Abstract

Consider a vertically integrated input monopolist, $M$, that supplies to its downstream unit, division 1, and a downstream rival, firm 2. Downstream competition may be differentiated-products Bertrand or Cournot. We compare two organizational structures for the integrated firm. Under Centralization, $M$ sets an input price only to firm 2, and division 1’s choice (price or quantity) maximizes integrated profit, including the upstream profit from total input sales. With Delegation, $M$ sets observable input prices, $w_1$ and $w_2$, to both downstream competitors, division 1 maximizes only its own (downstream) profit, and $M$ internalizes division 1’s profit when setting input prices. We show that these structures are not equivalent: the integrated firm generically earns higher profit with Delegation. This result holds whatever the nature of downstream competition.

Charging an observable input price $w_1$ is valuable solely because—by signaling a change in division 1’s downstream choice—it provides the integrated firm with an additional instrument for altering the choice of firm 2, beyond relying solely on the input price $w_2$. Indeed, we show that Delegation yields the same profit as the integrated firm could earn if it remained centralized but could act as a Stackelberg leader downstream and, hence, could influence firm 2’s choice directly through its own downstream choice (instead of indirectly via $w_1$ under Delegation).

The desired change in firm 2’s choice is a priori more complex than in standard two-stage models of competition because here firm 2 is both a downstream rival and an input customer. Seemingly, there is a tradeoff: e.g. with Bertrand competition, an increase in firm 2’s price benefits division 1 but reduces the profitable input sales to firm 2. Despite the seeming ambiguity, we establish a sharp result: starting from the level of $w_2$ under Centralization, with Delegation the integrated firm wants to move firm 2’s choice in the opposite direction to the prediction of standard two-stage games with no supply relationship: it wants to induce a reduction in firm 2’s price under Bertrand competition or an increase in firm 2’s quantity under Cournot competition. These results do not hinge on whether, in each case, the downstream variables are strategic substitutes or strategic complements (though the direction of change in $w_1$ will vary).

If the integrated firm charges firm 2 a two-part tariff for the input instead of a linear price, the first two results extend—Delegation still dominates Centralization and achieves the same outcome as Stackelberg leadership—and we show how the incentives to shift firm 2’s choice are altered.
1. Introduction

In the familiar successive monopoly setting with linear input pricing, vertical integration eliminates double marginalization and yields equal profit under two alternative organizational structures for the integrated firm: the downstream variable (price or quantity) is chosen to maximize overall profit, and there is no economically meaningful input price; or the downstream division maximizes its own profit and buys the input from the upstream division at marginal cost. Instead of downstream monopoly, consider strategic competition—duopoly for simplicity—and partial integration: the input monopolist, $M$, is vertically integrated with a downstream unit, division 1, and continues supplying firm 2. Does the choice of organizational structure now matter?

We compare two structures. Under Centralization, $M$ sets an input price only to firm 2, and then downstream variables are chosen simultaneously, with division 1 acting to maximize integrated profit, including the upstream profit from total input sales. Under Delegation, $M$ sets observable input prices to both downstream competitors, then downstream choices are made simultaneously, with division 1 acting to maximize only its own profit (as explained later, this assumption can be relaxed substantially); here, $M$ internalizes division 1’s profit when setting input prices. We show that the integrated firm generically earns higher profit with Delegation — whatever the downstream competition (in prices or quantities, and strategic substitutes or strategic complements) and whether the integrated firm uses linear pricing for the input to the rival or a two-part tariff.

The logic is that commitment to an observable input price $w_1$ for the downstream division provides the integrated firm with an additional instrument for affecting the choice of firm 2 beyond relying solely on its input price $w_2$. Under Centralization, the integrated firm’s downstream choice is based on a shadow marginal cost equal to the input resource cost plus the opportunity cost arising if division 1’s output reduces input sales to firm 2. In equilibrium, firm 2 correctly anticipates division 1’s shadow marginal cost and, hence, its downstream choice. Under Delegation, the upstream supplier can set an observable and economically relevant $w_1$ different from the (ex post) shadow marginal cost. The value of this additional instrument $w_1$ derives entirely from using it to signal a change in division 1’s downstream choice in order to alter firm 2’s choice. Indeed, we show that Delegation
yields the same profit as the integrated firm could earn if it remained centralized but could act as a Stackelberg leader downstream and, hence, could influence firm 2’s choice directly through its own choice (instead of indirectly via $w_1$).

More surprisingly, the change in firm 2’s choice desired by the integrated firm runs opposite to that in standard two-stage models of competition with no supply relationship (e.g. Fudenberg and Tirole, 1984; Bulow, Geanakoplos, and Klemperer, 1985), where the goal is to make the rival’s behavior “softer.” Because firm 2 here is both a downstream competitor and an input customer, its choices have opposing effects on the integrated firm’s profits. For example, when firm 2 and division 1 engage in (differentiated) Bertrand competition, a rise in firm 2’s price would increase division 1’s profit but lower the upstream division’s profit from input sales to firm 2. Despite these opposing effects, we establish a strong result: starting from the level of $w_2$ under Centralization (with linear input pricing), under Delegation the integrated firm wants to induce (via $w_1$) a decrease in firm 2’s price if the downstream competition is Bertrand, or an increase in firm 2’s output if the competition is Cournot. These results hold whether the downstream variables (prices or quantities) are strategic substitutes or strategic complements (though the required level of $w_1$ will differ). We also discuss how the incentives change if the integrated firm charges firm 2 a two-part tariff.

The closest work to ours is by Arya, Mittendorf, and Yoon (2008). Our analysis is broader in several important respects, e.g. allowing general non-linear demands, Bertrand competition (not only Cournot), and two-part tariffs; but they consider a richer set of potential organizational arrangements, as we discuss in the concluding section. There, we also contrast our work to that of Bonanno and Vickers (1988) on the strategic advantage of vertical separation by rival suppliers, and to the literature on strategic advantages of establishing competing divisions (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). We also discuss briefly the plausibility of Delegation. Empirically, vertically integrated firms have broad latitude in how they structure their inter-division dealings, so the strategic advantages of the Delegation structure may have practical implications.

In the rest of the paper, Section 2 describes the model and Section 3 presents the results, first for linear pricing of the input and then for two-part tariffs. Section 4 concludes.
2. The Setting

Consider an input monopolist, $M$, that supplies to its downstream unit, division 1, and to an independent downstream rival, firm 2. The downstream choice variables $x_1$ and $x_2$ are either per-unit prices ($p_1$ and $p_2$) or quantities ($q_1$ and $q_2$), thereby allowing either Bertrand or Cournot competition downstream.\(^1\) We consider two games, representing alternative organizational structures of the partially integrated firm.

**Centralization:** First, $M$ publicly sets a per-unit input price $w_2$ to firm 2. (Two-part tariffs are discussed in Section 3.2.) Then, division 1 and firm 2 simultaneously set downstream variables $x = (x_1, x_2)$, consumers purchase, and firm 2 pays for $M$'s input. Firm 2 chooses $x_2$ to maximize its profit $\Pi_2(x; w_2)$. There is no explicit input price to division 1, and $x_1$ is chosen to maximize integrated profit, i.e. division 1’s profit plus the profit from input sales. $M$ sets $w_2$ to maximize integrated profit. This is a standard representation of behavior by an integrated firm.\(^2\)

**Delegation:** First, $M$ publicly commits to a pair of input prices $w = (w_1, w_2)$ where $w_1$ denotes the price to division 1. Given these prices, division 1 and firm 2 choose downstream variables $x$ simultaneously, consumers purchase, and $M$ receives input payments. Division 1 chooses $x_1$ to maximize only its profit $\Pi_1(x; w_1)$ as would an independent firm, but supplier $M$ now sets both $w_1$ and $w_2$ to maximize integrated profit. (As explained later, the assumption that division 1 maximizes only its profit can be relaxed substantially.)

For any pair of publicly observed input prices $w$ under Delegation, assume there exists a unique downstream equilibrium in pure strategies, with the choice of downstream rival $k$ ($k = 1, 2$) denoted $X_k^*(w)$ and its corresponding output level denoted $Q_k^*(w)$. As a

\(^1\) The integrated input monopolist prefers to maintain firm 2 as an active buyer of the input rather than foreclose firm 2 (under both Bertrand and Cournot downstream competition) if firm 2 is a sufficiently efficient competitor. See Arya, Mittendorf, and Sappington (2008).

function of the downstream variables $x$, a firm’s output level is denoted $Q_k(x)$. To simplify, we assume for each firm fixed proportions, one-to-one, between the output and input quantities, so the function $Q_k^*(w)$ is also $k$’s input demand (and $Q_k(x)$ is the input demand conditional on the downstream variables). We also assume that the marginal cost of producing the input is constant at $c$, and that all the relevant functions are differentiable.\footnote{We thus have $Q_k^*(w) \equiv Q_k(X_1^*(w), X_2^*(w))$. Similarly, we define $\Pi_k^*(w) \equiv \Pi_k(X_1^*(w), X_2^*(w); w_k)$. Under Cournot competition, $Q_k(x) = x_k$ and $Q_k^*(w) = X_k^*(w)$.}

The integrated firm’s total profits are given by:

$$V(x; w_2) = \Pi_1(x; w_1) + (w_1 - c)Q_1(x) + (w_2 - c)Q_2(x).$$

(1)

Note that this integrated profit function depends on the downstream variables $x$ and the input price $w_2$, but not directly on $w_1$ since input payments from division 1 are a pure transfer (though under Delegation $w_1$ will affect $V$ indirectly, by changing the equilibrium values of $x$).

Under Delegation, supplier $M$ sets $w_1$ and $w_2$ to maximize

$$V^D(w) \equiv V(X_1^*(w), X_2^*(w); w_2).$$

(2)

Under Centralization, denote the downstream equilibrium choices by the functions $X_1^C(w_2)$ and $X_2^C(w_2)$. Supplier $M$ now sets $w_2$ to maximize

$$V^C(w_2) \equiv V(X_1^C(w_2), X_2^C(w_2); w_2).$$

(3)

Let $w_2^C$ denote the profit-maximizing choice; $x_1^C$ and $x_2^C$ denote the resulting equilibrium downstream choices; and $V^C$ the corresponding maximized profit of the integrated firm.

\footnote{We thus are implicitly assuming there are no binding capacity constraints and the downstream products are differentiated (i.e. imperfect substitutes). We also assume the downstream firms’ reaction functions are either strictly increasing (strategic complements) or strictly decreasing (strategic substitutes), and have slopes smaller than unity in absolute value. In addition, we assume $\partial \Pi_k / \partial x_k$ is either strictly increasing or strictly decreasing in $w_k$, so that $X_k^*$ is either strictly increasing or strictly decreasing in $w_k$ (for $k=1,2$). All these conditions are assumed to hold over the relevant range of the variables.}
We now offer two useful observations. First, the best response function of firm 2 is the same under both regimes:

\[ R_2(x_1; w_2) \equiv \arg \max_{x_2} P_2(x; w_2). \] (4)

Second, although there is no explicit input price \( w_1 \) under Centralization, the integrated firm faces a shadow marginal cost (\( C_1 \) below) of supplying the input to its downstream division. This shadow marginal cost is equal to the resource cost \( c \) plus any opportunity cost due to reduced input sales to firm 2 resulting from 1’s expansion:

\[ C_1(w_2) \equiv c + (w_2 - c)D_{21}(w_2), \] (5)

where \( D_{21} \equiv -\frac{\partial q_2}{\partial q_1} \) is the input diversion ratio—i.e. decreased input sales to firm 2 per extra unit of input to division 1. (The input diversion ratio \( D_{21} \) is evaluated at \( x_1 = X^F_1(w_2) \) and \( x_2 = X^C_2(w_2) \), hence is a function of \( w_2 \).) Note that \( D_{21} > 0 \) if downstream competition is Bertrand, since division 1’s output expansion (induced by cutting price) displaces some sales of firm 2; and \( D_{21} = 0 \) if competition is Cournot, since division 1 then takes firm 2’s output, and thus also firm 2’s input purchases, as given.\(^5\)

3. Results

We now compare the two regimes, starting with the case where the integrated firm uses linear pricing for the input to firm 2, and then considering also two-part tariffs.

3.1 Linear Pricing of the Input

Under Delegation, let \( W_1^D(w_2) \) denote the integrated firm’s optimal choice for \( w_1 \) given \( w_2 \), i.e. \( W_1^D(w_2) \equiv \arg \max_{w_1} V^D(w) \). We establish the following result.

\(^5\) Arya, Mittendorf, and Sappington (2008) invoke this distinction to show that, unlike in standard duopoly, Bertrand competition downstream can yield higher prices than Cournot when a partially integrated input monopolist sells also to a downstream rival, because the monopolist internalizes a higher opportunity cost under Bertrand than under Cournot.
**Proposition 1.** (i) Under Delegation, the integrated firm can achieve the same profit as under Centralization by setting \( w_2 = w_2^C \) and \( w_1 = C_1(w_2^C) \), i.e. \( V^C = V^D(C_1(w_2^C), w_2^C) \).

(ii) Therefore, the integrated firm’s equilibrium profit is (weakly) higher with Delegation, i.e. \( V^C = V^D(C_1(w_2^C), w_2^C) \leq V^D(W_1^D(w_2^C), w_2^C) \leq V^D(w_1^D, w_2^D) = V^D \).

**Proof.** (i) Under Delegation, suppose the integrated firm were to set \( w_2 = w_2^C \) and \( w_1 = C_1(w_2^C) \), i.e. the same input price to firm 2 as with Centralization, and an input price to division 1 equal to the shadow marginal cost prevailing with Centralization. This would induce the same downstream equilibrium choices as with Centralization:

\[
X_1^*(C_1(w_2^C), w_2^C) = x_1^C \quad \text{and} \quad X_2^*(C_1(w_2^C), w_2^C) = x_2^C.
\]

To see this, observe first that under Delegation, division 1’s best response function maximizes \( N_1(x; w_1) \) and, hence, is implicitly determined by:

\[
\frac{\partial N_1}{\partial x_1} = 0,
\]

whereas with Centralization, division 1’s best response function maximizes \( V(x; w_2) \) and, hence, using (1), is implicitly determined by

\[
\frac{\partial N_1}{\partial x_1} + (w_1 - c) \frac{\partial Q_1}{\partial x_1} + (w_2 - c) \frac{\partial Q_2}{\partial x_1} = 0.
\]

Setting \( w_1 = C_1(w_2) \) from (5) makes (7) identical to (8), so division 1’s best response will be the same function of \( w_2 \) and \( x_2 \) in both cases. Since firm 2’s best response to \( w_2 \) and the expected \( x_1 \) is also the same function in both cases, setting \( w_2 = w_2^C \) and \( w_1 = C_1(w_2^C) \) under Delegation would replicate the Centralization outcome.

(ii) Under Delegation, the integrated firm’s optimal choice for \( w_1 \) conditional on \( w_2 = w_2^C \) is typically not \( C_1(w_2^C) \). Profit could be (weakly) increased by setting \( w_1 \) at the optimal level \( W_1^D(w_2^C) \). (The direction of desired deviation in \( w_1 \) will be discussed later in Corollary 1.) The actual optimum will generally involve changing both \( w_1 \) and \( w_2 \), thereby further increasing profit compared to Centralization.
It is useful to clarify why the ability to charge division 1 an observable input price \( w_1 \neq C_1(w_2) \) under Delegation benefits the integrated firm. The direct effect of changing \( w_1 \) (holding \( x \) constant) on integrated profit is zero because it is a pure transfer; the change in \( x_1 \) induced by a small change in \( w_1 \) also has no effect on integrated profit because, given \( w_2 \) and \( x_2 = X_2^*(w_2) \), setting \( w_1 = C_1(w_2) \) already induces the \( x_1 \) that maximizes integrated profit. Therefore, a small change in \( w_1 \) affects integrated profit solely by altering firm 2’s choice, \( x_2 \), in response to the expected change in \( x_1 \). The same outcome could be achieved if, under Centralization, the integrated firm could act as a Stackelberg leader in the downstream competition, as in the following game.\(^6\)

*Leadership:* As with Centralization, \( M \) publicly commits only to \( w_2 \). But now downstream choices occur sequentially, with first the integrated firm choosing its downstream variable \( x_1 \) to maximize integrated profit (not division 1’s profit) and firm 2 then choosing its best response, \( x_2 = R_2(x_1; w_2) \).

**Proposition 2.** The integrated firm’s profit is the same under Delegation or Leadership.

**Proof.** Under Leadership, the integrated firm sets \( w_2 \) and \( x_1 \) to maximize:

\[
V(x_1, R_2(x_1; w_2); w_2).
\]  
(9)

Under Delegation, let \( w_1 = W_1(x_1; w_2) \) be the inverse function of \( x_1 = X_1^*(w_1, w_2) \). Setting \( w_1 \) and \( w_2 \) to maximize \( V(X_1^*(w), X_2^*(w); w_2) \) is equivalent to setting \( w_2 \) and \( x_1 \) to maximize:

\[
V(x_1, X_2^*(W_1(x_1; w_2), w_2); w_2).
\]  
(10)

Since \( X_2^*(W_1(x_1; w_2), w_2) \equiv R_2(x_1; w_2) \), the two maximization problems are identical.

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\(^6\) Lu, Moresi, and Salop (2007) show the same result as the ensuing Proposition 2 assuming Bertrand competition. On the general connection between strategic delegation and Stackelberg leadership in the competition game, see Vickers (1985, sections I and II). Adapted to our setting, his agent appointment game—that precedes downstream competition—corresponds to whether the integrated firm initially adopts the Centralization structure and sets only an input price \( w_2 \) or the Delegation structure and sets an additional input price \( w_1 \). The agent appointment game maximizes the integrated firm’s profit if and only if it implements the same downstream outcome as Stackelberg leadership by the integrated firm.
Our Delegation structure in which division 1 maximizes solely its downstream profit offers one way to implement the downstream Stackelberg leader outcome for the integrated firm. The same outcome could be implemented under alternative objective functions for division 1, as long as division 1’s downstream choice is affected by the input price \( w_1 \) (and not entirely independent of it as occurs when division 1 treats \( w_1 \) as a pure transfer within the integrated firm). We will expand on this point in the Conclusion.

Delegation and public commitment to an input price \( w_1 \), therefore, are a means of shifting division 1’s best response function in the ensuing downstream game so as to change firm 2’s behavior. The general strategic incentives for commitment through vertical delegation are well known (e.g. Vickers, 1985; Bonnano and Vickers, 1988; Caillaud and Rey, 1994). However, the analysis of incentives is seemingly more complex here, because here firm 2 is both a downstream rival and an input customer. Nevertheless, we can establish a fairly general result. Recalling that \( x_k = p_k \) under Bertrand competition and \( x_k = q_k \) under Cournot \((k = 1,2)\), we have:

**Proposition 3.** At \((x_1, x_2, w_2) = (x_1^C, x_2^C, w_2^C)\), \( V \) is decreasing in \( x_2 \) if downstream competition is Bertrand and increasing in \( x_2 \) if downstream competition is Cournot.

**Proof.** From (2) and (1), we have:

\[
\frac{\partial V^C}{\partial w_2} = \frac{\partial V}{\partial x_1} \frac{\partial x_1^C}{\partial w_2} + \frac{\partial V}{\partial x_2} \frac{\partial x_2^C}{\partial w_2} + Q_x^C = \frac{\partial V}{\partial x_2} \frac{\partial x_2^C}{\partial w_2} + Q_x^C = 0, \tag{11}
\]

where the second equality follows from \( \partial V / \partial x_1 = 0 \) at \((x_1, x_2, w_2) = (x_1^C, x_2^C, w_2^C)\). It then follows from (12) and \( Q_x^C > 0 \) that \( (\partial V / \partial x_2)(\partial X_2^C / \partial w_2) < 0 \). If downstream competition is Bertrand, we have \( \partial X_2^C / \partial w_2 > 0 \), implying \( \partial V / \partial x_2 < 0 \); if it is Cournot, \( \partial X_2^C / \partial w_2 < 0 \), implying \( \partial V / \partial x_2 > 0 \).

Observe that Proposition 3 identifies opposite incentives to those in a standard duopoly environment where firm 1 does not supply firm 2.\(^7\) There, starting from the

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\(^7\) See, for example, Fudenberg and Tirole (1984), Bulow, Geanakoplos, and Klemperer (1985), Fershtman and Judd (1987), or the survey by Shapiro (1989).
equilibrium with simultaneous downstream choices and holding $x_1$ constant, firm 1 would gain from a rise in 2’s price under Bertrand competition or a fall in 2’s output under Cournot. This is the usual “softening downstream competition” effect. It is present also in our context, but is necessarily dominated by the opposing input supply effect. Given that the integrated firm under Centralization sets the input price to firm 2 at the profit-maximizing level, $w_2^C$, a small reduction in $w_2$ would have the following equal but opposite effects: profit from inframarginal input sales would fall, implying that profit must rise (equally) from increased input sales—even after incorporating the loss in division 1’s downstream profit caused by firm 2’s expansion. Thus, holding $w_2$ constant at $w_2^C$, the integrated firm would gain on balance, despite the loss to division 1, if firm 2 were exogenously to increase its input purchases and downstream sales.

Building on the above observation, the next result identifies the integrated firm’s incentive to change $w_1$ under Delegation, relative to the shadow marginal cost under Centralization, $C_1(w_2^C)$.

**Corollary 1.** Holding $w_2$ constant at $w_2^C$, under Delegation:

(i) With Bertrand competition, the integrated firm wants to induce a fall in 2’s price, implying $W_1^D(w_2^C) < (>) C_1(w_2^C)$ if prices are strategic complements (substitutes).

(ii) With Cournot competition, the integrated firm wants to induce a rise in 2’s quantity, implying $W_1^D(w_2^C) > ( <) C_1(w_2^C)$ if quantities are strategic substitutes (complements).

**Proof.** Starting at $w_2 = w_2^C$ and $w_1 = C_1(w_2^C)$, under Delegation the integrated firm’s desired change in $w_1$ is determined by the sign of (from (3)):

$$
\frac{\partial V^D}{\partial w_1} = \frac{\partial V}{\partial x_1} \frac{\partial x_1^*}{\partial w_1} + \frac{\partial V}{\partial x_2} \frac{\partial x_2^*}{\partial w_1} = \frac{\partial V}{\partial x_2} \frac{\partial x_2^*}{\partial w_1}.
$$

(12)

Observe that $\partial X_2^*/\partial w_1 = (\partial R_2/\partial x_1)(\partial X_1^*/\partial w_1)$, and we assumed $\partial X_1^*/\partial w_1 > (<) 0$ if downstream competition is Bertrand (Cournot), while by definition $\partial R_2/\partial x_1 > (<) 0$ if downstream variables are strategic complements (substitutes).

As noted, fixing $w_1$ in our setting is akin to an “investment” that affects marginal cost in a standard two-stage game and thereby shifts division 1’s best-response function in the
downstream competition. The change in \( w_1 \) needed to induce the desired change in \( x_2 \) will therefore depend on familiar issues — whether downstream competition is in prices or quantities and whether these choice variables are strategic complements or substitutes. Since the basic logic is the same for parts (i) and (ii) of Corollary 1, we will explain only (i).

For Bertrand competition, we know from Proposition 3 that the integrated firm’s overall profit would increase if firm 2 were to reduce its price \( p_2 \) (holding \( p_1 \) and \( w_2 \) constant at their Centralization equilibrium values). Under Delegation, the integrated firm can induce firm 2 to cut its price by lowering \( w_1 \) to signal a reduction in division 1’s price \( p_1 \) in the “normal” case where prices are strategic complements (or via an increase in \( w_1 \) if prices are strategic substitutes). A small change in \( p_1 \) from the initial equilibrium has a zero first-order effect on the integrated firm’s profit and, hence, the change in \( w_1 \) is profitable. Essentially, lowering \( p_1 \) is an imperfect way to shrink firm 2’s margin and reduce double marginalization in input sales to firm 2.

Finally, we note that Proposition 3 identifies the incentives regarding \( w_1 \) under Delegation while holding \( w_2 \) constant at the Centralization solution \( w_2^C \). In equilibrium, the integrated firm would typically adjust also \( w_2 \). However, since the adjustment in \( w_2 \) is only in response to the change in \( x_2 \) (induced by \( w_1 \)), one would expect the equilibrium change in \( x_2 \) to track the direction indicated by Proposition 3 and Corollary 1. This expectation is confirmed for the linear example reported in the Appendix.

### 3.2 Two-Part Tariff for the Input

Suppose that instead of linear pricing of the input, the integrated firm uses a two-part tariff: a pair \((w_2, f_2)\), where \( f_2 \) is a fixed upfront fee. Proposition 1 continues to hold—the integrated firm still earns (weakly) higher profit under Delegation. The logic is as follows. Denote the downstream choices that maximize overall industry profit (downstream plus

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9 However, because the decrease in \( p_2 \) is induced by lowering \( p_1 \), firm 2’s output is likely to decline on balance relative to Centralization (still holding \( w_2 = w_2^C \)). The linear demand example in the Appendix supports this intuition.
upstream) as the “monopoly solution,” $x_1^m$ and $x_2^m$. Under Delegation, the integrated firm can attain this monopoly profit, by setting the marginal input prices at the levels $w_1^m$ and $w_2^m$ that will induce division 1 and firm 2 to choose (unilaterally) $x_1^m$ and $x_2^m$, and setting $f_2$ to extract firm 2’s profits.$^{10}$

With Centralization, the integrated firm generally cannot implement the monopoly solution even with two-part tariffs. The ability to charge firm 2 an upfront fixed fee will typically lead the integrated firm to reduce $w_2$ below the optimal level for linear pricing. However, the fixed fee does not affect the shadow marginal cost of supplying to division 1 as a function of $w_2$, which remains $C_1(w_2)$ from (5). Generally, this will not be the integrated firm’s optimal $w_1$ conditional on $w_2$ under Delegation. For example, suppose that maximizing industry profit requires a symmetric outcome downstream, $x_1 = x_2 = x^m$, and equal input marginal prices under Delegation, $w_1 = w_2 = w^m$. Under broad conditions, we have $C_1(w_2) < w_2$. Thus, Centralization cannot achieve the downstream monopoly outcome: if the integrated firm sets $w_2 = w^m$, its shadow marginal cost will be $C_1(w^m) < w^m$, yielding asymmetric downstream choices skewed to favor division 1.

Since Delegation attains the monopoly outcome, Proposition 2 also continues to hold. Under Leadership, the integrated firm can achieve the monopoly outcome by setting $w_2 = w_2^m$ and $x_1 = x_1^m$, which will induce firm 2 to set $x_2 = x_2^m$. And by definition, it can do no better than the monopoly outcome. Thus, Delegation and Leadership yield the same payoff.

Unlike Propositions 1 and 2 that extend to two-part tariffs for the input, the direction in which the integrated firm wants to shift firm 2’s choice when moving from Centralization to Delegation can differ with two-part tariffs from the pattern shown in Proposition 3 with linear pricing of the input. Under Centralization, if firm 2 accepts the

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$^{10}$ When division 1 and firm 2 are competitors, to implement $x_1^m$ and $x_2^m$ the integrated firm typically must set the marginal input prices above the marginal cost $c$. The qualitative results do not change if firm 2 has an alternative source of supply and, as a result, the integrated firm cannot extract all the profits of firm 2.

$^{11}$ From (5), $C_1 = c$ with Cournot competition (since $D_{21} = 0$) and, typically, $C_1 < w_2$ with differentiated Bertrand competition (since typically we have $w_2 > c$ and $D_{21} < 1$ when both firms use the input in the same fixed proportion to output).
two-part tariff offer \((w_2, f_2)\) (as it will in equilibrium), the downstream outcome is given by
the functions \(X_1^C(w_2)\) and \(X_2^C(w_2)\) as with linear pricing. Denote firm 2’s profit gross of the
fixed fee as \(\Pi_2^C(w_2) = \Pi_2(X_1^C(w_2), X_2^C(w_2); w_2)\). In equilibrium, the integrated firm will
extract firm 2’s profit by setting \(f_2 = \Pi_2^C(w_2)\). Therefore, its profit function is:
\[ V^{C,T}(w_2) = V^C(w_2) + \Pi_2^C(w_2), \]
where \(V^C(w_2)\) is the same as with linear pricing (see (3)). The integrated firm chooses
\(w_2^{C,T} \equiv \arg\max_{w_2} V^{C,T}(w_2)\), and induces the equilibrium
downstream values \(x_1^{C,T}\) and \(x_2^{C,T}\).

**Proposition 4.** At \((x_1, x_2, w_2) = (x_1^{C,T}, x_2^{C,T}, w_2^{C,T})\):

(i) \(V\) is decreasing in \(x_2\) if downstream competition is Bertrand and prices are strategic
complements (and increasing or decreasing in \(x_2\) if prices are strategic substitutes).

(ii) \(V\) is decreasing in \(x_2\) if downstream competition is Cournot and quantities are strategic
substitutes (and increasing in \(x_2\) if quantities are strategic complements).

**Proof.** Let \(R_1^C(x_2, w_2)\) denote division 1’s reaction function under Centralization. The
analogue to (12) is now:

\[
\frac{\partial V^{C,T}}{\partial w_2} = \frac{\partial V^C}{\partial w_2} + \frac{\partial \Pi_2^C}{\partial w_2} = \frac{\partial \Pi_2}{\partial x_2} \frac{\partial x^C_2}{\partial w_2} + Q^C_2 - \frac{\partial \Pi_2}{\partial x_1} \frac{\partial x^C_1}{\partial w_2} = 0, \quad \text{or} \tag{13a}
\]

\[
\frac{\partial V^{C,T}}{\partial w_2} = \frac{\partial V}{\partial x_2} \frac{\partial x^C_2}{\partial w_2} + \frac{\partial \Pi_2}{\partial x_1} \left( \frac{\partial R^C_1}{\partial x_2} \frac{\partial x^C_1}{\partial w_2} + \frac{\partial R^C_1}{\partial w_2} \right) = 0. \tag{13b}
\]

(i) For Bertrand competition: \(\partial X_2^C/\partial w_2 > 0, \partial \Pi_2/\partial x_1 > 0,\) and \(\partial R_1^C/\partial w_2 > 0\) (since the
shadow marginal cost \(C_1(w_2)\) in (5) increases with \(w_2\)); thus, (13b) implies \(\partial V/\partial x_2 < 0\) if
prices are strategic complements (\(\partial R_1^C/\partial x_2 > 0\)). (If \(\partial R_1^C/\partial x_2 < 0\), the term in parentheses
in (13b) has an ambiguous sign, hence so does \(\partial V/\partial x_2\).)

(ii) For Cournot competition: \(\partial X_2^C/\partial w_2 < 0, \partial \Pi_2/\partial x_1 < 0,\) and \(\partial R_1^C/\partial w_2 = 0\) (there is no
input diversion in Cournot, so the shadow marginal cost \(C_1(w_2)\) is always equal to \(c\)); thus,
(13b) implies \(\partial V/\partial x_2 < 0\) if quantities are strategic substitutes (\(\partial R_1^C/\partial x_2 < 0\), and
\(\partial V/\partial x_2 > 0\) if instead \(\partial R_1^C/\partial x_2 > 0\). (The sign is now unambiguous in both cases because
the shadow marginal cost does not depend on \(w_2\), unlike for Bertrand competition.)
Thus, for Bertrand competition and the “normal” case of strategic complements, we have the same pattern as shown in Proposition 3 with no fixed fee (i.e. \( f_2 = 0 \)): starting at the Centralization solution, the integrated firm wants to induce a reduction in \( p_2 \). But for Cournot competition and the “normal” case of strategic substitutes, the pattern is now reversed since here the integrated firm wants to induce a *decrease* in \( q_2 \).

These patterns can be understood by comparing the optimal input pricing conditions (13a) or (13b) to condition (11) for linear pricing. Rearranging (11) gives
\[
\frac{\partial V}{\partial x_2}(\partial X_C^f / \partial w_2) = - Q_C^{2,12} \text{With two-part tariffs, rearranging (13a) gives}
\]
\[
(\partial V/\partial x_2)(\partial X_C^f / \partial w_2) = -(\partial \Pi_2 / \partial x_1)(\partial X_C^f / \partial w_2). \text{ Thus, term } Q_C^2 \text{ is absent: changing } w_2 \text{ no longer affects the integrated firm’s net profit from inframarginal input sales to firm 2, due to the required adjustment in the fixed fee which fully extracts firm 2’s profit. Instead, the novel effect under a two-part tariff is } (\partial \Pi_2 / \partial x_1)(\partial X_C^f / \partial w_2): \text{ changing } w_2 \text{ alters the integrated firm’s equilibrium downstream choice, } X_C^1, \text{ which impacts firm 2’s expected profit—an effect now internalized by the integrated firm via its fixed fee to firm 2. Since } Q_C^2 > 0, \text{ if } (\partial \Pi_2 / \partial x_1)(\partial X_C^f / \partial w_2) > 0 \text{ then } \partial V / \partial x_2 \text{ will have the same sign as under linear pricing; while if } (\partial \Pi_2 / \partial x_1)(\partial X_C^f / \partial w_2) < 0, \text{ the sign of } \partial V / \partial x_2 \text{ will switch.}

For Bertrand competition, \( \partial \Pi_2 / \partial x_1 > 0 \). Thus, \( \partial V / \partial x_2 \) will have the same sign as under linear pricing if \( \partial X_C^f / \partial w_2 = [(\partial R_C^f / \partial x_2)(\partial X_C^f / \partial w_2) + \partial R_C^1 / \partial w_2] > 0 \), which holds if \( \partial R_C^f / \partial x_2 > 0 \) (since \( \partial X_C^f / \partial w_2 > 0 \) and \( \partial R_C^1 / \partial w_2 > 0 \) for price competition). In words: a small cut in \( w_2 \) and, hence, in firm 2’s equilibrium downstream price will—in the “normal” case where prices are strategic complements (\( \partial R_C^f / \partial x_2 > 0 \))—induce a reduction in firm 1’s downstream price, which lowers firm 2’s profit and the attainable fixed fee \( f_2 \). At the optimal \( w_2 \), this loss in \( f_2 \) due to a small cut in \( w_2 \) is just offset by a *gain* in variable profits from the induced reduction in \( p_2 \). Thus, starting from the two-part tariff equilibrium and assuming prices are strategic complements, the integrated firm would gain from an exogenous reduction in \( p_2 \), as under linear pricing of the input.

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12 With Bertrand competition \( \partial X_C^f / \partial w_2 > 0 \), implying \( \partial V / \partial x_2 < 0 \), whereas both inequalities are reversed under Cournot competition, as stated in Corollary 1.

13 Under linear pricing, a reduction in \( w_2 \) decreases profits, instead, from inframarginal input sales to firm 2.
For Cournot, \( \partial \Pi_2 / \partial x_1 < 0 \), so \( \partial V / \partial x_2 \) will switch sign compared to linear pricing if \( \partial X^C_2 / \partial w_2 = (\partial R^C_2 / \partial x_2)(\partial X^C_2 / \partial w_2) > 0 \). This holds if \( \partial R^C_2 / \partial x_2 < 0 \) (since \( \partial X^C_2 / \partial w_2 < 0 \) for quantity competition). An increase in \( w_2 \) causing a decrease in firm 2’s equilibrium downstream quantity \( q_2 \), will—in the “normal” case where quantities are strategic substitutes—lead to an increase in firm 1’s quantity \( q_1 \), which lowers firm 2’s profit and, hence, the attainable fixed fee \( f_2 \). At the optimal \( w_2 \), this loss term caused by a small rise in \( w_2 \) is just offset by a gain in variable profits from the induced decrease in \( q_2 \). Thus, the integrated firm would gain from an exogenous reduction in firm 2’s quantity, unlike with linear pricing of the input.

4. Discussion and Conclusions

We conclude with a brief discussion of related work, and of the plausibility of Delegation by a vertically integrated firm. Bonanno and Vickers (1988) noted the strategic advantage of commitment to observable input prices. They consider differentiated Bertrand competition between two suppliers where prices are strategic complements. If both suppliers are vertically integrated, each provides the input to its downstream retail unit at marginal cost. If both are vertically separated, each sells through a different single retailer and captures the latter’s profit with a two-part tariff. For any input price set by one supplier (including marginal cost if integrated), the other supplier prefers to vertically separate and raise the input price somewhat above marginal cost, so as to coax an increase in the rival’s downstream price. We showed that the incentive to influence a downstream rival through an observable input price to one’s own downstream unit extends to a vertically integrated firm that supplies to the rival. However, because the rival is also an input customer, the incentive is to induce a reduction in its downstream price.\(^{15}\)

\(^{14}\)Recall that \( \partial R^C_1 / \partial w_2 \) is zero under Cournot, since the integrated firm’s shadow marginal cost of supplying division 1 under Centralization is unaffected by \( w_2 \) because there is no input diversion.

\(^{15}\)Also, because our input supplier is a monopolist, it benefits from Delegation whatever the downstream competition. Bonanno and Vickers observe that if downstream variables are strategic substitutes, vertical separation will lead both firms to cut input prices, which intensifies competition and harms both firms (though separation remains unilaterally profitable).
Our assumption that under the Delegation structure the downstream division maximizes its own profit is reminiscent of the literature on the strategic advantage of creating autonomous competing divisions (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). There, divisional autonomy is strategically beneficial because it makes the firm a tougher competitor, in order to deter entry or induce output contraction by oligopoly rivals when choice variables are strategic substitutes. Here, autonomy involves a vertical division and the strategic gain does not hinge on presenting a tough posture, e.g. under Cournot competition, the integrated firm uses Delegation to make itself a softer competitor in the downstream market.

Observe that our organizational Delegation framework embodies two assumptions: the downstream division acts as a profit center to maximize solely its own profit; and the integrated firm can credibly commit to charge the downstream division an input price observable by downstream rivals. The role of the profit center assumption is to prevent the downstream division from “undoing” the effects of an observable input price charged to it by fully internalizing how its choice affects the upstream division’s profit. As explained earlier, all our results will extend to any objective function of the downstream division for which the division’s downstream choice is affected by its input price. For example, if the division maximizes a weighted average of its own profit and the integrated firm’s profit, the integrated firm can implement its preferred downstream outcome by suitably raising the input price to the division compared to the case where the division only maximizes its own profit (Arya, Mittendorf, and Yoon, 2008, Proposition 3). In practice, integrated firms may find it feasible and beneficial to establish divisions as profit centers for incentive reasons; our analysis suggests the benefits can also be strategic in partial integration settings, provided a commitment can be made to charge one’s division an observable input price.

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16 These authors further show that decentralization (our Delegation) increases the integrated firm’s profit if the input price to the downstream division is determined by bargaining, for a range of assumptions about the relative bargaining power.

17 For a general discussion on firms’ ability to make commitments, see Shapiro (1989, p. 382).
The latter assumption is more questionable for unregulated firms.\textsuperscript{18} If the downstream division is an independent profit center but under common ownership with the upstream supplier, can the integrated firm commit to an arm’s length input price? One consideration that may aid such commitment is the supplier’s interest in maintaining a reputation for not acting opportunistically to disadvantage independents by secretly favoring its division (e.g. McAfee and Schwartz, 1994). To preserve such a reputation, it may adopt a policy of public and transparent pricing.

\textsuperscript{18} Regulated firms may be subject to rules governing input pricing to subsidiaries as well as the subsidiaries’ behavior through imputation requirements (e.g. Laffont and Tirole, 2000).
References


Appendix: Example

Assume that the marginal cost of the input is constant, \( c = 1 \), and that there are no other downstream costs. Consumer demand for the products of firms 1 and 2 is given by \( q_i = 500 - 200p_i + 100p_j \), where \( q_i \) denotes the output of firm \( i \), and \( p_i \) and \( p_j \) denote the prices of firms \( i \) and \( j \), respectively \((i, j \in \{1, 2\}, i \neq j)\).19

We begin with the case of Bertrand competition. Table 1 below shows the equilibrium profits, outputs and prices under Centralization and Delegation.

Table 1: Bertrand Competition Downstream

<table>
<thead>
<tr>
<th></th>
<th>Centralization</th>
<th>Delegation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit of Integrated Firm</td>
<td>696.97</td>
<td>700.00</td>
</tr>
<tr>
<td>Profit of Downstream Rival</td>
<td>59.50</td>
<td>50.00</td>
</tr>
<tr>
<td>Output of Integrated Firm, ( q_1 )</td>
<td>227.27</td>
<td>250.00</td>
</tr>
<tr>
<td>Output of Downstream Rival, ( q_2 )</td>
<td>109.09</td>
<td>100.00</td>
</tr>
<tr>
<td>Output Price of Integrated Firm, ( p_1 )</td>
<td>3.12</td>
<td>3.00</td>
</tr>
<tr>
<td>Output Price of Downstream Rival, ( p_2 )</td>
<td>3.52</td>
<td>3.50</td>
</tr>
<tr>
<td>Input Price Charged to Downstream Rival, ( w_2 )</td>
<td>2.97</td>
<td>3.00</td>
</tr>
<tr>
<td>Shadow Cost of Supplying Input to Division 1, ( C_1(w^C_2) )</td>
<td>1.98</td>
<td>NA</td>
</tr>
<tr>
<td>Input Price Charged to Division 1, ( w_1 )</td>
<td>NA</td>
<td>1.75</td>
</tr>
<tr>
<td>( W_1^D(w^C_2) )</td>
<td>NA</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Table 1 illustrates Proposition 1: Delegation allows the integrated firm to increase its profit relative to Centralization. It also illustrates Proposition 3 and Corollary 1(i): starting from the Centralization solution, the integrated firm wants to induce a reduction in firm 2’s

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19 In this example where firms 1 and 2 are symmetric, vertical integration of firm 1 and the input monopolist does not lead to foreclosure of firm 2 under Centralization. See Arya, Mittendorf, and Sappington (2008). Our results imply that this also is true under Delegation.
downstream price $p_2$; and since linear demands imply that prices are strategic complements, inducing a reduction in $p_2$ requires signaling a reduction in $p_1$ by lowering $w_1$: $W_1^D(w_2^C) < C_1(w_2^C)$. That is, at the input price charged to firm 2 under Centralization (2.97), the profit-maximizing input price to division 1 under Delegation is lower than the shadow marginal cost under Centralization, $1.74 < 1.98$. In the actual Delegation equilibrium, the integrated firm raises $w_2$ slightly, from 2.97 to 3.00, and reduces $w_1$ to 1.75 (instead of 1.74) from the shadow marginal cost of 1.98. The reduction in $w_1$ and increase in $w_2$ lead to a (substantial) decrease in the profit of the downstream rival.

We now turn to the case of Cournot competition.

**Table 2: Cournot Competition Downstream**

<table>
<thead>
<tr>
<th></th>
<th>Centralization</th>
<th>Delegation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit of Integrated Firm</td>
<td>682.76</td>
<td>685.71</td>
</tr>
<tr>
<td>Profit of Downstream Rival</td>
<td>45.66</td>
<td>48.98</td>
</tr>
<tr>
<td>Output of Integrated Firm, $q_1$</td>
<td>279.31</td>
<td>257.14</td>
</tr>
<tr>
<td>Output of Downstream Rival, $q_2$</td>
<td>82.76</td>
<td>85.71</td>
</tr>
<tr>
<td>Output Price of Integrated Firm, $p_1$</td>
<td>2.86</td>
<td>3.00</td>
</tr>
<tr>
<td>Output Price of Downstream Rival, $p_2$</td>
<td>3.52</td>
<td>3.57</td>
</tr>
<tr>
<td>Input Price Charged to Downstream Rival, $w_2$</td>
<td>2.97</td>
<td>3.00</td>
</tr>
<tr>
<td>Shadow Cost of Supplying Input to Division 1, $C_1$</td>
<td>1.00</td>
<td>NM</td>
</tr>
<tr>
<td>Input Price Charged to Division 1, $w_1$</td>
<td>NM</td>
<td>1.29</td>
</tr>
<tr>
<td>$W_1^D(w_2^C)$</td>
<td>NM</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Table 2 also illustrates Proposition 1—the integrated firm’s profit again is higher under Delegation. It also illustrates Proposition 3 and Corollary 1(ii): starting from the Centralization solution, the integrated firm wants to induce an *increase* in firm 2’s quantity $q_2$; and since linear demands imply that quantities are strategic substitutes, this requires signaling a reduction in $q_1$ by raising $w_1$. Thus, moving to Delegation the integrated firm wants to raise the input price to division 1 above the shadow marginal cost under Centralization (which, under Cournot competition, simply equals the resource marginal
cost, $C_1 = c = 1$, independent of $w_2$): $W_1^D(w_2^C) = 1.28 > 1.00 = C_1$. In the actual Delegation equilibrium, $w_1$ rises substantially, to 1.29 (from $C_1 = 1$), while $w_2$ rises slightly, from 2.97 to 3.00, causing the integrated firm’s output $q_1$ to fall and the rival’s output $q_2$ to rise, consistent with the incentives described above. This time, the rival’s profit increases, unlike in the Bertrand case.