Investment, Subsidies, and Universal Service: Broadband Internet in the United States

Kyle Wilson*

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Abstract

Access to the internet is critical for participating in modern society, and yet 17% of Americans lack access to broadband internet, according to the Federal Communications Commission (FCC). A key objective of the FCC is to promote policies that advance the availability of quality telecommunications services across the United States. To that end, the FCC has recently secured funding to provide subsidies to internet service providers on a massive scale, and has been given considerable flexibility in the distribution of these funds.

The aim of this paper is to identify the determinants of internet service providers’ entry and infrastructure upgrade decisions, observe how these determinants vary across markets, and to use this information to provide policy recommendations about how to target subsidies in order to best accomplish the longstanding goal of Universal Service.

I develop a dynamic model, which encapsulates potential entrants’ decisions to enter new markets as a low-speed or high-speed provider, as well as incumbents’ decisions to upgrade their infrastructure. I then estimate this model using data from the National Broadband Map, a recent initiative to track availability of broadband internet across the United States. Then, I use this model to perform counterfactuals, which generate predictions of firm behaviors under a variety of potential subsidy structures.

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1 Introduction

Since its introduction to U.S. households in the early 1990s, internet access has undergone a transformation from a luxury service to an absolute necessity. Access to fast broadband internet is critical for communication, business, education, entertainment, and to having an informed society. And yet, as of 2015, only 83% of Americans have access to broadband internet. Those who live in rural areas are especially disadvantaged, as 53% of rural Americans but only 8% of urban Americans lack access to broadband internet speeds (FCC).

Universal service, the practice of providing a baseline level of service to every resident in the country, has been a longstanding policy goal for many sectors in the United States, and for telecommunications in particular. The 1996 Telecommunications Act set goals of promoting the availability of quality telecommunications services at reasonable prices, and advancing the availability of these services to all consumers. In keeping with this directive, the American Recovery and Reinvestment Act of 2009 instructed the Federal Communications Commission (FCC) to create a plan to improve internet access in the United States. In response, the FCC released the U.S. National Broadband Plan in March 2010. Among other provisions, this plan proposed the creation of the Connect America Fund, which would provide subsidies to internet service providers (ISPs) who increase the broadband speeds they offer in underserved areas of the United States. The goal of this fund was initially to make download speeds of 4 Megabits-per-second (Mbps) and upload speeds of 1 Mbps available to all Americans. During Phase 1 of the program, $269 million was distributed to three ISPs \(^1\) to expand their broadband offerings in 37 states. Moving forward, however, the FCC has determined that ISPs will be required to deliver download speeds of at least 10 Mbps and upload speeds of at least 1 Mbps. To this end, $9 billion in funding has been allocated to the now-underway Phase 2 of the program. With a program of this magnitude, the FCC is in a position to make considerable strides in improving the broadband infrastructure of the United States, but choosing where to allocate funds is not a simple task.

Motivated by the goal of the Connect America Fund, the aim of this paper is to provide insight into the question of how to best distribute funds intended to subsidize internet service

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\(^1\)These three ISPs were Frontier Communications, CenturyLink, and AT&T.
providers’ entry and investment, with the objective of extending high-speed internet availability to as many consumers as possible. In order to do this, I develop a dynamic model of the internet service provision industry, wherein ISPs make entry, exit, and infrastructure investment decisions. Because firms are forward-looking when they make entry and investment decisions, a dynamic framework is needed to capture all of the relevant tradeoffs faced by firms in the industry. We expect firms’ actions to be motivated by expectations about future market conditions, including changes in demand and future entry or investment by other firms. For instance, firms may enter markets or upgrade their infrastructure before it is actually profitable to do so, in order to ensure that the market will support them when it does become profitable. A dynamic model is capable of capturing this tradeoff, while a static model would erroneously attribute observed entry and investment to current profitability, thus highlighting its inability to estimate sunk costs. Without reliable estimates of the sunk costs of entry and upgrade, it would be difficult to say much about the effectiveness of various subsidy policies.

To estimate this model, I use data from the National Broadband Map, a comprehensive catalog of broadband availability from every firm in each census block in the United States. I use this data to track entry, upgrade, and exit behavior of firms over the time period from 2010 through 2013. The observed patterns in firms’ behavior, working in tandem with the structural model I develop, identify the relative importance of a number of market and firm characteristics in determining firms’ entry, upgrade, and exit decisions. The model’s estimates provide insight into how market population, the distance from firms’ central offices, and market structure influence firms’ decision-making. These factors influence the cost advantages that firms have in certain markets and therefore the likelihood of an ISP upgrading their infrastructure. These conditions have direct implications for subsidy design, as they tell us where funding should be targeted in order to get the most “bang for the buck.”

A targeted subsidy is likely to extend broadband availability more than would a non-discriminatory subsidy, as a well-designed targeted subsidy would provide funding to areas where it is most likely to induce entry or investment that would not otherwise occur. To this end, I conduct a number of counterfactual analyses in order to predict outcomes under various subsidy structures. These
results provide information useful to designing effective subsidies for internet service providers, and may have more general implications for designing subsidies with similar technology-adoption goals in other settings as well.

1.1 Related Literature

This paper adds to the literature on entry and investment, and in particular on dynamic models of firms’ investment decisions. For instance, Mazzeo (2002) considers entry and investment in a static model of firms’ product choices in the context of the motel industry. Similarly, Seim (2006) models endogenous product-differentiation decisions in a static framework, and estimates a model of video retail stores’ location decisions. The disadvantage of these, and other similar static models, is that they cannot reliably estimate the sunk costs associated with entry and investment. Around the same time, a number of papers emerged, proposing various methods for estimating dynamic structural models of entry and investment. This paper most closely follows the work of Pakes, Ostrovsky, and Berry (2007), who propose a method for estimation of a simple entry and exit model. Other approaches due to Aguirregabiria and Mira (2007), Pesendorfer and Schmidt-Dengler (2008), and Bajari et al. (2007), have been applied to similar settings as well. For instance, Ryan (2012) and Collard-Wexler (2013) model dynamic games in the cement and concrete industries to investigate the true costs of the Clean Air Act Amendments and the impact of demand smoothing on market structure, respectively. Igami (2015) also studies the determinants of entry and investment in a dynamic model of creative destruction in the hard disk drive industry.

In addition, this paper adds to the body of literature related to the telecommunications sector. Previous policies, such as the 1996 Telecommunications Act, have been analyzed by Economides (1999) and Hausman and Taylor (2012). Other work, such as that of Greenstein and Mazzeo (2006), Economides et al. (2008), and Goldfarb and Xiao (2011) has focused on understanding firms’ entry behavior in the telecommunications sector.

This paper follows the most directly in the line of research pursued by Fan and Xiao (2015), who explore the effectiveness of different subsidy structures in encouraging entry of local telephone providers into new markets. This paper builds on their findings by applying a similar model to a
related industry in the telecommunications sector. Specifically, I extend their model by allowing firms to choose to enter as one of two incumbent types, and by modeling incumbents’ investment decisions.

2 The Basics of Internet Access

In order to properly motivate the model I develop, it is important to provide a baseline level of knowledge about the internet architecture. The goal of this section is to give a layperson’s explanation of how internet access is provided to households in the United States.

The term broadband refers to a transmission utilizing a wide bandwidth capable of transferring multiple signals and traffic types. Broadband is not unique to any particular technology, and does not convey any information about the speed of data transmission. The term high-speed refers to data transmitted at a rate above a certain threshold, one which is arbitrarily defined and which changes over time. This difference between broadband and high-speed is often blurred (sometimes intentionally), but is important to this paper, which focuses on firms’ deployment of broadband internet access, and specifically on the decision to provide high-speed access.

There are currently two dominant technologies used to provide broadband internet access to American households: cable modems and digital subscriber lines (DSL). ISPs using either technology perform an identical function. Data is transmitted between the customer’s premises and the ISP's central office for the market. This stretch of wiring is referred to as the last-mile. The ISP then interconnects with a high-speed fiber backbone provider, who then connects to last-mile networks around the world.

Cable modem service is provided by firms who traditionally have provided cable television service to households. The cable operator dedicates the equivalent of one or more television channels of bandwidth to providing internet access. Data is then transmitted between a cable modem located in the customer’s home and a cable modem termination system located in the ISP’s central office. A defining feature of a cable internet network is that the bandwidth of a single wire is shared by many subscribers. This means that the more users on the network that simultaneously access the internet, the slower each of their connections will be. This problem only worsens as consumers shift
toward greater use of data-intensive applications such as video-streaming and online gaming.

DSL service is provided by firms who traditionally have provided telephone service to households. These telephone networks were necessarily set up so that each subscriber had a dedicated line running between their house and the provider’s central office. The biggest drawback of DSL service is that the signal decays over the distance it must travel between the customer and the ISP’s central office. This means that the further a customer is from the central office, the slower their connection will be.

The solution for each of these technologies to overcome their fundamental issues in providing high-speed internet turns out to be the same. An ISP can install remote terminals throughout a market, closer to their subscribers’ premises. The ISP then runs high-speed fiber-optic cable from their central office to each of their remote terminals, and runs coaxial cable or copper wire from the remote terminal to their customers’ homes. This type of network is called a hybrid-fiber network. For cable providers, this has the effect of reducing the number of customers who share a wire, as they now only share bandwidth with other customers connected to the same remote terminal, thereby increasing their speed. For DSL providers, this allows them to provide high-speed access to subscribers further from their central office. Since the signal does not decay with distance when it travels over fiber-optic cable, the relevant distance is now the shorter distance from the customer’s home to the remote terminal. This has the effect of increasing the speed of DSL customers. For DSL providers, the need for remote terminals depends on the distance from the market to its central office, while for cable providers, the need for remote terminals depends on the market’s population.

The deployment of a hybrid-fiber network for cable and DSL providers is illustrated in figure 1. First, note that a DSL provider runs a cable directly from its remote terminals to each subscriber’s home, while a cable provider connects its remote terminal to all homes using a single cable. Next, notice that the neighborhoods depicted in the upper-left corner can be served using just one remote terminal, regardless of whether the provider uses cable or DSL technology, because those neighborhoods are close together and do not contain a large population. The neighborhood in the bottom-left corner, however, requires an additional remote terminal if the provider uses DSL technology. This is because this neighborhood is far away from the provider’s central office and all
other remote terminals. A cable provider can serve this neighborhood from the upper-left remote terminal because the population of these five neighborhoods does not exhaust the line’s bandwidth. Finally, the neighborhoods in the upper-right corner can all be served by a single remote terminal if the provider uses DSL technology, due to their close proximity to one another. However, these neighborhoods require two remote terminals if the provider uses cable technology, since their large population will overwhelm the bandwidth available from a single remote terminal.

Another solution would, of course, be to run fiber-optic cables directly to each customer’s house, in what is known as a fiber-to-the-home (FTTH) network. This would mitigate the bandwidth-sharing issue of cable networks and the distance issue of DSL networks. This solution, is, however significantly more expensive for ISPs to implement, and therefore has seen limited use in the United States. As consumers’ demand for greater bandwidth and faster speeds continues to increase, we should expect to see greater use of FTTH networks.
3 Data

The primary data I use come from the National Broadband Map, a project undertaken by the National Telecommunications and Information Administration, to catalog broadband availability in the United States. This dataset contains the universe of all broadband internet service providers and all census blocks in the U.S. The data provides details on the technology used to provide service and the maximum advertised download and upload speeds for each provider in every census block in the United States. This dataset has been updated every six months since June 2010, allowing me to construct a bi-annual panel of data from June 2010 through December 2013. In addition, I supplement this with data from TelcoData.us, a consulting firm providing geographic data on the locations of the central offices of all DSL providers in the United States. Finally, I combine this with demographic data from the 2010 through 2013 American Community Survey (ACS), and with geographic data from the 2010 Census Gazetteer files.

Upon merging together all of the available iterations of the National Broadband Map, I track firms’ actions over time in each market, allowing me to observe whether firms enter as slow or fast providers, whether slow incumbents upgrade to become fast incumbents, and whether incumbents exit. From the ACS and Census Gazetteer, I gather data on population, land area, and the geographic coordinates of each census tract. Incorporating the locations of DSL central offices allows me to calculate the distance from the centroid of each census tract to the nearest central office. Though the data specifies the firm that owns each central office, I choose to calculate the distance to the nearest central office regardless of ownership. This is reasonable, as the 1996 Telecommunications Act requires that incumbent local exchange carriers allow competitive local exchange carriers to co-locate within their facilities if the cost of setting up their own facilities would be cost prohibitive. Therefore, the relevant distance for a DSL provider is well-approximated by the distance to the nearest central office, regardless of ownership.

The National Broadband Map recently released data that is current as of June 2014, but I have not yet incorporated this data.
3.1 Definitions of the Market, Firm Types, and the Sample

I define a market in this industry to be a census tract. This allows me to combine the broadband data with demographics data that varies over time\(^3\). More importantly, the census tract seems to be the geographic level that best approximates the level at which internet service providers make decisions. Census tracts are subdivisions of counties with an average of 4,000 inhabitants, and are designed to be relatively homogeneous in demographic characteristics. A broader market definition, such as a city or county, would give rise to concerns about heterogeneity within markets, as ISPs often do not serve entire cities or offer the same services to all areas within a city. A smaller market definition such as a census block, which contain about 50 people on average, seems unreasonable, as one would not expect that ISPs tailor their offerings to such a small group. Indeed, I find that variation in download speeds is substantially higher across tracts than across blocks within tracts\(^4\).

In the model I estimate, firms exist as one of three types in each market: potential entrant, slow incumbent, or fast incumbent. I define a potential entrant to be any firm who offers service within the state, does not currently serve a particular market, and did not previously exit this market. I define a slow incumbent to be an incumbent firm who offers a maximum advertised download speed of less than 10 Mbps, and a fast incumbent to be an incumbent firm who offers a maximum advertised download speed of at least 10 Mbps, since this is the minimum download speed required for firms to be eligible for funding from the Connect America Fund\(^5\). I interpret the firm’s maximum advertised download speed as the level of the firm’s infrastructure in that location at that time, since it is the highest speed that the firm is able and willing to provide.

For the time being, I restrict the sample to providers of residential internet access in the state of California who offer service to a population of at least 250,000. I exclude providers who only provide business-class internet because it is typically provided quite differently from residential

\(^3\)The ACS, which is available annually, does not make data available at any geographic level finer than the census tract, and the decennial census does not vary over the time period of my sample.

\(^4\)The standard deviation of download speeds across tracts (calculated for each ISP and then averaged) is 4.73 times higher than the standard deviation of download speeds across blocks within a tract (calculated for each ISP in each tract and then averaged).

\(^5\)The Connect America Fund also requires an upload speed of 1 Mbps, but this constraint is non-binding in almost all cases. 94% of all observations had an upload speed of greater than 1 Mbps as early as June 2010.
internet access. Providers often use different technology and negotiate the terms of service on a business-to-business basis. My model is therefore unlikely to accurately reflect these features. For firms that serve both residential and business customers, residential customers typically account for the vast majority of the firms’ revenue\textsuperscript{6}. I chose to limit the sample to markets within California because it is the largest state by population, the third largest state by land area, and contains both large urban areas and open rural areas. These features suggest that results from the state of California are likely to generalize well to the rest of the United States.

3.2 Descriptive Statistics

Tables 1 and 2 provide some descriptive statistics that give a general picture of the typical market in the sample and an overview of the action taking place during the sample period. Table 1 shows that the average market’s population and population density slowly increase over the sample period. There is considerable variation in population across markets. The average market in the sample is very densely populated, with over 8,400 people per square mile in 2013. The average distance to the nearest central office is about six miles, which is consistent with the range at which a DSL provider can provide service to a customer. This distance does not change over time, since I do not have a panel of central office locations, but there is substantial variation across markets, with a standard deviation larger than the mean. There is a downward trend in the average number of slow incumbents, with a spike at the end of the sample, and there is a corresponding upward trend in the average number of fast incumbents.

Table 2 shows that the number of distinct DSL firm-market observations rises initially before decreasing toward the end of the sample. A similar pattern holds for cable providers. There is no clear trend in entry, upgrade, and exit period, with years alternating between periods of high and low activity.

\textsuperscript{6}For example, in 2013, 83\% of Comcast’s revenue was derived from its residential customers, while just 8\% of its revenue was derived from its business customers (SEC).
Table 1: Mean Values (Std. Dev.) of Market Characteristics

<table>
<thead>
<tr>
<th></th>
<th>June 2010</th>
<th>June 2011</th>
<th>June 2012</th>
<th>June 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>4573.85</td>
<td>4615.33</td>
<td>4659.41</td>
<td>4701.28</td>
</tr>
<tr>
<td></td>
<td>(1913.75)</td>
<td>(1951.26)</td>
<td>(1996.48)</td>
<td>(2031.30)</td>
</tr>
<tr>
<td>Population Density</td>
<td>8298.88</td>
<td>8336.16</td>
<td>8401.13</td>
<td>8462.04</td>
</tr>
<tr>
<td></td>
<td>(9285.43)</td>
<td>(9294.12)</td>
<td>(9356.25)</td>
<td>(9404.65)</td>
</tr>
<tr>
<td>Distance to nearest CO</td>
<td>6.28</td>
<td>6.28</td>
<td>6.28</td>
<td>6.28</td>
</tr>
<tr>
<td></td>
<td>(8.09)</td>
<td>(8.09)</td>
<td>(8.09)</td>
<td>(8.09)</td>
</tr>
<tr>
<td># of Potential Entrants</td>
<td>15.22</td>
<td>14.54</td>
<td>14.45</td>
<td>14.22</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
<td>(.69)</td>
<td>(.69)</td>
<td>.94</td>
</tr>
<tr>
<td># of Slow Incumbents</td>
<td>.74</td>
<td>.37</td>
<td>.28</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>(.65)</td>
<td>(.55)</td>
<td>(.48)</td>
<td>(.33)</td>
</tr>
<tr>
<td># of Fast Incumbents</td>
<td>1.04</td>
<td>2.05</td>
<td>2.16</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>(.81)</td>
<td>(.74)</td>
<td>(.68)</td>
<td>(.64)</td>
</tr>
<tr>
<td># of Observations</td>
<td>7,992</td>
<td>7,992</td>
<td>7,992</td>
<td>7,992</td>
</tr>
</tbody>
</table>

Table 2: Summary Statistics of Firm Actions

<table>
<thead>
<tr>
<th></th>
<th>June 2010</th>
<th>June 2011</th>
<th>June 2012</th>
<th>June 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td># of DSL observations</td>
<td>8,158</td>
<td>10,952</td>
<td>10,897</td>
<td>9,619</td>
</tr>
<tr>
<td># of Cable observations</td>
<td>6,101</td>
<td>8,323</td>
<td>8,605</td>
<td>8,378</td>
</tr>
<tr>
<td># of Potential Entrant observations that Enter Slow</td>
<td>78</td>
<td>17</td>
<td>1,491</td>
<td>11</td>
</tr>
<tr>
<td># of Potential Entrant observations that Enter Fast</td>
<td>427</td>
<td>181</td>
<td>330</td>
<td>58</td>
</tr>
<tr>
<td># of Incumbents that Exit</td>
<td>30</td>
<td>314</td>
<td>1597</td>
<td>60</td>
</tr>
<tr>
<td># of Slow Incumbent observations that Upgrade</td>
<td>4,470</td>
<td>60</td>
<td>123</td>
<td>27</td>
</tr>
</tbody>
</table>

Note 1: An observation is a firm-market combination.
Note 2: Number of entries or upgrades at time $t$ refers to the number of observations that enter or upgrade between time period $t$ and time period $t+1$, where the time step is a 6 month interval.
4 Model

In the following section, I develop a model of ISPs’ decisions about which markets to enter and exit, and whether or not to offer high-speed internet service. I allow ISPs in the model to observe the most salient market features, which are likely to be the predominant factors affecting their choices.

4.1 Overview

Time is discrete and infinite, and there are a finite number of geographic markets. Players in the game are a finite set of firms who provide internet access, each of which is predetermined to be either a DSL provider or a cable provider. In each market, in each time period (up until it exits the market), each firm’s type is either a potential entrant, an incumbent providing slow service, or an incumbent providing fast service. The objective of all players is to maximize the expected net present value of their infinite stream of profits.

In each period and market, a potential entrant observes its sunk cost of entry, and can then choose to enter the market or remain a potential entrant in the next period. Upon choosing to enter the market, it observes the additional sunk cost of entering as a fast provider, and it then chooses whether to enter the market as a slow provider or a fast provider.

In each period and market, a slow incumbent observes its sell-off value of exiting, and can then choose to remain in the market or exit the market forever. If a slow incumbent chooses to remain in the market, it observes its sunk cost of upgrading to become a fast provider, and it then chooses whether to remain a slow provider or become a fast provider.

In each period and market, a fast incumbent observes its sell-off value of exiting, and can then choose to continue operating as a fast provider or exit the market forever.

The sunk costs of firms’ available actions are randomly drawn from a distribution that is common knowledge to all players. A firm’s own sunk cost draw is private information.

Modeling a potential entrant’s decision as two sequential choices is rather natural, as one would expect a potential entrant to decide whether it wishes to enter a market before it decides what speed of service it will offer there. Likewise, it is natural to model a slow incumbent’s decision as two sequential choices, as one would expect the firm to decide whether or not it wishes to continue
serving a market before it decides whether or not to upgrade. With that said, the assumption is somewhat strong, as it does assume that a potential entrant (slow incumbent) does not know the additional sunk cost of entering as a fast provider (upgrading to become a fast provider) at the time they make the decision to enter or wait (exit or not), but rather only knows the distribution from which it will be drawn.

Finally, in each period and market, each firm knows its own technology, its current type, the number of firms in each technology-type combination \(^7\), and the payoff-relevant characteristics of the market. A firm’s per-period profits are then determined by the number of slow providers in the market, the number of fast providers in the market, the firm’s own type, and the market’s characteristics.

4.2 Notation

To formalize notation of the model:

- There are infinite discrete time periods, indexed by \(t\).
- There are \(J\) firms, indexed \(j = 1, \ldots, J\).
- There are \(M\) markets, indexed \(m = 1, \ldots, M\).
- There is a common discount factor, \(\beta\).
- The state space includes a firm’s own technology \((T_j)\) and type \((\tau_{jtm})\), market characteristics \((Z_{tm})\), and the number of firms in each technology-type combination \((N_{T,\tau}^{T,\tau} \text{ for } T \in \{\text{DSL, cable}\} \text{ and } \tau \in \{\text{potential entrant, slow incumbent, fast incumbent}\})\). For ease of notation, I define the state space to be \(S_{jtm} \equiv (T_j, \tau_{jtm}, Z_{tm}, N_{T,\tau}^{T,\tau} | T \in \{\text{DSL, cable}\} \text{ and } \tau \in \{\text{potential entrant, slow incumbent, fast incumbent}\})\).
- A potential entrant’s sunk cost of entry is denoted \(\kappa_e\). The additional sunk cost (on top of \(\kappa_e\)) of entering as a fast provider is denoted \(\kappa_f\). A slow incumbent’s sunk cost of upgrading

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\(^7\)The technology-type combinations are DSL potential entrant, cable potential entrant, DSL slow incumbent, cable slow incumbent, DSL fast incumbent, and cable fast incumbent.

\(^8\)For the purpose of estimation, \(Z_{tm}\) will include the market’s population and the distance from the market’s centroid to the nearest DSL central office.
to become a fast incumbent is denoted $\kappa_u$. An incumbent’s exit value is denoted $\kappa_x$. The sunk costs and value of exit ($\kappa_e, \kappa_f, \kappa_u, \kappa_x$) are private information and are randomly drawn from a distribution with means $\mu \equiv (\mu_e, \mu_f, \mu_u, \mu_x)$.

- Per-period profits are denoted $\pi(S_{jtm}; \alpha)$, where $\alpha$ is a vector of parameters affecting profits.
- I define the vector of all parameters to be $\theta = (\alpha, \mu)$.

To formalize the timing of the game:

1. At the start of each period, each firm in each market observes $S_{jtm}$.
2. Firms earn per-period profits according to the current state, $S_{jtm}$.
3. All firms move simultaneously, but in the following manner:
   - Potential entrants observe $\kappa_e$ and choose to enter the market or wait. Next, if a potential entrant chooses to enter the market, it observes $\kappa_f$ and chooses to enter as a fast provider or to enter as a slow provider.
   - Slow incumbents observe $\kappa_x$ and choose to exit or remain in the market. Next, if a slow incumbent chooses to remain in the market, it observes $\kappa_u$ and chooses to upgrade to become a fast incumbent or remain a slow incumbent.
   - Fast incumbents observe $\kappa_x$ and choose to exit or remain in the market.
4. Market structure state variables evolve according to the actions chosen by the players - potential entrants who enter become incumbents (either slow or fast), slow and fast incumbents that exit drop out of the market, and slow incumbents who upgrade become fast incumbents.

Market characteristics evolve exogenously.

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9 The next period’s per-period profits will reflect actions taken by firms in the current period.
10 To be clear, since firms move simultaneously, they do not observe the actions of their rivals until the next period. Potential entrants and slow incumbents have a two-step decision process, but both steps occur consecutively within the current period.
4.3 Value Functions

In what follows, I will suppress the subscripts for ease of notation. This setup generates the following value functions for the firms:

\[
V_p(S, \kappa_e; \theta) = \max\{VC_p(S; \theta), VC_e(S; \theta) - \kappa_e\} \tag{1}
\]

\[
V^s(S, \kappa_x; \theta) = \pi^s(S; \alpha) + \max\{\kappa_x, VC^s(S; \theta)\} \tag{2}
\]

\[
V^f(S, \kappa_x; \theta) = \pi^f(S; \alpha) + \max\{\kappa_x, VC^f(S; \theta)\} \tag{3}
\]

where \(VC_p(\cdot)\) is a potential entrant’s continuation value associated with waiting, \(VC_e(\cdot)\) is a potential entrant’s continuation value associated with entering, \(VC^s(\cdot)\) is a slow incumbent’s continuation value associated with remaining in the market, \(VC^f(\cdot)\) is a fast incumbent’s continuation value associated with remaining in the market as a fast incumbent, \(\pi^s(S; \alpha)\) is the per-period profit earned by a slow provider in state \(S\), and \(\pi^f(S; \alpha)\) is the per-period profit earned by a fast provider in state \(S\).

Specifically,

\[
VC_p(S; \theta) = \beta \mathbb{E}_{S'}[\mathbb{E}_{\kappa'_e}[^*\{VC_p(S'; \theta), VC_e(S'; \theta) - \kappa'_e\}|S']|S] \tag{4}
\]

\[
VC^e(S; \theta) = \mathbb{E}_{S'}[\mathbb{E}_{\kappa'_u}[^*\{\beta \pi^s(S'; \alpha) + \beta \mathbb{E}_{\kappa'_e}[^*\{VC^s(S'; \theta, \kappa'_x)\}|S'] - \kappa'_s|S'|}|S] \tag{5}
\]

\[
\beta \pi^f(S'; \alpha) + \beta \mathbb{E}_{\kappa'_u}[^*\{VC^f(S'; \theta, \kappa'_x)\}|S'] - \kappa'_u|S'||S] \tag{6}
\]

\[
VC^s(S; \theta) = \mathbb{E}_{S'}[\mathbb{E}_{\kappa'_u}[^*\{\beta \pi^s(S'; \alpha) + \beta \mathbb{E}_{\kappa'_e}[^*\{VC^s(S'; \theta, \kappa'_x)\}|S'] - \kappa'_u|S'|}|S] \tag{6}
\]

\[
\beta \pi^f(S'; \theta) + \beta \mathbb{E}_{\kappa'_u}[^*\{VC^f(S'; \theta, \kappa'_x)\}|S'] - \kappa'_u|S'||S] \tag{7}
\]

Equation (5), which represents the continuation value of a potential entrant who has chosen to enter the market, is interpreted in the following manner: ignoring both outer expectations for the moment, upon choosing to enter, the firm faces the choice to become a slow provider or a fast provider. Accordingly, after the max operator, the top line of equation (5) is the discounted value of what the firm would receive in the next period if it enters as a slow provider - it would earn
slow incumbent profits according to next period’s state \((S')\), and it would receive the expected maximum (conditional on next period’s state) of next period’s exit value and next period’s slow incumbent continuation value, which is given by equation (6). Likewise, the bottom line of equation (5) is the discounted value of what the firm would receive in the next period if it enters as a fast provider - it would receive fast incumbent profits according to next period’s state \((S')\), and it would receive the expected maximum (conditional on next period’s state) of next period’s exit value and next period’s fast incumbent continuation value, which is given by equation (7) - minus the sunk cost of becoming a fast provider (which is paid in the current period). Returning to the two outer expectations, the expectation is taken over \(\kappa_f\) because at the time the firm has chosen to enter the market, it has not yet observed the cost of entering as a fast provider. This expectation is also taken conditional on next period’s state, \(S'\). Finally, the outermost expectation is taken over all possible states for next period, and is taken conditional on the current state, \(S\).

Equation (6) can be interpreted in a similar fashion, and equations (4) and (7) are much simpler, as firms who have decided to remain potential entrants or fast incumbents do not have a two-step decision to make.

An important point to make is that the expectations over future states reflect the relevant firm’s perceptions about the likelihood of observing each future state conditional on the current state, and \textit{conditional on the firm taking the specified action.} For example, the expectation over the future number of slow and fast providers (which are elements of \(S\)) in \(VC^f(S; \theta)\) is conditional on the firm itself remaining a fast provider in the next period.

### 4.4 Equilibrium

I assume that the observed data are generated by a Markov-perfect equilibrium (MPE) of the model described in this section. A MPE requires that in equilibrium, all policy functions (entry, upgrade, and exit probabilities) be optimal for each firm, given the continuation values and given that all other firms follow these same policy functions. Given the construction of the model, there is no guarantee that there is a unique equilibrium. However, the estimation procedure allows the data
to “pick out” the equilibrium that was played\textsuperscript{11}.

### 4.5 Parameterization of the Profit Function

In order to estimate the model, I assume the following functional form for the per-period profit function:

\[
\pi(S; \alpha) = \alpha_0 + \alpha_1 pop {\text{ulation}} + \alpha_2 (N^{DSL, slow} + N^{cable, slow}) + \alpha_3 (N^{DSL, fast} + N^{cable, fast}) + \alpha_4 1\{\tau = fast\}
\]  

(8)

This functional form assumes that per-period profits are affected by the market’s population, the number of slow providers in the market, the number of fast providers in the market, and whether or not the firm itself is a fast provider. Implicit in this choice of functional form is the assumption that all firms offering the same speed within the same market earn the same profit. I argue that this assumption is reasonable, as internet service is typically viewed as a homogeneous good, conditional on speed.

### 4.6 Parameterization of Sunk Costs

Rather than assuming that the means of the sunk cost distributions are the same across all markets and firms, I leverage the information about each technology type described in section 2 in order to build a more complete model of entry and upgrade behavior. To this end, I parameterize the sunk cost distributions in the following manner:

\[
\mu_e = \lambda_e^c + \lambda_e^f 1\{T = cable\}
\]  

(9)

\[
\mu_u = \lambda_u^c + \lambda_u^f 1\{T = DSL\} \ast CODistance + \lambda_u^f 1\{T = cable\} \ast pop {\text{ulation}}
\]  

(10)

\[
\mu_f = \mu_u
\]  

(11)

where \textit{CODistance} represents the distance from the market’s centroid to the nearest DSL provider’s central office.

\textsuperscript{11}See Pakes, Ostrovsky, and Berry (2007) for a detailed discussion.
This allows the expected upgrade sunk cost paid by a slow incumbent to become a fast incumbent to vary with the distance from the market’s centroid to the nearest central office if the firm is a DSL provider, but vary with the population of the market if the firm is a cable provider. This reflects the fact that cable internet speeds are limited by the number of subscribers sharing a line, while DSL speeds are limited by the length of the line. In addition, the expected sunk cost of entering as a slow provider is constant across markets, but is allowed to differ depending on the firm’s technology. This reflects the fact that the cost of setting up the central office and laying the initial wiring is different for DSL and cable providers. Finally, I assume that the additional expected cost of entering as a fast provider is equal to the expected cost of upgrading to become a fast provider.

5 Estimation

In order to estimate the model, I use a modified version of the estimation procedure proposed by Pakes, Ostrovsky, and Berry (2007). This procedure is a two-step estimation method which first computes non-parametric estimates of the conditional choice probabilities, and then uses these estimates to construct estimates of the continuation values. Finally, the estimated continuation values are used to predict probabilities of firms’ actions conditional on alternative parameter values, and these predictions are fitted to the data. A key advantage of using this estimation method is that the continuation values are linear functions of the variables, thus eliminating the need to perform computationally burdensome nested fixed-point algorithms or matrix inverses for each alternative parameter vector. This permits the use of a large state space in the estimation. Another attractive feature of this estimation method is its intuitive design. It uses the actual average of the realized continuation values as estimates for the firms’ expected continuation values, providing a very natural estimator.

In order to apply this estimation procedure to my model, I make several modifications. First, as in Fan and Xiao (2015), I allow potential entrants to weigh the value of waiting against the value of entry, rather than forcing potential entrants to make a now-or-never entry decision. This necessitates the estimation of the continuation value of waiting for potential entrants. Second, I
allow firms to enter as one of two different types of incumbents, and I allow slow incumbents to choose to become fast incumbents. This requires the estimation of a continuation value for slow incumbents as well as a continuation value for fast incumbents. The timing assumptions of the model are critical to the estimation, as they allow for the construction of closed-form representations of these additional continuation values.

Following Pakes, Ostrovsky, and Berry (2007), I assume that $\kappa_e$, $\kappa_f$, $\kappa_u$, and $\kappa_x$ are exponentially distributed and i.i.d. over markets, firms, and time. This allows me to compute the following closed form solutions for each of the continuation values in the model$^{12}$

\[
VC^f(\theta) = (I - \beta M^f)^{-1}\beta M^f[\pi^f(\alpha) + P_x^f\mu_x]
\]

\[
VC^s(\theta) = (I - \beta M^s)^{-1}M^s[\beta \pi^s(\alpha) + \beta P^s_x\mu_x + P^s_u[\beta \pi^f(\alpha) + \beta VC^f(\theta) + \beta P^f_x\mu_x - \mu_u]]
\]

\[
VC^e(\theta) = M^e[\beta \pi^s(\alpha) + \beta VC^s(\theta) + \beta P^s_x\mu_x + P^e_f[\beta \pi^f(\alpha) + \beta VC^f(\theta) + \beta P^f_x\mu_x - \mu_f]]
\]

\[
VC^p(\theta) = (I - \beta M^p)^{-1}\beta M^p P_e[VC^e(\theta) - \mu_e]
\]

Equations (12) through (15) above are matrix representations of equations (4) through (7). Each of the continuation values above are vectors whose components represent the continuation value associated with each possible state. Each of the $M$ matrices are Markov transition matrices representing the probabilities of transitioning to each of the possible states, conditional on currently being in each of the possible states. The superscripts $f$, $s$, $e$, and $p$ on the $M$ matrices denote that the transition probabilities are as perceived by a fast incumbent who has decided to remain in the market, a slow incumbent who has decided to remain in the market, a potential entrant who has decided to enter the market, and a potential entrant who has decided not to enter the market, respectively. These matrices differ because the transition probabilities for the number of potential entrants, slow incumbents, and fast incumbents are conditional on the action chosen by the relevant firm. The $P_x$ vectors represent the exit probabilities at each state, with the $f$ and $s$ superscripts denoting the exit probabilities for fast and slow incumbents, respectively. $P^s_u$ represents the probability at each state that a slow incumbent who has chosen to remain in the

$^{12}$Derivations of equations (12) through (15) can be found in the appendix.

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market upgrades to become a fast provider, $P_f^c$ represents the probability at each state that a potential entrant who has chosen to enter will enter as a fast provider, and $P_e$ represents the probability at each state that a potential entrant chooses to enter the market. The $\pi(\alpha)$ vectors represent per-period profits at each state, where

$$\pi(\alpha) = \alpha_0 + \alpha_1 \text{population} + \alpha_2 (N_{DSL,slow} + N_{cable,slow}) + \alpha_3 (N_{DSL,fast} + N_{cable,fast})$$

$$\pi_f(\alpha) = \pi(\alpha) + \alpha_4$$

The $\mu_x$, $\mu_u$, $\mu_f$, and $\mu_s$ vectors represent the expected sunk costs or values at each possible state of exiting, upgrading, entering as a fast provider, and entering as a slow provider, and are parameterized as in equations (9) through (11). With these definitions in mind, equations (12) through (15) can be interpreted as the expected discounted future returns that firms would earn on alternative future paths of the state variables, given their current state and action chosen.

In order to estimate the continuation values, I replace each of the $M$ matrices and $P$ vectors with their empirical counterpart. For example, I construct $\hat{M}^f$ such that the $(i,j)$th element of $\hat{M}^f$ is equal to the number of observations where an incumbent fast provider chooses to remain in the market and the state transitions from state $i$ to state $j$, divided by the number of observations where an incumbent fast provider chooses to remain in the market and the state is $i$. Similarly, for example, I construct $\hat{P}_x^f$ such that the $i$th element of $\hat{P}_x^f$ is equal to the average fraction of incumbent fast providers who exit the market when the state is $i$. The other $\hat{M}$ matrices and $\hat{P}$ vectors are constructed analogously. In order to construct the $\hat{M}$ matrices and $\hat{P}$ vectors, I must discretize many of the variables in the state space. The details of this procedure are in the appendix. Importantly, all $\hat{M}$ matrices and $\hat{P}$ vectors are functions of only the data, not of any parameters that are to be estimated. This means that they can each be constructed once and then held constant throughout the estimation procedure.

The estimation is carried out as follows: I do not estimate $\beta$, but rather assume a value of .99. Next, the $\hat{M}$ and $\hat{P}$ matrices and vectors are calculated. Then, for some possible parameter vector, $\tilde{\theta}$, I calculate $\hat{V}C^f(\tilde{\theta})$, $\hat{V}C^s(\tilde{\theta})$, $\hat{V}C^e(\tilde{\theta})$, and $\hat{V}C^p(\tilde{\theta})$. Then, given these estimates of the
continuation values, I calculate the model-predicted probabilities of each of the possible actions. Then I search over possible parameter vectors, $\hat{\theta}$, to maximize the following pseudo-log-likelihood function\(^\text{13}\):

$$L(\theta) = \sum_{S \in \mathcal{S}} \left\{ \sum_{a \in A_p} \#_S^a \times \log[Pr(a; \theta)_S] + \sum_{a \in A_s} \#_S^a \times \log[Pr(a; \theta)_S] + \sum_{a \in A_f} \#_S^a \times \log[Pr(a; \theta)_S] \right\}$$ (16)

where $S$ indexes states, $\mathcal{S}$ is the set of all states, $a$ indexes actions, $A_p$ is the set of actions available to potential entrants, $A_s$ is the set of actions available to slow incumbents, $A_f$ is the set of actions available to fast incumbents, $\#_S^a$ is the number of firms who take action $a$ in state $S$, and $Pr(a)_S$ is the model-predicted probability that a firm takes action $a$ in state $S$\(^\text{14}\). Note that the likelihood is summed only over states, as markets and time periods are pooled together.

5.1 Identification

Broadly speaking, the parameters of the model are identified by differences in the observed probabilities of entry, exit, and upgrades across markets with different characteristics and across the two firm technologies.

Specifically, $\alpha_1$ (the population profit effect) is identified by differences in firms’ entry and exit probabilities across high and low population markets, holding other characteristics constant. $\alpha_2$ (the slow competitors profit effect) is identified by differences in firms’ entry and exit probabilities across markets with many or few slow incumbents, holding other characteristics constant. Likewise, $\alpha_3$ (the fast competitors profit effect) is identified by differences in firms’ entry and exit probabilities across markets with many or few fast incumbents, holding other characteristics constant. $\lambda_1^e$ (the entry cost technology effect) is identified by differences in entry probabilities of DSL and cable providers in markets with the same characteristics. $\lambda_1^u$ (the upgrade cost distance effect) is identified by differences in upgrades and entry as a fast provider of DSL providers across markets that are near and far away from a central office. $\lambda_2^u$ (the upgrade cost population effect) is identified by differences in upgrades and entry as a fast provider of cable providers across high and low population

\(^{13}\)It is also possible to use a GMM or pseudo-minimum $\chi^2$ estimator as well. There is some debate in the literature about the advantages and shortcomings of each of these options.

\(^{14}\)The derivation of these model-predicted probabilities can be found in the appendix.
markets. Finally, with all other parameters identified, variation in the sunk costs of upgrade and entry as a fast provider pin down $\alpha_4$ (the own-speed profit effect), as firms will only upgrade or enter as a fast provider in markets where the expected sunk cost is less than the gain in profits from earning an additional $\alpha_4$ in each period.

6 Results

The estimation results of the structural parameters of the per-period profit function are presented in table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$ (Intercept)</td>
<td>155.356</td>
<td>15.364***</td>
</tr>
<tr>
<td>$\alpha_1$ (Population effect)</td>
<td>0.209</td>
<td>0.001***</td>
</tr>
<tr>
<td>$\alpha_2$ (Slow competitors effect)</td>
<td>-105.993</td>
<td>7.671***</td>
</tr>
<tr>
<td>$\alpha_3$ (Fast competitors effect)</td>
<td>-258.559</td>
<td>4.283***</td>
</tr>
<tr>
<td>$\alpha_4$ (Own speed effect)</td>
<td>61.692</td>
<td>7.086***</td>
</tr>
</tbody>
</table>

*** indicates significance at the 1% level, ** indicates significance at the 5% level, and * indicates significance at the 10% level.

All estimates are statistically significant at the 1% level and take on the expected sign. The population effect of .209 suggests that per-period profits are 401.54 higher in medium-sized markets than small markets, 561.30 higher in large markets than medium markets, and 962.85 higher in large markets than small markets. This effect is consistent with the idea that larger markets are typically more profitable for internet service providers. The slow competitors effect says that per-period profits decrease by 105.99 for each additional slow competitor in the market, while the fast competitors effect says that per-period profits decrease by 258.56 for each additional fast competitor in the market. One would certainly expect these competitive effects to diminish firms’ profits, and the stronger effect of fast competitors is unsurprising. Fast ISPs, who provide an unambiguously higher-quality service than slow ISPs, should have a greater effect on firm profits, other things.

---

15 The results presented in this section are very preliminary. The estimates are sensitive to the starting values passed to the optimization routine. The estimates reported are those that generate the highest value of the likelihood function, among all starting values attempted.

16 See the appendix for an explanation of the discretization of population into small, medium, and large markets.
equal. Finally, the own speed effect suggests that the per-period profits of high-speed ISPs are 61.69 greater than slow-speed ISPs. This is again an intuitive result, as high-speed providers can expect to earn higher revenues, while the majority of the cost differences manifest themselves as sunk costs.

Table 4 presents the estimation results of the structural parameters of the sunk cost distributions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0^e$ (Entry cost mean intercept)</td>
<td>10,000.00</td>
<td>7552.22**</td>
</tr>
<tr>
<td>$\lambda_1^e$ (Entry cost mean technology effect)</td>
<td>-20.965</td>
<td>33.903</td>
</tr>
<tr>
<td>$\lambda_0^u$ (Upgrade cost mean intercept)</td>
<td>-8.319</td>
<td>29.973</td>
</tr>
<tr>
<td>$\lambda_1^u$ (Upgrade cost mean DSL distance effect)</td>
<td>4.184</td>
<td>31.802</td>
</tr>
<tr>
<td>$\lambda_2^u$ (Upgrade cost mean cable population effect)</td>
<td>16.898</td>
<td>0.135***</td>
</tr>
<tr>
<td>$\mu_x$ (Exit value mean)</td>
<td>84.021</td>
<td>0.857**</td>
</tr>
</tbody>
</table>

** indicates significance at the 1% level, *** indicates significance at the 5% level, and * indicates significance at the 10% level.

The entry cost mean technology effect suggests that being a cable provider decreases the base cost of entry into a new market by 20.97, though this estimate is not statistically significant. *A priori*, I did not expect this parameter to take a particular sign, and the lack of statistical significance suggests that the cost of entering a market may not be significantly different for cable and DSL providers. The upgrade cost mean DSL distance effect of 4.184 says that the cost of upgrading in a market far away from a DSL central office is 35.88 higher than upgrading in a market that is nearby a DSL central office $^{17}$, though this estimate is also statistically insignificant. The upgrade cost mean cable population effect of 16.898 suggests that for cable providers, the cost of upgrading to become a fast provider is 32,465.47 higher in a medium-sized market than a small market, 45,382.29 higher in a large market than a medium market, and 77,847.76 higher in a large market than a small market. This estimate is consistent with the idea that the number of subscribers making use of the same cable determines the speed each receives. Finally, the exit value mean of 84.02 indicates that firms do indeed receive a positive value for selling off their assets upon exiting a market.

$^{17}$See the appendix for an explanation of the discretization of central office distance into nearby and far-away markets.
Table 5: Counterfactual results: The Effect of Subsidies on Market Structure

<table>
<thead>
<tr>
<th>Subsidy policy</th>
<th>$\Delta$ # fast providers in Dec. 13</th>
<th>% $\Delta$ # markets w/ 0 fast providers</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% on entry and upgrade costs</td>
<td>118</td>
<td>-2.84%</td>
</tr>
<tr>
<td>63% on DSL and cable upgrade costs</td>
<td>445</td>
<td>-29.38%</td>
</tr>
<tr>
<td>65.8% on cable upgrade costs</td>
<td>465</td>
<td>-29.62%</td>
</tr>
</tbody>
</table>

7 Counterfactuals

With estimates of the model’s parameters in hand, I now simulate the actions that would have occurred if a subsidy had been in place from 2010 through 2013. The results of these simulations are reported in table 5. I evaluate the effect of each subsidy by comparing market structures under the subsidy in December 2013 to market structures in the absence of any subsidy in December 2013 using two measures: the change in the total number of fast providers across all markets, and the percentage change in the number of markets with zero fast providers.

I first simulate a non-discriminatory subsidy that decreases the sunk cost of entry and the sunk cost of upgrading by 30%, regardless of whether the firm uses DSL or cable technology. I find that the number of fast providers serving markets in California by December 2013 increases by 118, relative to what would occur in the absence of any subsidy. Furthermore, I find that the number of markets with zero fast providers decreases by just 2.84%. The choice of a 30% subsidy is arbitrary, and in the counterfactuals that follow, I use this nondiscriminatory subsidy as a benchmark, calibrating the subsidy percentages so that the total amount paid out under the subsidy remains the same.

Since the goal of the subsidy policy is to induce firms to offer high-speed internet service, in the next counterfactual, I decrease only the sunk cost of upgrading (which is also the additional cost of entering as a fast provider, on top of the initial entry cost), again for both DSL and cable firms. The model then predicts that the number of fast providers serving in markets in California would increase by 445, relative to what would occur in the absence of any subsidy. Also, the number of markets without any fast providers would decrease by 29.38%. This is clear evidence that targeting the subsidy towards only those costs associated with providing high-speed service rather than supporting entry in general has a substantial impact upon the number high-speed providers.
Next, I use the information gained from estimating the structural parameters to attempt to further improve the effectiveness of the subsidy. The estimated parameters suggest that upgrading to become a fast provider is much more costly for a cable provider than for a DSL provider, and in particular, that this cost increases dramatically with the population of a market. Therefore, I next simulate a subsidy which decreases the sunk cost of upgrading by 65.8%, but only for cable providers. It is also important to note that because upgrade cost is a function of a market’s population for cable providers, this subsidy is much larger in absolute terms for markets with higher populations. The model predicts that the number of fast providers serving markets in California by December 2013 would increase by 465, relative to what would occur in the absence of any subsidy. This is significantly higher than what the model predicts for the policy that subsidizes both entry costs and upgrade costs, and slightly higher that what the model predicts for the policy that subsidizes only upgrade costs, but for both DSL and cable providers. In addition, the number of markets without any fast providers would decrease by 29.62%. This is significantly higher than what the model predicts for the policy that subsidizes both entry costs and upgrade costs, but nearly identical to what the model predicts for the policy that subsidizes only upgrade costs, but for both DSL and cable providers. This suggests that there are some gains in targeting the subsidy towards cable providers, but that nearly all of the additional entry that results from this occurs in markets that were already served by a fast provider.

8 Conclusions and Future Work

The model estimates indicate that characteristics of firms and markets dramatically influence the cost of providing high-speed internet access. For this reason, a subsidy policy with the goal of bringing high-speed access to as many households as possible should be aware of these cost differences and target funding accordingly. The results of the counterfactual subsidy simulations suggest that there are gains to be made by only subsidizing costs associated with high-speed provision and not entry in general, and by more heavily subsidizing cable providers in high-population areas.

The next step for this line of research is to explore more nuanced subsidy policies that further exploit the information contained in the structural parameter estimates. For example, it may be
more effective to limit subsidy funds to cable firms but only in markets with high populations, or DSL firms but only in markets that are far away from a central office.

In addition, the National Broadband Map provides data for the entire United States, so I will estimate the model using the entirety of the data and perform counterfactuals for the whole country, rather than just the state of California. This will provide a more accurate picture of what an effective national subsidy would look like.
References


metrica, 80(3), 1019-1061.


Appendix

Derivation of Continuation Values

To derive equation (12), I start with equation (7):

\[
VC^f(S; \theta) = \beta E_{S'}[\pi^f(S'; \alpha) + E_{\kappa_x'}[\max\{VC^f(S'; \theta), \kappa_x'\}]|S']|S
\]

where \( p_x^f(S') \equiv Pr(\kappa_x > \beta VC^f(S'; \theta)) \) is the probability that a fast firm exits when the state is \( S' \), and the value of the inner expectation is a property of the exponential distribution. Rewriting equation (18) in matrix notation, we have

\[
VC^f(\theta) = \beta M^f[\pi^f(\alpha) + P_x^f \mu_x + VC^f(\theta)]
\]

where \( M^f \) is the Markov transition matrix conditional on a fast incumbent remaining in the market, and \( P_x^f \) is a vector of probabilities that a fast incumbent exits the market at each state. This implies that

\[
VC^f(\theta) = (I - \beta M^f)^{-1} \beta M^f[\pi^f(\alpha) + P_x^f \mu_x]
\]

which is equation (12). This first derivation follows closely from Pakes, Ostrovsky, and Berry (2007). The remaining derivations require some adaptations to fit my model.

To derive equation (13), I start with equation (6):

\[
VC^s(S; \theta) = E_{S'}[E_{\kappa_x'}[\max\{\beta \pi(S'; \alpha) + \beta E_{\kappa_x'}[\max\{VC^s(S'; \theta), \kappa_x'\}]|S']|S],
\]

\[
\beta \pi(S'; \theta) + \beta E_{\kappa_x'}[\max\{VC^f(S'; \theta), \kappa_x'\}]|S'] - \kappa_u|S']|S]
\]
\[ \begin{align*}
&= \mathbb{E}_{S'}[\mathbb{E}_{\kappa_u}[\max\{\beta \pi^s(S'; \alpha) + \beta[p^*_u(S') \mu_x + VC^s(S'; \theta)]\}], \\
&= \mathbb{E}_{S'}[\mathbb{E}_{\kappa_u}[\max\{\beta \pi^f(S'; \alpha) + \beta[p^*_u(S') \mu_x + VC^f(S'; \theta)] - \kappa'_u\}|S']|S]
\end{align*} \]

where the value of the innermost expectations is again a property of the exponential distribution.

For ease of notation, I define the first term inside the max operator in equation (20) to be \( A \), and the second term to be \( B - \kappa'_u \). Then, I can rewrite equation (20) as:

\[ \begin{align*}
&= \mathbb{E}_{S'}[Pr(A > B - \kappa'_u)A + Pr(A \leq B - \kappa'_u)\mathbb{E}(B - \kappa'_u|A \leq B - \kappa'_u)] \\
&= \mathbb{E}_{S'}[(1 - p^*_u(S'))A + Ps_u(S')(A + B - \mu_u)|S] \\
&= \mathbb{E}_{S'}[\beta \pi^s(S'; \alpha) + \beta(p^*_u(S') \mu_x + VC^s(S'; \theta)) + p^*_u(S')\beta \pi^f(S'; \alpha) + \beta(p^*_u(S') \mu_x + VC^f(S'; \theta)) - \mu_u]|S]
\end{align*} \]

rewriting equation (24) in matrix notation, we now have

\[ VC^s(\theta) = M^s[\beta \pi^s(\alpha) + \beta(P^s_x \mu_x + VC^s(\theta)) + P^s_u[\beta \pi^f(\alpha) + \beta(P^f_x \mu_x + VC^f) - \mu_u]] \]

where \( M^s \) is the Markov transition matrix conditional on a slow incumbent remaining in the market, \( P^s_x \) is a vector of probabilities that a slow incumbent exits the market at each state, and \( P^s_u \) is a vector of probabilities that a slow incumbent upgrades at each state. This implies that

\[ VC^s(\theta) = (I - \beta M^s)^{-1} M^s[\beta \pi^s(\alpha) + \beta(P^s_x \mu_x + P^s_u[\beta \pi^f(\alpha) + \beta(P^f_x \mu_x + VC^f) - \mu_u]] \]

which is equation (13).

To derive equation (14), I start with equation (5):

\[ VC^e(S; \theta) = \mathbb{E}_{S'}[\mathbb{E}_{\kappa_u}[\max\{\beta \pi(S'; \alpha) + \beta\mathbb{E}_{\kappa_2}[\max\{VC^s(S'; \theta), \kappa'_x\}]|S'], \\
\beta \pi(S'; \theta) + \beta\mathbb{E}_{\kappa_2}[\max\{VC^f(S'; \theta), \kappa'_x\}]|S'] - \kappa'_f|S']\}|S] \]
\[
\begin{align*}
\beta \pi^f(S'; \alpha) + \beta [p_x^f(S') \mu_x + VC^f(S'; \theta)] - \kappa'_{f}][S]\vert |S]
\end{align*}
\]

where the value of the innermost expectations is again a property of the exponential distribution.

For ease of notation, I define the first term inside the max operator in equation (27) to be \(A\) and the second term to be \(B - \kappa_f\). Then, I can rewrite equation (27) as

\[
\begin{align*}
E_{S'}[Pr(A > B - \kappa'_f)A + Pr(A \leq B - \kappa'_f)E(B - \kappa'_f|A \leq B - \kappa'_f)]
\end{align*}
\]

\[
= E_{S'}[(1 - P^e_{n}(S'))A + P_{s_u}(S')(A + B - \mu_f)|S]
\]

\[
= E_{S'}[A + p_{u}^e(S')(B - \mu_f)]
\]

\[
= E_{S'}[\beta \pi^e(S'; \alpha) + \beta (p_x^e(S') \mu_x + VC^e(S'; \theta)) +
\]

\[
+ p_u^e(S')][\beta \pi^f(S'; \alpha) + \beta (p_x^f(S') \mu_x + VC^f(S'; \theta)) - \mu_f]|S]
\]

where \(p_f^e(S')\) is the probability that a potential entrant who has decided to enter chooses to enter as a fast provider when the next period’s state is \(S'\). Rewriting equation (5) in matrix notation, we now have

\[
VC^e(\theta) = M^e[\beta \pi^e(\alpha) + \beta (P_x^e \mu_x + VC^e(\theta)) + P_f^e[\beta \pi^f(\alpha) + \beta (P_x^f \mu_x + VC^f(\theta)) - \mu_f]]
\]

which is equation (14), where \(M^e\) is the Markov transition matrix conditional on a potential entrant entering the market, and \(P_f^e\) is a vector of probabilities that a potential entrant who has entered the market chooses to enter as a fast provider at each state.

To derive equation (15), I start with equation (4):

\[
\begin{align*}
VC^p(S; \theta) &= \beta E_{S'}[E_{\kappa'_e} [\max\{VC^p(S'; \theta), VC^e(S'; \theta) - \kappa'_e\}]|S'|]|S]
\end{align*}
\]

\[
= \beta E_{S'}[Pr(VC^p(S'; \theta) > VC^e(S'; \theta) - \kappa'_e)VC^p(S'; \theta) +
\]

\[
+ Pr(VC^p(S'; \theta) \leq VC^e(S'; \theta) - \kappa'_e)E[VC^e(S'; \theta) - \kappa'_e|VC^p(S'; \theta) \leq VC^e(S'; \theta) - \kappa'_e]|S]
\]

\[
= \beta E_{S'}[(1 - p_e(S'))VC^p(S'; \theta) + p_e(S')[VC^p(S'; \theta) + VC^e(S'; \theta) - \mu_e]|S]
\]

\[
= \beta E_{S'}[(1 - p_e(S'))VC^p(S'; \theta) + p_e(S')[VC^p(S'; \theta) + VC^e(S'; \theta) - \mu_e]|S]
\]

\[
32
\]
where the value of the expectation in equation (33) is again a property of the exponential distribution, and $p_e(S')$ is the probability that a potential entrant enters the market when the state is $S'$. Rewriting equation (5) in matrix notation, we have

$$VC^p(\theta) = \beta M^p[VC^p(\theta) + P_e(VC^e(\theta) - \mu)]$$

(36)

where $M^p$ is the Markov transition matrix conditional on a potential entrant not entering the market, and $P_e$ is a vector of probabilities that a potential entrant enters the market at each state. This implies that

$$VC^p(\theta) = (I - \beta M^p)^{-1}\beta M^p P_e[VC^e(\theta) - \mu]$$

which is equation (15).

**Pseudo-likelihood Probabilities**

The actions available to potential entrants are wait, enter as a slow provider, or enter as a fast provider. The probability that a potential entrant waits in state $S$ is

$$Pr(Wait)_S = Pr(VC^p(S; \theta) > VC^e(S; \theta) - \kappa_e)$$

(37)

$$= 1 - F_{\kappa_e}(VC^e(S; \theta) - VC^p(S; \theta))$$

(38)

where $F_{\kappa_e}(\cdot)$ is the CDF of an exponential distribution with mean $\mu_e = \lambda_0^e + \lambda_1^e I\{T = cable\}$. Therefore, the probability that a potential entrant enters (as either a slow or fast provider) in state $S$ is

$$Pr(Enter)_S = 1 - Pr(Wait)_S = F_{\kappa_e}(VC^e(S; \theta) - VC^p(S; \theta))$$

(39)
Then, the probability that a potential entrant enters as a slow provider in state \( S \), conditional on entering, is

\[
Pr(\text{EnterSlow}|\text{Enter})_S = Pr(\text{Enter})_S Pr(\text{EnterSlow}|\text{Enter})_S
\]

\[
= Pr(\beta \mathbb{E}_{S'}[\pi^s(S'; \alpha) + p^s_x(S')\mu_x + VC^s(S'; \theta)] > S) > (40)
\]

\[
= Pr(\kappa'_f > \beta \mathbb{E}_{S'}[\pi^f(S'; \alpha) + p^f_x(S')\mu_x + VC^f(S'; \theta)] - (41)
\]

\[
= Pr(\kappa'_f > \beta \mathbb{E}_{S'}[\pi^f(S'; \alpha) + p^f_x(S')\mu_x + VC^f(S'; \theta)] - (42)
\]

In vector form, the probability that a potential entrant enters as a slow provider in each state, conditional on entering, is

\[
Pr(\text{EnterSlow}|\text{Enter}) = Pr(\kappa'_f > \beta M^{ef}[\pi^f(\alpha) + p^f_x\mu_x + VC^f(\theta)] - (43)
\]

\[
= 1 - F_{\kappa'_f}(\beta M^{ef}[\pi^f(\alpha) + p^f_x\mu_x + VC^f(\theta)] - (44)
\]

where \( M^{ef} \) and \( M^{es} \) are Markov transition matrices conditional on a potential entrant entering as a fast and slow provider, respectively, and \( F_{\kappa_f}(\cdot) \) is the CDF of an exponential distribution with mean \( \mu_f = \lambda_0^u + \lambda_1^u \mathbb{1}\{T = DSL\} \ast CODistance + \lambda_2^u \mathbb{1}\{T = cable\} \ast population \). Therefore, the probability that a potential entrant enters as a fast provider in each state, conditional on entering, is

\[
Pr(\text{EnterFast}|\text{Enter}) = 1 - Pr(\text{EnterSlow}|\text{Enter})
\]

\[
= F_{\kappa'_f}(\beta M^{ef}[\pi^f(\alpha) + p^f_x\mu_x + VC^f(\theta)] - (46)
\]

\[
= \beta M^{es}[\pi^s(\alpha) + p^s_x\mu_x + VC^f(\theta)]
\]
The actions available to slow incumbents are exit, upgrade to become a fast provider, or remain a slow provider. The probability that a slow incumbent exits in state $S$ is

$$Pr(SlowExit)_S = Pr(\kappa_x' > VC^s(S; \theta))$$

$$= 1 - F_{\kappa_x'}(VC^s(S; \theta))$$

where $F_{\kappa_x'}$ is the CDF of an exponential distribution with mean $\mu_x$. Therefore, the probability that a slow incumbent does not exit in state $S$ is

$$Pr(SlowNoExit)_S = 1 - Pr(SlowExit)$$

$$= F_{\kappa_x}(VC^s(S; \theta))$$

Then, the probability that a slow incumbent upgrades in state $S$, conditional on not exiting, is

$$Pr(Upgrade|SlowNoExit)_S = Pr(\beta E_{S'}[\pi^f(S'; \alpha) + p_x^f(S')\mu_x + VC^f(S')|S] - \kappa_u > \beta E_{S'}[\pi^s(S'; \theta) + p_x^s(S')\mu_x + VC^s(S'; \theta)|S])$$

In vector form, the probability that a slow incumbent upgrades in each state, conditional on not exiting, is

$$Pr(Upgrade|SlowNoExit) = Pr(\kappa_u' < \beta M^u[\pi^f(\alpha) + p_x^f \mu_x + VC^f(\theta)] - \beta M^{rs}[\pi^s(\alpha) + p_x^s \mu_x + VC^s(\theta)])$$

$$= F_{\kappa_u}(\beta M^u[\pi^f(\alpha) + p_x^f \mu_x + VC^f(\theta)] - \beta M^{rs}[\pi^s(\alpha) + p_x^s \mu_x + VC^s(\theta)])$$

where $M^u$ and $M^{rs}$ are Markov transition matrices conditional on a potential entrant upgrading and remaining a slow provider, respectively, and $F_{\kappa_u}(\cdot)$ is the CDF of an exponential distribution.
with mean $\mu_u = \lambda_u^0 + \lambda_u^1 \mathbb{1}\{T = DSL\} \cdot \text{CODistance} + \lambda^2_u \mathbb{1}\{T = cable\} \cdot \text{population}$. Therefore, the probability that a slow incumbent remains a slow provider in each state, conditional on entering, is

$$\Pr(\text{RemainSlow}|\text{SlowNoExit}) = 1 - \Pr(\text{Upgrade}|\text{SlowNoExit})$$

$$= 1 - F_{\kappa_u'}(\beta M^u[\pi^f(\alpha) + p^f_x \mu_x + VC^f(\theta)] -$$

$$\beta M^{rs}[\pi^s(\alpha) + p^s_x \mu_x + VC^s(\theta)])$$

The actions available to fast incumbents are exit or remain a fast provider. The probability that a fast incumbent exits in state $S$ is

$$\Pr(\text{FastExit}_S) = \Pr(\kappa'_x > VC^f(S'; \theta))$$

$$= 1 - F_{\kappa'_x}(VC^f(S; \theta))$$

Therefore, the probability that a fast incumbent remains a fast provider is

$$\Pr(\text{RemainFast}_S) = 1 - \Pr(\text{FastExit}_S)$$

$$= F_{\kappa'_x}(VC^f(S; \theta))$$

**Discretization of the State Space**

Estimation of the model requires the state space to be finite. Therefore, I must first discretize any continuous variables in the state space. These variables include the market’s population and the distance from the market’s centroid to the nearest DSL central office. To do this, I assign each market’s population to a bin according to the quartile its population falls within. I create three bins, with populations in the first quartile labeled as small markets, populations in the second and third quartiles labeled as medium markets, and populations in the fourth quartile labeled as large markets. I then assign each bin a value according to the mean population within that bin, so that all small markets are assigned a population of 2,525.311, medium markets are assigned a population of 4,446.572, and large markets are assigned a population of 7,132.233. Similarly, I
assign each market’s distance to the nearest DSL central office to a bin according to whether it is above or below the median distance, thus creating two bins. I again assign each bin the value of the mean distance within that bin, so that all markets close to a central office are assigned a distance of 1.988 and all markets far from a central office are assigned a distance of 10.564.

In addition, in order to reduce the dimension of the state space to a feasible level, I further discretize the number of firms in each technology-type pair in \( N \). I assign the number of slow DSL firms in the market to one of two bins, according to whether \( N^d,s = 0 \) or \( N^d,s \geq 1 \). I assign the second bin a value equal to the mean of all observations that fall within the bin (1.091). I discretize the number of slow cable firms, fast DSL firms, and fast cable firms identically. Finally, I assign the number of DSL potential entrants to one of two bins, according to whether \( N^d,p \) is less than or greater than the median number (7) of DSL potential entrants. I assign the number of cable potential entrants to one of two bins, according to whether \( N^c,p \) is less than or greater than the median number (8) of cable potential entrants.