Wanted Dead or Alive: Satellite Depreciation and Spectrum Hoarding

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Abstract

To place an operational satellite in earth orbit, a firm must obtain spectrum and an orbital slot. We suggest incumbent satellite operators may have incentives to "warehouse" a fraction of their assigned spectrum and orbital slots, keeping marginally productive or non-operational assets in place, when fully operational satellites would generate more total social value. We model firms' incentives to warehouse, and show conditions under which firms choose to warehouse rather than replace marginally productive or non-functioning satellites.

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1 Introduction

The allocation of spectrum and orbital slots for commercial satellite service providers is achieved via a non-market regulatory process where national governments, typically at the behest of individual commercial firms within those nations, apply for spectrum and orbital slots from the United Nation's International Telecommunications Union (ITU), which is the prevailing international regulator. Commercial satellite services support and include television and radio broadcasts, broadband, mobile voice and data, business, government and military applications, global positioning services, and an enormous range of scientific endeavors.

Historically, the United States and the former Soviet Union were the initial grantees of orbital slots and spectrum in the 1950s. Over time, other nations gained the requisite technological capability and obtained spectrum and orbital slot allocations. Currently, there are a handful of large commercial geo-stationary satellite operators worldwide, and a group of smaller operators. The two largest international operators, Intelsat and SES, operate fleets of greater than fifty satellites each.

In the United States, the current allocation system for orbital slots and spectrum is first-come, first-served, with the first qualified applicant obtaining the license.[1] We suggest, however, that incumbents who had substantial first-mover advantages when licenses were initially allocated, and have some form of market power, may not be the socially desirable recipients of new allocations.

In what follows, we model the choice of a large incumbent firm and perfectly competitive entrants in placing satellites in orbital slots. We suggest that incumbent firms may have incentives to under-utilize spectrum and orbital slots by maintaining a partially-operational or non-operational satellite at the assigned orbital location or not utilize the spectrum or orbital slots for extended periods of time. In short, incumbents may benefit, in part, by holding under-utilized spectrum and orbital resources tactically to gain or maintain a competitive advantage over potential competitors. We refer to this tactic as "warehousing" and suggest that warehousing spectrum and orbital slots may be a form of market foreclosure.

[1] After the initial license allocation, a company is required to post a bond. The bond is a commitment mechanism implemented to discourage spurious applications, encourage quick build-out and deployment, and induce relinquishment of unneeded licenses in a timely fashion. The licensee is required to meet certain planning and infrastructure milestone deadlines; otherwise it forfeits the bond. Thus, the bond, if properly calibrated, might be useful in inducing satellite operators to release resources to higher-valued uses.

Satellites can be characterized by three states: fully functioning, partially functioning, or non-functioning. Satellites have two features that affect their functionality: their electronic array and their station-keeping fuel. Station keeping fuel is expended to maintain a "fixed" location in orbit. A satellite that has run out of station-keeping fuel drifts in an inclined orbit within its assigned orbital slot, reducing its ability to communicate with ground stations.[2] These satellites, assuming their electronics still function, provide per satellite revenue that is lower than a fully functioning satellite (i.e., a satellite with station-keeping fuel), ceteris paribus. A satellite whose electronic arrays are non-functional provides no direct revenue, irrespective of station keeping fuel.

In our model, firms can choose to place a fully functioning satellite in each assigned slot, or maintain a partially functioning or non-functioning satellite in some or all slots. A partially functioning or non-functioning satellite may provide less revenue per unit of cost than a fully functioning satellite, but even if there is no difference in overall profitability, we suggest a partially or non-functioning satellite still provides value for the incumbent operator via foreclosure. That is, if the incumbent maintains a partially or non-functioning satellite, the orbital slot and spectrum is not available to a new entrant to launch a fully-functional satellite, which may relax pricing pressure on the incumbent. In essence, the incumbent may raise endogenous barriers to entry.

A literature on sales of airport landing slots highlight regulatory problems somewhat analogous to the problems that arise in warehousing orbital slots. For example, Gale and O'Brien (2013) theoretically investigate the effects of "use-or-lose" requirements for landing slots in the airline industry using a dominant firm-competitive fringe model. The "use-or-lose" provision in the airline industry sets minimum utilization requirements for landing slots with the intention of restricting capacity hoarding. Using a three-stage model, the authors find this provision induces the dominant firm to acquire landing slots from the fringe. Under some conditions, aggregate output and social welfare may fall. The satellite industry has somewhat similar requirements for usage of orbital slots in that firms are expected to occupy their assigned orbital slot with a satellite within a reasonable period of time. While our model assumes that unused slots of the dominant firm are reallocated to the fringe, we do not analyze the effects of acquiring orbital slots by a dominant firm.

Reitzes et al. (2015) construct a theoretical model to analyze how sales of airport

[2] Over a longer period of time, these satellites will drift outside of their assigned orbital slots, increasing the risk of collision with other satellites.

landing slots affect social welfare and consumer surplus. The authors find that in a symmetric model, when there is an increase in the concentration of landing slots, consumer and social surplus might decrease because firms would serve fewer high-margin airline routes. However, the authors also find that their results may not hold if airlines face high fixed costs associated with serving airline routes. The model structure in Reitzes et al. differs from ours because they analyze a static model of airline slot allocations. The inefficiencies in their model come from serving lower margin routes when it was socially optimal to serve higher margin routes. Our model is dynamic and welfare inefficiencies come from under-utilizing orbital slots and from a failure to launch replacement satellites when it is socially desirable to do so.

Borenstein (1988), Brueckner (2009), and Starkie (1998) examine the efficiency of different mechanisms for allocating landing slots. Borenstein finds that allocation of licenses through market mechanisms does not necessarily yield efficient use of licenses. Brueckner concludes that uniform congestion tolls yield inefficient flight choices, while differential charges and auctioning of slots yield socially optimal results. Starkie argues that a secondary market for airline slots increases efficiency and raises average airline fares.

A similar problem is the practice of grocery slotting fees, in which manufacturers negotiate payments to grocery retailers in exchange for stocking, displaying and promoting their products. While some argue that this is an enhancement of efficiency, others argue that this is an exercise of market power, by the retailer and by larger manufacturers (Bloom et al. (2000); Commission et al. (2003)). This is similar to the problem of orbital slots analyzed in our paper, in that a limited number of slots for entry (in this case, shelf space in the grocery store) is being made available, and there is a cost to gaining and using these slots. One of the concerns that has been raised about this practice is that it can be used to foreclose entry or raise rivals costs. Klein and Wright (2007) find that there are pro-competitive business reasons for the use of slotting fees. But Shaffer (2005) finds that a dominant firm might outbid fringe firms for the slots, and that this yields a sub-optimal level of product variety which is socially undesirable.

The remainder of the paper is as follows: in Section 2 and its subsections we introduce the model and explore its predictions under two market structures. In section 3 we offer a conclusion. Longer proofs of propositions, and a table of parameter definitions, are contained in an Appendix.

2 Model

We model the satellite industry under a profit-maximizing objective. We explore a variety of market structures in order to investigate how the level of competition might impact the efficient use of bandwidth and spectrum. We begin with a relatively competitive structure.

2.1 Incumbent with Competitive Fringe

Assume that there is an incumbent provider of satellite services. The incumbent faces a competitive fringe that provides the same type of service. We assume a linear inverse demand function for satellite service given by:

$$P_t = a - bnQ_t \tag{1}$$

where the t subscript is an index of time, P_t is price, n is the number of consumers each satellite can serve, and Q_t is market output.[3] We assume that the incumbent can maintain three types of satellites: fully functional, given by y_t, partially functional, given by h_t and non-functional (or zombies) given by z_t. The quantity of output provided by the incumbent firm is given by:

$$q_t = y_t + (1 - \alpha)h_t \tag{2}$$

where α represents the fraction of output lost due to compromised performance from a partially functioning satellite. Notice that zombies do not produce any output. Market output is given by:

$$Q_t = q_t + u_t \tag{3}$$

where u_t is the quantity of entrants' satellites. By assumption the competitive fringe maintains only fully functioning satellites and occupies every slot received from the regulator.

There are four types of satellite launches. Total satellite launches in period t are given by $l_t = l_t^h + l_t^z + l_t^e + l_t^n$, where superscript h refers to launches to replace partially functioning satellites, z refers to launches to replace non-functioning satellites, e refers

[3]To aid the reader, we collect and introduce some notation in Table 1 of the appendix, noting here that subscript t represents the time period.

to launches into previously allocated but as yet unoccupied orbital slots, and n refers to launches into new unoccupied slots allocated in period t.

A satellite launch has a constant marginal cost given by c_l. We further assume that the launch of a satellite creates an adjustment cost which increases on the margin as more satellites are launched.[4] Adding a satellite to a fleet requires integrating that satellite with a network and we are assuming that doing so is more challenging as the volume of launches increases. We further assume that the marginal cost of operating a satellite is given by c_y, c_h, and c_z. respectively. Under these assumptions profits for the incumbent are given by:

$$\pi_t = P_t q_t - c_y y_t - c_h h_t - c_z z_t - c_l l_t - .5c_{2l}[(l_t^h)^2 + (l_t^n)^2 + (l_t^e)^2 + (l_t^z)^2] \quad (4)$$

where the last four terms represent adjustment costs.

We assume that satellites depreciate at a constant rate of δ so that the stock of the incumbent's fully functioning satellites follows

$$y_{t+1} = (1-\delta)y_t + l_t \quad (5)$$

We assume that depreciated satellites become partially functioning while partially functioning satellites become non-functioning at a rate of γ_h. Therefore the stock of these satellites follows

$$h_{t+1} = (1-\gamma_h)h_t + \delta y_t - l_t^h. \quad (6)$$

We assume that partially functioning satellites become non-functioning and non-functioning satellites disappear[5] at a constant rate of γ_z so that zombies follow

$$z_{t+1} = (1-\gamma_z)z_t + \gamma_h h_t - l_t^z. \quad (7)$$

The regulator assigns spectrum and satellite slots to the incumbent and the competitive fringe. The slots assigned to the incumbent are denoted by x. We let S represent total slots so that $S = x_t + u_t$. We let e represent empty slots held by the

[4] See Lucas (1967) and Hayashi (1982) for examples of these adjustment costs in a more general investment setting.

[5] A non-functioning satellite will disappear when forces push it out of its slot. In low earth orbit (LEO) this will usually result in the satellite burning up in the atmosphere while in geostationary orbit (GEO) the satellite will leave GEO.

incumbent. In period t, the regulator assigns new slots, ρ, to the incumbent so that $x_t = y_t + h_t + z_t + e_t + \rho_t$. We assume that the regulator will reallocate a fraction, ϕ of unused empty slots from the incumbent to the fringe. Further, we assume that all unused new slots are immediately reallocated to the fringe. This reallocation mechanism corresponds to the requirement that a launcher quickly meet benchmarks when it is assigned a new orbital slot. However, we are assuming that the regulator allows the incumbent to more slowly fill slots that unexpectedly become empty due to disappearance of zombie satellites. Under these assumptions the incumbent's slot allocation follows:

$$x_{t+1} = x_t - \phi(e_t - l_t^e) - (\rho_t - l_t^n) \tag{8}$$

The risk of losing a slot due to regulatory policy will heavily influence the incumbent's decisions regarding replacement of depreciated satellites.

In order to ensure that the stock of satellites is sensible we have a number of additional non-negativity constraints:

$$\rho_t \geq l_t^n \geq 0 \tag{9}$$

$$e_t \geq l_t^e \geq 0 \tag{10}$$

$$(1 - \gamma_h)h_t + \delta y_t \geq l_t^h \geq 0 \tag{11}$$

$$z_{t+1} = (1 - \gamma_z)z_t + \gamma_h h_t \geq l_t^z \geq 0 \tag{12}$$

The value function corresponding to maximization of (4) subject to (9)-(12) and the various laws of motion is:

$$\begin{aligned} V[y_t, h_t, z_t, x_t] \quad = \quad & \max_{l_e, l_n, l_h, l_z} \{\pi_t + \lambda_t^e(e_t - l_t^e) + \lambda_t^n(\rho_t - l_t^n) + \lambda_t^h[(1 - \gamma_h)h_t + \delta y_t - l_t^h] + \\ & \lambda_t^z[(1 - \gamma_z)z_t + \gamma_h h_t - l_t^z] - \mu_t^n l_t^n - \mu_t^e l_l^e - \mu_t^h l_t^h - \mu_t^z l_t^z\} + \\ & \beta V[y_{t+1}, h_{t+1}, z_{t+1}, x_{t+1}] \end{aligned} \tag{13}$$

where β is the discount factor and λ and μ are the relevant Lagrangian multipliers. The first-order conditions are:

$$l_t^h \quad : \quad -c_l - c_{2l} l_t^h - \lambda_t^h - \mu_t^h + \beta V_{y_{t+1}} - \beta V_{h_{t+1}} = 0 \tag{14}$$

$$l_t^z \quad : \quad -c_l - c_{2l} l_t^z - \lambda_t^z - \mu_t^z + \beta V_{y_{t+1}} - \beta V_{z_{t+1}} = 0 \tag{15}$$

$$l_t^n \quad : \quad -c_l - c_{2l} l_t^n - \lambda_t^n - \mu_t^n + \beta V_{y_{t+1}} + \beta V_{x_{t+1}} = 0 \tag{16}$$

$$l_t^e \quad : \quad -c_l - c_{2l} l_t^e - \lambda_t^e - \mu_t^e + \beta V_{y_{t+1}} + \phi\beta V_{x_{t+1}} = 0 \tag{17}$$

The condition for a launch to replace partially functioning satellites equates the marginal cost of a launch, including adjustment costs and the impact of the launch on constraints, to the discounted future marginal value of having a new satellite in orbit net of the discounted future value that is lost as a consequence of replacing a partially functioning satellite. The condition for a launch to replace a non-functioning satellite is similar. Note that the marginal value of a zombie satellite first appears in this expression.

The condition for launching a satellite into a newly available slot requires that the marginal cost of launching, including adjustment costs and the impact of the launch on constraints, to the discounted future marginal value a new satellite plus the future marginal value of a slot. Launching into an unoccupied new slot prevents the regulator from removing that slot and shifting it to the competitive fringe. This choice creates value by "reserving" the slot for the incumbent. The condition for launching into an empty slot is similar except the marginal return from a slot is discounted by an additional factor, ϕ. Because the regulator removes only a portion of empty slots, the value of reserving an empty slot by launching is correspondingly lower (there is some chance that the incumbent will not need to reserve a particular slot).

The envelope conditions associated with maximization of (13) are:

$$V_{y_t} = P_t - c_y + \delta\lambda_t^h + \beta[(1-\delta)V_{y_{t+1}} + \delta V_{h_{t+1}} + V_{x_{t+1}}] \tag{18}$$

$$V_{h_t} = (1-\alpha)P_t + \alpha bnq_t - c_h + \lambda_t^h(1-\gamma_h) + \lambda_t^z\gamma_h + \beta[(1-\gamma_h)V_{ht+1} + \gamma_h V_{z_{t+1}} + V_{x_{t+1}}] \tag{19}$$

$$V_{z_t} = bnq_t - c_z + \lambda_t^z(1-\gamma_z) + (1-\gamma_z)\beta V_{z_{t+1}} + \beta V_{x_{t+1}} \tag{20}$$

$$V_{x_t} = \beta V_{x_{t+1}} \tag{21}$$

These conditions can be obtained by substituting the expressions for incumbent and total slots into the inverse demand curve, yielding $P_t = a - bn(S - \alpha h_t - z_t - e_t - \rho_t)$.

From this expression it is clear that a fully functioning satellite has no impact on market prices. It expands output from the incumbent by the same amount as it decreases output by the fringe. On the other hand, partially functioning and zombie satellites increase market prices because they, net, reduce market output by preventing the fringe from deploying a fully functional satellite.

The first condition indicates that the marginal value of a fully functional satellite has three components: (1) the marginal revenue from satellite services, net of marginal operating cost, and the marginal value of relaxing the constraint on launching to replace partially functioning satellites, (2) the discounted value of having additional satellites in the future (fully and partially functional), and (3) the marginal value of having additional slots (a satellite prevents the regulator from removing a slot).

The second condition establishes that the marginal value of a partially functional satellite includes: (1) the marginal revenue, net of marginal operating costs, as well as the value of relaxing the launch constraints (for both partially functional as well as zombie satellites), (2) the discounted marginal value of future satellites (both partially and non-functional) and (3) the discounted marginal value of having additional slots in the future. It is worth noting that, other things equal and relative to a fully functional satellite, the marginal revenue from a partially functioning satellite is lower as a consequence of degraded service. However, there is an additional revenue term αbnq_t, which represents the price - increasing effect of maintaining a partially functioning satellite, as described above. We refer to this as strategic marginal revenue.

The third condition provides the marginal value of a zombie satellite. Zombies are a net loss in present terms, because they provide no direct revenue but involve operating costs. However, they involve greater strategic marginal revenue, relative to partially functioning satellites, because they deprive the market of more output. The incumbent also experiences future returns from maintaining non-functioning satellites in orbit. In particular, more zombie satellites in the present provides additional zombie satellites in the future, which has value. In addition, a non-functioning satellite serves as a "placeholder" and prevents the regulator from removing a slot and reallocating it to the competitive fringe. In this sense a non-functioning satellite creates value by preserving the incumbent's option to fill the slot with a functional satellite in the future.

The final envelope condition states that the marginal value of a slot is equal to the present discounted value of a slot in the next period. Other things equal a current slot gained represents an additional slot in the future. In addition to (14)-(21) the incumbent's choices are characterized by complementary slackness conditions corresponding to the inequality constraints (9)-(12).

These conditions can be solved in closed form for the volume of launches replacing non-functioning satellites. We are particularly interested in this result since, in our model, slots are scarce and they will be employed productively by the competitive fringe if they are not encumbered by the incumbent.[6] The competitive fringe's output is the opportunity cost of the incumbent's decision to hold a slot with an unproductive satellite. If the incumbent replaces nonfunctional satellites at a low rate then more output will be lost. We offer a solution to the model when none of the constraints bind:[7]

Proposition 2.1 *Assume $\lambda_t^j, \mu_t^j = 0$, $\forall j, t$. Then the optimal level of launches for an incumbent is given by:*

$$\begin{aligned} l_t^{z^*} =& (\delta^{-1}-1)^{-1}(1-\gamma_h)^{-1}(c_{2l})^{-1}\big[[1-\alpha-\delta^{-1}(1-\gamma_h)]P_t - c_h + \alpha bnq_t \\ &+ \delta^{-1}(1-\gamma_h)[V_{yt}+c_y-(1-\delta)c_l] + [1-\delta^{-1}(1-\gamma_h)] \\ &\left[\frac{(V_{z_t}+c_z-bnq_t)}{(1-\gamma_z)} - \gamma_z[(1-\phi)\beta(1-\gamma_z)]^{-1}c_{2l}(l_{t-1}^n - l_{t-1}^e)] - V_{h_t}\right] \end{aligned} \quad (22)$$

where the solutions for V_y^t, V_z^t, V_h^t are given by (33), (34) and (35) in the Appendix.

Proof See Appendix. ∎

Inspection of the equation for the optimal replacement of nonfunctional satellites indicates that the incumbent implements greater warehousing when there is a greater marginal cost of operating a zombie, depreciation of zombies, marginal cost of launching, price level, and productivity of partially functioning satellites (larger α). Moreover, when there are lower marginal costs of operating a partially functioning satellite, marginal costs of operating a fully functional satellite, lagged launches to replace partially functioning satellites, lagged launches to replace zombies, and satellite services there is greater warehousing.

[6]We are able to solve for all launch types, as described in the Appendix

[7]There are eight constraints that can bind or not bind. This yields 256 combinations of the constraints which could be explored. For this reason we begin by analyzing just one combination which represents the interior of all constraints.

These results indicate the impact of each factor on the replacement rate for non-functioning satellites. They do not ensure that warehousing actually takes place (that is, a strictly positive number of zombie satellites are in orbit). The following proposition provides the conditions under which the incumbent engages in warehousing:

Proposition 2.2 *If*

$$(1-\gamma_z)^t_{z_0}+\gamma_h h_0\sum_{j=0}^{t-1}(1-\gamma_z)^{t-1-j}(1-\gamma_h)^j > \sum_{j=1}^{t}(1-\gamma_z)^{j-1}l^{z*}_{t-j}+\gamma_h\sum_{j=0}^{t+2}\sum_{k=0}^{j}(1-\gamma_z)^{j-k}l^{h*}_{t-j-2}$$

then $z_t > 0$*, and warehousing takes place in period* t*.*

Proof See Appendix. $\blacksquare$

This proposition establishes initial values of non-functioning satellites and partially functioning satellites such that the rate of launching to replace these satellites is relatively low and zombies accumulate over time. We note that this is a sufficient condition for warehousing in a given period. There are additional values of launch rates which still involve warehousing. Moreover, we can combine the results from Propositions 2.1 and 2.2 to infer the underlying parameter values that will generate warehousing. For example, a high marginal cost for launching will reduce all launch types and increase the rate of warehousing.

We next turn to an analysis of launch choices by a monopolist in order to explore the relationship between market structure and warehousing.

2.2

Monopoly Market Structure

There are a few important differences when we assume that there is a single, monopoly, supplier of satellite services. First, market output, Q_t is equivalent to firm output q_t, so that $P_t = a - bnq_t$. Importantly, the accumulation of unused slots by the firm does not implicitly reduce output by the competitive fringe. Second, there is not a group of firms to which the regulator may reassign satellite slots. For this reason we assume there is no law of motion relating the use of empty and new slots to the slot allocation in the next period. The regulator does not "claw back" slots or spectrum when it is unused. The remainder of the model is the same.

The monopolist's maximization problem is now characterized by the value function

$$\begin{aligned} V[y_t, h_t, z_t] \quad = \quad & \max_{l_e, l_n, l_h, l_z} \{\pi_t + \lambda_t^e(e_t - l_t^e) + \lambda_t^n(\rho_t - l_t^n) + \lambda_t^h[(1-\gamma_h)h_t + \delta y_t - l_t^h] + \\ & \lambda_t^z[(1-\gamma_z)z_t + \gamma_h h_t - l_t^z] - \mu_t^n l_t^n - \mu_t^e l_l^e - \mu_t^h l_t^h - \mu_t^z l_t^z\} + \\ & \beta V[y_{t+1}, h_{t+1}, z_{t+1}] \end{aligned} \tag{23}$$

Notice that the number of slots is no longer a state variable since it does not evolve over time as a consequence of the firm's decision to leave slots empty.

The first order conditions for the monopolist's problem are:

$$l_t^h \quad : \quad -c_l - c_{2l} l_t^h - \lambda_t^h - \mu_t^h + \beta V_{y_{t+1}} - \beta V_{h_{t+1}} = 0 \tag{24}$$

$$l_t^z \quad : \quad -c_l - c_{2l} l_t^z - \lambda_t^z - \mu_t^z + \beta V_{y_{t+1}} - \beta V_{z_{t+1}} = 0 \tag{25}$$

$$l_t^n \quad : \quad -c_l - c_{2l} l_t^n - \lambda_t^n - \mu_t^n + \beta V_{y_{t+1}} = 0 \tag{26}$$

$$l_t^e \quad : \quad -c_l - c_{2l} l_t^e - \lambda_t^e - \mu_t^e + \beta V_{y_{t+1}} = 0 \tag{27}$$

Notice that the marginal cost of a launch into a new or empty slot is equated to the marginal benefit of fully functional satellites, per (26) and (27).

The monopolist's envelope conditions are:

$$V_{y_t} = P_t - bnq_t - c_y + \delta\lambda_t^h + \beta[(1-\delta)V_{y_{t+1}} + \delta V_{h_{t+1}}] \tag{28}$$

$$\begin{aligned} V_{h_t} = & (1-\alpha)(P_t - bnq_t) - c_h + \lambda_t^h(1-\gamma_h) + \lambda_t^z\gamma_h + \\ & \beta[(1-\gamma_h)V_{ht+1} + \gamma_h V_{z_{t+1}}] \end{aligned} \tag{29}$$

$$V_{z_t} = -c_z + \lambda_t^z(1-\gamma_z) + (1-\gamma_z)\beta V_{z_{t+1}} \tag{30}$$

These envelope conditions are very similar to the incumbent's with a few important exceptions. First, the marginal value of a slot is not taken into consideration by the monopolist. This reduces the marginal value of all satellite types, including zombies. Second, a non-functional satellite produces a negative marginal return in the present because there is no strategic marginal revenue from a zombie. Holding a zombie in orbit does not "prop up" prices by preventing competitor production. Third, the marginal return of a fully functional satellite is lower because it will lower prices.

These conditions provide a solution for the optimal launches to replace non-functional satellites:

Proposition 2.3 *Assume* $\lambda_t^j, \mu_t^j = 0$, $\forall j, t$. *The monopolist chooses the following rate for replacing non-functional satellites:*

$$\begin{aligned} c_{2l} l_t^{z**} &= (\delta^{-1} - 1)(1 - \gamma_h)^{-1} c_{2l}^{-1} \{[1 - \alpha - \delta^{-1}(1 - \gamma_h)](P_t - bnq_t) - c_h + \\ &\quad \delta^{-}1(1 - \gamma_h)[V_{y_t} - (1 - \delta)c_l + c_y] + \\ &\quad [1 - \delta^{-1}(1 - \gamma_h)](V_{z_t} + c_z)/(1 - \gamma_z) - V_{ht}\} \end{aligned} \tag{31}$$

Proof See the Appendix. ∎

With this result in hand we can contrast the rate of warehousing under the two different market structures. This comparison yields the following result:

Proposition 2.4 *Other things equal, a monopolist will engage in less warehousing if*

$$\frac{\alpha(1 - \gamma_z)}{1 - \gamma_h} < (\delta^{-1} - 1)c_{2l}[1 - \delta^{-1}(1 - \gamma_h)]$$

Proof Setting all values the same, with the exception of launches to replace non-functioning satellites, it becomes clear that the relative magnitude of warehousing is determined by the coefficient on bnq_t. The condition in the proposition ensures that this coefficient is negative so that an incumbent facing a competitive fringe replaces non-functioning satellites at a lower rate. ∎

We have demonstrated that the level of warehousing can vary with market structure. A monopolist may engage in less warehousing even when all other decisions would be equivalent in the presence of competition. This result is driven, in part, by the fact that a "use or lose" policy by the regulator can encourage an incumbent to occupy satellite slots in an unproductive manner. Further, there is no strategic marginal revenue for a monopolist. A non-functional satellite does not represent an implicit barrier to entry that lowers quantities and raises prices.

3 Conclusion

We modeled the choice of a large incumbent firm and perfectly competitive entrants in placing satellites in geo-stationary orbital slots. We also modeled the same choice for a monopolist. In the first case, we found that incumbent firms may have incentives to under-utilize spectrum and orbital slots by maintaining a partially-operational or non-operational satellite at the assigned orbital location or not utilize the spectrum or orbital slots for extended periods of time. In short, incumbents may benefit, in part, by holding under-utilized spectrum and orbital resources tactically to gain or maintain a competitive advantage over potential competitors. Conceivably, this can deprive the market of valuable output, raise prices, and alter welfare.

We showed conditions under which warehousing by the incumbent occurs and established factors that increase the incumbent's incentive to warehouse. We also noted the differences between this market structure and that of a monopolist, demonstrating that a monopolist may engage in less warehousing because the strategic motivation for encumbering slots is weaker. These results suggest that market structure is strongly related to the efficient use of a scarce resource, though in potentially non-intuitive ways.

Our analysis could easily be extended to other environments in which valuable capacity is limited and rationed to suppliers. These extensions include the airline industry, retailers, and the telecommunications industry. Potential regulatory and market-based solutions to the phenomenon of warehousing are worth additional exploration.

4 Appendix

4.1 Table of Parameters

Table 1: Some Notation

Q_t	:	quantity of market output generated by active satellites
q_t	:	quantity of output generated by incumbents satellites
n	:	number of consumers each satellite can serve
P_t	:	price of a unit of satellite service
S	:	total number of orbital slots
x_t	:	slots allocated to the incumbent firm
u_t	:	slots allocated to entrants
c_l	:	cost of a launch
c_x	:	marginal cost of a non-functioning satellite
c_h	:	marginal cost of a partially functioning satellite
c_y	:	marginal cost of a fully functioning satellite
y_t	:	quantity of incumbent's fully functioning satellites
α	:	fraction of output lost relative to a fully functioning satellite
z_t	:	non-functioning satellites
h_t	:	partially functioning satellites
l_t	:	total launches of new satellites
l_t^e	:	launch into an empty slot
l_t^n	:	launch into a newly provided empty slot
l_t^z	:	launch to replace a non-functioning satellite
l_t^h	:	launch to replace a partially functioning satellite
δ	:	depreciation rate for functioning satellites
u_t	:	quantity of entrants satellites
β	:	discount factor
γ_z	:	depreciation rate for non-functioning satellites
γ_h	:	depreciation rate for partially functioning satellites
π_t	:	profit

4.2 Proofs

4.2.1 Proposition 2.1

We begin by providing a proof of Proposition 2.1.

Combining (16) and (17) implies:

$$(1-\phi)\beta V_{x_{t+1}} = c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e \tag{32}$$

while combining (14) and (32) suggests:

$$\beta V_{y_{t+1}} = c_l + c_{2l}l_t^n + \lambda_t^n + \mu_t^n - (1-\phi)^-1[c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e] \tag{33}$$

Then, combining (14) and (33) yields:

$$\beta V_{h_{t+1}} = c_{2l}(l_t^n - l_t^h) + \lambda_t^n + \mu_t^n - \lambda_t^h - \mu_t^h - (1-\phi)^{-1}[c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e] \tag{34}$$

while combining (15) and (33) gives:

$$\beta V_{z_{t+1}} = c_{2l}(l_t^n - l_t^z) + \lambda_t^n + \mu_t^n - \lambda_t^z - \mu_t^z - (1-\phi)^{-1}[c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e] \tag{35}$$

Combining (18), (16), and (14) yields:

$$V_{y_t} = P_t - c_y + (1-\delta)c_l + c_{2l}(l_t^n - \delta l_t^h) + \lambda_t^n - \mu_t^n - \delta\mu_t^h \tag{36}$$

Combining (16) and (14) gives:

$$\beta(V_{h_{t+1}} + V_{x_{t+1}}) = c_{2l}(l_t^n - l_t^h) + \lambda_t^n - \lambda_t^h + \mu_t^n - \mu_t^h \tag{37}$$

while (15) and (14) yields:

$$\beta(V_{z_{t+1}} - V_{h_{t+1}}) = c_{2l}(l_t^h - l_t^z) + \lambda_t^h - \lambda_t^z + \mu_t^h - \mu_t^z \tag{38}$$

Combining (15) and (16) yields

$$\beta(V_{z_t+1} + V_{x_t+1}) = c_{2l}(l_t^n - l_t^z) + \lambda_t^n + \mu_t^n - \lambda_t^z - \mu_t^z \tag{39}$$

while combining (19), (37), and (39) gives:

$$V_{h_t} = (1-\alpha)P_t - c_h + \alpha bnq_t + c_{2l}(l_t^n - l_t^h) + \gamma_h c_{2l}(l_t^h - l_t^z) + \lambda_t^n + \mu_t^n - (1-\gamma_h)\mu_t^h - \gamma_h \mu_t^z \quad (40)$$

Next, combining (39), (35), and (20) gives:

$$V_{z_t} = bnq_t - c_z + c_{2l}(1-\gamma_z)(l_t^n - l_t^z) + (1-\gamma_z)(\lambda_t^n + \mu_t^n - \mu_t^z + \gamma_z(1-\theta)^{-1}[c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e] \quad (41)$$

Combining (21) and (32) gives:

$$\beta^{-1}[c_{2l}(l_{t-1}^n - l_{t-1}^e) + \lambda_{t-1)}^n + \mu_{t-1}^n - \lambda_{t-1}^e - \mu_{t-1}^e] = c_{2l}(l_t^n - l_t^e) + \lambda_t^n + \mu_t^n - \lambda_t^e - \mu_t^e \quad (42)$$

Together, (42) and (41) imply:

$$\begin{aligned} V_{z_t} =& bnq_t - c_z + c_{2l}(1-\gamma_z)(l_t^n - l_t^z) + (1-\gamma_z)(\lambda_t^n + \mu_t^n - \mu_t^z) \\ &+ \gamma_z(1-\theta)^{-1}\beta^{-1}[c_2 l(l_{t-1}^n - l_{t-1}^e) + \lambda_{t-1}^n + \mu_{t-1}^n - \lambda_{t-1}^e - \mu_{t-1}^e \end{aligned} \quad (43)$$

Consider a scenario where $\lambda_t^j, \mu_t^j = 0$, $\forall j, t$. From (43), we see:

$$c_{2l} l_t^n = (V_{z_t} + c_z - bnq_t)(1-\gamma)^{-1} - \gamma_z[(1-\phi)\beta(1-\gamma_z)]^{-1} c_{2l}(l_{t-1}^n - l_{t-1}^e) + c_{2l} l_t^z \quad (44)$$

and from (35):

$$c_{2l} l_t^h = \delta^{-1}[P_t - V_{y_t} - c_y + (1-\delta)c_l] + \delta^{-1} c_{2l} l_t^n \quad (45)$$

So:

$$\begin{aligned} c_{2l} l_t^h =& \delta^{-1}[P_t - V_{y_t} - c_y + (1-\delta)c_l] + \delta^{-1}(V_{z_t} + c_z - bnq_t)(1-\gamma_z)^{-1} \\ &+ \delta^{-1} c_{2l} l_t^z - \delta^{-1}\gamma_z[(1-\phi)\beta(1-\gamma_z)]^{-1} c_{2l}(l_{t-1}^n - l_{t-1}^e) \end{aligned} \quad (46)$$

Equation (40) implies:

$$\begin{aligned} V_{h_t} =& (1 - \alpha - \delta^{-1} + \gamma_h \delta^{-1})P_t - c_h + \alpha bnq_t + (\delta^{-1} - \gamma_h \delta^{-1})[V_{y_t} + c_y - (1-\delta)c_l] \\ &- (\delta^{-1} - \gamma_n \delta^{-1} - 1)c_{2l} l_t^n - \gamma_h c_{2l} l_t^z \end{aligned} \quad (47)$$

Substituting l_t^n:

$$\begin{aligned}V_{h_t} = &[1-\alpha-\delta^{-1}(1-\gamma_h)]P_t + \alpha bnq_t - c_h + \delta_h)[V_{y_t} + c_y - (1-\delta)c_l] \\ &+ [1-\delta^{-1}(1-\gamma_h)][(V_{z_t} + c_z - bnq_t)(1-\gamma_z)^{-1} + c_{2l}l_t^z \\ &- \gamma_z[(1-\phi)\beta(1-\gamma_z)]^{-1}c_{2l}(l_{t-1}^n - l_{t-1}^e)] - \gamma_h c_{2l} l_t^z \end{aligned} \tag{48}$$

Solving this equation for l_z^* provides the result provided in the proposition. Note that (42), (44) and (46) can be used with the solution for l^{z*} to solve for all of the launch rates. $\blacksquare$

4.2.2 Proposition 2.2

Lagging the law of motion for z by one period and substituting in the original expression yields:

$$z_t = (1-\gamma_z)^2_{z_{t-2}} + \gamma_h(1-\gamma_z)h_{t-2} - (1-\gamma_z)l_{t-2}^z + \gamma_h h_{t-1} - l_{t-1}^z \tag{49}$$

Repeated substitution of the law of motion for z, h gives:

$$z_t > (1-\gamma_z)^t_{z_0} + \gamma_h h_0 \sum_{j=0}^{t-1}(1-\gamma)^{t-1-j}(1-\gamma_h)^j - \sum_{j=1}^{t}(1-\gamma_z)^{j-1}l_{t-j}^z - \gamma_h \sum_{j=0}^{t+2}\sum_{k=0}^{j}(1-\gamma_z)^{j-k}l_{t-j-2}^h \tag{50}$$

Restricting z_t to strictly positive values results in the condition given in Proposition 2.2. $\blacksquare$

4.2.3 Proposition 31

Begin by equating the Lagrangian multipliers to zero. Equations (27) and (26) imply that $l_t^n = l_t^e$. We can also infer from these expressions that

$$\beta V_{y_{t+1}} = c_l + c_{2l} l_t^n \tag{51}$$

Combining this result with (24) and (25) yields

$$\beta V_{ht+1} = c_{2l}(l_t^e - l_t^h) \tag{52}$$

$$\beta V_{z_t+1} = c_{2l}(l_t^e - l_t^z) \tag{53}$$

Combining these results with (28), (29) and (30) implies

$$V_{y_t} = P_t - c_y + c_l + c_{2l} l_t^e - \delta c_{2l} l_t^h \tag{54}$$

$$V_{ht} = (1-\alpha)(P_t - bnq_t) - c_h + c_{2l}(l_t^e - l_t^h) + \gamma_h (c_{2l} l_t^z - c_{2l} l_t^h) \tag{55}$$

$$V_{z_t} = -c_z + (1-\gamma_z) c_{2l}(l_t^e - l_t^z) \tag{56}$$

Solving the last of these expressions for l_t^e yields

$$c_{2l} l_t^e = (V_{z_t} + c_z)/(1-\gamma_z) + c_{2l} l_t^z \tag{57}$$

Substituting this result into (54) and solving for l_t^h yields:

$$c_{2l} l_t^h = \delta^- 1[P_t - bnq_t - c_y + (1-\delta)c_l + (V_{z_t} + c_z)/(1-\gamma_z + c_{2l} l_t^z - V_{y_t}] \tag{58}$$

Substituting this result into (55) and solving for l_t^z provides the expression in the proposition. Note that the solutions to the other launch rates can be obtained by combining l_t^{z**} with the expressions in this proof. ∎

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