Are Estimates of Asymmetric First-Price Auction Models Credible?

Semi & Nonparametric Analyses

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Abstract

Structural first-price auction estimation methods, built upon Bayesian Nash Equilibrium (BNE), have provided prolific empirical findings. However, due to the unobserved nature of underlying valuations, the assumption of BNE is not feasibly testable with field data, a fact that evokes harsh criticism of the literature. To respond to skepticism regarding credibility, we provide a focused answer by analyzing estimates derived from experimental asymmetric auction data in which researchers observe valuations. We test the statistical equivalence between the estimated and true value distributions. The Kolmogorov-Smirnov test fails to reject the distributional equivalence, strongly supporting the credibility of structural asymmetric auction estimates.

Keywords and Phrases:
Empirical Auction, Asymmetric Auction, Risk Aversion, and Semi/Nonparametric Structural Estimation

JLE Classifications:
C13 - Estimation: General and D44 - Auctions

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1 Introduction

By scrutinizing estimates derived from the experimental auction data in which researchers observe laboratory-assigned valuations and by statistically testing the equivalence between estimated and observed valuations, this research establishes the credibility of broadly used asymmetric first-price auction estimates, both semi and nonparametric, a result that has not yet been reported in the literature.

In empirical first-price auction literature, researchers are interested in describing strategic interactions among bidders for understanding and designing auction markets based on underlying economic incentives. Traditionally, reduced-form regressions had been used, despite the fact that the usage of linear reduced-form regressions for analyzing bidders’ non-linear payoff-maximization problems was a hindrance in describing strategic behaviors and obtaining profound insights. In order to overcome this difficulty, semi and non-parametric structural estimation methods arose and, for the last twenty years, have been widely used for investigating testable implications and drawing sharp policy recommendations supported by counterfactual simulations.\(^1\) Specifically, model-based counterfactual policy analyses have become possible since empirical researchers are able to estimate the structural elements of auction theory, bidders’ value distributions and von-Neumann-Morgenstern (vNM) payoff functions. In addition, due to the ubiquity of asymmetry among bidders, the estimation methods have also been extended to asymmetric auctions. In such empirical auction research, estimated valuations are particularly important to both researchers and industry practitioners as market designs, such as setting reserve prices or detecting collusions,\(^2\) crucially depend on empirical estimates. As a result, asymmetric auction estimates now come to serve as the vital foundation of auction market research for addressing numerous positive and normative questions.

However, while more and more asymmetric first-price auction estimates are reported in the literature, there is a fundamental difficulty in evaluating the performance of these estimates. Structural methods, in which researchers strictly assume that observed bids are derived from Bayesian Nash Equilibrium, estimate the bidders’ valuations. However, despite the fact that the comparison between estimated and true valuations is essential in accurately measuring the performance of estimates, the truth is bidders’ valuations are unobserved in empirical first-price auctions. This latent nature of bidders’ valuations in empirics lead to an

\(^1\)The cornerstone work at the dawn of empirical and structural first-auction literature should be credited. To the best of our knowledge, the literature was initiated by the contribution made by the Ph.D. thesis of Paarsch (1992) [65] with parametric models. Donald and Paarsch (1993, 1996) [22] [23], Elyakime, Laffont, Loisel, and Vuong (1994) [24], and Laffont, Ossard, and Vuong (1995) [46] established statistically rigorous yet flexible parametric estimation methods. The survey paper of Hendricks and Paarsch (1995) [33] and Perrigne and Vuong (1999) [66] concisely illustrate the early contributions in the literature.

infeasible comparison between estimated and true valuations.\textsuperscript{3} Such an infeasible comparison, along with the rigid Bayesian Nash Equilibrium assumptions for describing bidders’ behavior, then becomes the target for harsh criticism and skepticism of empirical auction estimates.\textsuperscript{4}

Against such criticism and skepticism, Bajari and Hortaçsu (2005) \cite{Bajari2005} provide a concrete and focused response by using symmetric first-price auction data from laboratory study in which researchers observe experimentally-assigned true valuations, as depicted in Figure 1. By using the valuations assigned in the experiment as a benchmark, they compare the estimates generated by various structural models with semi-parametric estimation methods. Their analyses shows that estimates based on the Constant Relative Risk Aversion (CRRA) model can recover the distributions of latent valuations in symmetric auctions with a statistically acceptable degree of accuracy, at least under but not limited to a laboratory environment, and demonstrates the great potential of structural auction estimation methods.

\textsuperscript{3}This difficulty has been widely recognized from the earliest literature. The survey of Hendricks and Paarsch (1995) \cite{Hendricks1995} summarizes the difficulty this way: “The difficulty with field data.....is that neither the valuations of potential buyers nor the probability law determining these valuations is observed by the researcher.” In addition, McAfee and Vincent (1992) \cite{McAfee1992} comment that “The most obvious roadblock to test auction theory is the heavy use made of unobservables in the theory. Bidders choose optimal bids based on signals that are not observed by econometricians studying auction behavior.”

\textsuperscript{4}There are two major kinds of such criticism. The first is made by a group of robust mechanism design researchers who have a skeptical view on bidders’ abilities to find BNE, especially in asymmetric auctions. These theoretical researchers claim that bidders are not able to find BNE and suggest using weaker equilibrium concepts, such as the prior-distribution-free iterations of dominated-strategy eliminations in second price auctions, in order to design auction markets. See Wilson (1987) \cite{Wilson1987} for the critique on BNE. The second criticism comes from a group of applied micro economics researchers who claim that assumptions made for structural analyses are implausibly strong. For example, Angrist and Pischke (2010) \cite{Angrist2010} describe structural elements as a “superstructure of assumptions” and “industrial disorganization” and suggest using reduced-form analyses of field-experiment data to obtain policy implications. As Bajari and Hortaçsu (2005) \cite{Bajari2005} mention, these skepticisms are not without merit.
This research contributes to the empirical auction literature by extending the seminal symmetric first-price auction study of Bajari and Hortaçsu (2005) [11] to an asymmetric auction framework. Given the ubiquity of asymmetry among bidders in real-world auctions, prevalence of structural asymmetric first-price auction studies over the last 10 years,⁵ and difficulties reported in theoretical asymmetric auction literature,⁶ it is our belief that establishing the credibility of asymmetric auction estimates by testing the performance and pointing out potential improvements are invaluable, as they are crucial to empirical auction research and auction market designs. Following the precedent set by Bajari and Hortaçsu (2005) [11], we likewise use laboratory data as it is able to capture insightful views on the performance of auction estimates. Both strengths and shortcomings of estimates are explicitly detected with laboratory data, and such findings are essential for improving the performance of asymmetric first-price auction estimates. Also, despite the fact that empirical asymmetric auctions have been and continue to be actively investigated, to the best of our knowledge, direct evaluations of asymmetric auction estimates have not previously been investigated in the literature, and it is these estimated valuations that empirical auction market designs heavily rely upon.

Specifically, we investigate the performance of asymmetric first-price auction estimation methods introduced by Isabelle Perrigne, Quang Vuong, and their coauthors. We choose these methods since, due to the versatility in allowing asymmetry in value distributions and computational tractability, they are now the standard used by numerous empirical works for investigating auction markets.⁷

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⁶Krishna (2009) [45], one of the most popular auction-theory textbooks, describes the theoretical difficulties in extending symmetric auction results to asymmetric environments this way: “Much of the theory developed in the symmetric case is fragile and does not extend to situations in which bidders are asymmetric.”

⁷We study the accuracy of estimates derived by the structural and semi/nonparametric estimation methods for asymmetric auctions with exogenous variations. In the literature, these methods are proposed and evolve as follows: based on the cornerstone work of Guerre, Perrigne, and Vuong (2000) [27] that proposes a method for symmetric first-price auctions, Campo, Perrigne, and Vuong (2003) [17] extend the nonparametric estimation method to asymmetric auctions in which bidders draw their valuations from asymmetric distributions. In addition, Guerre, Perrigne, and Vuong (2009) [28], and Campo, Guerre, Perrigne, and Vuong (2011)
For this investigation, we use a unique dataset from the asymmetric private value first-price auctions collected in the laboratory experiment conducted by Chernomaz (2012) [20]. The data contains submitted bids and laboratory-assigned valuations for each bidder. Additionally, Chernomaz (2012) [20] investigates the effects caused by asymmetry among bidders under exogenously changing auction environments, while the majority of bidder valuations remain fixed before and after such exogenous changes.

Our estimation strategy takes advantage of exogenous changes in auction environments to identify both bidders’ payoff functions and underlying value distributions, as bid distributions vary before and after the exogenous change while the valuations that bidders hold remain unchanged. We construct, then estimate, the compatibility conditions derived by connecting first-order conditions of the payoff maximization problems before and after the exogenous change.

The primary analytic methodology employed in our research straightforwardly follows those implemented in Bajari and Hortacu (2005) [11], as depicted in Figure 1, yet we newly extend their analyses to three empirically important dimensions. The first extension is that we investigate asymmetric value distributions among bidders. Secondly, we allow for and test potential affiliations among underlying value distributions as empirical researchers seldom have a priori knowledge of the independence of underlying valuations. And the last but not least extension is that, in addition to the semi-parametric models, we investigate the nonparametric von-Neumann-Morgenstern (vNM) functions that allow flexibility of bidders’ risk preferences.

Based on comparisons between estimated and true private valuations, we report these main conclusions: (1) the risk-neutral model assumption, which is often made for simplicity and tractability in the literature, tends to inflate estimated valuations; (2) the assumption of risk-averse bidders is indispensable as it enables nonnegligible improvements in the accuracy of estimates; (3) among semi and nonparametric risk-averse models, the nonparametric model with conventional-wisdom-based shape restrictions provides the most accurate results; (4) when advanced risk-averse models are employed, the two-sample Kolmogoro-Smirnov test fails to reject the statistical equivalence between the estimated and true value distributions, positively supporting the

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Bajari and Hortacu (2005) [11] also conduct the analyses of Adaptive Learning and Quantal Response Models. We exclude these models from this research as they are not popularly used in empirical auction literature, although we recognize that these models have intriguing aspects for understanding bidding behavior.

We employ the nonparametric sieve estimation method with shape restrictions in which restrictions are based on commonly accepted empirical and experimental findings. Specifically, we place lower boundaries on slopes of nonparametrically estimated functions in the regions where identifications are challenging.
empirical findings reported in the empirical asymmetric auction literature; (5) estimated value distributions of stochastically-dominated bidders are relatively more accurate compared to those of stochastically-dominating bidders; and (6) if the true data generating process is independent private value, the assumption of affiliated or independent private value only creates a minor difference in accuracy, strongly encouraging the usage of affiliated private value models originally proposed by Li, Perrigne, and Vuong (2002) [52] in any empirical auction research. While degrees of accuracy and improvements differ by applications, the findings of this study are widely extendable to any empirical first-price asymmetric auction research.

The paper is organized as follows: Section 2 illustrates the experimental data that contains both valuation and bid information; Section 3 explains the theoretical auction models that are the basis for structural estimations; Section 4 describes the semi and nonparametric asymmetric auction estimation methods that are used to generate estimates; Section 5 visually reports the estimation results, then statistically tests the performance of asymmetric auction estimates; and lastly, Section 6 discusses the external validity and provides conclusions.

2 Data Descriptions

In this section, we illustrate the laboratory auction data used to obtain results described in the empirical section. The key feature of the data is that bidders in experiments participated in two exogenously varying formats of auctions while the majority (two out of three, as illustrated in Figure 2) of their valuations remained unchanged. Such exogeneity provides us with the exclusion restrictions and enables us to identify both risk-averse vNM payoff functions and value distributions in the empirical section. With the emphasis on such exogenous change, we first describe the laboratory auction procedures, then explicate the summary statistics for illustrating the differences in bidding behavior before and after the exogenous change.

The data is from Chernomaz (2012) [20], which investigates the results of joint bids in independent private value first-price auctions. The participants were recruited from undergraduate students at the Ohio State University and paid a $6 show-up fee. There were two experiment runs (denoted as Experiment Run I and II), conducted on different days, and participants were not allowed to join more than one experiment run.

A joint bid (also known as a consortium bid) is defined as two or more bidders who form a group and submit one joint (consortium) bid in an auction. Joint bids were allowed in Mexico and Louisiana Gulf Outer Continental Shelf (OSC) wildcat auctions, and as a result the implications of joint bids are now intensively scrutinized in empirical auction literature. Hendricks and Porter (1998) [35], Campo, Perrigne, and Vuong (2003) [17], and Hendricks, Pinkse, and Porter (2003) [38] investigate joint bids in wildcat auctions and the associated asymmetry in available economic resources. Note that our laboratory procedures in Figure 2 can be viewed as a miniature of hypothetical wildcat auctions in which both (non-collusively) individual and joint bids are allowed to submit to an auction where an auctioneer randomly determines whether or not to allow a joint bid.
run. A computer-based laboratory was used for this experiment, and participants interacted only through computer screens. By using computer-generated random numbers, valuations of an object were exogenously and randomly assigned to bidders. Table 1 summarizes the number of participating bidders and observed bids in each experiment run. At the beginning of each experiment run, bidder types (joint and solo) were randomly assigned, and every participant remained the same type throughout the experiment run. Thus, a participating bidder kept playing the same type throughout an entire experiment run. In each experiment run, bidders initially experienced two practice rounds, then they participated in twenty-four rounds involving monetary incentives. Figure 2 depicts the stages within each round. At the beginning of each round, participating bidders were randomly matched to form three-bidder groups. Within a group, two bidders were from the pool of joint-type bidders, and the remaining one was from the pool of solo-type bidders. Then, valuations were drawn from i.i.d. uniform distribution $U[0, 18.75]$, denoted by $v_1$ for a joint-type bidder, $v_2$ for another joint-type bidder, and $v_3$ for a solo-type bidder, as depicted in Figure 2. Within each round, there were symmetric- and asymmetric-auction stages. In a symmetric-auction stage, three bidders submitted one bid each (denoted as $b_1$, $b_2$, and $b_3$), yet the outcome of a symmetric-stage auction was not announced until the result-announcement stage. Next, at the beginning of an asymmetric-auction stage, the two joint-type bidders aggregated their valuations as $\max\{v_1, v_2\}$. In an asymmetric-auction stage, a solo-type bidders submitted a bid $b_{\text{Solo}}$, based on her valuation of $v_3$. On the other hand, each joint-type bidder submitted a respective bid, based on the aggregated valuation of $\max\{v_1, v_2\}$. At an asymmetric-auction stage, these two joint-type bids (denoted as $b_{1,\text{Joint}}$ and $b_{2,\text{Joint}}$) were separately submitted by each joint-type bidder; then the experiment organizer (i.e. the auctioneer) randomly chose one of them with equal probability (described as 50% and 50% in Figure 2) to be the chosen joint-type bid. After an asymmetric-auction stage, Chernomaz (2012) further conducted a communication-based asymmetric-auction stage in which within-a-group joint-type bidders were allowed to exchange messages via computers. In this research, we eliminate

\[\text{Chernomaz (2012) [20] conducted one more experiment run. However, in the middle of that run, one subject elected to leave and had to be replaced with a substitute subject. To avoid any resulting issues, such as discrepancies in bid distributions and bidders’ risk averse attitudes in structural estimations, we omit this experiment run from our analyses. See page 708 of Chernomaz (2012) [20] for details.}\]

\[\text{This way of value aggregation, adopting a maximum valuation among joint-type bidders, corresponds to the empirical observations that joint (consortium) bidders share their economic resources, such as the best available cost-saving technology, the closest geographical locations, and the information of best-available resale opportunities. In our experiment, this means that one of the joint-type bidders had the same valuation before and after the exogenous value aggregation, and this invariant nature is exploited in the estimation section.}\]

\[\text{In an asymmetric-auction stage, the two within-a-matched-group joint-type bidders are informed of their aggregated valuation (i.e. } \max\{v_1, v_2\} \text{) through their respective computer screens. However, verbal or textual communication between joint-type bidders was strictly forbidden at this stage. Therefore, a bid made by a joint-type bidder in an asymmetric stage (i.e. } b_{1,\text{Joint}} \text{ or } b_{2,\text{Joint}} \text{ in Figure 2) was derived from a single-agent payoff-maximization problem.}\]

\[\text{After an asymmetric-auction stage, Chernomaz (2012) [20] further conducted a communication-based asymmetric-auction stage in which within-a-group joint-type bidders were allowed to exchange messages via computers. In this research, we eliminate}\]
a-matched-group results, including assigned valuations \((v_1, v_2, \text{ and } v_3)\), aggregated valuation \((\max \{v_1, v_2\})\), bids in each stage \((b_1, b_2, b_3, \text{ chosen } b_{\text{Joint}}, \text{ and } b_{\text{Solo}})\), and winning/losing statuses in each stage were announced to the matched group members, yet the identities of bidders were kept hidden. Therefore, the participants in experiment played against anonymous opponent bidders. In addition, monetary payoffs were calculated and added to each participating bidder’s account. Monetary payoffs were calculated as follows. After a result-announcement stage, the experiment organizer (i.e. the auctioneer) randomly selected with equal probability an auction stage in which an outcome was actually paid. Note that since bidders’ vNM functions are additively separable, which is usually assumed and accepted in auction literature, this random selection does not affect bidders’ payoff-maximization problems in each auction stage.\(^{15}\)\(^ {16}\) Lastly, at the end of each round, a matched group was dissolved, and participants returned to the pool of bidders.

Note that results were announced only to matched-group members at the end of each round, and the results of a specific matched group were NOT available to members of any other groups. As a natural consequence, we observe sizable learning and adjusting behavior in the first half of the rounds in each experiment run. For investigating the strategic interactions and estimates of valuations without concern for the learning effect, this research excludes the data from the first half of the rounds, and only the data from the second half of the rounds is used in the empirical investigation. Table 1 summarizes the sample size in each auction stage.

Table 2 lists the summary statistics of observed bids in both symmetric- and asymmetric-auction stages. In theory, bidders are predicted to bid less in asymmetric-auction stages, as symmetric-auction stages consist of three bidders while asymmetric-auction stages consist of only two bidders (i.e. a chosen joint-type bidder and a solo-type bidder). At most bid quantiles, both joint- and solo-type bidders decreased their bids in asymmetric-auction stages, yet we observe a small increase in bids in Experiment Run I that could negatively affect the performance of structural estimations. Figure 3 depicts the pairs of symmetric-auction stage bids the communication-based asymmetric-auction stage data to focus our analyses on the single-agent payoff maximization problems. Nonetheless, we encourage interested researchers to see Chernomaz (2012) [20] for further details on the experiment design and the effect of communications.

\(^{15}\)This randomized selection process was empirically motivated by the factual observation of timber auctions in which the U.S. Forest Service randomized different auction rules. Lu and Perrigne (2008) [55] and Athey, Levin, and Seira (2011) [9] exploit such randomization of timber auctions for detailed investigations of identifications and bidding behavior.

\(^{16}\)In an asymmetric-auction stage shown in Figure 2, the outcome for a solo-type bidder was determined by comparing chosen \(b_{\text{Joint}}\) and \(b_{\text{Solo}}\). On the other hand, the outcome for a first joint-type bidder was determined by comparing \(b_{1,\text{Joint}}\) and \(b_{\text{Solo}}\), while the outcome of a second joint-type bidder was determined by comparing \(b_{2,\text{Joint}}\) and \(b_{\text{Solo}}\). Accordingly, to avoid the inequality of monetary payoffs across bidder types, a payoff of a joint-type bidder in an asymmetric-auction stage was halved. As this halved-payoff rule for joint-type bidders is a single-agent payoff maximization problem, we can straightforwardly apply structural estimations without concern for joint decision making. See Online Appendix for details.
and chosen (and also announced) asymmetric-auction stage bids in each experiment run for which we later apply kernel density estimations. For symmetric-auction stage bids, we also plot the highest of opponents’ bids, as it plays an important role in determining winning probability. Overall, although we observe the slight rounding-up/down effect (e.g. bidders tend to bid in increments of five: $0, $5, $10, and $15), this does not seem to severely affect the estimates of distributional functions as bids appear widely spread out.\textsuperscript{17}

Figure 2: Stages within a Round

Table 1: Sample Size

<table>
<thead>
<tr>
<th></th>
<th>Number of Participants</th>
<th>Symmetric-Auction Stage Bids Observed</th>
<th>Asymmetric-Auction Stage Bids Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Run I</td>
<td>Joint Type</td>
<td>16 Bidders</td>
<td>192 Bids</td>
</tr>
<tr>
<td></td>
<td>Solo Type</td>
<td>8 Bidders</td>
<td>192 Bids (96 Chosen Bids)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment Run II</td>
<td>Joint Type</td>
<td>12 Bidders</td>
<td>144 Bids</td>
</tr>
<tr>
<td></td>
<td>Solo Type</td>
<td>6 Bidders</td>
<td>144 Bids (72 Chosen Bids)</td>
</tr>
</tbody>
</table>

\textsuperscript{17}As this research emphasizes the empirical perspective of asymmetric auction data, we refrain from examining the data of laboratory-assigned valuations until Estimation Result section. Nonetheless, the plots of laboratory-assigned valuations and bids are reported in Online Appendix, and we observe large overbidding (bidding more than risk neutral BNE) in our laboratory auctions.
Table 2: Summary Statistics of Bid Data (in U.S. Dollars)

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Auction Stage</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Quantile 10th</th>
<th>Quantile 25th</th>
<th>Quantile 50th</th>
<th>Quantile 75th</th>
<th>Quantile 90th</th>
</tr>
</thead>
</table>
| Experiment Run I
| Joint Type 192 Symmetric | 9.54 | 3.75 | 5.00 | 6.28 | 9.99 | 12.40 | 14.98 |
| | 96 Asymmetric | 9.58 | 3.95 | 4.75 | 6.76 | 9.28 | 12.34 | 15.00 |
| Difference | 0.04 | -0.25 | 0.71 | -0.90 | 0.62 |
| Solo Type 96 Symmetric | 6.79 | 4.60 | 1.02 | 2.58 | 6.69 | 11.06 | 12.96 |
| | 96 Asymmetric | 6.92 | 4.64 | 0.75 | 2.56 | 7.22 | 10.93 | 13.75 |
| Difference | 0.13 | -0.27 | -0.01 | 0.54 | -0.14 | 0.79 |
| Experiment Run II
| Joint Type 144 Symmetric | 10.60 | 4.30 | 3.16 | 7.55 | 11.49 | 14.18 | 15.79 |
| | 72 Asymmetric | 10.22 | 4.39 | 3.16 | 7.38 | 11.25 | 15.64 | 15.63 |
| Difference | -0.38 | -0.00 | -0.24 | -0.54 | -0.17 |
| Solo Type 72 Symmetric | 8.52 | 4.88 | 2.03 | 4.53 | 8.61 | 12.78 | 15.27 |
| | 72 Asymmetric | 8.43 | 4.85 | 2.03 | 4.53 | 8.31 | 12.75 | 15.08 |
| Difference | -0.09 | -0.00 | -0.30 | -0.03 | -0.19 |

Figure 3: Observed Bids: LEFT - Symmetric Auction Stages; RIGHT - Asymmetric Auction Stages
3 Auction Models

This section describes the theoretical models of an affiliated private value (APV) auction that include an independent private value (IPV) auction as a special case. Although the bid data used in this research is generated from the experiments of IPV auctions, APV models are initially employed for the following empirically pragmatic considerations. In many empirical investigations, researchers seldom have enough prior evidence to determine the independence of underlying value distributions. Given the pervasiveness of insufficient initial information, therefore, it is empirically prudent for researchers to first estimate latent valuations with a model that can allow potential affiliations, then test independence. Formal and statistical tests for distributional independence are demonstrated in the Estimation and Test Results section. These cautious model-specification procedures are also based on the robustness requisition proposed by the well-known and influential critique by Leamer (1983) [47], which is largely concerned with the credibility of estimates in general empirical research due to the lack of robustness to modeling assumptions. With the emphasis on generality in modeling assumptions, we first explain the symmetric auction models, then illustrate the asymmetric auction models.

3.1 Symmetric Auction Models

A single and indivisible object is sold in an auction to \( N \) bidders who have the von-Neumann-Morgenstern (vNM) function \( U(\cdot) \) that is twice differentiable with \( U'(\cdot) > 0 \) and \( U''(\cdot) \leq 0 \) to allow potential risk aversion. As a vNM function is unique up to the positive-affine transformation, the normalization of \( U(0) = 0 \) and \( U(1) = 1 \) is imposed without loss of generality. For the sake of clear notations, we use a capital letter for describing a random variable and a lower-case letter for describing the realization of a random variable. Assuming that \( N = 3 \) bidders in an auction with index \( i \in \{1, 2, 3\} \) draw private valuations \( \{v_1, v_2, v_3\} \) from the potentially affiliated joint distribution \( F_{v_1,v_2,v_3}(v_1,v_2,v_3) \). The arguments of the joint distribution are exchangeable in its \( N = 3 \) elements, meaning that the model is distributionally symmetric. Given that

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18 Given the existence of behavioral bidders who may not strictly play BNE, the independence of observed bids does not imply the independence of valuations and vice versa.


20 This setting is called the symmetric affiliated private value (APV) model, and its econometric identification and rationalizability are intensively investigated in Li, Perrigne, and Vuong (2002) [52]. Using Monte Carlo simulations, they investigate the problem of misspecification, estimating distributions of valuations under the IPV assumption when the true data-generating process is APV. They report that such misspecification tends to result in overestimation of private values, specifically in the upper domain of private values. This overestimation is caused by the neglect of modeling a strategic behavior in an APV environment; if a bidder has a high valuation, other bidders are also likely to have high valuations, and she needs to bid aggressively. Conversely, we in this research investigate the empirically prudent misspecification with the laboratory data, estimating value distributions under
other bidders employ a symmetric equilibrium strategy, bidder \( i \)'s expected payoff maximization problem is

\[
\max_{b_i} U(v_i - b_i) \cdot \frac{F_{Y_{-i}|V_i}(\phi(b_i)|v_i)}{\text{Probability of Winning}}
\]

where \( Y_{-i} \) is a random variable of the highest valuations among opponent bidders with its realization \( y_{-i} = \max_{j \neq i} v_j \), and \( \phi(\cdot) \) is the inverse of a symmetric equilibrium bidding function. In addition, \( F_{Y_{-i}|V_i}(y_{-i}|v_i) \) denotes the conditional distribution function of \( Y_{-i} \) given \( v_i \). The first-order necessary condition\(^{21} \) of the above maximization problem, where a bidder equates marginal cost and benefit of changing her bid, can be written in the form of

\[
v_i = b_i + \lambda^{-1}(\frac{F_{Y_{-i}|V_i}(\phi(b_i)|v_i)}{F_{Y_{-i}|V_i}(\phi(b_i)|v_i)} \cdot \phi'(b_i))
\]

\( \lambda(\cdot) \equiv U(\cdot)/U'(\cdot) \) and \( \lambda^{-1}(\cdot) \) as the corresponding inverse function. We refer to a function \( \lambda^{-1}(\cdot) \) as a shading function after its role of describing the difference between a valuation and a bid. We also refer to the argument of the shading function as a R factor or R function.\(^{22} \) Since in field-auction data we cannot empirically observe latent valuations that appear in the argument of the shading function, we now need to replace unobserved valuations with observed bids. We denote \( B_{-i} \) as the random variable of the highest bid among opponent bidders with its realization \( b_{-i} = \max_{j \neq i} b_j \). In addition, we denote the conditional distribution of \( B_{-i} \) given \( b_i \) as \( G_{B_{-i}|B_i}(b_{-i}|b_i) \) and its derivative as \( g_{B_{-i}|B_i}(b_{-i}|b_i) \). By assuming bidder \( i \) also employs a symmetric equilibrium bidding strategy, the probabilistic relation between observable bids and latent valuations is \( G_{B_{-i}|B_i}(x|b_i) = F_{Y_{-i}|V_i}(\phi(x)|v_i) \). Moreover, by the fundamental theorem of calculus, the conditional density is obtained as \( g_{B_{-i}|B_i}(x|b_i) = (dF_{Y_{-i}|V_i}(\phi(x)|\phi(b_i))/dy_{-i}) \cdot \phi'(x) \). Then, by using these probabilistic relations that bridge the unobservable to the observable, the first-order necessary condition can be equivalently written as

\[
v_i = b_i + \lambda^{-1}\left(\frac{G_{B_{-i}|B_i}(b_i|b_i)}{g_{B_{-i}|B_i}(b_i|b_i)}\right), \tag{1}
\]

the APV assumption when the true data-generating process is IPV. We are happy to report that empirical asymmetric auction estimates in our research are robust (and even more accurate) against such empirically prudent misspecification, as we explain in the Estimation and Test Results section.

\(^{21}\)Throughout this research, we assume that second-order conditions are satisfied.

\(^{22}\)This naming comes after the fact that Guerre, Perrigne, and Vuong (2009) \(^{28} \) use the notation \( R \) to express this argument (see p.1202 of their paper for details), yet they do not provide a memorable name for this object. \( R \) factor represents a quotient of “probability of winning” divided by “marginal probability of winning.” It is actually the reciprocal of semi-elasticity \( \equiv [(dh(x)/dx)/h(x)] \) where \( h(x) \) is a winning probability and \( dx \) is a change in bid.
where components of the right-hand side of the equation are observable or estimatable. Furthermore, as the
distributional functions in the right hand side of equation (1) share the same conditional variable, we exploit
the definitions of conditional density and distribution functions, as suggested by Li, Perrigne, and Vuong
(2002) \[52\], \( g_{B_i|B_i}(z|b_i) = g_{B_i|B_i}(z,b_i)/g_{B_i}(b_i) \) and \( G_{B_i|B_i}(x|b_i) = \left( \int_b^x g_{B_i|B_i}(z,b_i)dz \right)/g_{B_i}(b_i) \) where \( b \)
is the lower bound of bid distribution. Thus, the equation (1) can be re-written in an unconditional fashion as

\[ v_i = b_i + \lambda^{-1} \left( \int_b^{\bar{b}} g_{B_i|B_i}(z,b_i)dz \right) \frac{g_{B_i|B_i}(z,b_i)}{g_{B_i}(b_i)}. \] (2)

For the sake of organized notations for later empirical use, we introduce the convenient terms of \( \Gamma(x,b_i|g_{B_i|B_i}) = \int_b^x g_{B_i|B_i}(z,b_i)dz \) and of \( R \) factor as \( R[x,y|g_{B_i|B_i},g_{B_i},g_{B_i|B_i}] = \Gamma(x,y|g_{B_i|B_i},g_{B_i},(x,y)) \). Given these simplified notations, we can denote the first-order necessary condition of symmetric auction as

\[ v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1} \left( \int_b^{\bar{b}} g_{B_i|B_i}(z,b_i)dz \right) \frac{g_{B_i|B_i}(z,b_i)}{g_{B_i}(b_i)} \] (i), \[ v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1} \left( \int_b^{\bar{b}} g_{B_i|B_i}(z,b_i)dz \right) \frac{g_{B_i|B_i}(z,b_i)}{g_{B_i}(b_i)}. \] (3)

where the upper indices of “Sym” and “Sym,Affi” emphasize that the auction model is symmetric and of
affiliated value. In addition, if a researcher further assumes the independence of private valuations, the bi-
variate functions are simplified as \( g_{B_i|B_i}(z,b_i) = g_{B_i}(z) \) and \( \int_b^x g_{B_i|B_i}(z,b_i)dz = G_{B_i}(x) \). Accordingly,
first-order necessary condition equation (3) becomes the well-known equation in empirical auction literature,

\[ v_i = b_i + \lambda^{-1} \left( G_{B_i}(b_i) \right) G_{B_i}(b_i). \] (4)

Lastly, by denoting a \( R \) factor as \( R[x|G_{B_i},g_{B_i}] = G_{B_i}(x)/g_{B_i}(x) \), we can write the first-order necessary condition as

\[ v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1} \left( \int_b^{\bar{b}} g_{B_i|B_i}(z,b_i)dz \right) \frac{g_{B_i|B_i}(z,b_i)}{g_{B_i}(b_i)} \] (ii), \[ v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1} \left( \int_b^{\bar{b}} g_{B_i|B_i}(z,b_i)dz \right) \frac{g_{B_i|B_i}(z,b_i)}{g_{B_i}(b_i)}. \] (5)

where the upper indices of “Sym” and “Sym,Inde” emphasize that the auction model is symmetric and of
independent value.

### 3.2 Asymmetric Auction Models

We next introduce asymmetric auction models. In order to be notationally minimalistic, we henceforth fo-
cus on the simplest environment, a two-type and two-bidder asymmetric auction on which our experimental
asymmetric auction data is based. We define the index of the bidder type as \( t \in \{ \text{Joint}, \text{Solo} \} \). By exploiting the two-type-two-bidder nature, we use a convenient notation of \(-t\) for representing the opponent bidder’s type. We denote \( U_t(\cdot) \) as a vNM function of type \( t \) bidder, as we allow for the possibility of joint- and solo-type bidders having different payoff functions. Bidders draw their private valuations from joint distribution \( F_{V_t}, v_t(v_{-t}, v_t) \) in which arguments are not exchangeable and valuations are potentially affiliated.\(^{23,24}\) Assuming a type \( t \) bidder draws her valuation of \( v_t \) and assuming the opponent bidder employs an equilibrium strategy \( \phi_{-t}(\cdot) \), her expected payoff maximization problem is\(^{25}\)

\[
\max_{b_t} U_t(v_t - b_t) \cdot F_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t), \tag{6}
\]

where \( F_{V_{-t}|V_t}(v_{-t}|v_t) \) denotes the conditional distribution function of \( V_{-t} \) given \( v_t \). By differentiating the above expected payoff function with respect to \( b_t \), a type \( t \) bidder equates the marginal cost and benefit of changing her bid, and we have the first-order necessary condition, written as

\[
v_t = b_t + \lambda_t^{-1} \left( \frac{F_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t)}{dF_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t)/dv_t} \cdot \phi'_{-t}(b_t) \right), \tag{7}
\]

where \( \lambda_t(\cdot) \equiv U_t(\cdot)/U'_t(\cdot) \) for each type of \( t \in \{ \text{Joint}, \text{Solo} \} \) and \( \lambda_t^{-1}(\cdot) \) is its inverse function, called as a shading function in this research. Next, similar to the symmetric case, we derive the relations between empirically unobservable valuations and observable bids. We denote the conditional distribution of an opponent type’s bid \( B_{-t} \) given \( b_t \) as \( G_{B_{-t}|B_t}(b_{-t}|b_t) \) and its derivative as \( g_{B_{-t}|B_t}(b_{-t}|b_t) \). By further assuming a type \( t \) bidder employs an equilibrium strategy \( \phi_t(\cdot) \), the probabilistic relation between latent valuations and observable bids is \( G_{B_{-t}|B_t}(x|b_t) = F_{V_{-t}|V_t}(\phi_t(x)|v_t) \). In addition, by the fundamental theorem of calculus, the conditional density is obtained as \( g_{B_{-t}|B_t}(x|b_t) = (dF_{V_{-t}|V_t}(\phi_t(x)|v_t)/dv_t) \cdot \phi'_t(x) \). Therefore, using these probabilistic relations, the first-order necessary condition can be equivalently written as the function of observable bids,

\[
v_t = b_t + \lambda_t^{-1} \left( \frac{G_{B_{-t}|B_t}(b_t|b_t)}{g_{B_{-t}|B_t}(b_t|b_t)} \right). \tag{7}
\]

\(^{23}\)The model explored in this subsection is the simplest version of an asymmetric and affiliated private value auction model, and its econometric identifications and rationalizability are established by Campo, Perrigne, Vuong (2003) [17].

\(^{24}\)We follow Milgrom and Weber (1982) [61] for the definition of affiliation among private values.

\(^{25}\)Technically speaking, as a joint-type bidder’s monetary payment is halved, the maximization problem for a joint-type bidder is \( \max_{b_{\text{Joint}}/2} U_{\text{Joint}}(v_{\text{Joint}} - b_{\text{Joint}})/2) \cdot F_{V_{\text{Solo}|V_{\text{Joint}}}(\phi_{\text{Solo}}(b_{\text{Joint}})|v_{\text{Joint}})} \). For simplicity’s sake, we keep using the simplified expression of the payoff maximization problem as seen in (6) and continue using it for maximization problems of both joint- and solo-type bidders. Detailed explanations are found in Online Appendix. Note that this halved-payment rule does not affect the discussion of theoretical auction models (as we can re-define a vNM function as \( U_{\text{Joint}}(x) \equiv U_{\text{Joint}}(x/2) \), yet it slightly affects the structural estimations.
Furthermore, similar to the symmetric case, by exploiting the definitions of conditional density and distribution functions, $g_{B^{-1}B_t}(z|b_t) = g_{B^{-1}B_t}(z,b_t)/g_{B^{-1}B_t}(b_t)$ and $G_{B^{-1}B_t}(x|b_t) = \left(\int_x^\infty g_{B^{-1}B_t}(z,b_t)dz\right)/g_{B^{-1}B_t}(b_t)$ where $\underline{b}$ is the lower bound of bid distributions, the first order necessary condition equation (7) can be written in an unconditional fashion as

$$v_t = \underline{b} + \lambda_t^{-1}\left(\int_{\underline{b}}^{b_t} g_{B^{-1}B_t}(z,b_t)dz\right).$$

(8)

To simplify notations, we again introduce the convenient terms of $\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q$ where $\underline{b}$ is the lower bound of bid distributions, the first order necessary condition equation (7) can be written in an unconditional fashion as

$$v_t = \frac{\Gamma_{B^{-1}B_t}(x,y|\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}{G_{B^{-1}B_t}(x, y|\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}.$$

(9)

where upper indices of “Asym” and “Asym,Inde” emphasize that the auction model is asymmetric and of independent value. In addition, if a researcher further assumes the independence of private valuations, the bivariate functions are simplified as $g_{B^{-1}B_t}(z,b_t) = g_{B^{-1}B_t}(z)$ and $\int_{\underline{b}}^{b_t} g_{B^{-1}B_t}(z,b_t)dz = g_{B^{-1}B_t}(x)$. Accordingly, equation (8) becomes

$$v_t = \frac{\Gamma_{B^{-1}B_t}(x,y|\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}{G_{B^{-1}B_t}(x, y|\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q\Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}.$$

(10)

Lastly, by denoting the R factor as $R[x|g_{B^{-1}B_t} x] = G_{B^{-1}B_t}(x)/g_{B^{-1}B_t}(x)$, we can write the first-order necessary condition as

$$v_t^{\text{Asym}} = \frac{\Gamma_{B^{-1}B_t} \Gamma_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}{G_{B^{-1}B_t} x, b_t|g_{B^{-1}B_t} z, b_t q_{g_{B^{-1}B_t} z, b_t q}}.$$

(11)

where the upper indices “Asym” and “Asym,Inde” emphasize that the auction model is asymmetric and of independent value.
4 Structural Estimation Methods

In this section, we illustrate the estimation methods for recovering valuations. Estimation procedures are summarized into three steps as depicted in Figure 4: Step 1 – nonparametrically estimating distributional functions; Step 2 – by applying semi or nonparametric methods, estimating shading functions (i.e. $\lambda^{-1}(\cdot)s$); Step 3 – estimating valuations based on estimated distributional functions and shading functions. As the main purpose of this research is to investigate the accuracy of asymmetric auctions estimates, we primarily recover valuations from bids observed in asymmetric auctions, and bid data from symmetric auctions is subsidiarily used solely for the purpose of estimating the shading functions. For the sake of clear notation, we introduce the following indices: $r \in \{1, \ldots, R\}$ as an auction round index; $m \in \{1, \ldots, M\}$ as a (within-a-round) matched group index; and $i \in \{1, \ldots, N\}$ where $N = 3$ as a bidder index in a symmetric-auction stage. In addition, as estimates are separately calculated for each experiment run, we omit an index for experiment runs.26

Given the goal of obtaining the estimates of valuations, this section is organized as follows. The first subsection illustrates Step 1 with descriptions of nonparametric estimation method for distributional functions. The following subsections illustrate Step 2 and Step 3 in the order of risk neutral, general estimation framework for risk-averse models, constant relative risk averse (CRRA), constant absolute risk averse (CARA), nonparametric vNM function, and heterogeneous risk averse attitude models, as the latter models require incrementally advanced estimation methods.

4.1 Nonparametric Estimations of Distributional Functions

Here, as Step 1, we nonparametrically estimate the distributional functions that are the basis for further estimations of shading functions and valuations. Nonparametric kernel estimation methods are employed throughout this research, and we use the Gaussian kernel with Silverman’s rule of thumb bandwidths. For

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26 Any estimation results reported in the rest of this research are separately calculated for each experiment run.
bi-variate distributional function estimations, we use a product kernel. In addition, we exploit the anonymous-identity nature of our experiment; a bidder did not know identities of opponents in each auction game. This anonymous-identity nature enables us to aggregate the distributional functions against which a bidder is best responding.

4.1.1 Affiliated Value Assumption

The distributional functions under affiliated private value assumption are estimated as follows. According to the tradition, we use $K(\cdot)$ for denoting a uni-variate kernel function. By using symmetric-auction stage bid data, we estimate (recall that $b_{-i}$ is the highest of opponents’ bids)\(^{27,28}\)

$$
\hat{\Gamma}_{B_{-i},B_i}^{\text{Sym, Affi}}(x,y) = \int_{\mathbb{R}} \hat{g}_{B_{-i},B_i}^{\text{Sym, Affi}}(z,y) dz = \frac{1}{h_{\text{Sym, Affi}}} \frac{1}{RMN} \sum_{r=1}^{R} \sum_{m=1}^{M} \sum_{i=1}^{N} \mathbb{I}(b_{r,m,-i}^{\text{Sym}} \leq x) \cdot K \left( \frac{b_{r,m,-i}^{\text{Sym}} - y}{h_{\text{Sym, Affi}}} \right),
$$

and by using asymmetric-auction stage bid data, for each type of $t \in \{\text{Joint, Solo}\}$, we estimate\(^{29}\)

$$
\hat{\Gamma}_{B_{-i},B_i}^{\text{Asym, Affi}}(x,y) = \int_{\mathbb{R}} \hat{g}_{B_{-i},B_i}^{\text{Asym, Affi}}(z,y) dz = \frac{1}{h_{\text{Asym, Affi}}} \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \mathbb{I}(b_{r,m,-i}^{\text{Asym}} \leq x) \cdot K \left( \frac{b_{r,m,-i}^{\text{Asym}} - y}{h_{\text{Asym, Affi}}} \right),
$$

\(^{27}\)In this subsection (and only in this subsection), we generically use $x$ for the first functional argument, $y$ for the second functional argument, and $z$ for an integrating variable. Note that the first function argument is a realized random variable of an opponent who has the highest valuation in a symmetric-auction stage or of an opponent-type bidder in an asymmetric-auction stage. The second functional argument is a realized random variable of a bidder $i$ in a symmetric-auction stage or of a type $t$ bidder in an asymmetric-auction stage.

\(^{28}\)Note that, in round $r \in \{1, \ldots, R\}$ within matched group $m \in \{1, \ldots, M\}$, we use $\{b_{r,m,1}, b_{r,m,2}, b_{r,m,3}\}$ in a symmetric auction stage and $\{b_{r,m,\text{Joint}}, b_{m,r,\text{Solo}}\}$ in an asymmetric-auction stage for estimating distributional functions. Although there are two observed joint-type bids in an asymmetric-auction stage (as depicted in Figure 2), only the chosen (and announced) joint-type bid in an asymmetric-auction stage is used for estimations of distributional functions. In other words, the non-chosen (and unannounced) joint-type bid in an asymmetric-auction stage is not used for estimations of distributional functions.

\(^{29}\)As there are only two bids in an asymmetric-auction stage used for estimations, (one is submitted by a solo-type bidder, and the other is submitted by a chosen joint-type bidder) and as we estimate asymmetric-auction stage distributional functions for each type of bidder, we drop summations over bidder types in the following equations.
where $h_{\text{Sym, Affi}}^* = c_T \cdot (RMN)^{-1/5}$ and $h_{\text{Asym, Affi}}^* = c_T \cdot (RM)^{-1/5}$ with $c_T = 1.06 \cdot \hat{\sigma}_b$, and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids.\footnote{Following Bajari and Hortacsu (2005) [11], in this research we use the unbounded-support Gaussian kernel with rule of thumb, $c = 1.06 \cdot \hat{\sigma}_b$ (see Li and Racine 2007 [53] page 26, for example). Note that, some of preceding and influential works (e.g. Li, Perrigne, and Vuong 2002 [52] and Campo, Perrigne, and Vuong 2002 [17]) use the bounded-support triweight kernel with the rule of thumb, $c = 2.978 \times 1.06 \cdot \hat{\sigma}_b$. See Härdle (1990, 1991) [30] [31] for the detailed description of bandwidth choices.} For the bi-variate density functions, we estimate

\[ \hat{g}_{B_{r,m,t},B_t}^{\text{Asym, Affi}}(x, y) = \frac{1}{h_{g_x}^{\text{Asym, Affi}} \cdot h_{g_y}^{\text{Asym, Affi}}} \frac{1}{RM} \sum_{r=1}^R \sum_{m=1}^M \sum_{t=1}^T K \left( \frac{b_{r,m,t}^{\text{Asym}} - x}{h_{g_x}^{\text{Asym, Affi}}} \right) \cdot K \left( \frac{b_{r,m,t}^{\text{Asym}} - y}{h_{g_y}^{\text{Asym, Affi}}} \right), \] (15)

and for each type of $t \in \{\text{Joint, Solo}\},$

\[ \hat{g}_{B_{r,m,t},B_t}^{\text{Sym, Affi}}(x, y) = \frac{1}{h_{g_x}^{\text{Sym, Affi}} \cdot h_{g_y}^{\text{Sym, Affi}}} \frac{1}{RMN} \sum_{r=1}^R \sum_{m=1}^M \sum_{t=1}^T K \left( \frac{b_{r,m,t}^{\text{Sym}} - x}{h_{g_x}^{\text{Sym, Affi}}} \right) \cdot K \left( \frac{b_{r,m,t}^{\text{Sym}} - y}{h_{g_y}^{\text{Sym, Affi}}} \right), \] (14)

where $h_{g_x}^{\text{Sym, Affi}} = c_g \cdot (RMN)^{-1/6}$, and $h_{g_y}^{\text{Asym, Affi}} = c_g \cdot (RM)^{-1/6}$ with $c_g = 1.06 \cdot \hat{\sigma}_b$ and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids. In addition, $h_{g_x}^{\text{Sym, Affi}}$ and $h_{g_y}^{\text{Asym, Affi}}$ are determined in the same manner. Given these estimated distributional functions with affiliated value assumption, we can calculate R factors as

\[ R_{\text{Sym, Affi}}^{\text{Sym,Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Sym, Affi}} \right], \] (16)

and for each type of $t \in \{\text{Joint, Solo}\},$

\[ R_{\text{Asym, Affi}}^{\text{Asym,Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \right] = \hat{R}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \left[ \hat{g}_{B_{r,m,t},B_t}^{\text{Asym, Affi}} \right]. \] (17)

The plots of these affiliated value distributional function estimates based on observed bids are found in Online Appendix.

### 4.1.2 Independent Value Assumption

Next, the distributional functions under independent private value assumption are estimated as follows. By using symmetric-auction stage bid data, we derive the empirical CDF as

\[ \hat{G}_{B_{r,m}}^{\text{Sym, Inde}}(x) = \frac{1}{RMN} \sum_{r=1}^R \sum_{m=1}^M \sum_{i=1}^N \mathbb{1}(b_{r,m,i}^{\text{Sym}} \leq x), \] (18)
and by using symmetric-auction stage bid data, for each type of $t \in \{\text{Joint, Solo}\}$, we derive

$$\hat{G}_{B_{t},t}^{\text{Asym,Inde}}(x) = \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \mathbb{1}(b_{r,m,t}^{\text{Asym}} \leq x).$$  \hfill (19)$$

For uni-variate density functions, we estimate

$$\hat{g}_{B_{t},t}^{\text{Sym,Inde}}(x) = \frac{1}{h_{g_{x}}^{\text{Sym,Inde}}} \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \sum_{i=1}^{N} K \left( \frac{b_{r,m,t}^{\text{Sym}} - x}{h_{g_{x}}^{\text{Sym,Inde}}} \right),$$  \hfill (20)$$

and, for each type of $t \in \{\text{Joint, Solo}\}$,

$$\hat{g}_{B_{t}}^{\text{Asym,Inde}}(x) = \frac{1}{h_{g_{x}}^{\text{Asym,Inde}}} \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} K \left( \frac{b_{r,m,t}^{\text{Asym}} - x}{h_{g_{x}}^{\text{Asym,Inde}}} \right),$$  \hfill (21)$$

where $h_{g_{x}}^{\text{Sym,Inde}} = c_{g} \cdot (RMN)^{-1/5}$, and $h_{g_{x}}^{\text{Asym,Inde}} = c_{g} \cdot (RM)^{-1/5}$ with $c_{g} = 1.06 \cdot \hat{\sigma}_{b}$ and $\hat{\sigma}_{b}$ is the empirical standard deviation of corresponding observed bids. Accordingly, we can calculate R factors as

$$R^{\text{Sym,Inde}} \left[ \hat{b}_{r,m,i}^{\text{Sym,Inde}}, \hat{G}_{B_{t},t}^{\text{Sym,Inde}}, \hat{g}_{B_{t},t}^{\text{Sym,Inde}} \right] = \hat{R}_{B_{t},t}^{\text{Sym,Inde}} \left[ \hat{b}_{r,m,i}^{\text{Sym,Inde}} \right] = \hat{G}_{B_{t},t}^{\text{Sym,Inde}} \left( \hat{b}_{r,m,i}^{\text{Sym,Inde}} \right),$$  \hfill (22)$$

and for each type of $t \in \{\text{Joint, Solo}\}$,

$$R^{\text{Asym,Inde}} \left[ \hat{b}_{r,m,t}^{\text{Asym,Inde}}, \hat{G}_{B_{t},t}^{\text{Asym,Inde}}, \hat{g}_{B_{t},t}^{\text{Asym,Inde}} \right] = \hat{R}_{B_{t},t}^{\text{Asym,Inde}} \left[ \hat{b}_{r,m,t}^{\text{Asym,Inde}} \right] = \hat{G}_{B_{t},t}^{\text{Asym,Inde}} \left( \hat{b}_{r,m,t}^{\text{Asym,Inde}} \right).$$  \hfill (23)$$

The plots of these independent value distributional function estimates based on observed bids are found in Online Appendix.

### 4.2 Estimation Method for Risk Neutral Model

We begin by assuming bidders behave according to the simplest model, risk neutral (RN) vNM function with $U_{t}(x) = x$, $\lambda_{t}(x) = U_{t}(x)/U'_{t}(x) = x$, and the shading function $\lambda_{t}^{-1}(y) = y$. Under the risk neutral model, by substituting estimated distributional functions, equilibrium first order condition equations (9) under the APV assumption and (11) under the IPV assumption become

$$\hat{v}_{\text{RN},r,m,t}^{\text{Asym,Affi}} = \hat{b}_{r,m,t}^{\text{Asym}} + R^{\text{Asym,Affi}} \left[ \hat{b}_{r,m,t}^{\text{Asym}}, \hat{G}_{B_{t},t}^{\text{Asym,Affi}}, \hat{g}_{B_{t},t}^{\text{Asym,Affi}} \right],$$  \hfill (24a)$$

$$\hat{v}_{\text{RN},r,m,t}^{\text{Asym,Inde}} = \hat{b}_{r,m,t}^{\text{Asym}} + R^{\text{Asym,Inde}} \left[ \hat{b}_{r,m,t}^{\text{Asym}}, \hat{G}_{B_{t},t}^{\text{Asym,Inde}}, \hat{g}_{B_{t},t}^{\text{Asym,Inde}} \right].$$  \hfill (24b)$$
for each type of $t \in \{\text{Joint}, \text{Solo}\}$. Accordingly, by substituting observed bids with estimated distributional functions in the right hand side of the above equations, we obtain the estimates of valuations $\{\hat{v}_{t}^{\text{Asym.Affi}}_{\alpha} \}_{m=1, \ldots, M}^{r=1, \ldots, R}$ under the APV assumption and $\{\hat{v}_{t}^{\text{Asym.Inde}}_{\alpha} \}_{m=1, \ldots, M}^{r=1, \ldots, R}$ under the IPV assumption for each type of $t \in \{\text{Joint}, \text{Solo}\}$.

4.3 Semi & Nonparametric Estimations of Risk Averse Models

In empirical auctions, the assumption of risk neutrality is justified when a bidder can be seen as a large firm whose wealth is relatively large compared to the value of an object under auction. However, in reality, a bidder is likely to be a representative of a firm whose personal incentives (e.g. individual bonus, promotion, or opportunity costs) depend on the result of an auction. In such situations, bidders are seen to have risk averse preferences, and derived model implications could be substantially different from those derived from the risk neutral model.\textsuperscript{31} Accordingly, for investigating bidders’ risk averse preferences, we now assume that bidders have preferences with type-homogeneous (i.e. bidders share the same risk attitude within a type) risk averse vNM functions, $U_{t}(\cdot)$. In Step 2, we use quantile restrictions to derive compatibility conditions, which in turn are used for semi and nonparametrically estimating the shapes of type-homogeneous shading functions, $\lambda_{t}(\cdot)$.\textsuperscript{32} We introduce the notation for $b_{t,\alpha}^{\text{Sym}}$ to denote the $\alpha$th quantile for distribution of observed symmetric-auction stage bids submitted by all types of bidders. Similarly, we denote $b_{t,\alpha}^{\text{Asym}}$ for the $\alpha$th quantile for distribution of observed asymmetric-auction stage bids submitted by type $t$ bidders. In addition, we denote $v_{t,\alpha}$ as $\alpha$th quantile of value distribution among all types of bidders and $v_{t,\alpha}$ as $\alpha$th quantile of value distribution among type $t$ bidders. Then, the quantile notations of equilibrium first order conditions (3) and (5) for the symmetric-auction models are

\begin{align*}
\hat{v}_{t,\alpha}^{\text{Sym.Affi}} &= b_{t,\alpha}^{\text{Sym}} + \lambda_{t}^{-1} \left( R_{t}^{\text{Sym.Affi}} \left[ b_{t,\alpha}^{\text{Sym}}, \lambda_{t} \left[ b_{t,\alpha}^{\text{Sym}}, G_{B_{t},B_{t}}^{\text{Sym.Affi}} \right] \right] \right) \\
\hat{v}_{t,\alpha}^{\text{Sym.Inde}} &= b_{t,\alpha}^{\text{Sym}} + \lambda_{t}^{-1} \left( R_{t}^{\text{Sym.Inde}} \left[ b_{t,\alpha}^{\text{Sym}}, G_{B_{t},B_{t}}^{\text{Sym.Inde}} \right] \right)
\end{align*}


\textsuperscript{32}Quantile restrictions with the resulting compatibility conditions are proposed by Guerre, Perrigne, and Vuong (2009) [28], Campo, Guerre, Perrigne, and Vuong (2011) [18], Campo (2012) [16] and also used by Bajari and Hortaçsu (2005) [11].
and those of equilibrium first order conditions (9) and (11) for the asymmetric-auction models are

\[
\begin{align*}
\nu_{t,\alpha}^{\text{Asym, Affi}} &= b_{t,\alpha}^{\text{Asym}} + \lambda_t^{-1} \left( R^{\text{Asym, Affi}} \left[ b_{t,\alpha}^{\text{Asym}}, \nu_{t,\alpha}^{\text{Asym, Affi}} \right] \right), \\
\nu_{t,\alpha}^{\text{Asym, Inde}} &= b_{t,\alpha}^{\text{Asym}} + \lambda_t^{-1} \left( R^{\text{Asym, Inde}} \left[ b_{t,\alpha}^{\text{Asym}}, \nu_{t,\alpha}^{\text{Asym, Inde}} \right] \right).
\end{align*}
\]  

(26a)  

(26b)

Next, by exploiting the fact that the majority of bidders in experiments did not change their valuations within a round while they submitted distinct bids in symmetric- and asymmetric-auction stages, we take advantage of the observed differences of bids between auction stages. Note that, as the valuations of a solo-type bidder and one of the joint-type bidders were unchanged in both symmetric- and asymmetric-auction stages, we have the equivalence of valuations, \( v_{t,\alpha}^{\text{Sym}} = v_{t,\alpha}^{\text{Asym}} \) with the notation of \( i = t \) for each type of \( t \in \{ \text{Joint, Solo} \} \), meaning that a type \( t \) bidder did not change her valuation across auction stages. Accordingly, by using this unchanged nature of valuations, we match the quantiles of bidders’ private value distribution \( v_{t,\alpha}^{\text{Sym}} = v_{t,\alpha}^{\text{Asym}} \), where \( v_{t,\alpha}^{\text{Sym}} \) and \( v_{t,\alpha}^{\text{Asym}} \) denote the \( \alpha \)th quantiles of type \( t \) bidders’ value distribution.\(^{33}\) Thus, for each type \( t \in \{ \text{Joint, Solo} \} \), we can equate the equilibrium first order condition equations (25a) and (26a) under the APV assumption and (25b) and (26b) under the IPV assumption. Then, by assuming that a type \( t \) bidder reveals the same preference \( U_i(\cdot) \) and \( \lambda_t^{-1}(\cdot) \) in different and exogenously changing auction stages, we have the following compatibility condition equations for each type of \( t \in \{ \text{Joint, Solo} \} \):

\[
\begin{align*}
\nu_{t,\alpha}^{\text{Sym}} &= \nu_{t,\alpha}^{\text{Asym}} - \lambda_t^{-1} \left( R^{\text{Asym, Affi}} \left[ b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}} \right] \right) - \lambda_t^{-1} \left( R^{\text{Asym, Inde}} \left[ b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}} \right] \right),
\end{align*}
\]  

(27a)  

\[
\begin{align*}
\nu_{t,\alpha}^{\text{Sym}} &= \nu_{t,\alpha}^{\text{Asym}} - \lambda_t^{-1} \left( R^{\text{Asym, Inde}} \left[ b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}} \right] \right) - \lambda_t^{-1} \left( R^{\text{Asym, Affi}} \left[ b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}} \right] \right),
\end{align*}
\]  

(27b)

where we use the notation of \( \nu_{i,t,\alpha}^{\text{Sym}} \) for the \( \alpha \)th quantile of observed bids made by type \( t \) bidders in symmetric-auction stages.\(^{34}\) We use \( \nu_{i,t,\alpha}^{\text{Sym}} \) to denote quantile points of bid distributions where \( Q \) is the number of quantile points. See Online Appendix for a detailed description of quantile points.\(^{35}\)

\(^{33}\)Note that, as we exploiting the invariance of laboratory assigned valuations, our quantile restrictions are unconditional. Guerre, Perrigne, and Vuong (2009) [28] Campo, Guerre, Perrigne, and Vuong (2011) [18] and Campo (2012) [16] establish identifications based on conditional quantile restrictions (that allow, observed characteristics of auction objects, endogenous participation, and unobserved heterogeneity) that accommodate a broad class of auction models and data generating processes. Our unconditional restrictions in this subsection are the special case of their conditional restrictions.

\(^{34}\)Precisely describing, within a matched group (of three bidders), a solo-type bidder and one of the joint-type bidders did not change valuations as depicted in Figure 2. For the semi and nonparametric estimations of risk-averse vNM functions, we abandon the bid data of joint-type bidders whose valuations were exogenously changed within a round.

\(^{35}\)In estimations, we use the sequence of equally spaced quantile points, \( \{\alpha_q\}_{q=0,\ldots,Q} \) where \( Q = 250 \) and \( \alpha_q \in [0.50, 0.75] \) for avoiding the boundary problem in kernel density estimations and the discontinuity problem in R functions. To the best of our knowledge, the literature has not settled on a method to choose quantile points as (i.e. how and how many as a researcher
Given the quantiles of bid distributions, in Step 2, the equation (27a) and (27b) can be estimated by the semi and nonparametric methods explained later in this subsection. Once we obtain the estimates of shading functions, \( \hat{\lambda}_{t}^{-1,\text{Affi}}(\cdot) \) from the equation (27a) and \( \hat{\lambda}_{t}^{-1,\text{Inde}}(\cdot) \) from the equation (27b), in Step 3, we can obtain the estimates of valuations by substituting the observed bids and estimated objects in Step 1 and Step 2 in first order necessary conditions as

\[
\begin{align*}
\hat{v}_{r,m,t}^{\text{Asym, Affi}} &= b_{r,m,t}^{\text{Asym}} + \hat{\lambda}_{t}^{-1,\text{Affi}}(\cdot) \left( R_{B_{r,t}, B_{t}}^{\text{Asym, Affi}} \left[ \hat{\lambda}_{t}^{-1,\text{Affi}}(\cdot) \right] \right) \\
\hat{v}_{r,m,t}^{\text{Asym, Inde}} &= b_{r,m,t}^{\text{Asym}} + \hat{\lambda}_{t}^{-1,\text{Inde}}(\cdot) \left( R_{B_{r,t}, B_{t}}^{\text{Asym, Inde}} \left[ \hat{\lambda}_{t}^{-1,\text{Inde}}(\cdot) \right] \right)
\end{align*}
\]

We now introduce the semi and nonparametric specifications of shading functions.\(^{36}\)

### 4.3.1 Semiparametric Estimation for CRRA Model

We assume that bidders have the preference of constant relative risk averse (CRRA) vNM functions \( U_t(x) = x^{\theta_t} \) where \( 0 < \theta_t \leq 1 \) for \( t \in \{\text{Joint, Solo}\} \). The CRRA model has an advantage, as it nests the risk neutral model as the special case of \( \theta_t = 1 \) that is empirically testable. Under the CRRA model, we have \( \lambda_t(x) = U_t(x)/U_t'(x) = x/\theta_t \) and the shading function, \( \lambda_t^{-1}(y) = \theta_t \cdot y \). Then, in Step 2, the compatibility condition equations (27a) and (27b) with estimated distributional functions (\( \hat{\Gamma}, \hat{G}, \) and \( \hat{g} \)) for each type of \( t \in \{\text{Joint, Solo}\} \) become

\[
\begin{align*}
\hat{\nu}_{t-1, t, \alpha}^{\text{Sym}} - \hat{\nu}_{t-1, t, \alpha}^{\text{Asym}} &= \theta_t \cdot \left\{ \hat{R}_{B_{t}, B_{t}}^{\text{Asym, Affi}} \left[ \hat{\lambda}_{t, \alpha}^{\text{Asym}} \right] - \hat{R}_{B_{t}, B_{t}}^{\text{Sym, Affi}} \left[ \hat{\lambda}_{t-1, t, \alpha}^{\text{Sym}} \right] \right\} + \varepsilon_{t, \alpha} \\
\hat{\nu}_{t-1, t, \alpha}^{\text{Sym}} - \hat{\nu}_{t-1, t, \alpha}^{\text{Inde}} &= \theta_t \cdot \left\{ \hat{R}_{B_{t}, B_{t}}^{\text{Asym, Inde}} \left[ \hat{\lambda}_{t, \alpha}^{\text{Asym}} \right] - \hat{R}_{B_{t}, B_{t}}^{\text{Sym, Inde}} \left[ \hat{\lambda}_{t-1, t, \alpha}^{\text{Sym}} \right] \right\} + \varepsilon_{t, \alpha},
\end{align*}
\]

where we use the shorthand notations for simplicity. With the bid quantile data of \( \{\nu_{i, t, \alpha}^{\text{sym}}\}_{q \in \{0, \ldots, Q \}} \) and \( \{\nu_{i, t, \alpha}^{\text{CRRA, Affi}}\}_{q \in \{0, \ldots, Q \}} \) we can apply the OLS estimation to the equations (29a) and (29b) to obtain \( \hat{\theta}^{\text{Affi}} \) and \( \hat{\theta}^{\text{Inde}} \) for each type of \( t \in \{\text{Joint, Solo}\} \). Consequently, in Step 3, we obtain the estimates of valuations

\[
\begin{align*}
\{\nu_{i, t, \alpha}^{\text{CRRA, Affi}}\}_{q \in \{0, \ldots, Q \}} \text{ under the APV assumption and } \{\nu_{i, t, \alpha}^{\text{CRRA, Inde}}\}_{q \in \{0, \ldots, Q \}} \text{ under the IPV assumption by sub-}
\end{align*}
\]

\(^{36}\)Once we estimate \( \hat{\lambda}_{t}^{-1}(\cdot) \), we can analytically or numerically recover a payoff function \( \hat{U}_{t}(x) \) by solving the differential equation of \( \hat{\lambda}_{t}(x) = \hat{U}_{t}(x)/\hat{U}_{t}'(x) \) with the normalized initial condition of \( \hat{U}(1) = 1 \); leading to the solution of \( \hat{U}(x) = \exp \left[ \int_{1}^{x} \hat{\lambda}(z) dz \right] \).

\[22\]
stituting $\hat{\lambda}_t^{-1,\text{Affi}}(\cdot)$ and $\hat{\lambda}_t^{-1,\text{Inde}}(\cdot)$ into equations (28a) and (28b).\footnote{The empirical drawback of the CRRA estimation model under the specification of $U(x) = x^\theta$ is that the estimated $\theta$ is not guaranteed to be in $(0,1)$. $\hat{\theta}$ could be negative (which is not compatible with economic theory) or larger than 1 (which indicates bidders are risk loving). Note that, if we affine-transform the CRRA vNM function into the form of $U(x) = \frac{1}{1+\theta}x^{1+\theta}$, the negative estimated coefficients could be interpreted as extreme risk aversion.}

### 4.3.2 Semiparametric Estimation for CARA Model

Next, we assume that bidders have the preference of constant relative risk aversive (CARA) vNM functions by modeling bidders' payoff functions as $U_t(x) - \frac{1}{\theta}x^{\frac{1}{\theta}}$. As decisions made by CARA-preference bidders are not affected by the level of their wealth (i.e. no wealth/income effect), the CARA model has an advantage of being able to control for bidders' heterogeneity in their wealth levels that are rarely observed in empirical auction research. Given the CARA model, we have $\lambda(x) = \frac{1}{\theta}x^{\frac{1}{\theta}}$, and the shading function has the form of $\lambda_t^{-1}(y) = \frac{1}{\theta_t} x^{1+\theta_t}$. Then, in Step 2, the compatibility condition equations (27a) and (27b) with estimated distributional functions ($\hat{F}_s$, $\hat{G}_s$, and $\hat{g}_s$) for each type of $t \in \{\text{Joint, Solo}\}$ become

\begin{align}
\hat{\beta}_{t,\alpha}^{\text{Sym}} - \hat{b}_{t,\alpha}^{\text{Asym}} &= \frac{1}{\theta_t} \left\{ \ln \left( 1 + \frac{1}{\theta_t} \cdot \hat{F}^{\text{Asym, Affi}}_{t,B_t,B_t} \left[ \hat{b}_{t,\alpha}^{\text{Asym}}, \hat{g}_{t,\alpha}^{\text{Sym}} \right] \right) - \ln \left( 1 + \frac{1}{\theta_t} \cdot \hat{F}^{\text{Asym, Affi}}_{t,B_t,B_t} \left[ \hat{g}_{t,\alpha}^{\text{Sym}}, \hat{b}_{t,\alpha}^{\text{Asym}} \right] \right) \right\} + \varepsilon_{t,\alpha} \tag{30a} \\
\hat{\beta}_{t,\alpha}^{\text{Sym}} - \hat{b}_{t,\alpha}^{\text{Asym}} &= \frac{1}{\theta_t} \left\{ \ln \left( 1 + \frac{1}{\theta_t} \cdot \hat{F}^{\text{Asym, Inde}}_{t,B_t,B_t} \left[ \hat{b}_{t,\alpha}^{\text{Asym}}, \hat{g}_{t,\alpha}^{\text{Sym}} \right] \right) - \ln \left( 1 + \frac{1}{\theta_t} \cdot \hat{F}^{\text{Asym, Inde}}_{t,B_t,B_t} \left[ \hat{g}_{t,\alpha}^{\text{Sym}}, \hat{b}_{t,\alpha}^{\text{Asym}} \right] \right) \right\} + \varepsilon_{t,\alpha} \tag{30b}
\end{align}

where we use the shorthand notations for simplicity. With the bid quantile data of $\{\hat{v}_{t,\alpha,q}^{\text{Sym}}\}_{q=0,\ldots,Q}$ and $\{\hat{v}_{t,\alpha,q}^{\text{Asym}}\}_{q=0,\ldots,Q}$, we can apply the non-linear least square (NLLS) estimation to the equations (30a) and (30b) to obtain $\hat{\zeta}_t^{\text{Affi}}$ and $\hat{\zeta}_t^{\text{Inde}}$ for each type of $t \in \{\text{Joint, Solo}\}$. Consequently, in Step 3, we obtain the estimates of valuations $\{\hat{v}_{t,\alpha,m,r,m,\cdot}^{\text{Asym, Affi}}\}_{r=1,\ldots,R}$ under the APV assumption and $\{\hat{v}_{t,\alpha,m,r,m,\cdot}^{\text{Asym, Inde}}\}_{r=1,\ldots,R}$ under the IPV assumption by substituting $\hat{\lambda}_t^{-1,\text{Affi}}(\cdot)$ and $\hat{\lambda}_t^{-1,\text{Inde}}(\cdot)$ into equations (28a) and (28b).\footnote{As a joint-type bidder’s monetary payment is halved, we need to make slight modifications to equations (28a), (28b), (30a), and (30b), for joint-type bidders. See Online Appendix for details of such slight modifications.}

### 4.3.3 Nonparametric Estimation Model

Finally, we estimate the nonparametric vNM function model that is proposed by Guerre, Perrigne, and Vuong (2009) [28] as it allows the most flexibility to the shapes of shading functions, $\lambda_t^{-1}(\cdot)$.\footnote{To the best of our knowledge, this is the first applied auction work of their nonparametric sieve method for estimating risk-averse preferences in auction research.} Based on the method proposed in their research, we use the sieve method to estimate the shading functions $\lambda_t^{-1}(\cdot) \in \Lambda^{-1}$, where...
\(\Lambda^{-1}\) is a set of differentiable and (strictly) monotonically increasing functions. In practice, we choose \(\Lambda^{-1}\) as the set of polynomial functions \(\text{Pol}(y; \eta, \pi) = \sum_{k=1}^{K} \eta_{t,k} y^k\) without intercept terms, where \(\eta, \pi\) stands for the coefficient vector of \(K\)th order polynomial. In addition, a polynomial order \(K\) flexibly changes. Then, in Step 2, as polynomials are linear in their coefficients, the compatibility condition equations (27a) and (27b) with estimated distributional functions (\(\hat{F}_s, \hat{G}_s, \hat{g}_s\)) for each type of element \(t \in \{\text{Joint, Solo}\}\) become

\[
\hat{b}_{\text{Sym}}^{t,\alpha} - \hat{b}_{\text{Asym}}^{t,\alpha} = \sum_{k=1}^{K} \eta_{t,k} \left( \left( \hat{\beta}_{\text{Sym},\alpha}^{t,\alpha}, \hat{\beta}_{\text{Asym},\alpha}^{t,\alpha} \right)^k \right) - \left( \hat{\beta}_{\text{Sym},\alpha}^{t,\alpha}, \hat{\beta}_{\text{Asym},\alpha}^{t,\alpha} \right)^k + \varepsilon_{t,\alpha} \quad (31a)
\]

\[
\hat{b}_{\text{Sym}}^{t,\alpha} - \hat{b}_{\text{Asym}}^{t,\alpha} = \sum_{k=1}^{K} \eta_{t,k} \left( \left( \hat{\beta}_{\text{Sym},\alpha}^{t,\alpha}, \hat{\beta}_{\text{Asym},\alpha}^{t,\alpha} \right)^k \right) - \left( \hat{\beta}_{\text{Sym},\alpha}^{t,\alpha}, \hat{\beta}_{\text{Asym},\alpha}^{t,\alpha} \right)^k + \varepsilon_{t,\alpha} \quad (31b)
\]

where we use the shorthand notations for simplicity. Under this polynomial specification, \(\Lambda^{-1}\) becomes the linear space, and we can solve the minimization problems by the least square method with the bid quantile data of \(\{\hat{y}_{t,q}\}_{q=0,\ldots, Q}\) and \(\{\hat{y}_{t,q}\}_{q=0,\ldots, Q}\). In practice, we have to select a number of polynomial terms. For selecting \(\hat{\eta}^{\text{Asym}}\) among \(\{\hat{\eta}_1^{\text{Asym}}, \hat{\eta}_2^{\text{Asym}}, \hat{\eta}_3^{\text{Asym}}, \ldots, \hat{\eta}_n^{\text{Asym}}\}\) and \(\hat{\eta}^{\text{Inde}}\) among \(\{\hat{\eta}_1^{\text{Inde}}, \hat{\eta}_2^{\text{Inde}}, \hat{\eta}_3^{\text{Inde}}, \ldots, \hat{\eta}_n^{\text{Inde}}\}\), we adopt the Akaike Information Criterion (AIC) (Akaike 1973 [2]). Consequently, in Step 3, we obtain the estimates of valuations \(\{\hat{\lambda}_{t,1}^{\text{Asym},\alpha}(\cdot)\}_{r=1,\ldots, M}^{m=1,\ldots, M}\) under the APV assumption and \(\{\hat{\lambda}_{t,1}^{\text{Inde},\alpha}(\cdot)\}_{r=1,\ldots, M}^{m=1,\ldots, M}\) under the IPV assumption by substituting \(\hat{\lambda}_{t,1}^{\text{Asym},\alpha}(\cdot)\) and \(\hat{\lambda}_{t,1}^{\text{Inde},\alpha}(\cdot)\) into equations (28a) and (28b). However, the fundamental difficulty of this sieve estimation method based on differentiated variables is that researchers identify \(\lambda^{-1}(\cdot)\) only on limited domains which are away from boundaries. This limitation comes from two facts: (1) we estimate nonlinear shading function \(\lambda^{-1}(\cdot)\) not by applying a nonlinear-recursive-projection estimator (as originally proposed by Guerre Perrigne Vuong (2009) [28]) but by applying a linear (in coefficients) difference estimator in which researchers cannot obtain data points in boundary areas of

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40 Chen (2007) [19] extensively surveys the recent developments of sieve estimations.

41 As we normalize a vNM function, for all sieve estimations, we impose the theoretical restrictions of Restriction1: \(\lambda^{-1}_t(0) = 0\) and Restriction2: \(0 < \frac{d}{dR} \lambda^{-1}_t(R) \leq 1\). In programming, for Restriction1, we remove the intercept terms in polynomials. For Restriction2, we impose the restrictions \(\varepsilon \leq \frac{d}{dR} \lambda^{-1}_t(R) \leq 1\) where \(\varepsilon = 10^{-8}\) on the MATLAB fmincon subroutine.

42 This research takes a view that sieve is a nonparametric estimation method. Some researchers use the terminology “semi-nonparametric” to describe sieve.

43 In programming, we choose the maximum number of polynomial terms as \(K_{\text{max}} = 24\).

44 We use the AIC, which assumes error terms are normally and homoskedastically distributed, for the computational tractability. The cross validation criterion, which is shown to be asymptotically optimal with heteroskedastic error terms by Andrews (1991) [3], is computationally expensive and is not used in this research.

45 Similar to the CARA model case, as a joint-type bidder’s monetary payment is halved, we need to make slight modification to equations (28a), (28b), (31a), and (31b) for joint-type bidders. See Appendix for details of such slight modifications.
domains; and (2) researchers practically need to trim the quantile points to avoid the well-known boundary problem in nonparametric density estimations. To overcome these difficulties, we apply conventional-wisdom-based shape restrictions to extrapolate sieve polynomial estimations to boundary domains while exploiting the additive-coefficient-preserving nature in domains where data variations are available. Appendix provides the details of these shape restrictions with figures. In the next section, we report the estimation results based on the following shape restrictions: (1) minimalistic slope restrictions based only on economic theory;\(^{46}\) (2) shape restrictions based on homogeneously-treated (across bidder types) lower bounds of slopes; and (3) shape restrictions based on heterogeneously-treated lower bounds of slopes.

5 Estimation and Test Results

This section reports the results of estimations and statistical tests under various modeling assumptions. Here, we analyze estimates of valuations derived from asymmetric auction models.\(^{47}\) We first visually compare laboratory-assigned true valuations and estimated valuations. Then, we statistically test their distributional equivalence, asymmetry, and independence. Regarding the model restrictions on vNM payoff functions, we start with the risk neutral model. Then, we discuss risk-averse models in the order of CRRA, CARA, and finally nonparametric models.

5.1 Estimation Results

The estimation results are plotted in Figure 5, 6, 7, 8, 9, and 10, which depict laboratory-assigned true valuations on the horizontal axis and estimated valuations on the vertical axis. For measuring the deviations from true valuations, a 45-degree line is added.\(^{48}\) In addition, to provide crude yet comparable measures of estimates’ accuracy, we calculate the \(L^1\) norm for measuring an averaged distance from the true valuation and \(L^2\) norm for measuring a dispersion, defined by

\[
\begin{align*}
L^1_t &= \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \left| \hat{v}_{r,m,t}^{\text{Asym}} - v_{r,m,t} \right| \\
L^2_t &= \left( \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \left( \hat{v}_{r,m,t}^{\text{Asym}} - v_{r,m,t} \right)^2 \right)^{\frac{1}{2}}
\end{align*}
\]

\(^{46}\)The minimalistic restrictions are \(\tilde{\lambda}^{-1}(0) = 0\) and \(\varepsilon < \frac{M}{2R} \tilde{\lambda}^{-1}(R) \leq 1\) on \(R \in [0, \tilde{R}_{\alpha-1.00}]\) where \(\varepsilon = 10^{-8}\).

\(^{47}\)In this research, symmetric-auction stage bid data is subsidiarily used solely for the purpose of recovering bidders’ risk-averse preferences.

\(^{48}\)Also, for creating equally-scaled figures (so that a 45-degree line is tilted exactly at 45 degrees), the estimated valuations are censored from above at $30. Note that a few of the estimated valuations, especially ones derived from the risk neutral model, exceed $100. Regarding the order, the plots are ordered from left to right as (a) joint-type bidders under APV, (b) joint-type bidders under IPV, (c) solo-type bidders under APV, and (d) solo-type bidders under IPV.
for each experiment run and for each bidder type. Averaged $L^1$ and $L^2$ measures are listed in Table 5 and Table 6.\(^{49}\)

There are four empirical findings across these figures. First, in general, more advanced estimation methods with risk averse vNM functions provide better model fits, accessed by both $L^1$ and $L^2$ measures. Second, within each estimation method, the estimates of solo-type bidders (i.e., valuations of stochastically dominated bidders) have smaller norms compared to those of joint-type bidders, indicating that estimated valuations are relatively more accurate for solo-type bidders. Third, as true valuations become larger, the distance from true to estimated valuations, on average, also gets larger. Fourth, the assumptions of independent and affiliated private valuations create minor difference in estimates in general.\(^{50}\) We now discuss the findings and details of each model.

### 5.1.1 Risk Neutral Model Estimates

The estimates of valuations derived from the asymmetric auction model are plotted in Figure 5. We discover a severe over-estimation of estimates, and this re-confirms the same finding detected in the symmetric model reported by Bajari and Hortacsu (2005) \cite{11}. This over-estimation result suggests that, beyond the commonly used risk neutral model, empirical auction researchers are recommended to investigate more advanced alternative models in order to achieve higher accuracy in estimates. We now discuss the gains in accuracy from such alternative models.

### 5.1.2 Semiparametric CRRA and CARA Model Estimates

The estimated valuations derived from the semiparametric CRRA model are plotted in Figure 6. The problem of over-estimation is largely resolved, yet we now have a noticeable systematic under-estimation problem, especially among joint-type bidders.\(^{51}\) The ordinal least square (OLS) estimates of risk averse parameters, $\theta_t$, derived from the CRRA model are reported in Table 3. All CRRA parameters are in the range of $(0, 1]$.\(^{49}\)

\(^{49}\)To exclude the well-known boundary problem of nonparametric density estimations, unless otherwise specified, $L^1$, $L^2$ and the two-sample Kolmogorov-Smirnov statistics are calculated on the domain between the 25th and 75th quantiles of true value distributions. They are $[9.375, 16.237]$ for joint-type bidders and $[4.6875, 14.0625]$ for solo-type bidders.

\(^{50}\)The surprising finding regarding the estimates of valuations in this research is that affiliated and independent private value models do not have large differences in estimates, although the true data generating process in our experiment is of independent private value. This finding strongly encourages the usage of the affiliated private value assumption in empirical auction research, as originally suggested by Campo, Perrigne, and Vuong (2003) \cite{17}.

\(^{51}\)This under-estimation problem primarily stems from these two factors: (1) we are here using a linear difference estimator instead of a nonlinear recursive estimator; and (2) quantile points are truncated to avoid the boundary problems. See Appendix for the detailed description of this under-estimation problem and the solution of nonparametric sieve estimation with shape restrictions.
as economic theory predicts, although the degree of Arrow-Pratt relative risk aversion \((1 - \theta_t)\) is remarkably large, and the null hypotheses of risk neutral bidders \((H_0: \theta_t = 1)\) are easily rejected.

Next, the estimated valuations derived from the semiparametric CARA model are plotted in Figure 7. The problem of systematic under-estimation has slightly deteriorated among the high valuation domains, as the shading function of the CARA model exhibits a concave shape compared to the linear shape of the CRRA model (see Figure ?? and ?? for the differences in functional forms). Table 4 lists the nonlinear least square (NLLS) estimates of risk averse parameters, \(\zeta_s\). While the estimates for solo-type bidders demonstrate large risk averse attitudes, remarkably, those of joint-type bidders are even larger. We find that these larger numbers in estimates among joint-type bidders are caused by two reasons: (1) the halved-payoff rule for joint-type bidders and (2) the scale variant nature of CARA payoff functions.\(^{52}\) Similar to the case of CRRA, the null hypotheses of risk neutral bidders \((H_0: \zeta_t = 0)\) are easily rejected. Regarding the performance of valuation estimates plotted in Figure 7, we again find a slight systematic under-estimation of valuations, and such under-estimation is particularly noticeable among joint-type bidders.

Note that these systematic under-estimation problems in our asymmetric auctions are consistent with the preceding finding of semiparametric estimates of symmetric auctions in Bajari and Hortaçsu (2005) \(^{53}\). From a practical point of view, the systematic under-estimations among joint-type (stochastically dominating) bidders require improvement, since they directly cause systematic biases in market designs, such as setting reservation prices and calculating expected revenues.\(^{54}\)

### 5.1.3 Nonparametric vNM Function Model Estimates with Shape Restrictions

For solving the systematic under-estimation problem in semiparametric estimates, we now progress to the nonparametric sieve estimation results. The estimates are plotted in Figure 8 (with minimalistic restrictions based on auction theory), Figure 9 (with homogeneous shape restrictions), and Figure 10 (with heterogeneous shape restrictions).\(^{55}\) We observe that, although nonparametric sieve estimations, in general, widen the dispersion of estimates (assessed by the \(L^2\) measure defined above) compared to those derived from semiparametric models, the problem of systematic under-estimation among joint-type bidders is improved, supporting

\(^{52}\) Although CARA preference models are invariant to wealth/income levels, it is variant to payoff scale changes. See Online Appendix for details.

\(^{53}\) See the histograms on pp. 723 of their work for the under-estimation problem of semiparametric estimates.

\(^{54}\) See Marshall, Meurer, Richard, and Stromquist (1994) [58]; Bajari (2001) [10], and Hubbard and Paarsch (2011) [41] for the numerical calculations of reserve prices and expected revenues.

\(^{55}\) Heterogeneous restrictions are motivated from the scale-variant nature of risk averse attitudes in nonparametric payoff models. See Appendix for details and validity of these shape restrictions.
the empirical usefulness of the shape-restricted nonparametric sieve estimation method.

Table 3:
OLS Estimates of CRRA Risk-Averse Parameters: $U_t(x) = x^{\theta_t}$

<table>
<thead>
<tr>
<th>Bidder Type</th>
<th>CRRA: $\theta_t$ under APV</th>
<th>CRRA: $\theta_t$ under IPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Run I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>0.050 (0.056, 0.044)</td>
<td>0.051 (0.059, 0.044)</td>
</tr>
<tr>
<td>Solo</td>
<td>0.271 (0.302, 0.239)</td>
<td>0.090 (0.185, -0.004)</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>0.026 (0.028, 0.024)</td>
<td>0.039 (0.042, 0.036)</td>
</tr>
<tr>
<td>Solo</td>
<td>0.056 (0.064, 0.049)</td>
<td>0.169 (0.196, 0.143)</td>
</tr>
</tbody>
</table>

* Inside of parentheses are 95% confidence intervals with heteroskedasticity-robust standard errors.

Table 4:
NLLS Estimates of CARA Risk Averse Parameters: $U_t(x) = \frac{1 - \exp(-\xi_t x)}{1 - \exp(-\xi_t)}$

<table>
<thead>
<tr>
<th>Bidder Type</th>
<th>CARA: $\xi_t$ under APV</th>
<th>CARA: $\xi_t$ under IPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Run I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solo</td>
<td>1.568 (0.653, 0.483)</td>
<td>1.519 (2.793, 0.246)</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solo</td>
<td>2.433 (2.763, 2.103)</td>
<td>0.804 (0.939, 0.669)</td>
</tr>
</tbody>
</table>

* Inside of parentheses are 95% confidence intervals with heteroskedasticity-robust standard errors.

Table 5: True vs Estimated Valuations: Averaged $L^1$ Norms

<table>
<thead>
<tr>
<th>Bidder Type</th>
<th>Risk Neutral</th>
<th>Semiparametric CRRA</th>
<th>Semiparametric CARA</th>
<th>Nonparametric Sieve: Minimalistic</th>
<th>Nonparametric Sieve: Homogeneous</th>
<th>Nonparametric Sieve: Heterogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
</tr>
<tr>
<td>Experiment Run I</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>5.106 5.583</td>
<td>1.271 1.272</td>
<td>1.255 1.251</td>
<td>1.420 1.403</td>
<td>1.123 1.240</td>
<td>0.924 1.275</td>
</tr>
<tr>
<td>Solo</td>
<td>1.839 2.167</td>
<td>0.615 0.655</td>
<td>0.837 0.614</td>
<td>1.105 1.208</td>
<td>0.842 0.882</td>
<td>0.936 1.005</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>5.573 4.459</td>
<td>1.071 1.021</td>
<td>0.996 0.983</td>
<td>1.155 1.091</td>
<td>0.948 0.902</td>
<td>0.468 0.824</td>
</tr>
<tr>
<td>Solo</td>
<td>2.969 2.088</td>
<td>0.489 0.452</td>
<td>0.440 0.625</td>
<td>0.519 0.449</td>
<td>0.705 0.654</td>
<td>0.569 0.809</td>
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<tr>
<td>Aggregated Runs (Run I and II, aggregated after estimations)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>5.306 5.101</td>
<td>1.185 1.164</td>
<td>1.144 1.136</td>
<td>1.306 1.269</td>
<td>1.048 1.095</td>
<td>0.728 1.082</td>
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<tr>
<td>Solo</td>
<td>2.323 2.133</td>
<td>0.561 0.568</td>
<td>0.667 0.619</td>
<td>0.854 0.883</td>
<td>0.783 0.784</td>
<td>0.732 0.921</td>
</tr>
</tbody>
</table>

* Averaged $L^1$ norms are calculated on $r^{\alpha_1}$, $v^{\alpha_1}$ that is $r^{9.375}$, $v^{16.237}$ for joint-type and on $r^{4.6875}$, $v^{14.0625}$ for solo-type bidders.

Table 6: True vs Estimated Valuations: Averaged $L^2$ Norms

<table>
<thead>
<tr>
<th>Bidder Type</th>
<th>Risk Neutral</th>
<th>Semiparametric CRRA</th>
<th>Semiparametric CARA</th>
<th>Nonparametric Sieve: Minimalistic</th>
<th>Nonparametric Sieve: Homogeneous</th>
<th>Nonparametric Sieve: Heterogeneous</th>
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<tr>
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<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
<td>APV IPV</td>
</tr>
<tr>
<td>Experiment Run I</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>7.513 8.877</td>
<td>2.268 2.266</td>
<td>2.228 2.222</td>
<td>2.439 2.395</td>
<td>2.093 2.218</td>
<td>1.843 2.262</td>
</tr>
<tr>
<td>Solo</td>
<td>2.950 3.644</td>
<td>1.594 1.731</td>
<td>1.667 1.548</td>
<td>1.821 2.006</td>
<td>1.743 1.878</td>
<td>1.683 1.852</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>10.577 7.082</td>
<td>2.254 2.209</td>
<td>2.163 2.151</td>
<td>2.339 2.259</td>
<td>2.244 2.124</td>
<td>2.703 2.069</td>
</tr>
<tr>
<td>Solo</td>
<td>4.896 3.323</td>
<td>1.024 0.892</td>
<td>0.849 1.046</td>
<td>1.076 0.862</td>
<td>1.252 1.121</td>
<td>0.925 1.307</td>
</tr>
<tr>
<td>Aggregated Runs (Run I and II, aggregated after estimations)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>8.956 8.156</td>
<td>2.262 2.242</td>
<td>2.199 2.191</td>
<td>2.397 2.338</td>
<td>2.159 2.178</td>
<td>2.252 2.181</td>
</tr>
</tbody>
</table>

* Averaged $L^1$ norms are calculated on $r^{\alpha_1}$, $v^{\alpha_1}$ that is $r^{9.375}$, $v^{16.237}$ for joint-type and on $r^{4.6875}$, $v^{14.0625}$ for solo-type bidders.
Figure 5: Risk Neutral Model: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV  (b) Joint Type: IPV  (c) Solo Type: APV  (d) Solo Type: IPV

Figure 6: CRRA Model: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV  (b) Joint Type: IPV  (c) Solo Type: APV  (d) Solo Type: IPV

Figure 7: CARA Model: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV  (b) Joint Type: IPV  (c) Solo Type: APV  (d) Solo Type: IPV
Figure 8: Nonparametric Model with Minimalistic Shape Restrictions: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV
(b) Joint Type: IPV
(c) Solo Type: APV
(d) Solo Type: IPV

Figure 9: Nonparametric Model with Homogeneous Shape Restrictions: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV
(b) Joint Type: IPV
(c) Solo Type: APV
(d) Solo Type: IPV

Figure 10: Nonparametric Model with Heterogeneous Shape Restrictions: True (horizontal axis) vs Estimated (vertical axis) Valuations in Asymmetric Auctions

(a) Joint Type: APV
(b) Joint Type: IPV
(c) Solo Type: APV
(d) Solo Type: IPV
5.2 True and Estimated Valuations - Test for Distributional Equivalence

Since auction market designs rely heavily on estimated valuations, the core question in the empirical auction research is whether researchers can achieve a high accuracy in estimated valuations. This is because if estimated valuations are largely different from true ones, researchers cannot expect to derive credible policy insights, such as optimal reserve prices. Accordingly, to test the statistical equivalence between true and estimated value distributions, we here use the two-sample Kolmogorov-Smirnov statistics that specifically test the equivalence between two empirical distributions.56

The test results are listed in Table 7 with a 5 percent significance level. In Table 7, “R” displays that the distributional equivalence is rejected, while “NR” indicates non-rejection. For the sake of increasing statistical power, we aggregate the estimated valuations after each method of structural estimation (denoted as “Aggregated Runs” in Table 7) to overcome the small sample size problem,57 and we here primarily analyze the test results of “Aggregated Runs.” As conjectured from Figure 5, the distributional equivalence in the risk neutral model is immediately rejected for both joint- and solo-type bidders, suggesting that empirical researchers should use more advanced risk-preference models. Next, by applying the CRRA and CARA semi-parametric models, we achieve the distributional equivalence among solo-type bidders. On the other hand, although they are improved compared to those in the risk neutral models, estimates among joint-type bidders still do not attain the equivalence, indicating the necessity of further improvements. Lastly, by applying the nonparametric sieve estimations, we obtain the highest accuracy of estimates, failing to reject distributional equivalences for both bidder types when heterogeneous restrictions are applied. Specifically, conventional-wisdom-based shape restrictions are empirically shown to be a great supporting tool in helping researchers to achieve higher accuracy. In summary, these distributional equivalence tests confirm that researchers attain higher accuracy when they use more advanced semi and nonparametric econometrics, strongly supporting the empirical usefulness and credibility of auction estimation methods developed in recent years.

56Note that the modified version of the two-sample Kolmogorov-Smirnov test is proposed by Haile, Hong, and Shum (2003) [29] and Bajari and Hortacsu (2005) [11] to account for the dependent nature between two samples with a subsampling method. We have tried their modified statistics, yet due to the computational burden associated with subsampling sieve estimations and our limited PC specs, we could not finish the computation within a reasonable time. Here, we simply present the results of the (original) two-sample Kolmogorov-Smirnov test as straightforward measures of distributional equivalence. Campo, Perrigne, and Vuong (2003) [17] also use the (original) two-sample Kolmogorov-Smirnov test.

57For example, the risk neutral model for solo type bidders using bid data from Experiment Run I (under both affiliated and independent assumptions) fails to reject the distributional equivalence due to the small sample size $N = 96$, despite the fact that Figure 5 visually shows the poor accuracy of estimates.
5.3 Estimated Joint- and Solo-Type Valuations - Test for Distributional (A)Symmetry

In empirical asymmetric auction studies, researchers often assume the asymmetry among bidders (or firms) based on a priori information, such as differences in firm size, capacity size, or observed bid distributions. In our experiment, the true data generating process is asymmetric, that is, joint-type and solo-type bidders draw valuations from different value distributions. Now, we investigate whether the estimated value distributions hold this asymmetric property. We again use the two-sample Kolmogorov-Smirnov statistics, yet we now test the hypothesis of distributional equivalence (or distributional symmetry) between an estimated joint-type value distribution and an estimated solo-type value distribution. The test results with the 5 percent significance level is described in Table 8. The test results show that, although distributional symmetry is not rejected under some modeling assumptions with Experiment Run II bid data, they are all rejected under all modeling assumptions when we aggregate the estimated valuations (the bottom row of Table 8). Thus, we confirm that the asymmetric distribution property is preserved in estimated value distributions, another evidence that supports the credibility of empirical asymmetric auction estimates.

5.4 Estimated Joint- and Solo-Type Valuations - Test for Independence

In many investigations of empirical auctions, the independence of valuations is assumed beforehand, yet it is empirically diligent to test the independence in order to avoid potential model-misspecification problems. In our experiment, joint- and solo-type bidders independently draw valuations. Thus, if estimates are accurate, independence should not be rejected. Accordingly, we here test the null hypothesis of independent valuations against the alternative of non-independent valuations. Specifically, we use the nonparametric test of independence proposed by Blum, Kiefer, and Rosenblatt (1961) [15] with the test statistic of \( \frac{1}{\pi} \cdot N \cdot B_N \) where \( B_N = N^{-4} \sum_{l=1}^{N} \{ N_1(l) \cdot N_4(l) - N_2(l) \cdot N_3(l) \}^2 \) with the index \( l \in \{1, \cdots, RM\} \) where \( R \) is the number of rounds in each experiment run and \( M \) is the number of (within-a-round) matched groups. Here, \( N_1(l) \), \( N_2(l) \), \( N_3(l) \), and \( N_4(l) \) are defined as the number of data points that fall into the region of \{\( x, y : x \leq X_l, y \leq Y_l \}\}, \{\( x, y : x \leq X_l, y \geq Y_l \}\}, \{\( x, y : x \geq X_l, y \leq Y_l \}\}, and \{\( x, y : x \geq X_l, y \geq Y_l \}\}, respectively. Table 9 reports the results of the BKR tests. In all experiment runs, including aggregated runs, and under all modeling assumptions, independence is not rejected, further supporting the credibility of empirical auction estimates.

58 A stochastic dominance test proposed by Barrett and Donald (2003) [14] could also be used if researchers are further interested in a specific order of stochastic dominance.

59 To avoid the boundary value problem, we calculate the Kolmogorov-Smirnov statistics on \([4.6875, 16.237]\) that is between the 25th quantile of true solo-type bidder value distribution and the 75th quantile of true joint-type bidder value distribution.

60 The non-rejection problem with Experiment Run II bid data is caused by the underestimation of joint-type bidder valuations.

61 Campo, Perrigne, and Vuong (2003) [17] use the same test statistics to investigate the affiliation of estimated valuations in Outer Continental Shelf (OCS) wildcat lease auction data and reject the null of independent valuations.

62 The \( p \)-values of the BKR statistic are listed in Table II (p.497) of Blum, Kiefer, and Rosenblatt (1961) [15].
Table 7: True and Estimated Valuations - Test for Distributional Equivalence* * †† (R = Rejected; NR = Not Rejected)

<table>
<thead>
<tr>
<th>Bidder Type</th>
<th>Risk Neutral</th>
<th>Semiparametric</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>APV IPV</td>
<td>CRRA CARA APV IPV</td>
<td>Minimalistic APV IPV</td>
<td>Homogeneous APV IPV</td>
<td>Heterogeneous APV IPV</td>
</tr>
<tr>
<td><strong>Experiment Run I</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>3.103 3.392</td>
<td>2.021 2.093</td>
<td>2.327 2.382</td>
<td>1.949 1.949</td>
<td>1.155 1.083</td>
</tr>
<tr>
<td></td>
<td>(0.000) (0.000)</td>
<td>(0.001) (0.000)</td>
<td>(0.000) (0.000)</td>
<td>(0.001) (0.000)</td>
<td>(0.139) (0.192)</td>
</tr>
<tr>
<td></td>
<td>R R R</td>
<td>R R</td>
<td>R R</td>
<td>R R</td>
<td>R R</td>
</tr>
<tr>
<td>Solo</td>
<td>1.010 1.010</td>
<td>0.722 0.722</td>
<td>0.722 0.650</td>
<td>0.577 0.650</td>
<td>0.505 0.505</td>
</tr>
<tr>
<td></td>
<td>(0.260) (0.260)</td>
<td>(0.706) (0.344)</td>
<td>(1.000) (0.706)</td>
<td>(1.000) (0.860)</td>
<td>(1.000) (1.000)</td>
</tr>
<tr>
<td></td>
<td>NR NR</td>
<td>NR NR</td>
<td>NR NR</td>
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<tr>
<td><strong>Experiment Run II</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>2.083 2.250</td>
<td>1.583 1.583</td>
<td>1.667 1.667</td>
<td>1.250 1.333</td>
<td>0.583 0.583</td>
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<tr>
<td></td>
<td>(0.000) (0.000)</td>
<td>(0.013) (0.013)</td>
<td>(0.008) (0.008)</td>
<td>(0.088) (0.057)</td>
<td>(1.000) (1.000)</td>
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<tr>
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<td>R R</td>
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<tr>
<td>Solo</td>
<td>1.917 1.583</td>
<td>0.750 0.583</td>
<td>0.750 0.500</td>
<td>0.583 0.500</td>
<td>0.583 0.833</td>
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<td>(0.013) (0.013)</td>
<td>(0.649) (1.000)</td>
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<td>NR NR</td>
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<tr>
<td><strong>Aggregated Runs (Run I and II, aggregated after estimations)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Joint</td>
<td>3.103 3.928</td>
<td>2.291 2.237</td>
<td>2.400 2.455</td>
<td>1.964 1.964</td>
<td>1.037 0.982</td>
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<td>(0.000) (0.000)</td>
<td>(0.000) (0.000)</td>
<td>(0.001) (0.001)</td>
<td>(0.233) (0.291)</td>
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<td>R R</td>
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<td>R R</td>
<td>R R</td>
<td>R R</td>
</tr>
<tr>
<td>Solo</td>
<td>2.019 1.746</td>
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<td>0.600 0.655</td>
<td>0.655 0.655</td>
<td>0.600 0.709</td>
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<td>(0.001) (0.005)</td>
<td>(0.358) (0.233)</td>
<td>(0.973) (0.849)</td>
<td>(0.849) (0.849)</td>
<td>(0.973) (0.731)</td>
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<td>NR NR</td>
<td>NR NR</td>
</tr>
</tbody>
</table>

* Kolmogorov-Smirnov statistics are calculated on $[v_{\alpha=0.25}, v_{\alpha=0.75}]$ that is $[9.375, 16.237]$ for joint-type and on $[84.6875, 114.0625]$ for solo-type bidders.

† Level of significance is $\alpha = 0.05$.

† Inside parentheses are $p$-values calculated by using the property of $4D^2[(n_1+n_2)/n_1 + 1]$, where $D$ is a Kolmogorov-Smirnov distance, which is asymptotically distributed as $\chi^2_{df=2}$.

†† Note that $p$-value = $\min(1, 2 [1 - \text{CDF}_\chi^2(4D^2(n_1+n_2)/[n_1 + n_2], df=2)])$. 
Table 8: Estimated Joint- and Solo-Type Valuations - Test for Distributional Symmetry*** (R = Rejected; NR = Not Rejected)

<table>
<thead>
<tr>
<th></th>
<th>Risk Neutral</th>
<th>Semiparametric</th>
<th>Semiparametric</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>CRRA</td>
<td>CARA</td>
<td>Minimalistic</td>
<td>Homogeneous</td>
<td>Heterogeneous</td>
</tr>
<tr>
<td></td>
<td></td>
<td>APV</td>
<td>IPV</td>
<td>APV</td>
<td>IPV</td>
<td>APV</td>
</tr>
<tr>
<td>Experiment Run I</td>
<td></td>
<td>2.815</td>
<td>3.175</td>
<td>1.949</td>
<td>2.021</td>
<td>1.804</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td>1.167</td>
<td>1.917</td>
<td>1.333</td>
<td>1.250</td>
<td>0.917</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.131)</td>
<td>(0.001)</td>
<td>(0.057)</td>
<td>(0.088)</td>
<td>(0.057)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Aggregated Runs (Run I and II,</td>
<td>2.619</td>
<td>3.491</td>
<td>2.073</td>
<td>2.182</td>
<td>1.800</td>
<td></td>
</tr>
<tr>
<td>aggregated after estimations)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>

Kolmogorov-Smirnov statistics are calculated on $r_{\text{Joint, } \alpha = 0.75}$. The level of significance is $\alpha = 0.05$. Inside parentheses are $p$-values.

Table 9: Estimated Joint- and Solo-Type Valuations - Test for Independence*** (R = Rejected; NR = Not Rejected)

<table>
<thead>
<tr>
<th></th>
<th>Risk Neutral</th>
<th>Semiparametric</th>
<th>Semiparametric</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
<th>Nonparametric Sieve:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CRRA</td>
<td>CARA</td>
<td>Minimalistic</td>
<td>Homogeneous</td>
<td>Heterogeneous</td>
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<tr>
<td></td>
<td></td>
<td>APV</td>
<td>IPV</td>
<td>APV</td>
<td>IPV</td>
<td>APV</td>
</tr>
<tr>
<td>Experiment Run I</td>
<td></td>
<td>0.033</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.257)</td>
<td>(0.228)</td>
<td>(0.228)</td>
<td>(0.228)</td>
<td>(0.124)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Experiment Run II</td>
<td></td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.450)</td>
<td>(0.450)</td>
<td>(0.450)</td>
<td>(0.450)</td>
<td>(0.450)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Aggregated Runs (Run I and II,</td>
<td>0.016</td>
<td>0.016</td>
<td>0.016</td>
<td>0.016</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>aggregated after estimations)</td>
<td>(0.798)</td>
<td>(0.818)</td>
<td>(0.809)</td>
<td>(0.878)</td>
<td>(0.807)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

Based on the test proposed by Blum, Kiefer, and Rosenblatt (1961). The level of significance is $\alpha = 0.05$. Inside parentheses are $p$-values.
6 Conclusion

To provide an answer to the criticism and skepticism aimed at empirical asymmetric auction studies, this research provides laboratory evidence to support the credibility of asymmetric first-price auction estimates that has not yet been reported. To establish the credibility, we provide new statistical evidence of estimate accuracy by demonstrating how well the estimates of valuations fit the true valuations. Also, we newly show that the usage of risk averse semi and nonparametric estimation methods lead to nonnegligible improvements in asymmetric auction estimates and that advanced estimation techniques, in general, achieve higher accuracy. Although the degree of such improvement will differ by applications, the fact of improvement is generalizable to other empirical auction research.

Finally, the external validity (i.e., generalizability and translatability) of our results to other auctions, especially empirical auction research with field auction data, must be addressed. We recognize that getting one good estimation result in one specific auction environment does not guarantee that researchers will get a similar result in other situations. However, one can deductively bring conservative yet practical insights by contrasting our experimental auctions with field auctions. Experimental auctions differ from field ones in, at least, three ways: (i) strategic intricacies of auctions; (ii) bidder sophistication; and (iii) bidder motivation due to monetary stakes. The discussion below, as depicted in Figure 11, breaks down (i) into two parts (i.e., low and high strategic intricacies compared to this research), then examines the generalizability of our results regarding (ii) and (iii).

Asymmetric auctions with high/advanced strategic intricacies, as compared to this research (right hand side of Figure 11): If an environment of empirical asymmetric auction research is more intricate than the one we have discussed in this study, such as endogenous and strategic participation in auctions or binding reserve prices, our results have limited external validity on the accuracy of estimates. Bidders who face such a high degree of strategic intricacies may behave differently than what we observe in our experiment. Further in-
vestigation on the accuracy of estimates derived from experimental data, or any field data that directly or indirectly contains information about underlying valuations, will extend the results of this study for such intricate auctions.⁶³

Asymmetric auctions with low (or similar) strategic intricacies, as compared to this research (left hand side of Figure 11): In our experimental asymmetric auctions, the participants were undergraduate students. Thus, given the lack of their real-world business experience, their degree of strategic sophistication is reasonably expected to be lower than the ones observed in real-world competitive business industry (i.e. low degree of (ii)). In addition, as the monetary stakes in our experiment are relatively low compared to the stakes observed in real-world auctions, the associated motivation among bidders in the laboratory is also expected to be low (i.e. low degree of (iii)). However, the positive finding of this research is that structural estimates derived from bids submitted by such strategically unsophisticated and relatively less financially motivated bidders are statistically shown to be accurate. Therefore, we can deductively translate the positive finding on the accuracy of estimates reported in this research into the estimates generated from bids submitted by professional industry bidders in field auctions for the following reasons: first, professional industry bidders must have a high degree of strategic sophistication in order to survive harsh industry competition (i.e. high degree of (ii)), and secondly, as monetary stakes in real-world auctions are high, the associated motivations among industry bidders are also high (i.e. high degree of (iii)). It stands to reason then, compared to our experiment participants, such professional bidders are more likely to profoundly recognize underlying strategic interactions in auctions as prescribed by BNE and less likely to make optimization errors. Accordingly, because structural estimations are rigidly based on BNE, the estimates derived from such sophisticated and seriously considered bids are likely to be more accurate than those reported in this research. Thus, we deductively conclude that, as long as the strategic intricacy of underlying asymmetric auction market is not vastly different from the one discussed in this research and as long as industry bidders are maximizing expected payoffs, what holds accurate in our laboratory auctions also holds accurate in a real-world industry setting.

For these reasons, this research not only contributes to providing support for estimates previously reported in the literature but also pushes the credibility of present and future empirical auction research further.

References


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