Health Insurance and Competition in Markets for Differentiated Medical Products

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April 2015

Abstract

I study duopolistic market for differentiated medical products. Medical providers decide whether to sell on the spot market to sick consumers or to sell through competitive insurance market to healthy consumers. While shopping for insurance consumers know only the distribution of possible medical needs they may have if they get sick. Only when getting sick their actual medical need reveals and diagnosed. Hence consumers on the insurance market have lower taste differentiation than the sick consumers who are shopping on the spot market. I find that in equilibrium providers sell only on the insurance market, even though this intensifies competition because of lower taste differentiation. Competition between providers under insurance sales brings premiums low enough to motivate consumers buying insurance for both products. Insurance sales generate efficient horizontal product differentiation, lower prices, and efficiently higher quality.

JEL Classification: : I11, I13, L1

Key-words: Insurance, Non-linear Pricing, Option Demand, Differentiation

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†Thanks go to Randy Beard and Aditi Sengupta for helpful comments and to Adeet Handel for challenging discussions. I also benefited from participants comments on presentations at Auburn University, the spring 2014 Midwest-Theory Conference at UIPUI and the ASHE 2014 conference at USC, and discussion comments by Ted Frech.
1 Introduction

I study duopolistic competition in Hotelling’s (1929) linear market for differentiated medical goods. Providers of medical products choose specialization (horizontal location) and then compete in quality and price. In addition they choose whether to sell on spot market to sick consumers or through competitive insurance market to healthy consumers. When selling through insurance the prices set by medical providers translate to up-front (actuarially fair) premiums.

Healthy consumers face a probability distribution of possible medical needs. In case of sickness each consumer will draw one medical need from this distribution. In Hotelling’s setup this means that consumers don’t know their location on the differentiated medical care market before getting sick. Therefore, healthy consumers are also uncertain regarding which medical product they will prefer to utilize before their actual medical need reveals (i.e. before getting sick).

This uncertainty decreases consumers taste differentiation on insurance market, which in turn intensifies competition compared with spot market sales. Moreover, it generates demand for including both medical products under insurance coverage by each consumer. The two medical products that are substitutes for sick consumers on the spot-market may be perceived as complements when sold through insurance. In other words, ex-ante uncertainty on future medical needs generates option-demand for multiple medical products.

The option value of each product depends on the risk of getting sick, the distribution of medical needs, and technological differentiation. The latter factor implies that the option values of both technologies are co-determined. For given products and prices, consumers may either choose buying an option for utilizing one technology only or to buy options for utilizing both of them. Hence, when opting to sell through insurance innovators shift from spot-price competition over the marginal consumers, to competition in the option market over marginal inclusion under insurance coverage.

I find that in the equilibrium with insurance sales premiums are low enough to motivate consumer buying insurance for both products. In addition providers (privately) choose their locations and qualities as to maximize the joint-value from covering both technologies under insurance, thereby maximizing welfare. This insurance-sales equilibrium has "prisoner’s dilemma" flavor: the choice to sell through insurance is beneficial for each provider privately, but profits are ultimately lower when both innovators do so.

2 Related Literature and Contributions

Several works have studied markets for differentiated medical products under insurance sales\(^1\). Special attention was given in this literature to the implications of alternative payment schemes (Nell et al. 2009 and Bardey et al. 2012), vertical relations between providers and payers (as in

\(^{1}\text{In this context horizontal differentiation on the products space has two common interpretations: product differentiation within a given therapeutic category (e.g. pharmaceuticals as in Grossman 2013) and hospitals differentiation with respect to area of specializations (as in Gal Or 1997, Lyon 1999 and Brekke et al. 2007b). In both cases sellers’ locations defines the effectiveness of their product in treating different medical conditions (within or across therapeutic category). The present analysis applies to both interpretations, and for consistency I will follow the first.}\)

Most aforementioned studies assumed consumers face a specific medical risk. That is each consumer knows the exact medical need she will have if she gets sick. In Hotelling’s setup this means that healthy consumers know what would be their exact location on the medical-needs line, in case they will get sick.

In reality however, patients’ specific medical needs most commonly reveal and diagnosed after getting sick³. This means that while consumers are shopping for insurance they know only the distribution of possible medical needs each one of them may have. I will refer to this type of medical risk as "indeterminate", because consumers are uncertain which of all possible medical needs will be realized in case of sickness. The implications of this plausible uncertainty to market outcomes are the focus of this study.

The few works that accounted for indeterminate medical risks by Gal-Or (1997, 1999) and Douven et al. (2014) focus on bargaining and network formation among differentiated providers and differentiated insurers, abstracting from providers choices of specialization and quality⁴.

In this paper I provide a complementary market analysis under indeterminate medical risks. I study duopolistic competition in location, quality, and price. In addition providers choose whether to sell through competitive insurance markets or sell on the spot market. When selling through insurance providers compete in the option market: the inclusion of medical product under insurance coverage gives an option to utilize it at lower marginal price when sick.

Under full reimbursement (zero co-pays), the option value of a given medical product defines the highest insurance-premium consumer is willing to pay (for getting that product under insurance coverage). However, the option value of a given medical product sold on the insurance market depends on the type of medical risk consumers are facing.

Consider first the case of specific medical risk. Here ex-ante healthy consumers on the insurance market and ex-post sick consumers on the spot market have the same degree of taste differentiation for alternative products. In this case the option value of each technology depends solely on its effectiveness (quality and location) and the probability of becoming sick. Then, each consumer prefers buying only one insurance - for the most cost effective product⁵.

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²This literature followed an earlier line of research on the effect of mark-up pricing in health care markets on insurance markets efficiency. See Gaynor et al. (2000).

³Yet, some inherited medical conditions can be highly predicted through genetic checks and family history for example. Furthermore diagnoses are also subject to errors. Thus some uncertainty regarding the effectiveness of each medical treatment may be revealed only after utilization, making medical technologies an experience-good. This type of uncertainty however is common to spot-market and insurance consumers and thus should not alter our main results, as long as diagnosis is somewhat informative. Brekke et al. (2007b) model diagnosis inaccuracy to study the role of gatekeepers, but without repeated purchases. Crawford and Shum (2005) model the dynamics of learning pharmaceuticals’ effectiveness through experiencing.

⁴Another related exception, is the work by Ma (1997) which employs a different setup. In this study consumer’s utility from one of two products is ex-ante uncertain, and the two products can be bundled by insurers, under option price contracts. Here, differentiation and mode of provision choices are also abstracted. Brekke et al. (2007b) model consumer’s uncertainty regarding medical needs, to study the effect of Gatekeepers on differentiation and quality choices, in spot-market with regulated prices.

⁵In this case, if consumers are risk neutral market outcomes would not change merely buy selling each product
Under indeterminate medical risk consumers taste differentiation on the insurance market is lower than on the spot market. If they all face the same distribution of medical risks, healthy consumers on the insurance market have identical (ex-ante) preferences over all medical products. In this case the option value of each product depends on the distribution of medical needs and distribution of available products. Here consumers may like to include either one product or both under insurance coverage, depending on their prices and differentiation degree.

To better understand consumers considerations in this case, suppose two differentiated medical product are equally priced and symmetrically located on the preference line (which represents the range of possible medical needs). Consumers would buy insurance for both products only if the additional premium payment justifies the additional variation products effectiveness over all different medical needs. If the products are only slightly differentiated, only a correspondingly low price shall motivate consumers to buy insurance for both.

However, so long as prices are not low enough to motivate buying insurance for both products, the two providers are engaged in intense price competition, a-la Bertrand. This is because they have the same stand along option value: symmetrically located and equally priced the two products are perceived as perfect substitutes to consumers on the insurance market.

Hence, under insurance sales demand faced by each provider is extremely elastic for high prices. However, it becomes perfectly inelastic once consumers buy insurance for both products. Then demand cannot increase with further price cut. This price level defines the equilibrium in this analysis. I show that insurance sales are chosen in equilibrium by both providers and that competition under insurance sales results in efficient differentiation and quality provision.

Among the aforementioned references the works by Lyon (1999) and Katz (2011) are most closely related to mine. Both works study the implications of vertical integration between competitive insurers and differentiated providers, under specific medical risks. My analysis is directly comparable with their market analyses under no vertical integration (see Section 4 in both papers).

Lyon (1999) assumes that consumers are uncertain about products qualities when buying insurance, and insurers bundle both providers under one unified premium. Katz (2011) abstracts from quality uncertainty, but also assumes employers offer their employees uniform insurance plans.

In both works bundling providers under pooled insurance policy softens price competition and imposes complementarity effects among providers. It is not well justified however, why would competitive insurers bundle technologies if consumers are better off having only one product under coverage. If consumers are risk averse medical prices could rise due to consumers’ additional willingness to pay for the bundled insurance services (elimination the financial risk of medical bills).

That is they have very similar therapeutic impact in treating different medical needs.

Note however that even if consumers buy options for both technologies, they eventually utilize only one of them. Hence the ex-post substitutable treatments can be ex-ante compliments. Dranove and White (1996) also study the implications of option demand in health-care markets, under the same type of ex-ante uncertainty regarding medical needs. However, they focus on competition among different physicians groups and hospitals, and the effect of labor specialization on prices. They do not study differentiation, quality and mode of provision choices. In a related paper, Capps et al. (2003) formulate and estimate measures of market power held by networks of medical providers.

As explained above under specific medical needs as in Katz (2011) consumers cannot gain from having more than
Unlike Lyon (1999) and Katz (2011) I assume that (1) consumers face indeterminate medical risk\(^9\) (2) Perfectly competitive insurers can price and sell separated policies for each product (3) Sellers choose specialization and provision mode along with quality and price\(^{10}\). The analysis yields the following distinctive results.

(a) I start with solving competition in location and price with simultaneous moves. I define unique symmetric equilibrium in which both products sold through insurance, all consumers buy insurance for both products, medical prices are lower than under spot market competition, and horizontal differentiation is efficient.

These results are distinctive to this setup, as differentiation is typically inefficient under spot-price competition and under insurance sales with specific medical risk (see for example d’Aspremont et al. 1979 and Brekke et al. 2007a). They also contrast with the anti-competitive effect of insurance derived under specific medical risk and pooling insurance policy (Lyon 1999 Nell et al. 2009, Katz 2011, and Grossman 2013)\(^{11}\).

(b) Allowing innovators a costly quality choice (i.e. vertical differentiation) does not change equilibrium prices and locations. Equilibrium quality level is efficient and is higher than under spot-price competition. This result is consistent with the common view on health insurance as spurring medical R&D (see Lyon 1999, Katz 2011, and Grossman 2013). Nonetheless, in this literature quality provision is typically inefficient (see Brekke et al. 2006 for discussion)\(^{12}\).

(c) For the case of sequential entry (that is abstracted in the aforementioned literature), I characterize asymmetric equilibrium in insurance sales: the leader located at the middle of the preference range, and the follower located close to the market end. In comparison, the maximal-differentiation equilibrium under spot-price competition is neutral to entry-timing (See Tabuchi and Thisse 1995)\(^{13}\). Furthermore, I find that sequential entry yields first-mover gains, and utility gains to consumers - compared with simultaneous moves equilibrium.

This result is relevant to the study on follow-on drugs, considered also as "me-too" drugs. Unlike generic drugs, "me-too" drugs are patented drugs developed by following innovators in a given therapeutic category, meaning they are differentiated from the drug provided by the incumbent. Recent papers have estimated significant gains to consumers from follow-on drugs, for recent examples see Bokhari and Fournier (2013) and Arcidiacono et al. (2013).

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9 In his concluding remarks Lyon (1999) suggests exploring the implications of uncertain medical needs ("uncertain health status") as a worthwhile extension of his work research.

10 Location and provision mode choices are abstracted in Lyon (1999) and Katz (2011).

11 Vaithianathan (2006) derive the same results in oligopolistic market under Cournot competition. Dranove and White (1996) suggest that option demand for specialists-groups works to increase prices set by each specialist, and thereby overall medical spending.

12 By exception, in Katz (2011) quality provision is also efficient due to the complementarity effect imposed by bundling technologies under one insurance policy. In the present work quality choices are efficient because sellers realize that competition under insurance sales will equalize the option value of each product to their joint value. Note also that consumer's uncertainty regarding quality typically works to decrease investment in quality. For recent examples see Brekke et al. (2007b) and Gravelle and Sivey (2010).

13 ImPLYING THAT COMPETITION THROUGH INSURANCE UNDER CERTAIN MEDICAL NEEDS IS ALSO NEUTRAL TO ENTRY TIMING.
Other theoretical works study the implications of different price regulations ("reference prices") to competition in pharmaceuticals markets, but assumed exogenous symmetric differentiation (see for example Brekke et al. 2007a, Miraldo 2009 and Bardey et al. 2010). The analysis here suggests that in face of perfectly competitive insurers sequential entry implies inefficiently low differentiation, but yet higher gains for consumers compared to simultaneous moves.

To summarize, this work shows that accounting for consumers’ uncertainty regarding future medical needs, changes the way health insurance affects markets for medical products. To this end it abstracts from the conventional informational imperfections associated with health insurance - i.e. moral hazards and adverse selection (following Gal-or 1997, Lyon 1999, Katz 2011 and Douven et al. 2014).

On a final note, the present work relates also to studies on competition in non-linear pricing written outside the realm of the health-care industry (See for example Rochet and Stole 2002, Yin 2004, and Armstrong and Vickers 2010). There however, non-linear pricing does not have the dynamic structure and consumers’ needs are certain. Consequently, in this literature non-linear pricing commonly promotes welfare and shifts surplus in favor of producers, whereas in the present work welfare gains shift in consumers’ favor.

The remainder of the paper develops as follows: Section 3 introduces the detailed setup. Section 4 studies Duopolistic location-price competition, under simultaneous and sequential entry. Section 5 incorporates quality choice, and Section 6 concludes this study.

3 The Model

The market is populated with a unit mass of ex-ante identical consumers. Each consumer faces the probability $\pi$ to develop a medical need denoted $x$, that is becoming sick. All medical needs are uniformly distributed over an interval of length $L : x \sim U[0,L]$. The distribution $x$ is a common knowledge. However, the exact medical need of each consumer is revealed only once getting sick. Each sick consumer draws one medical need from the distribution $x$, which is correctly diagnosed at no cost and then becomes a common knowledge.

When healthy, consumer utility is $v$, and when sick it droops to zero if not treated. Once treated, consumer’s utility depends on the effectiveness of the medical product utilized and the price paid. The effectiveness of a medical product is decreasing with horizontal distance from the treated medical need.

Hence the expected utility from using product $y$ for treating medical need $x$ is

$$ E(u) = (1 - \pi) v + \frac{\pi}{L} \left[ v - \theta (x - y)^2 \right] $$

and the expected utility from using product $y_j$ to treat all alternative medical needs is

$$ E(u) = (1 - \pi) v + \frac{\pi}{L} \int_0^L \left[ v - \theta (x - y)^2 \right] dx $$  \hspace{1cm} (1)
The expected utility function (1) is conventional to the related literature\(^\text{14}\). It implies neutrality with respect to the financial risk associated with medical expenses. Abstracting from risk aversion simplifies the analysis. More importantly however, it allows focusing on the effects of insurance on market outcomes that are solely due to its non-linear dynamic structure, which corresponds to option-pricing. Adding risk-aversion should not alter our main results. The parameter \(\theta\) measures the degree of differentiation over medical conditions, or the cost of mismatch between medical need and treatment that are non-insurable.

Providers choose in which medical need to specialize by picking location \(y \in L\). I assume zero marginal cost of provision (as in Lyon 1999 and Katz 2011), and \(\frac{4\Sigma}{\phi} > L^2\)\(^\text{15}\). In addition to choosing location, providers choose their marketing method: they can sell to sick diagnosed consumers for the spot price \(p_i\), or sell through perfectly competitive insurers.

Insurers offer separated policies for each product and consumers decide which policy to buy (possibly both)\(^\text{16}\). I assume full reimbursement (as in Lyon 1999 and Katz 2011) implying that the price \(p_i\) translates into an actuarially fair premium denoted \(op_i \equiv \pi_i p_i\), where \(\pi_i\) is consumers probability to utilize technology \(i\). To simplify exposition I will describe provider’s pricing considerations in terms of the insurance premiums - i.e. the option price \(op\).

For \(v > p_i + \theta (L)^2\) consumer’s expected utility from buying product \(i\) on the spot market is

\[
(1 - \pi) v + \pi \int_0^L \left[ v - p_i - \theta (x - y_i)^2 \right] dx
\]

And when buying an insurance for that product at the premium \(op_i\) her expected utility is

\[
(1 - \pi) v + \pi \int_0^L \left[ v - \theta (x - y_i)^2 \right] dx - op_i
\]

Risk-neutrality implies that consumers’ expected utility is not affected by the payment scheme - spot or insurance sales - so long as insurance premiums are actuarially fair. I will show however that the different payment schemes alter providers’ strategic considerations.

4 Equilibrium

4.1 Simultaneous Entry

The conventional utility function (1) implies the well-known maximal-differentiation principle derived by d’Aspremont et al. (1979): providers locate at the two ends of the preference line sharing

\(^{14}\)Except that the literature typically assumes consumers know their specific ex-post medical need before getting sick, implying deterministic value of \(x\). See for example Lyon (1999) Brekke et al. (2007b) and Bardey et al. (2012).

\(^{15}\)To assure a fully served market in spot sales equilibrium, and that all product utilizations are beneficial to consumers (at zero price) and socially desired.

\(^{16}\)One can equivalently think of a public insurer that maximizes consumers’ utility, by choosing which technologies to include under coverage given their prices.
the market equally. Each provider charges $p_s^* = \theta L^2$ to earn a profit $R_s^* = \frac{\pi \theta L^2}{2}$, with total health spending of $\pi \theta L^2$. Hence consumers’ expected utility is\textsuperscript{17}

$$E\{u\} = v - \frac{13}{12} \pi \theta L^2 \tag{2}$$

Suppose now, that under maximal differentiation seller 2 switches to insurance-sales. If consumers buy this insurance for the premium $o p_2$ they could utilize product 2 at no additional cost when sick. Therefore they will buy product 2 on the spot market only for the range of medical needs $\bar{x} < \frac{L}{2} - \frac{p_1}{\theta L}$. Thus, if product 2 is sold through insurance for $p_1 = p_s^* = \theta L^2$ the spot demand for product 1 goes down to zero, and consumers expected utility becomes:

$$E(u) = v - \frac{\pi \theta}{L} \int_0^L x^2 dx - o p_2 = v - \frac{\pi \theta L^2}{3} - o p_2 \tag{2a}$$

**Lemma 1** There is no equilibrium in spot sales.

**Proof.** equalizing (2) and (2a) yields $o p_2 = \frac{3 \pi \theta L^2}{4} > R_s^*$. Hence unilateral deviation from spot price to insurance sales is profitable for each provider. \hfill \blacksquare

Recall that $o p_2$ is the option value of product 2 when product 1 is sold at the spot price $p_1 = p_s^* = \theta L^2$. This the highest premium consumers will pay for getting product 2 under (full) insurance coverage. This premium implies that in order to motivate consumers buying insurance for product 2 its per unit price should be reduced from $p_2 = p_s^* = \theta L^2$ at least to $p_2 = \frac{3 \theta L^2}{4}$. However, this lower price will be paid for all sick consumers (of measure $\pi$) through insurance. Hence overall profit is higher. Note that provider 2 could reduce the price down to $p_2 = \frac{\theta L^2}{2}$ to maintain the spot-market equilibrium profit.\textsuperscript{18}

Although spot-sales equilibrium does not sustain when insurance sales are available, I will keep considering the spot-competition outcomes (absent insurance) for later comparisons.

**Lemma 2** There is no hybrid equilibrium under which one technology is sold through insurance and the other is sold on spot market. Proof is in the Appendix.

The proof for Lemma 2 (see in the Appendix) shows that provider 2 would to choose locating close to 1, whereas provider 1 would try to get away from 2. Hence, equilibrium does not exist. Hence, only equilibrium with both products sold through insurance may exist. I proceed now to prove the existence of this equilibrium and characterizing it.

\textsuperscript{17} $E\{U\} = v - \pi \left( \frac{\theta}{L} \int_0^{L/2} (x^2) \, dx - p^* \right)$

\textsuperscript{18} Note also that if consumers face specific medical risk, provision through insurance does not affect equilibrium prices: Seller 2 cannot gain from switching to insurance sales because the median consumer will still be indifferent between getting technology 2 through insurance and getting technology 1 on the spot market the same price.
Suppose that both providers are selling through insurance by setting option-price for their products given their locations. Define consumers expected utility from having one option only - $O_i$, with $i = \{1, 2\}$, and consumer’s expected utility from buying both options $O_{1+2}$.

Note that so long as the representative consumer buys buys option for only one product, providers are engaged in a tough price competition a-la Bertrand: by slightly cutting down each other’s option-price, each provider can steal the entire market. Thus, equilibrium can be reached only when consumers buy options for both products. Then, both providers have no incentive to further cut prices down. This intuition is formally stated in the following Theorem.

**Theorem 1** Insurance sales equilibrium must satisfy $O_{1+2} = O_2 = O_1$.

**Proof.** $\forall O_i \geq O_j > O_{1+2}, \exists \ op_j' \equiv \ op_i - \epsilon$, where $\epsilon \rightarrow 0$, such that $O_j \geq O_i > O_{1+2}$. Thus in equilibrium it must be that $O_{1+2} \geq O_1$ and $O_{1+2} \geq O_2$. Profit maximization implies that in equilibrium providers will set prices as to satisfy both conditions with equality. Thus, in insurance sales equilibrium consumers are indifferent between having either product or both products under insurance. ■

Applying Theorem (1) I obtain the following equilibrium condition for the option value of product 2 given the locations chosen by both providers and the option price of product 1:

$$O_1 \leq O_{1+2} \implies \frac{\pi}{L} \int_0^L \left[ V - \theta (x - y_1)^2 \right] dx - \opi \leq \left( \frac{y_1 + y_2}{2} \right)^{\frac{3}{2}} \left[ V - \theta (y_2 - y_1)^2 \right] dx - \opi - \op_2$$

Rearranging and elaborating (3) I obtain the following explicit expression for the option value of product 2:

$$\opi \leq \frac{\pi \theta}{3L} \left( (L - y_1)^3 - (L - y_2)^3 - 2 \left( \frac{y_2 - y_1}{2} \right)^3 \right)$$

In the first stage, provider 2 chooses optimal location as to maximize the option value (3a) implying the following first order condition:

$$y_2^* = \frac{2L + y_1}{3}$$

Due to the symmetry of the problem the F.O.C condition (4) implies efficient horizontal differentiation: $y_2^* = \frac{3L}{4}$, $y_1^* = \frac{L}{4}$. Under these locations, equilibrium option prices (and profits) are: $\opi^* = \frac{\pi \theta L^2}{8}$. The latter results are summarized in the following proposition$^{19}$.

**Proposition 1** The simultaneous game has unique equilibrium in which both providers are selling through insurance, differentiation is efficient, and prices are lower compared spot-price competition.

$^{19}$The efficient locations minimize expected mismatch costs - see d’Aspremont et al. (1979).
Proof. To complete the proof I show that given the chosen locations providers cannot benefit from switching to spot market sales. If provider 1 switches to spot sales while 2 is selling through insurance, she faces the demand: 
\[ \frac{d}{L} = \frac{L}{2} - \frac{p_1}{\theta L} \]. Given this demand optimal price is 
\[ p_1^* = \frac{\theta L^2}{4} \], and the corresponding profit is 
\[ R_1 = \frac{\pi \theta L^2}{16} \], which is smaller than the one under insurance-sales competition.

Consumers’ expected utility under insurance sales is 
\[ E\{U\} \approx V - \pi \theta L^2 \theta 0.305 \], that is higher than under spot-price competition (i.e. if insurance sales are not available). In light of these novel results I turn to explore the implication of insurance-sales to market outcomes under sequential entry.

4.2 Sequential Entry

Suppose now that provider 1 has the opportunity to choose location first. Next, provider 2 enters the market by choosing location and then they both compete in prices. Such a Stakelberg-Location competition in spot prices was solved by Tabuchi and Thisse (1995). They showed that when location is bounded within the preference interval \( L \), the maximum differentiation principal applies\(^{20}\). That is, under spot-price competition the timing of entry does not affect equilibrium.

However, I already showed that when insurance sales are available there is no spot-price equilibrium under maximal differentiation. It can be verified that this result holds also for lower horizontal differentiation. Hence, I turn now to solve the option price equilibrium. Assume, without loss of generality, that in equilibrium: \( y_2 > y_1 \). For given locations and option price set by the leader, the value of a second option should still satisfy equation (3), and thus the optimal location for seller 2 is still given by equation (4): 
\[ y_2^* = \frac{2L + y_1}{3} \]. Then however, the Theorem defines the following equilibrium option-value for the leader, accounting for the follower’s location choice

\[
\frac{\pi}{L} \left\{ \frac{2L + y_1}{3} \sum_{0}^{L} \left[ V - \theta (x - y_1)^2 \right] dx + \int_{\frac{2L + y_1}{3}}^{L} \left[ V - \theta \left( x - \frac{2L + y_1}{3} \right)^2 \right] dx \right\} - op_1 - op_2 \leq (5)
\]

\[
\leq \frac{\pi}{L} \int_{0}^{L} \left[ V - \theta \left( x - \frac{2L + y_1}{3} \right)^2 \right] dx - op_2
\]

Rearranging and elaborating condition (5) I write its following explicit form

\[
\frac{\pi \theta}{3L} \left[ \left( \frac{2L + y_1}{3} \right)^3 - 2 \left( \frac{L - y_1}{3} \right)^3 - y_1^3 \right] \geq op_1 \tag{5a}
\]

\(^{20}\)Nonetheless, for sequential entry with unbounded location they show that the leader locates in the middle of preferences range, and the follower locates outside the range. Anderson (1987) solved a similar game allowing also for sequential price setting. However, as noted by Tabuchi and Thisse (1995) prices are usually considered flexible and thus not likely to be effectively pre-set.
Maximizing the option value (5a) with respect to \( y_1 \), one gets that in equilibrium the leader locates at the middle of the preference range, and the follower locates at five sixth of the range correspondingly:

\[
y_1^* = \frac{L}{2}, y_2^* = \frac{5L}{6} \tag{6}
\]

Applying (6) to (3a) and (5a) yields the corresponding option values:

\[
op_1^* = \frac{4\pi \theta L^2}{27}, \; \nop_2^* = \frac{\pi \theta L^2}{27} \tag{6a}
\]

Note that the leader’s option price in (6a) is greater than the one obtained under simultaneous moves, and the follower’s is smaller. Under the locations and prices given in (6)-(6a) consumer expected utility is \( E \{ U \} = V - \pi \theta L^2 \left( \frac{25}{108} \right) \), which is greater than under simultaneous entry - for both spot and option price competition. Proposition 2 concludes the latter results.

**Proposition 2** Insurance-sales equilibrium with sequential entry is defined by (6)-(6a), implying asymmetric locations, first-mover gains, and higher expected utility to consumers compared with simultaneous moves equilibrium.

**Proof.** To complete the proof I show that under (6) unilateral switch to spot sales is not beneficial. The demand faced by each seller when switching to spot-price sales is given by:

\[
\bar{x}_1 = \frac{\pi}{L} \left( \frac{2L}{3} - \frac{3\eta_1}{2\pi L} \right), \; \bar{x}_2 = \frac{2L}{3} + \frac{3\eta_2}{2\pi L}.
\]

Given these demands the optimal spot-prices that maximize profit for each seller are:

\[
p_1^* = \frac{2\theta L^2}{9}, \; p_2^* = \frac{\theta L^2}{9},
\]

and the corresponding expected profits are:

\[
R_1^* = \frac{2\pi \theta L^2}{27} < \nop_1^*, \; R_2^* = \frac{\pi \theta L^2}{54} < \nop_2^*.
\]

That is deviation from insurance sales prices (6a) is not beneficial.

**5 Quality choice – Vertical competition**

Here I add vertical differentiation through product-quality choice to the analysis. Denoting \( q_i \) the quality of each technology I conventionally modify the expected utility function (1):

\[
E \{ U \} = (1 - \pi) V + \frac{\pi}{L} \int_0^L \left[ V + q_i - p_i - \theta (y_i - x) \right]^2 dx \tag{7}
\]

I assume the provision of quality is subject to quadratic cost function that is fixed with respect to output level:

\[
C(q_i) = \frac{\mu}{2} q_i^2 \tag{8}
\]

I also assume \( \theta L^2 > \frac{\pi}{\mu} \) to ensure non-negative profit under insurance-sales competition. I solve a two-stage game in simultaneous moves. Location and quality are chosen in the first stage, and in

\[\text{See for examples:} \; \text{Lyon (1999) and Katz (2011) and Bardey et al. (2012).}\]

\[\text{Both Brekke et al. (2007b) and Bardey et al. (2012) quadratic cost function, but the latter assume also that marginal cost of output production is increasing with quality.}\]
the second stage price competition prevails. The equilibrium under insurance sales competition is still subject to the Theorem presented in section 4. Accommodated for utility from quality, condition (3) becomes:

$$\int_0^L \left[ V + q_1 - \theta (x - y_1)^2 \right] dx - op_1$$

(9)

$$\leq \frac{\pi}{L} \left\{ \int_0^{\bar{x}} \left[ V + q_1 - \theta (x - y_1)^2 \right] dx + \int_{\bar{x}}^L \left[ V + q_2 - \theta (y_2 - x)^2 \right] dx \right\} - op_1 - op_2$$

Where $\bar{x} = \frac{1}{2} \left[ \frac{q_1 - q_2}{y_2 - y_1} + y_1 + y_2 \right]$. Simplifying (9) and incorporating quality cost I obtain the profit function for seller 1:

$$\frac{\pi}{L} \left[ (q_1 - q_2) \bar{x} - \frac{\theta (y_1 - y_2)^3}{3} - \frac{\theta y_1^3}{3} + \frac{\theta (y_2 - y_1)^3}{3} + \frac{\theta y_2^3}{3} \right] - \frac{\mu}{2} q_1^2$$

(10)

**Lemma 3** Under equal qualities equilibrium locations and option prices are as in proposition 1: $y_1^* = \frac{1}{4}$, $y_2^* = \frac{3}{4}$, $op^* = \frac{\pi \theta L^2}{8}$.

**Proof.** for $q_1 = q_2$, the first addend in (10) coincides with (3). Hence the derived equilibrium locations and option prices also coincide.

Under the locations in Lemma 3 equation (10) becomes:

$$\frac{\pi}{L} \left[ (q_1 - q_2) \left( \frac{q_1 - q_2}{\theta L} + \frac{L}{2} \right) - \frac{\theta (\frac{q_1 - q_2}{\theta L} + \frac{L}{4})^3}{3} - \frac{\theta \left( \frac{L}{3} \right)^3}{3} + \frac{\theta \left( \frac{q_1 - q_2}{\theta L} - \frac{L}{4} \right)^3}{3} + \frac{\theta \left( \frac{L}{4} \right)^3}{3} \right] - \frac{\mu}{2} q_1^2$$

(11)

Differentiating (11) for $q_1$ I obtain the first order condition for optimal quality:

$$q_1^* = \frac{\pi}{2\mu}$$

(12)

**Lemma 4** Under the efficient locations quality choices are equal and efficient.

**Proof.** Condition (12) implies that under symmetric locations $q_1^*, q_2^* = \frac{\pi}{2\mu}$. Solving

$$Max \quad q_i : \int_0^{\frac{L}{2}} \left[ V + q_i - \theta \left( x - \frac{L}{4} \right)^2 \right] dx - \frac{\mu}{2} q_i^2$$

proves that equilibrium qualities are efficient for $y_1^* = \frac{1}{4}$, $y_2^* = \frac{3}{4}$, as they maximize consumers expected utility net of quality cost.

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23 One can show that under spot-price competition there is no symmetric equilibrium in pure strategies when location and quality are simultaneously chosen. Nonetheless, in the three stage game “location-quality-price” principle of maximum differentiation still applies in this setup: See Economides (1989) and Piga and Poyago-Theotoky (2005) in the case of zero R&D spillovers.
Here again providers are maximizing the option value of their own product by maximizing the joint value of both products, thereby choosing locations and qualities that are socially efficient. Proposition 3 summarizes the latter results.

**Proposition 3**  There exists unique symmetric equilibrium with efficient locations and quality choices, and option prices as defined in proposition 1. Proof is in lemmas 3-4.

6 Conclusions

I have studied Duopolistic competition in a market for differentiated medical products. The novel results stem from incorporating consumers’ uncertainty regarding their specific future medical need, and allowing perfectly competitive insurers to price and sell each product separately.

I showed how the lack of *ex-ante* differentiation in medical needs intensifies competition between medical providers under insurance sales, compared with spot price competition. Consequently, under insurance-sales prices are lower and investment in quality is higher, implying lower profitability. Yet, each provider unilaterally opts into insurance sales.

In equilibrium horizontal differentiation and quality provision are efficient, and prices are low enough to motivate consumers buy both options. Sequential entry gives rise to asymmetric equilibrium with gains to the leader and to consumers, in contrast to spot price competition that is neutral to entry timing.

The results imply that insurance sales can promote efficiency in medical care markets if consumers face indeterminate medical risk. Furthermore, in comparison to Lyon (1999) and Katz (2011) the results imply that vertical integration cannot be welfare improving if insurance markets are indeed perfectly competitive. That is if insurer can price and sale separated policies for each medical product.
Appendix: proof for Lemma 2

Under general locations, given that provider 2 is selling through insurance, spot demand for technology 1 is given by

\[ V - \sigma_2 (\bar{x} - y_2)^2 < V - \sigma_2 - p_1 - \theta (\bar{x} - y_2)^2 \implies \bar{x} = \frac{y_1 + y_2}{2} - \frac{p_1}{\theta_2 (y_2 - y_1)} \]

Given this demand, the optimal spot price is \( p_1^* = \frac{\theta(y_2^2 - y_1^2)}{2} \), and actual market share is \( x = \frac{y_1 + y_2}{4} \), for \( y_2 > y_1 \). Therefore, optimal location for provider 1 is \( y_1^* = \frac{y_2}{2} \) for \( y_2 > \frac{L}{2} \). Given the locations of both providers and the corresponding spot price set by provider 1, The option value \( \sigma_2 \) is defined by the increase in expected utility due to all \( ex-post \) utilizations of product 2, compared with relying on spot market utilization of product 1 only:

\[ \sigma_2 \leq \frac{\pi}{L} \int_{\frac{y_1 + y_2}{4}}^{L} \left[ V - \theta (y_2 - x)^2 \right] dx = \frac{\pi}{L} \left[ V \left( L - \frac{y_2 - y_1}{4} \right) - \theta (L - y_2)^3 - \theta \left( \frac{3y_2 - y_1}{4} \right)^3 \right] \]

Maximizing the option value (2) with respect to \( y_2 \) I derive the first order condition: \((L - y_2^*)^2 - \frac{3}{4} \left( \frac{3y_2^* - y_1}{4} \right)^2 = \frac{V}{46} \). Plugging in \( y_1^* = \frac{y_2}{2} \) I obtain the optimal location for provider 2: \( y_2^* = \frac{3}{4} L - \sqrt{\frac{3L^2}{4} + \frac{3V}{86}} \). However, the fully-served-market assumption \( 4V r > L^2 \) implies a corner solution \( y_2^* < 0 \). This violates the best response function \( y_1^* = \frac{y_2}{3} \), which is valid only for \( y_2^* > \frac{L}{2} \). Q.E.D.

References


