Price-Cap Regulation of Firms That Supply Their Rivals*

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Abstract

We study the effects of price-cap regulation in a market in which a vertically integrated upstream monopolist sells a requisite input to a downstream competitor. In the absence of regulation, entry benefits both firms, but may be detrimental to (downstream) consumers because the upstream monopolist can set a high input price that pushes downstream prices above the monopoly level. However, if a regulator imposes price caps that constrain the incumbent’s upstream and downstream prices, consumers—as well as both firms—benefit from entry. Using a dynamic extension of the model, we also explore the concern that price caps may induce incumbents to forgo cost-reducing investments.

Keywords: Bertrand Competition, Foreclosure, Innovation, Outsourcing, Price Cap, Wholesale Pricing

JEL Classification Codes: D43, L13, L50

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1 Introduction

We study markets in which a vertically-integrated incumbent firm sells essential inputs of production to unintegrated, entrant rivals who engage in differentiated product competition with the incumbent in a downstream setting. Examples of such markets include food and beverages, fuel refining, mobile phones, semiconductors, video content distribution (Arya et al. 2008, Moresi and Schwartz 2015, Koning and Yankelevich 2016), and as we expound in much greater detail below, telecommunications. As we show in this manuscript, under fairly standard assumptions, when these markets are left unregulated, whereas entry is beneficial to firms, it will always lead to “price-increasing competition,” (Chen and Riordan 2008) whereby downstream customers pay higher prices than if no entry had occurred. The effect on consumer and total welfare is ambiguous and depends on the tradeoff between higher prices and greater variety that consumers face.

In certain industries, recognition of some of the potential anti-competitive effects of markets where rivals outsource production—monopoly absent entry; foreclosure concerns when entry is possible—has led government regulators to step in. This manuscript is motivated by price-cap regulation as it occurs in the telecommunications sector in such markets, with particular focus on high-capacity Internet broadband capable connections used by downstream businesses. In many parts of the United States, these services are subject to price-cap regulation whereby vertically integrated incumbent wholesale (or input) as well as downstream retail prices are subject to price caps, whereas entrant services are not. Intuitively, such a regime halts price-increasing competition by forbidding a previously capped monopolist to raise its price. Moreover, we find that in this case, both firms and downstream consumers are better off following entry than they would be in the presence of a price capped monopoly.

However, two major concerns can arise when markets are capped: incentives for regulated firms to foreclose entry (e.g., via margin squeeze) and disincentives for regulated firms to invest. Interestingly, we find that by regulating both the upstream and downstream price of the incumbent, while avoiding regulating the entrant, U.S. regulators at least in principal,
quash incentives to foreclose because the regulated incumbent’s only means to earn above zero profit is through sales to the entrant. This static efficiency notwithstanding, it could be argued that regulation can discourage long-term investment. The idea is that if a regulated incumbent now relies on entry for its exclusive source of profits, such an incumbent may be less interested in marginal cost reductions for their own services than if they remained unregulated. As we show, under fairly standard assumptions, this intuition turns out to be incomplete and regulated incumbents invest more in equilibrium than they would sans regulation. Thus, price-cap regulation in the U.S. telecommunications sector is rightfully referred to as incentive regulation: the incentive referring to regulators’ intent to induce incumbents to invest in cost reducing technology.

1.1 Institutional Background

This manuscript was initially motivated by the authors’ work to better understand existing regulation in the market for business data services at the Federal Communications Commission (FCC). Business data services (known as “special access” until 2016) lines are dedicated high-capacity connections used by businesses and institutions to transmit voice and data traffic. Typical uses include the connection of different business units within large enterprises, wireless backhaul, and importantly, wholesale access services. Because competition in the provision of business data services is limited in the United States, these services have a history of regulation by the FCC, initially on a “cost-plus” basis, and according to price-cap “incentive” regulation since the early 1990s. The market for business data services has grown from approximately $2.6 billion in revenue when the FCC first implemented price-cap regulation to a market that may exceed $75 billion in revenue today (Rysman 2016).

One notable feature of U.S. markets for business data services is the presence of competitive local exchange carriers (“CLECs”) that obtain business data services on a wholesale basis to compete with incumbent local exchange carriers (“ILECs”) downstream. Competition be-

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1Yankelevich was previously affiliated with the FCC.
tween the ILECs and CLECs in the market for business data services has frequently been under FCC scrutiny and was most recently the subject of a May 2016 Order and Further Notice of Proposed Rulemaking that, among other things, sought to adopt a technology-neutral framework that could extend price-cap regulation to IP-based business data services in parts of the U.S. that the FCC deems as not competitive (FCC 2016). In light of the FCC’s plan to continue to apply price-cap regulation, and possibly to extend price-cap regulation to a presently unregulated service, it is important to understand how FCC-style price caps are likely to impact welfare, both in the short run as well as in the long run.

Although the FCC has broadly simplified its incentive regulation methodology from what was initially effectively a hybrid of price-cap and rate-of-return regulation, the category of business data services has been subject to particular attention from the FCC since 2000, and its regulation via price caps remains an evolving and highly contested issue.  

In 2000, special access was separated from the FCC’s trunking basket and began to be regulated under its own price cap rate adjustment mechanism (X-Factor) separate from other price-cap regulated services (see FCC 2000 ¶¶30, 132, 149, 150). In 2005, special access regulation was subjected to an individual rule making proceeding (see FCC 2005), with additional proceedings initiated exclusively for different aspects of business data services regulation in 2015 and 2016. Among other things, the special access proceedings initiated an extensive data collection undertaken to reevaluate competition and price-cap regulation in the business data services market (see FCC 2012a, 2012b, 2013, 2014), an investigation of certain business data services pricing practices by ILECs (FCC 2015), and plans to adopt a technology-neutral framework that could extend price-cap regulation to IP-based business data services (FCC 2016). As recently articulated in a USTelecom-funded study undertaken by Hal Singer, notwithstanding its static impact on welfare, price-cap regulation can have

\[^2\]For example, in its 1997 Price Cap Performance Review Order, the FCC eliminated a “sharing” regime (implemented as part of its initial 1990 LEC Price Cap Order and further developed in its 1995 Price Cap Review Order) whereby profits that were substantially above the FCC’s pre-1990s permitted rate of return had to be shared in part or in whole with customers in spite of the fact that prices were bound by price cap indices (see FCC 1990 ¶¶5, 7, 123-125; 1995 ¶¶19-20; and 1997 ¶1).
major implications for network investment (Singer 2016).³

Although Singer (2016) finds a potential detrimental effect of expanded FCC price-cap regulation to investment in IP-based business data services via a nationwide extrapolation of a geospatial counterfactual study of Ethernet expansion in the Charlotte, North Carolina market, a crucial current aspect of FCC price-cap regulation is regulatory ("Phase II") relief from price-cap regulation in areas deemed sufficiently competitive (e.g., see FCC 2012a ¶¶23-25). Ironically, the Charlotte-Gastonia, NC market was granted regulatory relief in 2000 (FCC 2012a Appendix D). Although additional grants of regulatory relief were suspended in 2012 pending the FCC’s reevaluation of the business data services market, the FCC’s 2016 Further Notice of Proposed Rulemaking suggests that “where competition is sufficient in a relevant market [the FCC would rely] upon market forces to constrain rates, terms, and conditions” (FCC 2016 ¶270). We read this as applying to both historically regulated time-division multiplexing (TDM) services and presently unregulated IP-based services.

Our primary research interest relates to those markets expected to be deemed insufficiently competitive so as to possibly warrant regulation under the FCC’s price caps. Such markets would be expected to include those with a single vertically integrated incumbent with potential downstream wholesale competitors, but no existing facilities-based (i.e., vertically integrated) competitors—that is, markets where rival entrants are forced to outsource. Various studies submitted to the FCC pursuant to its 2016 proceeding suggest that a substantial geographic portion of the United States consists of such markets (FCC 2016 ¶¶173-185). Our “static” analysis is intended to gauge the welfare impact of wholesale entry in such monopoly markets with and without price-cap regulation, and our “dynamic” investment analysis seeks to assess incentives for further investment by price-cap regulated firms. Although our research is largely motivated by the market for business data services, because the core of our research is theoretical, our findings are applicable to regulated and unregulated markets where firms supply their rivals more broadly.

³USTelecom is the nation’s leading trade association representing broadband service providers and suppliers. See http://www.ustelecom.org/.
1.2 Related Literature

Our research is broadly related to the literature on one-way access pricing in network industries (e.g., Armstrong et al. 1996, Sappington 2005). Unlike in the framework of Armstrong et al. (1996), we suppose that the entrant maintains some market power via its product differentiation in the downstream market (that is, in light of the switching costs inherent in the business data services market—see for instance FCC 2016 ¶198—we do not view CLECs as fringe competitors). From conversations with network engineers, the authors have learned that a major differentiator between different business data service is in the quality of support that customers face in light of network congestion or outages. Other differences that apply across many industries include brand, quality, and consumer tastes. Moreover, we assume that the entrant does not have the alternative to produce the essential upstream input as in Sappington (2005). More importantly, in contrast to Sappington, we do not rely on full displacement in competition, which arises distinctly within the Hotelling framework.

Our model is most closely related to the frameworks of Arya et al. (2008) and Moresi and Schwartz (2015). Arya et al. (2008) is primarily concerned with how results in markets where firms sell to their rivals compare under Bertrand and Cournot competition downstream as well as when and how firms would want to vertically integrate. Moresi and Schwartz focus on the idea that under various circumstances, a vertically integrated incumbent would wish to induce expansion by the rival it is supplying. They also show how delegation could foster such expansion. In contrast to these works, our focus is on price-cap regulation of such markets. Moreover, neither Arya et al. (2008) nor Moresi and Schwartz (2015) compare the potential welfare outcomes of wholesale entry to those under a single product monopoly, an important consideration for the business data service market. The potential for price-increasing competition, which such a comparison highlights, lends support to the case for

\footnote{Related to this is a strand of literature on vertical integration by a dominant or incumbent firm that ends up selling to rivals that compete with it on quantity downstream. See Riordan (1998), Loertscher and Reisinger (2014), Reisinger and Tarantino (2015).}
price-cap regulation.\textsuperscript{5} In one part of their work, Koning and Yankelevich (2016) are also interested in caps in markets where firms supply their rivals, but their focus is on data caps set by vertically integrated Internet Service Providers, which they liken to an ability for a vertically integrated incumbent to cap its rival’s quantity. This is quite different from the type of price-cap regulation considered here.

Bourreau et al. (2011) consider a framework similar to that of Arya et al., but with two vertically integrated firms competing with a single downstream rival who can purchase from either firm. The focus is on scenarios in which competition between the integrated firms nevertheless leads to above-marginal-cost input pricing. Another recent related article by Mandy et al. (2016) considers a vertically integrated firm that can sabotage either of two downstream competitors that purchase from the integrated firm at a regulated price. However, the focus of that work is on the optimal method of sabotage and not on the endogenous determination or means of regulation of the input price.

Our theoretical model is also related to the literature on margin squeeze, a recent overview of which is provided by Jullien et al. (2014). A margin squeeze may occur in a setting where firms supply their rivals when the vertically integrated incumbent adopts retail and wholesale prices that together make downstream competition unprofitable even when downstream entry would enhance welfare. Bouckaert and Verboven (2004) study margin-squeeze regulation in a setting with homogenous downstream competition and propose various margin squeeze tests. More recently, Petulowa and Saavedra (2014) study margin-squeeze regulation in a framework very similar to ours, focusing attention on the European Union’s margin-squeeze test, in which an incumbent that sets its price below the sum of its wholesale price and downstream cost is found to be in violation of the test. Notably, the authors find that an unregulated vertically integrated incumbent that is at least as efficient as the downstream competitor—as we believe is often true in markets for business data services—will not engage

\textsuperscript{5}We note that our interest here is in whether price-cap regulation could be potentially welfare improving, and not on the regulatory mechanism used to maintain such regulation. Discussions of the latter topic are found in Braeutigam and Panzar (1993), Bernstein and Sappington (1999), Uri (1999), and Laffont and Tirole (2001).
in a margin squeeze.

Crucially, other differences aside, only a few of the works cited above consider investment incentives by firms that sell to rivals, and none are concerned with the impact of price-cap regulation on investment, which is one of the primary focuses of this work. A number of empirical peer-reviewed publications do consider the impact of regulation on investment in telecommunications, though the focus is generally not on price-cap regulation of the business data services market.

Previous empirical work generally focuses on the investment impact of mandatory unbundling requirements imposed on ILECs by the Telecommunications Act of 1996. The evidence is mixed. Using a case-based approach, Hausman and Sidak (2005) suggest that, in the United States, mandatory unbundling requirements did not appear either to decrease prices or to increase investment by either incumbents or CLECs that were able to share the incumbent competitors’ facilities. Similarly, Crandall et al. (2004) found that U.S. states with lower rental rates for unbundled network elements had lower relative levels of facilities-based investment by CLECs, whereas Chang et al. (2003) found that lower rates were associated with higher levels of ILEC investment. Moreover, in a study submitted to the FCC, Willig et al. (2002) found that lower rates corresponded with higher levels of ILEC and CLEC investment, and that CLEC investment reinforces ILEC investment. Finally, Grajec and Röller (2012) examine the potential trade-off between access regulation and investment in telecommunications using a data set covering over 70 fixed-line operators in 20 EU member states. Access regulation is found to discourage investment by incumbents, but to increase entrants’ total investment.

The remainder of this manuscript is organized as follows. Section 2 develops a model of duopoly competition in a market in which a vertically integrated firm sells to a downstream rival. Section 3 uses the model to derive short-run theoretical predictions in such a market with and without price-cap regulation. Section 4 considers the potential impact of price-cap regulation.

6The regulatory variables include the existence of accounting separation obligation, regulation regarding full unbundling, line sharing, bitstream access, and subloop unbundling of fixed-line incumbents local loops.
regulation on long-run investment incentives. Section 5 concludes. The Appendix contains formal proofs.

2 Baseline Model

Our model follows the framework of Arya et al. (2008). An incumbent, \( I \), is the sole producer of both a downstream consumer product and an essential input for that product. There is also a potential entrant, \( E \), in the consumer product market. Because the input is essential for production, \( E \) can enter the downstream market only if it purchases from \( I \) in the upstream market. Each unit of downstream output requires exactly one unit of upstream input. \( I \) produces the input at zero cost and sets a fixed, nonnegotiable unit price, denoted by \( w \), for it. In addition to the cost (which is zero for \( I \)) of obtaining the upstream input, each firm \( i \in \{I, E\} \) faces a constant marginal cost of \( c_i \geq 0 \) per unit of downstream output sold.\(^7\) In this baseline model, we abstract away from fixed costs, treating them as zero.\(^8\)

The representative consumer obtains the following surplus from consuming quantities \( q_I \) and \( q_E \) of retail service from firms \( I \) and \( E \), respectively, at unit prices \( p_I > 0 \) and \( p_E > 0 \):\(^9\)

\[
U(q_I, q_E; p_I, p_E) \equiv \alpha (q_I + q_E) - \left(\frac{q_I^2 + q_E^2}{2}\right) - \beta q_I q_E - p_I q_I - p_E q_E. \tag{1}
\]

In Equation (1), we assume that \( \max \{c_I, c_E\} < \alpha \) to rule out cases in which the provision of retail service by either firm is trivially inefficient and \( 0 < \beta < 1 \), which implies that the two marginal costs are different.

\(^7\)The marginal cost \( c_i \) includes downstream production and retail costs. We follow Arya et al. (2008) in normalizing the marginal cost of the input to zero. With respect to the motivating application of business data services, one may view the marginal costs associated with supplying the business data services market as comprising two components: (1) routine activities in the provision of service; and (2) activities associated with sales and customer satisfaction. The former component arguably applies equally to upstream wholesale access and downstream retail service and may be normalized, and the latter should be expected to be more costly in a downstream retail setting, where the incumbent might have to deal with multiple small buyers instead of a single wholesale buyer. One, therefore, might interpret \( c_I \) as the difference between the marginal cost of retail and wholesale sales activities. See Armstrong et al. (1996) for discussion of a somewhat more flexible cost structure.

\(^8\)Because \( I \) is, by assumption, already operating, its fixed cost is sunk and can be treated as zero. As a non-facilities-based entrant, \( E \) has a fixed cost that reasonably can be treated as negligible.

\(^9\)In the business data service context, we view the downstream consumer as a large enterprise that might choose to diversify its purchases among the two competitors.
firms’ retail services are imperfect substitutes and that own-effects dominate cross-effects. This specification induces a linear demand structure, in which the inverse demand for firm \( i \neq j \in \{ I, E \} \) is given by \( p_i = \alpha - q_i - \beta q_j \) in the region, \( Q \subset \mathbb{R}_+^2 \) of the quantity space in which prices are positive. Solving the system of linear equations characterized by the inverse demand equations yields the direct demands for the two firms’ downstream products in the region, \( P \subset \mathbb{R}_{++}^2 \) of the price space in which quantities are positive:

\[
q_I = \frac{\alpha}{1 + \beta} - \frac{p_I - \beta p_E}{1 - \beta^2},
\]

\[
q_E = \frac{\alpha}{1 + \beta} - \frac{p_E - \beta p_I}{1 - \beta^2}.
\]

Firms \( I \) and \( E \) play the following extensive form game:

1. \( I \) announces the input price, \( w \).

2. \( E \) decides whether to enter the downstream market.

3. If \( E \) enters, then \( I \) and \( E \) engage in differentiated Bertrand competition in the downstream market. Otherwise, \( I \) operates as a monopolist in the downstream market.

Note that the firms’ profits can be written as

\[
\pi_I \equiv wq_E + (p_I - c_I)q_I,
\]

\[
\pi_E \equiv (p_E - w - c_E)q_E.
\]

Given production levels \((q_I, q_E) \in Q\), consumer surplus can be computed as

\[
CS \equiv U(q_I, q_E; \alpha - q_I - \beta q_E, \alpha - q_E - \beta q_I) = \frac{(q_I^2 + q_E^2)}{2} + \beta q_I q_E.
\]

When \( E \) does not enter, \( CS \) reduces to \( q_I^2/2 \). Total welfare is defined as \( W \equiv CS + \pi_I + \pi_E \).

For notational convenience, following Arya et al. (2008), we define “value margin” \( \alpha_i \) as \( \alpha_i \equiv \alpha - c_i \) for each \( i \in \{ I, E \} \).

We will apply the standard solution concept—subgame-perfect Nash equilibrium—for games in this class.
3 Theoretical Short-Run Predictions

3.1 Unregulated Equilibrium

We first consider, as a benchmark, the case without price caps. Arya et al. (2008) show that, under both Bertrand and Cournot retail competition, the incumbent will foreclose the entrant if and only if $\alpha_E/\alpha_I \leq \beta$. Note that the left-hand side term of this foreclosure condition represents the relative efficiency of the entrant’s provision of the downstream product compared to the incumbent’s provision of the product, whereas the term on the right-hand side represents the degree to which the two firms’ downstream products are substitutes. In particular, foreclosure occurs if the entrant’s provision of the product is relatively inefficient, or if the two firms’ products are close substitutes.

In their analysis, Arya et al. (2008) generally assume that $a_I \geq a_E$ (i.e., that the incumbent is at least as efficient a provider as the potential entrant), which, in the case of business data services, appears to be realistic. Although we will emphasize this case as well, for completeness we also consider the complementary case of a more efficient entrant. It is worth noting, in particular, that the incumbent will never foreclose a more efficient entrant, because, in that case, we would have $\alpha_E/\alpha_I > 1 > \beta$.

One striking result is that the incumbent’s price under duopoly is higher than that under monopoly. Furthermore, for intermediate levels of the entrant’s efficiency relative to the incumbent’s (and even for a fairly efficient entrant if downstream products are not highly differentiated), consumer surplus is higher under foreclosure than under entry. A simple revealed-preference argument demonstrates that neither firm is worse off following entry than under foreclosure. The impact of entry on total welfare is, therefore, ambiguous. The above insights are stated formally in Proposition 1 and are illustrated in Figure 1.

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10It seems plausible that ILECs typically have advantages in technical and business expertise over CLECs.
11For the entrant, declining to enter the market leads to zero profit, so entry implies that the entrant expects to earn nonnegative profit: at least as much as under foreclosure. For the incumbent, because foreclosure (by setting the input price to an exorbitant level) is always an available option, entry indicates that the incumbent expects to earn weakly higher profit under entry than under foreclosure.
Proposition 1. Suppose that $\alpha_E/\alpha_I > \beta$. Then entry occurs, and, compared to the case of monopoly, the incumbent charges a higher price for its downstream product. The consumer and total welfare effects of entry can be summarized as follows:

(i) Consumer surplus declines if

$$\frac{\alpha_E}{\alpha_I} < \frac{12\beta - 2\beta^3 - \beta^5}{4 + 5\beta^2}$$

and increases if the inequality is reversed. In particular, for a sufficiently low degree of downstream product differentiation, consumer surplus increases only if the entrant is more efficient than the incumbent.

(ii) Total welfare declines if

$$\frac{\alpha_E}{\alpha_I} < \frac{36\beta + 8\beta^3 + \beta^5}{28 + 15\beta^2 + 2\beta^4}$$

and increases if the inequality is reversed. In particular, total welfare increases if the entrant is more efficient than the incumbent.

Proposition 1 illustrates that Bertrand competition between a vertically integrated incumbent and an entrant that purchases an essential input from the incumbent can reduce consumer surplus relative to the case of a vertically integrated monopoly. In particular, although the entry of a firm that offers a new, horizontally differentiated product tends to raise consumer surplus by inducing the consumer to purchase a mix of products, entry—perhaps surprisingly—also raises the incumbent’s downstream price above its monopoly level. In this model, entry always raises the incumbent’s downstream price, but, as shown in the Appendix, the overall welfare effects depend on the firms’ relative efficiency levels and on the degree to which products are differentiated. The question is then, why competition leads to higher prices.

The theoretical possibility of so-called “price-increasing competition” is explored by Chen and Riordan (2008), who note that the entry of a firm that offers a product that is an imperfect substitute for a product that is currently offered by an incumbent firm has two (possibly opposing) effects on the price of the incumbent firm’s product. The first effect, which Chen
Figure 1: The incumbent forecloses the entrant and continues to operate as a monopolist when $\alpha_E/\alpha_I < \beta$. Otherwise, both firms benefit from entry, but the consumer surplus and total welfare effects of entry depend on the values of $\alpha_E/\alpha_I$ and $\beta$.

and Riordan call the “market share effect,” refers to the incumbent firm’s incentive to lower its product’s price due to the fact that, all else being equal, the incumbent loses some of its market share to the entrant. The second effect, which Chen and Riordan call the “price sensitivity effect,” refers to the incumbent firm’s incentive to modify its product’s price due to the effect of product differentiation on the slope of the demand curve for the incumbent’s product. In particular, when the rival’s entry leads to a steepening of the demand curve that the incumbent faces, the price sensitivity effect will create upward pricing pressure on the incumbent firm’s product. Price-increasing competition, as Chen and Riordan point out, occurs when the price sensitivity effect opposes and dominates the market share effect.\(^\text{12}\)

Extrapolating from Chen and Riordan’s (2008) discrete choice framework, let us first suppose that $w = 0$ following entry, which leads to the standard differentiated Bertrand

\(^{12}\)Chen and Riordan’s model does not contemplate firms selling to their rivals; indeed, price-increasing competition is a theoretical possibility even in a standard differentiated product duopoly framework.
duopoly result. Let \( p_i(w), i \in \{ I, E \} \) represent firm \( i \)'s price as a function of \( w \). In particular,

\[
p^D_I(w) = \alpha - \frac{\beta(\alpha_E - 3w)}{4 - \beta^2},
\]

where the \( D \) superscript represents the duopoly equilibrium (as a function of \( w \)). Note that Equation (2) is increasing in \( w \). Setting \( w = 0 \) yields:

\[
p^M_I - p^D_I(w)|_{w=0} = \frac{\beta(2\alpha_E + \alpha_I \beta)}{2(4 - \beta^2)} > 0,
\]

where the \( M \) superscript represents the monopoly equilibrium. Expression (3) indicates that price-increasing competition does not occur when \( w = 0 \) in a linear demand framework.

In order to identify the market share and price sensitivity effects, define

\[
\psi^M_I(p_I) \equiv (\alpha - p_I) - (p_I - c_I) \times 1,
\]

\[
\psi^D_I(p_I)|_{w=0} \equiv \left( \frac{\alpha}{1 + \beta} - \frac{p_I}{1 - \beta^2} + \frac{\beta p_E(p_I; w)|_{w=0}}{1 - \beta^2} \right) - (p_I - c_I) \left( \frac{1}{1 - \beta^2} \right).
\]

Observe that \( \psi^M_I(p_I) \) is the first-order derivative of monopoly profit equation \( \pi^M_I = (p_I - c_I)(\alpha - p_I) \) and equals zero at \( p^M_I \). Note that the rightmost term is multiplied by one, which indicates the slope of the monopolist’s demand curve. Similarly, \( \psi^D_I(p_I)|_{w=0} \) defines the first-order condition of the incumbent duopolist’s problem when \( w = 0 \), with the entrant’s best response at \( w = 0 \), \( p_E(p_I; w)|_{w=0} \), substituted for \( p_E \). The rightmost parenthetical term, \( 1/(1 - \beta^2) \), represents the inverse of the slope of the incumbent duopolist’s demand curve—thus, the slope of demand is less steep in the duopoly case. Moreover, \( \psi^D_I(p^D_I(w))|_{w=0} = 0 \).

It is easily verified that the applicable second-order conditions in the second stage pricing game hold at any \( w \), so that \( p^M_I \geq p^D_I(w)|_{w=0} \) if and only if \( \psi^M_I(p^M_I) - \psi^D_I(p^M_I)|_{w=0} \geq 0 \). Writing this expression out, we have:

\[
\psi^M_I(p^M_I) - \psi^D_I(p^M_I)|_{w=0} = \left\{ (\alpha - p^M_I) - \left( \frac{\alpha}{1 + \beta} - \frac{p^M_I}{1 - \beta^2} + \frac{\beta p_E(p^M_I; w)|_{w=0}}{1 - \beta^2} \right) \right\} + (p^M_I - c_I) \times \left( \frac{1}{1 - \beta^2} - 1 \right).
\]

\[^{13}\text{We define the market share effect slightly differently from Chen and Riorden (2008) to permit us to appropriately deal with the case of asymmetric costs, which Chen and Riorden do not consider.}\]
The market share effect equals zero when $\beta = 0$ and substituting $p_E(p_I; w)|_{w=0} = (\beta p_I + \alpha(1 - \beta) + c_E)/2$, it can be verified that it is increasing in $\beta$. That is, following entry, the incumbent always has an incentive to lower price relative to the monopoly case to regain lost market share. Moreover, when $w = 0$, the price sensitivity effect reinforces the market share effect—the difference in inverse demand slopes is positive—so that price-increasing competition could never occur.

Thus, as Equation (2) indicates, for price-increasing competition to occur, it must be that $w > 0$. Defining $\psi_I^D(p_I)$ as:

$$\psi_I^D(p_I) = \left( \frac{\alpha}{1 + \beta} - \frac{p_I}{1 - \beta^2} + \beta p_E(p_I; w) \right) - (p_I - c_I) \left( \frac{1}{1 - \beta^2} \right) + w \left( \frac{1}{1 - \beta^2} \right),$$

and noting that $p_I^M \geq p_I^D(w)$ if and only if $\psi_I^M(p_I^M) - \psi_I^D(p_I^M) \geq 0$, we can decompose the difference in first-order conditions as follows:

$$\psi_I^M(p_I^M) - \psi_I^D(p_I^M) = \left\{ (\alpha - p_I^M) - \left[ \frac{\alpha}{1 + \beta} - \frac{p_I^M}{1 - \beta^2} + \beta p_E(p_I^M; w)|_{w=0} + \frac{\beta w}{2(1 - \beta^2)} \right] \right\}$$

$$+(p_I^M - c_I) \times \left( \frac{1}{1 - \beta^2} - 1 \right) - w \times \frac{\beta}{1 - \beta^2}.$$
shown in the Appendix, at the incumbent’s optimal input price $w^*$, $p^D_I - p^M_I > 0$, where $p^D_I \equiv p^D_I(w^*)$. That is, the cross slope effect dominates the market share and price sensitivity effects, such that price-increasing competition always occurs in the subgame-perfect Nash equilibrium of this game.

Put another way, the incumbent exploits its control over the input price to force the entrant to set a high retail price, which mitigates the negative impact of entry on the incumbent’s retail sales (market share effect). In addition, when $w$ is higher, the incumbent incurs a greater opportunity cost of lowering its price through lost unit sales to the entrant (cross slope effect). Thus, the entrant’s reliance on the incumbent in the upstream market undoes the typical effect of entry, which normally is a disciplining force on the incumbent’s retail price.\footnote{Through its control over the input price, the incumbent attains a Stackelberg-style leadership position that allows it to signal its intention to set a higher retail price. Related work examining this effect includes Fershtman and Judd (1987), Bonanno and Vickers (1988), and Aguelakakis and Yankelevich (2016).}

As Figure 1 indicates, the adverse welfare impact of price-increasing competition dominates the benefit from product diversity when $\alpha_E/\alpha_I - \beta$ is positive, but closer to zero. In fact, if $\alpha_E/\alpha_I - \beta$ is sufficiently close to zero, the decline in consumer surplus can outweigh the increase in cumulative firm profits, such that total welfare declines.

Before moving on to analyze how price-cap regulation alters the welfare results, we consider comparative statics with respect to $c_I$ and $c_E$ (or equivalently, with respect to $\alpha_I$ and $\alpha_E$). Proposition 2 shows that both firms’ profits, but not necessarily consumer surplus, are increasing in the entrant’s efficiency.

**Proposition 2.** Holding other factors fixed, and assuming that the entrant is not foreclosed, both firms’ profits under duopoly are increasing in the entrant’s efficiency. Consumer surplus is initially decreasing but eventually increasing in the entrant’s efficiency, and the threshold ratio of entrant efficiency to incumbent efficiency for consumer surplus to be increasing in entrant efficiency decreases with the level of product differentiation. Further, if the entrant is at least as efficient as the incumbent, consumer surplus is unambiguously increasing in the entrant’s efficiency.
entrant’s efficiency.

\[ CS \text{ increasing in } \alpha_E \]
\[ CS \text{ decreasing in } \alpha_E \]

**Figure 2:** As stated in Proposition 2, consumer surplus is decreasing in the entrant’s efficiency for intermediate levels of the entrant’s efficiency relative to the incumbent’s.

The equilibrium input price, \( w^* \), and the incumbent’s duopoly equilibrium downstream price, \( p^D_I \), are respectively:

\[
w^* = \frac{8\alpha_E + \alpha_I \beta^3}{16 + 2\beta^2},
\]

\[
p^D_I = c_I + \frac{(8 - \beta^2) \alpha_I + 2\beta\alpha_E}{2(8 + \beta^2)}.
\]

Note that both are increasing in \( \alpha_E \)—that is, the incumbent can charge a higher input price to a more efficient entrant, which also induces the incumbent to charge a higher downstream price. Although, this price-increasing effect is counteracted by a decrease in the entrant’s equilibrium price:

\[
p^D_E = \alpha - \frac{4\alpha_E + \alpha_I \beta(4 + \beta^2)}{2(8 + \beta^2)},
\]

as the proof of Proposition 2 shows in the Appendix, the latter effect can dominate in terms of
decreasing consumer surplus. Figure 2 shows that this occurs when \(\alpha_E/\alpha_I - \beta\) is sufficiently close to zero.

The final result of this section, Proposition 3, describes the analogous comparative statics with respect to the incumbent’s efficiency level. The result is depicted in Figure 3.

**Proposition 3.** Holding other factors fixed, and assuming that the entrant is not foreclosed, the entrant’s profit is decreasing in the incumbent’s efficiency. The incumbent’s profit is increasing in its own efficiency unless the entrant is substantially more efficient than the incumbent. The threshold factor by which the entrant’s efficiency needs to exceed the incumbent’s efficiency for the incumbent’s profit to be increasing in its own efficiency increases with the level of product differentiation. The comparative statics for consumer surplus with respect to the incumbent’s efficiency follow a qualitatively similar pattern.

![Figure 3](image-url)

**Figure 3:** As shown in Proposition 3, for most parameter values, both consumer surplus and the incumbent’s profit level are increasing in the incumbent’s efficiency.

Observe that \(w^*\) and \(p_E^D\) respond in the same direction to \(\alpha_I\) as they do to \(\alpha_E\), whereas \(p_I^D\) moves in the reverse direction. However, using Equation (2) it can be readily shown that \(p_I^D\)
does not fall as much as $p^D_I(w)|_{w=0}$ does in response to an increase in incumbent efficiency. This is because in contrast to the case when $w = 0$, a decrease in $p^D_I$ lowers entrant demand, which decreases incumbent profits from upstream sales to the entrant. On the other hand, a more efficient incumbent does earn higher incremental profits from own unit sales and so raises $w^*$ to induce the entrant to increase its price and raise incumbent demand. If the entrant is very efficient relative to the incumbent at the outset, this could turn out to hurt the incumbent through lost upstream sales. The impact of a lower incumbent price and a higher entrant price on consumer surplus is likewise ambiguous.

3.2 Equilibrium with Price Caps

We now turn to the case of price caps. Given our focus on the market for business data services, we assume that the regulator imposes a common cap on the incumbent’s retail and wholesale prices, and that this cap does not apply to potential entrants. In other words, let us suppose that the regulator caps $p_I$ and $w$ at $c_I$, which is typically referred to as the “efficient” cap, but that there is no cap on $p_E$. This is effectively the framework imposed by the FCC. The benefit of this framework is that it avoids margin squeeze situations. That is, by not capping $p_E$ and simultaneously capping $w$, the FCC prevents an ILEC from adopting a retail and wholesale price combination that makes entry unprofitable. At the same time, the retail price cap on the ILEC acts as a competitive constraint on the CLEC’s potential price.

We assume that consumer demand and the firms’ costs are known not just to both firms, but also to the regulator. This may be justified within the business data service context by FCC efforts to collect relevant market information directly from the LECs. For instance, in 1987, the FCC initiated the Automated Reporting Management Information System (ARMIS) for collecting financial and operational data from the largest exchange carriers. Although in 2008, the FCC issued a number of conditional forbearance orders that limited the information available to it, as part of its special access/business data service
proceeding, the FCC collected extensive data from LECs to be able to reevaluate and update its price caps.\textsuperscript{15}

In actuality, price capped LECs sell multiple business data service products and price caps are imposed on quantity adjusted indices rather than on individual products. To simplify matters, we abstract away from multi-product pricing, effectively treating business data services as a single product category and also assuming that demands for any other products produced by the incumbent and entrant are independent of those for business data services. Moreover, in this section, we abstract from fixed costs and therefore do not consider the principles of Ramsey-Boiteux pricing (Laffont and Tirole 2001, pp. 60–96).\textsuperscript{16}

Finally, we suppose that in all scenarios, the incumbent stands by ready to sell. That is, by assumption, we rule out the idea that an incumbent may choose to exit the downstream market, even if participation in that market lead to zero sales. Although seemingly strong, this assumption is meant to emulate reality—that is, a regulator does not engage in price-cap regulation only to permit the incumbent to recreate a monopolistic market via other means. Moreover, in the business data services context, the reality is that ILECs dominate the market in their areas—they do not simply stand by ready to sell (FCC 2016 ¶¶160-165, Rysman 2016). In Lemma 3 we discuss conditions necessary for this to occur and largely focus on this case.

We begin our analysis by noting that the incumbent capped at $c_I$ will never set a negative price for the input.

\textbf{Lemma 1.} \textit{Regardless of whether or not the incumbent faces price caps, $w \geq 0$ in equilibrium.}

\textit{Proof.} The proof is by contradiction. Suppose that $w < 0$. Assume, first, that $p_I(w) < c_I$. In this case, even if there is a price cap, it does not bind. Then the incumbent loses money


\textsuperscript{16}Under Ramsey pricing, a regulator sets higher prices for lower elasticity components of a multi-product basket while making sure that the regulated entity can cover fixed costs of investment.
on both upstream sales of the input and downstream sales of the product; which leads to negative profit. The incumbent can do strictly better by setting $w = 0$ and $p_I(w) = c_I$.

Now assume that $p_I(w) = c_I$. Consider a deviation to a slightly higher input price, $w' \in (w, 0)$, while keeping the incumbent’s downstream price fixed. This deviation has three effects:

1. The incumbent incurs a less severe loss on each inframarginal sale of the input.

2. The entrant’s marginal cost rises. In response, the entrant raises its downstream price, and the quantity demanded of the entrant’s downstream product falls. As a result, the entrant purchases less of the input. This effect further reduces the incumbent’s upstream loss.

3. The rise in the entrant’s downstream price increases the incumbent’s downstream sales. The first two effects identified above strictly increase the incumbent’s profit, whereas the third effect does not change the incumbent’s profit. Thus, deviating to $w'$ is profitable.

Finally, consider the case in which $p_I(w) > c_I$, which can hold only if the incumbent does not face a price cap. As in the previous case, consider a deviation to a slightly higher $w' \in (w, 0)$. The three effects identified above also arise here. The only difference is that the third effect—the increase in the incumbent’s downstream sales—now results in a strict increase in the incumbent’s profit, because the incumbent prices above marginal cost. The reasoning behind Lemma 1 is intuitively clear: by setting a negative input price, the incumbent incurs losses on upstream sales and also allows the entrant to reduce its price downstream, which makes it more difficult for the incumbent to compete downstream. Thus, going forward, we assume that $w \geq 0$.

Suppose that the regulator sets the cap at “efficient” level $c_I$. Let $\overline{p}_I \equiv \min \{p_I, c_I\}$, and $\overline{w} \equiv \min \{w, c_I\}$.\(^{17}\) We solve the game by backward induction. In particular, given input price $w \in [0, c_I]$, we have

\(^{17}\)Observe that the regulator permits the incumbent to profit from input sales to the entrant.
\[ p_I^D(w) = \alpha - \frac{2\alpha_I + \beta \alpha_E - 3\beta w}{4 - \beta^2}, \]
\[ \bar{p}_I(w) = \min \{ p_I^D(w), c_I \}, \]
\[ \bar{p}_E(w) = \frac{\alpha(1 - \beta) + \beta \bar{p}_I(w) + w + c_E}{2}. \]

Note that the efficient cap typically binds the incumbent’s retail price.

**Lemma 2.** For every input price that leads to positive demand for the incumbent’s retail product (i.e., for every \( w \) such that \( q_I(p_I(w), p_E(w)) > 0 \)), \( p_I(w) \geq c_I \).

**Proof.** Suppose that \( q_I(p_I(w), p_E(w)) > 0 \) and \( p_I(w) < c_I \). Then the incumbent earns negative profits from retail sales. Observe that, by unilaterally increasing its retail price to \( c_I \), it increases its retail profits to zero. Furthermore, this unilateral increase in the incumbent’s retail price increases the volume of the entrant’s retail sales, which (because \( w \geq 0 \)) at least weakly increases the incumbent’s upstream profit as well. In short, the incumbent has a profitable deviation from \( p_I(w) \): a contradiction. \( \square \)

Lemma 3, which is proven in the Appendix, provides sufficient conditions for the hypothesis of Lemma 2 to be satisfied.

**Lemma 3.** If the incumbent is more efficient than the entrant or retail products are sufficiently differentiated, the price-capped incumbent achieves positive sales of its retail good for every \( w \geq 0 \).

The intuition behind Lemma 3 is straightforward. If the incumbent is at least as efficient as the entrant, or the firms’ retail products are highly differentiated, the incumbent will generally be able to compete quite effectively with the entrant in the retail market, even if the upstream input price is very low.

Setting \( \bar{p}_I(w) = c_I \) yields the following quantities and profits as functions of \( w \):

\[ q_I(w) = \frac{\alpha_I (2 - \beta^2) - \beta (\alpha_E - w)}{2 (1 - \beta^2)}, \quad q_E(w) = \frac{\alpha_E - \alpha_I \beta - w}{2 (1 - \beta^2)}, \]
\[ \pi_I(w) = \frac{w (\alpha_E - \alpha_I \beta - w)}{2 (1 - \beta^2)}, \quad \pi_E(w) = \frac{(\alpha_E - \alpha_I \beta - w)^2}{4 (1 - \beta^2)}. \]
In the absence of a cap on the input price (but given \( \overline{p}_I(w) = c_I \)), and assuming that \( \alpha_E > \alpha_I \beta \) (which implies that foreclosure does not occur), the incumbent would set \( w = (\alpha_E - \alpha_I \beta)/2 \) to maximize \( \pi_I(w) \). This input price level may be either above or below \( c_I \).

Suppose, first, that \( (\alpha_E - \alpha_I \beta)/2 \leq c_I \), so that the input price cap does not bind. Then equilibrium prices, quantities, profits, and consumer surplus are

\[
\begin{align*}
\overline{p}^D_I &= c_I, \\
\overline{p}^D_E &= \frac{4\alpha - \alpha_E - 3\alpha_I \beta}{4}, \\
\overline{q}^D_I &= \frac{\alpha_I (4 - 3\beta^2) - \alpha_E \beta}{4(1 - \beta^2)}, \\
\overline{q}^D_E &= \frac{\alpha_E - \alpha_I \beta}{4(1 - \beta^2)}, \\
\overline{\pi}^D_I &= \frac{\pi^D_E = \frac{(\alpha_E - \alpha_I \beta)^2}{8(1 - \beta^2)}}, \\
\overline{\pi}^D_E &= \frac{(\alpha_E - \alpha_I \beta)^2}{16(1 - \beta^2)}, \\
\overline{CS}^D &= \frac{\alpha^2 + 16\alpha_I^2 - 2\alpha_E \alpha_I \beta - 15\alpha_I^2 \beta^2}{32(1 - \beta^2)}.
\end{align*}
\]

From this point, assume that \( \alpha_E / \alpha_I < 4/\beta - 3 \beta \), so that \( \overline{q}^D_I > 0 \) holds. As Figure 4 shows, this condition holds unless the entrant is significantly more efficient than the incumbent, and the factor by which the entrant’s efficiency needs to exceed the incumbent’s efficiency for the condition to fail increases with the level of product differentiation. From the above expression for \( \overline{q}^D_E \), it is evident that, if the upstream price cap does not bind, the foreclosure condition \( (\alpha_E \leq \alpha_I \beta) \) from the unregulated case is also the foreclosure condition here.

Suppose, now, that \( (\alpha_E - \alpha_I \beta)/2 > c_I \), so that the input price does bind at \( c_I \). In this case, equilibrium prices, quantities, profits, and consumer surplus are

\[
\begin{align*}
\overline{p}^D_I &= c_I, \\
\overline{p}^D_E &= \frac{3\alpha - \alpha_E - \alpha_I (1 + \beta)}{2}, \\
\overline{q}^D_I &= \frac{2\alpha_I + \beta (\alpha - \alpha_E - \alpha_I - \alpha_I \beta)}{2(1 - \beta^2)}, \\
\overline{q}^D_E &= \frac{\alpha_E - \alpha_I \beta - c_I}{2(1 - \beta^2)}, \\
\overline{\pi}^D_I &= \frac{c_I (\alpha_E - \alpha_I \beta - c_I)}{2(1 - \beta^2)}, \\
\overline{\pi}^D_E &= \frac{(\alpha_E - \alpha_I \beta - c_I)^2}{4(1 - \beta^2)}, \\
\overline{CS}^D &= \frac{\alpha^2 + \alpha_E^2 + 5\alpha_I^2 + 2\alpha_E \alpha_I - 2 \alpha (\alpha_I + \alpha_E - \alpha_I \beta) - \beta(2\alpha_E \alpha_I + 2\alpha_I^2 + 3\alpha_I^2 \beta)}{8(1 - \beta^2)}.
\end{align*}
\]

Note that \( \overline{q}^D_E \leq 0 \) if and only if \( \alpha_E - \alpha_I \beta \leq c_I \). However, by assumption, we have \( \alpha_E - \alpha_I \beta > \)

\[\text{[Footnote 18]}\]
2c_I > c_I. Thus, foreclosure does not occur. Observe that the introduction of price caps, then, does not lead to foreclosure that would not have occurred in the absence of price caps.

Now consider the case of a price-capped monopolist:

\[ \bar{p}^M_I = c_I, \quad \bar{q}^M_I = \alpha_I, \]
\[ \pi^M_I = 0, \quad CS^M = \frac{\alpha_I^2}{2}. \]

Clearly, regardless of whether the input price cap binds, both the incumbent and entrant are better off competing in the price-capped regime than when the incumbent operates as a price-capped monopolist in the downstream market. In the latter case, both earn zero profits. By capping the wholesale and retail prices of the incumbent at the same level, the regulator allows the incumbent to profit in the upstream market as long as the marginal cost of downstream production is higher than the marginal cost of input provision.

Consumers are unambiguously better off following entry. If the input price cap does not
bind (i.e., if \( w = (\alpha_E - \alpha_I\beta) / 2 \)), then

\[
\tilde{C}S^D - \tilde{C}S^M = \frac{(\alpha_E - \alpha_I\beta)^2}{32(1 - \beta^2)} > 0,
\]

\[
\tilde{W}^D - \tilde{W}^M = \frac{7(\alpha_E - \alpha_I\beta)^2}{32(1 - \beta^2)} > 0.
\]

If, instead, the input price cap does bind (i.e., if \( w = c_I \)),

\[
\bar{C}S^D - \bar{C}S^M = \frac{(\alpha_E - \alpha_I\beta - c_I)^2}{8(1 - \beta^2)} > 0,
\]

\[
\bar{W}^D - \bar{W}^M = \frac{(\alpha_E - \alpha_I\beta - c_I)^2}{8(1 - \beta^2)} 
\left[ 3(\alpha_E - \alpha_I\beta) + c_I \right] > 0.
\]

Unlike in the unregulated regime, price caps that bind the incumbent’s retail price at marginal cost do not permit the incumbent to use the input price to raise its retail price above the retail price that prevails under monopoly. Consumers are better off because of increased product differentiation. Firms are also better off, because they share the benefits of increased product differentiation between themselves and with consumers.

**Proposition 4.** Suppose that \( \beta < \alpha_E / \alpha_I < 4 / \beta - 3\beta \), and that downstream competition is in prices. Suppose that a regulator caps incumbent input and final product prices at \( c_I \). Then both firms and consumers are better off under Bertrand competition between the incumbent and the entrant than when a price-capped monopolist serves the downstream market. Furthermore, foreclosure of the entrant occurs under price caps only if it also would have occurred without price caps.

### 4 Theoretical Long-Run Effects of Price Caps: Investment Incentives

In justifying its transition to price-cap regulation, the FCC in 1990 wrote, “by establishing limits on prices carriers can charge for their service, and placing downward pressure on those limits or ‘caps,’ we create a regulatory environment that requires carriers to become more productive.” In particular, the FCC noted that, “carriers that can substantially increase
their productivity can earn and retain profits at reasonable levels above those we allow for rate of return carriers” (FCC 1990 ¶22). The logic is that price caps will only be reevaluated and reset sporadically, giving regulated firms that reduce their costs the opportunity to earn positive economic profits in the interim.\footnote{The basic principles underlying price-cap regulation are set out in Bernstein and Sappington (1999).} Tellingly, FCC special access price caps have been largely frozen at 2003 levels, giving regulated firms substantial time to reduce their costs.

However, as we point out in Section 1, it is unclear how incumbent investment under price-cap regulation compares to investment without regulation. To accommodate the price-cap adjustment mechanism and to be able to compare investment across the regulated and unregulated cases, we modify the original game by considering an earlier stage in which the incumbent can incur a one-time cost to reduce its marginal cost, $c_I$ (or, equivalently, to increase its efficiency level, $\alpha_I$) after the price-cap is fixed at $c_I$. In particular, suppose that the incumbent can incur investment cost $\kappa(k)$, where $k \in [0, 1]$, to change its marginal cost to $(1-k)c_I$. Equivalently, by incurring the cost of $\kappa(k)$, the incumbent improves its efficiency level to $\alpha_I(k) \equiv \alpha - (1-k)c_I = \alpha_I(0) + kc_I$. Assume that $\kappa(0) = 0$ and $\lim_{k \to 0} \kappa'(k) = 0$, and that $\kappa$ is strictly increasing, strictly convex, thrice differentiable, and unbounded on $[0, 1)$. Assume further that $\kappa'$ is weakly convex, which as will become evident below, guarantees a unique equilibrium level of investment.\footnote{An example of a function that satisfies each of these conditions is $\kappa(k) = k^2/(1-k)$.}

To simplify matters, going forward, let us assume that foreclosure does not happen even if the incumbent manages to reduce its marginal cost to zero. The relevant condition may be written as

$$c_E < \alpha(1 - \beta).$$ \hfill (5)

Inequality (5) is a way to rewrite the assumption that the ratio of the entrant value margin to the incumbent value margin is greater than $\beta$ even when $\alpha_I = \alpha$. Then, in the case without price caps, the incumbent’s problem in the investment stage can be written as

$$\max_{k \in [0,1)} \pi_D^I(k) - \kappa(k).$$ \hfill (6)

The optimal level of cost reduction, $k^*$, in the case without price caps is characterized by
the first-order condition:

\[ \kappa'(k^*) = (\pi^D_I)'(k^*) \]
\[ = \frac{\partial \pi^D_I}{\partial \alpha_I} \cdot \alpha_I(k^*) \]
\[ = \frac{c_I [4\alpha E \beta + \alpha_I(k^*) \cdot (8 - 3\beta^2 - \beta^4)]}{2 (8 + \beta^2) (1 - \beta^2)}. \]  

(7)

The second-order condition for \( k^* \) to be a solution to Expression (6) is

\[ (\pi^D_I)^{(\prime\prime)}(k^*) - \kappa''(k^*) < 0, \]

which can be written as

\[ \frac{c^2_I \cdot (8 - 3\beta^2 - \beta^4)}{2 (8 + \beta^2) (1 - \beta^2)} < \kappa''(k^*). \]  

(8)

Note that the left-hand side of Inequality (8) is increasing in \( \beta \). Thus, going forward, we suppose that products are differentiated enough (\( \beta \) low enough) or the cost of investment is convex enough for the second-order condition to hold.\(^{21}\)

Next, consider the case with price caps. From the analysis of the previous section, we know that, when the incumbent chooses investment level \( k \) and input price \( w \in [0, c_I] \), the equilibrium of the resulting subgame has the two firms pricing according to

\[ p_I(w, k) = \min \left\{ c_I, \alpha - \frac{2(\alpha_I + kc_I) + \beta(\alpha_E - 3w)}{4 - \beta^2} \right\}, \]
\[ p_E(w, k) = \frac{\alpha(1 - \beta) + \beta p_I(w, k) + w + c_E}{2} \]
\[ = \min \left\{ \frac{\alpha(1 - \beta) + \beta c_I + w + c_E}{2}, \frac{\alpha[4 - \beta(k + \beta)] - \alpha_I \beta(1 - k) - 2\alpha_E + w(2 + \beta^2)}{4 - \beta^2} \right\}. \]

Our primary interest is when downstream price caps bind. Although unlike in the previous section, this is no longer necessarily the case—when \( k > 0 \) the incumbent can earn positive profits from its own downstream sales—if we show that a regulated incumbent invests more under a binding downstream cap, we might anticipate that an incumbent that invests to reduce marginal costs so substantially as to price below the cap likewise invests

\(^{21}\)As an example, if the cost function in footnote 20 is used and for simplicity, say \( c_I = 1 \), then even if \( k = 0, \kappa''(k)|_{k=0} = 2 \), so that the second-order condition holds for any \( \beta < 0.93 \).
no less than an unregulated incumbent.\textsuperscript{22}

Thus, to simplify the exposition, suppose that the incumbent’s downstream price binds even if \( w = 0 \) and \( k = 1 \). Note that when below \( c_I \), \( p_I(w, k) \) is increasing in \( w \) and decreasing in \( k \), so that if it binds at \( w = 0 \) and \( k = 1 \), it binds in equilibrium as well. A sufficient condition for this to occur is

\[
\alpha \left( 2 - \beta - \beta^2 \right) > c_I \left( 4 - \beta^2 \right) - c_E \beta.
\]

In other words, given \( \beta \), if \( \alpha \) is sufficiently large relative to an adjusted difference of \( c_I \) and \( c_E \), the price cap binds at any level of investment. When products are less differentiated, in order for this inequality to hold, either \( \alpha \) needs to be higher, or the entrant needs to be relatively less efficient than the incumbent. Going forward, we assume that Inequality (9) holds.

Even if the downstream cap binds, as was the case in Subsection 3.2, there are two cases of interest: \( w^*(k) < c_I \) and \( w^*(k) = c_I \), which we consider in turn. Suppose first that \( w^*(k) < c_I \). The incumbent’s problem in the investment stage can be written as

\[
\max_{k \in [0,1]} \tilde{\pi}_D I(k) - \kappa(k).
\]

The optimal level of cost reduction, \( \tilde{k}^* \), is characterized by the first-order condition:

\[
\kappa'(\tilde{k}^*) = \frac{c_I[\alpha_E \beta - 4\alpha_I(1 + \beta^2) + \beta^2\alpha_I(\tilde{k}^*)]}{4(1 - \beta^2)}.
\]

The second-order condition for \( \tilde{k}^* \) to be a solution to Expression (10) is

\[
\frac{c_I^2 \beta^2}{4(1 - \beta^2)} < \kappa''(\tilde{k}^*).
\]

Note that as with Inequality (8), the left-hand side of Inequality (12) is increasing in \( \beta \) and as in the unregulated case, we suppose that products are differentiated enough for the

\textsuperscript{22}Note that naturally, in the case that neither the incumbent’s downstream nor upstream prices bind, investment will be the same as if the incumbent were unregulated. Investment in cost reduction can have two potential effects. It increases incumbent profit from all (inframarginal) units sold by the incumbent downstream. Additionally, if the incumbent prices below the cap, it can induce “business stealing” from the entrant relative to the no-investment case. We are primarily concerned with the first effect.
second-order condition to hold.\footnote{Again letting \( c_I = 1 \), if we rely on the cost function in footnote 20, then even if \( k = 0 \), the second-order condition holds for any \( \beta < 0.94 \).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Suppose that \( \alpha = 10, c_I = 1, c_E = 2, \beta = 0.75, \) and \( \kappa(k) = k^2/(1-k) \). These parameter values imply that \( w^*(k) = 0.625 + 0.375k \), so that the condition \( w^*(k) < c_I \) is satisfied for any \( k \in [0, 1) \). Moreover, Inequalities (5), (8), (9), and (12) all hold and a regulated incumbent invests more than an unregulated one in equilibrium.\footnote{An additional, but necessary check required us to confirm that for these parameter values, the downstream incumbent price would bind regardless of whether the equilibrium value of \( w^*(k) \) binds or not. The calculations are available upon request from the authors.}

Although we can only solve for \( \tilde{k}^* \) implicitly, our interest here is on whether or not \( \tilde{k}^* \) is greater than \( k^* \), the arg max for Equation (6). If so, then investment in cost reduction is higher under regulation than without it. What simplifies this problem tremendously is the fact that the right-hand sides of both Equations (7) and (11) are affine in \( k \), coupled with our assumptions on \( \kappa \), particularly that \( \lim_{k \downarrow 0} \kappa'(k) = 0 \) and \( \kappa''(k) \geq 0 \). Then, as shown in Figure 5, for a sample set of parameters that satisfies Conditions (Inequalities) (5), (8), (9), and (12), and which assumes that \( \kappa(k) = k^2/(1-k) \), investment is greater in the regulated case if the line representing the right-hand sign in Equation (11) crosses the curve representing \( \kappa'(k) \) above the line representing the right-hand sign in Equation (7). As we show in the Appendix, under the assumptions that we have laid out in this section, this
turns out to always be the case.

Next, suppose that \( w^*(k) = c_I \). As we show in the Appendix, the analysis of this case is analogous to that when the input price does not bind.\(^{25}\) Moreover, as in the previous analysis, under the assumptions laid out in this section, a regulated incumbent invests more in cost reduction than an unregulated one.

**Proposition 5.** Consider the extended game with investment cost function \( \kappa \) as characterized in this section. Then, assuming that the entrant is never foreclosed by incumbent investment in marginal cost reduction and that a regulated incumbent’s downstream price binds in equilibrium, a regulated incumbent will invest more in equilibrium than an unregulated incumbent would.

Recall from Subsection 3.1 that an unregulated incumbent responds to an increase in efficiency by increasing its upstream price while lowering its downstream price. Suppose that under price-cap regulation \( w^*(k) = c_I \). Moreover, by assumption, the downstream price binds regardless of \( k \). Thus, on the one hand, under regulation, efficiency gains are not passed through to the consumer as they are without regulation, which favors investment under regulation. On the other hand, unregulated incumbents increase their demand when they become more efficient, which favors investment when there is no regulation. Proposition 5 states that the former effect dominates. The result is strengthened when \( w^*(k) < c_I \) because then investment facilitates business stealing by the incumbent under regulation, as it would sans regulation.

### 5 Conclusion

In this manuscript, we have shown that the type of price-cap regime that is set out by the FCC has a number of desirable characteristics. In particular, it impedes price-increasing

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\(^{25}\)Interestingly, one difference is that \( k \) drops out of the component of the incumbent’s profit equation that does not include \( \kappa'(k) \), so that in this case the optimal investment level can be solved for explicitly for any \( \kappa \) that satisfies our assumptions above. Moreover, this implies that the second-order condition needed for a unique level of investment to exist holds regardless of the value of \( \beta \).
competition while also encouraging incumbent investment in cost reduction without giving the regulated incumbent an incentive to attempt to foreclose its rival. In part, this is accomplished by allowing the incumbent to procure above zero profits via its sales to the entrant and allowing the entrant to share in the potential benefits of entry by not capping it as well. This profit sharing regime avoids margin squeeze, giving the entrant motivation to participate in the market, but also constrains the entrant’s price by capping the price of its rival.

One important consequence of price-cap regulation that we have not addressed in the current manuscript, but which we hope to investigate in future work, is the impact that it has on entrant incentives to invest in their own facilities. Without undertaking further analysis, this effect is ex-ante ambiguous to us because of two opposing effects. Entry via owned facilities is equivalent to fixing the input price to zero in our baseline model. If the incumbent’s input price is capped, entrant investment is favored in the unregulated case because there is less of a cost decline to be achieved by building own facilities under regulation. On the other hand, because the incumbent’s downstream price was bound under price cap regulation and would continue to bind at \( w = 0 \), a facilities based entrant is not harmed by a downstream price decline by the incumbent following entry in the regulated case, which favors entrant investment under regulation.

Ultimately, the impact of price caps on investment—either by incumbents or entrants—in a market where firms supply their rivals should be addressed empirically. One possibility for undertaking this analysis is to use ARMIS data between 1990 and 2003 (after which substantially less data is available for ILECs), during which time the price-cap was reset multiple times. The ARMIS data contains various barometers of individual ILEC network investments (available, for instance in FCC Reports 43-07 and 43-08) at a level that is substantially granular that some components of investment could be more closely attributed to special access (business data services), while others are attributed to services that are not regulated in the same way as special access. Patterns of cost reducing investment could then
be gauged over groups of periods between price cap adjustments. Our results in Proposition 5
indicate that we should expect higher levels of investment closer in time to a prior price cap
adjustment.

Appendix

Proof of Proposition 1.

Proof. Assume that \( \alpha_E/\alpha_I > \beta \). Arya et al. (2008) show that entry occurs. Denote the in-
cumbent’s downstream price under duopoly by \( p^D_I \). Solving the game by backward induction,
we find that

\[
p^D_I = c_I + \frac{(8 - \beta^2) \alpha_I + 2\beta\alpha_E}{2(8 + \beta^2)}.
\]

On the other hand, when the incumbent operates as a monopolist, its downstream price is
given by \( p^M_I = (\alpha + c_I)/2 \). Moreover,

\[
p^D_I - p^M_I = \frac{\beta(\alpha_E - \beta\alpha_I)}{8 + \beta^2} > 0,
\]

so that the incumbent sets a higher downstream price under duopoly than under monopoly.

(i) Solving the game by backward induction shows that consumer surplus under duopoly
is given by

\[
CS^D = \frac{(64 - 23\beta^4 - 5\beta^6) \alpha_I^2 + 4\alpha_E^2 (4 + 5\beta^2) - 4\alpha_I\alpha_E (16\beta + 3\beta^3 - \beta^5)}{8 (1 - \beta^2) (8 + \beta^2)^2}.
\]

On the other hand, consumer surplus under monopoly (and hence under foreclosure)
is given by \( CS^M = \alpha_I^2/8 \). Thus, the consumer-welfare gain from entry is

\[
CS^D - CS^M = \frac{(\alpha_E - \beta\alpha_I) (4\alpha_E - 12\alpha_I\beta + 5\alpha_E\beta^2 + 2\alpha_I\beta^3 + \alpha_I\beta^5)}{2 (1 - \beta^2) (8 + \beta^2)^2}.
\]

Because \( \alpha_E/\alpha_I > \beta \), the sign of \( CS^D - CS^M \) is the same as that of the second parenthe-
sized expression in the numerator of the term on the right-hand side of Equation (13):

\[
4\alpha_E - 12\alpha_I\beta + 5\alpha_E\beta^2 + 2\alpha_I\beta^3 + \alpha_I\beta^5.
\]

For fixed values of \( \alpha_I \) and \( \beta \), let \( \varphi(\cdot; \alpha_I, \beta) \) denote
the function (of \( \alpha_E \)) that this expression defines. Note that \( \varphi(\cdot; \alpha_I, \beta) \) is continuous.
and increasing. Furthermore,
\[
\lim_{\alpha_E \downarrow \alpha_I} \phi(\alpha_E; \alpha_I, \beta) = -\alpha_E \left(1 - \beta^2\right) (8 + \beta^2) < 0.
\]
Thus, entry of a relatively inefficient provider decreases consumer surplus relative to the case of monopoly. In particular,
\[
\text{sgn} \left( CS^D - CS^M \right) = \text{sgn} \left[ \alpha_E - \frac{\alpha_I (12\beta - 2\beta^3 - \beta^5)}{4 + 5\beta^2} \right].
\]
Furthermore, when \( (12\beta - 2\beta^3 - \beta^5) / (4 + 5\beta^2) \geq 1 \) (which holds for \( \beta > 0.42 \)), \( CS^D > CS^M \) only if \( \alpha_E > \alpha_I \).

(ii) Under duopoly, the firms’ equilibrium profits are
\[
\pi_I^D = \frac{(8 - 3\beta^2 - \beta^4) \alpha_I^2 + 4\alpha_E^2 - 8\alpha_I \alpha_E}{4 (1 - \beta^2) (8 + \beta^2)},
\]
\[
\pi_E^D = \frac{(2 + \beta^2)^2 (\alpha_E - \beta \alpha_I)^2}{(1 - \beta^2) (8 + \beta^2)^2}.
\] (14)
Under monopoly, the incumbent’s equilibrium profit is \( \pi_I^M = \alpha_I^2 / 4 \). The difference in welfare levels between duopoly and monopoly is given by
\[
W^D - W^M = \frac{(\alpha_E - \beta \alpha_I) (28\alpha_E - 36\alpha_I \beta + 15\alpha_E \beta^2 - 8\alpha_I \beta^3 + 2\alpha_E \beta^4 - \alpha_I \beta^5)}{2 (1 - \beta^2) (8 + \beta^2)^2}.
\] (15)
Because \( \alpha_E / \alpha_I > \beta \), the sign of \( W^D - W^M \) is the same as that of the second parenthesized expression in the numerator of the term on the right-hand side of Equation (15):
\[
28\alpha_E - 36\alpha_I \beta + 15\alpha_E \beta^2 - 8\alpha_I \beta^3 + 2\alpha_E \beta^4 - \alpha_I \beta^5.
\]
For fixed values of \( \alpha_I \) and \( \beta \), let \( \phi(\cdot; \alpha_I, \beta) \) denote the function (of \( \alpha_E \)) that this expression defines. Note that \( \phi(\cdot; \alpha_I, \beta) \) is continuous and increasing. Furthermore,
\[
\lim_{\alpha_E \downarrow \alpha_I} \phi(\alpha_E; \alpha_I, \beta) = -\alpha_E \left(1 - \beta^2\right) (8 + \beta^2) < 0.
\]
Thus, entry of a relatively inefficient provider decreases total welfare relative to the case of monopoly. In particular,
\[
\text{sgn} \left( W^D - W^M \right) = \text{sgn} \left[ \alpha_E - \frac{\alpha_I (36\beta + 8\beta^3 + \beta^5)}{28 + 15\beta^2 + 2\beta^4} \right].
\]
Because \( (36\beta + 8\beta^3 + \beta^5) / (28 + 15\beta^2 + 2\beta^4) < 1 \) for all \( \beta \in (0, 1) \), entry unambigu-
ously increases total welfare if the entrant is more efficient than the incumbent. □

**Proof of Proposition 2.**

*Proof.* It is clear from Equation (14) that \( \pi^D_E \) is increasing in \( \alpha_E \). Observe that

\[
\frac{\partial \pi^D_E}{\partial \alpha_E} = \frac{2 (\alpha_E - \beta \alpha_I)}{(1 - \beta^2) (8 + \beta^2)} > 0.
\]

Hence, both firms’ profits increase with \( \alpha_E \). Finally, we have

\[
\frac{\partial CS}{\partial \alpha_E} = \frac{8 \alpha_E - 16 \alpha_I \beta + 10 \alpha_E \beta^2 - 3 \alpha_I \beta^3 + \alpha_I \beta^5}{2 (1 - \beta^2) (8 + \beta^2)^2},
\]

which implies that

\[
\text{sgn} \left( \frac{\partial CS}{\partial \alpha_E} \right) = \text{sgn} \left( 8 \alpha_E - 16 \alpha_I \beta + 10 \alpha_E \beta^2 - 3 \alpha_I \beta^3 + \alpha_I \beta^5 \right).
\]

For given values of \( \alpha_I \) and \( \beta \), let \( \xi(\alpha_E; \alpha_I, \beta) \equiv 8 \alpha_E - 16 \alpha_I \beta + 10 \alpha_E \beta^2 - 3 \alpha_I \beta^3 + \alpha_I \beta^5 \). Note that \( \xi(\cdot; \alpha_I, \beta) \) is continuous and increasing, and that

\[
\lim_{\alpha_E \downarrow \alpha_I} \xi(\alpha_E; \alpha_I, \beta) = \alpha_E (\beta^2 + 8) (\beta^2 - 1) < 0.
\]

However, \( \xi(\alpha; \alpha_I, \beta) > 0 \) for

\[
\frac{\alpha_E}{\alpha_I} > \frac{16 \beta + 3 \beta^3 - \beta^5}{8 + 10 \beta^2}.
\]

The expression on the right-hand side of Inequality (16) is strictly increasing in \( \beta \) and bounded above by 1. □

**Proof of Proposition 3.**

*Proof.* It is clear from Equation (14) that \( \pi^D_I \) is decreasing in \( \alpha_I \). Note that

\[
\frac{\partial \pi^D_I}{\partial \alpha_I} = \frac{4 \alpha_E \beta + \alpha_I (\beta^4 + 3 \beta^2 - 8)}{2 (\beta^2 + 8) (\beta^2 - 1)}.
\]

Because the denominator in Equation (17) is negative for \( \beta \in (0, 1) \),

\[
\text{sgn} \left( \frac{\partial \pi^D_I}{\partial \alpha_I} \right) = -\text{sgn} \left[ 4 \alpha_E \beta + \alpha_I (\beta^4 + 3 \beta^2 - 8) \right].
\]

Thus, \( \partial \pi^D_I / \partial \alpha_I > 0 \) if and only if

\[
\beta \leq \frac{\alpha_E}{\alpha_I} < \frac{8 - 3 \beta^2 - \beta^4}{4 \beta}.
\]
The function of $\beta$ that is defined by the expression on the right-hand side of Inequality (18) is unbounded above and decreasing in $\beta$. It approaches 1 as $\beta$ approaches 1.

For consumer surplus,
\[
\frac{\partial CS^D}{\partial \alpha_I} = 64\alpha_I - 32\alpha_E\beta - 6\alpha_E\beta^3 - 23\alpha_I\beta^4 + 2\alpha_E\beta^5 - 5\alpha_I\beta^6
\]
so that
\[
\text{sgn} \left( \frac{\partial CS^D}{\partial \alpha_I} \right) = \text{sgn} \left( 64\alpha_I - 32\alpha_E\beta - 6\alpha_E\beta^3 - 23\alpha_I\beta^4 + 2\alpha_E\beta^5 - 5\alpha_I\beta^6 \right).
\]
Therefore, $\partial CS^D / \partial \alpha_I > 0$ if and only if
\[
\beta \leq \frac{\alpha_E}{\alpha_I} < \frac{64 - 23\beta^4 - 5\beta^6}{32\beta + 6\beta^3 - 2\beta^5}.
\] (19)
Just as in Inequality (18), the function of $\beta$ that is defined by the expression on the right-hand side of Inequality (19) is unbounded above and decreasing and approaches 1 as $\beta$ approaches 1.

Proof of Lemma 3.

**Proof.** A price-capped incumbent achieves positive sales of its retail good at input price $w$ if and only if
\[
q_I(p_I(w), p_E(w)) > 0.
\] (20)
In view of Lemma 2, we can substitute $c_I$ in for $p_I(w)$ into Inequality (20), which can then be rewritten as
\[
\frac{\alpha(2 + \beta)(1 - \beta) + (\beta^2 - 2)c_I + \beta w + \beta c_E}{2(1 - \beta^2)} > 0.
\] (21)
Because $0 < \beta < 1$, Inequality (21) holds if and only if the numerator in the left-hand side term is positive. That condition can be rewritten as
\[
w > \alpha_E - \alpha_I \cdot \left( \frac{2}{\beta} - \beta \right).
\] (22)
The result now follows by observing that the expression on the right-hand side of Inequality (22) is negative (and hence Inequality (22) holds for every $w \geq 0$) if and only
if $\alpha_E/\alpha_I < 2/\beta - \beta$. In particular, this latter condition holds whenever $\alpha_I > \alpha_E$, and, for fixed values of $\alpha_E$ and $\alpha_I$, it holds for $\beta$ sufficiently close to zero (i.e., when the retail products are sufficiently differentiated).

**Proof of Proposition 5.**

Proof. Suppose that $w^*(k) < c_I$. Then, if we show that the difference between the derivatives of regulated and unregulated incumbent profits with respect to $k$, $(\tilde{\pi}_I^D)'(k) - (\pi_I^D)'(k)$ is positive when $k = 0$ and $k = 1$, then because both $(\tilde{\pi}_I^D)'(k)$ and $(\pi_I^D)'(k)$ are affine functions of $k$, this proves that the unique equilibrium level of $k$ is higher under regulation than without.

$$
(\tilde{\pi}_I^D)'(0) - (\pi_I^D)'(0) = \frac{c_I[\alpha_I(16 - 14\beta^2 - \beta^4) - \alpha_E\beta^3]}{4(8 - 7\beta^2 - \beta^4)}.
$$

Equation (23) is increasing in $c_E$ and $\alpha$, so that if it is positive for $c_E = 0$ and the lowest value of $\alpha$ that satisfies our assumptions, then it holds for all $c_E$ and $\alpha$. Note that $c_E = 0$ implies that Inequality (5) holds, so that foreclosure does not occur. Moreover, Inequality (9) stipulates an infimum for $\alpha$. Substituting $c_E = 0$ into Inequality (9) yields the following infimum for $\alpha$

$$
\alpha = \frac{c_I(2 - \beta)}{(1 - \beta)}.
$$

Substituting $c_E = 0$ and Equation (24) into the right-hand side of Equation (23) yields

$$
\frac{c_I^2[8 + \beta(8 + \beta)]}{2(8 - 7\beta^2 - \beta^4)} > 0,
$$

which proves that $(\tilde{\pi}_I^D)'(0) - (\pi_I^D)'(0) > 0$ when $w^*(k) < c_I$.

$$
(\tilde{\pi}_I^D)'(1) - (\pi_I^D)'(1) = \frac{c_I[4\alpha_I(8 - 7\beta^2 - \beta^4) - \alpha_I(16 - 14\beta^2 - \beta^4) - \alpha_E\beta^3]}{4(8 - 7\beta^2 - \beta^4)}.
$$

Equation (25) is also increasing in $c_E$ and $\alpha$, so that we may proceed as above, by substituting $c_E = 0$ and Equation (24) into the right-hand side of Equation (25). This yields

$$
\frac{c_I^2\beta(16 + 16\beta + 3\beta^3)}{4(8 - 7\beta^2 - \beta^4)} > 0,
$$

completing the proof for the case of $w^*(k) < c_I$. 

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Now suppose that $w^*(k) = c_I$. The proof of this case is analogous. In particular, the first-order condition analogue to Equation (11) in this case is

$$
\kappa'(k^*) = \left(\pi^D_I\right)'(k^*) = \frac{c_I[\alpha_I(2 - \beta - \beta^2) + c_E\beta]}{2(1 - \beta^2)}.
$$

It is readily shown that both $\left(\pi^D_I\right)'(0) - \left(\pi^D_I\right)'(0)$ and $\left(\pi^D_I\right)'(1) - \left(\pi^D_I\right)'(1)$ are increasing in $c_E$ and $\alpha$. Substituting $c_E = 0$ and Equation (24) into these expressions yields respectively

$$
c^2_I[8 + \beta(8 + \beta)] > 0, \quad c^2_I\beta(8 + 4\beta + \beta^3) > 0,
$$

completing the proof. \hfill \Box

References


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