Platform Competition with Endogenous Homing

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April 13, 2015

Abstract

We consider two-sided markets in which consumers and firms endogenously determine whether they single-home (patronize only one platform), or multi-home (join competing platforms). While platform competition yields a unique pricing equilibrium there are potentially three distinct equilibrium allocations of consumers and firms. An allocation in which all consumers single-home while firms multi-home always exists (mirroring smartphones). The second allocation has a mix of multi-homing and single-homing on both sides of the market (akin to game consoles). In the third allocation firms single-home and some consumers multi-home (as happens with rideshare services). Competition leads to lower prices and greater market access, but results in cost duplication and disaggregates network effects compared to a monopoly platform. Moreover, cross-subsidization that increases total welfare only takes place in monopoly. Thus, which market structure results in more welfare depends on the interplay of these factors. Lastly, endogenous multi-homing can prevent markets from tipping, even when there are exclusive contracts or a platform has favorable beliefs or is considered focal.

Keywords: two-sided markets, platforms, platform competition, multi-homing, single-homing, endogenous homing decisions, network effects, smartphones, video games and game consoles, rideshare services.


∗We thank John Asker, Irina Baye, Daniel Belton, Emilio Calvano, Jay Pil Choi, Andrei Hagiu, Andres Hervas-Drane, Justin Johnson, Byung-Cheol Kim, Tobias Klein, Marius Schwartz, Liad Wagman, Jay Wilson, Lei Xu, and the audiences at the Fall 2013 Midwest Theory Meetings, the 12th Annual International Industrial Organization Conference, the Fifth Annual Conference on Internet Search and Innovation, the 2015 DC IO day and the Royal Economics Society 2015 Conference for their helpful comments.

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1 Introduction

Video game consoles and smartphones are two platforms that have grown in importance in many people’s lives: Two thirds of U.S. households own a video game console, and the average time spent by gamers on their consoles is eight hours a week—the equivalent of a full work day. While this is a considerable amount of time, it pales in comparison to college students who spend as much as eight to ten hours on their phones every day—amounting to half their waking hours. Nearly 85 percent of 18–29 year olds in the U.S. have smartphones, and Nielson data show a 65 percent increase in time spent using apps by Android and iPhone users over the last two years, with 18–44 year olds using close to thirty different apps each month. These trends are not confined to the U.S., they are present in Europe and Asia as well.

The market structures in which platforms operate vary considerably in terms of the participation decisions that people make. When several platforms offer competing services, participants may join only one platform (called single-homing)—e.g., most consumers own only one smartphone—or they may patronize several platforms (called multi-homing)—e.g., many app developers make their apps available across competing smartphones.

We consider platform competition in which consumers and firms endogenously choose which and how many platforms to join. We show that in equilibrium different allocations of consumers and firms emerge, mirroring the configurations found on many platforms, including those for smartphones and game consoles. We further identify under which circumstances a monopoly platform generates higher welfare than competing platforms.

The literature on platforms can be traced back to work on markets where network externalities are prominent. In the case of platforms, network effects carry over across the

\footnote{The data on gaming come from ESRP (2010), those on college students’ phone usage from Roberts et al. (2014), and the data on prevalence of smartphones at Smith (2012). The Nielson data is at Nielsen (2014a,b).}
platform from one side of the market to the other; see, e.g., Evans (2003), Ellison and Fudenberg (2003), Rochet and Tirole (2003, 2006), Armstrong (2006); and sometimes also within one side of the platform, e.g., Deltas and Jeitschko (2007). While much of the literature considers monopoly platforms, competition between platforms is recognized to be an important characteristic that shapes these markets; and this is a necessary focus in order to understand endogenous homing decisions.

As agents choose which platforms to join, the availability of agents on the other side of the platform is critical in determining the equilibrium and potential coordination issues across the two sides of the platform can arise. Caillaud and Jullien (2003) assume that with platform competition coordination favors the incumbent platform; otherwise platforms may fail to gain a critical mass, i.e. “fail to launch.” They argue this solves the “chicken and egg” problem of each side’s action depending on the other side’s action. Hagiu (2006) shows the chicken and egg problem does not occur when sides join platforms sequentially; and Jullien (2011) investigates this further over a broader class of multi-sided markets. Ambrus and Argenziano (2009) show how prices can endogenize heterogeneity and steer agents to asymmetric allocation configurations; and Lee (2013) investigates the video game market, and shows that Xbox was able to enter the video game market because exclusive contracts with game developers allowed Microsoft to overcome the coordination issue. Moreover, the role of beliefs and information play an important role in determining the equilibria as examined by Hagiu and Halaburda (2014), who consider ‘passive’ price expectations on one side in contrast to complete information about prices on the second side, and Gabszewicz and Wauthy (2014) who also consider active and passive beliefs in determining platform allocations, or Halaburda and Yehezkel (2013), who show how multi-homing alleviates coordination issues tied to asymmetric information.2

2 In normative analyses Weyl (2010) and White and Weyl (2013) fully mitigate coordination issues through insulating tariffs in which prices are contingent upon participation, thus resolving failure-to-launch and multiple equilibrium concerns.
These models of platform competition generally fix exogenously whether agents single-home or multi-home. However, a common feature of many of these markets is that there are a mix of single-homers and multi-homers. In order to understand how this affects competition, agents’ allocation decisions must be derived endogenously as part of the equilibrium.

One of the early papers that allows for endogenous homing is Rochet and Tirole (2003), where buyers and sellers engage in a matched transaction that takes place on a platform. The model is best illustrated by the credit card market: it is assumed that card-issuers only charge per transaction and do not charge card-users or merchants any membership fees, so all agents can costlessly multi-home. However, because card-users choose which card to use when they make a purchase, merchants may wish to single-home in order to limit the customers’ options of which card to use. In contrast to per-transaction fees, many platform markets—including smartphones and gaming systems—are characterized by access or membership fees. Although we allow for both usage and stand-alone membership benefits, we consider platforms that compete by setting membership/access fees, as this relates more closely to the markets that we are concerned with.

There is also a nascent literature on multi-homing in media markets. There the focus is on determining the pricing structure on the advertising-side of the platform. Thus, in Anderson et al. (2013) consumers are not charged to join a platform, so the only cost is a nuisance cost of advertising. However, as consumers do not observe the prices platforms charge to advertisers, platforms cannot affect consumer participation. In Athey et al. (2014) the focus is on endogenous homing on the ad-side, while assuming that consumer allocations are exogenously fixed; and in a related model Ambrus et al. (2014) allow platform pricing to affect consumer participation; however, participation on any given platform does not affect demand on other platforms, so there is no competition between platforms for consumers. In contrast to the studies of media markets, we explicitly model the competition that takes place between platforms to attract agents on both sides of the market and we allow agents
from both sides to endogenously make their homing decisions.

In allowing for endogenous homing decisions, we find a unique pricing equilibrium that emerges in platform competition. However, even with unique prices, the allocation equilibria of firms and consumers need not be unique. Interestingly, though, the case where agents on both sides of the market choose to single-home and the case where all agents multi-home do not actually arise in equilibrium, even though the former is a constellation that is frequently postulated in the literature. Instead, there are as many as three possible types of equilibrium allocation configurations, each reflecting a commonly found market structure.

The first allocation equilibrium—which can always arise in equilibrium—has all consumers single-homing, whereas all firms multi-home. This is the allocation generally observed in the market for smartphones. Indeed, Bresnahan et al. (2014) find that the practice of multi-homing by app producers—that is their simultaneous presence on competing platforms—insures against a tipping in the market that would concentrate all economic activity on a single platform.

The second equilibrium allocation is one in which there is a mix of single-homing and multi-homing on both sides. This is the division found in the market for game consoles (Lee 2013). Many consumers have only one system, but others will buy competing platforms; and while some games are available only on one system, others can be purchased for multiple machines.

In the third equilibrium allocation all firms single-home, whereas some or all consumers multi-home. An example of this is found in ride-sharing services such as Uber and Lyft or the nascent restaurant reservations market with opentable.com and the anticipated launch of Yelp’s seatme. Here firms (drivers or restaurants) are members of one system, whereas consumers either have a preferred platform and single-home or they make use of both platforms and multi-home.

When comparing welfare between competing platforms and a monopoly platform, we find
that lower prices and stronger platform differentiation favors competition, whereas more concentrated network effects and potential cost savings from not having to make apps compatible across multiple platforms tend to favor a monopoly platform. In addition to these factors, we find that when firms’ presence on the platform greatly benefit consumers, then a monopoly platform may increase welfare by subsidizing firm entry. Due to non-appropriability if firms multi-home in competition, competing platforms forgo such welfare enhancing investments, which tends to also favor the monopoly platform market in terms of total welfare.

Given the interplay of these factors, we show that a monopoly platform can generate more welfare than that obtained in any of the three competitive equilibrium allocations. Conversely, however, any of the three competitive allocations may also result in competition leading to greater welfare.

Lastly we sketch how entry into the platform market or declining marginal network effects impact welfare; how exclusive deals can lead to asymmetric equilibrium configurations or even tipping of the market to monopoly; and how endogenous multi-homing can prevent a market from tipping even if there is a focal platform.

2 The Model

Two groups of agents can benefit from interaction, but require an intermediary in order to do so. The benefits from the interaction to an agent in one group depends on the number of agents of the other group that are made available through the intermediary. This intermediary—the platform—charges agents in each group a price to participate on the platform and in exchange brings these groups together. We consider two platforms, indexed by $X \in \{A, B\}$. 
Platforms

Agents on each side of the platform are described by continuous variables. Agents on Side 1 are consumers or buyers, and agents on Side 2 are firms or sellers. The number of consumers that join Platform $X$ is $n_1^X \in [0, N_1]$, and the number of firms on Platform $X$ is $n_2^X \in [0, N_2]$.

The cost to the platform of accommodating an agent on side $i \in \{1, 2\}$ who joins the platform is $f_i \geq 0$, and there are no fixed costs. Platform $X$ has profits of

$$
\Pi^X = n_1^X (p_1^X - f_1) + n_2^X (p_2^X - f_2),
$$

where $p_i^X$ is the price that platform $X$ charges to the agents on side $i$.

Once we discuss consumers and firms it will be clear that from the agents’ perspectives the only differences between the two platforms are solely due to either different prices being charged, $p_i^A \lesssim p_i^B$, or due to different participation rates on the opposing side, $n_i^A \lesssim n_i^B$, which affect the attractiveness of a platform.

Side 1: Consumers

Consumers on Side 1 are indexed by $\tau \in [0, N_1]$. The utility for a consumer of type $\tau$ from joining Platform $X$ is

$$
u_1^X(\tau) = v + \alpha_1(\tau) \cdot n_2^X - p_1^X.
$$

Here $v \geq 0$ is the membership value every consumer receives from joining the platform. This is the stand alone utility of being a member of the platform that one gets even if no firms join the platform. Note that it is possible for $v = 0$, but for smartphones and video game consoles $v > 0$. For smartphones $v$ is the utility from using a smartphone as a phone, including the preloaded features, and for video game consoles $v$ is the utility from using the console to watch Blu-ray discs. Consumers are homogeneous in their membership benefit to
the platform; so \( v \) does not depend on consumer type \( \tau \); and the stand-alone value of joining a platform is the same regardless of which platform is joined.

Consumers are heterogeneous in their marginal benefit from firms. The network effect or the marginal benefit to a consumer of type \( \tau \) for an additional firm on the platform is constant and given by \( \alpha_1(\tau) \); and the number of firms that join the platform is \( n_2^X \). We focus on the case when network effects are positive so \( \alpha_1(\tau) \geq 0 \) for all \( \tau \), where \( \alpha_1(\cdot) \) is a decreasing, twice continuously differentiable function. Since \( \alpha_1(\tau) \) is decreasing, it orders consumers by their marginal benefits. Consumers whose type \( \tau \) is close to zero have marginal benefits that are high relative to those consumers whose type is located far from zero.

The platform knows \( v \) and \( \alpha_1(\cdot) \) but cannot distinguish the individual values for each consumer \( \tau \). Thus, it cannot price discriminate between consumers, so the price or membership fee that consumers pay the platform is a uniform price given by \( p_1^X \).

With there being two platforms in the market, consumers and firms can either join a single platform (single-home) or join multiple platforms (multi-home). A consumer who multi-homes has utility

\[
    u_{1AB}(\tau) = (1 + \delta)v + \alpha_1(\tau) \cdot N_2 - p_A^1 - p_B^1. 
\]  

(3)

Notice that if a consumer participates on two platforms the intrinsic benefit from membership to the second platform diminishes by \( \delta \in [0, 1] \) so that the total stand-alone membership benefit from the two platforms is \( (1 + \delta)v \). If \( \delta = 0 \), then there is no additional membership benefit from joining the second platform, and when \( \delta = 1 \) the membership benefit is unaffected by being a member of another platform.\(^3\)

Apart from the positive membership value of being on a second platform, the main gain

\(^3\)One may also consider the possibility that owning a second platforms is tedious so that \( \delta < 0 \), but this doesn’t impact the main results; and depreciation in network benefits, \( \alpha_1 \), is also a possibility, see, e.g., Ambrus et al. (2014).
to joining a second platform is access to additional firms. Letting $n_m^2$ denote the number of multi-homing firms, a consumer that multi-homes has access to $N_2 := n_m^A + n_m^B - n_m^m$ distinct firms: these are all the firms that join at least one platform. The above utility function implies a multi-homing firm provides only a one-time gain to a consumer that multi-homes. Having a firm available on both platforms to which the consumer has access provides no added benefit.

**Side 2: Firms**

On the other side of the platform, Side 2, are firms that are indexed by $\theta \in [0, N_2]$. A firm’s payoff from joining Platform $X$ is

$$u^X_2(\theta) = \alpha_2 \cdot n_1^X - c\theta - p^X_2.$$  

(4)

The marginal benefit firms receive from an additional consumer on the platform is $\alpha_2$—which is the same for all firms, so firms’ marginal benefits for an additional consumer are homogeneous across firm type. The logic here is that an additional consumer will (in expectation) shift the demand curve for a firm’s app upward in the same way for all firms. The assumption we are making here is that each consumer sees firm products—their app, or game—as homogeneous, but consumers differ in their preferences, resulting in different willingness to purchase apps and games.

Firms incur a cost of $c > 0$ to join the platform. This cost reflects development and synchronization costs associated with programming and formatting their product to fit the

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4Note that we abstract from the transactions costs that may occur between consumers and firms. Thus, the benefits that accrue to consumers from interacting with firms—apps and games—can be viewed as being net of prices paid to firms. [Deltas and Jentschko (2007)](Deltas2007) consider auctioneers setting optimal reserves on an auction hosting platform, and [Reisinger (2014)](Reisinger2014) generalizes [Armstrong (2006)](Armstrong2006) and considers tariff-pricing with heterogeneous trading. See [Tremblay (2015)](Tremblay2015) for a more detailed analysis of pricing across the platform in a framework that is more similar to our current setting.
platform. Firms are heterogeneous with respect to their development and synchronization costs. Firms with type $\theta$ close to zero have lower costs compared to those with higher $\theta$.\footnote{The model is isomorphic to assuming there is a limited number of app developers with increasing costs of developing apps so the equilibrium number of apps brought to market is endogenous.}

The platform knows the firm’s profit structure but cannot identify firms individually; hence, it cannot price discriminate between firms and the price or membership fee the firms pay to the platform is given by $p^X_2$ for all firms.

A firm that multi-homes has payoff

$$u^A_B(\theta) = \alpha_2 \cdot N_1 - (1 + \sigma) c \theta - p^A_2 - p^B_2,$$ \hspace{1cm} (5)

where $N_1 := n^A_1 + n^B_1 - n^m_1$ is the number of distinct consumers to which the firm gains access; these are all the consumers that join at least one platform. As noted above, when a firm’s product is available to the multi-homing consumer on both platforms, a consumer will only purchase the product at most once. Therefore a firm only cares about the number of distinct consumers that are available to it through the platforms.

When a firm participates on two platforms its development and synchronization cost for joining the second platform diminishes to $\sigma \in [0, 1]$. Thus, $\sigma$ represents the amount of ‘duplication economies’ that exist when synchronizing an app or game to a second platform. If $\sigma = 1$ then there are no economies of duplication and as $\sigma$ decreases, there exists economies of duplication.\footnote{The relative lack of duplication economies played a role in providing an app for Facebook in the tablet market. For some time the app ‘Friendly for Facebook’ was used by Facebook users because Facebook itself had not developed an app for the tablet. It was rumored that Apple later ‘assisted’ Facebook in developing the official Facebook app.}
Allocation Decisions

By allowing consumers and firms to make homing decisions there are potentially many allocations that can occur for a given set of prices. Agents’ beliefs about the allocation decision on the opposite side of the platform play a critical role in determining their membership decisions. We make a few basic assumptions about agents’ beliefs and allocation decisions that are particularly salient in our context. We then determine all possible equilibrium allocations of consumers and firms, for arbitrary prices, that are consistent with these minimal assumptions.

First, we impose a tie-breaking rule for the case when agents have beliefs such that they are indifferent between joining Platform A or B.

**Assumption 1 (Tie-Breaking Rule).** If, for given beliefs about the allocation on the opposite side and for given prices, an agent (consumer or firm) is indifferent between joining platforms A and B, i.e. \( u^A_i = u^B_i > 0 \), then the agent either multi-homes, or chooses to join one of the platforms with equal probability.

Assumption 1 is made for ease of exposition (it precludes certain allocations, but does not actually affect the final equilibrium constellations). To be sure, the assumption does not rule out a clustering of agents on one platform due to, say, successful ‘viral marketing’ efforts, because the assumption only addresses agent behavior for a given set of prices and beliefs that the agent has. A successful marketing campaign is successful precisely in affecting (i.e., tipping or skewing) beliefs to achieve the desired outcome.

Second, we preclude dis-coordinated allocation configurations in which despite having worse (i.e., higher) prices a platform corners the market on the firm side.

**Assumption 2 (No Dis-Coordination).** If \( p^X_i \leq p^Y_i, \forall i \) then \( n^X_2 \neq 0 \).

7In Section 5 we dispense with this assumption when we consider focal platforms and favorable beliefs.
Assumption 2 states that a platform that offers a price advantage on at least one side and is no worse than its rival in terms of the price it charges on the other side will attract at least some firms. Note in particular, however, that Assumption 2 says nothing about the equilibrium allocation of consumers. And, importantly, it says nothing about consumers or firms for the case that one platform has a lower price on one side, but the rival platform has the lower price on the other side.

The rationale for why the assumption pertains only to a minimum participation of the firms—rather than guaranteeing minimum participation by consumers—is twofold. First, a platform can always attract some consumers when it prices sufficiently low, because the platform offers a stand-alone value to consumers; and second, in the contexts we have in mind it is reasonable to assume that firms are aware of pricing on both sides of the platform, whereas consumers are likely to only observe prices on Side 1. Hence, firms are able to observe any price-advantages regardless of the side on which they are offered.

Lastly, we include the standard equilibrium requirement that all agents’ beliefs about allocations are consistent with equilibrium actions taken by agents on the other side. That is, when an agent makes a participation decision based on an expectation of the number of agents on the other side, then in equilibrium this expectation must coincide with the actual decisions of the agents on the other side so that expectations are correct.

Our basic assumptions are used to characterize the set of all allocations that are possible in equilibrium for any arbitrarily given price constellations. To be sure, these generally do not generate unique equilibrium configurations. However, there is enough structure in order to derive meaningful pricing strategies for the platforms that yield clear equilibrium implications.

[Hagiu and Halaburda (2014)] use this fact to differentiate between information that firms have in contrast to beliefs that consumers have about prices. [Jullien and Pavan (2014)] further investigate responses by platforms to imperfect information within a two-sided market, however their main focus is restricted to single-homing consumers.
3 Equilibrium

The sequence of play is as follows: first the platforms simultaneously (and non-cooperatively) choose consumer and firm prices, $p^X_i$ for $X = A, B$ and $i = 1, 2$. Then consumers and firms simultaneously choose whether and which platforms to join, yielding $n^X_i$ and $N_i = n^A_i + n^B_i - n^m_i$, $i = 1, 2$.

We first investigate the allocation subgame of consumers and firms in joining platforms for given (arbitrary) prices charged by the platforms; and then we determine the equilibria for the entire game by considering price competition between the two platforms, in light of the profits obtained in the allocation subgame equilibrium.\(^\text{[10]}\)

The Allocation Equilibrium for Arbitrary Prices

We first suppose platforms choose symmetric pricing strategies.

**Proposition 1** (Allocations under Symmetric Pricing). If $p^X_i = p^Y_i = p_i$ then $n^X_i = n^Y_i = n_i$.

The set of multi-homing consumers is given by $\tau \in [0, n^m_1]$, and the set of single-homing consumers is given by $\tau \in [n^m_1, N_1]$, with

$$n^m_1 = \alpha_1\left(\frac{p_1 - \delta v}{n_2 - n^m_2}\right) \text{ and } N_1 = \alpha_1\left(\frac{p_1 - v}{n_2}\right). \quad (6)$$

The set of multi-homing firms is given by $\theta \in [0, n^m_2]$, and the set of single-homing firms is given by $\theta \in (n^m_2, N_2]$, with

$$n^m_2 = \min\left\{\frac{\alpha_2 \cdot N_1 - 2p_2}{(1 + \sigma)c}, \frac{\alpha_2 \cdot (n_1 - n^m_1) - p_2}{\sigma c}\right\} \text{ and } N_2 = \min\left\{\frac{\alpha_2 n_1 - p_2}{c}, n^m_2\right\}. \quad (7)$$

\(^{10}\)For simplicity, we focus exclusively on the cases when prices are sufficiently low for at least some participation to exist. Constellations in which a platform prices itself out of the market are easily derived, but are merely a distraction as they do not arise in any of the pricing equilibrium configurations of the entire game. An implication of this is that the total participation of agents on each side across both platforms is positive, $N_i > 0$ for $i = 1, 2$.\(^{12}\)
For each side of the market at least one set is non-empty, and it is possible for both sets to be non-empty; as a result there exist multiple equilibrium allocations.

Proposition 1 says that when platforms set equal prices, the platforms split both sides of the market equally. However, this equal division does not determine the extent to which consumers and firms multi-home in equilibrium. In fact, the allocation of one side of the market depends on the allocation on the other side; and this results in the possibility of multiple equilibrium allocations—depending on parameter values as well as the platform prices chosen.

Consider first consumers. Consumers always obtain an added benefit from joining a second platform, namely $\delta v$. Hence, if prices to consumers are low enough, $p_1 < \delta v$, then all consumers join both platforms: $n_1^m = N_1 = \overline{N}_1$. For consumer prices above this threshold, but still below the stand-alone utility from a single platform membership, $\delta v < p_1 < v$, all consumers will join one platform, $N_1 = \overline{N}_1$; but whether any consumers join a second platform (multi-home) depends on whether firms multi-home. In particular, if the number of multi-homing firms is large ($n_2 - n_2^m$ is small), then consumers have access to many firms when joining the first platform and so the number of multi-homing consumers is small, or even zero. For even higher consumer prices, consumers with large values of $\tau$ even refrain from joining a single platform, $N_1 < \overline{N}_1$.

Unlike consumers, firms do not obtain a stand-alone benefit from joining a platform. However, they experience duplication economies in production when joining a second platform. This implies that a firm will multi-home only when the marginal gain from joining a second platform and the total payoff from being on two platforms are both positive. And hence, the set of multi-homing firms depends on the number of consumers that multi-home. If all consumers multi-home then $n_1^m = n_1$ and no firm will multi-home, unless they are paid

\[\text{Note that since } \alpha(\cdot) \text{ is positive and decreasing, so is } \alpha^{-1}(\cdot) \text{ and therefore when } p_1 - \delta v < 0 \text{ the corner solution obtains in which } n_1^m = N_1 = \overline{N}_1.\]
to do so (which requires $p_2 < 0$). Second, the firms that choose to multi-home instead of single-home are the firms with sufficiently low synchronization costs, $\theta$ close to zero. As the synchronization cost gets larger the marginal cost for joining another platform becomes larger than the marginal gain from having access to additional consumers. Hence, for firms with higher synchronization costs, $\theta$ farther from zero, it becomes too costly to join more than one platform. Thus, a firm is more likely to multi-home if it faces a lower synchronization cost to join a platform.

Note finally that if few consumers multi-home ($n_2^m$ is small) and there are strong duplication economies ($\sigma$ small), then it is possible that no firms single-home and all firms multi-home.

![Figure 1: Homing Decisions by Consumers and Firms According to Type $\tau, \theta$](image)

We now determine the allocations that occur with unequal price constellations.

**Proposition 2** (Allocations with Price-Undercutting). If $p_i^Y \leq p_i^X$ with at least one strict inequality then there exists a unique allocation equilibrium. In this equilibrium $n_i^Y = N_i$, $i = 1, 2$, with $n_1^X = n_1^m > 0$ only when $p_1^X \leq \delta v$ and $n_2^X = n_2^m > 0$ only when $p_2^X < 0$.

Proposition 2 shows that when one platform has better prices (at least one better price, and the other price no worse), then all agents—consumers and firms alike—will join the platform with the price advantage. Whether agents also join the second platform (and, thus, multi-home) depends on the prices on the second platform. Consumers will join the second platform only if the price is below their marginal stand-alone benefit from joining a second platform, $p_1^X < \delta v$, because they already have access to all active firms through the first platform so that any firm presence on the second platform is of no value to consumers. Sim-
ilarly, because firms already have complete market access to all consumers on one platform, they will only join the second platform if they are paid to do so, \( p_2^X < 0 \).

Lastly, consider the case when prices are unequal and neither platform has a clear price advantage.

**Proposition 3** (Allocations under Orthogonal Pricing). If \( p_1^X > p_1^Y \) and \( p_2^X < p_2^Y \) for \( X \neq Y \in \{A, B\} \) then the following are possible equilibrium allocations:

- \( n_Y^1 = N_1 > n_X^1 = n_1^m > 0 \) with \( n_2^X = n_2^Y \),
- \( n_Y^1 = N_1 \) and \( n_X^1 = 0 \), with \( n_2^Y = N_2 \) and \( n_2^m = n_2^X > 0 \) only when \( p_2^X < 0 \). This only exists when \( p_1^X > \delta v \).
- \( n_2^X = N_2 \) and \( n_2^m = n_2^Y > 0 \) only when \( p_2^Y < 0 \) with
  - \( n_X^1 = N_1 \) and \( n_Y^1 = 0 \) when \( p_1^Y > v \),
  - \( n_X^1, n_Y^1 > 0 \) with no multi-homing when \( v \geq p_1^Y > \delta v \), and
  - \( n_Y^1 = N_1 \) with \( n_X^1 > 0 \) multi-homers when \( \delta v \geq p_1^Y \).

When prices are unequal and neither platform has the lower price on both sides of the market there are three possible ways consumers and firms can divide themselves onto the two platforms—these depend on the relative magnitude of prices, but are not mutually exclusive. Thus, in this case it is possible to have multiple equilibrium allocations.

In the first two cases listed in the proposition, all consumers join the platform that has the better consumer price (\( n_Y^1 = N_1 \)). Beyond that, in the first case platforms capture an equal number of firms, which comes about either due to some consumers multi-homing and the difference in the price on the firm side is large, or because firms are being subsidized.

In the second case listed, if the platform with the higher price for consumers doesn’t price low enough to capture consumers seeking the marginal stand-alone benefit of the second
platform, i.e., $p_1^X \not< \delta v$, then no consumers will multi-home, and firms only multi-home when they are subsidized to do so, $p_2^X < 0$.

The third possibility differs markedly from the other two in that all firms congregate on the platform with the higher consumer price. All consumers will join the firms when the platform with the better consumer price is not attractive enough to make the stand-alone value worth capturing, $p_1^X > v$. However, consumers who have little value for apps, will switch to the otherwise empty platform in order to capture the stand-alone value when $v \geq p_1^Y > \delta v$; and when prices are even lower, then all consumers will join this platform—many of whom will also remain members of the other platform and thus multi-home.

**Equilibria of the Pricing Game**

A recurring theme in the allocation configurations was whether a platform sets prices low enough to attract consumers merely for the marginal stand-alone value. This pricing decision often plays a special role in determining whether consumers multi-home. In particular, if a platform sets $p_1^X < \delta v$, then it is sure to capture all consumers—regardless of all other prices and homing decisions. In light of this, when determining the platforms’ pricing decisions it is important to consider the relationship between the cost of providing service to a consumer and the consumer’s marginal stand-alone value for the second platform, i.e., $f_1 \not< \delta v$.

We first suppose that $f_1 < \delta v$. In this case a platform can charge a consumer price of $p_1^X = \delta v > f_1$ and guarantee itself positive profits since consumers will either single-home on platform $X$ or if a consumer is already on platform $Y \neq X$ then they will be willing to multi-home even absent any firms on platform $X$. Hence, both platforms are guaranteed profits and, in equilibrium, all consumers $\tau \in [0,N_1]$ join at least one platform.

**Theorem 1** (Weak Competition; $f_1 < \delta v$). There exists a unique symmetric equilibrium with $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$. All consumers multi-home, $n_1^m = N_1$, and firms
that join a platform single-home on each platform with equal probability, \( n_2^A = n_2^B \), \( n_2^m = 0 \). Platform profits are \( \Pi^A = \Pi^B = N_1(\delta v - f_1) > 0 \).

The case of ‘weak competition’ implies that failure to launch issues are not encountered, since both platforms are able to establish themselves on the consumer side of the market. This occurs because platforms are differentiated from the consumers’ perspectives—there is a positive marginal value from joining a second platform—and so consumers are willing to join a platform even if there are no apps available on that platform.

However, for many products the membership benefit depreciates almost to zero when a consumer multi-homes, \( \delta \approx 0 \). This implies that even for small \( f_1 \) the marginal cost of accommodating an additional consumer on the platform is greater than the additional membership benefit from joining another platform. In the smartphone case, for example, the membership benefit is the ability to make calls and use the phone’s preloaded features. Since most phones have similar pre-loaded features, \( \delta \) is close to zero and any additional benefit from a second phone would not overcome the production cost of an additional phone.

We now consider the case \( f_1 \geq \delta v \), so the cost of attracting a consumer who has already joined the rival platform exceeds the platform’s stand-alone value to the consumer. As a result, platforms compete head-to-head for single-homers, rather than trying to attract multi-homers. We refer to this as ‘strong competition,’ which leads to fierce price-competition resulting in a form of Bertrand Paradox. While this implies a unique price equilibrium, there are potentially three different allocations of consumers and firms in equilibrium.

**Theorem 2** (Strong Competition; \( f_1 \geq \delta v \)). The unique equilibrium prices are \( p_1^A = p_1^B = f_1 \) and \( p_2^A = p_2^B = f_2 \) so that \( \Pi^A = \Pi^B = 0 \).

There exists at least one and possibly as many as three types of equilibrium allocations:

1. All active consumers single-home and all active firms multi-home: \( n_1^m = 0 \), \( n_2^m = N_2 \). This is always an equilibrium.
II. A mix of multi-homing and single-homing consumers with multi-homing and single-homing firms: \( n_i^m \in (0, N_i) \).

III. All active firms single-home and many, potentially all, active consumers multi-home: 
\[ n_2^m = 0, \ n_1^m \in (0, N_1) \].

Allocation II mirrors the two-sided market for smartphones. Almost all consumers single-home, they own only one phone; and almost all firms multi-home, the vast majority of apps are available across all types of smartphones. This is also the ‘competitive bottleneck’ allocation in Armstrong (2006). There, however, the homing decisions are exogenously assumed, rather than endogenously derived. As a result, firms face high prices, whereas in our model—in which platforms compete to attract firms—firms face marginal cost pricing.

Allocation III resembles current allocations seen in many two-sided markets, including those for game consoles: For video game platforms, there exist consumers who multi-home—buying several game consoles—and others that single-home; and there exists game designers whose games are available across platforms, i.e., they multi-home, while others are available on only one system, i.e., they single-home.

For Allocation III \( v = 0 \) is a sufficient condition for existence when \( f_2 > 0 \), whereas when \( f_2 = 0 \) existence requires \( v = 0 \). The allocation is best characterized when considering the sufficient condition. An example of this type of configuration is seen with the ride sharing companies Uber and Lyft. These are platforms that connect drivers (i.e., firms) with passengers seeking transportation (consumers). Drivers offer their services through one ride sharing company (i.e., they single-home); whereas many customers seeking rides use both companies and compare availability and prices (i.e., they multi-home). Since there is no benefit from linking to a ride sharing company that has no drivers \( v = 0 \).

Another example are antique malls with many individual stalls each rented out to individual antiques dealers (i.e., firms), and consumers who visit the mall to browse the individual stalls. Vendors sell their antiques in only one mall (single-home), yet consumers browse at different malls (multi-home). There is no

\[12\text{Another example are antique malls with many individual stalls each rented out to individual antiques dealers (i.e., firms), and consumers who visit the mall to browse the individual stalls. Vendors sell their antiques in only one mall (single-home), yet consumers browse at different malls (multi-home). There is no}\]
It is worth noting that the equilibrium allocation configurations in Theorem 2 are exhaustive; that is, there are no other equilibrium allocations. In particular, while many papers on platform competition assume single-homing, there does not exist an equilibrium in which all active consumers and all active firms single-home. When all consumers single-home, then firms optimally multi-home in order to reach all consumers. Also, there is no equilibrium allocation in which all active firms and consumers multi-home. If all consumers are multi-homing, firms optimally respond by single-homing.

4 Monopoly versus Strong Competition

Does strong competition between two platforms result in higher welfare when compared to a monopoly platform? The answer is not readily apparent. On the one hand competition results in lower prices and additional stand-alone membership benefits to consumers who multi-home. On the other hand, however, competition can increase synchronization costs, as well as destroy network surplus by fragmenting the market. Moreover—as we show in this section—competition may also undermine welfare-increasing cross-subsidization that takes place in the monopoly setting.

Since weak competition leads to all consumers multi-homing due to the positive incremental value of joining another platform, we consider the case of strong competition, and show that even in this case the monopoly equilibrium may welfare-dominate.

To obtain closed form solutions and welfare we assume that $\tau$ is distributed uniformly on $[0, a/b]$, which implies that $\alpha_1(\cdot)$ is linear: $\alpha_1(\tau) = a - b\tau$. The number of potential consumers is then $N_1 = \frac{a}{b}$. We assume that $\theta$ is distributed uniformly, with $N_2$ sufficiently large so that the platform can always attract more app producers. That is, there exists many potential

benefit from going to a vacant antique mall so $v = 0$. Also, Yelp’s seatme restaurant reservation system may enter into competition with opentable.com. In this case, restaurants would work with one or the other system, but patrons could search either.
app producers, many of which end up not developing an app because their synchronization costs are too high. To simplify calculations, we further assume \( v = f_1 \) and \( f_2 = 0 \) (which implies the case of strong competition since \( f_1 = v > \delta v \)).

**Monopoly**

For given prices, the agents’ participation decisions are implied by the marginal agents on both sides being indifferent between participation and opting out, in light of their expectations about the participation decisions on the opposite side of the platform. Thus, on the consumer side \( u_1(\tau = N_1) \equiv 0 \) implies \( p_1 = v + (a - bN_1) \cdot N_2 \) (see 2); and on the firm side \( u_2(\theta = N_2) \equiv 0 \) yields \( p_2 = \alpha_2 \cdot N_1 - cN_2 \) (see 4). We maintain the natural assumptions of consistent beliefs and no coordination failure.

Using these relations between participation and prices in conjunction with the platform’s profit function \( \Pi \), the monopolist’s objective is to choose the implied participation levels, \( N_1 \) and \( N_2 \) to maximize

\[
\Pi^M = N_1(v + (a - bN_1) \cdot N_2 - f_1) + N_2(\alpha_2 \cdot N_1 - cN_2 - f_2).
\]

With \( \alpha_1(\tau) = a - b\tau \), the highest marginal benefit any consumer has (namely a consumer of type \( \tau = 0 \)) for firm participation is \( a \). If the firms’ constant marginal valuation of consumer participation exceeds that of consumers, \( \alpha_2 > a \geq \alpha_1(\tau) \), then firms’ gross willingness to pay (gross of the synchronization costs \( c\theta \)) exceeds that of all consumers. Hence the optimal platform strategy is to attract as many consumers as possible in order to make the platform as valuable as possible to firms. In turn, this allows the platform to extract a larger

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\( ^{13} \)These assumptions are not that critical and they make computations straightforward: In the market for smartphones and video game consoles it would be the case that both the marginal cost to produce the platform and the membership gains consumers receive are positive and approximately equal; and the cost to platforms of adding an additional app or game is nearly costless.
surplus from firms than was the cost of attracting consumers. Hence, whenever \( \alpha_2 \geq a \) a corner solution is obtained in which the platform prices consumer participation such that all consumers join. The highest price to consumers that still attracts all consumers is \( p_1 = v \). Given this price and the implied consumer participation of \( N_1 = a/b = \bar{N}_1 \), the platform maximizes profits with respect to \( N_2 \) with \( p_2 = p_2(N_1 = a/b, N_2) = \alpha_2 b - cN_2 \). This yields

\[
P_{1,MC}^M = v, \quad P_{2,MC}^M = \frac{a\alpha_2}{2b}, \quad \text{and} \quad N_{1,MC}^M = \frac{a}{b} = \bar{N}_1, \quad N_{2,MC}^M = \frac{a\alpha_2}{2bc}; \quad (9)
\]

where \( M_C \) is a mnemonic that denotes the monopoly corner solution (with respect to consumers).

In contrast, when \( \alpha_2 \leq a \), the consumers with the highest marginal benefit from firm participation have a higher willingness to pay than any firm has for consumers. As a result, consumers are charged higher prices and an interior equilibrium emerges, in which some consumers do not join the platform.

The second order conditions hold for this problem and it is straightforward to show that for the interior equilibrium the prices and allocations are

\[
P_{1,MI}^M = v + \frac{1}{16bc}(a + \alpha_2)^2(a - \alpha_2), \quad P_{2,MI}^M = \frac{1}{8b}(a + \alpha_2)(3\alpha_2 - a), \quad \text{and} \quad (11)
\]

\[
N_{1,MI}^M = \frac{1}{2b}(a + \alpha_2), \quad N_{2,MI}^M = \frac{1}{8bc}(a + \alpha_2)^2; \quad (12)
\]

where \( M_I \) denotes the interior monopoly solution.

There are two things worth noting in this equilibrium. First, recall the usual monopoly problem with linear inverse demand \( P = a - bQ \) and marginal cost equal to zero, yielding the monopoly output of \( Q^M = \frac{a}{2b} \). Now notice that \( N_{1,MI}^M > Q^M \), so in equilibrium, a monopoly platform will price to have more consumers than a traditional (one-sided) monopolist—even
when there is no corner solution on the consumer side. This is because the added consumers generate additional surplus on the platform which makes the platform more attractive to firms.

More important is the second observation, namely that prices charged to firms can be negative, \( p^M_2 < 0 \). That is, firms may be subsidized to join the platform. This occurs when consumers with a high willingness to pay for apps (small \( \tau \)) value firm presence on the platform especially high compared to the value of a consumer to the firm, i.e., \( a > 3\alpha_2 \). The monopolist then invests in the attractiveness of the platform to consumers by paying firms to join. This investment is then recouped through higher prices to consumers.

**Welfare Comparison**

Consider now competing platforms. Because we are dealing with the case of strong competition, Theorem 2 holds and so \( p^A_1 = p^B_1 = f_1 = v \geq 0 \) and \( p^A_2 = p^B_2 = f_2 = 0 \). Moreover, from Theorem 2 we know that there can be up to three distinct allocations of consumers and firms in equilibrium.

Allocation I is an equilibrium allocation that exists for all parameter values. All consumers single-home and all firms multi-home: \( n^m_1 = 0 \) and \( n^A_2 = n^B_2 = n^m_2 = N_2 \). Given the prices from Theorem 2 in conjunction with our functional form assumptions, \( n_1 := n^A_1 = n^B_1 = \frac{1}{2}N_1 = \frac{1}{2}a \) and \( n_2 := n^A_2 = n^B_2 = n^m_2 = \frac{\alpha_2}{(1+\sigma)c}b \). From this follows:

**Theorem 3 (Allocation I v. Monopoly).** Whenever \( \alpha_2 \geq a \) all consumers join a platform regardless of the market structure (corner monopoly solution); and there exists \( \sigma^C := \frac{a+2\alpha_2}{3a+2\alpha_2} \in (\frac{3}{5}, 1) \) such that monopoly generates more welfare than competition iff \( \sigma \geq \sigma^C \).

When \( \alpha_2 < a \) competition serves all consumer types, whereas the monopoly limits consumer participation (interior monopoly solution); and yet there exists \( \sigma^I := \frac{64\alpha_2a^2}{5(a+\alpha_2)^3} - 1 \in (-1, 1) \) such that monopoly generates more welfare than competition iff \( \sigma \geq \sigma^I \).
Notice that competition always leads to all consumers joining a platform, $N_{1}^{Al} = N_{1}$, whereas a monopoly excludes some when the willingness to pay is sufficiently large for the consumers with the highest marginal benefits from firm participation (the interior solution), $N_{1}^{MI} < N_{1}$.

Despite the (weakly) greater market coverage on the consumer side when platforms compete, whenever $\sigma > \max\{\sigma^{I}, \sigma^{C}\}$ the monopoly generates higher surplus than competition. The intuition for a superior outcome under monopoly when duplication costs are high is quite straightforward: in the competitive equilibrium of Allocation \[ all firms that join a platform end up multi-homing, and therefore incur the duplication cost of $\sigma c \theta$. Hence, the larger are the duplication costs, the more costly is the competitive solution—so much so that for sufficiently high duplication costs the monopoly platform welfare-dominates; even when it reduces participation on the consumer side.

Notice, however, that it is possible that a monopoly generates greater welfare even when there are no duplication costs ($\sigma = 0$) so there are no additional costs associated with firms joining a second platform. This can happen when the monopoly interior solution obtains $(\alpha_{2} < a)$ and $\sigma^{I} \leq 0$. The reason for this is that whenever $\sigma^{I} \leq 0$ then it must be that $3\alpha_{2} < a^{14}$ and for this case the monopolist subsidizes firm entry (i.e., $p_{2}^{MI} < 0$, see \[11]). The reason for the firm subsidy is to increase the total welfare that the platform generates so that the monopolist can appropriate this through higher prices to consumers.

In contrast, in the competitive equilibrium the welfare-enhancing investment in firm-entry does not take place: a platform that subsidizes firms must make up for the cost of doing so by charging higher consumer prices. However, when $\sigma = 0$, all firms that obtain the investment subsidy will join both platforms; and so the platform that doesn’t make the investment in firm participation reaps the same reward as does the rival platform that does.

\[14\] For $\alpha_{2} < a$ (the interior solution) $\sigma^{I}$ is strictly increasing in $\alpha_{2}$. However, when $\alpha_{2} = \frac{1}{3} a$, $\sigma^{I} > 0$, so whenever $\sigma^{I} \leq 0$, it follows that $\alpha_{2} < \frac{1}{3} a$. 

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subsidize firm entry—thus placing the non-investing platform at a competitive advantage. Therefore welfare-increasing firm subsidies do not occur when platforms compete.\footnote{An obvious implication of this is that having firms enter into exclusive deals with a platform can be welfare enhancing as it solves the non-appropriability problem associated with firm subsidies when $\sigma = 0$, see Section 5.}

Turning to the comparison between monopoly and Allocation II under competition, note from Theorem 2 that a type-II Allocation may not exist. Indeed, it only occurs when network effects are sufficiently strong, viz., $\alpha_2 > \frac{(1-\delta)\eta}{8\delta c a^2}$.\footnote{This follows from Equations 13 and 14 and since $x$ in the proof of Theorem 2 must be in $[0,1]$. Also see the proof of Theorem 4.}

**Theorem 4 (Allocation II v. Monopoly).** When duplication costs are zero ($\sigma = 0$) competition always generates more total surplus; but regardless of whether the monopoly has an interior or a corner solution, there exists mixed allocations such that the welfare from the competitive mixed allocation equilibrium is greater than the welfare with the monopoly platform; even when there are no savings in duplication ($\sigma = 1$).

Thus, when under competition the mixed allocation emerges in equilibrium then sufficiently strong duplication economies (small enough $\sigma$) will assure greater welfare from competition than in monopoly, because firms are able to cheaply multi-home. However, Theorem 4 also makes clear that the converse need not hold. That is, even when it is costly for firms to multi-home in terms production and synchronization costs ($\sigma = 1$), competition can generate greater welfare in the mixed allocation.

Taken together, Theorems 3 and 4 show that when duplication economies are small (large $\sigma$) monopoly is preferred to competition, unless under competition a mixed allocation emerges in which not all firms multi-home. Notice also that for the case of an interior monopoly solution and parameters such that $\sigma^I < 0$, then for sufficiently large duplication effects ($\sigma$ small) competition generates less welfare than monopoly if with competition all consumers single-home, but competition generates more welfare if a mixed-homing equilib-
rium emerges, because multi-homing is not expensive and the lower prices increase network effects.

Consider now Allocation III. To assure existence of this equilibrium, let \( v = f_1 = 0 \). Prices are \( p_1 = 0 \) and \( p_2 = 0 \), all firms single-home, and all consumers multi-home.

**Theorem 5** (Allocation III v. Monopoly). Whenever \( \alpha_2 \geq \frac{1}{4}a \) the competitive equilibrium in which all firms single-home and all consumers multi-home generates greater welfare than the monopoly outcome.

Notice that the welfare comparison between monopoly and competition does not involve duplication economies \( \sigma \), since in either case firms only single-home. Also, when \( \alpha_2 \in (\frac{1}{4}a, \frac{1}{3}a) \), then the monopoly platform subsidizes firms to increase the value of the platform and increase overall welfare, but the lower prices on the consumer side that occur under competition more than offset this so that competition leads to greater welfare.

Lastly, because Allocation III does not involve any multi-homing by firms it is possible that it generates greater welfare than Allocation II. This then implies that welfare can be non-monotone in duplication economies across the degree of competitiveness in the market: In particular, there are parameters under which for small enough \( \sigma \) competition with Allocation II generates the highest welfare, whereas increases in \( \sigma \) lead to the monopoly generating the greatest welfare, only to be dominated by the competitive Allocation III upon further increases in \( \sigma \).

### 5 Extensions

Before concluding we briefly address issues that the model can easily account for. First, we consider entry beyond two platforms, decreasing marginal network effects, and exclusive

\[^{17}\text{This occurs when both equilibrium configurations exist and } \sigma > \frac{\alpha_2}{3\alpha_2 + 4a}.\]
deals; and then we consider other issues addressed in the literature, in particular the case of one platform having an inherent advantage by being “focal.”

**Entry** If additional platforms enter the market, then multi-homing becomes more complex, as agents may join any number of platforms. However, welfare implications are unambiguously worse for entry beyond two platforms for the case of strong competition, where the cost of accommodating another consumer costs more than the marginal value of the second platform, \( f_1 \geq \delta v \): in this case even just two platforms compete prices down to marginal costs, and so there are no additional beneficial price effects from more entry. Moreover, since prices are above the marginal value of additional platforms to consumers, there are no gains through platform differentiation. Hence there is no increased participation in the market on the consumer side. However, if consumers fragment across platforms, then firms’ additional replication costs destroy welfare, and network effects that accrue to firms are also diminished.

The case of weak competition \( (f_1 < \delta v) \) is less clear. In this case competition beyond the duopoly level will also drive prices on the consumer side down, as now there is strong competition for multi-homing consumers. However, it requires a great deal of platform differentiation (large \( \delta \) that doesn’t decrease (too much) upon consumer participation on additional platforms), in order to add to the overall welfare in the market.

**Decreasing marginal network effects** Marginal network effects are assumed to be constant in the base model, i.e., \( \partial^2 u_i / \partial n_j^2 = 0 \). However, in many settings it is likely the case that the value of additional interactions with agents from the other side of the platform decrease in the number of available agents, i.e., \( \partial^2 u_i / \partial n_j^2 < 0 \). For instance, consumers have decreasing marginal benefits from the number of apps available on their smartphone, and similarly the value of additional games declines as enjoying each game requires an investment in time.
Decreasing marginal valuations reduce the total surplus generated in both monopoly and competition. However, in the competitive market there are no price effects, as prices are already competed to the lowest levels. In contrast, the monopoly will raise prices, because the value of the marginal firm is decreased, and so the change in prices further destroys surplus (see Tremblay (2015)). Indeed, if consumers’ marginal network effects from firms’ participation are strongly declining, then it is unlikely that subsidizing firm entry is welfare improving, and so one of the possible reasons for a monopoly to welfare-dominate the case of competition becomes weaker.

In sum, decreasing marginal network effects reduce welfare in the monopoly market more than in competition, so the stronger are deceasing marginal network effects, the more likely it is that competition welfare-dominates the monopoly platform.

**Exclusive deals** In some instances firms enter into exclusivity contracts with platforms postulating that they cannot offer their service on competing platforms. In our model this has two implications. First, there exist equilibrium configurations in which the market tips and only one platform survives. That is, one platform can use exclusive deals to lock in enough firms to have the market tip. To see this, recall from the discussion of Theorem 4 that when $\alpha_2 < \frac{1}{3} a$ subsidizing firm entry increases welfare in the market. In competition, in this case, this welfare-increasing subsidy does not take place because of the non-appropriability of the investment when firms subsequently multi-home. However, under exclusive deals, the investment in welfare can be appropriated allowing a single platform to corner the market. Of course, in this case overall welfare is actually greater than under competition, because the investment in welfare overcomes adverse price effects.

Exclusive deals can also be used to successfully enter into the platform market, as was studied by Lee (2013) for the case of the successful entry of the Xbox in the video game market. In our model there also exists exclusive contract equilibrium configurations without
market tipping. These are distortions of the Allocation I equilibrium where in Theorem 2 all firms multi-home, all consumers single-home, and prices are $p_i^A = p_i^B = f_i$. To see this, take these prices as given on Platform $X$ and suppose that Platform $Y$ sets $p_2^Y = p_2^X = f_2$, but also offers some of the firms an exclusive contract with $p_{2,\text{excl}}^Y < f_2$. The firms who are offered the exclusive deal will join Platform $Y$ and the remaining firms will multi-home so long as there are the same number of consumers on each platform. This can be the case, although with $p_{2,\text{excl}}^Y < f_2$, Platform $Y$ must charge $p_1^Y > f_1 = p_1^X$. Now, with $p_1^Y > f_1 = p_1^X$ and $n_2^Y = N_2 > n_2^X$ consumers are indifferent when $u_1^Y(\tau) = v + \alpha_1(\tau) \cdot N_2 - p_1^Y = v + \alpha_1(\tau) \cdot n_2^X - p_1^X = u_1^X(\tau)$, so $n_1^Y = n_1^X$, and both platforms have zero profits, $\Pi^Y = \Pi^X = 0$; and so this characterizes an equilibrium.18

**Favorable Beliefs/Focal Platform** In many settings one platform may have an inherent advantage over another that is not tied to superior technology or pricing. This may be due to incumbency, greater name recognition, successful marketing campaigns, or the like. We call Platform $X$ *focal* when all agents believe that $n_1^X = N_1$, that is, all agents believe that all consumers that join at least one platform will join Platform $X$.19

In the case of weak competition ($f_1 \leq \delta v$), having Platform $X$ be focal does not affect the previous results. The non-focal Platform $Y$ can fully establish itself in the market and Theorem 1 holds: prices are $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$, all consumers multi-home, the firms that join a platform single-home, and platform profits are $\Pi^A = \Pi^B = N_1(\delta v - f_1) > 0$.

However, under strong competition ($f_1 > \delta v$), having Platform $X$ be focal makes it harder for Platform $Y$ to compete. Nevertheless, Platform $Y$ may be able to compete by using a muted divide-and-conquer strategy. Caillaud and Jullien (2003) note that by subsidizing one side of the market a platform “divides” that side, and by subsequently “conquering”

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18It is worth pointing out that a similar asymmetric equilibrium can also be derived for the case that platforms differ in their stand-alone values, $v^A \leq v^B$.

19Alternatively, we could have beliefs be that $n_1^X > n_1^Y$ instead of $n_1^X = N_1$. Either framework is largely consistent with the models developed by Caillaud and Jullien (2003), Hagiu (2006) and Jullien (2011) on favorable beliefs and focal platforms.
the other side of the market it can recover the loss it made through subsidization. \cite{jullien2011} demonstrates this in a model where a second mover platform uses divide-and-conquer to compete against a first mover platform. However, all agents are assumed to single-home. With endogenous homing decisions, the divide-and-conquer strategy is muted: the non-focal platform’s ability to attract consumers by pricing low is enhanced through the possibility of these choosing to multi-home. However, this need not lead a ‘divide’ (because the multi-homers remain on the focal platform as well), and hence the ability to recover the costs of the subsidy may be harder to achieve as the ‘conquering’ is less effective (firms that access multi-homing consumers on the focal platform have no need to join the ‘conquering’ platform).

In order for Platform \( Y \) to compete with the focal Platform \( X \), Platform \( Y \) sets \( p_Y^1 = \delta v < f_1 \). This guarantees \( n_Y^1 = N_1 = \overline{N}_1 \) and, so long as \( f_1 \) is not too large relative to \( \delta v \) and \( p_X^2 \) is not too low, Platform \( Y \) can charge a mark up on the firm side of the market that makes up for its losses on the consumer side. Thus, if Platform \( X \) attempts to use its advantage as a focal platform and charge very high prices then Platform \( Y \) can use this divide and conquer strategy to compete.\footnote{Platform \( Y \) is unable to divide and conquer by pricing below marginal cost on the firm side, \( p_Y^2 < 0 = f_2 \), since this only guarantees a small mass of firms will find this sufficient to multi-home. If \( p_Y^2 < f_2 \) it must charge a consumer price such that \( p_Y^1 > f_1 \) in order to have non-negative profits. However, \( p_Y^1 > f_1 \geq \delta v \) implies \( n_Y^1 = 0 \) since Platform \( X \) is focal; this will result in negative profits for Platform \( Y \).}

The limits of this strategy are captured in the following theorem.

**Theorem 6** (Focal Platform Equilibrium). Let \( f_1^L := \delta v + \frac{\alpha^2 N_1}{8c} < \delta v + \frac{\alpha^2 N_1}{4c} =: f_1^H \). Then,

- if \( f_1 < f_1^L \), there is a competitive equilibrium in which \( p_1^X = p_Y^1 = \delta v < f_1 \), \( p_2^X = p_Y^2 > f_2 \), \( n_1^X = n_Y^1 = n_Y^m = N_1 \), \( n_2^m = 0 \), and \( \Pi^X = \Pi^Y = 0 \);

- if \( f_1 \in [f_1^L, f_1^H] \), Platform \( X \) is a monopoly constrained to \( p_1^X = \delta v < f_1 \) and \( p_2^X > f_2 \);

- if \( f_1 > f_1^H \), Platform \( X \) is an unconstrained monopoly.
Theorem 6 shows that there exists two critical levels of marginal cost to the platform for an additional consumer, $f_1$. In particular, if $f_1$ is relatively close to $\delta v$ competition is strong and profits are zero. For a moderately large $f_1$ relative to $\delta v$ the presence of the non-focal platform will prevent monopoly prices by the focal platform, but participation will only occur on the focal platform (akin to a contestable market). Lastly, if $f_1$ is large relative to $\delta v$ then the focal platform will charge monopoly prices.

Thus, when agents make endogenous homing decisions a focal platform cannot necessarily tip the market and become a monopoly platform; in fact, competition can be strong between the focal and non-focal platforms. Agents’ ability to either single-home or multi-home allows the non-focal platform to attract one side of the market to multi-home, which enables it to compete with the focal platform.

6 Conclusion

In many markets in which platforms compete against each other agents choose to join either one platform (single-home) or they join several platforms (multi-home). While most of the previous literature on platform competition has assumed this decision to be given exogenously, we allow participants on both sides of the platform to endogenously make an optimal homing decision.

When platforms are sufficiently differentiated in the sense that membership at a second platform bestows an additional stand-alone benefit apart from access to more agents on the other side, then platforms set prices to attract multi-homers, blunting head-to-head price competition between platforms (weak competition). In contrast, when added stand-alone values are small relative to the cost of providing membership benefits, platforms engage in fierce competition for single-homers (strong competition), which leads to a zero-profit equilibrium with marginal-cost pricing, akin to the Bertrand Paradox. While we find that
there is a unique pricing equilibrium, there are potentially multiple equilibrium allocations concerning how consumers and firms dived themselves onto the competing platforms.

One equilibrium allocation that always exists entails all consumers single-homing and all firms multi-homing. This mirrors the allocation in the market for smartphones, where virtually all consumers own only a single phone, but virtually all apps are available across competing smartphones.

When network effects are strong enough, another type of equilibrium allocation emerges in which there is a mix of multi-homing and single-homing on both sides of the platforms. This constellation is found in the market for video game consoles: while many consumers have only one console, serious gamers often have more than one system; and while some games are available across providing platforms, others are exclusive to one system.

The third possible equilibrium constellation has all firms single-homing, whereas some or all consumers multi-home. This occurs when there is no stand-alone benefit to consumers of accessing the platform, that is, consumers are exclusively interested in the service provided by firms. This market structure is found with the rideshare services Uber and Lyft, where drivers sign up with one or the other platform (single-home), but some consumers multi-home to compare prices and availability across the services.

Compared to the lower prices and possibly greater access to services provided by competition between platforms, a monopoly platform may (but need not) generate higher welfare compared to any of the three possible allocations under competition. This is because the monopoly may better concentrate network effects and prevents cost-redundancies when firms multi-home. Moreover, the monopolist may invest in the value of the platform by subsidizing firm entry and thereby increase total welfare—whereas such investments are not undertaken in competition due to non-appropriability of the investments.

We find that decreasing marginal network effects adversely affect welfare under monopoly more than with competition, but that increased entry beyond two platforms is unlikely to
increase welfare. Finally we show that due to endogenous homing exclusive contracts and focal platforms may skew market shares, but a complete tipping of the market is often prevented as disadvantaged platforms can set prices so as to attract some multi-homers, which proves to be a sufficiently strong foothold to preserve competition.

**Appendix of Proofs**

The following lemma is used to prove the equilibrium allocations.

**Lemma 1.** In equilibrium $n_2^X \in \{0, n_2^Y, N_2\}$, $Y \neq X$ whenever $p_2^X, p_2^Y \geq 0$; and $n_2^A = n_2^B$ if and only if $u_2^A(\theta) = u_2^B(\theta)$ for all $\theta \in [0, N_2]$.

**Proof of Lemma 1** Note that if for some $\theta$, $u_2^A(\theta) \preceq u_2^B(\theta)$, then this holds for all $\theta$; and because $u_2(\cdot)$ is linear, there exist three mutually exclusive and exhaustive relations in comparing $u_2^A(\theta)$ to $u_2^B(\theta)$, for all $\theta$, which are covered by the following two cases:

1. $u_2^X(\theta) > u_2^Y(\theta)$ which implies that firms that join a platform will only join Platform $X$: $n_2^X = N_2$ and $n_2^Y = 0$; $X, Y = A, B$; $X \neq Y$ for all $p_2^X, p_2^Y \geq 0$.

2. $u_2^A(\theta) = u_2^B(\theta)$ for all $\theta$ which by Assumption 1 implies $n_2^A = n_2^B$ for all $p_2^X, p_2^Y \geq 0$.

Thus, $n_2^X \in \{0, n_2^Y, N_2\}$ for $X = A, B$ and $Y \neq X$ for all $p_2^X, p_2^Y \geq 0$.

As for $n_2^A = n_2^B$, the ‘only if’ follows directly from Assumption 1 and for the ‘if’ part, notice from above that $n_2^A = n_2^B$ can only occur when $u_2^A(\theta) = u_2^B(\theta)$.

**Proof of Proposition 1** If $p_i^X = p_i^Y$ then by Assumption 2 $n_2^X, n_2^Y > 0$. By Lemma 1 $n_2^X = n_2^Y$ in the allocation equilibrium. This implies $u_1^X(\tau) - u_1^Y(\tau) = 0, \forall \tau$. By Assumption 1 then $n_1^X = n_1^Y$. Thus, $n_1^X = n_1^Y$ is the unique allocation equilibrium.

Consider the homing decisions. All $\tau$ with $u_1^{AB}(\tau) > u_1^X(\tau)$ and $u_1^{AB}(\tau) > 0$ multi-home. Equations (2) and (3) imply $0 < \delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1$ and $0 < v + \alpha_1(\tau)N_2 - p_1$. 

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However, $\delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1 \leq v + \alpha_1(\tau)N_2 - p_1$, so the first equation implies the second. So all $\tau < \alpha_1^{-1}(\frac{p_1 - \delta v}{n_2^Y - n_2^m})$ multi-home.

All $\tau$ with $u_1^X(\tau) > u_1^{AB}(\tau)$ and $u_1^X(\tau) > 0$ single-home. Equations (2) and (3) now imply $0 > \delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1$ and $0 < v + \alpha_1(\tau)n_2^X - p_1$ and, if $p_1 > \delta v$ then for all $\delta < 1$ the set of single-homing consumers is nonempty since $v + \alpha_1(\tau)n_2^X > \delta v + \alpha_1(\tau)(n_2^Y - n_2^m)$.

Thus, the set of single-homing consumers is $\tau \in \left[\alpha_1^{-1}(\frac{p_1 - \delta v}{n_2^Y - n_2^m}), \alpha_1^{-1}(\frac{p_1 - v}{n_2^Y})\right]$.

All $\theta$ with $u_2^{AB}(\theta) > u_2^X(\theta)$ and $u_2^{AB}(\theta) > 0$ multi-homing. Equations (4) and (5) now imply $\theta < \frac{\alpha_2(N_1) - 2p_2}{(1 + \sigma)c}$ and $\theta < \frac{\alpha_2(n_1^Y - n_1^m) - p_2}{\sigma c}$. Which inequality dominates depends on the parameters. Thus, the set of firms who multi-home is $\theta < \min\left\{\frac{\alpha_2(n_1^Y + n_1^m - n_1^m) - 2p_2}{(1 + \sigma)c}, \frac{\alpha_2(n_1^Y - n_1^m) - p_2}{\sigma c}\right\}$.

All $\theta$ with $u_2^X(\theta) > u_2^{AB}(\theta)$ and $u_2^X(\theta) > 0$ single-homing. Equations (4) and (5) imply this occurs when $\theta \geq n_2^m$ and when $\theta \leq \frac{\alpha_2n_1^Y - p_2}{c}$. Thus, the total number of firms that single-home on platforms is $\theta \in \left[n_2^m, \frac{\alpha_2n_1^Y - p_2}{c}\right]$. □

**Proof of Proposition 2** If $p_1^X = p_1^Y$ and $p_2^X > p_2^Y$ then by Assumption 2 $n_2^Y > 0$. By Lemma 1 there are two possible allocations in equilibrium, $n_2^Y = n_2^X$ and $n_2^Y = N_2$ for $p_2^X \geq 0$.

1. Suppose $n_2^Y = n_2^X$. This with $p_2^X < p_2^Y$ and the Lemma implies $n_1^Y < n_1^X$. For consumer, $n_2^Y = n_2^X$ implies $u_1^Y(\tau) - u_1^X(\tau) = p_1^X - p_1^Y = 0$ for all $\tau$ since $p_1^X = p_1^Y$. By Assumption 1 this implies $n_1^Y = n_1^X$, a contradiction. Thus, $n_2^Y = n_2^X$ is not possible.

2. Suppose $n_2^Y = N_2$. For consumers, $u_1^Y(\tau) - u_1^X(\tau) = \alpha_1(\tau) \cdot N_2 + p_1^X - p_1^Y > 0$ for all $\tau$. Thus, $n_1^Y = N_1$. For firms, $u_2^Y(\theta) - u_2^X(\theta) = \alpha_2 \cdot N_1 - p_2^Y + p_2^X > 0$ for all $\theta$.

Thus, $n_2^Y = N_2$ and $n_1^Y = N_1$ is the unique allocation equilibrium.

Arguments for the allocations prescribed in $p_1^X > p_1^Y$ with $p_2^X > p_2^Y$ and $p_1^X > p_1^Y$ and $p_2^X = p_2^Y$ follow similarly. □

**Proof of Proposition 3** If $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ then by Lemma 1 there are three possible allocations in equilibrium, $n_2^Y = n_2^X$, $n_2^Y = N_2$, and $n_2^Y = 0$ for $p_2^X, p_2^Y \geq 0$. 33
1. Suppose \( n_2^Y = n_2^X \). For consumers, \( n_2^Y = n_2^X \) implies \( u_1^X(\tau) - u_1^Y(\tau) = p_1^Y - p_1^X < 0 \) for all \( \tau \) since \( p_1^X > p_1^Y \). This implies \( n_1^Y = N_1 \) and \( n_1^X \geq 0 \) depending on prices some consumers may join \( X \) and multi-home. For firms, the Lemma implies \( u_2^X(\theta) = u_2^Y(\theta) \) for all \( \theta \). This implies \( n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \geq 0 \). Thus, if \( n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \geq 0 \) holds then \( n_2^Y = n_2^X \), \( n_1^Y = N_1 \), and \( n_1^X \geq 0 \) is a possible equilibrium allocation.

2. Suppose \( n_2^Y = N_2 \). For consumers, \( u_1^Y(\tau) - u_1^X(\tau) = \alpha_1(\tau) \cdot N_2 - p_1^Y + p_1^X > 0 \) for all \( \tau \). Thus, \( n_1^Y = N_1 \). Since all single-homing consumers and all firms join platform \( Y \), consumers will join platform \( X \) and multi-home when \( u_1^X(\tau) > u_1^{AB}(\tau) \). This occurs when \( \delta v \geq p_1^X \). If \( \delta v \geq p_1^X \) then all consumers join platform \( X \), otherwise none will, so either \( n_1^X = 0 \) or \( n_1^X = N_1 \). For firms, the Lemma implies \( u_2^X(\theta) < u_2^Y(\theta) \) for all \( \theta \). This implies \( n_1^X < N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} < N_1 \). Thus, the only allocation equilibrium that can exist is \( n_2^Y = N_2 \), \( n_1^X = 0 \), and \( n_1^Y = N_1 \) with \( \delta v < p_1^X \).

3. Suppose \( n_2^Y = 0 \). Since \( n_2^Y = 0 \) it must be that \( n_2^X = N_2 \) for \( p_2^X, p_2^Y \geq 0 \). If \( p_1^Y > v \) then consumers that join a platform will join only platform \( X \), so \( n_1^X = N_1 \) and \( n_1^Y = 0 \). When \( v \geq p_1^Y > \delta v \) we have \( u_1^X(\tau) - u_1^Y(\tau) = \alpha_1(\tau) \cdot N_2 - p_1^X + p_1^Y > 0 \). Thus there exists \( \tau' \) such that consumers \( \tau \in [0, \tau'] \) join Platform \( X \) and consumers \( \tau \in (\tau', N_1] \) join Platform \( Y \). This implies \( n_1^X, n_1^Y > 0 \) with no multi-homing. Lastly, when \( \delta v \geq p_1^Y \) we have \( n_1^Y = N_1 \). For firms, since \( n_2^Y = 0 \) it must be that \( u_2^X(\theta) > u_2^Y(\theta) \) for all \( \theta \). This implies \( n_1^X > N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \) join platform \( X \) and multi-home. Thus, this allocation equilibrium is possible and for all price levels of \( p_1^Y \).

Thus, with these prices we have three possible allocation equilibria. \( \square \)

**Proof of Theorem** Given these prices it is clear that both platform’s profits are \( \Pi^X = \Pi^Y = N_1(\delta v - f_1) > 0 \).

When the platforms set equal prices on both sides of the platform then \( p_1^X = p_1^Y = \delta v \) and \( p_2^X = p_2^Y = f_2 \) is the only equal price constellation where neither platform has an incentive
to deviate. At any \( p_1 < \delta v \) both platforms will increase their price. If \( p_1 > \delta v \) then both platforms have an incentive to undercut the other platform. Similarly for any \( p_2 \neq f_2 \).

When \( p_i^X > p_i^Y, i = 1, 2 \), with at least one inequality being strict, if \( \Pi^Y > 0 \) then Platform X will undercut its prices and if \( \Pi^Y = 0 \) then it will increase its price but still undercut Platform X’s prices. Thus, a deviation occurs.

When \( p_1^X > p_1^Y \) and \( p_2^X < p_2^Y \) there are three possible allocations we must check from Proposition 3. Some equations used below are from the proof of Proposition 3.

1. When \( n_2^X = n_2^Y = n_2, n_1^Y = N_1, \) and \( n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \). This is an equilibrium when \( \Pi^X = \Pi^Y \) since otherwise the lower profit platform will deviate. Thus, \( n_2(p_2^Y - p_2^X) = N_1(p_1^Y - f_1) - (N_1 - \frac{p_2^Y - p_2^X}{\alpha_2})(p_1^X - f_1) \). However, both platforms have an incentive to deviate by raising their lower price to just undercutting the other platforms price on that side of the market. Thus, this cannot be an equilibrium.

2. When \( n_1^Y = N_1 \) and \( n_2^Y = N_2 \). In this case when \( \Pi^X \geq 0 \) it must be that \( \Pi^Y > \Pi^X \geq 0 \). Thus, Platform X always has an incentive to deviate. This allocation cannot be an equilibrium.

3. When \( n_2^Y = 0 \) and \( n_2^X = N_2 \). If \( p_1^Y > \delta v \) then \( n_1^Y = 0 \) and either \( \Pi^X > 0 \) and Platform Y has an incentive to deviate or \( \Pi^X \leq 0 \) and Platform X has an incentive to deviate.

If \( v \geq p_1^Y > \delta v \) then \( n_1^Y, n_1^X > 0 \) with no multi-homing consumers. However, for all \( \Pi^X \geq \Pi^Y \) both platforms have an incentive to raise their lower price to just less than the other platforms price on that side. Thus, this cannot be an equilibrium.

If \( \delta v \geq p_1^Y \) then \( n_1^Y = N_1 \) and \( n_1^X > N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \) and Platform X always has an incentive to increase its price \( p_2^X \) to just below \( p_2^Y \). This allocation cannot be an equilibrium.

Thus, the unique set of prices that occurs in equilibrium is \( p_1^X = p_1^Y = \delta v \) and \( p_2^X = p_2^Y = f_2 \).
The allocation follows since, \( p_1 = \delta v \) all consumers will multi-home which implies all the firms that participate will single-home.

**Proof of Theorem 2** Given these prices it is clear that both platforms make zero profits.

Prices must be set equally in equilibrium follows as in Theorem 1. The only price constellation where neither platform has an incentive to deviate is \( p^X_i = p^Y_i = f_i \).

We now show the equilibrium allocations for general symmetric prices \( p_1 \) and \( p_2 \).

**Allocation I:** Allocations (7) imply all firms multi-home when all consumers single-home, since \( n^m_2 > N_2 \) i.e. \([n^m_2, N_2]\) is empty when \( n^m_1 = 0 \). Furthermore, when \( n^m_2 = n^A_2 = n^B_2 \), allocation (6) implies no consumer multi-homes. Hence, all consumers single-home if and only if all firms multi-home. Thus, the allocation where all firms multi-home and all consumers single-home is a Nash Equilibrium.

**Allocation II:** Since \( p_1 > \delta v \), allocation (6) implies the set of multi-homing consumers is non-empty when the number of multi-homing firms is not to large. Let \( x \in [0, 1] \) be the percent of consumers who multi-home of those \( n^X_1 \) who join platform \( X \) so that in expectation \( n^m_1 = xn^X_1 = xn^Y_1 \). This implies \( N_1 = (2-x)n^X_1 \) since \( N_1 = n^X_1 + n^Y_1 - n^m_1 \) and in expectation \( n^X_1 = n^Y_1 \). From the Allocation I \( x > 0 \) occurs when not all of the firms are multi-homing. This occurs when \( \min \left\{ \frac{\alpha_2(2-x)n^X_1 - 2p_2}{(1+\sigma)c}, \frac{\alpha_2(1-x)n^Y_1 - p_2}{\sigma c} \right\} < \frac{\alpha_2 n^X_1 - p_2}{c} \).

In the remainder of this proof we assume \( \sigma = 1 \), no economies of duplication. Using allocation (7) there exists \( x^m \) such that for \( x > x^m \) no firm will multi-home. Allocation (7) implies \( 0 = \alpha_2(1 - x^m)n^Y_1 - p_2 \). Thus, \( x^m = 1 - \frac{p_2}{\alpha_2 n^Y_1} \). And for all \( x > x^m \) no firm multi-homes. Note, \( p_2 < \alpha_2 n^Y_1 \) since otherwise the market collapses, hence \( x^m \in (0, 1) \).

If \( 0 < x < x^m \) then some firms will single-home and some firms will multi-home. Allocation (7) implies \( n^m_2 = \frac{\alpha_2 (1-x)n^Y_1 - p_2}{c} \) and allocation (7) implies \( n^Y_2 = (1/2)(N_2 + n^m_2) = (1/2c)[\alpha_2(2-x)n^X_1 - 2p_2] \). Similarly, allocation (6) defines the number of multi-homing consumers: \( 0 = \delta v + \alpha_1(n^m_1)(n^Y_2 - n^m_2) - p_1 \).
and \( n_1^m = x n_1^X = x n_1^Y \) we can characterize \( x \) by:

\[
0 = \delta v + \alpha_1 (x n_1^X) \left( \frac{1}{2c} \right) \left[ \alpha_2 (2 - x) n_1^X - 2 p_2 - 2 \alpha_2 (1 - x) n_1^Y + 2 p_2 \right] - p_1 \\
= \delta v + \alpha_1 (x n_1^X) \left( \frac{1}{2c} \right) \alpha_2 \cdot x n_1^X - p_1,
\]  \hspace{1cm} (13)

Furthermore, allocation \((6)\) defines \( N_1 \), the number of consumers on Platform \( X \): \( 0 = v + \alpha_1 (N_1) n_2^X - p_1 \). Thus we have:

\[
0 = v + \alpha_1 (N_1) n_2^X - p_1 = v + \alpha_1 ((2 - x) n_1^X) \left( \frac{1}{2c} \right) \left( \alpha_2 \cdot (2 - x) n_1^X - 2 p_2 \right) - p_1.
\]  \hspace{1cm} (14)

Thus, we have two equations \((13)\) and \((14)\) and two unknowns, \( x \) and \( n_1^X \). If the solution is \( x \in (0, x^m) \) then we have a Nash Equilibrium. Note, this equilibrium does not exist when \( x \notin (0, x^m) \).

**Allocation III**: Allocation \((7)\) implies all firms single-home when the number of multi-homing consumers is relatively large, \( n_1^Y \leq n_1^m + \frac{p_2}{\alpha_2} \). If \( p_2 = 0 \), then this holds when all consumers multi-home. By allocation \((6)\), this will only be an equilibrium when \( v = 0 \). If \( p_2 > 0 \), then allocation \((6)\) implies there exists an equilibrium where all firms single-home and a large portion of consumers multi-home given prices such that \( N_1 - n_1^m = \alpha_1^{-1} \left( \frac{p_1 - v}{n_1^X} \right) - \alpha_1^{-1} \left( \frac{p_1 - \delta v}{n_1^X} \right) \leq \frac{2 p_2}{\alpha_2} \).

Thus, there exists at least one and potentially three allocations that occur in equilibrium with unique equilibrium prices \( p_1^X = p_1^Y = f_1 \) and \( p_2^X = p_2^Y = f_2 \). \( \square \)

**Proof of Theorem 3** Given the monopoly corner solution, we calculate the standard
welfare measures.

\[
\Pi^{MC} = N_1(p_1^{MC} - f_1) + N_2(p_2^{MC} - f_2) = N_1 \times 0 + N_2 p_2^{MC} = \frac{a^2 \alpha_2^2}{4b^2 c}, \tag{15}
\]

\[
CS^{MC} = \int_0^{N_1^{MC}} (v + \alpha_1(\tau)N_2^{MC} - p_1^{MC}) d\tau = \frac{a^3 \alpha_2}{4b^2 c}, \tag{16}
\]

\[
FS^{MC} = \int_0^{N_2^{MC}} (\alpha_2 N_1^{MC} - c \theta - p_2^{MC}) d\theta = \frac{a^2 \alpha_2^2}{8b^2 c}, \tag{17}
\]

\[
W^{MC} = \frac{a^2 \alpha_2^2}{8b^2 c} (3\alpha_2 + 2a). \tag{18}
\]

For the monopoly interior case: given equilibrium prices, the number of consumers, and the number of firms, we calculate platform profits, consumer, firm, and welfare.

\[
\Pi^{MI} = N_1(p_1^{MI} - f_1) + N_2(p_2^{MI} - f_2) = \frac{(a + \alpha_2)^4}{64b^2 c}, \tag{19}
\]

\[
CS^{MI} = \int_0^{N_1^{MI}} (v + \alpha_1(\tau)N_2^{MI} - p_1^{MI}) d\tau = \frac{(a + \alpha_2)^4}{64b^2 c}, \tag{20}
\]

\[
FS^{MI} = \int_0^{N_2^{MI}} (\alpha_2 N_1^{MI} - c \theta - p_2^{MI}) d\theta = \frac{(a + \alpha_2)^4}{128b^2 c}, \tag{21}
\]

\[
W^{MI} = \frac{5(a + \alpha_2)^4}{128b^2 c}. \tag{22}
\]

Lastly, the welfare results for Allocation \(\text{I}\) with competing platforms.

\[
\Pi^A = \Pi^B = 0, \tag{23}
\]

\[
CS^{AI} = \int_0^{a/b} (a - b\tau) \frac{2\alpha_2 \cdot n_1}{(1 + \sigma)c} d\tau = \frac{a^2 \alpha_2}{2(1 + \sigma)cb^2}, \tag{24}
\]

\[
FS^{AI} = \int_0^{n_2} (\alpha_2 N_1 - 2c \theta - 2p_2) d\theta = \frac{a^2 \alpha_2^2}{2(1 + \sigma)cb^2}, \tag{25}
\]

\[
W^{AI} = \frac{a^2 \alpha_2}{2(1 + \sigma)cb^2} (\alpha_2 + a); \tag{26}
\]

where the superscript \(\text{AI}\) denotes Allocation \(\text{I}\).
A monopoly corner solution occurs when $\alpha_2 \geq a$. Using the welfare equations (18) and (26), $W^{AI} < W^{MC}$ occurs when $\sigma > \frac{a+2\alpha_2}{3a+2\alpha_2}$. A monopoly interior solution occurs when $\alpha_2 < a$. Using welfare equations (22) and (26), $W^{AI} < W^{MI}$ occurs when $\sigma > \frac{64a^2\alpha_2^2}{5(a+\alpha_2)^3} - 1$.

□

Proof of Theorem 4 We first show that Allocation II in Theorem 2 exists when $b(1-\delta)v_c^{a_2} \in (0, \frac{1}{8})$: Equations (13) and (14) imply we have two equations and two unknowns, $x$ and $n_1^A$. Solving these equations implies $x$ is implicitly defined by: $t \equiv \frac{b(1-\delta)v_c^{a_2}}{a^2\alpha_2} = \frac{(1-x)x}{(2-x)^2}$. This implies $0 = (1+t)x^2 - (1+4t)x + 4t$. Solving for $x$ as a function of $t$ and using the quadratic formula such that $x \in (0, 1)$ implies we must have $t \in (0, \frac{1}{8})$.

Consider now the Theorem. When $x = 1/2$, equations (13) and (14) imply half of firms and a third of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly interior solution if and only if $0 > 135a^4 - 484a^3\alpha_2 - 150a^2\alpha_2^2 + 540a\alpha_2^3 + 135\alpha_2^4$. This occurs when $\alpha_2 \in [r_1 \cdot a, r_2 \cdot a]$ where $r_1 \cdot a = \alpha_2$ and $r_2 \cdot a = \alpha_2$ are the roots of the preceding polynomial. However, the welfare for $x = 1/2$ is never greater than the monopoly corner solution.

When $x = .9$, equations (13) and (14) imply a tenth of firms and $(8/11)s$ of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly corner solution for all $\alpha_2 \geq a$ since $3.3388\alpha_2 + 3.5823a > 3\alpha_2 + 2a$.

□

Proof of Theorem 5 In Allocation III all firms single-home and all consumers multi-home; we have $n_2^m = 0$ and $n_1^A = n_1^B = n_1^m = N_1 = \frac{a}{b}$. With $p_1 = 0$ and $p_2 = 0$ we have
\[ n_2^A = n_2^B = (1/2)N_2 = \frac{\alpha_2a}{bc}; \text{ resulting in} \]

\[
\Pi^A = \Pi^B = 0, \quad (27)
\]

\[
CS^{AIII} = \int_0^{\alpha_2a} (\alpha_2a)^{\frac{a}{b\alpha}} d\tau = \frac{a^3\alpha_2}{2(bc)^2}, \quad (28)
\]

\[
FS^{AIII} = \int_0^{\alpha_2a} \alpha_2N_1 - c\theta d\theta = \frac{3a^2\alpha_2^2}{8cb^2}, \quad (29)
\]

\[
W^{AIII} = \frac{a^2\alpha_2}{8cb^2}(3\alpha_2 + 4a); \quad (30)
\]

where the superscript \( AIII \) denotes Allocation III.

By comparing equations (18) and (30) we see that \( W^{AIII} > W^M \) always holds. The corner solution for the monopoly platform is implemented when \( \alpha_2 \geq a \); thus, \( W^{AIII} > W^M \) when \( \alpha_2 \geq a \). When an interior solution occurs, equations (22) and (30) imply \( W^{AIII} > W^M \) if and only if

\[
0 > 5a^4 - 44a^3\alpha_2 - 18a^2\alpha_2^2 + 20a\alpha_2^3 + 5\alpha_2^4.
\]

This occurs when \( \alpha_2 > (1/4)a \). Thus, \( W^{AIII} > W^M \) when \( \alpha_2 > (1/4)a \).

\[ \square \]

**Proof of Theorem 6** The proof follows in two parts:

**Lemma 2.** When \( f_1 \leq f_1^L = \delta v + \frac{\alpha_2N_1}{8c} \) we have \( p_1^X = p_1^Y = \delta v \), all consumers multi-home, \( N_1 = n_1^m = n_1^X = n_1^Y = \overline{N}_1 \), \( p_2^X = p_2^Y > f_2 \), all firms that participate single-home, \( n_2^X = n_2^Y = (1/2)N_2 \), and profits are zero for each platform.

**Proof of Lemma 2** The fact that neither platform has an incentive to deviate given such prices with profits equal to zero follows in the same manner as Theorems 3 and 4 using Propositions 1, 2, and 3. To determine the cutoff point we have two equations and two unknowns. First, \( u_2(N_2) = 0 \) gives the last firm to join a platform. This implies \( N_2 = \frac{\alpha_2N_1 - p_2}{c} \) which implies \( p_2 = \alpha_2\overline{N}_1 - cN_2 \). The second equation is the zero profit condition, \( \Pi^X = \Pi^Y = 0 \), which implies \( (1/2)N_2 + \overline{N}_1(\delta v - f_1) = 0 \). Substituting \( N_2(p_2) \) implies \( p_2 \) is given by \( (\alpha_2\overline{N}_1 - p_2)p_2 = 2\overline{N}_1c(f_1 - \delta v) \). Similarly, substituting \( p_2(N_2) \) implies
\( N_2 \geq 0 \) is given by \( N_2(\alpha_2 N_1 - cN_2) = 2N_1(f_1 - \delta v) \). The most \( N_2 \) can be is \( N_2 = \frac{\alpha_2 N_1}{2c} \); hence, profits can only be nonnegative when \( f_1 \leq \delta v + \frac{\alpha_2 N_1}{8c} \equiv f_1^L. \)

When the marginal cost gets larger, \( f_1 > f_1^L \), the pricing strategy in Lemma 2 is not sustainable as profits are negative if both platforms stay and participate. Thus, one platform will drop out of the market but its presence will still keep prices low. Since Platform \( Y \) is non-focal, it is forced to not participate but its presence affects the focal platforms prices.

Lemma 3. When \( f_1 \) is such that \( f_1^L < f_1 < f_1^H \equiv \delta v + \frac{\alpha_2 N_1}{4c} \) we have \( p_1^X = \delta v \), all consumers multi-home, \( N_1 = n_1^X = N_1 \), \( p_2^X > f_2 \), all firms that participate single-home, \( n_2^X = N_2 \), and \( 0 < \Pi^X < \Pi^M \). Platform \( Y \) does not participate.

Proof of Lemma 3 The fact that neither platform has an incentive to deviate given such prices follows in the same manner as Theorems 3 and 4 using Propositions 1, 2, and 3. To determine the cutoff point we have two equations and two unknowns. First, \( u_2(N_2) = 0 \) gives the last firm to join a platform. This implies \( N_2 = \frac{\alpha_2 N_1 - p_2}{c} \) which implies \( p_2 = \alpha_2 N_1 - cN_2 \). The second equation is profits which must be maximized by Platform \( X \): \( \Pi^X = N_2(\alpha_2 N_1 - cN_2) - N_1(f_1 - \delta v) \). Maximizing implies which implies \( N_2^* = \frac{\alpha_2 N_1}{2c} \). Profits are positive with these prices only when \( f_1 < \delta v + \frac{\alpha_2 N_1}{4c} \equiv f_1^H. \)

When the marginal cost on the consumer side is too large, \( f_1 > f_1^H \), the divide and conquer strategy is not profitable and the presence of the non-focal platform has no competitive affect on the focal platform. Thus, the focal platform is a monopoly platform.

References


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