Merger Remedies in Oligopoly under a Consumer Welfare Standard*

Markus Dertwinkel-Kalt† Christian Wey‡

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Abstract

We analyze the welfare effects of structural remedies on merger activity in a Cournot oligopoly when the antitrust agency applies a consumer surplus standard. We derive conditions such that otherwise price-increasing mergers become externality free by the use of remedial divestitures. In this case, the consumer surplus standard ensures that mergers are only implemented if they raise social welfare. If the merging parties can extract the entire surplus from the asset sale, then the socially optimal buyer will be selected under a consumer standard.

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†Heinrich-Heine University Düsseldorf, Düsseldorf Institute for Competition Economics (DICE), Universitätsstr. 1, 40225 Düsseldorf, Germany. Email: dertwinkel@dice.hhu.de, Telephone: +4917683038485.

‡Heinrich-Heine University Düsseldorf, Düsseldorf Institute for Competition Economics (DICE), Universitätsstr. 1, 40225 Düsseldorf, Germany. Email: wey@dice.hhu.de
1 Introduction

Remedies are increasingly applied by antitrust agencies (in short: AA) in the US and EU to clear merger proposals which are otherwise subject to serious anticompetitive concerns (see FTC, 1999, EU, 2006, and OECD, 2011, for recent remedy reviews).\(^1\) The US Horizontal Merger Guidelines and the EU Merger Regulation allow for remedial offers to address competitive concerns (see DOJ, 2010, and EU, 2004, respectively). The EU Remedy Notice states that “the most effective way to restore effective competition, apart from prohibition, is to create the conditions for the emergence of a new competitive entity or for the strengthening of existing competitors via divestiture” (EU, 2008, Article 22). Accordingly, remedies are offered by the merging parties to effectively protect competition and to remove any competition concern the AA may have.

Our focus is on oligopolistic industries which are characterized by barriers to entry where the amount of some critical productive assets can be regarded as fixed for some time period. In those industries, divestitures of those critical resources can be used to increase market competition by reallocation. For instance, in the retailing sector suitable property and branches are largely fixed. A similar role is taken by gasoline stations in the petroleum industry, by landing slots in the airline industry, and by spectrum in the mobile phone industry.\(^2\)

The following principles in association with remedies are stated both in EU and US regulations concerning remedies (see, EU, 2008, and DOJ, 2011, respectively): First, the remedy is designed and proposed to the AA by the merging firms, while the AA can either reject or accept the offer.\(^3\) Second, a remedial divestiture may go to an already existing competitor or to a new

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\(^1\) Early accounts of remedies are Parker and Balto (2000) and the volume edited by Leveque and Shelanski (2003).

\(^2\) In all those examples, the critical assets are largely fixed for some time period, but may alter in the longer run because of innovations are entry. Antitrust authorities consider the “foreseeable” future in their decisions (which is typically confined to the next 1-2 years), so that the capital stock in the mentioned industries is usually regarded as fixed. As a consequence, the respective assets also qualify as divestitures which can counter anticompetitive merger effects already right after the execution of the merger.

\(^3\) This is particularly true for fix-it-first remedies in the US and phase 1 merger proposals in the EU. The rules are somewhat different in the next stage of the merger processes in the US and the EU (see, for instance, Wood, 2003, for a comparison of the US and EU merger control systems and the role of remedies therein, and Farrell,
entrant firm. Third, the remedy must be proportional to the competitive concern (see EU, 2004, Article 30).

Taking care of those features, we analyze the impact of remedies on (horizontal) merger activity in oligopoly. We take a consumer surplus standard for granted according to which the AA disapproves mergers which lower consumer surplus.4 Mergers are assumed to produce synergies, which may make mergers desirable also from a consumer perspective.5 If, however, synergies are not sufficient, the AA may condition a merger clearance on structural remedies, i.e., physical asset sales to rival firms (“divestitures”).6 The possibility to clear a merger conditional on remedies is shown to enlarge the set of profitable and acceptable mergers.7

More importantly, if a divestiture is necessary to keep the price from rising, then under reasonable conditions the merging parties will propose a divestiture which is price-fixing. Therefore, any merger which involves such a structural divestiture is “externality-free”, i.e., it leaves consumer surplus and outsiders’ profits unchanged.8 This insight sheds a new light on the seemingly inefficient consumer surplus standard (or, “price test”) which ignores changes in profits. First, 

4This is in line with the literature as recent Industrial Organization literature (e.g., Nocke and Whinston, 2010) takes the consumer surplus standard for granted. For instance, Whinston (2007) states that the AA’s “enforcement practice in most countries (including the US and the EU) is closest to a consumer surplus standard.” Davies and Lyons (2007) emphasize that AAs have no mandate to use merger review for industrial policy purposes. Hence, remedies should be only applied if there is a threat to competition.

5Our analysis is placed in a Cournot setting in which synergies are necessary to make consumers not worse off after the merger (see Farrell and Shapiro, 1990a; Spector, 2003; Vergé, 2010).

6Remedies are distinguished into structural and behavioral remedies (see EU, 2008, and DOJ, 2011). Structural remedies involve asset sales to counter anticompetitive effects of a merger, while behavioral remedies target the merged firms’ after merger business conduct (see DOJ, 2011, p. 6). Overall, by far the most mergers are horizontal and divestitures are the most frequently applied type of remedy (Davies and Lyons, 2007).

7The takeover of Veba Oel/Aral by BP in Germany in 2003 is an example of a merger which was cleared subject to divestitures sold to an entrant firm. Precisely, the German Federal Cartel Office required a divestiture of 4% of the joint market share of BP and Aral. PKN Orlen (a major Polish oil refiner and petrol retailer) then entered the German market by taking over 494 stations.

8In our model, outsider firms anticipate that the merged firm sells just enough assets to render the price unaffected. Hence, outsider firms are also unaffected as their optimal quantities do not change.
proposals are in equilibrium proportional to the competitive concern. That is, a lower merger
synergy level must induce a larger divestiture proposal, which endogenously ensures that the
guideline’s third principle holds. More importantly, the merging firms have a strong incentive to
search for the buyer which is preferred from a social welfare perspective if they can extract the
entire gains from the asset sale.\footnote{If this is a rather efficient or inefficient rival firm depends on the following trade-off. The more efficient a firm is, the more efficient the acquired assets can be run and the lower the inframarginal production costs are. More efficiency, however, induces a larger quantity, such that an additional unit shrinks the revenue earned on the other units further.} Since under price-fixing and perfect selling-power the social
surplus is proportional to the gains generated by the merging firms, these are residual claimants
and act in a socially optimal way.

We extend our model by comparing the merger outcomes under different types of rent-
extraction. If the merging firms’ rent-extraction is limited such that the divestiture is sold at
a fixed price, then in equilibrium is sold to the weakest competitor. If a price-fixing remedy
is sold through an auction it will be acquired by the incumbent competitor with the highest
willingness to pay. In general, only perfect selling power (or efficient bargaining between the
parties) ensures that the divestiture is acquired by the socially efficient buyer. Thereby, we give
a novel rationale for the efficiency of \textit{fix-it-first remedies} which are favored both by the EU and
the US merger guidelines. We also examine remedy-dependent synergies such that the realized
synergy of the merged firm decreases when assets have to be divested and/or the acquirer of the
assets realizes synergies on its own. While the scope for mergers is reduced when divestitures
reduce the merged firm’s synergies, we show that our main results remain valid.

Our paper contributes to the analysis of mergers in Cournot oligopoly when productive
capital in an industry is fixed (Perry and Porter, 1985; Farrell and Shapiro, 1990a,b; McAfee
and Williams, 1992). That approach was applied to structural remedies in Medvedev (2007),
Vergé (2010), and Vasconcelos (2010), all of whom, however, used specific models and functional
forms. In the absence of synergies, Vergé (2010) proves that only under very restrictive conditions
a re-allocation of productive assets through structural remedies may increase consumer surplus.
Medvedev (2007) shows for a three-firm oligopoly that remedies in association with merger
synergies extend the scope for acceptable mergers. Vasconcelos (2010) analyzes remedies for the
case of a four firm oligopoly when merger synergies are possible. Each firm owns one unit of capital and capital is indivisible. He assumes that the AA restructures the industry optimally in order to maximize consumer surplus, which is crucial when at least three firms are involved in a merger. In those instances he shows that the over-fixing problem associated with remedial divestitures (see also Farrell, 2003) may emerge. Over-fixing unfolds adverse effects because a firm may abstain from proposing a (socially desirable) merger with two other firms as the acquirer expects (correctly) that the AA will use its power to sell one of the acquired firms to the remaining competitor. Consequently, the acquirer may strategically propose a one-firm takeover which can be worse from a consumer point of view than allowing a takeover of the two other firms.

Cabral (2003) analyzes mergers in a differentiated industry with free entry. When assets are sold to an entrant firm as a remedy, then a “buy them off” effect follows, which means that an entrant firm is dissuaded from opening a new store (or introducing a new product variant). That effect may work against the interest of consumers, who are better off the more variants are offered in the market. Chen (2009) analyzes mergers in a three-firm oligopoly model of dynamic capital accumulation. A merger may then have long-run effects that are worse than its short-run effects. We disregard the issue of endogenous entry and endogenous capacities as the capital is assumed to be fixed in the industry.

The impact of remedies on the effectiveness of merger control has also been investigated empirically (see Duso et al., 2011, and Duso et al., 2012, for the EU and Clougerthy and Seldeslachts, 2012, for the US). Those works use an event study approach which identifies the anticompetitive effect of a merger by abnormal stock market returns of competing firms. Overall, the results appear to indicate that an upfront-buyer remedy tends to restore the pre-merger competitive situation.

We proceed as follows. Section 2 presents the basic model. In Section 3 we conduct the merger analysis for two different merger control regimes depending on whether or not remedies are feasible. Section 4 presents extensions of our model before Section 5 concludes.

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10 Ormosi (2012) analyzes major EU merger cases and shows that remedial offers and efficiency claims are often strategic to avoid costly delay in litigation processes.
2 The Model

We analyze the effects of remedies in a Cournot oligopoly with homogeneous products by extending the analysis in Farrell and Shapiro (1990a). There are \( n \geq 3 \) firms indexed by \( i \in I = \{1, \ldots, n\} \). All firms produce a homogenous good with inverse market demand given by a twice differentiable function \( p(X) \), where \( p \) is price, \( X \) is industry output, and \( p'(X) < 0 \). Firm \( i \)'s production costs depend on its output level, \( x_i \), and the capital stock, \( k_i \), it uses for production. Total productive capital of the industry, \( K \), is fixed and distributed among the firms in the industry; i.e., \( k_i > 0 \) for all \( i \in I \) and \( \sum_{i \in I} k_i = K \). Firm \( i \)'s total production cost function is given by \( c^i := c^i(x_i, k_i) \).\(^{11}\) We invoke the standard assumption that additional capital lowers the marginal cost curve; i.e., \( c^i_{xx} < 0 \). Firms set their output levels simultaneously (Cournot competition).

Each firm \( i \) maximizes its profit \( \pi_i = p(X)x_i - c^i(x_i, k_i) \) given its rivals’ outputs, which yields the first-order conditions

\[
p(X) + x_i p'(X) - c^i_x(x_i, k_i) = 0, \text{ for all } i \in I.
\]

In a Cournot equilibrium, (1) holds for all firms \( i \in I \). From (1) it follows that firm \( i \) produces a larger quantity than firm \( j \) if and only if its marginal production costs are lower; i.e., \( c^i_x < c^j_x \) holds. We assume that each firm’s reaction function slopes downward with slope between \(-1\) and \( 0 \), for which it is sufficient to assume that\(^{12}\)

\[
p'(X) + x_i p''(X) < 0, \text{ holds for all } i \in I.
\]

The AA applies a consumer standard when evaluating a merger proposal. Therefore, a merger is approved if and only if the post-merger price level does not exceed the pre-merger equilibrium price \( p^* \). We distinguish two different merger control regimes, depending on whether or whether not remedies are feasible.\(^{13}\)

\(^{11}\)We abbreviate a function’s partial derivative by indexing the respective variable; for instance \( c^i_x \equiv \partial c^i(x_i, k_i)/\partial x_i \).

\(^{12}\)The following inequality holds if the industry demand curve satisfies \( P''(Q)Q + P'(Q) < 0 \).

\(^{13}\)Throughout the analysis we assume that the AA can only impose a remedy on the merging firms that the
• **No-remedy regime** (in short: NR): If the merger guidelines do not allow for a remedial divestiture, then the AA can either approve or block the merger proposal altogether.

• **Remedy regime** (in short: R): The merger guidelines allow for an approval conditional on a divestiture to a competitor if it counters any price-increasing effects of the proposed merger.

We examine a bilateral merger with firm $i$ being the acquirer and firm $j$ the target firm. Firms $i$ and $j$ merge if the merged entity’s profit does not fall short of the sum of the pre-merger profits. A merger allows to re-combine the capital of the merging firms to explore economies of scale.\(^{14}\) In addition, a merger between firms $i$ and $j$ may lead to a synergy, which is measured by the parameter $s = s(i, j) \in [0, 1]$. The synergy rotates the cost function downward such that marginal costs for a given level of output are lowered. Precisely, the merged firm $M = M(i, j)$ (which combines the assets of firms $i$ and $j$) produces with the cost function $c_M(x, k, s)$, where $k$ denotes the merged firm’s capital possibly reduced by divested assets. Let $c_M(x, k, s)$ be continuous in $s$ with $c_M(x, k, 1) = c^i(x, k)$ and $c_M(x, k, 0) = 0$. Perfect synergies ($s = 0$) imply that the firm’s costs are reduced to zero, while the absence of any synergies ($s = 1$) implies that the merged firm produces with the pre-merger cost function $c^i(x, k)$. In addition, we assume

$$\frac{\partial c_M(x, k, s)}{\partial s} > 0 \text{ for all } x, k > 0,$$

so that the impact of a marginal change in capital on marginal costs increases in the synergy level.

Let $0 \leq \sigma \leq k_j$ denote the share of firm $j$’s capital which stays under control of the merged firm $M$ after a possible divestiture.\(^{15}\) Accordingly, $k_j - \sigma$ is the share of firm $j$’s capital which parties themselves propose. This mirrors legal practice in the EU and in the US (see EU, 2006/2008, and DOJ, 2011).

\(^{14}\)After the merger it is optimal to bring all the new entity’s capital together rather than leaving it divided among the plants of the pre-merger configuration. This is optimal because of economies of scale; formally, because of assuming $c_{xk} < 0$ (see also Farrell and Shapiro, 1990b, p. 113). In that sense, we suppose that productive capital is mobile (at least to some extent) which is also a necessary prerequisite to analyze remedial divestitures of assets.

\(^{15}\)That is, we suppose that the acquirer will divest parts of the target firm’s assets in case the AA requires a
goes as a divestiture to another firm, say firm \( l \). Let \( I_M \) denote all firms which are active after the merger; i.e., \( I_M := I\setminus\{i, j\} \cup M \). Furthermore, denote the total pre-merger equilibrium quantity by \( X^* \) and the post-merger equilibrium quantity by \( X^*(k_j - \sigma) \) depending on the divestiture level \( k_j - \sigma \) and synergy level \( s \).

We impose two independence conditions on the interplay between the synergy level and the remedy. First, we assume that the synergy level \( s(i, j) \) is unaffected by the size of the divestiture. Second, the buyer of the assets does not realize any synergies.\(^{16}\) Consequently, the merged entity faces overall costs of \( c^M(x_M, k_i + \sigma, s) \), while firm \( l \) operates with the cost function \( c^l(x_l, k_l + k_j - \sigma) \).\(^{17}\)

In addition, we invoke two more assumptions concerning firms’ cost functions. First, all firms (except the merged firm) have access to the same technology.\(^{18}\) Second, all firms (including the merged firm) have constant marginal costs; i.e., \( \frac{\partial c}{\partial x} = 0 \) holds always.\(^{19}\) Specifically, we suppose that each firm \( i \in I \) produces with the cost function

\[
c^i(x_i, k_i) = \frac{x_i}{k_i} \tag{4}
\]

prior to the merger. If firms \( i \) and \( j \) merge to form the merged entity \( M \), they realize the synergy level \( s \) and (possibly) divest \( k_j - \sigma \) to firm \( l \). The merged firm’s cost function is then given by

\[
c^M(x, k_i + \sigma, s) = \frac{sx}{k_i + \sigma}, \tag{5}
\]

while the acquiring firm \( l \) produces with the pre-merger cost function

\[
c^l(x, k_l + k_j - \sigma) = \frac{x}{k_l + k_j - \sigma}. \tag{6}
\]

\(^{16}\)Below, we discuss how our results change if we relax each of these requirements.

\(^{17}\)In our model, the effects of a structural remedy are not burdened with uncertainties which may play a role in practical merger control (see, Davies and Lyons, 2007).

\(^{18}\)This assumption implies that asymmetries among firms (apart from the merged firm) only depend on the amount of productive assets they own.

\(^{19}\)Even though this assumption restricts the generality of our results, the constant-marginal cost case is important in itself. In fact, many results obtained in the merger literature (for instance, Salant, Switzer, and Reynolds, 1983, and Nocke and Whinston, 2010) are based on constant marginal costs.
Note that the synergy level \( s \) enters the merged firm’s cost function (5) in multiplicative form, so that \( c^M(x, k, s = 1) = c^i(x, k) \) and \( c^M(x, k, s = 0) = 0 \) follow. We analyze the following merger game under the \( NR \) and the \( R \) regime: In the first stage, firms \( i \) and \( j \) decide whether or not to propose a merger to the AA. If they decide to merge, they can also specify a divestiture under regime \( R \) which they sell to a competing firm \( l \in I \setminus \{i, j\} \). In the second stage, the AA either approves or blocks the merger proposal according to a consumer standard. In the third stage, firms compete à la Cournot.

3 Merger Analysis and Main Results

We first examine how a change in capital \( dk := (dk_1, ..., dk_n) \) affects equilibrium quantities \( dx := (dx_1, ..., dx_n) \). Following Farrell and Shapiro (1990a), the total derivative of firm \( i \)’s first-order condition with respect to \( X \) and \( k_i \) can be written as\(^{20}\)

\[
\frac{dx_i}{dX} = -\lambda_i dX + \delta_i dk_i,
\]

where

\[
\delta_i := \delta_i(k) := \frac{c^i_{xk}}{p'(X)} > 0
\]

and

\[
\lambda_i := \frac{p'(X) + x_i p''(X)}{p'(X)} > 0.
\]

The variable \( \delta_i \) gives the direct effect of capital \( k_i \) on firm \( i \)’s output \( x_i \) and \( \lambda_i \) denotes firm \( i \)’s equilibrium responsiveness to changes in price. There is a direct relationship between \( \lambda_i \) and the slope of firm \( i \)’s reaction function \( R_i \) which is given by \( \lambda_i = -R_i/(1 + R_i) \) (see Farrell and Shapiro, 1990a). Summing up (7) over all firms yields the following lemma (see Farrell and Shapiro, 1990a, Prop. 2).\(^{21}\)

**Lemma 1 (Effects of selling units of capital).** A sale of a small amount of capital from firm \( j \) to firm \( l \) raises industry output and reduces price if and only if \( \delta_l > \delta_j \).

\(^{20}\)Note that we assume \( c_{xx} = 0 \) which simplifies the expressions below when compared with the corresponding expressions in Farrell and Shapiro (1990a).

\(^{21}\)We present all omitted proofs in the Appendix.
Lemma 1 gives a necessary and sufficient condition under which small asset sales to a rival firm increase consumer surplus. Our main analysis builds on this local result and investigates under which circumstances divestitures of a merged firm can restore the pre-merger industry output.

Second, we impose a condition under which a merger satisfies the consumer standard at least if the realized synergy is perfect ($s = 0$). A merger between firms $i$ and $j$, which creates synergies $s$, leads to an output level of the merged firm that is weakly larger than the sum of the firms’ outputs before the merger if and only if

$$c^i_x + c^j_x - c^M_x \geq p(X^*).$$

Using the cost functions (4) and (5), condition (10) becomes

$$\frac{1}{k_i} + \frac{1}{k_j} - \frac{s}{k_i + k_j} \geq p(X^*).$$

This condition is satisfied if the reservation price $p(X \to 0)$ is not too large or if the merging firms’ capital stocks, $k_i$ and $k_j$, are not too large. Furthermore, the lower the merger synergy parameter $s$ is, the more likely condition (11) is to hold. In the following, we assume that condition (11) is fulfilled at $s = 0$.

**Assumption 1.** A merger between firms $i$ and $j$ weakly increases the industry output if the merger synergies are maximal, i.e., $s = 0$ holds.

### 3.1 Merger Outcomes With and Without Remedies

**No-remedy regime ($NR$).** Under the no-remedy regime, the AA can only clear or reject a merger proposal in its entirety. Hence, if a merger is approved, then $\sigma = k_j$ holds. We obtain the following lemma according to which mergers are approved if and only if the generated synergy does not fall short of a certain threshold value.

**Lemma 2 (Full mergers).** Suppose a no-remedy regime ($NR$). Then, there exists a critical synergy level $\bar{s} := \bar{s}(i, j) \in [0, 1]$, such that a merger of firms $i$ and $j$ does not decrease the

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22 See Farrell and Shapiro (1990b, Prop. 1, p. 112).
overall industry output $X^*$ if and only if $s \leq \bar{s}$. Hence, the AA approves a merger proposal between firms $i$ and $j$ if and only if $s \leq \bar{s}$. Such a merger is strictly profitable for the merging firms.

Lemma 2 makes use of the fact that for $s \leq \bar{s}$ the merged firm produces more than both firms $i$ and $j$ together before the merger (with equality holding at $s = \bar{s}$). This implies that the market price decreases and consumers strictly benefit. Moreover, profitability follows from noticing that the merged firm’s production costs are reduced over all output levels because of the increased productive capital. According to Lemma 2, only mergers with relatively large synergies can pass the decision screen of the AA. If the synergy level falls short of the critical value ($s > \bar{s}$), then the post-merger price increases in which case the AA blocks the merger proposal altogether.

**Remedy regime** ($R$). With a remedy rule at hand, the AA can make a merger proposal conditional on structural remedies. According to the consumer surplus standard, the AA accepts all remedial offers which off-set the price-increasing effect of reduced competition. It follows from Lemma 2 that remedies become relevant if the synergy parameter $s$ is larger than $\bar{s}$. In those instances, the merged firm may offer to divest a share of the target firm’s capital, $k_j - \sigma$, which suffices to lower the consumer price or to fix it at the pre-merger level.

**Lemma 3 (Approvability).** Suppose a remedy regime $R$. If a merger between firms $i$ and $j$ yields relatively large synergies with $s \leq \bar{s} := \bar{s}(i, j)$, it is approved without a remedy. For lower synergy levels, $s > \bar{s}$, there exists a unique threshold value $s^R \geq \bar{s}$, such that any merger proposal with $s \in (\bar{s}, s^R]$ is approvable with a certain divestiture level. For merger proposals with $s > s^R$, no divestiture levels exists which induces the AA to approve the merger.

Typically, $s^R > \bar{s}$ holds, such that the feasibility of remedies strictly increases the scope for mergers. Then there is an interval of synergies $(\bar{s}, s^R]$ for which divestitures exist that resolve the AA’s anticompetitive concerns such that a merger between firms $i$ and $j$ becomes approvable. If, however, the created synergy is too low, i.e., $s > s^R$, divestitures cannot outweigh the merger’s anticompetitive effects.

In fact, the proof of Lemma 3 reveals that remedies strictly increase the scope for mergers if
and only if
\[ k_i + k_j > \sqrt{s} \min_{l \in \mathcal{I} \setminus \{i, j\}} \{k_l\} \]
holds. Condition (12) ensures that there is a firm \( l \) for which \( \delta_l(k_i) > \delta_M(k_i + k_j) \) holds, i.e., for which the direct effect of capital on output is larger than for \( M \), where \( M \) represents the full merger between firms \( i \) and \( j \). In that case, selling capital of \( M \) to firm \( l \) increases industry output and therefore increases the scope for mergers. In particular, (12) holds if the merged entity \( M \) is not the smallest firm in the market. If, however, there is a firm which is smaller than \( M \), then competition can be strengthened through selling capital to this firm.

So far we have discussed the approvability of mergers involving divestitures. We examine next the profitability of a merger which involves divestitures to an incumbent competitor. Therefore, we assume that the merging firms have full power in the asset sales process for the remainder of this section.

**Assumption 2.** The merged firm can make take-it or leave-it offers to each firm in the market. It can also tailor the divestiture level to each buyer.

Suppose that a merger involves divestitures to ensure its approval by the AA. Then, the following lemma states that there exists a unique threshold value of the synergy level \( \bar{s}^R \) up to which such a merger is also profitable.

**Lemma 4 (Profitability).** Suppose a remedy regime \( R \). There exists a unique threshold \( \bar{s}^R \in (\bar{s}, \bar{s}^R] \), such that a profitable and approvable merger generating synergy \( s \) (possibly with a remedy) exists if and only if \( s \leq \bar{s}^R \).

If the synergy level \( s \) falls short of the threshold value \( \bar{s}^R \), i.e., \( s > \bar{s}^R \), then a merger which requires divestitures to ensure acceptance by the AA is not profitable. As the merged firm’s cost savings are already relatively low if the synergy is small, the obligation to divest assets may frustrate the incentives to propose a merger in the first place. These results are summed up in the following proposition.

**Proposition 1 (Implementation).** Suppose a remedy regime \( R \). Then all mergers with relatively large synergies, \( s \leq \bar{s} \), are profitable and approved without a remedy. For lower synergy levels \( s \in (\bar{s}, \bar{s}^R] \), a merger with a certain divestiture is proposed and approved by the AA. For
merger proposals with \( s > \bar{s}^R \), no merger will be implemented since either the merged firm’s profitability condition is violated or there exists no remedy which could fix the AA’s competitive concerns.

Proposition 1 states that there is a monotone relationship between a merger’s synergy level and the AA’s final decision. Precisely, there exists a unique threshold value of the synergy level, \( \bar{s}^R \), up to which mergers (possibly including a certain divestiture) are implementable. Given that Condition (12) holds, this threshold value strictly exceeds \( \bar{s} \). Only if the merger synergies are too low (i.e., \( s > \bar{s}^R \)), then allowing for remedies does not change the AA’s decision when compared with the NR regime.\(^{23}\)

The identity of the buyer for which an appropriate remedy exists cannot be decided in general. The existence of such a buyer depends on its incentive to raise its output, which involves the following trade-off. On the one hand, a larger firm runs capital more efficiently than a smaller firm and produces additional units at lower costs. On the other hand, a large buyer firm does not necessarily raise its output as much as a smaller firm because producing more output lowers the revenues earned on inframarginal units. Since a firm’s equilibrium quantity is increasing in its capital stock, an additional unit sold lowers the revenue earned on the other units more for a larger than for a smaller firm. Therefore, given a divestiture of a fixed size, a smaller buyer may raise its output by more as the negative effect on revenues earned on the other units is lower. However, a larger buyer may also raise its output by more through the acquisition of a remedy of a fixed size as its marginal production costs are lower. Due to this trade-off it is also ambiguous to which firm the remedy is sold if there are several potential buyer firms for which an appropriate remedy exists.

### 3.2 Social Welfare

According to Proposition 1, the introduction of remedies may change the market structure and, therefore, also social welfare (the sum of consumer surplus and producer surplus), which we

\(^{23}\) Note that if the merged firm is forced to sell the assets at a fixed price, then Lemma 4 and Proposition 1 still hold, however, with a potentially different profitability threshold value \( \bar{s}^R \). Proofs are analogous. Therefore, our qualitative insights do not depend on Assumption 2 of perfect selling power.
denote by $SW$. We compare social welfare if the merger control regime allows for remedies and if it does not. The following analysis is restricted to the synergy levels $s \in (\bar{s}, \bar{s}^R]$ for which remedies strictly increase the scope for approvable and profitable mergers.

**Binding consumer surplus standard.** If there is a divestiture such that the consumer surplus constraint is satisfied, then (due to continuity of the cost function) there also exists at least one divestiture level such that the consumer surplus standard binds (i.e., pre- and post-merger prices are the same). We call such a divestiture level a *price-fixing divestiture*. Such divestitures are externality-free as consumer surplus and profits of those firms not involved in the merger process are unaffected and stay put at their pre-merger levels. In particular, the smallest divestiture level $\min_{\sigma \in [0, 1]} \{k_j - \sigma\}$ which satisfies the consumer surplus standard condition is such a price-fixing divestiture. While it appears to be intuitive that the merged firm should prefer not to divest more than required by the AA, this is not always the case. The next lemma states conditions under which the merged firm proposes to divest the smallest approvable divestiture which we denote by $k_j - \hat{\sigma}$.

**Lemma 5 (Price-fixing divestitures).** Suppose a remedy regime $R$ and assume $s \in (\bar{s}, \bar{s}^R]$. Then each of the following conditions is sufficient to ensure a price-fixing divestiture.

i) Independent of the merged firm’s selling power, the merged firm proposes the minimal price-fixing divestiture $k_j - \hat{\sigma}$ if $\delta_l(k_l + k_j - \hat{\sigma}) \leq \delta_M(k_i + \hat{\sigma})$.

ii) If the divestiture is sold at a fixed price, then the merged firm proposes the minimal, price-fixing divestiture $k_j - \hat{\sigma}$.

iii) If the merged firm has the entire selling power, then it will propose the minimal, price-fixing divestiture $k_j - \hat{\sigma}$ if

$$2 \left( \frac{x_l}{(k_l + k_j - \sigma)^2} - \frac{s x_M}{(k_i + \sigma)^2} \right)$$

$$< \left( \frac{1}{(k_l + k_j - \sigma)^2} - \frac{s}{(k_i + \sigma)^2} \right) \frac{(1 + \lambda_l)x_l + (1 + \lambda_M)x_M}{1 + \sum_{m \in I_M} \lambda_m}$$

holds for all $\sigma > \hat{\sigma}$, where $k_j - \hat{\sigma}$ is the remedy which induces the lowest possible post-merger price over the interval $\sigma \in [0, k_j]$.\(^{24}\)

\(^{24}\)The threshold value $\hat{\sigma}$ is derived in the proof to Lemma 3 in the Appendix.
Parts i)-iii) of Lemma 5 can be explained as follows. Part i): From Lemma 1 we observe directly that the remedy must be price-fixing of size \( k_j - \sigma \) if \( \delta_l(k_l + k_j - \sigma) \leq \delta_M(k_i + \sigma) \) holds as \( \delta_l(k_i + k_j - \sigma) \) and \( \delta_M(k_i + \sigma) \) denote the direct effects of capital on \( M \)'s and \( l \)'s output levels after \( l \) has acquired the assets \( k_j - \sigma \). Selling more assets implies a market price which is above \( p^* \), so that the consumer surplus condition is violated.\(^{25}\) Therefore, in equilibrium, the consumer surplus standard must be binding.

Part ii): If the merged firm sells the assets at a fixed price, it has no incentive to divest more than the AA requires. Hence, it proposes to divest the minimal price-fixing divestiture \( k_j - \sigma \), so that consumer surplus condition binds.

Part iii): If the merged firm has the entire selling power, then (13) is the condition under which it cannot profitably sell more to firm \( l \) than the minimal required asset package \( k_j - \sigma \). Each of the following two requirements \( a) \) and \( b) \) is sufficient for (13) to hold.

\[ a) \text{ If } \delta_l(k_l + k_j - \sigma) > \delta_M(k_i + \sigma) \text{ (i.e., the bracket of the right-hand side of (13) is positive) and if } c_k^l > c_k^M \text{ (i.e., the bracket of the left-hand side of (13) is negative) hold for all } \sigma > \tilde{\sigma}, \text{ then the implemented remedy will be price-fixing of size } k_i + \sigma. \]

\[ b) \text{ If } \delta_l(k_l + k_j - \sigma) > \delta_M(k_i + \sigma), \text{ the proposed merger will involve the price-fixing divestiture } k_j - \sigma \text{ if } 2 \left( 1 + \sum_{m \in M} \lambda_m \right) < (1 + \lambda_l)x_l + (1 + \lambda_M)x_M. \]

For instance, a linear demand function, \( p(X) = a - bX \), implies \( \lambda_i = 1 \), so that the preceding condition is equivalent to \( a < x_l + x_M \).

\(^{25}\) In the proof of Lemma 3 (see Appendix), we see that the function \( \delta_l(k_l + k_j - \sigma) - \delta_M(k_i + \sigma) \) has at most one zero on \( \sigma \in [0, 1] \). Therefore, it suffices to require that \( \delta_l(k_l + k_j - \sigma) \leq \delta_M(k_i + \sigma) \) holds at \( \sigma = \tilde{\sigma} \).

\(^{26}\) Note that \( \delta_l(k_l + k_j - \sigma) = -1/[p'(X)(k_l + k_j - \sigma)^2] \) and \( \delta_M(k_M + \sigma) = -s/[p'(X)(k_i + \sigma)^2] \). Furthermore, \( c_k^l = -c_a = -x_l/(k_i + k_j - \sigma)^2 \) and \( c_k^M = c_a^M = -sx_M/(k_i + \sigma)^2 \).
This holds if the reservation price $a$ is sufficiently large.

If none of the conditions listed in Lemma 5 holds, then the merging parties may divest more than required by the AA. In that case, prices may strictly decrease and consumers may be strictly better off when remedies are feasible.\footnote{Price-decreasing divestitures may also exist if the buyer of the divested assets also experiences synergies (see Section 4 below).} In the following, we restrict our analysis to externality-free mergers. Therefore, we invoke Assumption 3 for the remainder of our analysis.

**Assumption 3.** Suppose $s \in (\bar{s}, \bar{s}^R]$. All proposed divestitures are price-fixing.

Given Assumption 3, we can easily derive the proportionality-principle claimed in the remedy guidelines; namely that the remedy’s size should be proportional to the anticompetitive concern. If the merged firm’s synergy level increases, then the merger’s anticompetitive effects are smaller such that it has to divest less assets in order to satisfy the consumer surplus standard.

**Lemma 6 (Proportionality principle).** Suppose that $s \in (\bar{s}, \bar{s}^R]$ and Assumption 3 hold. Then the size of the price-fixing divestiture sold to some firm $l$ increases in $s$.

If a merger is externality-free, then the first-order conditions of the outsider firms remain unaffected by the merger. As a consequence, the social welfare effect of remedies depends only on a comparison of total production costs for the firms involved in the merger (firms $i$ and $j$) and firm $l$ which buys the divested assets. This yields the following proposition.

**Proposition 2 (First efficiency result).** Suppose that $s \in (\bar{s}, \bar{s}^R]$ and Assumption 3 hold. Given a consumer surplus standard, firms merge if and only if the merger raises social surplus. Assume that the merging parties can choose to divest assets to any incumbent competitor. If the merging parties can extract the entire gains from the asset sales (e.g., through a take-it or leave-it offer), then they select the socially optimal buyer.

Proposition 2 shows that a merger control regime which allows for remedies under a consumer surplus standard is always preferable from a social welfare perspective when compared with regime $NR$. The reason for this result is quite general: given that consumer surplus is held fixed, under Cournot competition the market price must be held fixed and therefore the profits
of any firm to whom the assets are not divested. Then, the merger only affects the profits of
the merged firm and the firm which buys the divested assets. Firms’ incentives to search for the
most efficient buyer are fully aligned with the social welfare maximizing choice. The merging
firms are residual claimants and act socially optimally as they maximize the gains from trade
under the remedy constraint. We can generalize this reasoning to any oligopoly market with
homogenous products.

**Corollary 1.** Suppose an arbitrary oligopoly market and assume that the merging parties propose
a price-fixing remedy to the AA (which uses a consumer surplus standard). Then the following
efficiency result holds: If the merging parties can extract the entire gains from the asset sales,
then they will pick the socially optimal buyer.

This efficiency result critically depends on the fact that the AA applies a consumer standard.
If, instead, the AA imposes a social welfare standard, according to which a merger must not lower
social surplus, a similar efficiency result cannot be obtained. Given a social welfare standard,
all mergers are allowed which do not lower the overall welfare in the market. Hence, the merger
is typically not externality-free which implies that the merger and its divestiture decision do
not maximize social welfare. Moreover, as the merger must be profitable, market participants
other than the merging parties (including consumers) are made worse off as a whole if the social
welfare standard binds.

### 4 Extensions and Discussion

We analyze three extensions of our basic setup. *First*, we investigate the equilibrium outcomes
under different selling mechanisms and find that our efficiency result concerning the consumer
surplus standard (Proposition 2) depends crucially on the merged firm’s ability to extract all
rents from selling the assets. *Second*, we examine remedy-dependent synergies according to which
the size of the divestiture lowers the merged firm’s synergy or creates a synergy for the buyer
firm. *Third*, we discuss structural remedies in an oligopolistic market with price competition.
4.1 Different Selling Mechanisms

Different selling mechanisms for a divestiture might induce different post-merger market structures and outcomes. When there are several possible buyer candidates, then depending on the selling mechanism a different buyer may be chosen. Suppose that $s \in (\bar{s}, \bar{s}^R]$. We examine remedial asset sales for three different selling mechanisms to show how distortions from the socially optimal choice (according to Proposition 2) can occur. First, the divestiture may be sold at a fixed price. Second, the divestiture may be auctioned off. Third, the merged firm has perfect seller power, so that it can make a take-it or leave-it proposal to a pre-selected buyer. In each case, we assume that the divested remedy is price-fixing. We define $W(i, j, s)$ as the set of potential buyers for which a price-fixing remedy exists, where firms $i$ and $j$ are the merging firms which realize synergies $s$. Note that any other buyer not in $W(i, j, s)$ will be disregarded by the AA as the consumer surplus standard would be violated for any divestiture level in those instances. Furthermore, note that the size of the price-fixing remedy depends on the buyer itself, that is, each remedy is buyer-specific.

**Selling at a fixed price.** Assume that the divestiture is sold at a fixed price. In order to ensure that no potential buyer is excluded, we assume that the selling price is zero. As a consequence, the merged firm selects the buyer which leads to the highest post-merger profit level. As a firm produces a larger quantity the lower its marginal costs are, its output will also be the larger the less capital it divests. Therefore, the merged firm selects a buyer firm to minimize the size of the asset sale.

**Bidding for the divestiture.** For simplicity, we take it for granted that the merged firm can extract the entire willingness to pay for a divestiture from the winning bidder; e.g., by setting a reserve price. If the divestiture is sold through an auction in which all buyers bid their maximum willingness to pay, then the divestiture goes to the buyer for which the profit-differential through the acquisition of the remedy is largest. A firm $l$’s maximum willingness to pay equals the difference between its post-acquisition and its pre-merger profit as the sale of a price-fixing remedy to an incumbent competitor is externality-free so that firm $l$’s profit is

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28 This assumption can be relaxed if many potential entrants bid for the divestiture.
not affected if it does not acquire the assets. The winner of the auction is likely to be a firm for which the price-fixing divestiture is rather large. A large divestiture weakens the merged firm’s market position and lowers its equilibrium output, but enables the acquirer to steal a rather large proportion of the merged firm’s market share. Consequently, a larger price-fixing divestiture shifts equilibrium output to the acquirer, at the cost of the merged firm. Therefore, the winner of the auction may not be the firm which is preferred by the seller or from a social welfare point of view as a relatively large output-share is reallocated to the buyer firm.

**Perfect selling power.** If the merged firm can commit to make a take-it or leave-it offer to a pre-selected firm, it extracts all gains from trade as we have shown in the previous section.

**Proposition 3 (Second efficiency result).** Suppose firms $i$ and $j$ propose to merge with synergy level $s \in (\bar{s}, \bar{\bar{s}}]$, so that the AA requires a divestiture in order to approve a merger proposal. Suppose the divestiture is price-fixing. The outcome of the sales process depends critically on the selling mechanism.

i) If the divestiture is sold at a fixed price which does not exclude any potential buyer (thus is assumed to be zero), then the merged firm sells the remedy to a firm for which the size of the divestiture is minimized. For a linear demand function, $p(X) = a - bX$, this is the smallest firm within $W(i, j, s)$.

ii) If the divestiture is sold through an auction in which all buyers bid their maximum willingness to pay, then the merged firm sells the remedy to a firm with the largest post- and pre-merger profit differential. For a linear demand function, $p(X) = a - bX$, this is the largest firm within $W(i, j, s)$.

iii) If the merged firm can make a take-it or leave-it offer to a pre-selected buyer, then the divestiture is sold to the socially optimal buyer within $W(i, j, s)$.

Proposition 3 shows that the merged firm’s ability to extract rents from the asset sale determines the divestiture level and the buyer’s identity. If, for some reason, potential buyers can avoid to get absorbed in a bidding race, so that rent extraction is severely limited, then the merging parties minimize the amount of assets to be sold (part i) of Proposition 3). If rent extraction is enhanced, for instance, when the asset sale is structured through an auction-type selling process, then the divestiture should be expected to go to a firm which can run the ad-
ditional assets most profitably (part \textit{ii}) of Proposition 3). Even though such a buyer may not be preferred by the merged firm as it may “steal” its market share, it cannot avoid such an outcome if the remedy is sold through an auction. Finally, part \textit{iii}) of Proposition 3 shows that the merged firm’s divestiture decision is perfectly aligned with the social welfare maximizing rule whenever it can commit to make a take-it or leave-it offer to a pre-selected buyer. The merged firm is then able to extract the entire surplus created by the divestiture process. As the sale of a price-fixing remedy is externality-free, it follows that the merged firm makes the socially optimal choice.

In Dertwinkel-Kalt and Wey (2012) we have shown that it depends on the specific setup and the synergy level $s$ whether the socially optimal buyer is more likely to be an efficient (i.e., large) firm or an inefficient (i.e., small) competitor. A relatively inefficient firm can be regarded as an “entrant” firm, which has not yet acquired sufficient capital to get a substantial market share. In contrast, efficient firms can be regarded as incumbent competitors which are established in the market and have build up a considerable capital stock. Therefore, our analysis mirrors a feature of the remedy guidelines, according to which remedies might be sold to an entrant firm or an incumbent competitor. Per se, it cannot be determined which buyer type is optimal from a social-welfare perspective.

The message of Proposition 3 is that the merging parties should have a maximum of power in the asset sales process, because this must lead to the selection of the socially preferred buyer type. It is noteworthy that remedy guidelines mirror our findings. For instance, the merger remedy guidelines of the DOJ distinguish between “fix-it-first remedies” and “post consummation sales” (DOJ, 2011, pp. 22-25). Successful fix-it-first remedies eliminate the competitive concerns and allow the AA to clear the merger without the need to file the case in court. In contrast, post-consummation sales induce the AA to file the case in court to obtain a consent decree, which allows to enforce and monitor the remedial provisions because of the court’s contempt power. The guidelines clearly favor an adequate fix-it-first remedy, while the post-consummation sale is much more restrictive (and costly) for the merging parties.\footnote{With regard to the fix-it-

\footnote{Quite bluntly the remedy guidelines state: “For the parties, resolving a merger’s competitive issue with an upfront buyer can shorten the divestiture process, provide more certainty about the transaction than if they (…)}
first remedy, the guidelines “provide the parties with the maximum flexibility in fashioning the appropriate divestiture” (DOJ, 2011, p. 22). Accordingly, the merging parties can adjust the divestiture freely, so that the assets can be “tailored to a specific proposed purchaser” (DOJ, 2011, p. 22). In contrast, if a consent decree is needed for a post-consummation sale, then the guidelines build up a credible threat of force. First, a package of assets to be divested must be identified in advance, and second, “crown-jewels” must be offered “to increase the likelihood that an appropriate purchaser will emerge” (DOJ, 2011, p. 24).

Those rules increase the commitment value of the merging parties when proposing an asset sale to a potential purchaser to obtain a fix-it-first remedy. First, the guidelines give a maximum of flexibility in adjusting the asset sale to the competitiveness of the purchaser. Second, entering into a consent decree is costly, full of uncertainty, and further burdened with the crown-jewel provision. Those additional costs may make the entire merger unattractive, adding to the commitment value necessary to extract rents in the fix-it-first sales process.

4.2 Remedy-dependent Synergies

So far we have assumed that only the merging firms realize a fixed synergy level. We discuss two plausible extensions: first, the firm which buys the to be divested assets may realize synergies itself, and second, the merged firm’s synergy level may depend negatively on the amount of the to be divested assets.

In the first case, the acquirer of the assets may generate a synergy \( t \), which effect is analogous to that of \( s \) (i.e., \( t \) enters the acquirer’s cost function as a multiplicative factor). The vector \((k_l, t_l)\) describes the efficiency of an acquiring firm \( l \). Given that the price-fixing condition is fulfilled, the merging firms’ incentives to search for the most efficient buyer are fully aligned with the social welfare maximizing choice. As before, the reason for this result is that the merged firm maximizes the gains from trading the remedy as long as it has perfect selling power.

**Corollary 2.** Suppose an arbitrary oligopoly market and assume that the merging parties propose a price-fixing remedy to the AA (which uses a consumer surplus standard). Assume that buyers must seek a buyer for a package of assets post-consummation, and avoid the possibility of a sale dictated by the Division in which the parties might have to give up a larger package of assets” (DOJ, 2011, p. 22).
of different efficiencies exist. Then the following efficiency result holds: If the merged firm has perfect selling power, then it selects the most efficient buyer.

Note also that a very small divestiture (“\(\varepsilon\)-divestiture”) which is not price-fixing, but price-decreasing may become possible. Even a small divestiture may have significant impact on competition if the divested assets create a considerable synergy level \(t\). As such a divestiture raises the competitor’s efficiency significantly while it lowers the merger’s efficiency only marginally, consumer surplus may strictly increase through the merger.

In the second case, the divestiture of assets may lower the synergy level of the merging firms. If remedies reduce synergies, i.e., if the synergy \(s = s(k_j - \sigma)\) is a decreasing function of the divested capital \(k_j - \sigma\), then the scope for mergers shrinks. If, however, there is a remedy which suffices to fix the AA’s concerns, then this is even more likely to be price-fixing than in our basic model. This is so because the merged entity has less incentives to sell more assets than necessary if this affects its realized synergies negatively. While our efficiency results concerning the consumer surplus standard remain true, a social welfare standard has now the advantage that more full mergers, realizing also full synergies, could pass the decision screen of the AA. If the effect of remedies on the realized synergy level \(s\) is very small, then the consumer surplus standard fully unfolds the advantageous effects which we have shown. However, in settings where the negative effect of assets sales, \(\partial s/\partial \sigma\), is relatively large, a social surplus standard may induce higher social welfare levels than the consumer surplus standard does.

### 4.3 Price competition

It is not likely that our results concerning the efficiency effects of remedies under a consumer surplus standard carry over to the case of price competition. To see this, consider a differentiated products Bertrand model, for instance, a Salop-circle model. If capital is distributed unevenly before the merger, then different equilibrium prices prevail (even if products are symmetrically differentiated). If two non-neighboring firms merge, then there is no need for divestitures as the merging firms do not compete directly against each other before the merger.\(^{30}\) In this case, a merger leads to an efficiency gain of the merged firm which reduces the prices it charges. As

\(^{30}\)Note that it is typically optimal to keep the product variants of merging after the merger.
prices are strategic complements all other firms will also reduces their prices and consumers gain from such a merger.

If two neighboring firms merge, then three different constellations have to be considered after a full merger: 1) both prices go down, 2) both prices go up, 3) one price does up, the other one goes down. In case 1), there is obviously no need for a remedy, because all prices in the market will be reduced after the merger which follows from the fact that prices are strategic complements. Hence, in this case consumer surplus increases and there is not need for a remedial divestiture. Regarding case 2), if the synergy is relatively small, so that the merged firm would increase its prices, then remedies are unlikely to be helpful. A divestiture of assets would increase the merged firm’s costs and hence further increase its incentive to raise prices. Because of strategic complementarity, the outsiders’ optimal reaction is to raise their prices as well. Therefore, the merged firm would have to divest a share of its capital that would induce the buyer firm to lower its price in equilibrium. Under standard assumptions, this seems to be unlikely and only possible if the buying firm generates extremely large synergies.

Moreover, in case 3) a merger reduces one price and increases the other one. This can happen if the merging firms have different capital endowments prior to the merger. If this is true, then an asset sale of the merging parties cannot fix consumer surplus in the way as we have discussed in our main analysis. In a homogenous goods Cournot oligopoly fixing consumer surplus is achieved by fixing the market price through an appropriate divestiture. In this case all first-order conditions of all outsider firms remain unaffected by the merger. This logic does not hold under product differentiation in case 3) because there is no one-to-one relationship between consumer surplus and the “price level” which is an aggregate of all prices charged. Even if a remedy is found which fixes consumer surplus, then it is unlikely that the merger becomes externality-free because individual prices adjust differently.

In sum, remedies are less effective to enlarge the scope of acceptable mergers in a differentiated products Bertrand competition model and even if consumer-surplus fixing remedies exist, then it is unlikely that the merger becomes externality-free.
5 Conclusion

We analyzed the effects of remedies on merger activity in a Cournot oligopoly model with homogeneous products under a consumer welfare standard. In general, remedies increase the scope for profitable mergers that do not harm consumers. In particular, if the consumer surplus standard binds, the merger does not change the equilibrium market price and is therefore externality-free. Accordingly, the profits of firms not involved in the merging process do not change. We derive fairly general conditions under which the consumer surplus standard binds and obtain that remedial offers must be larger when the merger’s synergy level is smaller, which mirrors the proportionality principle in the US- and the EU-remedy guidelines.

Furthermore, we derive several efficiency properties concerning current merger control regimes. The ability of the merging firms to extract the gains from trading the divested assets is critical when the purchaser is endogenously determined. If the merging parties’ ability to extract these gains is maximal, i.e., if they can make a take-it or leave-it offer to a pre-selected buyer, then the socially optimal buyer is selected. The merging firms have strong incentives to search for the socially optimal buyer as this tends to increase the feasible set of mergers and, at the same time, maximizes the gains from trading the divestiture. The consumer surplus standard together with the formulation of merger remedy guidelines yield efficient outcomes with respect to two features. First, a remedy regime in combination with a consumer surplus standard ensures that only those mergers are implemented which are strictly social welfare-enhancing in such a way that no market participant is worse off after the merger. Second, as endorsed by the guidelines, firms should have a maximum of power in the asset sales process concerning whom to divest, what assets to divest and at which price to divest.

However, our model has also limitations. It crucially relies on the fact that consumer surplus is proportional to the market price. Under price competition with heterogenous products mergers with remedies are usually not externality-free. We also took it for granted that claimed synergies are verifiable, i.e., the AA can fully anticipate the size of synergies created through a merger which may be not the case in reality. Finally, we regarded industry capital as fixed and abstracted from a long run perspective where the industry’s capital stock may be endogenous because of innovations and entry.
Appendix

In this Appendix we provide the omitted proofs.

**Proof of Lemma 1.** This proof is analogous to the proof of Proposition 2 in Farrell and Shapiro (1990a). In order to assess how a change in capital $d\mathbf{k} = (dk_1, ..., dk_n)$ affects equilibrium quantities $d\mathbf{x} = (dx_1, ..., dx_n)$, we take the total derivative of (1) with respect to $k_j$ and $x_j$ which gives

$$[p'(X) + x_jp''(X)]dX + p'(X)dx_j - c^j_{xk} dk_j = 0.$$  

Using (7) and (9) and defining $\Lambda := \sum_i \lambda_i$ we obtain

$$dX = \sum_i \delta_i dk_i / (1 + \Lambda). \quad (14)$$

Let capital $dk$ be sold from firm $j$ to firm $l$, so that the preceding formula simplifies to

$$\frac{dX}{dk} = \frac{\delta_l - \delta_j}{1 + \Lambda}. \quad (15)$$

This proves the lemma.

**Proof of Lemma 2.** Let $x^*_i$ and $X^*$ denote the pre-merger equilibrium levels of firm $i$’s output and of the industry output, respectively. Let $x^*_M$ and $X^*$ denote the merged firm’s equilibrium output and equilibrium industry output, respectively, after firms $i$ and $j$ have merged and have realized synergy level $s$. By Assumption 1, industry output increases strictly at $s = 0$. Note that the industry output $X^*$ is strictly monotonically decreasing in the sum of firms’ marginal costs. As the merged firm’s cost function is monotone and continuous in $s$, it follows that industry output is also monotonically and continuously decreasing in $s$. If there is no synergy parameter $s \in [0, 1)$ for which the post-merger industry output falls short of the pre-merger output, i.e., for which $X^* < X^*$ holds, then we define $\bar{s} := 1$. Otherwise, there exists a unique threshold value $\bar{s} \in [0, 1)$ such that industry output increases, $X^* > X^*$, if and only if $s < \bar{s}$, while it decreases, $X^* < X^*$, if and only if $s > \bar{s}$, with equality holding at $s = \bar{s}$. Note that all approvable mergers (for which $s \leq \bar{s}$ holds) are profitable as the joint output of the merging firms (weakly) increases while marginal and infra-marginal production costs decrease. \[\square\]
Proof of Lemma 3. Let firms $i$ and $j$ be the merger candidates. Note that merging and selling the divestiture simultaneously is formally equivalent to a two-stage procedure where firms $i$ and $j$ merge before they divest $k_j - \sigma$ to a rival firm $l$.

If $s \leq \bar{s}$, then consumers will not be harmed by the merger, and the AA applying a consumer-surplus standard will approve a full merger. For lower synergy levels ($s > \bar{s}$), however, a full merger cannot be approved by the AA since the industry’s post-merger equilibrium quantity falls below the pre-merger industry quantity. In order to assess to which extend remedies can improve the scope for approvable mergers, we first show that there exists a unique threshold value of the synergy parameter $s$ for each potential buyer $l$ up to which a divestiture increases industry output locally. Second, we show that remedies increase the scope for mergers. Third, we show that the existence of an approvable remedy is monotone in the realized synergy level. Fourth, we generalize our findings for all potential buyers $l$ and thus obtain a unique synergy threshold value up to which remedies enlarge the scope of mergers. Fifth, we derive a weak condition under which remedies strictly enlarge the scope of mergers.

Step 1 (Local effects of a divestiture on industry output). Let $x^s(\sigma)$ and $X^s(\sigma)$ denote the equilibrium quantities for a given divestiture level $k_j - \sigma \geq 0$ and a given buyer firm $l \in I \setminus \{i, j\}$. Given the specification of firms’ cost functions (4)-(6), we obtain

\[
c^M_x(x, k, s) = \frac{s}{k}, \quad c^M_{xk}(x, k, s) = -\frac{s}{k^2}.
\]

After divesting $k_j - \sigma$ to firm $l$, the direct effects of capital on output (8) for firms $M$ and $l$ are given by

\[
\delta_M(k_i + \sigma) = \frac{c^M_{xk}}{p^f} = \frac{s/(k_i + \sigma)^2}{-p'(X^s(\sigma))} \quad \text{and} \quad \delta_l(k_l + k_j - \sigma) = \frac{c^l_{xk}}{p^f} = \frac{1/(k_l + k_j - \sigma)^2}{-p'(X^s(\sigma))},
\]

respectively. Therefore, the difference of the direct effects of capital on output between the merged firm $M$ and the acquirer $l$ of the assets $k_j - \sigma$, which is given by

\[
\delta_M(k_i + \sigma) - \delta_l(k_l + k_j - \sigma) = \frac{1}{-p'(X(\sigma, l))} \left[ \frac{s}{(k_i + \sigma)^2} - \frac{1}{(k_l + k_j - \sigma)^2} \right].
\]
is continuous and has at most one zero on the interval $\sigma \in [0,1]$. If there is no such zero and $\delta_M(k_i + \sigma) > \delta_l(k_l + k_j - \sigma)$ for all $\sigma \in [0,1]$, then set $\tilde{\sigma} = 1$. If there is no such zero and $\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)$ for all $\sigma \in [0,1]$, then set $\tilde{\sigma} = 0$. Otherwise, there is a unique threshold value

$$\tilde{\sigma} = \tilde{\sigma}(s) = \frac{1}{\sqrt{s} + 1} \left(-k_i + \sqrt{s}k_j + \sqrt{s}k_l\right)$$  \hspace{1cm} (16)$$

for which the direct effect of capital on output is the same for the merged firm $M$ and the acquirer of the assets $l$; i.e., for which $\delta_M(\tilde{\sigma}, l) = \delta_l(k_l + k_j - \tilde{\sigma})$ holds. The threshold $\tilde{\sigma}$ gives the unique maximum divestiture up to which a divestiture can increase the industry output. We find that

$$\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)$$  \hspace{1cm} (17)$$

holds if and only if

$$\sigma > \tilde{\sigma}. \hspace{1cm} (18)$$

This means that the direct effect of capital on output is larger for the merged firm $M$ than for buyer $l$ if and only if the divestiture’s size exceeds the threshold value $k_j - \tilde{\sigma}$. Note that

$$\frac{\partial \tilde{\sigma}}{\partial s} = \frac{1}{2\sqrt{s}} \frac{(k_i + k_j + k_l)}{(\sqrt{s} + 1)^2} > 0,$$

such that the threshold value $\tilde{\sigma}$ is strictly increasing in $s$. Therefore, the range of remedies $\{k_j - \sigma|\sigma > \tilde{\sigma}\}$ for which $\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)$ strictly increases with a higher synergy level, i.e., a lower parameter value $s$. Equivalently, the range of divestiture sizes to a certain firm $l$, where divesting assets increases the industry post-merger output locally is strictly enlarged through stronger synergies.

**Step 2 (Remedies increase the scope for acceptable mergers).** In order to prove that remedies increase the scope for mergers, we have to investigate those potential buyers $l$ for which $\delta_M(k_i + \sigma) < \delta_l(k_l + k_j - \sigma)$ holds for a small divestiture $k_j - \tilde{\sigma}$ and for some $s > \tilde{s}$. For a certain buyer $l$, Condition (17) holds if and only if $\sigma \in (\tilde{\sigma}, k_j]$, while this interval may be empty. Fix $l$ and define $\varepsilon(s) = \max\{k_j - \tilde{\sigma}(s), 0\}$. Since $\tilde{\sigma}$ is strictly monotonically increasing in $s$, the function $\varepsilon(s)$ is monotonically decreasing in $s$.

For the moment, we assume that $\varepsilon := \varepsilon(\tilde{s}) > 0$ (which holds if $k_j - \tilde{\sigma} > 0$). According to
(16), this is equivalent to assuming that
\[ k_i + k_j > \sqrt{ss}k_l \]
holds. Given \( \varepsilon > 0 \), (17) holds for \( s = s \) and for all \( \sigma \in (k_j - \varepsilon, k_j] \). Since \( \delta_i(k_l + k_j - \sigma) - \delta_{M}(k_i + \sigma) \) is continuous in \( s \) and since \( d\bar{\sigma}/ds > 0 \), for each \( \varepsilon' < \varepsilon \) we can define \( s' = s'(\varepsilon') > s \) to be the largest \( s \) such that (17) holds for all \( s \in [\bar{s},s] \) and for all \( \sigma \in (k_j - \varepsilon', k_j] \). For each \( \varepsilon' \), define \( \varepsilon'' = \varepsilon''(\varepsilon') = \min_{s \in [\bar{s},s']}[X^s(k_j - \varepsilon') - X^s(0)] \). First, \( X^s(k_j - \varepsilon') - X^s(0) > 0 \) holds for all \( s \in (\bar{s}, s') \). Second, \( \lim_{s \to \bar{s}} X^s(k_j - \varepsilon') - X^s(0) = X^\bar{s}(k_j - \varepsilon') - X^{\bar{s}}(0) > 0 \) holds and third, \( \lim_{s \to s'} X^s(k_j - \varepsilon') - X^s(0) > 0 \), holds. Therefore, \( \varepsilon'' \) is well defined and \( \varepsilon'' > 0 \). As for each diverstiture \( k \), the equilibrium quantity \( X^s(k) \) is continuous in \( s \), there exists a largest \( s \in (\bar{s}, s' \varepsilon') \) which satisfies \( X^s - X^s(0) \leq \varepsilon'' \); we denote this synergy \( s'' = s''(\varepsilon') \). Then, \( X^s - X^s(0) \leq \varepsilon'' \) holds for all \( s \in [\bar{s}, s''] \).

As a consequence, \( X^s(k_j - \varepsilon') - X^s = [X^s(k_j - \varepsilon') - X^s(0)] - [X^s - X^s(0)] \geq \varepsilon'' - \varepsilon'' = 0 \) for all \( s \in [\bar{s}, s''] \). Thus, for synergy \( s \in (\bar{s}, s'') \) there exists a divergence which can offset the merger’s negative effect on aggregate output. Consequently, as long as our initial assumption \( \varepsilon(\bar{s}) > 0 \) holds, for all such \( s \in (\bar{s}, s'') \) there exists a remedy which fixes the AA’s concerns. We will call such a remedy an approvable remedy.

**Step 3 (Monotonicity and uniqueness).** Clearly, \( \partial X^s(k, l)/\partial s < 0 \) holds as a larger \( s \) implies a lower synergy and therefore a higher sum of firm’s marginal production costs. If there is an approvable remedy sold to firm \( l \) for a merger which realizes synergy \( s \), then there is an approvable remedy also for higher synergies, i.e., lower \( s \). As a consequence, there is a threshold synergy value \( s^R_l \) such that there exists an acceptable remedy if and only if the merger synergy satisfies \( s \leq s^R_l \). Precisely, this threshold can be defined as \( s^R_l := \sup_{\varepsilon' < \varepsilon(\bar{s})} s''(\varepsilon') \in [\bar{s}, 1] \) if \( \varepsilon(\bar{s}) > 0 \) and \( s^R_l := \bar{s} \) if \( \varepsilon(\bar{s}) = 0 \).

**Step 4 (Extending towards all potential buyers).** We can repeat the analysis with all potential buyer firms \( l \neq i, j \). Allowing for remedies to any competitor further increases the scope where remedies can induce a merger’s approval. We define \( s^R := \max_{l \in \{i, j\}} s^R_l \in [\bar{s}, 1] \). Therefore, the synergy range where mergers do not harm consumers is strictly increased through remedies if there is a firm \( l \) such that \( s^R_l > \bar{s} \).

**Step 5 (Condition such that remedies increase the scope for mergers strictly).** Extending con-
dition (19) towards all potential buyers yields that divestitures strictly increase the scope for mergers if and only if

\[ k_i + k_j > \sqrt{s} \min_{l \in I \setminus \{i,j\}} \{ k_l \}. \]

This proves the lemma. \( \square \)

**Proof of Lemma 4.** Assume that for synergy \( s \) there is a merger (potentially involving a remedy sold to some firm \( l \)) which does not increase the final good’s price and which is profitable for the merged firm. For instance, this holds for \( s = \overline{s} \). As we will show, this implies that there exists also a profitable merger for all synergies \( s - \varepsilon \) with \( \varepsilon > 0 \). First, a merger’s infra-marginal production costs per unit are lower the lower if a merger realizes synergy \( s - \varepsilon \) instead of \( s \) for any fixed \( \sigma \) and any fixed quantities \( x_M \) and \( x_l \). Second, the equilibrium output of the merged firm \( M \) and the joint equilibrium output of firms \( M \) and \( l \) are larger if the merger’s synergy level is \( s - \varepsilon \) instead of \( s \), provided a fixed share \( \sigma \). As the merged firm \( M \) is assumed to have perfect selling power, which means that it can extract the entire surplus from the asset sale (i.e., firm \( l \)’s post-merger profit minus its pre-merger profit), for any fixed \( \sigma \) the merger with divestiture \( k_j - \sigma \) is more profitable for synergy \( s - \varepsilon \) than for synergy \( s \). Furthermore, as the industry output is decreasing in the sum of firms’ marginal costs, a merger with divestiture \( k_j - \sigma \) is approvable for synergy level \( s - \varepsilon \) if it is approvable for synergy level \( s \). \( \square \)

**Proof of Lemma 5.** Part i) is immediate while part ii) follows directly from Lemma 1. In order to prove part iii), we derive condition (13). Using Farrell and Shapiro (1990a, Equation 13) the derivative of the sum of firm \( M \)’s and firm \( l \)’s profits with respect to \( \sigma \) can be written as\(^{31}\)

\[
\frac{d(\Pi_M + \Pi_l)}{d\sigma} = -p'(X) \left( \delta_M x_M - \delta_l x_l + \frac{\delta_l - \delta_M}{1 + \Lambda} ((1 + \lambda_l)x_1 + (1 + \lambda_M)x_M) \right) - c^M_k + c^l_k.
\]

Substituting \( \delta_M, \delta_l, c^M_k \) and \( c^l_k \) and re-arranging yields

\[
\frac{d(\Pi_M + \Pi_l)}{d\sigma} = 2 \left( \frac{sx_M}{(k_i + \sigma)^2} - \frac{x_l}{(k_l + k_j - \sigma)^2} \right) + \left( \frac{1}{(k_l + k_j - \sigma)^2} - \frac{s}{(k_i + \sigma)^2} \right) \left( \frac{1 + \lambda_l)x_1 + (1 + \lambda_M)x_M}{1 + \Lambda} \right).
\]

\(^{31}\)The following equation can be derived by using the total derivatives of firm \( l \)’s and firm \( M \)’s first-order conditions with respect to \( x \), \( X \) and \( \sigma \).
This proves the lemma.

**Proof of Lemma 6.** For \( s \in (\bar{s}, s^R) \) let \( k_j - \hat{\sigma} \) be the price-fixing equilibrium divestiture to be sold to firm \( l \). Suppose the synergy parameter \( s \) falls marginally. Holding \( \hat{\sigma} \) fixed, the final good price decreases. Due to Assumption 3, the merging firms will adjust the remedy in order to keep the remedy price-fixing. As \( \delta_l(k_l + k_j - \sigma) \geq \delta_M(k_i + \sigma) \) for synergy \( s \) and all \( \sigma > \hat{\sigma} \), this inequation holds also for a synergy parameter slightly below \( s \) and all \( \sigma > \hat{\sigma} \). Therefore, the respective price-fixing remedy is smaller than \( k_j - \hat{\sigma} \).

**Proof of Proposition 3.** We proof each part of the proposition separately.

*Part i)* As the merged firm does not earn any revenues from selling the assets, it maximizes its own market profit. The market profit is maximal if the size of the divestiture is minimal. This must be so as additional capital lowers marginal production costs and as the own equilibrium quantity strictly increases with lower marginal costs. We consider the linear demand function \( p = a - bx \) with parameters \( a, b > 0 \). In order to assess the impact of a divestiture to firm \( l \) on the industry output, we analyze condition (15) and obtain

\[
\frac{dX}{d\sigma} = \frac{\delta_M(\sigma) - \delta(l)(\sigma)}{(1 + \Lambda)} = \frac{s}{(k_i + \sigma)^2} - \frac{1}{nb}, \tag{20}
\]

which is strictly monotonically decreasing in \( k_l \) for all admissible \( \sigma \). Therefore, the size of the price-fixing divestiture, i.e., the divestiture which suffices to restore the pre-merger industry quantity, is smallest if the merged firm divests to that firm \( l \in W(i, j, s) \) which holds the smallest capital stock.

*Part ii)* Let \( \tilde{l} \) be the firm with the largest capital stock within \( W(i, j, s) \); i.e., \( k_{\tilde{l}} \geq k_l \) for all \( l \in W(i, j, s) \). *First*, note that firm \( \tilde{l} \) produces the largest pre-merger equilibrium quantity, i.e., \( \tilde{l} \in \arg \max_{l \in W(i, j, s)} \{x_l^*\} \). *Second*, Equality (??) implies that the price-fixing divestiture \( k_j - \sigma \) is weakly larger for firm \( \tilde{l} \) than for all other firms \( l \in W(i, j, s) \). *Third*, as the pre-merger industry quantity is required to be restored through the divestiture process and as the quantity produced by the merged firm \( M \) is strictly monotonically decreasing in the size of the divestiture \( k_j - \sigma \), firms of size \( k_l \) record the largest increase in equilibrium output through the acquisition of the price-fixing divestiture. This means that \( \tilde{l} \in \arg \max_{l \in W(i, j, s)} \{x_l - x_l^*\} \), where \( x_l \) denotes firm \( l \)'s equilibrium output after the acquisition of a price-fixing divestiture. *Fourth*, a firm
l’s willingness to pay for a price-fixing divestiture, \( WTP(l) \), equals the difference between its profit after the asset’s acquisition, \( \Pi_l \), and its pre-merger profit, \( \Pi_l^* \). For the demand function \( p = a - bX \) with parameters \( a, b > 0 \), a firm \( l \) producing quantity \( x_l \) earns profit \( \Pi_l = bx_l^2 \) (which can be easily seen from inspecting a firm’s first order condition). It then follows that \( \hat{l} \in \arg \max_{l \in W(i,j,s)} WTP(l) \), which proves the claim.

\( \text{Part iii)} \) This follows from Proposition 2.

6 References


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