Rational Buyers Search When Prices Increase

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Abstract. Motivated by observed patterns in business-to-business transactions, we develop a dynamic model of pricing and buyer search. Seller costs are perfectly correlated and evolve according to a Markov process. In every period, each buyer observes (for free) the price set by their current supplier, but not the other sellers’ prices or the sellers’ (common) cost level. Moreover, by paying a cost \( s \) the buyer becomes a searcher and benefits from (Bertrand) competition among sellers.

We show that there exists a semi-separating equilibrium whereby sellers increase price very rapidly when costs increase but decrease price very slowly when costs decrease. Moreover, buyers search when prices increase but not otherwise. These patterns are consistent with two stylized facts; (a) prices adjust to cost changes asymmetrically (“rockets and feathers”); and (b) buyers search in response to large price increases.

Finally, contrary to the intuition that more information is beneficial, we find that pooling equilibria (where prices reveal no information) Pareto dominate separating equilibria.
1. Introduction

Many sellers routinely purchase inputs from regular suppliers. For example, Blinder et al. (1998) report that out of sample of GDP representative sellers, on average 85% of sales are to regular buyers. In this business-to-business (B2B) context, buyers are faced with a dilemma: either they do “business as usual” with their current supplier (that is, pay the quoted price); or, alternatively, they try to obtain a better deal. A better deal can be gotten in several ways, including negotiating with the current supplier or searching for better prices currently offered by rival suppliers. For example, typically automakers such as General Motors have long-term relationships with parts suppliers, but every so often solicit bids from outside suppliers as well.

In this paper, we develop a dynamic model of pricing and buyer search which is motivated by examples like the one above. Seller costs are perfectly correlated across sellers and evolve according to a Markov process. In each period, each buyer observes (for free) the price set by their current seller, but not the other sellers’ prices or the sellers’ (common) cost level. By paying a cost $s$, the buyer benefits from (Bertrand) competition among sellers (for example, the buyer convenes a second-price auction among suppliers).

Although we present our model as a search model, it differs from existing search models in the assumption that sellers can price discriminate between searchers and non-searchers. (In this sense, a more appropriate name for searchers might be “active” buyers.) We believe this assumption is quite reasonable in many B2B markets, as well as in some consumer markets such as cable TV services.

Although parsimonious in its setup, our model produces rich dynamics. As in many infinite-period models, there exists a plethora of equilibria. We focus on equilibria that produce “pooling” or “separating” patterns in price dynamics.

First, we provide conditions for the existence of a pooling equilibrium: price converges to a constant level and is invariant to cost shocks. This equilibrium provides an explanation for sticky prices that differs from the standard menu-costs explanation: in our equilibrium, sellers refrain from changing price for fear of “rocking the boat” and inducing buyers to search (which would reduce seller profit from such buyers).

Second, we provide conditions for the existence of a separating equilibrium: price changes between low and high levels as cost itself alternates between low and high levels. Although in this equilibrium prices are fully informative, social welfare is lower than in a pooling equilibrium. This is due to the fact that, unlike the pooling equilibrium, a separating equilibrium requires that search take place along the equilibrium path.\footnote{Our assumption of unit demands implies that there are no output distortions. This biases the welfare comparison in favor of the pooling equilibrium. More generally, the message is that there is a trade-off between informative prices and the equilibrium search costs required for prices to be informative. In this sense, our result is related to Benabou and Gertner (1993). They show that higher uncertainty in the aggregate component of costs leads to less informative prices but may increase welfare; we show that the same effect may result from switching from a separating to a pooling equilibrium.}

Finally, we provide conditions for the existence of a semi-separating equilibrium, an equilibrium that is separating for price increases and pooling for price decreases: when costs increase, sellers immediately increase price; and subsequently price decreases gradually, in a pattern that induces buyers not search. This equilibrium fits two stylized facts from price and search dynamics: first, prices increase more rapidly than they decrease, a pattern sometimes known as “rockets and feathers” (e.g., Bacon (1991), Peltzman (2000), Lewis...
We are by no means the first to provide a theoretical explanation for asymmetric price adjustment. Tappata (2009), for example, proposes a Varian (1980) type model of mixed-strategies and price dispersion. The model produces rockets-and-feathers asymmetric price adjustment. Intuitively, when costs are high, price dispersion is lower; lower price dispersion implies less buyer search; and less buyer search reduces the sellers' incentives to lower prices when costs decrease. Our story for asymmetric price adjustment is very different. We show that, starting from a low cost state, there is equilibrium separation (prices increase if and only if costs increase); whereas, starting from a high-cost state, there is equilibrium pooling (prices gradually decrease as the buyers’ beliefs about costs change). Unlike Tappata (2009), our sellers play pure strategies; the price dispersion that induces buyer search results from the assumption that sellers can discriminate between searchers and non-searchers.

In contrast with Tappata (2009), Yang and Ye (2008) propose a Salop and Stiglitz (1977) type bargains-and-ripoffs model. Capacity constrained sellers mix between a high and a low price. Some buyers have very low search cost and always search; some buyers have very large cost and never search; and an intermediate set of “critical” buyers search with some probability. When prices increase, searchers learn that cost is high and will likely remain high for some time. As a result, they stop searching, which in turn implies a slow decline in prices.

Although Yang and Ye (2008) and Tappata (2009) differ in the details of seller pricing, they share a similar story for asymmetric price adjustment. They also share the same prediction regarding buyer behavior: there is more search when prices are low and less search when prices are high. By contrast, our semi-separating equilibrium features search when prices increase. As often is the case in industrial organization, the question is not so much which model is right as which model better fits each industry. Our assumption that sellers can discriminate between searchers and regulars makes our model better applicable to ongoing services such as cable TV or B2B customer markets than for repeated “anonymous” purchases such as gasoline.

To the best of our knowledge, the only dynamic model predicting that a price increase leads to search is Lewis (2011). He assumes that buyers form expectations about the price distribution based on the average price level from the previous period. In this context, when prices increase buyers expectations of the price distribution tend to be too low, causing them to search more than they otherwise would. Our paper differs from his in that we assume buyers are rational and hold correct beliefs regarding seller prices. In other words, we present a complete equilibrium story for the prediction that buyers search when prices increase.

The other dynamic equilibrium search model that we are aware of is by Cabral and Fishman (2012), who propose a dynamic version of a Diamond (1971) type of model. Unlike Cabral and Fishman (2012), we assume that sellers can discriminate with respect to searchers. As a result, our equilibrium does not suffer from the well-known Diamond paradox (that there is no search in equilibrium even if the search cost is arbitrarily small and sellers set monopoly prices). Similarly to Cabral and Fishman (2012), we obtain asymmetric price adjustments to costs changes, though for a different reason.

The literature on search and price dispersion extends well beyond the above papers, including Burdett and Judd (1983), Stahl (1989), Benabou and Gertner (1993), Janssen and
Moraga-González (2004). These papers develop static models, which makes the comparison to ours difficult. In particular, these papers are silent with respect to the question of buyer behavior in reaction to a price change.

**Roadmap.** The rest of the paper is structured as follows. In Section 2, we set up the basic model components, which include a cost level that evolves according to a two-state Markov process. Section 3 deals with the particular case when the high-cost state is absorbing (the increasing-cost case). Section 4, by contrast, deals with the particular case when the low-cost state is absorbing (the decreasing-cost case). Section 5 deals with the case when costs follow a stationary stochastic process. The reason for Sections 3 and 4 is twofold: First, much of the intuition for the general case can be obtained from the simpler cases when costs either increase or decrease. Second, some results from the simpler, absorbing-state cases will be used as building blocks in the derivation of the stationary-cost-process case.

We discuss possible extensions of the model in Section 6 and conclude in Section 7.

## 2. Model

Consider a discrete time, infinite period model with two sellers \((i = 1, 2)\) and a measure \(m\) of buyers. Both sellers and buyers discount the future according to the factor \(\delta\). Sellers produce the same product and face the same unit cost, \(c\). We assume \(c \in \{c_L, c_H\}\) and that \(c\) follows a Markov process with transition matrix

\[
M = \begin{bmatrix}
1 - \gamma_L & \gamma_L \\
\gamma_H & 1 - \gamma_H
\end{bmatrix}
\]

For \(i \in \{L, H\}\), \(\gamma_i\) is the probability of a cost change when in state \(i\). Each buyer has a per period unit demand with choke price \(\bar{u}\), which we assume is very large, so that all buyers make a purchase in equilibrium.

The timing within each period runs as follows. Sellers observe the value of \(c\) and simultaneously set prices \(p_i\). Each buyer is assigned to the seller it purchased from in the previous period. (In the first period, buyers are randomly allocated across sellers.) Each buyer observes the price (but not the cost) of the seller they are assigned to and chooses between two options: (a) buy from the current seller at the going price; or (b) search for a better deal by paying a cost \(s\). We make the important assumptions that, by paying \(s\), the buyer is perceived by both sellers as a searcher; and that sellers are able to set different prices to searchers, prices that we denote by \(q_i\). Finally, sellers simultaneously set prices for searchers; searchers make their choices of seller; searchers and non-searchers make their choices of whether to make a purchase; and period payoffs are received by sellers and buyers.

A strategy for seller \(i\) consists of prices \(p_i(t), q_i(t)\) to set in each period, possibly as a function of a history \(\{c(\tau), p_i(\tau), q_i(\tau)\}_{\tau=0}^{t-1}\) of costs and prices. A strategy for a buyer consists of (a) a choice of searching or not (where by search we mean paying cost \(s\) to get quotes \(q_i\)); and (b) a decision of whether to purchase or not given the available prices \((p_i(t)\) if the buyer does not search and \(\{p_i(t), q_1(t), q_2(t)\}\) if the buyer searches); where both decisions (a) and (b) are a function of a history \(\{p_i(\tau), q_i(\tau)\}_{\tau=0}^{t-1}\) of prices and a belief \(\beta(t)\) regarding the current cost level; and where \(\beta(t)\) is the probability at time \(t\) that \(c = c_H\). An equilibrium is a set of strategies for sellers and buyers, as well as a belief by buyers.
regarding seller cost such that (a) no player can improve payoffs by unilaterally changing their strategy and (b) beliefs are consistent with strategies.

As frequently happens in games of this kind, there are multiple history-dependent equilibria. We focus on simpler equilibria where buyer strategies are only a function of current price levels, differences in price levels from the previous period, and their prior on the underlying cost. We first consider a separating equilibrium, where seller prices are a function of cost; and then a pooling equilibrium, where seller prices are a function of time but not cost level.

For ease of notation, when there is no danger of confusion we drop the argument $t$ from prices and cost levels. Specifically, in our separating equilibrium price is only a function of the cost state, not of time. We thus denote by $p_L$ (resp. $p_H$) the price level when $c = c_L$ (resp. $c = c_H$); and so forth.

3. The increasing-cost case

In this section, we assume that $c(0) = c_L$ and that $\gamma_H = 0$: firm cost starts off at a low level and increases with probability $\gamma_L$; and $c = c_H$ is an absorbing state. We call this the increasing-cost case; later we consider the case when $c$ decreases to a low cost absorbing state; and the case when $c$ follows a stationary process.

3.1. Separating equilibrium

We first consider the possibility of a (symmetric) separating equilibrium: sellers set $p_L$ when $c = c_L$ and $p_H$ when $c = c_H$; buyers, in turn, search with probability $\alpha$ when price switches from $p_L$ to $p_H$, and do not search otherwise; and always make a purchase. Buyers who become searchers at time $t$ pay price $q = q_L$ if $c = c_L$ and $q = q_H$ if $c = c_H$ (both sellers set the same price for searchers, that is, $q_1 = q_2 = q_k$, where $k \in \{L, H\}$ as the case may be). Since we’re considering a separating equilibrium, the information obtained by searchers at time $t$ has no value in future periods, so in the next period searchers face the same problem as other buyers.

Let $v_k$ be seller value per customer attached to the seller, measured at the beginning of the period, when the cost state is $k$ ($k \in \{L, H\}$) and the customer does not search (which in equilibrium is true always except when cost switches from $c_L$ to $c_H$). When $c = c_H$, we have

$$v_H = \frac{p_H - c_H}{1 - \delta}$$

In state $c = c_H$ a buyer is indifferent between searching and not searching in a given period if and only if

$$p_H = q_H + s$$

In fact, after getting a special deal by searching, price reverts back to $p_H$. At this point, it may be worth recalling that our sense of search differs from the conventional sense. In a separating equilibrium, searchers do not learn anything about prices, that is, buyers hold precise estimates of each seller’s price. The benefit form being a searcher is then to “force”

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2. Note that the values $v_k$ typically depend on buyer beliefs about $c$. In a separating equilibrium buyers are fully informed, so the information structure is irrelevant; but this is, of course, an equilibrium outcome.
sellers to offer a better deal \(q\) instead of \(p\). Specifically, when competing for a searcher, sellers lower prices to the point where discounted profit is zero (Bertrand competition). This implies:

\[
q_H - c_H + \delta v_H = 0 \tag{3}
\]

Together, (1)–(3) imply

\[
\begin{align*}
 p_H &= c_H + (1 - \delta) s \\
 q_H &= c_H - \delta s \tag{4}
\end{align*}
\]

To understand the intuition for these values, suppose that \(\delta = 0\). Then buyers have no future value for a seller and \(q_H = c_H\), that is, competing for a searcher is like static Bertrand competition, yielding price equal to marginal cost. Knowing this, buyers are willing to accept a price of \(c_H + s\), one that exactly makes them indifferent between searching and not searching. At the opposite extreme, if \(\delta = 1\) then a buyer is an extremely valuable asset if \(p_H > c_H\). Therefore, it must be that Bertrand competition implies that \(p_H = c_H\).

Substituting (4) for \(p_H\) in (1) we get

\[
v_H = s
\]

In words, given the buyer’s ability to force sellers to compete head to head by paying a cost \(s\), the value of \(s\) is also the measure of the rent that a seller earns from a loyal buyer.

Suppose that buyers search with probability \(\alpha\) when price increases from \(p_L\) to \(p_H\). Then, when \(c = c_L\), value per customer is given by

\[
v_L = p_L - c_L + \delta \left( \gamma_L (1 - \alpha) v_H + (1 - \gamma_L) v_L \right) \tag{5}
\]

(Recall that a customer who searches is effectively a customer lost, revenue wise; in other words, the seller is indifferent between keeping and losing a customer who is offered price \(q_k\).) Similarly to the case when \(c = c_H\), when \(c = c_L\) a buyer is indifferent between searching and not searching in a given period if and only if

\[
p_L = q_L + s \tag{6}
\]

Moreover, when \(c = c_L\) competition for searchers implies

\[
q_L - c_L + \delta \left( \gamma_L (1 - \alpha) v_H + (1 - \gamma_L) v_L \right) = 0 \tag{7}
\]

Equations (5)–(7) can be solved to obtain

\[
\begin{align*}
p_L &= c_L + s \left( 1 - \delta (1 - \alpha \gamma_L) \right) \\
q_L &= c_L - s \delta (1 - \alpha \gamma_L) \\
v_L &= s \tag{8}
\end{align*}
\]

The intuition for \(v_L = s\) is similar to the intuition for \(v_H = s\). The intuition for \(p_L, q_L\) is also similar to the intuition for \(p_H, q_H\): In the limit when \(\delta = 0\), we obtain \(p_L = c_L + s\) and \(q_L = c_L\). In the opposite extreme, when \(\delta = 1\), we get \(p_L = c_L + s \alpha \gamma_L\) and \(q_L = c_L - s (1 - \alpha \gamma_L)\). These latter expressions differ from their high-cost counterpart because,
at state $c = c_L$, there is always the chance that cost changes and a buyer is lost to search (which, along the equilibrium path, happens with probability $\alpha$ when cost changes from $c_L$ to $c_H$, which in turn happens with probability $\gamma_L$).

Next, we consider whether there may be profitable deviations from the above strategies. First note that, by construction, buyers do not have a profitable deviation: they are always indifferent between searching and not searching, between purchasing and not purchasing. The binding constraint is therefore that firms do not want to deviate.

A deviation for a $c_L$ type would be to masquerade itself as a $c_H$ type and raise price before marginal cost increases. This increases markup ($p_H - c_L > p_L - c_L$), but also results in the loss of an $\alpha$ fraction of buyers. This gives us a lower bound on the value of $\alpha$ required for the above strategies to be equilibrium strategies.

Conversely, a deviation for a $c_H$ type would be to masquerade itself as a $c_L$ type and keep price at $p_L$ when cost increases to $c_H$. By doing so, the seller retains the $\alpha$ fraction of buyers who search when price increases, but at a cost of a lower markup ($p_L - c_H < p_H - c_H$). This gives us an upper bound on the value of $\alpha$ required for the above strategies to be equilibrium strategies.

The next result summarizes the above discussion regarding a separating equilibrium:

**Proposition 1.** There exist bounds $\underline{\alpha}$ and $\overline{\alpha}$, such that, if $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, then a separating equilibrium exists. Non-searchers (resp. searchers) pay $p_L$ (resp. $q_L$) when $c = c_L$; and $p_H$ (resp. $q_H$) when $c = c_H$, where the values of $p_k$ and $q_k$ are given by (4) and (8). Non-searchers’ beliefs that $c = c_H$ are given by $\beta = 1$ if $p = p_H$ and $\beta = 0$ otherwise. Buyers search with probability $\alpha$ in the first period that $p = p_H$.

The proof of this and subsequent results may be found in the Appendix. The separating equilibrium seller strategies are defined by Equations (2), (3), (6) and (7). As to buyers, their strategy is to search with probability $\alpha$ when price switches from $p_L$ to $p_H$, where

$$\alpha \in \left[ \frac{c_H - c_L}{c_H - c_L + (1 - \delta(1 - \gamma_L))s}, \min \left\{ 1, \frac{c_H - c_L}{(1 - \delta(1 - \gamma_L))s} \right\} \right]$$

As frequently happens, we have a continuum of separating equilibria. A natural criterion is to select the equilibrium corresponding to the lowest value of $\alpha$. This is a Pareto optimal equilibrium, since it minimizes search costs along the equilibrium path. Since seller prices in $\alpha$ in $p_L$, their profit is $s$ independent of the fraction of searchers. However as the search cost gets passed on to buyers, their highest value is achieved when $\alpha$ is the lowest in the interval given by Proposition 1. We later return to issues of equilibrium selection.

### 3.2. Pooling Equilibrium

We now consider the possibility of a pooling equilibrium, that is, an equilibrium where the price that a seller charges to non-searchers does not depend on the seller’s cost. Rather, this price is such that buyers are indifferent between searching and not searching; and buyers

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3. Note that some of these buyers may remain with the seller. However, to the extent that buyers become searchers, their net present value is zero — just as if they actually left the seller for the rival seller.
do not search along the equilibrium path. Given this equilibrium strategy, at time \( t \) buyers holds a belief \( \beta \) that cost is high, where

\[
\beta(t) = 1 - (1 - \gamma_L)^t \tag{9}
\]

Although in equilibrium buyers do not search, the value of search determines their indifference condition and thus the highest price that sellers can charge to avoid search. Suppose that a buyer decides to search and that \( c = c_H \) in that period. Sellers compete for the buyer’s business, and so the discounted payoff from serving a searcher is zero. Since sellers must pay \( c_H \) in each period to serve the customer in the future, it follows that the net present value of all prices sold to a searcher is equal to the net present value of \( c_H \) in each period. It follows that the buyer’s expected discount value is given by

\[
\frac{\bar{u} - c_H}{1 - \delta}
\]

In other words, the buyer receives utility \( \bar{u} \) and pays a sequence of prices which, in terms of net present value, is equivalent to \( c_H \) in each period. Note that this value implies that buyers’ utility is only a function of prices they will be facing in the future and it is implied that they will never incur search costs again to get those prices. In fact, observation of the price set by sellers provides the buyer with the information that \( c = c_H \). This means that the buyer is more “pessimistic” about the value of \( c \) than non-searching buyers (that is, the buyer’s belief that \( c = c_H \) is greater than \( \beta(t) \)). Since non-searching buyers are indifferent between searching and not searching, it follows that a searcher who finds that \( c = c_H \) strictly prefers not to search in the future.

Note that, in equilibrium, buyers purchase one unit in each and every period. This is true both along the equilibrium path and along the deviation path where a buyer decides to search. This implies that buyer value at every subgame takes the form \( \bar{u}/(1 - \delta) \) plus an expression that does not depend on \( \bar{u} \). (This is immediately evident in the above expression for expected value conditional on search and conditional on \( c = c_H \).) For this reason and for simplicity, we will compute buyer value functions net of willingness to pay \( \bar{u} \). Specifically, if a buyer searches and cost happens to be high then we have

\[
u_H = \frac{-c_H}{1 - \delta} \tag{10}\]

Suppose instead that the buyer searches when \( c = c_L \). Now the opposite is true about their search behavior in the future: the buyer knows with certainty that the state is \( c_L \) at \( t > 0 \) and thus is more optimistic about the value of \( c \) than non-searchers in the future. This implies that a buyer who searched at time \( t > 0 \) will continue to search as long as he doesn’t discover \( c = c_H \). Given such search behavior, sellers correctly anticipate that the value in period \( t + 1 \) of a buyer who searches in period \( t \) is zero. Finally, Bertrand competition implies that sellers offer to sell at \( q_L = c_L \).

The buyer’s discounted value can be computed recursively as follows:

\[
u_L = -c_L + \delta \left( (1 - \gamma_L) u_L + \gamma_L u_H - s \right)
\]

In the current period (and after having paid the search cost \( s \)), the buyer pays \( c_L \) for a unit worth \( \bar{u} \). Next period, the buyer again searches (thus paying \( s \)) and two things may
happen: with probability $1 - \gamma_L$ cost remains at $c_L$, in which case the buyer’s search value remains at $u_L$; and with probability $\gamma_L$ cost switches to $c_H$, in which case the buyer receives $u_H$, as given by (10). The above recursive equation can be solved to

$$u_L = \frac{-c_L + \delta (\gamma_L u_H - s)}{1 - (1 - \gamma_L) \delta}$$ \hspace{1cm} (11)$$

When $t = 0$, however, the searchers do not gain any additional information from search and so their beliefs align with the average buyer. Therefore, following the equilibrium path, they will not search in the periods that follow.

$$u_L(0) = \frac{-c_L + \delta (\gamma_L u_H)}{1 - (1 - \gamma_L) \delta}$$ \hspace{1cm} (12)$$

As mentioned earlier, along the pooling equilibrium path buyers are indifferent between searching and not searching. Let $u(t)$ be the buyer’s value (along the equilibrium path). By recursion, we have

$$u(t) = -p(t) + \delta u(t+1)$$

which implies

$$p(t) = -u(t) + \delta u(t+1)$$ \hspace{1cm} (13)$$

The indifference condition (buyers are indifferent between searching and not searching) may be written as

$$u(t) = (1 - \beta(t)) u_L + \beta(t) u_H - s$$

$$= u_L - s - \beta(t) (u_L - u_H)$$ \hspace{1cm} (14)$$

In other words, given buyer indifference between searching and not searching, we can compute buyer value by computing their value of search. By searching, the buyer pays $s$ and expects, in the best possible case, $u_L$ (where we note that $u_L > u_H$). To the extent that the buyer is more pessimistic about the cost state, that is, to the extent that $\beta(t)$ is higher, the expected value from searching is lower, at the rate $-(u_L - u_H) < 0$.

Substituting (14) for $u(t)$ and $u(t+1)$ in (13), we derive price along the pooling equilibrium path:

$$p(t) = (1 - \delta) (s - u_L) + \left( \beta(t) - \delta (\beta(t) + 1) \right) (u_L - u_H)$$ \hspace{1cm} (15)$$

Substituting in (10) and (11) we get an expresion for the price at $t > 0$.

$$p(t) = c_H + (1 - \delta) s - (1 - \gamma_L)^t (c_H - c_L - \delta s)$$

At $t = 0$, we plug in (10) and (12) into (15)

$$p(0) = c_L + s - \frac{\delta s}{1 - \delta (1 - \gamma_L)}$$

In deriving the pooling equilibrium we did not need to characterize the seller’s value. However, an argument similar to the separating equilibrium shows that, at $t = 0$, the seller’s value is simply given by $s$. In fact, a buyer who searches at $t = 0$ receives a discounted stream of prices equal to the discounted stream of costs (by virtue of Bertrand competition)
and pays \( s \). Since the buyer is indifferent between searching and not searching at \( t = 0 \), it follows that, along the equilibrium path, the buyer pays a stream of prices equal to the discounted sum of costs. Finally, this implies that the seller's value is equal to \( s \), the same value that the seller obtains under a separating equilibrium. In both cases, the argument is that the buyer is \( s \) dollars away from forcing sellers to bring discounted expected price down to discounted expected cost level, which implies that \( s \) is the measure of the rent earned by a seller from an attached buyer.

The next result summarizes the above derivation and completes the characterization of the pooling equilibrium:

**Proposition 2.** If \( s \geq (1 - \gamma_L)(c_H - c_L) \), then there exists a pooling equilibrium: non-searchers pay

\[
p(0) = c_L + s - \frac{\delta s}{1 - \delta (1 - \gamma_L)}
\]

\[
p(t) = c_H + (1 - \delta) s - (1 - \gamma_L)^t (c_H - c_L - \delta s), \forall t > 0
\]

whereas buyers who search at time \( t \) pay

\[
q_L(0) = c_L - \frac{\delta s}{1 - \delta (1 - \gamma_L)}
\]

\[q_L = c_L, \forall t > 0\]

\[
q_H(t) = c_H - \delta s + \frac{\delta (1 - \gamma_L)^{t+1}}{1 - \delta (1 - \gamma_L)} (c_H - c_L - \delta s)
\]

Along the equilibrium path, non-searchers believe that \( c = c_H \) with probability

\[
\beta(t) = 1 - (1 - \gamma_L)^t
\]

If a different price is observed at time \( t \), then \( \beta(t) = 0 \). Buyers do not search in the equilibrium.

Notice that, while the equilibrium establishes the values of the prices set to searchers, there is no search in equilibrium. Observable equilibrium prices are therefore given by \( p(t) \).

### 3.3. Comparing the pooling and separating equilibria

Figure 1 shows the values of \( p \) and \( q \) that are observed along the pooling and the separating equilibrium, assuming that \( c \) switches from \( c_L \) to \( c_H \) at time \( t' \). Consider first the separating equilibrium. While \( c = c_L \) (that is, for \( t < t' \)), price (paid by non-searchers) is given by \( p = p_L \) (black line). At \( t = t' \), cost switches from \( c_L \) to \( c_H \) and price increases from \( p_L \) to \( p_H \). A fraction \( \alpha \) of buyers searches at this point and is offered price \( q_H \). After than, no more search takes place and all buyers pay price \( p_H \).

Contrast this to the pooling equilibrium, where there is no search and the price paid by non-searchers is given by the gray line in Figure 1. Initially price is set at \( p(0) \), which is smaller than \( p_L \), the initial price in the separating equilibrium. Then the price increases sharply and follows a smooth path eventually converging to \( p_H \). As is the case in the separating equilibrium, in a pooling equilibrium sellers take advantage of the switching cost.
Figure 1
Separating and pooling equilibria in increasing-cost case
\( c(t), p(t), q(t) \)

\[ \begin{align*}
& \text{pooling} \\
& \text{separating}
\end{align*} \]

to extract profit from the buyers. However in a pooling equilibrium sellers have an additional benefit from asymmetric information. There are two forces at play here: searching becomes more attractive for buyers, because there is an additional benefit of learning the state, which in theory makes firms offer lower prices to match this additional value. However learning that the state is low means that a searcher becomes more optimistic than the average consumer and so will search in the period that follows with probability one. This makes seller profits from a searcher lower, thus increasing the introductory price \( q_L \) and making search less attractive to buyers. The second effect dominates, so in periods \( t > 0 \) (when these dynamics are relevant due to information asymmetry) seller is expected to make extra profits. However in period \( t = 0 \) the zero profit condition implies that a search will be able to get all those profits back from the seller (since in the future they are expected to follow an equilibrium path), and so the low price at \( t = 0 \) reflects the seller essentially giving away the profits from the information asymmetry to the buyer. Then in an equilibrium seller’s welfare is \( s \), same as in the separating equilibrium.

Unlike the separating equilibrium, price in the pooling equilibrium increases gradually. The upper bound on \( p(t) \) corresponds to the high price in the separating equilibrium: \( p(t) \rightarrow p_H \). Also unlike the separating equilibrium, price under the pooling equilibrium may fall below cost. This happens when cost switches from \( c_L \) to \( c_H \) very early on while the price is still low and potentially below \( c_H \).

Why would a seller accept setting price below cost? Given the equilibrium strategies, the alternative of setting a higher price (e.g., a price above cost) leads all buyers to search; and as we showed earlier a searcher is worth zero. This implies that any upward deviation from the pooling equilibrium price implies a payoff of zero. Therefore, in order for the putative price path to be an equilibrium, we require that the seller receive positive profits. Although early on the seller may make negative period profits, its expect discounted profit is positive; in fact, it’s positive with certainty, since after the cost switch there is no additional uncertainty. Naturally, the condition that discounted profit be positive implies limits on the relevant parameter values, which corresponds to the condition in the text of Proposition 2.
**Equilibrium selection.** We showed earlier that there exists a continuum of separating equilibria (with different values of $\alpha$). A simple Pareto criterion selects a unique separating equilibrium, the one with the least search required for $c_L$ sellers not to mimic $c_H$ sellers. Still, we have (at least) two possible equilibria: the pooling and the separating one. We don’t see this necessarily as a worrisome outcome: different industries may feature different equilibria. In fact, each equilibrium implies a testable prediction that seems to be borne by the data: the separating equilibrium implies that buyers search when prices increase (and only when prices increase); whereas the pooling equilibrium implies that there is no search even though prices increase gradually.\(^4\)

The above notwithstanding, the question may be asked as to whether standard Nash equilibrium refinements may indicate that one equilibrium is more reasonable than the other one. In Appendix B we show that both equilibria survive an intuitive criterion, but fail a natural extension of the divinity refinement (extension to dynamic games with a continuum strategy space).

**Welfare analysis.** Throughout the paper, we assume buyers demand exactly one unit of the good up to a choke price $\bar{u}$. Since in equilibrium consumers always make a purchase, the only source of variation in the level of social welfare is the cost of search. It follows that the pooling equilibrium is socially optimal, as it features no search along the equilibrium path. As discussed before, sellers’ welfare is the same in both equilibria and is equal to $s$. Then all the loss in the social welfare is absorbed by buyers. We can compute this loss as the discounted sum of expected search costs in every period.

\[
W_{\text{pool social}} - W_{\text{sep social}} = W_{\text{pool buyer}} - W_{\text{sep buyer}} = \frac{\delta \gamma_L s \alpha}{1 - \delta(1 - \gamma_L)} \tag{17}
\]

\[
W_{\text{pool seller}} - W_{\text{sep seller}} = 0 \tag{18}
\]

4. The decreasing-cost case

In this section, we assume that $c(0) = c_H$ and that $\gamma_L = 0$: firm cost starts off at a high level and decreases with probability $\gamma_H$; and $c = c_L$ is an absorbing state. We call this the decreasing-cost case.

4.1. Separating equilibrium

As in the case where $\gamma_H = 0$, the separating equilibrium is characterized by regular prices $p_L$ ($p_H$) and $q_L$ ($q_H$) when $c = c_L$ ($c = c_H$). However the search on the equilibrium path must happen at all times when $p = p_H$. This is an equilibrium outcome that follows directly from the incentive compatibility constraint of the seller. If the seller does not lose any customers at $p_H$ it will have no incentive to lower the price to $p_L$. Let the fraction of searchers in a high state be $\alpha$, following the same notation.

As in the increasing-cost case, the equilibrium prices are solving the following system of

\[^4\text{Later we will show that, when costs follow a stationary stochastic process, the pooling equilibrium also implies that, in the limit, prices are sticky with respect to cost changes.}\]
equations

\[ p_L = q_L + s \quad (19) \]
\[ p_H = q_H + s \quad (20) \]
\[ 0 = q_L - c_L + \frac{p_L - c_L}{1 - \delta} \quad (21) \]
\[ 0 = q_H - c_H + \frac{\delta (1 - \alpha)(1 - \gamma_H)(p_H - c_H)}{1 - \delta(1 - \alpha)(1 - \gamma_H)} + \frac{\delta \gamma_H (p_L - c_L)(1 - \delta)}{1 - \delta(1 - \alpha)(1 - \gamma_H)} \quad (22) \]

Where the first two equations come from the buyers’ indifference condition and the last two from sellers’ zero profit condition. Solving the system we obtain expressions for equilibrium prices.

\[ p_L = c_L + (1 - \delta)s \]
\[ q_L = c_L - \delta s \quad (23) \]
\[ p_H = c_H + (1 - \delta)s + \alpha \delta s - \alpha \delta \gamma_H s \]
\[ q_H = c_H - s \delta + \alpha \delta s - \alpha \delta \gamma_H s \quad (24) \]

Incentive compatibility conditions for the sellers then define conditions on \( \alpha \) for the equilibrium existence.

**Proposition 3.** There exist bounds \( \bar{\alpha} \) and \( \underline{\alpha} \), such that, if \( \alpha \in [\underline{\alpha}, \bar{\alpha}] \), then a separating equilibrium exists. Non-searchers (resp. searchers) pay \( p_L \) (resp. \( q_L \)) when \( c = c_L \); and \( p_H \) (resp. \( q_H \)) when \( c = c_H \), where the values of \( p_k \) and \( q_k \) are given by (23) and (24). Non-searchers’ beliefs that \( c = c_H \) are given by \( \beta = 1 \) if \( p = p_H \) and \( \beta = 0 \) otherwise. Buyers search with probability \( \alpha \) when the \( p = p_H \).

We construct this equilibrium in the Appendix using the same steps as in the increasing-cost case. As in the case with the increasing costs, seller’s welfare in this equilibrium is simply equal to \( s \). Even though the seller looses a fraction of his customers at every point when \( c = c_H \), the loss that he incurs gets passed on to consumers through a higher price.

**4.2. Pooling equilibrium**

The pooling equilibrium is constructed in a similar way to Section 3. Now beliefs are characterized by

\[ \beta(t) = (1 - \gamma_H)^t \]

The expected value of a buyer who searches is now given by

\[ u_H = -\frac{c_L}{1 - \delta} + \frac{c_L - c_H}{1 - \delta(1 - \gamma_H)} \quad (25) \]
\[ u_L = -\frac{c_L}{1 - \delta} - \frac{\delta s}{1 - \delta} \quad (26) \]

Note that there is not discontinuity at time 0, because \( c \) starts off in a high state. And thus conditional on finding themselves in a low state, it must be that \( t > 0 \). It follows
Proposition 4. If \( s = (c_H - c_L) \frac{1 - \delta}{1 - \delta + \delta \gamma_H} \), then there exists a pooling equilibrium: non-searchers pay
\[
p(t) = c_L + s + (1 - \gamma_H)^t \left( c_H - c_L - s \delta \frac{1 - \delta + \delta \gamma_H}{1 - \delta} \right)
\]
whereas introductory prices in period \( t \) are
\[
q_H(t) = c_L + (1 - (1 - \gamma_H)^{t+1} \delta) \left( \frac{c_H - c_L}{1 - \delta + \delta \gamma_H} + \frac{\delta s}{1 - \delta} \right)
\]
\[q_L = c_L\]
Along the equilibrium path, non-searchers believe that \( c = c_H \) with probability
\[
\beta(t) = (1 - \gamma_H)^t
\]
If a different price is observed in period \( t \), then \( \beta(t) = 0 \). Buyers do not search in the equilibrium.

The proof of (4) is included in the Appendix. As before, depending on the actual cost realization, there may be a period where sellers price below cost which imposes some restrictions on the parameters to ensure positive expected profits for the seller at all times.
4.3. Comparing the pooling and separating equilibria

Figure 2 depicts the pooling and separating equilibria in the decreasing-cost case. While the pooling equilibrium delivers no search on the equilibrium path, in the separating equilibrium search occurs at every period while \( c = c_H \) (in contrast with the increasing-cost case where search occurs only when prices increase).

**Welfare analysis.** As in the increasing-cost case the only source of welfare variations are search costs paid along the equilibrium path. Similarly, we find that in both equilibria sellers make the same profit equal to the search cost \( s \). Since in the pooling equilibrium there is no search, it is social welfare improving by exactly the amount of discounted expected search costs. In addition, since sellers are indifferent between the two equilibria, we find that all of the welfare loss is again absorbed by the buyers.

\[
W_{\text{pool}}^{\text{social}} - W_{\text{sep}}^{\text{social}} = W_{\text{pool}}^{\text{buyer}} - W_{\text{sep}}^{\text{buyer}} = \frac{\delta(1 - \gamma_H)s\alpha}{1 - \delta(1 - \gamma_H)} \tag{28}
\]

\[
W_{\text{pool}}^{\text{seller}} - W_{\text{sep}}^{\text{seller}} = 0 \tag{29}
\]

5. Stationary cost dynamics

We now come to the core of our paper, where we consider the more general case when cost follows a stationary stochastic process: \( \gamma_H > 0 \) and \( \gamma_L > 0 \), so that there is no absorbing state. The first part of the section follows a sequence similar to the previous sections: we consider the possibility of a separating as well as a pooling equilibrium. After that, we consider the additional possibility of a hybrid equilibrium, specifically a semi-separating equilibrium.

5.1. Separating equilibrium

We can construct a separating equilibrium where in state \( c = c_H \) (\( c_L \)) sellers charge \( p_H \) (\( p_L \)) to their existing consumers and \( q_H \) (\( q_L \)) to the searchers. As in the previous case this will necessarily result in search at all times the state is high (with probability \( \alpha_H \)) and when the price increases (with probability \( \alpha_L \)). As before we can solve the system of equations

\[
p_L = q_L + s \tag{30}
\]
\[
p_H = q_H + s \tag{31}
\]
\[
0 = q_L - c_L + \delta ((1 - \gamma_L)v_L + \gamma_L(1 - \alpha_L)v_H) \tag{32}
\]
\[
0 = q_H - c_H + \delta ((1 - \gamma_H)(1 - \alpha_H)v_H + \gamma_H v_L) \tag{33}
\]
\[
v_L = p_L - c_L + \delta((1 - \gamma_L)v_L + \gamma_L(1 - \alpha_L)v_H) \tag{34}
\]
\[
v_H = p_H - c_H + \delta((1 - \gamma_H)(1 - \alpha_H)v_H + \gamma_H v_L) \tag{35}
\]

Here the first two equation correspond to buyers’ indifference condition, the second pair of equation defines the zero profit condition and, finally, the last two equations represent sellers’ value function at each state of the marginal cost. Solving the system we obtain
We also find that $v_H = v_L = s$, so just as in the absorbing case, the seller always extracts profit $s$ from the buyer.

The existence of this equilibrium depends on where there exist $\alpha_L$ and $\alpha_H$ that make these prices incentive compatible. Not surprisingly, since sellers’ value for a buyer at any point in time is $s$, the bound on the search probabilities are exactly the same as in simple cases discussed.

**Proposition 5.** There exist bounds $\overline{\alpha}_L$, $\overline{\alpha}_L$, $\overline{\alpha}_H$ and $\overline{\alpha}_H$ such that, if $\alpha_L \in [\overline{\alpha}_L, \overline{\alpha}_L]$ and $\alpha_H \in [\overline{\alpha}_H, \overline{\alpha}_H]$, then a separating equilibrium exists. Non-searchers (resp. searchers) pay $p_L$ (resp. $q_L$) when $c = c_L$; and $p_H$ (resp. $q_H$) when $c = c_H$, where the values of $p_k$ and $q_k$ are given by (36)-(39). Non-searchers’ beliefs that $c = c_H$ are given by $\beta = 1$ if $p = p_H$ and $\beta = 0$ otherwise. Buyers search with probability $\alpha_L$ in the first period that $p = p_H$ and with probability $\alpha_H$ at each period when the price stays at $p_H$.

### 5.2. Pooling equilibrium

Here prices are independent of the state, so do not convey any information to buyers. Then buyer beliefs evolve solely based on the law of motion of the cost. Of course the beliefs will depend on the last state that is known. Following the same notation, define buyer beliefs to be $\beta(t) = \text{Prob}(c(t) = c_H|c(0) = c_H)$. We will define these recursively as follows

$$\beta(t) = (1 - \gamma_H)\beta(t - 1) + \gamma_L(1 - \beta(t - 1))$$
Figure 4
Pooling equilibrium in dynamic case

\[ c(t), p(t), q(t) \]

We compute the value of searching conditional on the state of the marginal cost from the sellers' zero profit condition. Defining we values recursively.

\[
\begin{align*}
    u_L &= -c_L + \delta \left( (1 - \gamma_L)u_L + \gamma_L u_H - s \right) \\
    u_H &= -\frac{c_H (1 - \delta + \delta \gamma_L) + c_L \delta \gamma_H}{(1 - \delta + \delta \gamma_H)(1 - \delta + \delta \gamma_L) - \delta^2 \gamma_H \gamma_L}
\end{align*}
\]

Recall that when a searcher finds himself in a low state, he will become more optimistic than the average buyer and so will search again in the next period. If the search, instead, finds himself in a high state, they will never search again in the future, thus their value of being at that state is simply the discounted expected sum of marginal costs.

The indifference condition for the pooling equilibrium defines the pooling prices

\[
p(t) = (s - u_L)(1 - \delta) + (\beta(t) - \delta \beta(t + 1))(u_L - u_H)
\]

As before \( q_L = c_L \) as sellers expect to make no profits from a searcher who is expected to search again in the next period. Then \( q_H(t) \) is computed as \( c_H - \sum_{\tau=t+1}^{\infty} E [p(\tau) - c(\tau)] \).

To insure that the prices above are incentive compatible for the seller, we need that sellers' profits are positive at all times.

We solve for this equilibrium numerically. Figure (4) plots a pooling equilibrium price and the underlying sample path of the marginal costs.

5.3. Semi-separating equilibrium

Although fully separating equilibria do not exist, we can construct a semi-separating equilibrium where buyers know when the marginal cost goes up from \( c_L \) to \( c_H \), but otherwise have no information. Buyers only search when the price goes up (the fraction of searchers depends on the amount of the price increase), sellers charge a the highest price as soon as the marginal cost goes from low to high and then slowly decrease the prices until the price jumps to high again.
We begin constructing this equilibrium by describing buyer beliefs. Let \( \tau_i \) be the \( i \)th time that the marginal cost went from \( c_L \) to \( c_H \). At \( t = \tau_i \) \( (\forall i) \) buyers know with certainty that the marginal state is high. In the periods following the jump to \( c_H \) but before the next jump occurs (in other words in periods between \( \tau_i \) and \( \tau_{i+1} \)), the buyers do now know when specifically the marginal cost dropped down to \( c_L \) again, however they know that if at some point it did, it stayed at \( c_L \). During this period the problem resembles the absorbing state problem with decreasing marginal cost. Beliefs are then summarized by two equations

\[
\beta(\tau_i) = 1 \quad (44)
\]
\[
\beta(t + \tau_i) = (1 - \gamma_H)^t, \quad \forall \tau_i < t < \tau_{i+1} \quad (45)
\]

Using the values of searching in each state (54) and (26) and the buyer’s indifference conditions, we can solve for prices in a standard way. The indifference condition is

\[
\beta(\tau_i + t)u_H + (1 - \beta(\tau_i + t))u_L - s = -p_{\tau_i + t} + \delta \left( \beta(\tau_i + t + 1)u_H + (1 - \beta(\tau_i + t + 1))u_L - s \right), \quad (46)
\]

where \( \tau_i \leq t + \tau_i < \tau_{i+1} \) (Note that this restriction on \( t \) doesn’t guarantee that \( \tau_i + t + 1 < \tau_{i+1} \), in fact the next period the marginal cost can potentially jump from \( c_L \) to \( c_H \) and then \( \tau_i + t + 1 = \tau_{i+1} \) implying that \( \beta(\tau_i + t + 1) = 1 \)). We compute prices in a standard way. Figure 5 plots a sample path of the marginal cost and the corresponding realization of the equilibrium price. We see that the price path is characterized by sharp increases (following transition from \( c_L \) to \( c_H \)) and slow declines. We refer to each of the periods between \( \tau_i \) and \( \tau_{i+1} \) as “ignorance spell”.

Although the prices are easy to compute because of the resemblance to the absorbing case, some equilibrium conditions are less straightforward to calculate. We do not have a clean expression for the introductory prices, sellers’ participation constraint and the equilibrium search following a price increase. We can, however, compute all of these numerically. In particular, given parameter values, we compute sellers’ value in each state. States are characterized by the underlying marginal cost and the time since the last price increase. That is \( v_H(t) \) denotes sellers’ value when the marginal cost is high and has been high for exactly \( t \) periods. We then find that in equilibrium, when a price increase occurs at time \( t \) into the “ignorance spell” the lower bound on the fraction of searchers following a price increase is \( \alpha(t) = 1 - \frac{v_L(t)}{v_L(t+1)} \). The positive expected profit condition is summarized by the equation below.

\[
\lim_{t \to \infty} v_H(t) > 0 \quad (47)
\]

Intuitively sellers have to earn positive profits in the worst possible scenario, being the case where the prices are at their lowest (have converged to the lowest value for a particular “spell”), but the marginal cost is still high. The difference between \( v_H(t) \) and \( v_H(t+1) \) is simply the difference in the pooling price going forward for the period before the next jump. When the pooling price converges these two are practically identical, and so the limit exists.

Refer to the Appendix for the detailed explanation of the algorithm for computing \( v_L(t) \) and \( v_H(t) \).

**Proposition 6.** Given \( (\gamma_L, \gamma_H, \delta, s, c_L, c_H) \) and condition (47) a semi-separating equilibrium exists, and is characterized by sellers charging

\[
p(t) = c_L + s + (1 - \gamma_H)^t \left( c_H - c_L - s \delta \frac{1 - \delta + \delta \gamma_H}{1 - \delta} \right) + \frac{(1 - (1 - \gamma_H)^t) \delta \gamma_L (c_H - c_L)}{1 - \delta + \delta \gamma_H + \delta \gamma_L} (48)
\]
where \( t \) is now redefined as time since the marginal cost last increased from \( c_L \) to \( c_H \). Buyers don’t search unless the price increases (each time the marginal cost increases, prices revert back to \( p(1) \)), in which case they search with probability \( \alpha(t) = 1 - \frac{v_L(t)}{v_L(1)} \) (where \( t \) is the time since last price increase).

\[
q_L = c_L
\]

\[
q_H(t) = c_H - \sum_{\tau=1}^{\infty} \delta^\tau p(t) + \frac{c_H(1 - \delta + \delta \gamma_L) + c_L \delta \gamma_H}{(1 - \delta)(1 - \delta + \delta \gamma_H + \delta \gamma_L)}
\]

\( v_L(t) \) and \( v_H(1) \) are computed numerically for this paper.

5.4. Welfare Comparison

Because of the more complex state transitions, we compute welfare numerically. Although this method does not provide us with clean closed form solutions, we can nevertheless gain important insights and even perform some comparative statics analysis. In addition to computing welfare of separating, pooling and semi-separating equilibria, we also included welfare comparison to full information equilibrium. Although differing in the basic assumption, we thought it would be of interest to get a sense of the value of information.

Welfare computations for the pooling and full-information equilibria are straightforward. Because there is no search in a pooling equilibrium, social welfare is just a discounted sum of expected future marginal costs. Buyer’s welfare is simply a sum of discounted prices that they face in a pooling equilibrium. Seller’s welfare is then the difference between the two.

Computing welfare for the separating equilibrium is equally as straightforward. We already know that sellers’ welfare is \( s \). Social welfare is computed as usual by taking the negative sum of discounted expected costs and subtracting the expected search costs in each period.

Semi-separating equilibrium values are trickier to compute and we describe the basic steps in the appendix.
Interestingly, all the insights from the simple case with absorbing state hold in this more complicated setting. We find that pooling equilibrium is always social welfare improving (as it avoids search) and the welfare gain is fully passed on to the buyers. Sellers are always indifferent between equilibria and are always able to extract the same profit \( s \).

6. Discussion

An interesting extension would be to introduce heterogeneity in sellers. Thus, ex-ante identical sellers have their own realizations of the underlying marginal cost, while the stochastic process generating those values is the same. This extension is interesting in the presence of small number of sellers, otherwise non-sequential search ensures that at any time there is a very high probability of at least one seller with marginal cost still low, thus the option value of search doesn’t adjust over time.

In this paper we do not address price dispersion. The only time when two different prices are offered is the time when a fraction of the buyers search, thus non-searchers are facing posted prices, while searchers face the introductory prices. However, search doesn’t occur on the equilibrium path in the pooling equilibria and only happens at a one point in time in a separating one. By introducing some heterogeneity in search costs, however, we could potentially create a group of buyers who search at all times. Since they would bring zero profits for the sellers, they would not change any major results in the paper.

The one plausible equilibrium that we did not consider yet in this paper is one symmetric to the semi-separating one in section (5.3) where prices are pooling upward and separating downward. In this equilibrium there must be search happening at every period in time, although the probability of search may vary. We conjecture that this equilibrium would lead to the worst social welfare as search costs will be paid in every period in time.

7. Conclusion

In markets with inelastic demand the sole role of a price is distributing the gains from trade. Then the only channel by which pricing affects the social welfare is the amount of search it generates. The best possible outcome then is an equilibrium with no search. This can only be achieved if at each point in time buyers are indifferent between searching and not. To make buyers believe that the underlying marginal cost is high is then beneficial to sellers because it makes the search option less attractive and hence allows for more profit to be extracted from non-searchers. When prices act as signals, sellers are always tempted to charge the highest price they can to signal high marginal cost. As all the prices reach the ceiling they become uninformative and result in buyer search for better deals. We can see that the only equilibrium that can be sustained without search is one where prices are uninformative. The pooling equilibrium presented in this paper is one that maximizes the surplus.

However pooling equilibria imply restrictions on parameter values and are not always feasible. In addition, since the sellers’ are price-setters, it is reasonable to assume that it is the sellers’ welfare that determines which equilibrium is selected. In our setting sellers are indifferent between all equilibria, so all of them have a potential to arise in the data.

Although we find many potential equilibria, one in particular reflect patterns observed in the data. In a semi-separating equilibrium we see that buyers search for a better deal
only when prices increase by a substantial amount (independent on the level). In addition we find the “rockets and feathers” pattern of responding to marginal costs - prices increase instantly following the cost increase, but fall gradually after the costs go down.
A. Proofs

Proof of Proposition 1: First consider a $c_L$ type trying to mimic a $c_H$ type. This will change the value function only in the period where the marginal cost is still low. The incentive compatibility (IC) condition is that the value of deviating is lower than the value of following an equilibrium strategy in that period:

$$(1 - \alpha) \left( \frac{p_H - c_L}{1 - \delta(1 - \gamma_L)} \right) < \left( \frac{p_L - c_L}{1 - \delta(1 - \gamma_L)} \right)$$

The LHS decreases in $\alpha$ while the RHS is constant. Equating the two we find a lower bound on $\alpha$ for this IC condition to be satisfied:

$$\alpha = \frac{c_H - c_L}{c_H - c_L + (1 - \delta(1 - \gamma_L))s}$$

A $c_H$ type deviating to mimic a $c_L$ obtains a different value only insofar as it keeps deviating. The one-step deviation rule applies, and so the IC condition simply becomes

$$p_L - c_H < (1 - \alpha)(p_H - c_H)$$

This gives us an upper bound on $\alpha$

$$\bar{\alpha} = \frac{c_H - c_L}{(1 - \delta(1 - \gamma_L))s}$$

To make sure that $\alpha \in [\max(0, \alpha), \min(1, \bar{\alpha})]$ is a well defined interval we need $\alpha < 1, \alpha < \bar{\alpha}$, and $\bar{\alpha} > 0$. The three conditions are always satisfied under an admissible calibration.

Note that $\bar{\alpha} > 0$ always holds, while $\bar{\alpha}$ is not necessarily less than one. For example when the cost gap is really large, $p_L$ might be less than $c_H$, thus deviating by charging a low price when the costs are high actually leads to negative profits. ■

Proof of Proposition 2: For buyers who have not searched in the past, the no-deviation constraint holds by construction: in each period, they are indifferent between searching and not searching. Next we consider the sellers’ incentives. First, we need to confirm that the seller’s expected future profit is positive for all $t$: if that is not the case, then the seller is better off by charging an infinite price thus making zero profits. Clearly the worst possible scenario for the seller is when the marginal cost switches to $c_H$ at $t = 1$. It follows that, if the expected payoff for a seller in this case is positive, then all other state realizations lead to a positive equilibrium payoff as well.

$$\sum_{t=1}^{\infty} \delta^{t-1}(p(t) - c_H) > 0$$

This imposes a lower bound on the search costs

$$s \geq (1 - \gamma_L)(c_H - c_L)$$

To exclude any other profitable deviations, we simply define off-equilibrium beliefs of buyers to be “worst-case” beliefs. That is, once a buyer observes a price that is off the equilibrium
path, it updates its beliefs to $\beta = 0$ (or $c = c_L$ with probability 1). Consistent with the new beliefs, if the new price is above $p(0)$ — that is, above the indifference point of search — then all buyers search. Then deviating to a price above the equilibrium path results in a loss of all the seller’s buyers, so can not be profitable. Deviating to a lower price is equally not profitable: although it does not result in a loss of buyers, it lowers the profit margin on all transactions.

To complete the equilibrium description, we must include the introductory prices as part of the sellers strategy. As discussed earlier, searchers who discover the true state to be $c_L$ are more optimistic than the average buyer and so keep searching until they find that the state is $c_H$. A searcher who identifies the state to be $c_H$ is less optimistic than the regular buyer, and so never searches again. These buyer strategies, together with the sellers’ zero profit condition, imply the values of $q$ in the proposition.

**Proof of Proposition 3:** Note that the buyers’ equilibrium strategy is optimal by construction of prices. However the sellers’ optimal strategy depends on $\alpha$: the fraction of searchers when the price is high. If $\alpha$ is really large, then the seller might prefer to cut their price to $p_L$, accepting a lower margin but retaining all existing buyers. If $\alpha$ is too low, then it might be optimal for the seller to keep the price high when the cost transitions to a low state. That way he can extract a higher markup from buyers even though a small fraction of them leave every period. Using the prices derived in equations (23) and (24) we can construct the incentive compatibility conditions

\[
(1 - \alpha) \left( \frac{p_H - c_H}{1 - \delta(1 - \gamma_H)(1 - \alpha)} + \frac{\delta \gamma_H p_L - c_L}{1 - \delta(1 - \gamma_H)(1 - \alpha)} \right) \geq \frac{p_L - c_L}{1 - \delta} - \frac{c_H - c_L}{1 - \delta(1 - \gamma_H)}
\]

\[
\frac{p_L - c_L}{1 - \delta} \geq (1 - \alpha) \left( p_H - c_L + \delta \frac{p_L - c_L}{1 - \delta} \right)
\]

The two conditions lead to bounds on $\alpha$

\[
\bar{\alpha} = \frac{c_H - c_L}{s(1 - \delta(1 - \gamma))}
\]

\[
\underline{\alpha} = \frac{\sqrt{((c_H - c_L) + s(1 - \delta(1 - \gamma)))^2 + 4\delta(1 - \gamma)(c_H - c_L) - ((c_H - c_L) + s(1 - \delta(1 - \gamma)))}}{2s\delta(1 - \gamma)}
\]

Since $\underline{\alpha} < 1$, $\bar{\alpha} > 0$ and $\underline{\alpha} < \bar{\alpha}$, this interval $\{\underline{\alpha}, \min(1, \bar{\alpha})\}$ is non-empty and thus the equilibrium exists. ■

**Proof of Proposition 4:** Buyers who have never searched have no profitable deviation by construction of prices. One deviation for the seller is to exit the market all together, so the first thing to check is that buyers make positive expected profits at all times. Consider the worst case scenario for the buyer: the price has converged to a low to $p_{lim} = \lim_{t \to \infty} p(t) = c_L + s$, but the marginal cost remains high. If the seller is profitable in this realization of the marginal cost, then it’s also profitable everywhere on the equilibrium path. Thus we
can construct a participation constraint for the seller.

$$\sum_{t=0}^{\infty} \delta^t \left( p_{\text{lim}} - (1 - \gamma_H)^t c_H - (1 - (1 - \gamma_H)^t) c_L \right) > 0$$

Leading to a lower bound on the difference in marginal costs

$$s \geq (c_H - c_L) \frac{1 - \delta}{1 - \delta + \delta \gamma_H}$$

To exclude any other deviation from the equilibrium path we assume that buyers who observe an off-equilibrium price infer that the marginal cost is low. This makes them more optimistic about prospects of outside options and so the increase in price induces all buyers to search resulting in sellers’ loss of all customers. Since staying on the equilibrium path always leads to positive profits, it must be that this deviation is not profitable.

To complete the equilibrium, we compute the introductory prices that a seller will offer to a searching buyer. This price, of course, depends on the state of the marginal cost and depends on what actions a seller expects from this new buyer. As in the set up with increasing marginal costs, a buyer who discovers $c_H$ is less optimistic about search than an average buyer and so will not search in the future. However a buyer who discovers $c_L$ is more optimistic about the future than the average buyer and so will search every period going forward. The zero profit condition then pins down the equilibrium introductory prices. ■

B. Equilibrium refinement

All the equilibria described in this model satisfy the intuitive criterion. This is easily verified, since increasing price is potentially beneficial for both types of sellers (both when marginal cost is high and low) and decreasing price is never optimal for any seller. Since both types are potentially benefitting from deviation, buyers are not restricted on their off-equilibrium beliefs.

However the equilibria we find in this paper are not “divine”. Divinity requires that the off-equilibrium beliefs attribute probability one to a type that is most profitable from the deviation. Consider a deviation of raising a price above the equilibrium path. The change in markups (the difference between the new price and the equilibrium price) is the same for both $c_L$ and $c_H$ sellers. However the loss from deviating, if it induces searching, is different, because loosing customers is more harmful for low cost sellers as their markup level is higher. This means that buyers observing an increase in price off the equilibrium path should infer that it is the high cost seller that must be deviating. Which is not what we have in our equilibra. Moreover, we can show that there does not exist a “divine” equilibrium in this model. We outline the main idea in the context of the simplest example.

Consider the separating equilibrium of the absorbing state model with decreasing marginal cost. Sellers then charge different prices in different states of the marginal costs, in particular, they charge $p_L$ when $c = c_L$ and $p_H$ when $c = c_H$. We already established that it is

5. The only conceivable reason for a seller to lower its’ price is to potentially signal that it is a lower cost seller. However this will result in more optimistic beliefs by buyers, so the seller would have to charge lower prices going forward than it would have in the equilibrium.
unprofitable for a seller to lower its price no matter the believes the deviation induces. Also raising the price above $p_H$ induces all buyers to show as $p_H$ is the highest indifference price for the most pessimistic beliefs. But suppose a buyer observes a price increase $p_H > \hat{p} > p_L$ from $p_L$. If this deviation does not induce search, then it is clearly profitable for all sellers, including low cost sellers, as it increases a seller’s margin by $\hat{p} - p_L$. Then it must be that this deviation induces a fraction $\alpha > 0$ of consumers to search. If $\alpha > 0$ search, then it must be that the high cost seller is deviating most from this deviation and so buyers should infer that it is the high cost seller that deviated. Recall that a buyer who believes the underlying marginal cost is high has an indifference price $p_H$, which is higher than $\hat{p}$, so it must be that buyers strictly prefer not to search when they observe $\hat{p}$. This is a contradiction to $\alpha < 0$.

\textbf{C. Full information equilibrium}

Consider the case where marginal cost starts at a high state and is switching to low with probability $\gamma_H$. In a full information equilibrium buyers know exactly which state they are in. Then the strategies can be solved in a usual way assuming that $\beta(t) = 1$ when $c = c_H$ and $\beta(t) = 0$ when $c = c_L$. As before we define buyer searching values in each state of the marginal cost

$$u_H = -\frac{c_H}{1 - (1 - \gamma_H)\delta} - \left(\frac{1}{1 - \delta} - \frac{1}{1 - (1 - \gamma_H)\delta}\right)c_L.$$  \hfill (54)

$$u_L = -\frac{c_L}{1 - \delta}$$  \hfill (55)

Sellers always charge a price to make buyers indifferent between searching or not. Then, we can solve for a price from the following two indifference conditions corresponding to high and low state

$$u_H + s = -p_H + \delta ((1 - \gamma_H)u_H + \gamma_H u_L + s)$$  \hfill (56)

$$u_L + s = -p_H + \delta (u_L + s)$$  \hfill (57)

We obtain prices with constant markups

$$p_H = c_H + (1 - \delta)s$$  \hfill (58)

$$p_L = c_L + (1 - \delta)s$$  \hfill (59)

Welfare analysis between pooling and full information equilibrium is straightforward. Normalizing buyer utility from the good to 0, social welfare in both cases is the same and is equal to a negative discounted sum of all marginal costs. buyer welfare is a negative discounted sum of prices that sellers charge and sellers’ welfare is the difference between the two. Recall the pooling equilibrium price is $p_t = (1 - \gamma_H)^t(c_H - c_L) + c_L + \delta s$.
\[ W_{social}^{fi} = W_{social}^{pool} = -\sum_{t=0}^{\infty} \delta^t ((1 - \gamma_H)^t c_L + (1 - (1 - \gamma_H)^t) c_H + (1 - \gamma_H)^t \gamma_H \alpha s) \] (60)

\[ W_{buyer}^{fi} = -\sum_{t=0}^{\infty} \delta^t ((1 - \gamma_H)^t p_H + (1 - (1 - \gamma_H)^t) p_L) \] (61)

\[ W_{seller}^{fi} = W_{social}^{fi} - W_{buyer}^{fi} \] (62)

\[ W_{buyer}^{pool} = -\sum_{t=0}^{\infty} \delta^t p_t \] (63)

\[ W_{seller}^{pool} = W_{social}^{pool} - W_{buyer}^{pool} \] (64)

We compute the differences in welfare for both equilibria

\[ W_{social}^{fi} - W_{social}^{pool} = 0 \] (65)

\[ W_{buyer}^{fi} - W_{buyer}^{pool} = 0 \] (66)

\[ W_{seller}^{fi} - W_{seller}^{pool} = 0 \] (67)

Turns out that both buyers and sellers are ex-ante indifferent about this equilibrium to play. This is surprising, as intuitively we would expect buyers to prefer more information to avoid sellers taking advantage of their ignorance.

### D. Semi-Separating Equilibrium: Value Function and Welfare Computations

Recall a semi-separating equilibrium is one where prices perfectly reveal the time when marginal cost jumps from \( c_L \) to \( c_H \), but do not reflect any information until the next such jump occurs.

Define \( v_H(t) \) the value of the seller with a unit customer base when the marginal state is high and \( t \) period into the last price increase (in our previous notation \( v_H(1) \) will correspond to periods \( \tau_i \forall i \)). Similarly, let \( v_L(t) \) be the value of a seller with unit customer base with low marginal cost \( t \) times into the last price jump (here \( v_L(1) \) is never realized, because in the first period of the price change the state is \( c_H \) by the construction of the equilibrium, however this value will be important for computing incentive compatibility conditions). Also, because in the equilibrium \( p(\tau_i) = p(\tau_j) \) for all \( i, j \), we can refer to any such price as simply \( p(1) \), similarly for any time \( \tau_i \) \( \tau_i - 1 + t < \tau_i + 1 \) the equilibrium price will be \( p(t) \).

In equilibrium, seller’s value function satisfy the following equations

\[ v_H(t) = p(t) - c_H + \delta ((1 - \gamma_H)v_H(t+1) + \gamma_H v_L(t+1)) \] (68)

\[ v_L(t) = p(t) - c_L + \delta ((1 - \gamma_L)v_L(t+1) + \gamma_L (1 - \alpha(t+1))v_H(1)) \] (69)

Thus from \( v_H(t) \) a seller either transitions to \( v_H(t+1) \) where price remains pooling and state remains high, or to \( v_L(t+1) \) where state transitions to \( c_L \) and the price remains that of a pooling equilibrium. From \( v_L(t) \), however one either remains with the pooling equilibrium at \( v_L(t+1) \) or, if the marginal cost increases, restarts the price at \( p(1) \). Because it must
be incentive compatible for firms to not raise their price earlier than they should, we need to introduce search following a price increase, hence \( v_L(t) \) is expecting to lose \( \alpha(t+1) \) customers once the marginal cost increases.

Let us first describe the incentive compatibility constraint for sellers that pins down the optimal search in the equilibrium (really the lower bound of the fraction of searchers, but we will assume the least amount of search for welfare computations, slightly abusing the notation we will say that \( \alpha(t) \) is this lower bound). First note that both high and low cost sellers benefit from deviating to \( p(1) \) signaling the cost jump from \( c_L \) to \( c_H \). However the profit from deviating is larger for the low cost sellers, because they make a larger margin on their buyers. Thus to compute \( \alpha(t) \) we only need to consider the deviations of the low cost seller, the lower bound being resulting in indifference between deviating and not.

\[
v_L(t) = (1 - \alpha(t)) v_L(1) \tag{70}
\]

This gives us the fraction of searchers following a price increase at time \( t \) (recall, \( t \) refers to time into the last increase in price).

\[
\alpha(t) = 1 - \frac{v_L(t)}{v_L(1)} \tag{71}
\]

Since the price in decreasing in time, the \( v_L(t) \) is also decreasing, and thus a fraction of searchers has to go up in order to keep the seller indifferent between deviating or not.

Now that we established all the parameters, the algorithm for solving these value functions is straightforward and is described in the five steps.

1. Guess \( \hat{v}_L(T) \) for a large \( T \). Because \( p(t) \) converges to a single number, we can assume that \( v_H(T) = v_H(T+1) \) and similarly \( v_L(T) = v_L(T+1) \).

2. Solve for \( \hat{v}_H(T) \) from equation (68) assuming that \( v_H(T) = v_H(T+1) \) and \( v_L(T) = v_L(T+1) = \hat{v}_L(T) \).

\[
\hat{v}_H(T) = \frac{p(T) - c_H + \delta \gamma_H v_L(T)}{1 - \delta (1 - \gamma_H)} \tag{72}
\]

3. From equations (69) and (71), solve for \( \frac{v_H(1)}{v_L(1)} \), assuming that \( v_L(T) = v_L(T+1) = \hat{v}_L(T) \).

\[
\frac{v_H(1)}{v_L(1)} = \frac{\hat{v}_L(T) - (p(T) - c_L) - (1 - \gamma_L) \hat{v}_L(T)}{\gamma_L \hat{v}_L(T)} \tag{73}
\]

We solve for all other values recursively. That is, given \( \hat{v}_L(t+1), \hat{v}_H(t+1), \frac{v_H(1)}{v_L(1)} \), compute \( \hat{v}_L(t), \hat{v}_H(t) \) as in steps 4 and 5.

4. Given \( \hat{v}_L(t+1), \hat{v}_H(t+1) \), compute \( \hat{v}_H(t) \) from equation (68)

5. Given \( \hat{v}_L(t+1), \frac{v_H(1)}{v_L(1)} \), compute \( \hat{v}_L(t) \) from equation (69)

6. Compare \( \frac{v_H(1)}{v_L(1)} \) from step 3 to \( \frac{\hat{v}_H(1)}{\hat{v}_L(1)} \) that we get from iterating backward. If not sufficiently small, update the guess for \( \hat{v}_L(T) \) and go back to step 1.
This algorithm gives us not only the values of sellers at each point along the equilibrium path, but it also gives us values for fractions of buyer searching in each scenario.

We compute social welfare by summing up all the discounted expected marginal costs and subtracting the search costs incurred by all searching buyers. The first statistic we already computed in $u_H$. The second is tricky, because for each time period we need to multiply the probability of $c_L$ to $c_H$ transition (which is when search occurs) and then for all $t$ sum up $\alpha(t)$ times the probability that that particular period happened $t$ periods after the last time change.

Seller’s welfare is $v_H(1)$ from our numerical solution (assuming that $c(1) = c_H$). Buyer’s welfare is simply the difference between the two.
References


