0 otherwise.
disclose information, but do so in order to discourage the certifier from acquiring more information by conducting the test.

We first show that the net impact of the discouragement effect on the aggregate amount of information available to the certifier worsens as the applicant pool improves, yielding Result 1. We then show that the net impact of the discouragement effect is negative for extreme values of the signal accuracy, but positive for intermediate values; this yields Result 2.

Building on Results 1 and 2 allows us to show that even a regulator with the expertise needed to acquire information could benefit from delegating certification (Result 3). The basic idea is that delegating allows the regulator to credibly reduce the number of tests performed, and to boost the encouragement effect highlighted above. On the positive side, our results thus help explain the prevalence of delegated certification.

Finally, we show that, perhaps counter to intuition, more accurate signals can worsen certification quality (Result 4). On the one hand, when signal accuracy is low, avoiding the test is ‘easy’ for the candidates, since the certifier does not expect to learn much from running the test. On the other hand, when signal accuracy is high, avoiding the test is ‘crucial’ for each candidate to maintain the ability of manipulating beliefs about his type. For intermediate signal accuracy however, avoiding the test is neither crucial nor easy. In this case doing nothing to avoid the test can be part of a candidate’s equilibrium strategy. This implies that information voluntarily disclosed by the candidates is non-monotonic with respect to signal accuracy. Hence, more accurate signals can reduce the aggregate amount of information available to the certifier and lower certification quality.

Below, we provide examples of delegated certification, and discuss the related literature. Section 2 introduces the model. Section 3 contains the preliminary analysis, while Section 4 compares rules and discretion. In Section 5 we endogenize the signal accuracy. Section 6 extends the analysis by considering certification without delegation (Subsection 6.1) and transfers between the regulator and the certifier (Subsection 6.2). Section 7 concludes.

Examples of Delegated Certification. In this paragraph and the next we provide a few examples of regulators relying on private experts for certification. These examples show that regulators frequently delegate certification. While the details of the regulatory framework vary across markets, the insights we gain shed light on each of the examples considered here. Whereas in some markets certifiers take decisions that have regulatory value (as in the case of organic food), others are meant to inform the decisions of the regulators (as in the case of toy safety), while yet others have the task of providing information to the public, though
the certificates they produce have no regulatory value (as in the case of car safety). Finally, expectations of information disclosure by sellers can vary substantially across markets (e.g., in the case of toy safety, sellers are required to demonstrate directly that toys comply with certain safety regulations).

Both the U.S. Department of Agriculture and the authorities in charge of the control system for organic production in the E.U. leave the testing as well as the final decision as to whether a food product qualifies as organic in the hands of independent accredited certifiers. Sellers of toys in the U.S. and the E.U. obtain approval from the regulators to market their products by submitting those products to testing by independent certifiers or by disclosing safety information themselves. The following quote from a directive of the European Parliament sheds light on the position of European authorities with regard to the role of toy manufacturers in the safety certification of their products: “The manufacturer, having detailed knowledge of the design and production process, is best placed to carry out the complete conformity assessment procedure for toys.” Finally, authorities regulating car safety in the U.S. and the E.U. leave the task of informing the public about the safety of different models almost entirely in the hands of independent certifiers, which provide a rating system somewhat comparable to the one used by credit rating agencies.

Related Literature. Our paper relates to two broad strands of literature: on certification (in particular the work focusing on credit rating agencies), and on information disclosure.

The bulk of the literature studying certification either focuses on information acquisition by certifiers (e.g., Kashyap and Kovrijnykh (2016), Bizzotto (2016)), emphasizes information disclosure by certified entities (e.g., Kolotilin (2015), Quigley and Walther (2016)), or takes the information of certifiers as given (e.g., Lizzeri (1999), Faure-Grimaud, Peyrache and Quesada (2009)). To the best of our knowledge, Cohn et al. (2016) is the only paper apart from ours which investigates the interplay between information disclosure from certified entities and

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6For the list of certifiers accredited in the U.S. see the [USDA Certifier Locator](https://www.ams.usda.gov/ams/plain/index). Approved control bodies in the E.U. can be found in the [List of Control Bodies and Control Authorities in the Organic Sector](https://ec.europa.eu/food/sites/food/files/organics/file/2018-09-21_list_control_bodies_en.pdf) provided by the European Commission.


9In the U.S., the Insurance Institute for Highway Safety, a nonprofit organization funded by auto insurer, rates car models according to their safety: see [www.iihs.org](http://www.iihs.org). In the E.U., Euro NCAP, a consortium of transportation authorities, publishes car safety ratings based on tests run by independent facilities: see [www.euroncap.com](http://www.euroncap.com).
information acquisition by certifiers. The modelling approach is however very different in the two papers. The model we propose makes it possible to calculate precisely the equilibrium certification quality achieved. This in turn allows us to compare the performance of different regulatory frameworks, based on the certification quality they induce, and to examine the merits of delegating certification.

Our paper borrows techniques developed by the recent literature exploring Bayesian persuasion, starting with Rayo and Segal (2010) and Kamenica and Gentzkow (2011). That strand of research examines sender-receiver games where the sender’s goal is to manipulate the receiver’s beliefs about a state variable in order to induce the receiver to choose the action preferred by the sender. To do so, the sender is able to design in advance how to filter information about the state. There are various ways to model disclosure of hard information. One class of models exhibits ‘unraveling’ (Grossman (1981) and Milgrom (1981)). However, unraveling is a rare phenomenon in practice (see Dranove and Jin (2010)). To address this issue, different approaches have been suggested: the model of Jovanovic (1982) assumes that disclosing information has substantial costs; Dye (1985) assumes that candidates may be uninformed about their type (or, equivalently, that candidates may be unable to disclose information); Kamenica and Gentzkow (2011), finally, assume that candidates can commit to disclosure rules before observing private information. In a recent working paper, Best and Quigley (2016) microfound Bayesian persuasion through reputational incentives. Within Bayesian persuasion, the paper closest to us is Bizzotto, Rüdiger and Vigier (2016), which explores a dynamic model of Bayesian persuasion with exogenous public news. The passing of time in that model allows the receiver to accumulate information. This is akin to the present paper, where the certifier can acquire information by running a costly test on the candidate.

Finally, we are related in spirit to Henry and Ottaviani (2016) and Quigley and Walther (2016). The former paper explores the impact of different organizational structures on the equilibrium quality of decisions made by a receiver who obtains information from experiments run by a sender. There is however no information acquisition by the receiver. The latter paper examines the optimal course of action for financial supervisors when banks are able to disclose information about themselves. However, the objective of supervisors is not to obtain more information about the banks; instead, the problem is how to induce strong banks not to disclose information about themselves.

\footnote{In Cohn et al. (2016), the candidate chooses whether to misreport his type, and lying is costly.}
2 The Model

2.1 General Setting

Agents. There are three agents: the regulator, the certifier (she) and the candidate (he). The candidate is either good or bad. The certifier has the expertise to retrieve information about the candidate’s type which, in the baseline model, the regulator does not. The regulator thus delegates that task to the certifier. In Section 6 we consider the case where the regulator can acquire information herself and chooses whether or not to delegate.

State of the World. The state of the world is \((\lambda, \omega)\); \(\lambda \in [0, 1]\) denotes the mean of the applicant pool from which the candidate is picked, while \(\omega \in \{0, 1\}\) denotes the actual type of the candidate (\(\omega = 1\) if he is good, and \(\omega = 0\) if he is bad):

\[E[\omega | \lambda] = \lambda.\]

For simplicity, \(\lambda\) is uniformly distributed on the unit interval. The realization of \(\omega\) is private information of the candidate, while \(\lambda\) is public. In the context of our lead example on credit rating, \(\lambda\) can be thought of as the state of the economy. In good times (resp. bad times), the prior probability that issuers are distressed is low (resp. high).

Information Acquisition. The certifier can obtain information about \(\omega\) by conducting a test on the candidate. In our lead example for instance, credit rating agencies can develop their own models for making forecasts, or content themselves with information publicly available. Let \(e = 1\) if the certifier conducts the test, and \(e = 0\) if she does not. The test costs \(c \in (0, \min\{\frac{1}{2}, \tau\})\) to the certifier and reveals a signal \(s \in \{0, 1\}\) such that \(\Pr(s = \omega | \omega) = p\). The parameter \(p \in [\frac{1}{2}, 1]\) is the signal accuracy. Hence, if \(p = \frac{1}{2}\) the signal is uninformative about \(\omega\), whereas \(s = \omega\) if \(p = 1\). In the baseline model, \(p\) is exogenous. Later, in Section 5, \(p\) is endogenized.

\(^{11}\)This assumption simplifies the exposition and can be relaxed.

\(^{12}\)Kashyap and Kovrijnykh (2016) document evidence showing that rating mistakes are in part due to insufficient effort by rating agencies. See Opp, Opp and Harris (2013) or Kashyap and Kovrijnykh (2016) for analogous models of information acquisition by credit rating agencies.

\(^{13}\)Where \(\bar{\tau}\) is some endogenously determined upper bound assuring that the certifier wants to conduct the test for some values of signal accuracy. Specifically, \(\bar{\tau}\) is determined by \(p(\bar{\tau}, c_b) = \bar{p}(\bar{\tau}, c_b)\), with \(\underline{p}\) and \(\bar{p}\) as defined in Lemma 2.
Information Disclosure. Before learning $\omega$, the candidate selects information structures to filter information about $\omega$ to the certifier. An information structure $\pi$ consists of a finite realization space $\chi_\pi$ and a family of probability distributions $\{\pi(x|\omega)\}_{\omega\in\{0,1\}}$ over $\chi_\pi$. Given a prior belief $m = \mathbb{P}(\omega = 1)$, to each realization $x \in \chi_\pi$ corresponds a posterior belief $m'(x) = \mathbb{P}(\omega = 1|x)$ calculated using Bayes’ rule. Thus $\pi$ induces a distribution over posterior beliefs. Moreover, if $\tau^m$ denotes this distribution, then $\sum_{m'} \tau^m(m')m' = m$. Let $T(m)$ denote the set of distributions with mean $m$. Any element in $T(m)$ can be generated by some information structure $\pi$. Therefore, to simplify the exposition, given a prior $m$, we henceforth identify the information structure $\pi$ with $\tau^m \in T(m)$.

If $\text{supp}(\tau^m) = \{m'_1, m'_2, ..., m'_{k}\}$, we say that the candidate splits $m$ on $m'_1, m'_2, ..., and m'_{k}$. A disclosure rule is a family of information structures, $\{\tau^m\}_{m\in[0,1]}$, specifying an element in $T(m)$ for each $m \in [0,1]$.

Certification and Payoffs. To fix ideas, we assume that the decision whether or not to certify the candidate is made by the certifier. As will become clear, nothing would change if we instead assumed that the regulator delegated information acquisition, all the while keeping the decision whether or not to certify the candidate in her own hands (see the remarks in Subsection 2.3). Let $a = 1$ if the certifier certifies the candidate, and $a = 0$ if she rejects the candidate. The candidate aims to be certified and may incur a cost from being subjected to the test by the certifier. His payoffs are given by

$$v := a - c_b e,$$

where $c_b \in [0, \overline{c}_b])$. If $c_b$ were arbitrarily large then the candidate would reveal $\omega$ just to avoid the cost $c_b$ from the test; to make the analysis interesting, we thus impose an upper bound on $c_b$. Certification quality, denoted $\Phi$, is defined by

$$\Phi := \omega a + (1 - \omega)(1 - a).$$

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14 An alternative interpretation of the model is to suppose that the candidate has no private information concerning $\omega$. See Kamenica and Gentzkow (2011) and Perez-Richet (2014) for a discussion. Given the applications we have in mind, we feel that the interpretation we give in the text is more natural.

15 See, e.g., Kamenica and Gentzkow (2011).

16 Specifically, $\overline{c}_b$ is determined by $p(c, \overline{c}_b) = \overline{p}(c, \overline{c}_b)$, with $p$ and $\overline{p}$ as defined in Lemma 3.
Hence $\Phi = 1$ if the certifier makes the correct certification decision, and $\Phi = 0$ if she does not. The payoffs $u$ of the certifier have two parts. First, the certifier incurs the cost $c$ from running the test on the candidate. Second, we assume that the certifier cares about making the correct certification decision. Implicit behind our model is the idea that the certifier’s revenue depends on her reputation, and that reputation is a function of her track record of making correct decisions. Hence

$$u := \Phi - ce.$$ 

In Section 6, we consider transfers between the regulator and the certifier.

In this paper, the regulator is the social planner. Let $N$ denote the total number of agents benefiting from certification other than the certifier or the candidate. The goal of the regulator is to maximize $u + v + N\Phi = (N + 1)(\Phi - \frac{c + cb}{N + 1}e + \frac{1}{N + 1}a)$. Henceforth, we assume that $N$ is large relative to $c$ and $c_b$, such that the payoffs of the regulator can be approximated by $\Phi$. While we state our results in the limit as $N$ tends to infinity, importantly, all our results hold for $N$ sufficiently large.

### 2.2 Regulatory Frameworks

The regulator chooses the regulatory framework, i.e., whether to require the certifier to conduct the test on the candidate or leave that decision in the hands of the certifier. Each regulatory framework induces an extensive-form game played between the certifier and the candidate. The equilibrium concept is perfect Bayesian equilibrium. We next describe the details specific to each of these games.

**The Mandatory Framework.** In the mandatory framework, the certifier is bound to run the test. The candidate first chooses two disclosure rules, $\{\tau^m_1\}_{m \in [0,1]}$ and $\{\tau^m_2\}_{m \in [0,1]}$. Nature then draws the state of the world, $(\lambda, \omega)$. The ‘operative’ phase of the game unfolds as follows. The candidate first discloses information using the information structure $\tau^1_1$, inducing the belief...
A strategy for the candidate is a choice of two disclosure rules. A strategy for the certifier is a decision rule whether to certify or reject the candidate after all information concerning \( \omega \) has been revealed, that is, as a function of the belief \( \xi \).

Figure 1 shows the timing of the operative phase in the mandatory framework. The sequence \( \lambda, \mu, \nu \) and \( \xi \) shown above the timeline keeps track of the certifier’s belief concerning \( \omega \) at each point in time. Below the timeline, we specify at each point in time the source generating information next.

The Discretionary Framework. In the discretionary framework, the certifier is free to decide whether or not to run the test on the candidate. Other aspects of the game are as in the mandatory framework. The candidate first chooses two disclosure rules, \( \{\tau_1^m\}_{m \in [0,1]} \) and \( \{\tau_2^m\}_{m \in [0,1]} \). Nature then draws the state of the world, \( (\lambda, \omega) \). The ‘operative’ phase of the game starts with the candidate disclosing information using \( \tau_1^\lambda \), inducing the belief \( \mu \) that \( \omega = 1 \). Given \( \mu \), the certifier chooses whether to run the test or to select \( a \) based on \( \mu \) alone. In the former case, let \( \nu \) denote the resulting posterior belief that \( \omega = 1 \). At that point, the candidate discloses information using \( \tau_2^\nu \), inducing belief \( \xi \). The certifier then selects \( a \). The game ends as soon as the certifier picks \( a \).

Figure 1

\[ \lambda \quad \mu \quad \nu \quad \xi \]

- candidate discloses information with \( \tau_1^\lambda \)
- certifier runs the test
- candidate discloses information with \( \tau_2^\nu \)
- certifier selects \( a \)

\( \mu \) that \( \omega = 1 \). The certifier runs the test, inducing belief \( \nu \). The candidate then discloses information using the information structure \( \tau_2^\nu \), inducing belief \( \xi \). The certifier ends the game by selecting \( a \).

In particular, we implicitly assume that, at each point in time, the candidate knows what information concerning \( \omega \) the certifier has. This implies that the candidate can track down the beliefs of the certifier concerning \( \omega \). This is to guarantee tractability of the model.
A strategy for the candidate is a choice of two disclosure rules. A strategy for the certifier specifies for each choice of disclosure rules, and each realization $\lambda$ of the applicant pool: (i) a decision rule whether or not to conduct the test as a function of the belief $\mu$ and, if not, whether to certify or reject the candidate; (ii) a decision rule whether to certify or reject the candidate as a function of the belief $\xi$.

Figure 2 shows the timing of the operative phase in the discretionary framework. The structural break between the second and third node indicates that the game may terminate at the second node.

2.3 Scope

Here, we briefly discuss three modeling assumptions. First, the regulator cannot mandate disclosure from the candidate. It is easy to see that if the regulator could, she would mandate disclosure, since there are no constraints on the amount of information that the candidate can disclose. The interesting case is therefore when that option is not available to the regulator. Second, in the model, the certifier chooses $a$. An alternative is to assume that $a$ is chosen by the regulator, or even by a third party (e.g. investors, or buyers) outside the current model. As long as whoever chooses $a$ aims to ‘match’ $\omega$ and has access to the same information available to the certifier then all results in the paper carry through. Third, we assume that the certifier maximizes certification quality net of the cost of acquiring information. In the context of our lead example, the bias of credit rating agencies is widely believed to have played an important role in the build up to the 2007-09 financial crisis. Importantly, all results in this paper remain qualitatively the same if the certifier exhibits a small bias in favor of either $a = 1$ or $a = 0$. 
When the bias is large however, then unsurprisingly delegating certification is never optimal from the regulator’s viewpoint. It is worth noting that in the U.S., the Dodd-Frank Act of 2010 intended among other things to address credit rating bias. For instance, Section 939A requires federal regulators to remove references to Nationally Recognized Statistical Rating Organizations credit ratings and find alternatives.\footnote{In the words of Thomas McGuire, then VP at Moody’s: “By using securities ratings as a tool of regulation, governments fundamentally change the nature of the product agencies sell. Issuers then pay rating fees to purchase, not credibility with the investor community, but a license from a government.”}

3 Preliminary Analysis

A regulatory framework may fare well for certain applicant pools but less well for others. This remark motivates the next definitions.\footnote{From the law of iterated expectations, if a regulatory framework is interim optimal for all \( \lambda \), then it is also ex ante optimal.}

**Definition 1.** Interim certification quality is given by \( \mathbb{E}[\Phi|\lambda] \). Ex ante certification quality is given by \( \mathbb{E}[\Phi] \).

**Definition 2.** A regulatory framework is interim optimal if it maximizes interim certification quality. A regulatory framework is ex ante optimal if it maximizes ex ante certification quality.

**Definition 3.** The maximum (interim or ex ante) certification quality is the certification quality achieved in the (interim or ex ante) optimal regulatory framework.

3.1 The Mandatory Framework

This subsection studies the mandatory framework. Our first result shows that interim certification quality is as if the candidate disclosed no information to the certifier.

**Lemma 1.** In the mandatory framework, \( \mathbb{E}[\Phi|\lambda] = \max\{1 - \lambda, p, \lambda\} \).

The basic ideas behind the lemma are as follows. In the mandatory framework, the candidate has no incentives to disclose information before the certifier conducts the test. Therefore, in equilibrium, any relevant information is disclosed after the certifier conducted the test. Such disclosure must leave the certification quality unchanged: either the test outcome is sufficient to induce certification, in which case the candidate has no incentive to disclose more information, or the test outcome induces the certifier to reject the candidate. In the latter case,
standard arguments due to Kamenica and Gentzkow (2011) show that the candidate discloses information so as to make the certifier just indifferent between certifying and rejecting. Combining the remarks above shows that interim certification quality is as if the certifier based her certification decision on the outcome of the test alone.

3.2 The Discretionary Framework

This subsection studies the discretionary framework. The distinguishing feature is the possibility that the certifier forgoes the test on the candidate. This affects the candidate’s incentives to disclose information and, ultimately, also the certification quality.

Lemma 2. In the discretionary framework, there exist \( p(c, c_b) \) and \( \bar{p}(c, c_b) \) such that, in equilibrium, if \( p < p \) or \( p > \bar{p} \), the candidate splits \( \lambda \) on 0 and \( p - c \) for all \( \lambda \in (0, p - c) \), whereas if \( p \in (p, \bar{p}) \), the candidate:

- splits \( \lambda \) on 0 and \( 1 - p + c \) for all \( \lambda \in (0, 1 - p + c) \);
- splits \( \lambda \) on \( 1 - p + c \) and \( p - c \) for all \( \lambda \in (1 - p + c, p - c) \).

All equilibria of the discretionary framework induce the same expected payoffs for both players. Furthermore, an equilibrium exists in which the candidate chooses his second disclosure rule, \( \{\tau_2^m\}_{m \in [0,1]} \), as we described it to be in the equilibrium of the mandatory framework (call this Property A): either the test outcome is sufficient to induce certification, in which case the candidate discloses no more information, or the test outcome induces the certifier to reject the candidate in which case the candidate discloses information so as to make the certifier just indifferent between certifying and rejecting. Equilibria satisfying Property A exhibit intuitive features, which help shed light on Lemma 2 in the rest of this subsection, we therefore restrict attention to those equilibria (this is purely for the sake of exposition; none of our results impose this restriction). In this case, the best response of the certifier is characterized by two threshold beliefs about \( \omega \), respectively equal to \( 1 - p + c \) and \( p - c \), which together determine the region of beliefs for which the certifier chooses to run the test. Figure 3 illustrates. Either \( \mu < 1 - p + c \) (resp. \( \mu > p - c \)), in which case the certifier is sufficiently certain that the candidate is bad (resp. good) and prefers to reject the candidate (resp. certify), or \( \mu \in [1 - p + c, p - c] \), in which case the certifier chooses to run the test.

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24For \( p = \bar{p} \) and \( p = \bar{p} \), certification quality is not uniquely determined. As they are non-generic, we ignore these cases.

25The same is true for the mandatory framework.
Observe that $(1 - p + c, p - c) \subset (1 - p, p)$. Hence for some beliefs the certifier chooses not to conduct the test although doing so would improve the expected certification quality. This reflects the fact that the certifier incurs a cost $c > 0$ for running the test.

![Figure 3](image)

From the candidate’s perspective, the former considerations have two principal implications. First, if the certifier’s belief that $\omega = 1$ is less than $1 - p + c$, the candidate is forced to disclose at least some information in order to mitigate the risk of being rejected. Second, by disclosing a still greater amount of information the candidate could in fact avoid being subjected to the test: the candidate just needs to induce the belief $p - c$ or above that he does in fact belong to the good type.

Lemma 2 shows that the candidate’s equilibrium disclosure rules depend on the signal accuracy as follows:

- if $p$ is either high or low the candidate discloses information so as to avoid being subjected to the test, independently of the certifier’s belief about his type;

- by contrast, for intermediary values of $p$ the candidate (a) discloses information so as to avoid being subjected to the test only if $\lambda > 1 - p + c$, and (b) discloses the least possible amount of information mitigating the risk of being rejected if $\lambda < 1 - p + c$.

Figure 4 illustrates the candidate’s disclosure of information in the discretionary framework, for various values of the signal accuracy $p$. In the top row (resp. the bottom row) $p$ is small (resp. large), and in the middle row $p$ is intermediate. On the left the mean of the applicant pool is chosen below $1 - p + c$, and on the right $\lambda$ is chosen in the interval $(1 - p + c, p - c)$. On the top and bottom rows the candidate splits the certifier’s prior belief on 0 and $p - c$. Either the resulting posterior belief about $\omega$ is 0 and the certifier rejects the candidate, or the resulting posterior belief is $p - c$ and the candidate is certified. Either way, as a result of the candidate’s disclosure, the certifier chooses never to run the test. In the middle row on the left, the candidate splits the certifier’s prior belief on 0 and $1 - p + c$. 13
Either the resulting posterior belief about $\omega$ is 0 and the certifier rejects the candidate, or the resulting posterior belief is $1 - p + c$, at which point the certifier chooses to run the test. In the middle row on the right, the candidate splits the certifier’s prior belief on $1 - p + c$ and $p - c$. Either the resulting posterior belief is $1 - p + c$ and the certifier runs the test, or the posterior belief is $p - c$ and the candidate is certified.

The basic intuition is the following. On the one hand, if $p$ is low avoiding the test is easy, since the certifier does not expect to learn much from running the test. On the other hand, if $p$ is high avoiding the test is crucial in order for the candidate to manipulate beliefs about $\omega$. For intermediate $p$, avoiding the test is neither crucial nor easy. In this case doing nothing to avoid the test can be part of the candidate’s equilibrium strategy.

We conclude the subsection by recording the implications of Lemma 2 concerning certification quality.

**Lemma 3.** In the discretionary framework,

- if $p < \underline{p}$ or $p > \bar{p}$ then

$$
\mathbb{E}[\Phi | \lambda] = \begin{cases} 
1 - \left(\frac{1 - p + c}{p - c}\right)\lambda & \text{for } \lambda \in [0, p - c] \\
\lambda & \text{for } \lambda \in [p - c, 1];
\end{cases}
$$

- if $p \in (\underline{p}, \bar{p})$ then
\[ E[\Phi | \lambda] = \begin{cases} 
1 - \left( \frac{1 - p}{1 - p + c} \right) \lambda & \text{for } \lambda \in [0, 1 - p + c] \\
1 - \left( \frac{c}{2(p - c) - 1} \right) (\lambda - (1 - p + c)) & \text{for } \lambda \in [1 - p + c, p - c] \\
\lambda & \text{for } \lambda \in [p - c, 1] 
\end{cases} \]

The dashed lines in Figure 8 show $E[\Phi | \lambda]$ in the discretionary framework for different values of $p$. The solid lines in all panels show $E[\Phi | \lambda]$ in the mandatory framework.

### 4 Rules vs Discretion

In this section we evaluate the merits of rules vs discretion with regard to certification quality, by comparing the analysis of the regulatory frameworks examined in Section 3.

The paper’s first main result establishes that no regulatory framework is interim optimal for all kinds of applicant pools: when the pool of applicants is bad, discretion fares better than rules, and vice versa when the pool of applicants is good.

**Proposition 1.** There exists $\hat{\lambda}$, which depends on the parameters of the model, such that:

- the discretionary framework is interim optimal if $\lambda \in [0, \hat{\lambda}]$,
- the mandatory framework is interim optimal if $\lambda \in [\hat{\lambda}, p]$.

If $\lambda \geq p$ then both frameworks are interim optimal.

The intuition behind Proposition 1 is based on the following remarks: the certifier’s preferences are aligned with those of the candidate when $\omega = 1$; if $\omega = 0$ on the other hand, the candidate and the certifier have opposed preferences. Therefore, the conflict of interest between the candidate and the certifier is severe when the applicant pool is bad, but moderate when the applicant pool improves. Leaving much discretionary power in the hands of the certifier in the former case tends to be efficient, as the candidate is then forced to disclose information in order to mitigate the risk of being rejected. When the applicant pool is good however, the former benefits from giving the certifier discretionary power vanish. Instead, the focus ought to be on providing the certifier with maximum incentives to run the test.
When the applicant pool is good, rules are therefore better than discretion in order to assure certification quality.

We next turn to the paper’s second main result, establishing that, ex ante (i.e. before the applicant pool is known), rules are optimal if the signal accuracy is either very accurate or very inaccurate. Otherwise, discretion is optimal.

**Proposition 2.** There exists $\hat{c} < \bar{c}$ such that, if $c < \hat{c}$, the ex ante optimal regulatory framework depends on $p$. Specifically, there exist $\hat{p}(c, c_b)$ and $\bar{p}(c, c_b)$ such that:

- the mandatory framework is ex ante optimal if $p \leq \hat{p}$ or $p \geq \bar{p}$,
- the discretionary framework is ex ante optimal if $p \in [\hat{p}, \bar{p}]$.

Furthermore $p \in [\underline{p}, \bar{p}]$. If $c \geq \hat{c}$, the mandatory framework is ex ante optimal for all $p$.

Figure 5 shows the threshold signal accuracies from Lemma 2, $\hat{p}$ and $\bar{p}$, those appearing in Proposition 2, $\hat{p}$ and $\bar{p}$, and the various parametric regions of the model in $(c, p)$-space when $c_b = 0$. In the dashed region the set of beliefs for which the certifier conducts the test is empty. In that region, our model reduces to the setting analyzed by Kamenica and Gentzkow (2011). The gray region (resp. white region) is where the discretionary framework (resp. mandatory framework) is ex ante optimal.

$^26$The $\hat{p}$ appearing in the figure refers to Proposition 5.
The first claim of the proposition states that the discretionary framework is ex ante optimal for an intermediate range of $p$, whereas the mandatory framework is ex ante optimal for extreme values of $p$. To shed light on this finding we next decompose the effects of the discretionary framework into two parts, illustrated on Figure 6. There is first an *encouragement effect* of the discretionary framework, due to the fact that for pessimistic beliefs the certifier rejects the candidate outright (i.e. without first running the test). This forces the candidate to disclose some information to the certifier. There is also a *discouragement effect* of the discretionary framework, due to the fact that for optimistic beliefs the certifier certifies outright. This incentivizes the candidate to disclose information with a view to avoid the test.

![Figure 6](image)

The encouragement effect unambiguously improves the aggregate amount of information based on which the certifier makes her certification decision. The impact of the discouragement effect is more ambivalent, since the candidate discloses information but does so in order to discourage the certifier from acquiring more information through the test. We show that the net impact of the discouragement effect is positive for intermediate values of $p$, but negative for extreme values of $p$. This is illustrated in Figure 7. The different rows correspond to different values of $p$. The gray arrows depict information disclosed by the candidate in order to avoid the test. The dashed black arrows depict information generated by the test (if a test were to be conducted). Observe that in the middle row the gray arrow overshoots the black one. This overshooting captures the net gain of information due to the discouragement effect of the discretionary framework occurring for intermediate values of $p$.

Figure 8 summarizes the results of this section. The mean of the applicant pool, $\lambda$, is shown on the horizontal axis. The vertical axis shows interim certification quality, $E[\Phi|\lambda]$. The different panels correspond to different values of $p$. The solid lines show interim certification

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27 In fact, if $p$ is close to 1 then the mandatory framework is trivially ex ante optimal. As the test in this case all but reveals $\omega$, the mandatory framework cannot possibly be improved upon.

28 It follows from Lemma 2 that if $p \in (p, \overline{p})$ the discouragement effect only bites for $\lambda \in (1 - p + c, p - c)$ (see Figure 4). This is why, in Figure 7, the middle row corresponds to $p \in (p, \overline{p})$. This remark also explains why, in Figure 5, the bulk of the gray area lies above $p$. 

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quality in the mandatory framework, and the dashed lines show interim certification quality in the discretionary framework. First, observe that, on the left hand side of all three panels, $E[\Phi|\lambda]$ is largest in the discretionary framework, whereas, on the right hand side of all three panels, $E[\Phi|\lambda]$ is largest in the mandatory framework. This is Proposition 1. Second, observe that the difference between the dark gray area and the light gray area is negative in panels (a) and (c), but positive in panel (b). This is Proposition 2.

5 Signal Accuracy

In this section we examine the equilibrium impact on certification quality of the signal accuracy, $p$.

Lemma 4. In the mandatory framework, interim certification quality is monotonically increasing in $p$, for all $\lambda \in [0, 1]$. By contrast, if $c_b > 0$ then, in the discretionary framework, interim certification quality is non-monotonic in $p$.

The logic behind the non-monotonicity of interim certification quality arising in the discretionary framework is the following. We showed with Lemma 2 that, as $p$ crosses $\underline{p}$ from below, the candidate’s equilibrium disclosure policy undergoes a qualitative change: for $p$ below $\underline{p}$, the candidate discloses information so as to avoid being subjected to the test, whereas as $p$ crosses $\underline{p}$ the candidate discloses less information, inducing the certifier to conduct the test.
Figure 8

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with some probability (see Figure 4). By virtue of equilibrium, at $p = \underline{p}$ the candidate is exactly indifferent between these two disclosure policies. If being subjected to the test imposed no direct payoff externalities on the candidate ($c_b = 0$) we would conclude that the probability of $a = 1$ is the same in both scenarios. The only difference being that in one scenario the information based on which the certifier chooses $a$ comes from the candidate, whereas in the alternative scenario the source of that information is the test. When $c_b > 0$ on the other hand, the candidate must be compensated for the negative externality suffered from being subjected to the test. In consequence, when $c_b > 0$, the probability that the candidate is certified rises as $p$ crosses $\underline{p}$ from below. This, in turn, implies that the certification quality falls as $p$ crosses $\underline{p}$ from below.

As $p \in [\underline{p}, \bar{p}]$, combining Lemma 4 with Proposition 2 yields the next result:

**Proposition 3.** If $c_b > 0$ and $c < \hat{c}$ then the maximum ex ante certification quality is non-monotonic in $p$.

More accurate signals can thus worsen the certification quality achieved. In particular, if tests generating perfectly revealing signals are infeasible, then the ex ante optimal $p$ can be less than the maximum feasible signal accuracy. Hence, if signal accuracy is a policy instrument, the regulator could decide to choose $p$ below the maximum feasible signal accuracy.

### 6 Extensions

#### 6.1 Certification without Delegation

Here we briefly examine the case where the regulator is able to conduct the test herself (at the same cost $c$), but still has the option to delegate certification using either of the frameworks described in the baseline model.

**Proposition 4.** Let $c < \hat{c}$. If the regulator is able to conduct the test herself then, ex ante, for $p \in (\underline{p}, \bar{p})$, the regulator strictly prefers delegating certification.

The basic idea behind the proposition is as follows. Suppose the regulator is able to conduct the test herself. The (exact, i.e. not approximated) payoffs of the regulator are

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29 Formally, this is Lemma 13 in Appendix A.

30 This result is reminiscent of Elliott, Golub and Kirilenko (2014) and Kolotilin (2016), since in both settings giving an expert access to better information leads to worse decisions. However, the mechanisms involved in those papers are very different from the mechanism operating in our model.
\((N + 1)(\Phi - \frac{c + \alpha}{N+1} e + \frac{1}{N+1} a)\). The setting without delegation thus amounts to the discretionary framework of the baseline model with \(c' = \frac{c + \alpha}{N+1}\) (and a small bias in favor of certification).

Next, one shows that, as \(c\) tends to 0 in the baseline model, certification quality in the discretionary framework converges to certification quality in the mandatory framework. As certification quality in the mandatory framework does not depend on \(c\), to prove Proposition 4 one just needs to find parameters such that, ex ante, certification quality is higher in the discretionary framework than in the mandatory framework. Using Proposition 2 now concludes the argument.

Note finally that it emerges from the argument above that if the regulator prefers delegating certification, the regulator then also selects the discretionary framework over the mandatory one. The basic idea is that delegating certification within the discretionary framework allows the regulator to credibly reduce the number of tests performed, and to boost the encouragement effect described Section 4.

### 6.2 Transfers

In this subsection, we drop the assumption that the regulator is able to set rules requiring the certifier to conduct the test on the candidate. Instead, the regulator chooses transfers \(t(\Phi, e)\) to the certifier, where

\[
t(\Phi, e) = t_0 + t_\Phi \Phi + t_e e;
\]

\(t_0\) is a lump-sum transfer, \(t_\Phi\) a transfer rewarding the certifier for making the correct certification decision, and \(t_e\) a transfer to subsidize the test run by the certifier on the candidate. The payoffs induced are \(\Phi - \kappa t(\Phi, e)\) for the regulator and \(u + t(\Phi, e) = t_0 + (1 + t_\Phi) \Phi - (c - t_e) e\) for the certifier, where \(\kappa\) is a parameter capturing the regulator’s marginal utility for money. The payoffs of the candidate are as in the baseline model, \(v = a - c_b e\).

The timing of the game with transfers is as follows. First, the regulator publicly selects transfers \(t(\Phi, e)\), and the certifier chooses whether to accept the task of certifying the candidate. If the certifier accepts, the candidate then commits to two disclosure rules, \(\{\tau_1^m\}_{m \in [0,1]}\) and \(\{\tau_2^m\}_{m \in [0,1]}\). Next, nature draws the state of the world \((\lambda, \omega)\). The ‘operative’ phase of the game unfolds as described in the discretionary framework (see Section 2).

Observe that even though in this setting the regulator is unable to mandate the test, the use of transfers allows the regulator to mimic the mandatory framework by fully subsidizing the certifier for conducting the test, i.e. by choosing \(t_e = c\). This subsection’s main result
shows that, if the signal accuracy is not too large, then the regulator chooses not to fully subsidize the certifier for running the test, even when the regulator’s marginal utility for money is zero.

**Proposition 5.** There exists \( \hat{p} > \frac{1}{2} \) such that, for all \( \kappa \geq 0 \), if \( p < \hat{p} \) then in equilibrium \( t_e < c \).

The threshold signal accuracy \( \hat{p} \) below which the regulator prefers not to fully subsidize the test is illustrated in Figure 5. The logic underlying Proposition 5 is the same we discussed in connection to Proposition 2. Subsidizing the test creates incentives for the certifier to conduct the test on the candidate but, indirectly, also eliminates part of the candidate’s incentives to disclose information about his type. In consequence, unless signals from the test are very accurate, optimal transfers are ‘interior’, ensuring a net positive cost of running the test on the part of the certifier.

We conclude with two brief remarks. First, a corollary of Proposition 5 is that if the regulator could invest in a technology reducing the cost incurred by the certifier to run the test, the regulator might forgo that opportunity even if that investment cost her nothing. Second, by making transfers contingent on \( a \), the regulator could induce ex ante certification quality arbitrarily close to 1. Effectively, by promising a large reward to the certifier for choosing \( a = 0 \), the regulator forces the candidate to disclose large amounts of information to mitigate the risk of being rejected by the certifier. The contracts involved however are not renegotiation-proof since they require the certifier to reject the candidate even when the belief that the candidate belongs to the good type is larger than \( \frac{1}{2} \).

7 Conclusion

The literature on certification has up to now either focused on information disclosure from certified entities, or focused on information acquisition by certifiers. This paper is the first to examine the rich interplay between the two. We identify a key trade-off facing the regulator: stringent rules imposed on certifiers can raise the amount of information acquired by certifiers, but could crowd out information voluntarily disclosed by certified entities. We show that, as a result, inflexible rules imposed on certifiers could worsen certification. Our results also shed light on the pervasiveness of delegation in certification, by showing that even a regulator with the expertise needed to acquire information herself could benefit from delegating certification.
The basic idea is that delegating certification enables the regulator to credibly diminish information acquisition, which in turn can boost information disclosure from the part of the candidates.

Our paper builds on recent theoretical advances on Bayesian persuasion and makes a methodological contribution, by proposing a tractable model combining information disclosure by candidates and tests conducted by the certifier. This model could prove useful for a range of applications. For instance, versions of the model could be used to explore the hiring processes of firms, the optimal design of stress tests by central banks, or even to study patenting.
Appendix A: Analysis of the Discretionary Framework

In this appendix, we examine the set of equilibria of the discretionary framework.

**Lemma 5.** Given any pair of disclosure rules from the candidate, $E[\Phi|\mu] \geq p$ for $\mu \in (1 - p + c, p - c)$. Moreover, in that belief range the certifier chooses to run the test.\(^{31}\)

**Proof:** First, the minimum expected payoff of the certifier from conducting the test is $p - c$, since the certifier can always blindly follow the outcome of the test, and the test reveals $\omega$ with probability $p$.

Second, the maximum expected payoff of the certifier without conducting the test is $\max\{1 - \mu, \mu\}$. Hence, for $\mu \in (1 - p + c, p - c)$, conducting the test is always best. The result of the lemma is now immediate since, conditional on running the test, $E[\Phi|\mu] \geq p$.

---

**Lemma 6.** Given any pair of disclosure rules from the candidate,

$$E[a|\mu] = 2\mu E[a|\mu, \omega = 1] + 1 - \mu - E[\Phi|\mu]$$

**Proof:** We have

$$E[a|\mu] = E[a|\mu, \omega = 1]P(\omega = 1|\mu) + E[a|\mu, \omega = 0]P(\omega = 0|\mu)$$
$$= \mu E[a|\mu, \omega = 1] + (1 - \mu) E[a|\mu, \omega = 0],$$

and

$$E[\Phi|\mu] = E[\Phi|\mu, \omega = 1]P(\omega = 1|\mu) + E[\Phi|\mu, \omega = 0]P(\omega = 0|\mu)$$
$$= \mu E[a|\mu, \omega = 1] + (1 - \mu)(1 - E[a|\mu, \omega = 0]).$$

Hence

$$(1 - \mu) E[a|\mu, \omega = 0] = \mu E[a|\mu, \omega = 1] + 1 - \mu - E[\Phi|\mu].$$

Plugging the last equation into the first yields the lemma.\(^{31}\)

---

\(^{31}\)Here and throughout this appendix we simplify notation by omitting the dependence of continuation payoffs on the candidate’s disclosure rule after the test.
In what follows define

\[ V(t) := \begin{cases} 
0 & \text{for } t \in [0, 1 - p + c) \\
1 - p + t - c_b & \text{for } t \in [1 - p + c, p - c) \\
1 & \text{for } t \in [p - c, 1]. 
\end{cases} \]

**Lemma 7.** Given any pair of disclosure rules from the candidate, \( \mathbb{E}[a - c_b e | \mu] \leq V(\mu) \) for \( \mu \in [1 - p + c, 1] \).

**Proof:** The result is obvious for \( \mu \in [p - c, 1] \), since in that case \( V(\mu) = 1 \). For \( \mu \in (1 - p + c, p - c) \), the result follows from combining Lemma 5 and Lemma 6. Finally, for \( \mu = 1 - p + c \), either the certifier rejects the candidate outright or the certifier chooses to conduct the test. In the former case, the inequality is trivial since \( \mathbb{E}[a - c_b e | \mu = 1 - p + c] = 0 \). In the latter case, the results from Lemmas 5 and 6 extend and can again be applied to yield the desired inequality.

**Lemma 8.** In equilibrium, for \( \mu \in [0, 1 - p + c] \):

\[ \mathbb{E}[a - c_b e | \mu] \leq \frac{\mu}{1 - p + c} V(1 - p + c) = \frac{\mu}{1 - p + c} (2(1 - p) + c - c_b). \]

**Proof:** If given belief \( \mu \) the certifier rejects outright then \( \mathbb{E}[a - c_b e | \mu] = 0 \), and the statement of the lemma holds trivially.

Next, suppose that given belief \( \mu \) the certifier conducts the test. As the certifier could obtain \( 1 - \mu \) from rejecting outright, we must then have

\[ \mathbb{E}[\Phi | \mu] - c \geq 1 - \mu. \]

Hence, Lemma 6 gives

\[ \mathbb{E}[a | \mu] \leq 2\mu - c. \]

The result of the lemma ensues, since

\[ (2\mu - c - c_b)(1 - p + c) \leq \mu(2(1 - p) + c - c_b) \iff (c + c_b)\mu \leq (c + c_b)(1 - p + c), \]

and we assumed \( \mu \leq 1 - p + c \).
Henceforth, let cav $V$ denote the concavification of $V$.

**Lemma 9.** In equilibrium, $\mathbb{E}[a - c_b e|\lambda] \leq \text{cav} V(\lambda)$.

**Proof:** We have $\mathbb{E}[a - c_b e|\lambda] \leq \text{cav} \mathbb{E}[a - c_b e|\mu]$. Furthermore, Lemma 7 gives $\mathbb{E}[a - c_b e|\mu] \leq V(\mu)$ for $\mu \in [1 - p + c, 1]$, while Lemma 8 gives $\mathbb{E}[a - c_b e|\mu] \leq \frac{\mu}{1 - p + c} V(1 - p + c) + (1 - \frac{\mu}{1 - p + c})V(0) \leq \text{cav} V(\mu)$ for $\mu \in [0, 1 - p + c]$. Hence, we obtain $\text{cav} \mathbb{E}[a - c_b e|\mu] \leq \text{cav} V(\mu)$ and, ultimately, $\mathbb{E}[a - c_b e|\lambda] \leq \text{cav} V(\lambda)$.

**Lemma 10.** Either

$$(1 - p + c) \geq (p - c)[2(1 - p) + c - c_b],$$

in which case $\text{cav} V(\lambda) = \min \{\frac{\lambda}{p - c}, 1\}$, or

$$\text{cav} V(\lambda) = \begin{cases} 
\frac{\lambda}{1 - p + c} V(1 - p + c) & \text{for } \lambda \in [0, 1 - p + c] \\
\frac{\lambda - (1 - p + c)}{2(p - c) - 1} + \frac{(p - c) - \lambda}{2(p - c) - 1} V(1 - p + c) & \text{for } \mu \in [1 - p + c, p - c] \\
1 & \text{for } \mu \in [p - c, 1],
\end{cases}$$

**Proof:** Immediate from the definition of $V$, and noting that $V(1 - p + c) \leq \frac{1 - p + c}{p - c}$ if and only if (1) holds.

**Lemma 11.** For all $\epsilon > 0$, there exists a pair of disclosure rules from the candidate such that $\mathbb{E}[a - c_b e|\lambda] \geq \text{cav} V(\lambda) - \epsilon$.

**Proof:** We consider two cases separately: Case 1 where (1) holds, and Case 2 where it does not.

Start with Case 1, and consider the following disclosure rules from the candidate, where
η > 0:

\[
\begin{cases}
\tau^\nu_2 & \text{is uninformative if } \nu = 0 \text{ or } \nu \geq \frac{1}{2}, \\
\tau^\nu_2 & \text{splits } \nu \text{ on } 0 \text{ and } \frac{1}{2} \text{ if } \nu \in (0, \frac{1}{2}).
\end{cases}
\]

\[
\begin{cases}
\tau^\lambda_1 & \text{splits } \lambda \text{ on } 0 \text{ and } \lambda \leq \frac{1}{2} \text{ if } \lambda \in (0, \frac{1}{2}), \\
\tau^\lambda_1 & \text{splits } \lambda \text{ on } 0 \text{ and } \frac{1}{2} \text{ if } \lambda \in (0, \frac{1}{2}).
\end{cases}
\]

Case 2 is treated similarly. Choose

\[
\begin{cases}
\tau^\nu_2 & \text{is uninformative if } \nu = 0 \text{ or } \nu \geq \frac{1}{2} + \eta, \\
\tau^\nu_2 & \text{splits } \nu \text{ on } 0 \text{ and } \frac{1}{2} \text{ if } \nu \in (0, \frac{1}{2} + \eta).
\end{cases}
\]

\[
\begin{cases}
\tau^\lambda_1 & \text{splits } \lambda \text{ on } 0 \text{ and } 1 - \frac{1}{2} + \eta \text{ if } \lambda \in (0, 1 - \frac{1}{2} + \eta), \\
\tau^\lambda_1 & \text{splits } \lambda \text{ on } 1 - \frac{1}{2} + \eta \text{ and } \frac{1}{2} + \eta \text{ if } \lambda \in (1 - \frac{1}{2} + \eta, \frac{1}{2} + \eta), \\
\tau^\lambda_1 & \text{is uninformative if } \lambda = 0, \lambda = 1 - \frac{1}{2} + \eta, \text{ or } \lambda \geq \frac{1}{2} + \eta.
\end{cases}
\]

Combining the definition of \( V(\cdot) \) with Lemmas 5 and 6 now yields the result.

Proposition 6. There exists an equilibrium. Moreover, in any equilibrium, \( \mathbb{E}[a - c_e|\lambda] = cav V(\lambda) \).

Proof: The second part of the lemma follows from Lemma 11. So we only need to show that a pair of disclosure rules exists, together with a best response for the certifier, such that the candidate achieves expected payoff \( \mathbb{E}[a - c_e|\lambda] = cav V(\lambda) \). Just choose, e.g., \( \{\tau^x_1\}_{x \in [0,1]} \) and \( \{\tau^x_2\}_{x \in [0,1]} \) as in Lemma 11 but with \( \eta = \eta' = 0 \), together with the following best response for
the certifier: (a) reject if $\mu < 1 - p + c$, (b) conduct the test if $\mu \in [1 - p + c, p - c)$, (c) certify if $\mu \geq p - c$.

Corollary 1. Unless (4) holds with equality, in any equilibrium, the certifier (on path):

1. conducts the test if $\mu = 1 - p + c$,
2. certifies outright if $\mu = p - c$.

Moreover, if (4) is a strict inequality then the candidate splits $\lambda$ on 0 and $p - c$ for all $\lambda \in (0, p - c)$; if (4) is violated then the candidate:

1. splits $\lambda$ on 0 and $1 - p + c$ if $\lambda \in (0, 1 - p + c)$,
2. splits $\lambda$ on $1 - p + c$ and $p - c$ if $\lambda \in (1 - p + c, p - c)$.

Proof: Immediate from combining Proposition 6 and Lemma 10.

Lemma 12. In equilibrium, $\mathbb{E}[a|\lambda, \omega = 1] = 1$, except possibly given a set of $\lambda$’s with measure zero.

Proof: First, Suppose we can find an equilibrium and $\lambda > 0$ such that $\mathbb{E}[a|\lambda, \omega = 1] < 1$. Then either (a) or (b) below holds: (a) the candidate splits $\lambda$ on $\mu_1$, ..., $\mu_k$ and the certifier rejects the candidate for some $\mu_j > 0$; (b) there exists $\nu$ (occurring with strictly positive probability on the equilibrium path) such that the candidate splits $\nu$ on $\xi_1$, ..., $\xi_k$ and the certifier rejects the candidate for some $\xi_j > 0$.

In case (a), the candidate can always redistribute the probability mass allocated on $\mu_j$ to the posterior beliefs 0 and 1. As $\mu_j > 0$, doing this would strictly improve the candidate’s expected payoff, contradicting equilibrium. Case (b) is treated similarly.

Lemma 13. In equilibrium,

$$\mathbb{E}[\Phi|\lambda] = 1 + \lambda - \mathbb{E}[a|\lambda],$$

except possibly given a set of $\lambda$’s with measure zero.
Proof: Combining Lemma 6 and Lemma 12 shows that in any equilibrium:

\[ E[a|\mu] = 1 + \mu - E[\chi|\mu]. \]

Integrating over \( \mu \) for given \( \lambda \) and rearranging yields the lemma.

Lemma 14. Let \( f(p, c) \) denote the maximum ex ante certification quality. Either the mandatory framework is ex ante optimal, in which case \( f(p, c) = (1 - p)^2 + p \), or the discretionary framework is ex ante optimal, in which case:

- \[ f(p, c) := \begin{cases} 
\frac{c}{2} \left[ 2 - p - 2(p - c) \right] + (1 - p)^2 + p & \text{for } c \leq \tilde{c}(p) \\
3p - 1 - 2p^2 - c + (1 - p)^2 + p & \text{for } c \geq \tilde{c}(p) 
\end{cases} \]

- \( \tilde{c}(p) \) is defined implicitly by \( p(\tilde{c}(p)) = p \).

Furthermore, there exists \( \hat{p} \) such that for all \( p < \hat{p} \): \( \arg \max_c f(p, c) = \tilde{c}(p) \).

Proof: The first part of the lemma follows immediately from the analysis of the mandatory and discretionary frameworks.

For the second part, observe that (a) in the mandatory framework ex ante certification quality is independent of \( c \) and (b) for \( c = 0 \) ex ante certification quality is the same in the mandatory and discretionary frameworks. The first part of the lemma thus yields the second part.

Appendix B: Proofs of Results in the Text

Proof of Lemma 14: Note first that in the mandatory framework the candidate cannot gain from disclosing information using the first disclosure rule, since any information disclosed before the test could be disclosed after it. So, without loss of generality, suppose the candidate only discloses information after the certifier runs the test.
The continuation game after the test outcome corresponds to the basic example analyzed in Kamenica and Gentzkow (2011). In particular, in any equilibrium: \( \mathbb{E}[\Phi | \nu] = \max\{1 - \nu, \nu\} \). Hence the certifier does not benefit from information disclosed by the candidate. Lemma 1 ensues.

**Proof of Lemma 2:** This is Corollary 1 in Appendix A. The thresholds \( p(c, c_b) \) and \( \overline{p}(c, c_b) \) correspond to the roots of the second order polynomial obtained from taking equality in (1).

**Proof of Lemma 3:** The proof has two parts. We first express the equilibrium probability that the candidate is certified. We then deduce from this the interim certification quality.

If (1) is a strict inequality, then combining Lemma 10, Proposition 6 and Corollary 1 shows that, in any equilibrium, \( \mathbb{E}[a | \lambda] = \min\{\frac{\lambda}{p-c}, 1\} \). If (1) is violated, then combining the same results shows that, in any equilibrium:

\[
\mathbb{E}[a | \lambda] = \begin{cases} 
\frac{\lambda}{1 - p + c} (2(1 - p) + c) & \text{for } \lambda \in [0, 1 - p + c] \\
\frac{\lambda - (1 - p + c)}{2(p - c) - 1} + \frac{(p - c) - \lambda}{2(p - c) - 1} (2(1 - p) + c) & \text{for } \lambda \in [1 - p + c, p - c] \\
1 & \text{for } \lambda \in [p - c, 1],
\end{cases}
\]

Lemma 3 now follows from Lemma 13 and straightforward algebraic manipulations.

**Proof of Proposition 1:** Immediate from Lemmas 1 and 3.

**Proof of Proposition 2:** Comparing the areas highlighted on Figure 8 shows that, for \( p \in (p, \overline{p}) \), the dark gray area is greater than the light gray area if and only if \( p < \frac{2(1+c)}{3} \). For \( p < p \) or \( p > \overline{p} \), the condition becomes

\[
\int_{0}^{p-c} \left(1 + \lambda - \frac{\lambda}{p - c}\right) d\lambda + \int_{p-c}^{p} \lambda d\lambda > \int_{0}^{1-p} (1 - \lambda) d\lambda + \int_{1-p}^{p} p d\lambda.
\]
This simplifies to $1 + 2p^2 - 3p + c < 0$. 

Proof of Proposition 3: By corollary 1 in equilibrium, for $p$ just below $\bar{p}$ (which we denote $p = p^-$), the candidate uses an uninformative information structure given $\lambda = 1 - p + c$, whereas for $p$ just above $\bar{p}$ (which we denote $p = p^+$), the candidate splits $\lambda = 1 - p + c$ on 0 and $p - c$. In the first case, the certifier runs the test with probability 1; in the second case the probability that the certifier runs the test is 0. Hence:

$$\mathbb{E}_{p=p^-}[a_i - c_b e_i | \lambda = 1 - p + c] = \mathbb{E}_{p=p^+}[a_i | \lambda = 1 - p + c] - c_b,$$

whereas

$$\mathbb{E}_{p=p^+}[a_i - c_b e_i | \lambda = 1 - p + c] = \mathbb{E}_{p=p^+}[a_i | \lambda = 1 - p + c].$$

By choosing $p^-$ and $p^+$ close enough to one another, $\mathbb{E}_{p=p^-}[a_i - c_b e_i | \lambda = 1 - p + c]$ and $\mathbb{E}_{p=p^+}[a_i - c_b e_i | \lambda = 1 - p + c]$ can be made arbitrarily close. Hence, for $c_b > 0$, by choosing $p^-$ and $p^+$ close enough we obtain

$$\mathbb{E}_{p=p^-}[a_i | \lambda = 1 - p + c] < \mathbb{E}_{p=p^+}[a_i | \lambda = 1 - p + c].$$

Lemma 13 now shows that $\mathbb{E}_p[\Phi | \lambda = 1 - p + c]$ drops as $p$ crosses $\bar{p}$ from below.

Proof of Proposition 4: One checks that $\lim_{c \to 0} p = \frac{1}{2}$, whereas $\lim_{c \to 0} \bar{p} = 1$. Hence, for all $p \in (\frac{1}{2}, 1)$: $p \in (\bar{p}, \bar{p})$ if $c$ is small enough. Hence, Lemma 3 yields $\mathbb{E}[^{\Phi | \lambda}] \to_{c \to 0} \max\{1 - \lambda, p, \lambda\}$ in the discretionary framework. In other words, the interim certification quality in the discretionary framework converges to the interim certification quality in the mandatory framework as $c$ tends to 0.

Now fix $c > 0$, and consider the setting without delegation. Recall that the (not approximated) payoffs of the regulator are $(N+1)(\Phi - \frac{c + c_0}{N+1} \epsilon + \frac{1}{N+1} a)$. The setting without delegation is thus equivalent to the discretionary framework of the baseline model with $c' = \frac{c + c_0}{N+1}$ (and a small bias in favor of certification). As $N$ tends to infinity, $c'$ tends to 0. Hence, the arguments of the first paragraph above show that the certification quality without delegation tends to the certification quality in the mandatory framework of the baseline model. Proposition 4 now follows from Proposition 2.
Proof of Proposition 5: The proof consists in showing that for $\kappa = 0$ the regulator can do better than any transfer scheme fully subsidizing the test. The same must then evidently be true for all $\kappa > 0$.

Using notation from Lemma 14, let $\tilde{c}(p)$ maximize $f(p,c)$ (i.e., given $p$, $\tilde{c}(p)$ maximizes the maximum ex ante certification quality of the baseline model).

Factorizing gives

$$u = (1 + t\Phi)\left(\Phi - \frac{c - t_e}{1 + t\Phi}e\right) + t_0 = (1 + t\Phi)(\Phi - c'e) + t_0,$$

where $c' = \frac{c - t_e}{1 + t\Phi}$. Next, choose $t\Phi = 0$, $c - t_e = \tilde{c}(p)$, and $t_0$ such that the (ex ante) participation constraint of the certifier binds. The proof is concluded by noting that $\tilde{c}(p) > 0$ for all $p < \hat{p}$, by Lemma 14.
References


