Measuring the Effectiveness of Anti-Cartel Interventions: 
A Conceptual Framework

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Abstract

This paper develops a model of cartel behaviour in the presence of enforcement activities by a Competition Authority (CA). We distinguish three sets of interventions: (a) detecting, prosecuting and penalising cartels; (b) actions to bring cartel activity to an end in the short-term, immediately following successful prosecution; (c) actions that aim to prevent the longer term re-emergence of collusive activity in industries where a cartel has been successfully prosecuted. The last two intervention activities have not been previously analysed. In addition, we take account of the structure and toughness of penalties. We identify the impact of CA enforcement activities on various outcomes - the levels of disruption and deterrence of cartel activity and cartel pricing – which we then map into welfare measures of CA performance. We derive measures of both the total and marginal welfare effects of enforcement activities and decompose these into a Direct Effect (due to cartel activity being disrupted) and two Indirect/Behavioural Effects – on deterrence and pricing. Finally, we calibrate the model and estimate the fraction of the harm that CAs remove as well as the total and marginal welfare effects of interventions. We show that these effects can be substantial and also that estimate of these effects in the existing literature depend crucially on the special assumptions made.

JEL Classification: L4 Antitrust Policy, K21 Antitrust Law, D43 Oligopoly and Other Forms of Market Imperfection, C73 Stochastic and Dynamic Games; Repeated Games

Keywords: Antitrust Enforcement, Antitrust Law, Cartel, Oligopoly, Repeated Games.
1. Introduction

European and UK Competition Authorities (hereafter CAs) use a range of methodologies for calculating the direct impact of competition policy interventions on consumers. However, they still struggle to find proper methodologies to assess and estimate the indirect/behavioural effects of their work on deterrence and pricing. Such methodologies are highly relevant for CAs as they would allow them to assess both deterrence and price effects as well as the total effect of their interventions, and also provide information that assists in determining the prioritisation of resources. It is also very important to understand better how the effectiveness of various interventions varies depending on differences in competition policy enforcement. The aim of this paper is to provide a conceptual framework that addresses all these issues. In particular, it aims to examine the total impact on aggregate welfare of interventions by a competition authorities. While the analysis is conducted in the context of anti-cartel interventions, much of it will be applicable to other forms of intervention.

Our work is close to that by Harrington and Chang (2009, 2015). Harrington and Chang (2009) in particular is a major contribution since it is the first paper to establish conditions under which observable changes in the number of discovered cartels and the duration of cartels can be effective proxies for the change in the unobservable total number of cartels in existence and so can help us understand how this is affected by specific antitrust policies. Our work differs from theirs since our objective is to provide a general framework for evaluating the effects on welfare of a wider range of policy instruments than they consider, and since it captures a more comprehensive set of effects, encompassing: the direct effect of detecting and stopping cartels; the indirect-deterrence effect; an indirect-price effect which captures the effect of policies on the price set by those cartels that do form. Moreover we consider both the total and marginal

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6 The issues are also of considerable relevance for other enforcement agencies – e.g. tax authorities.

7 A comprehensive survey of the methodologies currently used for assessing the effectiveness of a CA in enforcing competition policy, including policy towards cartels, is contained in Davies and Ormosi (2010 and 2012).

8 Evaluations of Competition Policy in general are contained in Baker (2003) and Werden (2008). Davies and Ormosi (2010), review a range of methodological approaches to evaluating competition policy. Studies using particular approaches to evaluate specific aspects of competition policy are set out in Harrington and Chang (2009, 2015), Miller (2009) and Ormosi (2014) – the first three of which are concerned with the effects of antitrust fines and leniency programs on deterrence and the last one offering an alternative methodology to identifying the fraction of detected cartels. For experimental work trying to obtain a direct measurement of deterrence in a controlled laboratory environment see Bigoni et al. (2012, 2015). Finally, there is work using survey methodologies such as that of Deloitte (2007) and London Economics (2011) commissioned by OFT showing, respectively, that for each cartel detected by a CA at least another 5 (28) are deterred.


10 Harrington and Chang (2015) extend this approach to analysis of the effects of leniency programs.

11 The policy instruments they examine is a tougher penalty regime and an increase in the detection rate. As we will see below we consider these instruments and an additional two instruments that affect the re-emergence of collusive behaviour following successful prosecutions. Essentially, Harrington and Chang (2009) do not examine policy instruments that affect their parameter κ, the cartel rebirth rate.

12 We measure this by considering the fraction of those cartels that would have existed in the absence of a Competition Authority that continue to exist in the presence of a CA and its enforcement activities.
effects of a CA’s interventions and show how these can be decomposed into these three effects.

Recent work by Davies and Ormosi (2014) has similar objectives to ours in that they want to measure the welfare effects of a general range of interventions and show how these can be broken down into direct and indirect effects. The question they address is: “how much harm can cartels cause to an economy and how successful are CAs in rectifying that harm?” While we also address this question, their conceptual framework and modeling approach is purely statistical and so is very different to ours. They start from the amount of harm that a CA removes as a result of investigating and prosecuting cartels – essentially the Direct Effect that CAs think they can measure that was referred to in our opening quotation. Davies and Ormosi call this Detected Harm. They take as exogenous both the proportion of cartels that are deterred as a result of CA interventions and the proportion of undeterred cartels that are detected, and, assuming certain distributions of these two parameters, work backwards to obtain the distributions of potential harm; of undetected harm and of deterred harm and so various measures of the effectiveness of the CA. Their approach differs fundamentally from ours since they have no model of cartel behaviour and so are unable to examine: (i) the effect of CA interventions on cartel pricing; (ii) marginal effects of CA interventions; (iii) the effect of other enforcement parameters, such as the penalty rate, on the effectiveness of CA interventions.

Davies and Ormosi (2014) treat detection of harm by a CA as equivalent to the removal of harm by a CA. Indeed on p.8 of their paper they talk about the harm a CA “removes in the case it detects and intervenes”. Other papers in the literature – e.g. Harrington (2004, 2005) or Houba et al. (2010) – also assume that the detection and successful prosecution of a cartel (leading to the imposition of a penalty) brings it to an end. The polar opposite assumption - that cartels continue in existence even if they are detected, successfully prosecuted and penalised - is made in other papers in the literature, e.g. Motta and Polo (2003) or Katsoulacos et al. (2015). However, this assumption also seems implausible, because if, following a successful

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13 Davies and Ormosi (2014), page 3.
14 Bos et al. (2016) contains analysis of the effects of a CA’s penalty regime on cartel pricing. The results largely replicate those in the literature – e.g. Katsoulacos and Ulph (2013) or Houba et al. (2010) – in showing that, when the penalty is imposed on profits then, if the probability of detection increases with the price set by the cartel, the cartel price will be below the monopoly price. In addition, Bos et al. (2016) also analyse the restrictions imposed by cartel stability condition on collusive prices and compare the impact of these restrictions on pricing by both legal and illegal cartels.
15 It is interesting to note that when we set the values of our key intervention parameters to those implicitly assumed by Davies and Ormosi (2014), our calculations of the fraction of potential harm removed by a CA are consistent with theirs, showing that a CA can remove between 55 and 85% of all potential harm depending on the effort devoted to detection and prosecution. However, we also show that this conclusion is at the top end of the range and is very sensitive to Davies and Ormosi’s implicit assumptions about the key intervention parameters. Moreover our calculations of other measures of the effectiveness of CA interventions such as the ratio of the Total Effect to the Direct Effect also differ from theirs, showing that the difference in methodologies really matters.
16 A justification for this assumption is that, when cartels form they anticipate the possibility of being detected, prosecuted and penalised; consequently, when this actually happens, it is treated as just a realisation of one of many anticipated risks they face, and, if nothing else has changed, if it was profitable to be in a cartel before prosecution it would remain profitable after prosecution.
17 Note also that under this assumption (that cartels continue in existence after prosecution) CA interventions would have no Direct Effect, so it is an inappropriate basis for studying the relationship between the Direct Effect and Indirect Effects of CA interventions.
prosecution, CAs implement certain interventions, some things might change.\textsuperscript{18} For example, depending on the sanctioning regime in force, all the key personnel involved in operating the cartel might be jailed, or even debarred, so the capacity to form a cartel might be removed. In addition, CAs might intensively monitor prices and other activities in that industry, increasing the risk of detection, and so reducing the incentive to form a cartel.

Empirical evidence on this issue is mixed. Davies and Ormosi (2010) cite evidence that cartels re-emerge in industries in which a cartel had previously been prosecuted by a CA.\textsuperscript{19} The evidence reviewed by Werden et al. (2011) suggests that it is not a serious phenomenon in the US, but there is no clear indication about its existence in other countries. Connor (2015) argues that “recidivism is rampant”.\textsuperscript{20,21} Ormosi (2014) fits a capture-recapture model to data on European firms involved in prosecuted cartels. He shows that in the year after prosecution 75\% of prosecuted firms are no longer likely to be caught in collusive activity, but that this figure drops to 10\% in the long run.

The paper closest to ours on this issue is that of Harrington and Chang (2009) who assume that successful prosecution of a cartel in an industry will bring cartel activity to a stop immediately after prosecution but allow for the possibility that a cartel may re-emerge in the same industry at a future date. What none of the existing literature allows for is the possibility that, following a successful prosecution, a cartel might come to an immediate end (but also might not), and that, if it does, collusive activity might subsequently re-emerge in the same industry at a later date. This is precisely the possibility that underpins the framework developed in this paper. To capture this set-up we introduce three intervention parameters:

- the probability that in any given period a cartel will be successfully prosecuted and penalised.\textsuperscript{22}
- the probability that, in the period immediately following a successful prosecution the industry reverts to competitive behaviour;\textsuperscript{23}
- the probability that, if an industry in which a cartel was previously prosecuted is acting competitively at the start of any period, it continues to act competitively at the start of the

\textsuperscript{18}Houba et al. (2012) make an intermediate assumption that, following a successful prosecution, cartels either stop or continue in existence with some probability.
\textsuperscript{19}Aguzzoni et al. (2013) note that many European firms from the sample they analyse are repeat offenders in the sense that these firms have infringed competition law in different product markets and different countries.
\textsuperscript{20}Connor (2015) slide 44 asserts that in his sample there are: “at least 70 companies with ten or more violations between 1990-2015; scores of same market/same nation cases; many sequential violations, some of which are overlapping and so may not meet the start/stop/start legal definition of recidivism”.
\textsuperscript{21}One has to treat this evidence with considerable caution since Connor defines recidivism as a given firm being involved in more than one cartel even though the cartel may be contemporaneous and in a different industry. We are concerned with the possibility that, in industries in which cartels have been detected and prosecuted, collusive behaviour re-emerges at a later stage, something completely different to recidivism in Connor’s sense.
\textsuperscript{22}As in Harrington (2011) one could think of the probability of being successfully prosecuted as being the product of three probabilities: probability of detection, probability of investigation if detected; probability of successful prosecution if investigated.
\textsuperscript{23}This probability can be viewed as a behavioural parameter, but it is affected by the activities of CAs. In practice, industries where cartels have recently been discovered are monitored more closely by CAs. These additional interventions effectively should change cartel behaviour and the likelihood of cartel formation in the period following a successful prosecution.
next period.24

The first parameter reflects resources that a CA puts into detection, investigation and prosecution - activities on which much of the literature has focussed.25

The second and third parameters reflect resources that CAs puts into preventing re-emergence of collusive behaviour following successful prosecutions. To justify having two parameters, it is essential to distinguish between:

- **short-term interventions** which are implemented for a limited period of time immediately following the successful prosecution of a cartel and which aim to prevent the continuation of price-fixing behaviour by the prosecuted cartel,26 and
- **longer-term interventions** potentially involving sustained monitoring of activities in industries in which a cartel has previously been prosecuted.

The reason for making the distinction is as follows. For the relatively small number of industries in which cartels have recently been prosecuted, it is plausible to assume that, for a limited period of time, CAs can monitor activities with an intensity that ensures that price-fixing activity is brought to an end in the short-run with a very high probability – possibly even unity, as assumed by Davies and Ormosi (2014) and Harrington and Chang (2009). Extending their framework and treating this probability parametrically allows us to capture more realistic settings where the successful prosecution of a cartel by a CA does not always bring cartel activity to an end, even in the short-run. In addition, such an extension also allows to test the sensitivity of conclusions to this assumption. However it is implausible to assume that CAs could indefinitely sustain this intense level of monitoring in every industry in which they have ever prosecuted a cartel. Consequently, in the long term the probability of collusive activity re-emerging is likely to be higher than in the short-term. We capture this through the third intervention parameter above, for which the only restriction we impose is that the probability is less than unity.

It should be noted that these interventions relating to efforts that the CA puts into preventing cartels from re-establishing have not received much if any attention in the literature. As we will show, the strength of interventions that affect the re-emergence of collusive behaviour has substantial effect on the strength of other interventions on which previous literature focuses (such as Davies and Ormosi (2014)).27

These three intervention parameters give us a very rich framework within which to conceptualise the intervention activities of a CA, and one that encompasses the existing literature as special cases.

A final feature of our framework is that we explicitly take into account the nature of the

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24These last two parameters reflect in part the behaviour of firms facing certain post-prosecution activities by CAs, rather than being direct measures of such intervention activity. For tractability reasons we treat them as exogenous.
25Following standard approach we assume that this is strictly less than 1 to capture the fact that CAs don’t have the resources to investigate every industry all the time.
26Examples of such short term interventions are: closer control and monitoring of CAs in the markets, where cartels are just discovered; the imposition of remedies for a limited period of time in order to restore competitive environment; and, in some jurisdictions, jail or debar officials that were responsible for price-fixing.
27Bos et al. (2016) also show that both the amount of effort put into detecting and prosecuting cartels and the toughness of the penalty regime have a significant impact. However, our analysis shows that depending on how successful CAs are in preventing re-emergence of collusive behaviour following prosecutions, the effect of those other interventions may be substantially lower than that predicted in that paper as well.
penalty regime, both the penalty base\textsuperscript{28} and the penalty rate - which is our fourth enforcement parameter.\textsuperscript{29} This enables us to address the final issue of concern to CAs as captured in our opening paragraph – how the effectiveness of interventions depends on differences in policy enforcement. We can also examine the effectiveness of intervention activities relative to that of raising the penalty rate.

We show that the Direct Effect that CAs measure is an imperfect measure of the true Direct Effect, essentially because it fails to reflect the dynamic nature of the environment within which CAs operate where cartel activity is only being disrupted by CA interventions not brought to a complete halt. While the true Direct Effect can sometimes be a little lower than what CAs can measure –up to 10% – it can also be substantially higher – up to 3 times higher. The Total Effect of CA interventions can be between 7.5 – 8.5 times as large as the Direct Effect that CAs measure. However, the magnitude of this effect will be substantially smaller the less effective are CAs in preventing the re-emergence of collusive activity.

We show that there are complex interactions between the various enforcement parameters which can act through very different channels. In addition, we derive explicit expressions showing the complex interaction between the disruption and deterrence of cartel activity, such that the magnitude of the Direct Effect is greatest when the level of disruption is 50%. Another observation is that if penalties are based on revenue then both the total and marginal Indirect Price Effects of interventions can be negative.\textsuperscript{30}

Our numerical analysis also shows that the marginal effect of an increase in the penalty rate is around ten times smaller than the marginal effect of an increase in the probability of successful prosecution.\textsuperscript{31} Potentially the most powerful marginal effect is that of longer-term interventions aiming at preventing re-emergence of cartels – but this is very sensitive to the existing level of CA efforts. These results on marginal effects should be helpful in determining the prioritization of resources and are highly relevant for CAs as was announced in our opening paragraph.

The next section sets out a formal model. In section 3 we map our four enforcement parameters into three outcome measures of the impact of CA activity on cartel behaviour; the level of disruption of collusive activity, the level of deterrence and the cartel price. In section 4 we map these outcomes into welfare measures of CA performance and decompose these welfare effects into direct and indirect effects. Section 5 analyses marginal effects of changes in enforcement parameters. Section 6 provides numerical calculations of the direct and indirect effects. Section 7 concludes.

\textsuperscript{28} Katsoulacos et. al. (2015) show that the cartel price can be above or below the monopoly price depending on which penalty based is used. We show this matters for the sign of both the total and marginal indirect price effect.

\textsuperscript{29} We refer to this as an enforcement parameter rather than an intervention parameter because the level of the penalty rate does not depend on the ongoing deployment of resources by the CA.

\textsuperscript{30} It should be noted that this negative effect result emerges for sure only in the case of revenue-based penalties where the penalty rate is constant. A cartel price lower than the monopoly price may emerge even with revenue based penalty if the penalty rate is an increasing function of the price-overcharge. In setting out our model we allow a very general specification of the penalty function, which encompasses both of the above alternatives as well as penalties on other bases such as those considered in Katsoulacos, et al. (2015), Houba et al. (2010). It is only in the simulation analysis that we consider revenue-based penalties with a fixed penalty rate.

\textsuperscript{31} It is important to be clear that this statement refers solely to the percentage impact on harm of increasing certain enforcement parameters by a given percentage. It takes no account of the resource costs of bringing about this percentage change.
2. The Model

There is a continuum of industries, each producing a homogeneous product with identical constant unit cost $c$ and demand function $Q(p)$. Industries differ in the exogenous number of firms, $n$, that operate in each industry, as well as in some other parameters as specified below. Within each industry $n$ symmetric firms compete in prices.\(^{32}\) We let $\pi(p) = (p-c)Q(p)$, $p \geq c$ denote industry profits when the price is $p$. It will also be useful to let $p^M(c)$ (resp. $\pi^M(c) \equiv \pi(p^M(c))$) denote the monopoly price (resp. profits) which would prevail if the market were served by a monopolist with constant unit costs $c$. This price (resp. profit) is taken to be a strictly increasing (resp. decreasing) function of $c$.

We let $p^C > c$ denote the price that will prevail if the industry is cartelized. We model the cartel as an infinitely-repeated collusive game supported by a simple grim trigger strategy profile in the presence of antitrust enforcement.\(^{33}\) In every period, the $n$ symmetric firms decide whether to collude and, if so, at what price. So long as firms are colluding, then, with constant\(^{34}\) probability $\beta$, $0 < \beta \leq 1$, in each period the CA may detect and successfully prosecute the cartel, in which case it imposes a penalty at the constant rate $\rho > 0$ on a strictly positive penalty base $B(p^C) > 0$, which may vary with the cartel price - so it can be a fixed penalty or a penalty based on revenue, profits, or overcharge.\(^{35}\)

We generalize the assumptions in the literature about the behaviour of cartels after detection and prosecution.\(^{36}\) We assume that there is a probability $\sigma$, $0 \leq \sigma \leq 1$ that collusive activity will be shut down following detection and prosecution and so the industry will move to the competitive state at the start of the next period. In addition, following prosecution there is a constant per-period probability $\lambda$, $0 \leq \lambda \leq 1$ that if the industry is competitive at the start of a period, it remains competitive at start of next period. We can think of $\sigma$ (resp. $\lambda$) as measuring the short–term (resp. long-term) efforts by a CA to prevent the re-emergence of collusion following a cartel prosecution.\(^{37}\)

\(^{32}\)There is Bertrand competition, so when an industry acts competitively price is driven down to costs and profits are zero. Also if a cartel is to be able to raise price above costs all $n$ firms have to be in it.

\(^{33}\)This approach is widely accepted and has been employed in e.g. Motta and Polo (2003), Harrington (2005), Houba et al. (2012, 2015), or Katsoulacos et al. (2015).

\(^{34}\)The assumption of a constant probability means that the probability is independent of the past history of detection.

\(^{35}\)The penalty base in our framework does not depend on cartel duration. In a stationary infinitely-repeated game framework it is not feasible to incorporate cartel duration explicitly, though, to the extent that, in a stationary setting, duration depends on the probability of conviction, we could capture this by allowing the penalty base to depend also on $\beta$. Notice also that our framework is sufficiently general to capture the possibility that the penalty rate varies with the cartel price.

\(^{36}\)Harrington (2004, 2005) and Houba et al. (2010) assumes that the cartel stops after detection and prosecution, while Motta and Polo (2003) and Katsoulacos et al. (2015) have an opposite assumption, i.e. the cartel restarts with probability 1 following detection and prosecution. Harrington and Chang (2009) introduce parameter $\kappa$, the cartel rebirth rate. However, they do not examine policy instruments that affect this parameter.

\(^{37}\)Strictly speaking $\sigma$ and $\lambda$ are not pure intervention parameters. They reflect both intervention activities of CA’s and behavioural responses to those interventions. However, standard repeated games framework does not allow endogenizing these responses.
Given these assumptions it is straightforward to show that the expected present value of profits to a single firm from participating in a collusive agreement with price \( p^c \), when facing intervention parameters \( \beta, \sigma, \lambda \) and a penalty imposed at rate \( \rho > 0 \) on a base \( B(p^c) > 0 \) is:

\[
V(p^c) = (1 - d) \left[ \pi \left( p^c \right) - \beta \rho B(p^c) \right] \frac{\delta}{n(1 - \delta)}
\]

where \( \delta, 0 < \delta < 1 \) is the discount factor, and

\[
d = \frac{\delta \beta \sigma}{(1 - \delta \lambda) + \delta \beta \sigma} \Rightarrow 0 \leq d \leq 1,
\]

measures the level of disruption of collusive activity in an industry in which a cartel forms. It can be interpreted in two ways. Note first that if cartels continue in existence even after being detected and penalised, then \( \sigma = 0 \), which implies \( d = 0 \) and the corresponding expression in (1) is then precisely the expression given for the expected present value of profits of a cartel member in Katsoulacos et al. (2015). So \( d \) is the fractional reduction in the expected present value of cartel profits brought about by the disruptive interventions by the CA which may force the industry to behave competitively at least for some periods. The alternative interpretation is that \( d \) measures the fraction of future time that a cartel that has been detected and penalised will be forced to behave competitively because of the disruptive interventions of a CA.

If a firm defects from the collusive agreement in a particular period it sets a price \( p^D(p) < p^c \) that undercuts the cartel price and, for a single period, gets the entire industry profits at that lower price. Additionally, the defector incurs no penalty that would arise should the cartel be investigated and prosecuted in that period.\(^{39}\) The defector will wish to obtain the greatest profits it can, so, if the cartel sets a price above the monopoly price \( p^M(c) \) the defector will set the monopoly price, while, if the cartel sets a price at or below the monopoly price, the defector slightly undercut that price and so effectively takes the cartel profits. Consequently, the one-period defection profits are:\(^{40}\)

\[
\pi^D(p^c) = \begin{cases} 
\pi^M(c), & p^c > p^M(c) \\
\pi(p^c), & c \leq p^c \leq p^M(c).
\end{cases}
\]

\(^{38}\)This expression is obtained by solving the system of two recursive equations which define the expected present value of industry profits if the industry is in a cartelised/collusive (respectively competitive) at the start of any period, \( \bar{V}^{\text{CART}} \) (resp. \( \bar{V}^{\text{COMP}} \)):

\[
\bar{V}^{\text{CART}} = (1 - \beta)\left[ \pi \left( p^c \right) + \delta \bar{V}^{\text{CART}} \right] + \beta \left[ \pi \left( p^c \right) - \rho B(p^c) \right] + \delta \left[ \sigma \bar{V}^{\text{COMP}} + (1 - \sigma)\bar{V}^{\text{CART}} \right]
\]

and

\[
\bar{V}^{\text{COMP}} = \delta \left[ \lambda \bar{V}^{\text{COMP}} + (1 - \lambda)\bar{V}^{\text{CART}} \right]
\]

\(^{39}\)This assumption is also adopted in Motta and Polo (2003). Alternative assumptions are examined in Spagnolo (2004), Buccirossi and Spagnolo (2007), Chen and Rey (2012), Jansen and Sorgard (2014). However, adopting these different assumption would not affect the main results developed in our paper. In particular, the opposite assumption, where price deviating firms are liable, only relaxes the ICC in (4) and does not have any impact on the value function in (1). This implies that level of cartel disruption and collusive prices will not be affected and only level of cartel deterrence can change. However, it will not cause any changes in the measure of cartel harm and CA performance measures developed in section 4.

\(^{40}\)This formula for defection profits was first given in Katsoulacos, Motchenkova and Ulph (2015), who also showed that when penalties are imposed on revenue then the cartel price can be above the monopoly price.
Following the standard grim-trigger strategies, we assume that the other members of the cartel punish the defector by reverting to Nash behaviour for ever more. Then for a cartel to be stable it is necessary that

\[
(1-d) \left[ \pi(p^c)-\tau B(p^c) \right] \geq \pi^D(p^c),
\]

(4)

where, as in Katsoulacos, Motchenkova and Ulph (2015), \( \tau = \beta \rho \) denotes the toughness of the penalty regime\(^1\) and \( \Delta = n(1-\delta) \) is the intrinsic difficulty of holding the cartel together. In what follows we treat \( \tau \) rather than \( \rho \) as a primitive enforcement parameter.\(^2\)

We assume that the cartel sets a price that maximises the expected present value of profits, \( V(p) \), subject to the stability condition (4). So we formally define the cartel price as

\[
p^c = \arg \max_p (1-d) \left[ \pi(p)-\tau B(p) \right] \text{ subject to } (1-d) \left[ \pi(p^c)-\tau B(p^c) \right] \geq \pi^D(p^c) \tag{5}
\]

3. Effects of CA Enforcement Activities

In this section we analyse how the four enforcement parameters (\( \beta, \sigma, \lambda \) and \( \tau \)) affect the degree of competitiveness of an economy with an active CA. For that we model the impact of CA enforcement activities on three central outcomes measures. The first measure is the level of cartel disruption. It is denoted by \( d \). It is the fraction of time when initially cartelized industries end up acting competitively. The second measure is the level of cartel deterrence. It is denoted by \( D \). It determines the fraction of industries in which cartels would have formed in which they do not form given the presence of a CA. The third measure is the level of the cartel/collusive price, \( p^c \), that will prevail in an industry when collusive behaviour is taking place.\(^3\) In section 4 we will map these outcomes into various welfare measures.

3.1. Cartel Disruption

Comparing the expression for expected present value of collusive profits in (1) to that in Katsoulacos et al. (2015), which is given by \( V(p^c) = \left[ \pi(p^c)-\beta \rho B(p^c) \right] / \Delta \),\(^4\) implies that the variable \( d \) measures the fractional reduction in the value that a cartel would have made had it stayed in existence for ever that it actually makes given disruptive interventions by a competition

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\(^1\) However, given the dependence of \( \tau \) on \( \beta \) readers will have to bear in mind that, if the penalty rate \( \rho \) is treated as the primitive parameter, then changes in \( \beta \) will give rise to additional effects through their impact on \( \tau \).

\(^2\) Notice that \( \Delta \) can vary across industries with \( n \). We can also allow for some variation in \( \delta \) across industries. In addition, we can allow for the possibility of some variation in \( \delta \) away from the “typical” value assumed in the rest of the analysis. Given the analysis of deterrence that we undertake below, this may help explain why some specific industries have stable cartels while in others they fall apart very quickly.

\(^3\) The latter two measures relate to cartel behavior and so to the indirect effects of CA enforcement activities.

\(^4\) Note in Katsoulacos et al. (2015) it is assumed that cartels always remain in existence following a successful prosecution. So, in that paper we effectively assumed that \( \sigma = 0 \), which, from (2), implies \( d = 0 \).
authority. Recall these interventions can disrupt/impede cartel activity through prosecution and subsequent interventions/monitoring to prevent re-emergence of collusion. Collectively these interventions cause the industry to act competitively for certain periods of time, and one can think of \( d \) as measuring the fraction of time in which the industry is competitive.

In the presence of an active CA carrying out investigations and prosecutions, i.e. when \( \beta > 0 \), expression (2) implies the following comparative-static results:

**Proposition 1**

(i) \( \sigma = 0 \Rightarrow d = 0 \) \textit{whatever the values of} \( \beta \) \textit{and} \( \lambda \)

(ii) \( \sigma > 0 \) \textit{then} \( d \) \textit{is strictly increasing in} \( \beta, \sigma \) \textit{and} \( \lambda \), \textit{taking a maximum value} \( \tilde{d} = \delta > 0 \) \textit{when} \( \beta = \sigma = \lambda = 1 \).\(^{45}\)

(iii) \textit{the toughness of the penalty regime,} \( \tau \), \textit{has no effect on} \( d \).

Part (i) of Proposition 1 implies that in the absence of interventions aimed at shutting down cartels in the short run, the level of cartel disruption will also be zero. Part (ii) of the proposition implies that the greater the values of enforcement parameters \( \beta, \sigma \) \textit{and} \( \lambda \) the larger will be the increase in cartel disruption level \( d \). Also (2) implies that there are strong complementarities between the intervention parameters in terms of their impact on cartel disruption. While the penalty rate, \( \rho \), has no effect on cartel disruption.

3.2. Cartel Pricing

If a cartel exists it will choose the price to maximise the expected present value of its profits subject to the ICC in (5). As shown in Katsoulacos, Motchenkova and Ulph (2015) in a wide range of cases of interest – e.g. fixed penalties, penalties that are based on revenue or profits – the optimum price is just that which maximises the expected value of profits without any constraint, so, drawing on that analysis we will take it that the cartel price is the unconstrained profit-maximising collusive price:

\[
\begin{align*}
p_C^* &= \arg \max_{\rho \geq c} \left[ \pi(p) - \tau B(p) \right].
\end{align*}
\] (6)

We then have the following results regarding the cartel price:\(^{46}\)

**Proposition 2**

(i) \textit{The collusive price depends solely on} \( \tau \), \textit{the toughness of the penalty regime}.

(ii) \( \tau = 0 \Rightarrow p_C^* = p^M(c) \).

\(^{45}\) Notice that it then follows from (1) that \( V(p_C^*) = \frac{\pi(p_C) - \rho B(p_C)}{n} \) since, under these conditions, a cartel will only last for a single period, during which it will definitely be penalised.

\(^{46}\) See technical Appendix for detailed proof.
(iii) If $\tau > 0$ then, depending on what penalty base is used, the cartel price can be above or below the monopoly price;

(iv) The cartel price is a strictly increasing (resp. decreasing) function of $\tau$ depending on whether the cartel price is above (resp. below) the monopoly price. Formally,

$$\frac{dp^C}{d\tau} < 0 \quad \text{as} \quad p^C > p^M.$$ 

The first implication of this proposition is that the cartel price is a function purely of the toughness of the penalty regime, $\tau$, and it is independent of the intervention parameters $\sigma$ and $\lambda$ which reflect CAs efforts to prevent re-emergence of collusive behaviour following successful prosecutions. Part (ii) of Proposition 2 implies that in the absence of an active CA or if penalties are zero (i.e. when $\tau = 0$) cartel will set monopoly price. While part (iii) shows that in the presence of an active CA (i.e. when $\tau > 0$) depending on what penalty base is used, the cartel price can be above or below the monopoly price. Finally, part (iv) shows that depending on the penalty base cartel price can be either increasing or decreasing function of $\tau$. This last result is a generalization of related results in Katsoulacos et al. (2015).

3.3. Cartel Deterrence

Given the cartel price we can define the critical level of difficulty, $\Delta$, for which the ICC just holds. From (4) this is defined by:

$$\Delta = (1 - d) \frac{\pi(p^C) - \tau B(p^C)}{\pi^D(p^C)}. \quad (7)$$

It then follows that for industries, in which $\Delta \leq \bar{\Delta}$, the inequality in (4) holds and so stable cartels exist. However, the disruptive interventions of the CA reduce the expected cartel profits by a fraction $d$. While for those industries, in which $\Delta > \bar{\Delta}$, the inequality in (4) fails to hold, no stable cartels exist and these industries remain in a state of permanent competition.

Now we can re-write (7) as

$$\Delta = (1 - d) \bar{\Delta}_0, \quad (8)$$

where

$$\bar{\Delta}_0 = \frac{\pi(p^C) - \tau B(p^C)}{\pi^D(p^C)} \quad (9)$$

is the critical level of difficulty that would arise if $\sigma = 0 \Rightarrow d = 0$ and so CA interventions never disrupt collusive activity, even temporarily.$^{47}$

$^{47}$ $\bar{\Delta}_0(\tau)$ is the expression for the critical level of difficulty that appears in Katsoulacos, Motchenkova and Ulph (2015), where, as pointed out above, it is assumed that $\sigma = 0$. 
Notice that if there were no active CA (i.e. $\beta = 0$ and $\tau = 0$), then we would have $\bar{\Delta} = 1$.\(^{48}\) On the other hand if a CA is active (i.e. $\beta > 0$ and $\tau > 0$), then $\bar{\Delta} < 1$.\(^{49}\) So we can interpret $\bar{\Delta}$ as the fraction of industries in which stable cartels would have existed had there been no active CA for which stable cartels continue to exist when there is an active CA. Equivalently we can say that $D = 1 - \bar{\Delta}$ is the fraction of industries in which stable cartels would have existed in the absence of a CA in which cartels are deterred from forming because of the actions of a CA. Thus $D$ measures the level of cartel deterrence brought about by a CA. From (8) we have:

\[
D = 1 - \bar{\Delta} = D^0(\tau) + d\left(1 - D^0(\tau)\right) = (1-d)D^0(\tau) + d ,
\]

where

\[
D^0(\tau) = 1 - \bar{\Delta}^0(\tau)
\]

is the level of deterrence that the CA would achieve if it never disrupted collusive behaviour.

This shows that deterrence, $D$, grows linearly with the level of disruption, $d$, achieved by the CA, and the rate at which it grows is proportional to the fraction of industries in which cartels would not be deterred if they were never disrupted. Given Proposition 1, $d$ is in turn a function of the three intervention parameters $\beta, \sigma, \lambda$ but not of $\tau$.

Now it follows from (9) and (11), that $D^0(\tau)$ is a function solely of the toughness of the penalty regime, $\tau$, which therefore influences deterrence solely through its effects on $D^0(\tau)$. Notice that the effect of $\tau$ on $D^0(\tau)$ comes in part through its effect on the cartel price, and so, from Proposition 2, to obtain comparative static predictions we have to distinguish cases where the cartel price is above the monopoly price from those where it is below.\(^{50}\)

We have the following lemma:

**Lemma 1:** (i) If $p^c \geq p^m$ then $\frac{dD^0(\tau)}{d\tau} = \frac{B\left(p^c\right)}{\pi\left(p^m\right)} > 0$

(ii) If $p^c < p^m$ then $\frac{dD^0(\tau)}{d\tau} = \frac{B\left(p^c\right)}{\pi\left(p^m\right)} - \frac{\left[p\left(p^c\right) - \tau B\left(p^c\right)\right]}{\left[p\left(p^c\right)\right]^2} p'\left(p^c\right)\left(-\frac{dp^c}{d\tau}\right)$,

where the term in curly brackets is unambiguously positive.

\(^{48}\) If $\beta = 0$ and $\tau = 0$, from Proposition 1 (i) we would have $d = 0$, and also, from Proposition 2 (ii) we have $p^c = p^m$, from which it follows that $\pi\left(p^c\right) = \pi^0\left(p^c\right) = \pi^m \Rightarrow \bar{\Delta}^0(0) = 1$. So if there were no CA in existence then we would have $\bar{\Delta} = 1$.

\(^{49}\) If $\beta > 0$ and $\tau > 0$, then $d \geq 0$ and $\bar{\Delta}^0(\tau) < \frac{\pi\left(p^c\right)}{\pi^0\left(p^c\right)} \leq 1$ where the first inequality follows from the fact that $\tau > 0$, and the second from (3). Taken together with fact that $d \geq 0$ this implies that when there is an active CA then $\bar{\Delta} < 1$.

\(^{50}\) As shown in Katsoulacos, Motchenkova and Ulph (2015) if the penalty rate is independent of the cartel price then the former case arises when penalties are based on revenue – as is the case for most jurisdictions - and the latter when they are based on the cartel overcharge.
Proof: See technical appendix.

Although in the second case the effect of penalty toughness on deterrence is in general ambiguous, in a wide range of circumstances it will be positive. So in all that follows we assume:

$$\frac{\partial D}{\partial \tau} = (1-d) \frac{dD^0(\tau)}{d\tau} > 0.$$  

(12)

Now based on (10) and (12) we can summarise our comparative static predictions about the drivers of deterrence in Proposition 3:

**Proposition 3:**

(i) The level of deterrence depends on both the level of disruption achieved by the CA and on the toughness of the penalty regime through the formula given in (10)

$$D = 1 - \tilde{\Lambda} = D^0(\tau) + d \{1 - D^0(\tau)\}.$$ 

(ii) Actions taken by a CA to increase the level of disruption, \(d\), produced by its interventions also increase the level of deterrence, \(D\). Formally \(\frac{\partial D}{\partial d} = (1 - D^0(\tau)) > 0\).

(iii) For any given level of disruption, \(d\), achieved by the CA, an increase in the toughness of the penalty regime increases deterrence. Formally \(\frac{\partial D}{\partial \tau} = (1-d) \frac{dD^0(\tau)}{d\tau} > 0\).

Finally, combining the results of Proposition 1(ii) and Proposition 3(ii) we obtain that if \(\sigma > 0\), then \(\partial D / \partial i > 0\), \(i = \beta, \sigma, \lambda\). Hence, we can conclude that increases in all three forms of intervention improve deterrence.

The following table summarises all our comparative static predictions about the effects of CA enforcement activities on the three outcomes measures: disruption, deterrence and collusive price.

**Table 1: Summary Comparative Static Predictions**

<table>
<thead>
<tr>
<th></th>
<th>(d)</th>
<th>(D)</th>
<th>(p^C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>+</td>
<td>+</td>
<td>[+, \text{ if } p^C &gt; p^u] [−, \text{ if } p^C &lt; p^u]</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0</td>
<td>+</td>
<td>[+, \text{ if } p^C &gt; p^u] [−, \text{ if } p^C &lt; p^u]</td>
</tr>
</tbody>
</table>

**4. Measuring the Effectiveness of CA Enforcement Activities**

In this section we show how the three outcome measures set out in section 3, namely, the level of cartel disruption \(d\), the level of cartel deterrence \(D\), and the level of collusive price \(p^C\) can be
used to determine the harm created by collusion and the measures of the effectiveness of CA enforcement actions.

Before proceeding we note that the analysis that follows is undertaken in relation to the mass of industries in which cartels would form if there were no CA enforcement activity, since we obviously cannot attribute the lack of collusive activity in any other industries to the action of the CA. We normalise the mass of such industries to 1. We assume that the relevant welfare objective for a CA is consumer surplus and let \( CS(p) = \int_{p}^{1} Q(x) dx \) denote the flow of consumer surplus in a period when the price is \( p \).

### 4.1 Cartels, Welfare and Harm

In this sub-section we measure the harm that is suffered from the collusive activity that arises despite the efforts of a CA that generates outcomes \( (d, D, p^C) \) by comparing the flow of consumer surplus that arises in these circumstances with that which would have arisen under perfect competition.

By an argument analogous to that used to establish (1), the constant per-period flow of consumer surplus generated in any industry in which cartels emerge is

\[
CS = (1 - d)CS\left(p^C\right) + dCS(c).
\]

(13)

Taking account of the fact that stable cartels exist in only a fraction \( 1 - D \) of industries, while the remaining fraction of industries will be competitive and generate a flow of consumer surplus \( CS(c) \), we can define the consumer surplus of the collusive industry that arises in the presence of an active CA that generates outcomes \( (d, D, p^C) \) as:

\[
\overline{CS}\left(d, D, p^C\right) = (1 - D)CS + DCS(c)
\]

\[
= (1 - D)\left( (1 - d)CS\left(p^C\right) + dCS(c) \right) + DCS(c).
\]

(14)

If every industry were perfectly competitive all the time, then the flow of consumer surplus would be \( CS(c) \). So the harm actually suffered, despite the active intervention of a CA that generates outcomes \( (d, D, p^C) \) is:

\[
H\left(d, D, p^C\right) = CS(c) - \overline{CS}\left(d, D, p^C\right).
\]

(15)

Substitute (14) and we get:

\[
H\left(d, D, p^C\right) = (1 - D)(1 - d)\left[ CS(c) - CS\left(p^C\right) \right].
\]

(16)

---

51 Everything that follows will go through in a completely analogous fashion if the CA uses a total welfare standard, and just replace the function \( CS(p) \) with \( TW(p) = CS(p) + \pi(p) \).
The harm $H(d, D, p^C)$ that is actually suffered is just the loss of consumer surplus due to collusion $\left[ CS(c) - CS(p^C) \right]$ multiplied by the fraction of industries $(1 - D)$ in which cartels are not deterred from forming, multiplied again by $(1 - d)$ which is the fraction of time for which, in industries where cartels emerge, collusive activity is not being disrupted by the interventions of a CA.

4.2 Direct and Indirect Effects of Anti-Cartel Enforcement

In this sub-section we measure the effectiveness of a CA that generates outcomes $(d, D, p^C)$, by comparing the harm $H(d, D, p^C)$ actually suffered from the collusive activity that exists despite the CA’s enforcement activities with what that harm would have been had there been no CA. This gives us what we call the total effect (on welfare) of the CA’s enforcement actions. We then address two related questions:

- how to decompose the total effect into a direct effect plus two indirect effects – on price and deterrence;
- what is the size of the currently unmeasured indirect effects relative to the direct effect that CAs think they can currently measure – at least to some degree.\(^{52}\)

Answering the second question helps CAs understand by how much they can scale up their measures of the direct effect to obtain a measure of the total effect. We consider the two issues in turn.

4.2.1 The Total Welfare Effect of a Competition Authority

To measure the welfare effects of having an active CA in place we need to compare the total harm that arises when there is an active competition authority in place - as given by (16) - with that which would have arisen had there been no CA – which, following Davies and Ormosi (2014), we will call Potential Harm.

Now if there were no CA – and so $\beta = 0$ - then, as shown in Section 3.3 we would have $d = D = 0; p^C = p^M$, and so, from (16), the Potential Harm that could have been suffered by the economy from collusive activity would be

$$H^0 = \left[ CS(c) - CS\left(p^M\right) \right].$$  \hspace{1cm} (17)

Consequently the Total Effect (TE) of having an active competition authority is:

$$TE = H^0 - H(d, D, p^C)$$
$$= \left[ CS(c) - CS\left(p^M\right) \right] - (1 - d)(1 - D) \left[ CS(c) - CS\left(p^C\right) \right].$$  \hspace{1cm} (18)

\(^{52}\) See e.g., Ilzkovitz and Dierx (2016).
4.2.2 Decomposing the Total Effect into Direct and Indirect Effects

To decompose this Total Effect into a Direct Effect and two Indirect/behavioural Effects (on price and deterrence) we can re-write (18) as

\[
TE = \left\{ d(1-D) \left[ CS(c) - CS\left( p^c \right) \right] \right\} + \\
\left\{ D \left[ CS(c) - CS\left( p^c \right) \right] \right\} + \\
\left\{ CS\left( p^c \right) - CS\left( p^H \right) \right\}
\]

(19)

The first term on the RHS of (19) is the Direct Effect (DE) of having an active CA in place. For those industries in which cartels are NOT deterred it shows the gain in consumer surplus from blocking cartel activity for a certain fraction of time through disruptive interventions by the CA. We call this Disrupted Harm. It is a close analogue of what Davies and Ormosi (2014) call Detected Harm\(^{53}\) - although, as we will explain below, what they and competition authorities actually measure as the direct effect/detected harm is somewhat different from the Direct Effect as defined above. Notice that the Direct Effect is larger:

- the larger is \( d \) - so the more effective is the CA at disrupting cartel activity;
- the smaller is \( D \) - so the less effective is the CA in deterring cartels;\(^{54}\)
- the higher is \( p^c \) - so the less effective is the CA in driving down the price set by cartels.

So, as also noted by Davis & Ormosi (2014), Sorgard (2015)\(^{55}\) and others,\(^{56}\) a large Direct Effect may not always be a source of congratulation for a CA.

The second term on the RHS of (19) is the Indirect Deterrence Effect (IDE) of having a CA, for it measures the gain in consumer surplus that society obtains because the anticipated enforcement activities of the CA deter cartels from forming in a fraction \( D \) of the benchmark industries. As noted above, we can also refer to this as the amount of Deterred Harm. Notice that the expression for Deterred Harm does not contain \( d \), for the very good reason that, if a CA

---

\(^{53}\)As explained above in footnotes 22 and 24, theirs is a potentially misleading description of what the Direct Effect captures, since, as we stressed above in the introduction and in Proposition 1, in order to have a positive Direct Effect - i.e. in order to have \( d > 0 \) - CAs have to do more than just detect and prosecute cartels – they have to take actions to stop them – even if only temporarily. Davies and Ormosi (2014) partially recognise this because, on p.8, they talk about the harm a CA "removes in the case it detects and intervenes", so they implicitly assume that detection automatically leads to the removal of harm, and so, at the very least, are effectively assuming that \( \sigma = 1 \). To the extent that the removal of harm is permanent they may also be implicitly assuming that \( \lambda = 1 \).

\(^{54}\)This observation suggests there could be potential incentive problems for a CA trying to pursue a policy of directing resources to maximise the total effect of its actions. For if staff performance is assessed on effectiveness of clearing up cases, then there could be advantages in having a weak deterrent effect since there would be a larger pool of cases to be worked with some “low hanging fruit” to be picked.

\(^{55}\)However neither Davies and Ormosi (2014) nor Sorgard (2015) take any account of the indirect price effect, so their comments are based purely on the observation that a large direct effect could be consistent with a weak deterrent effect.

\(^{56}\)See also Ilzkovitz and Dierx (2016) who analyse the interactions between measures of direct and deterrent effects using EC DG Comp data.
deters cartels from forming in certain industries, society obtains all of the benefit from its doing so irrespective of what proportion of harm the CA would have disrupted had these cartels actually formed. However, as noted in Proposition 3 (ii) the better is the CA at disrupting cartel activity, the higher is the level of cartel deterrence, \( D \), and so the greater is the magnitude of Deterred Harm. Also, as with Disrupted Harm, the magnitude of Deterred Harm is greater the higher the price that is set by cartels.

The third term on the RHS of (19) is the Indirect Price Effect (IPE), for it measures the impact on consumer surplus that arises because, anticipating the intervention of the CA, cartels now set the price \( p^c \) rather than the price \( p^M \) that they would have set had there been no CA. Notice that the expression for this effect contains neither \( d \) nor \( D \), because we want to measure both Disrupted and Deterred Harm using the price that would have been set by the cartel had they not been deterred or disrupted, and so the welfare effects of having a price other than the monopoly price accrues not just for the harm suffered but also for both Deterred and Disrupted Harm.\(^{57}\)

So we have the following simple arithmetical decomposition\(^{58}\) of the Total Effect:

\[
TE = DE + IDE + IPE.
\]  

(20)

Figure 1 below illustrates Potential Harm, \( H^0 \), harm actually suffered, \( H \) and the decomposition of the Total Effect \( (TE = H^0 - H) \) into three effects. It is drawn for the case where \( p^c < p^M \).

We noted above that the magnitude of the Direct Effect increases in the level of disruption, \( d \), but decreases in the level of deterrence, \( D \). This is just a straightforward accounting point. But we also know from Proposition 3 (ii) that there is a powerful behavioural effect at work through which the level of deterrence is itself an increasing function of the level of cartel disruption. If we substitute (10) into (19) we get

\[
DE = d(1-d)(1-D^0(\tau))[CS(c) - CS(p^c)].
\]  

(21)

The first part of expression (21), \( d(1-d) \), implies inverted parabolic structure of \( DE \) as a function of \( d \) with maximum at \( d=1/2 \). This gives rise to the following proposition:

**Proposition 4:** For a given level of penalty toughness \( \tau \), and corresponding cartel price, \( p^c \), the magnitude of the Direct Effect is an inverse-U shaped function of the level of disruption, \( d \), and takes its maximum value when \( d = \frac{1}{2} \).

\(^{57}\)Notice also that if \( p^c > p^M \) the Indirect Price Effect will be negative.

\(^{58}\)Once again this is rather different from Davies and Ormosi (2014) whose analysis relies on a multiplicative decomposition.
Notice that, even though there are complex interactions between these three effects, as (20) shows, the total effect is obtained by a straightforward summation of the Direct Effect and the two Indirect Effects.

Figure 1: Total Effect, \( TE = H^0 - H \)

### 4.2.3 Performance Measures

To get a sense of just how large is the Total Effect – and so how effective is a CA - there are two performance measures we could calculate.

First, we measure the Total Effect relative to Potential Harm, \( \frac{TE}{H^0} \). This shows how much of the potential harm the CA is removing.\(^{59}\)

Second, we measure the Total Effect and the Indirect Effects relative to the Direct Effect that CAs can measure. But this involves two rather separate issues:

- how large the Indirect Effects are relative to the Direct Effect as defined above in (19);
- how large the true Direct Effect as defined in (19) is relative to the Direct Effect that a CA might actually be able to calculate using data it can readily observe.

In an obvious notation the ratio of the two indirect effects to the direct effect as defined in (19), are given by:

\[
R_{IDE} = \frac{IDE}{DE} = \frac{D}{(1-D)d}; \quad R_{IPE} = \frac{IPE}{DE} = \frac{\left[ CS(p^c) - CS(p^{IM}) \right]}{(1-D)d\left[ CS(c) - CS(p^c) \right]}. \tag{22}
\]

\(^{59}\) This is also done by Davies and Ormosi (2014).
Now the *direct effect* as defined above in (19) depends on the two terms $D$ and $d$ which are going to be incredibly hard for a CA to actually measure, essentially because they both depend on counterfactuals that are not directly observable:

- the level of deterrence, $D$ depends on having a view about the industries in which cartels might have arisen in the absence of a CA;
- the level of disruption, $d$, is a forward-looking measure of the extent of reduction in the present value of profits that a cartel would have made had it stayed in existence for ever that is brought about by a CA’s disruptive interventions.

What a CA should be able to calculate using data that is readily observable is the number of cartels it actually investigates, penalises and stops in any given period. Within our framework this would be $\beta\sigma(1-D)$, for $(1-D)$ is the fraction of industries in which cartels have not been deterred from forming, $\beta$ is the fraction of these that are investigated and penalised in any given period, and $\sigma$ is the fraction of these that are brought to an end – even if only temporarily. A CA could observe this number even if it cannot observe the individual terms in this expression. If we multiply this by the gain in consumer surplus, $\left[ CS(c) - CS\left( p^e \right) \right]$, that arises from forcing an industry to set the competitive price rather than the cartel price we get what we can call the *Observable Direct Effect* ($ODE$):

\[
ODE = \beta\sigma(1-D)\left[ CS(c) - CS\left( p^e \right) \right].
\]  

(23)

This is the exact analogue of what Davies and Ormosi (2014) call *Detected Harm*. So, using (20) and (22), the amount by which a CA has to scale up its *Observable Direct Effect* to obtain a measure of the *Total Effect* of its actions is:

\[
\frac{TE}{ODE} = \frac{DE}{ODE} \frac{TE}{DE} = \frac{DE}{ODE} \left( 1 + R_{ide} + R_{ipe} \right),
\]  

(24)

where, from (19) and (23) we have:

\[
\frac{DE}{ODE} = \frac{d}{\beta\sigma}.
\]  

(25)

What (24) brings out is that there are two reasons why the *Total Effect* of a CA’s actions might differ from the *Direct Effect* that they can and do measure. The first, which everybody recognises and discusses, is the fact that the *Direct Effect* fails to capture the indirect/behavioural effects of a CA’s interventions. But the second, which has not received any attention, is that the *Direct Effect* that CA’s can measure (the *Observable Direct Effect*) may not actually be the true *Direct Effect*. To get a sense of whether CAs under or over measure the magnitude of the true *Direct Effect* substitute (2) into (25) to get
So if the long-term interventions of a CA to prevent re-emergence of collusive behaviour are weak relative to short term interventions – i.e. if $\lambda < \beta \sigma$ – then $\frac{DE}{ODE} < \delta < 1$ and so a CA’s observed measure of the Direct Effect will overstate the true measure. However, if a CA’s long-term interventions to prevent re-emergence of collusive behaviour are sufficiently strong relative to its short term interventions – specifically if $\lambda > \beta \sigma + \frac{1-\delta}{\delta}$ – then it’s observed measure of the Direct Effect will understate the true measure. In the calibration exercises of Section 6 we show that both outcomes are possible.

5 The Welfare Effects of Changes in Enforcement Parameters

In the previous section we provided various measures of the effectiveness of a CA’s enforcement activities starting from a knowledge of the outcome measures $d, D, p^C$. This is useful in order to get some overall external assessment of a CA’s performance. However CAs are also interested in knowing how best to deploy resources to various enforcement activities, and so may also want to know the effects on welfare of changes in the enforcement parameters $\beta, \sigma, \lambda$ and $\tau$. To do this we need to bring together the analysis of Section 3 which showed the effects of changes in $\beta, \sigma, \lambda$ and $\tau$ on the outcomes $d, D, p^C$ with the analysis in Section 4 which showed how these outcome measures affected welfare.\(^6\)

There are a number of ways of doing this – e.g. numerical simulations of the kind we report in the next section - but in this section we derive analytical results for the reduction in harm brought about by marginal changes in the enforcement parameters.

Before proceeding we draw attention to a change in terminology. In the previous section we obtained an expression for the total effect of a CA’s activities which we then decomposed into a Direct Effect and two Indirect Effects. As we showed, these various effects depend in a complex way on the three outcome measures $d, D, p^C$. We could in principle calculate the marginal impact of a change in each of our enforcement parameters on the Total Effect, the Direct Effect and the two Indirect Effects by first calculating the marginal effect of a change in the enforcement parameter on $d, D, p^C$ and then working out how the induced marginal changes in $d, D, p^C$ affect the Total Effect etc.

Rather than doing this we will calculate what we call the Total Marginal Effect (on harm reduction) of an increase in each of the enforcement parameters and decompose this into what

\(^6\) Of course to best understand how to deploy resources one also needs to take account of the marginal costs of changing various enforcement parameters. This is not something on which we have any expertise, so we confine attention to the marginal effects on welfare, which can still be highly informative when, for example one has two forms of enforcement that have very similar marginal costs.
we call a Marginal Direct Effect and two Marginal Indirect Effects (on deterrence and price) by which we will simply mean the marginal reduction in harm brought about by change in an enforcement parameter working through the induced changes in \( d, D \) and \( p^C \) respectively, keeping the other parameters constant.

Differentiating (16) we get the Total Marginal Effect (TME) of a small increase in enforcement parameter \( k = \beta, \sigma, \lambda, \tau \) as:

\[
-\frac{\partial H}{\partial k} = \left\{ -\frac{\partial H}{\partial d} \left( \frac{\partial d}{\partial k} \right) \right\} + \left\{ -\frac{\partial H}{\partial D} \left( \frac{\partial D}{\partial k} \right) \right\} + \left\{ -\frac{\partial H}{\partial p^C} \left( \frac{\partial p^C}{\partial k} \right) \right\},
\]

(27)

Given the discussion above the three terms on the RHS of (27) are, respectively, the Marginal Direct Effect, the Marginal Indirect Deterrence Effect (MIDE) and the Marginal Indirect Price Effect (MIPE). From Proposition 2(iv) the Marginal Indirect Price Effect is negative if \( p^C > p^M \)

So, analogous to (20), we get a simple decomposition of the total marginal effect of an enforcement action into a direct effect and two indirect effects:

\[
TME = MDE + MIDE + MIPE
\]

(28)

We can re-write (27) in elasticity form:

\[
\left( -\frac{\partial H \cdot k}{\partial k \cdot H} \right) = \left\{ -\frac{\partial H \cdot d}{\partial d \cdot H} \left( \frac{\partial d \cdot k}{\partial k \cdot d} \right) \right\} + \left\{ -\frac{\partial H \cdot D}{\partial D \cdot H} \left( \frac{\partial D \cdot k}{\partial k \cdot D} \right) \right\} + \left\{ -\frac{\partial H \cdot p^C}{\partial p^C \cdot H} \left( \frac{\partial p^C \cdot k}{\partial k \cdot p^C} \right) \right\}
\]

(29)

where, from (16), we have:

\[
\left( -\frac{\partial H \cdot \sigma}{\partial \sigma \cdot H} \right) = \frac{d}{1-d}; \quad \left( -\frac{\partial H \cdot D}{\partial D \cdot H} \right) = \frac{D}{1-D}; \quad \left( \frac{\partial H \cdot p^C}{\partial p^C \cdot H} \right) = \frac{p^C Q(p^C)}{[CS(c) - CS(p^C)]}.
\]

(30)

From Propositions 1-3 it is possible to work out the elasticities relating to the effects of all the various enforcement actions on our three outcome measures and then use (29) and (30) to calculate the relative effectiveness of different actions. As summarised in Table 1 above it is clear that the various actions work through different channels, so there are few simple conclusions at this level of generality. The next section reports some numerical calculations.

6. Illustrative Calculations of the Direct and Indirect Effects
As discussed above, the framework indicates that the direct and indirect deterrence effects are going to be extremely hard to measure in any direct way, since both $d$ and $D$ are defined in relation to counterfactuals that relate to what would have happened in the absence of a CA.

So one approach to trying to calculate the various direct and indirect effects is to use whatever empirical evidence is available to calibrate the model’s parameters; then use the formulae in Section 3 to calculate the three output measures $d, D, p^C$ and hence the formulae in Section 4 to calculate all our various measures of the effectiveness of a CA. In this section we take a very preliminary step in this direction. Our aim is not to provide an exhaustive analysis of how the various direct and indirect effects vary with the underlying parameters, but rather to show that this approach is feasible and to get a feel for the parameters to which our conclusions seems most sensitive.

The model is based on parameters: $\beta, \sigma, \lambda, \delta, \rho, c$, and two functions $Q(p)$ and $B(p)$.

To begin, it is helpful to think about the length of the period on which the model is built. The model is based on the assumption that a CA can carry out a successful cartel investigation in a period. For hard-core cartels it would be reasonable to assume that such an investigation might be successfully concluded in one year. While our model does not relate solely to hard-core cartels we will nevertheless assume that the appropriate interpretation of a period is a year.

With this in mind we discuss in turn the calibration of our various parameters. We begin with the four enforcement parameters, $\beta, \sigma, \lambda$ and $\rho$.

$\beta$: In their analysis Davies and Ormosi (2014), cite empirical evidence that the probability of detection ranges from 0.1 to 0.33, and assume a uniform distribution between these bounds. So we will report results for the three values $\beta = 0.1, 0.2, 0.3$. However, following the work of Bryant and Eckart (1991), a widely used value is $\beta = 0.15$. We have undertaken calculations for this value of $\beta$ as well. Obviously the resulting values for all our various measures of CA effectiveness lie between those obtained for $\beta = 0.1$ and $\beta = 0.2$, though, given the non-linearity of our performance variables such as $\sigma$, they do not lie exactly half way between, though a straightforward interpolation will give a good approximation.\footnote{Precise figures available from the authors on request.}

$\sigma$: As noted above, existing literature,\footnote{Harrington (2004, 2005) and Davies and Ormosi (2014).} effectively assumes that $\sigma = 1$. So, to facilitate comparison with their analysis, we will also allow $\sigma$ to take the value $\sigma = 1$. However, our framework allows us to test the sensitivity of the conclusions to this implicit assumption, so we also examine how our results change if $\sigma = 0.8$.

$\lambda$: As also mentioned above, it is implausible to assume that CAs have the resources to continuously monitor industries after a successful intervention. Moreover, both Davies and
Ormosi (2009) and Connor (2015) cite evidence that cartels do in deed re-emerge in industries following CA interventions. So it is implausible that \( \lambda = 1 \). However, existing literature seems to assume that \( \lambda = 1 \)\(^{63} \). So to understand what drives existing results we want to allow the possibility that \( \lambda \) can be quite large. Accordingly we will undertake calculations for three values of \( \lambda = 0.1, 0.5, 0.9 \).

\( \rho \): We assume throughout that penalties are based on revenue and that neither the penalty rate nor the probability of detection depend on the cartel price. In line with rates that apply in countries that use revenue as the penalty base for the bulk of our analysis we assume a 10% penalty – i.e. \( \rho = 0.1 \). However in order to address the question posed by CAs in the quotation at the start of the paper about how the effectiveness of interventions depends on “differences in competition policy enforcement” we discuss how our conclusions are affected if, instead, \( \rho = 0.2 \). Given our assumptions about the penalty structure, the cartel price will be above the monopoly price and so both the absolute and marginal indirect price effects will be negative.

Our framework also depends on a number of model parameters:

\( c \): We normalise prices by assuming that the competitive price \( c = 1 \).

Consistent with this we assume that the demand function is \( Q(p) = 1 + \varepsilon - p \)\(^{64} \) where \( \varepsilon = -\frac{dp}{dQ} \cdot \frac{Q}{p} > 0 \) is the inverse price elasticity evaluated at the competitive price. Connor and Lande (2012) suggest that the elasticity of demand lies in the range 0.95 to 1.65 with an average value of 1.3, which would imply values for \( \varepsilon \) of 1.05, 0.61 and 0.77 respectively. In what follows we allow \( \varepsilon \) to take the three values \( \varepsilon = 1.0, 0.8, 0.6 \). However it turns out that our results are not very sensitive to \( \varepsilon \), so in this section of the paper we report results just for the case \( \varepsilon = 0.8 \).

\( \delta \): The discount rate, \( \delta \), is both the rate at which cartels discount future profits and the rate at which CAs discount future consumer surplus. There is little agreement on what value this should take, with a wide variety of values being used in different studies.\(^{65}\) We use \( \delta = 0.9 \) as the central value in our calculations. However we report how our conclusions are affected if, instead, \( \delta \) takes a range of other values between \( \delta = 0.8 \) and \( \delta = 0.98 \).\(^{66}\)

---

\(^{63}\) So what Davies and Ormosi (2014) mean by removing harm is “permanent removal”.

\(^{64}\) Given the assumptions made above about the penalty base, this implies \( B(p) = p(1 + \varepsilon - p) \).

\(^{65}\) As is well known the value of this parameter will in practice be affected by considerations such as the frequency of interaction between cartel members and the evolution of demand, that will vary widely across industries, and from which we abstract here. We also performed a sensitivity analysis with respect to discount factor and other parameters of the model. The detailed results on sensitivity analysis are available from authors upon request.

\(^{66}\) We have performed several sensitivity checks with respect to discount factor. For a range of discount factors between \( \delta = 0.8 \) and \( \delta = 0.98 \) additional numerical simulations presented in the Appendix show that most of the effects are monotonic in \( \delta \). Namely, \( \bar{X} \) and ODE are decreasing in \( \delta \). On the other hand, TE, TE/H\(^0\), and
The Appendix to the Working Paper version of this paper, Katsoulacos, Motchenkova and Ulph (2016) gives more details of the formulae used in the calculations given the functional forms described above and a full list of the resulting calculations, including the sensitivity of the conclusions presented here to the assumed value of the penalty rate ($\rho = 0.1$) and to the model parameters $\delta$, $\varepsilon$.

6.1. The Total Effects of a Competition Authority’s Interventions

We start by considering how good a job a competition authority does in removing some of the potential harm that would have existed had there been no authority in existence. This is measured by $TE/H^0$ and Table 2 sets out the values for this that emerge from our framework.\(^6\)

**Table 2: $TE/H^0$**

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\sigma$</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>$\beta$</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.9</td>
<td>0.552</td>
<td>0.751</td>
<td>0.843</td>
</tr>
<tr>
<td>0.5</td>
<td>0.281</td>
<td>0.463</td>
<td>0.587</td>
</tr>
<tr>
<td>0.1</td>
<td>0.194</td>
<td>0.341</td>
<td>0.454</td>
</tr>
</tbody>
</table>

If we look at the left hand side of the table, where $\sigma = 1$ and the top row, where $\lambda = 0.9$ and so the set-up is effectively that which is implicitly assumed by Davies and Ormosi (2014) then we see that as the amount of effort that CAs put into investigations varies from a low value of $\beta = 0.1$ to the high value of $\beta = 0.3$ then the fraction of harm removed increases from 55.2% to 84.3% with the intermediate value of 75% when $\beta = 0.2$. These are almost exactly the figures produced in Table 2 of Davies and Ormosi (2014) where the results are, 55.5%, 86.9% and 76.7% respectively.

However the table also brings out just how sensitive these figures are to the implicit assumptions. So, for example, if the probability of keeping an industry competitive in the long-run, $\lambda$, drops from 90% to 50% then the fraction of potential harm removed by a CA varies from 28.1% to 58.7% with an intermediate value of 46.3%. So they have almost halved.

Comparing the LHS of Table 2 with the RHS we see that lowering the proportion of detected cartels that are immediately stopped, $\sigma$, from 100% to 80% also reduces the amount of potential harm removed by a Competition Authority.

TE/ODE are increasing in $\delta$. TE/ODE ratio ranges between 2 and 13 for the range of discount factors and other parameters we consider. Note also that RIDE is non-monotonic in $\delta$.

\(^6\)We are grateful to Khadija Lateef, an undergraduate student of Economics at the University of St Andrews for research assistance in undertaking the computations in this section.
So we have:

**Result 1:**

(i) **Our framework can support the finding of Davies and Ormosi (2014) that under the assumption that the cartel will be permanently shut down (i.e. $\sigma=1$ and $\lambda=1$) in the period following detection, the fraction of harm removed by a CA can vary from 55% to 85% depending on the probability of detection.**

(ii) **On the other hand, we show that this conclusion is at the top end of the range and is very sensitive to (implicit) assumptions about the key intervention parameters which affect the re-emergence of collusive behaviour, $\sigma$ and $\lambda$. Table 2 shows that if parameters $\sigma$ and $\lambda$ go down, i.e. CAs become less successful in preventing re-emergence of collusive behaviour following prosecutions, the effect of the probability of detection may be substantially lower than that predicted in Davies and Ormosi (2014).**

This shows that the impact of enforcement, identified in Davies and Ormosi (2014) and Bos et al. (2016) under the assumption that the cartel will be permanently shut down in the period following detection and prosecution, may overestimate the real impact in the situations when interventions affecting the re-emergence of collusive behaviour are less successful. Our theoretical model provides a framework which allows to relax this assumption and to take into account the strength of the interventions that affect the re-emergence of collusive behaviour.

The other aspect of CA performance is that CAs want to understand how large is the Total Effect of their interventions relative to the direct effect that they can measure – what we call the Observable Direct Effect. That is they want to understand how large is $\text{TE/ODE}$, but also how is this attributed to the indirect deterrence effect and the indirect price effect. From equation (24) we know that the magnitude of $\text{TE/ODE}$ depends on three other terms: how large are the Indirect Deterrence and Indirect Price Effects relative to the “true” Direct Effect ($\text{RIDE}$ and $\text{RIPE}$, respectively), and how large is $\text{DE/ODE}$ - the ratio of the “true” Direct Effect to the Observable Direct Effect (Davies and Ormosi’s Detected Harm).

Table 3 sets out the answers to these questions by presenting figures for $\text{TE/ODE}$ followed by its three components $\text{RIDE}$, $\text{RIPE}$ and $\text{DE/ODE}$. Once again all these calculations have been performed assuming $\delta = 0.9$, $\epsilon = 0.8$, $\rho = 0.1$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\sigma$</th>
<th>1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>8.359</td>
<td>7.729</td>
<td>7.416</td>
<td>8.649</td>
</tr>
<tr>
<td>0.5</td>
<td>3.365</td>
<td>3.251</td>
<td>3.179</td>
<td>3.475</td>
</tr>
<tr>
<td>0.1</td>
<td>2.193</td>
<td>2.159</td>
<td>2.139</td>
<td>2.279</td>
</tr>
</tbody>
</table>

**Table 3a: TE/ODE**
Table 3b: \( RIDE = IDE/DE \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>1.639</td>
<td>2.246</td>
<td>2.898</td>
</tr>
<tr>
<td>0.5</td>
<td>1.462</td>
<td>1.728</td>
<td>2.014</td>
</tr>
<tr>
<td>0.1</td>
<td>1.540</td>
<td>1.738</td>
<td>1.951</td>
</tr>
</tbody>
</table>

Table 3c: \( RIPE = IPE/DE \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.039</td>
<td>-0.068</td>
<td>-0.107</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.070</td>
<td>-0.092</td>
<td>-0.117</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.103</td>
<td>-0.124</td>
<td>-0.147</td>
</tr>
</tbody>
</table>

Table 3d: \( DE/ODE \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>3.210</td>
<td>2.432</td>
<td>1.957</td>
</tr>
<tr>
<td>0.5</td>
<td>1.410</td>
<td>1.233</td>
<td>1.098</td>
</tr>
<tr>
<td>0.1</td>
<td>0.900</td>
<td>0.826</td>
<td>0.763</td>
</tr>
</tbody>
</table>

Table 3e: \( IDE/ODE \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.9</td>
<td>5.258</td>
<td>5.453</td>
<td>5.671</td>
</tr>
<tr>
<td>0.5</td>
<td>2.061</td>
<td>2.135</td>
<td>2.209</td>
</tr>
<tr>
<td>0.1</td>
<td>1.386</td>
<td>1.435</td>
<td>1.489</td>
</tr>
</tbody>
</table>

One general conclusion that emerges from this is that although the indirect price effect is negative its absolute magnitude is small and doesn’t contribute much to our understanding of what drives the link between the Total Effect of a CA’s interventions and the Observable Direct Effect (ODE) that CAs can measure. So, in the discussion that follows we will say no more about it. The important effects that are at work are: the unobserved deterrence effect that everyone recognises and discusses, but also a second factor that has so far received no attention – the fact that the Observable Direct Effect significantly mis-measures the “true” Direct Effect.

If we start once again with the situation that is closest to that implicitly assumed by Davies and Ormosi (2014) – i.e. where \( \sigma = 1, \lambda = 0.9 \) - then we see from the first row on the LHS of Table 3a the total effect of a CA’s interventions may be between 7.5 and 8.5 times greater than the Observable Direct Effect. This is in part because the neglect of the Indirect Deterrence Effect, which, from Table 3b can vary between 1.6 and 2.9 times the Direct Effect, but also
because of the neglect of the fact that the “true” Direct Effect varies from 1.9 to 3.2 times the Observable Direct Effect. However, once again, these conclusions are very sensitive to what we assume about the parameter values – particularly the probability of keeping the industry competitive in the long run, \( \lambda \) - where the Total Effect will be just over twice as large as the Observed Direct Effect when \( \lambda = 0.1 \). But this sensitivity is largely driven by the fact that, as noted at the end of Section 4, the ratio of the true Direct Effect to the Observable Direct Effect is very sensitive to the assumed value of \( \lambda \). On the other hand the conclusions are not terribly sensitive to variations in either \( \beta \) or \( \sigma \).

Our figures are lower than those reported in Davies and Ormosi (2014), Table 1 even in their case where, as here, the probability of detection is constant. For they report that the Total Harm is 13.0 times the Detected Harm, while Deterred Harm is 7.8 times Detected Harm. So clearly the differences in methodology do matter when trying to put a figure on these important ratios.

Analysis of the ratio \( \frac{IDE}{ODE} \) presented in Table 3e can shed some light on how to assess the Indirect Deterrence Effect of CA interventions. Given that in practice it is very difficult (if not impossible) to design natural experiments that would give sharp predictions about deterrence effects, our approach of using model simulations to calculate this ratio could provide helpful complementary evidence. Table 3e implies that in the setting closest to that assumed by Davies and Ormosi (2014) – i.e. where \( \sigma = 1, \lambda = 0.9 \) - the Deterrence Effect of a CA’s interventions will be around 5.3 to 5.7 times larger than the Observable Direct Effect, which CAs are able to assess based on the information about cartels that they detect and prosecute.\(^{68}\) Our ratio presented in Table 3e is comparable to the ratio of Deterred Harm to Detected Harm obtained by Ilzkovitz and Dierx (2016) using EC DG Comp data. However, their estimate is higher than ours.

We summarise our findings in:

**Result 2:**

(i) The Total Effect is around 8 times as large as the Observable Direct Effect when \( \lambda = 0.9 \) but this drops to just over 3 when \( \lambda = 0.5 \) and just over 2 when \( \lambda = 0.1 \).

(ii) There are two important effects at work:
   a. the unmeasured Indirect Deterrence Effect which is between 1.5 and 2.9 times as large as the Direct Effect depending on the various parameter values;
   b. the fact that the true Direct Effect differs significantly from the Observable Direct Effect,

(iii) the ratio of the true Direct Effect to the Observable Direct Effect is very sensitive to the assumed value of \( \lambda \), taking values around 2 to 3 when \( \lambda = 0.9 \), but just 0.75 to 0.9 when \( \lambda = 0.1 \).

(iv) Because we have assumed that penalties are based on revenue and that both the probability of detection and the penalty rate are independent of the cartel price, the

\(^{68}\)Recall also that deterrence effect cannot be a simple multiple of the direct effect. It follows from Tables 3b and 3e that deterrence effect is bigger than direct effect, but the difference between these two effects depends on the size of intervention parameters and is non-linear in those parameters.
*Indirect Price Effect* is negative (as theory predicts), but our results show that it is very small in absolute magnitude.

### 6.1.1. Sensitivity to Model Parameters

We have undertaken calculations of how the measured values of the deterrence and total effects of a CA’s interventions vary with the two key model parameters, \( \varepsilon \) and \( \delta \). The Appendix to the Working Paper version of this paper, Katsoulacos, Motchenkova and Ulph (2016), shows how the values of \( \bar{\Lambda}, RIDE, TE/H^0 \) and \( TE/ODE \) that we presented above vary as we allow each of the two model parameters to vary over the ranges specified above.

We conclude that the results are not very sensitive to the assumed values of the inverse elasticity of demand, \( \varepsilon \). However, given the dynamic nature of our framework where we examine the way in which current interventions affect future behaviour of cartels, it is perhaps unsurprising that our conclusions are rather sensitive to the assumed value of the discount factor, \( \delta \). So for example, we reported above that, when \( \sigma = 1 \) and \( \lambda = 0.9 \) then the fraction of potential harm removed, \( TE/H^0 \) varies from 55% through 75% to 85% as the probability of successful prosecution, \( \beta \), varies from 0.1 through 0.2 to 0.3. However, these fractions rise to 71%, 87% and 93% respectively when \( \delta = 0.98 \), and fall to 41%, 62% and 73% respectively when \( \delta = 0.8 \). Similarly the values reported above for the ratio of the Total Effect to the Observable Direct Effect, \( TE/ODE \), were 8.4, 7.7 and 7.4 respectively. These values rise to 13.3, 12.2 and 11.7 respectively when \( \delta = 0.98 \), and fall to 5.5, 5.1 and 5.0 when \( \delta = 0.8 \).

So we see that considerable thought needs to be given to the appropriate choice of discount rate when placing numerical values on various measures of the total effectiveness of CA interventions.

### 6.1.2. Sensitivity to Penalty Rate

As indicated above Competition Authorities would like to know how the effectiveness of their interventions varies “depending on differences in competition policy enforcement.” To address this question we keep our model parameters at their central values: \( \delta = 0.9, \varepsilon = 0.8 \) but now allow the penalty rate to take the value \( \rho = 0.2 \), with consequent adjustments in the toughness of the penalty regime, \( \tau \). We know from the theory that an increase in the penalty rate (and hence \( \tau \)) will:

- have no effect on \( d \);
- will improve deterrence, \( D \);
- will cause cartels to increase the price.

Since the indirect price effects are typically very small we expect this latter effect to be dominated by the second.

We report below how the values for \( TE/H^0 \) and for \( TE/ODE \) that we presented above in Tables 2 and 3a respectively change when we use this new value of the penalty rate.
Table 2': \( \text{TE}/H^0, \delta = 0.9, \varepsilon = 0.8, \rho = 0.2 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>( \beta )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>( \lambda )</td>
<td>0.565</td>
<td>0.765</td>
<td>0.858</td>
</tr>
<tr>
<td>0.5</td>
<td>( \lambda )</td>
<td>0.302</td>
<td>0.495</td>
<td>0.626</td>
</tr>
<tr>
<td>0.1</td>
<td>( \lambda )</td>
<td>0.217</td>
<td>0.380</td>
<td>0.505</td>
</tr>
</tbody>
</table>

Table 3a': \( \text{TE}/ODE, \delta = 0.9, \varepsilon = 0.8, \rho = 0.2 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>( \beta )</th>
<th>1</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>( \lambda )</td>
<td>8.798</td>
<td>8.378</td>
<td>8.322</td>
</tr>
<tr>
<td>0.5</td>
<td>( \lambda )</td>
<td>3.711</td>
<td>3.693</td>
<td>3.737</td>
</tr>
<tr>
<td>0.1</td>
<td>( \lambda )</td>
<td>2.521</td>
<td>2.558</td>
<td>2.624</td>
</tr>
</tbody>
</table>

As we can see and as both ratios \( \text{TE}/H^0 \) and \( \text{TE}/ODE \) are indeed increasing in \( \rho \), though, particularly for \( \text{TE}/H^0 \), the effect is quite small.

6.2. The Marginal Welfare Effects of a Competition Authority's Interventions

As reported in Section 5 – particularly in equations (41)-(44) - the four enforcement parameters work through very different channels. As a consequence the magnitudes of the various marginal effects depend on rather different parameters. So, from (42), the marginal effect of an increase in \( \sigma \) depends solely on \( \sigma \), while the marginal effect of an increase in \( \lambda \) depends additionally on the associated value of \( \lambda \). However the marginal effects of an increase in the penalty rate, \( \rho \), depend on the values of \( \varepsilon \), \( \beta \) and \( \rho \), since these last two parameters determine \( \tau \). From (44) the marginal effect of an increase in \( \beta \) therefore depends on \( \sigma \), \( \varepsilon \), \( \beta \) and \( \rho \).

Now, from Table A1 in the Appendix to Katsoulacos et al. (2016) we find that if we set our 3 intervention parameters \( (\beta, \sigma, \lambda) \) to their maximum assumed values – namely \( (0.3, 1.0, 0.9) \) - then the associated value of \( d \) is 0.587; while if we set them to their minimum assumed values – namely \( (0.1, 0.8, 0.1) \) - then the associated value of \( d \) is 0.073. If we set the values of \( \beta \) and \( \lambda \) to their intermediate values of 0.2 and 0.5 respectively, and set \( \sigma = 1 \) we get the intermediate value of \( d = 0.247 \). Since the values of the associated intervention parameters are therefore uniquely associated with each of these three values of \( d \), in what follows we will report the results by referring solely to the value of \( d \). We have also used the central values for the remaining parameters – i.e. we have set \( \delta = 0.9, \rho = 0.1, \varepsilon = 0.8 \).
Table 4: Marginal Effects

<table>
<thead>
<tr>
<th>$d$</th>
<th>$-\frac{\partial H}{\partial \beta} \frac{\beta}{H}$</th>
<th>$-\frac{\partial H}{\partial \sigma} \frac{\sigma}{H}$</th>
<th>$-\frac{\partial H}{\partial \lambda} \frac{\lambda}{H}$</th>
<th>$-\frac{\partial H}{\partial \rho} \frac{\rho}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.587</td>
<td>1.263</td>
<td>1.174</td>
<td>5.005</td>
<td>0.089</td>
</tr>
<tr>
<td>0.247</td>
<td>0.557</td>
<td>0.494</td>
<td>0.404</td>
<td>0.057</td>
</tr>
<tr>
<td>0.073</td>
<td>0.174</td>
<td>0.146</td>
<td>0.014</td>
<td>0.028</td>
</tr>
</tbody>
</table>

In the Appendix to the Working Paper version of this paper, Katsoulacos et al. (2016) we report how these different marginal effects decompose into their associated Direct Effects and Indirect Deterrence and Price Effects. So we have:

**Result 3:** For the range of parameter values considered here:

(i) For all four enforcement parameters the elasticities relating to their total effect are smaller than 1;
(ii) Increasing the probability of successful investigation, $\beta$, has a more powerful impact than increasing the probability of closing cartels down following an investigation, $\sigma$;
(iii) The effectiveness of raising the probability of keeping an industry competitive once a cartel has been shut down, $\lambda$, rises sharply with the value of $\lambda$ itself.
(iv) Because it generates no direct effect, increasing the penalty rate has an effectiveness that is between 1/14th and 1/6th of that of increasing the probability of successful detection.

To summarize we can conclude that on average the marginal effect of an increase in the penalty rate is around ten times smaller than the marginal effect of an increase in the probability of successful prosecution. Secondly, potentially the most powerful marginal effect is that of longer-term interventions aiming at preventing re-emergence of cartels, $\lambda$—but this is very sensitive to the existing level of $\lambda$ and to the toughness of antitrust enforcement (i.e. the level of probability of successful prosecution and the amount of the resources the CA puts in preventing cartel re-emergence both in the short and in the long term). For example, for the case with the strongest (toughest) CA (see the first row in Table 4), marginal effect of long-term interventions aiming at preventing re-emergence of cartels (i.e. $\lambda$) can be about 5 times higher than the marginal effects of other two interventions. However, for the weakest CA (see the last row in Table 4), marginal effect of $\lambda$ is smaller than the marginal effects of other interventions.

7. Concluding Remarks

We have presented a model of cartel formation capturing the process of cartel re-emergence that allows an explicit characterization of the effects on welfare of a wide range of policy instruments. We distinguish between a direct effect of detecting and stopping cartels, as well as an indirect-deterrence effect of reducing the cartel rate (i.e. the fraction of cartels out of those
that would exist in the absence of a CA that exist in its presence and given its enforcement activities) and an indirect-price effect or effect on the price set by the cartels that form. We have used this model to obtain a number of results through which we characterize the total welfare effect of the presence of a competition authority and the marginal effects of the various enforcement interventions. Also given the difficulties of getting good well-designed natural experiments that would give sharp predictions about deterrence effects, our approach of using model simulations to calculate a ratio could provide helpful complementary evidence.

Though our contribution has related concerns and objectives to that of Davies and Ormosi (2014), their conceptual framework and modelling approach is very different to the one presented above. While some of our numerical results are very similar to theirs, others are significantly different, so the difference in methodologies really matters. One important difference is that previous contributions by Davies and Ormosi (2014) and Bos et al. (2016) focus only on the welfare effects of interventions related to detection and prosecution and the toughness of the penalties. While we introduce the impact of interventions aiming at preventing re-emergence of collusive behaviour and show that the strength of such interventions has substantial effect on the strength of other interventions, which were the focus of previous literature. So not paying proper attention to prevention of re-emergence of collusive behaviour (such as not investing in e.g. monitoring and screening in the industries, where cartels have recently been discovered) may reduce the effectiveness of CA efforts put in detection and prosecutions as well as efforts put in improving the design of the structure of antitrust penalties.

References


https://editorialexpress.com/cgibin/conference/download.cgi?db_name=EARIE43&paper_id=97


Available at: http://ec.europa.eu/competition/information/macroeconomy/index.html


Technical Appendix:

Proof of Proposition 2:

(i) Obvious from (6) in which $\sigma$ and $\lambda$ do not appear. (ii) again obvious from (6); (iii) follows from Katsoulacos, Motchenkova and Ulph (2015) where it is shown that if the penalty is on revenue and if both the penalty rate and the probability of successful prosecution are independent of the cartel price then the cartel price is unambiguously higher than the monopoly price, $^{69}$ while if the penalty is based on profits the cartel price equals then monopoly price and, if the penalty is based on the overcharge the cartel price is unambiguously lower than the monopoly price; (iv) since, from (6), $p^c$ satisfies the f.o.c. $\pi'(p^c) - \tau B'(p^c) = 0$, then, assuming the second-order conditions for a maximum are satisfied, it follows from standard comparative static analysis that

$$\text{sign}\left(\frac{dp^c}{d\tau}\right) = \text{sign}\left[-B'(p^c)\right] = \text{sign}[\pi'(p^c)].$$

The result then follows from the fact that $p^M = \text{arg max } \pi(p)$ and consequently $\pi'(p^c) > 0$ as $p^c < p^M$. ■

Proof of Lemma 1:

(i) From (3) $p^c \geq p^M$ implies $\Delta^0(\tau) = \frac{\pi(p^c) - \tau B(p^c)}{\pi(p^M)}$. Since $p^c$ is chosen to maximise $\pi(p) - \tau B(p)$ it follows from the Envelope Theorem and (11) that

$$\frac{dD^0(\tau)}{d\tau} = - \frac{d\Delta^0(\tau)}{d\tau} = \frac{B(p^c)}{\pi(p^M)} > 0.$$ ■

(ii) When $p^c < p^M$, then from (9) and (3) we have, $\Delta^0(\tau) = \frac{\pi(p^c) - \tau B(p^c)}{\pi(p^c)}$ and so now the denominator of the expression also depends on $\tau$. So if we differentiate first the numerator and then the denominator, then we get:

$$\frac{d\Delta^0(\tau)}{d\tau} = \left\{-\frac{B(p^c)}{\pi(p^c)} + \left[\frac{\pi(p^c) - \tau B(p^c)}{\pi(p^c)}\right] \pi'(p^c) \left(-\frac{dp^c}{d\tau}\right)\right\}, \quad (A1)$$

where the first term on the RHS of (A1) follows once again from the Envelope Theorem and is unambiguously negative. However given Proposition 2(iv) it follows that if $p^c < p^M$ then every component of the expression in the second term on RHS of (A1) is positive, so, at this level of generality, it is impossible to unambiguously determine the sign of $\frac{d\Delta^0(\tau)}{d\tau}$. However, it is clear that in a very wide range of circumstances it will be negative.

$^{69}$Katsoulacos and Ulph (2013) show that if the penalty is based on revenue but either the probability of investigation or the penalty rate are perceived to be positively related to the cartel price then the cartel will set a lower price than predicted by Katsoulacos, Motchenkova and Ulph (2015) – but it may still be higher than the monopoly price.