Mechanism Design for Designs: Principals with Good (and bad) Taste

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January 30, 2015

Abstract

This paper considers a moral hazard problem where a principal contracts with one or more agents to produce a design. The design refers to something whose value is subjective, may or may not be well understood by the principal, and can be returned if the principal deems it to have low value, which is sometimes called right of refusal. Correlation of the principal’s signal with the its true value plays a role, in contrast to standard principal agent problem; experience goods are different from credence goods. The principal’s ability to forecast value corresponds to a notion of taste for the principal that distinguishes taste from judgment. This measure of informativeness of the signal matters for many features of the contract design: the total cost, the number of agents contracted with, and the choice of agent quality. This measure of informativeness is related to subjectivity, and shows that privateness of the signal, which has been used synonymously with subjectivity in the literature, does not fully describe subjectivity. For examples where the subjectivity comes through the taste rather than lack of judgment, uncertainty in the outcome may make the incentive contract less costly, in contrast to both the canonical model of moral hazard and the case where signals are private but not necessarily truly subjective.

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1 Introduction

Technological change that comes through new products is driven by the process of design. Design is, more broadly, a quantitatively important part of the economy. Narrow measures of the contribution of “creative industries” to world output exceed four percent and are fast growing; authors have argued that a variety of “knowledge” work outside of the creative industries might be viewed as part of the design process, leading to estimates that employment in creative intensive jobs greater than 30% ([3]), a doubling in 50 years. Growing rates of patenting indicate an increase in the rate of change in the production of new designs. While important, design work may be difficult to monitor directly because effort is mental. Evaluating designs is often coarse and subjective, without an obvious way to pay based on a clear realization. When a new building is built or advertising campaign run, it can be difficult to assess, even ex post, the contribution of the project to output.1 Potential forms of output are difficult to describe, since the details of what would be contained in a good design are unknown ex ante; this is what necessitates the need for a designer.2 Moreover the designer may be better able to understand a good design than the principal.

Motivated by the design process, this paper studies a principal-agent model where the principal has a non-verifiable signal of the action of the agent or agents. The contract can specify not only payments, but an allocation of the object, which we call a design: if the principal reports that he does not like a particular design, he may not keep it. This contract term is used in some cases in practice with design contracts and is often called right of refusal. We describe the sense in which right of refusal is useful in this context, and show

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1When the military asks for designs for a new piece of military equipment, it is difficult to assess the value added of that equipment. One imagines that the design of the exterior of the iPhone is relevant to its value, but it is hard to determine the benefit of that vis a vis other unique characteristics of the product. When a marketing firm is hired to design an ad campaign, it is difficult to determine even ex post the financial impact of the ad campaign. Even road construction projects often include a design component that may be rated, subjectively, and the ratings used to decide which firm undertakes the project ([15]).

2In fact, even in cases like the well documented case of the prize for the naval chronometer, a detailed description of the design’s requirements was insufficient. Upon seeing a design that seemed to meet the criterion, the principal (the “Board of Longitude”) denied that the design was sufficient; the principal did not believe the previously unknown innovator nor the simple design could possibly solve the very hard problem at hand.
that its role is related to the principal’s ability to assess his own value, an element not relevant in canonical agency models. The key feature we study is how reflective the signal is of the true value of the design, which we associate with taste and judgment of the principal.

In the standard model of a principal and agent where the principal’s signal is verifiable, the relevant informativeness of a signal is its correlation with the agent’s action. In that textbook model it is irrelevant whether the payoff for the principal is exactly the signal, (for instance in the case where the signal is realized output of the agent’s effort or the realized utility of an experience good), or the payoff is exactly the agent’s action, so that the signal has no informativeness given the incentive compatible action (a credence good). Practically, the principal is merely trying to assess the magnitude of inputs; measuring output is just one possible route to that goal. Given the incentive compatible action, output is irrelevant for incentives, and should be assigned to maximize total surplus, without any regard to incentives. As a result, right of refusal, and more generally assessing output conditional on inputs, has no place in a standard model of incentive contracting.

When the principal’s signal is private, however, the allocation of the output can be useful in generating incentives for the principal to report his signal truthfully. In particular, it is useful to the extent that the principal’s signal has information content about the value of the output, so that the principal will be reluctant to refuse a project that he assesses to be valuable. This motivates the use of right of refusal, and allows the principal’s ability to assess output to play a role. The information is conditional on the action, which the principal infers from the contract; therefore, the important new element is the principal’s information about output, conditional on the agent’s action.

The first results fix the relationship between the signal and the value of the design, and discuss the value of the precision of the signal. In general there is value in coarser signals when right of refusal is used. This is a broad sense not surprising, since the model assumes the signal is private information of the principal, and more private information is often not socially desirable. Coarser signals are not, however, beneficial in the formulation without right of refusal studied in MacLeod; there a coarser signal always makes the principal worse off. The value of coarseness is specifically on the high end of signals: when the project is kept only in some (good) states, the principal prefers, among states where the project is to be retained with probability one, to be unable to further distinguish between the states. In other words, conditional on liking the project sufficiently well to keep it for sure, the prin-
Principal does not want precise information on exactly how much he “loves” the project.

The majority of the results are comparative statics on the impact of changing the degree of the principal’s ability to assess the value of output. The model has a parameter that allows the signal to vary from a credence good to a pure experience good. The credence good case, where the principal cannot judge output at all conditional on the agent’s input, is associated with the principal’s noisy signal reflecting his lack of judgment about the design. In the experience good case, the signal is fully reflective of value ex post, and the noisy signal reflects the idiosyncrasy of the principal’s taste. The closer the signal is to an experience good, the greater is the use of right of refusal is useful. This fits the casual evidence that it is common for commissioned artwork to include a “right of refusal” on the buyer side: if the buyer does not like the final product, he does not pay the full amount and, importantly, does not keep the artwork.\(^3\) On the other hand, right of refusal is less common in a variety of design contracts that involve subjective evaluation for the principal, but greater expertise of the principal, for instance in architecture contracts.\(^4\) When the agent is the expert on the value of the object, so that the signal provides relatively little information about the true value of the design, right of refusal is not a useful contract feature.

Being able to assess value lowers the principal’s cost because it enables the use of right of refusal as an incentive device. The closer the good is to an experience good, the less the principal may value high quality agents (defined to be those who produce more output on average with the same input), because high quality agents may actually interfere with the principal’s capability to assess value. Further, in the special case where the shock is Laplace, to the extent that right of refusal is useful, it can also be the case that noise in the principal’s signal can have incentive benefits. This is similar to the result on coarseness, and is in stark contrast to the canonical model

\(^3\)http://www.finearttips.com/2009/11/commissioned-art-%25E2%2580%2593-tips-to-make-it-a-success/, http://thepracticalartworld.com/2011/11/21/how-to-build-a-contract-for-commissioned-artworks/. A popular story of a commission being “refused” is the destruction of Diego Rivera’s fresco “Man at the Crossroads.” While clearly showing the possibility of refusal for art, it does not strictly fit the model in the sense that Rivera was paid the full commission even though it was destroyed. On the other hand, destroying the fresco clearly lowered Rivera’s utility, which would fit the notion that the principal takes an action which lowers both his payoff and the agent’s payoff in some states.

\(^4\)http://www.raic.org/practice/contract_documents/index_e.htm
of moral hazard, as well the case where the signal is private but, due to expertise on the agent’s side, right of refusal is not useful.

The comparative statics on the principal’s ability to assess output also provide insight into the use of multiple agents. Tournaments are a well known solution to verifiability on the principal’s side: the principal hires multiple agents and the commits to pay some agent, and therefore has no incentive to misreport the winner, as for instance in [7] and [11]. In the usual tournament model the principal benefits directly from both agents effort, for instance where both are selling a product or producing output, which the principal keeps. Designs are inherently somewhat indivisible. In these cases there is the matter of duplication: the designs that are not chosen in the contest are, essentially, wasted; they have value in generating incentives, but the effort is not directly beneficial to the principal. This tension between, on the one hand, competition as an incentive device and, on the other hand, redundant effort in competition, is a fundamental feature of the design process with multiple agents.

The comparative statics show that using two agents is more likely for cases where the principal cannot assess the value of output, as in a credence good, but less valuable for experience goods. In other words, one might commission fine art from a single agent, when the goal is simply to enjoy the art, or an exclusive supplier in a vertical relationship, where expertise is strong on the buyer’s side, but run a tournament for situations like architects and marketing firms, where the principal imagines that the designer is the best assessor of the quality of the design.

One interpretation of these comparative statics is as reflecting the type of subjectivity. Consider the example of having a building designed. Imagine that the principal wants the building to survive an earthquake of a particular magnitude, and simply does not know how to make a design that can survive that criterion. If the principal’s assessment of whether or not the building will survive the earthquake cannot be verified by the courts, it fits into the usual definition of subjective evaluation. The underlying question, however, is objective. Subjectivity reflects a lack of perfect judgment on the part of the principal; whether or not the principal can assess value, conditional on

5While one can, to some extent, take the best features from multiple designs, it is certainly costly to aggregate the work of multiple designs.

6Contests between agents are common in both marketing and architecture, where several designs are solicited. More generally, product innovations are pitched to firms before they are produced and sold.
the work of the agent, is driven by the relative expertise of the principal and agent. This does not imply that the principal has no judgment; the principal, however, has no extra judgment beyond what the agent’s effort offers. In this case noise reflects lack of judgment on the principal’s part.

Contrast that with the situation where the principal wants a building that looks beautiful to him. The assessment of beauty is inherently subjective: there is no objective test for the principal’s opinion of the beauty of the bridge other than his own signal. In this case the principal’s signal is a matter of taste and is unambiguously informative about output; beauty is in the eye of the beholder. Therefore the usefulness of right of refusal, and the value of uncertainty when right of refusal is used, is connected with cases where the subjective question is a matter of taste, or where the principal is the expert about the underlying objective question that is being subjectively evaluated. In this case noise reflects not a lack of judgment but rather strong individual preferences. Naturally, then, noise may play a different role in the two cases.

**Literature**

The fact that key features of design make it unusual relative to standard examples has been identified by Caves [1]. In his words, “Consumers’ reaction to a product are neither known beforehand, nor easily understood afterward.” Since we take the principal’s signal as private, a natural starting point is MacLeod’s ([10]) model of a principal-agent problem with a privately observed signal. We differ by allowing the contract to specify whether the agent’s output will be allocated to the principal, allows a role for the principal’s taste to play a role. We further depart from MacLeod and allow the principal to contract with multiple agents. In ([10]), as well as many other papers studying subjective evaluation, subjectivity is identical to unverifiable or private. This paper describes subjectivity in more detail, taking private information as one ingredient in subjectivity. In particular, the model draws a distinction between private signals of the principal that reflect the principal’s preference conditional on the agent’s action on the one hand, and private signals that reflect merely the principal’s inability to identify the action of the agent on the other. We describe below the sense in which the former might be more truly described as subjective.

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Note that an art buyer who wanted to buy a piece that would be widely thought to be beautiful is closer to the subjective evaluation case, since the underlying question “do most people like the way this painting looks?” is objective.
The model in this paper is also related to the model of up-or-out promotion in [6]. In that model, unlike [10], the principal can refuse to continue to employ the agent if productivity is low. In [6], the principal is assumed to perfectly assess the value of the worker. The model in this paper highlights the role of the principal’s ability to assess the value of output, conditional on inputs, in features of the optimal contract such as the use of right of refusal and multiple agents.

Right of refusal is similar to a money back guarantee. Money back guarantees have been extensively analyzed in double-moral hazard environments (see [12]) or as signals of quality ([9]). Here the role is different: the principal provides no effort to the project, and there is nothing to signal. Similarly to the double moral hazard case, however, there is private information on both sides. Our focus is different in that we describe how the contract varies with the principal’s ability to assess output.

In that sense our multiple agent setup is very much in the spirit of tournament models, such as [7] and [11]. These models incorporate multiple agents with an information structure that is the polar opposite of moral hazard in teams (as in [5]), where the principal sees a signal of team output but no signal of how that effort is distributed across team members. As in the tournament literature, the principal here gets a signal of relative inputs, but no signal of the absolute level of team effort. As a result, the structure with multiple agents is a sort of tournaments.

Our model with two agents is related to the problem in [2]. In that paper, a buyer contracts with potentially many sellers for an innovation. The buyer employs a contest, choosing one seller to buy from, at a price that the seller picked in conjunction with the investment decision. The quality is unverifiable, but, in strong contrast to this paper, is certain and observed by both parties, in the spirit of Hart and Moore (1988). In that paper, therefore, there is no real issue of subjectivity, since there is no difference of opinion. Even if an outsider cannot verify the ex post value of the object,[2] show that one can still use a sort of auction mechanism – make a quality, bid a price, effectively an auction in net value provided – if the parties can agree on what constitutes quality at the time that payments are made. The difference from here is that, in their paper, the true value of the project is know at the time when execution decisions must be made, and therefore execution

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8 Other papers that consider up-or-out structures include [4] who consider the informational content of promotions, and [16].
is essentially contingent on value. Che and Gale show that the ability to allocate the object based on the reported signal allows for an auction in net value: each design submits a price, and chooses effort; the principal chooses the best bid.

The dynamic contracting literature stresses the role of relational contracts in addressing subjectivity ([8]). Those models allow for a distinction between non-contractible signals and private signals that the static model does not have, since a non-contractible but observable signal can be used as a conditioning device by both players in the continuation subgame of a repeated interaction. That distinction is only relevant due to the repeated nature of the interaction studied in the relational contracting literature, which is focused on relationships like the employer-employee example which are ongoing. The approach to subjective contracting in this paper may be the most natural one in the context of art and architecture contracts that are often not repeated in the same way that as an employment relationship.

In the literature on rewarding innovation, the signal of quality is either viewed as contractible (in the classic patent race literature such as [14]) or ascertained through a reward tied to the ex post value of the innovation (for instance through market power that translates into profits depending on the quality of the design). These models share the feature of the multi-agent setting here, where inputs from multiple agents may be redundant. Here we imagine an innovation where an objective evaluation is impossible. Many innovations have this situation. Patent examiners make a subjective decision about the novelty and usefulness of an innovation when granting a patent, and courts do the same when determining validity and scope of a litigated patent. In many cases it is difficult to ascertain the value of a specific innovation even ex post. For instance, consider a patent contributing to a technology like a wireless network standard that contains thousands of patents, making individual value hard to assess. Deciding on compensation for the various patents is subjective, and fraught with disagreement. The model described here takes seriously the notion that rewards for innovators must confront the fundamentally subjective decision of evaluating an innovation ex post, where in many cases the details of the innovation were unknowable prior to the innovation. This is in contrast with views of innovation such as [13], where innovators need to be encouraged to take risks, but the nature of innovation can is well defined ex ante.
Paper Organization

Section 2 introduces the general model. It builds directly on [10], where a principal contracts with a single agent, but with the added possibility of “right of refusal.” We then turn to the two action, two outcome case to study both comparative statics on the informativeness of the principal’s signal, as well as the multi-agent model.

2 The Model

2.1 No reallocation of output: MacLeod (2003)’s Money Burning Contract

An agent takes an action denoted by $a \in R$, which is unobserved by the principal, at convex cost $c(a)$. The principal observes a signal $s$ of the agent’s action, which is unverifiable, because it is subjective. We take the signal to be distributed over a finite set; this follows MacLeod, and conforms to the notion that subjective views of a design project are unlikely to be very fine-grained. The signal is distributed according to $f(s|a)$. MacLeod allows for the principal to commit to destroy (burn) resources: the agent is paid $w(s)$, and an amount $b(s)$ is burned. The principal is risk neutral and allocated the value of the action $a$ regardless of the state, so it does not impact his reporting incentives. The agent values the wage by the concave $u(w)$ and has outside option normalized to zero.

The cost to the principal of implementing an action $a$ is

$$C(a) = \min_{w(s), b(s)} \sum (w(s) + b(s))f(s|a)$$

s.t.

$$a \in \arg\max \sum u(w(s))f(s|a) - c(a)$$

$$0 \leq \sum u(w(s))f(s|a) - c(a)$$

$$s \in \arg\min w(s) + b(s)$$

The solution must have $w(\hat{s}) + b(\hat{s}) = \max_s w(\hat{s}) + b(\hat{s})$, and therefore the cost is just equal to the maximum resources expended. This leads, under the usual monotonicity condition on $F$, to the planner using just two levels of payment.\(^9\)

\(^9\)Proofs are in the Appendix when not included in the text.
Proposition 1. (MacLeod 2003). Suppose that $F$ has the monotone likelihood ratio property, $\frac{df}{da}/f$ increasing. Then the optimal contract has $b = 0$ (and therefore a constant wage) for all but the lowest value of $s$.

Section 3 studies a model with only two signals; this both fits the coarse nature of tastes for design and, in many cases such as this one, may be without loss, in the sense that the optimal policy uses at most two outcomes for the contract. It is immediate from the fact that the principal is indifferent to all reports that a coarser signal must make the cost weakly higher; whatever is optimal given the coarser signal is incentive feasible for a finer signal, since the payments are the same for all signals in either case. It turns out that coarseness of the signal may be valuable when the principal chooses to discard a design; here, however, a coarser signal, in particular at the bottom of the distribution, makes the principal worse off.\footnote{For cases that do not involve the bottom state the coarseness has no effect since the principal takes the same action in those states.}

2.2 The Role of Taste and Judgment with Right of Refusal

One can interpret $s$ as a measure of output; however, in both the standard principal agent model and the model of MacLeod, $s$ is fundamentally an assessment of the agent’s input. The role of assessing output becomes clear when we consider the case where the principal can choose to refuse the design as a way to satisfy the constraint on reporting $s$. This feature matches the common ‘right of refusal’ included in some design contracts. One goal of the paper is to understand when such a feature might be useful to include in the contract.\footnote{This seems at least as reasonable as committing to money burning in some examples, where the output is a well defined object which can be returned.} One can interpret it either as the design being destroyed, or that the design is reallocated to the agent, who has a low value for discarded designs, in which case it relies only on allocation of objects between the two parties to the contract. Ex post allocation of output is also essential in environments like Che and Gale where the principal receives the signal, and then decides which project to select. We study multiple agents below.

Studying the impact of the option to “burn the design” rather than only burning money requires describing the planner’s valuation of designs, which is the sense in which taste matters. Since the principal will have to decide
whether to keep the project based on the signal, the principal’s reporting incentives are influenced by his valuation conditional on the signal; the principal also knows, for an incentive compatible contract, the agent’s choice of a. Let $E(s,a)$ denote the principal’s expected value of the design, given recommended action $a$ and signal $s$. States are ordered so that $E$ is strictly increasing in $s$ for all $a$.

With $s$ unverifiable the details of the mapping $E$ are relevant because they impact the incentive constraint describing $C(a)$. The interpretation of $E$ is related to whether the principal or the agent has better information about the principal’s valuation of the design. For example, if one interprets the design as a credence good, where the value is known by the agent but the principal has no information, $s$ is not informative and therefore $E(s,a) = a$. In that case it is immediate that the problem (2) is equivalent the the one in (1), i.e. similar to the one in MacLeod (2003); there is no value in a right of refusal when the principal knows that the agent is the expert, and the signal conveys no information given the recommended action. We discuss the benchmark case in the next section, but in the rest of this section we take $E(s,a)$ to be strictly increasing in $s$, to differentiate from [10]. For instance, a natural case is $E(s,a) = s$, so the signal is exactly the valuation of the principal of the object; we call this a pure experience good. This is often the interpretation in the canonical model principal-agent model, where the agent produces output for the principal stochastically by exerting effort.

Normalize the principal’s value, in the absence of the project, to be zero. In constructing the cost function, the costs of burning are taken relative to the case where the planner keeps the project, so that his gross expected value, before costs, is $\sum E(s,a)f(s|a)$. This expected payoff is assumed to be linear in $a$ and so can be normalized as $\sum E(s,a)f(s|a) = a$. We then include the impact of discarding the project in the cost function $C(a)$, so that the development follows the usual principal-agent approach where the principal’s chosen allocation impacts cost, but the gross return is determined purely by the action $a$, so that the problem can be studied as a cost minimization for any $a$.

The principal’s problem can be reduced to choosing a recommended action $a$ to maximize

$$\sum E(s,a)f(s|a) - C(a)$$

where $C(a)$ is the minimized cost of delivering action $a$; the objective is simply $a - C(a)$. If the principal discards the project with probability $p(s)$,
the cost of implementing $a$ is

$$
C(a) = \min_{w(s), b(s), p(s)} \sum (p(s)E(s, a) + w(s) + b(s)) f(s|a) \quad (2)
$$

s.t.

$$
a \in \arg\max \hat{a} \sum u(w(s))f(s|\hat{a}) - \hat{a}
$$

$$
0 \leq \sum u(w(s))f(s|\hat{a}) - c(a)
$$

$$
s \in \arg\min \hat{s} p(\hat{s})E(s, a) + w(\hat{s}) + b(\hat{s})
$$

Note that, since $E(s, a)$ could be negative for some $s$, if the project has negative value contingent on the signal, so that the term $p(s)E(s, a)$ could be negative.

Because $E$ and $f$ appear in this problem, we treat them as primitive. However one could think of modelling value explicitly. Specifically, suppose that value $v$ for the principal, which is not contractible, depends on the effort of the agent according to $g(v|a)$. Signals are associated with some combination of value (to the extent that the principal seeks to assess the final product) and inputs (to the extent that assessment is about inputs) by $h(s|a, v)$. Therefore

$$
f(s|a) = \int h(s|a, v)g(v|a)dv
$$

and

$$
E(s, a) = \int vh(s|a, v)g(v|a)dv
$$

An explicit discussion of this interpretation is developed in the next section.

The planner can use an increasing $p(s)$ to help solve the reporting incentive: the planner may pay less for lower $s$, but on the other hand gets to keep the design less often. This trade off may be attractive if the true state is low, but not if the true state is high. The reverse is never incentive compatible:

**Lemma 2.** $p(s)$ is increasing.

Discarding the project in states with $E(s, a) < 0$ is costless, and in fact is the optimal choice without reporting constraints. The only reason for refusing a positive value project, ex post, is because the planner is using the refusal to get incentives for a higher state. Lower states are not a problem: there is always a rate at which the planner could pay (through burning money) to
keep the project more often, such that lower states would not find the trade-off attractive. As a result, each project worth keeping \((E(s, a) > 0)\) must either be kept for sure, or be limited by a higher state \(s'\) that is indifferent to reporting \(s\).

**Lemma 3.** Each \(s\) with \(E(s, a) > 0\) has either \(p(s) = 1\) or

\[ p(s)E(s', a) + w(s) + b(s) = p(s')E(s', a) + w(s') + b(s') \]

for the smallest \(s' > s\) with \(p(s') > p(s)\). Further, for no \(s' < s\) does the equality hold.

Since every contract is strictly preferred to misreporting and getting a strictly higher \(p(s)\), one can always, if \(p(s) > 0\) and \(b(s) > 0\), lower \(p(s)\) and \(b(s)\), keeping the payoff the same for truthful reporting of \(s\) and maintain incentive compatibility for lower types (who strictly prefer not reporting \(s\)). This makes the contract for \(s\) strictly worse for \(s' > s\). In other words resources are only burned if the planner is certain to discard the project:

**Corollary 4.** \(p(s) > 0\) implies \(b(s) = 0\)

Whenever there are some states where the project is burned inefficiently (i.e. \(E(s, a) > 0\) and \(p(s) < 1\)), it must be the case that the principal would benefit from coarsening any states where \(p(s) = 1\). Moreover, the corollary implies that \(p(s) < 1\) for at least some \(s\), since otherwise \(b(s)\) is zero for all \(s\) and the only incentive compatible contract for the planner has \(w(s)\) constant, and therefore cannot be incentive compatible for the agent.

Denote by \(C^{VS}(a)\) the cost function when the signal is verifiable, i.e. without the incentive constraint on \(s\) in (2). This is a natural lower bound for cost in the subjective evaluation case. Let \(P = \{s | p(s) = 1\}\) for the problem in (2). Denote the conditional probabilities on \(P\) under \(f\) by \(f^c\). The optimal contract is constant on \(P\); if one makes information coarser, so that the principal cannot distinguish between \(P\), the policy remains feasible; moreover, it is incentive compatible since for the minimum \(s\) in \(P\):

\[ p(s - 1)E(s, a) + w(s - 1) + b(s - 1) \leq p(s)E(s, a) + w(s) + b(s) \]

\[ < p(s) \sum E(s, a)f^c(s|a) + w(s) + b(s) \]

**Proposition 5.** Let \(\tilde{C}(a)\) be the cost under the coarsening of information such that the principal cannot distinguish states in \(P\). Then \(\tilde{C}(a) \leq C(a)\).
If the constraint binds, then the coarsening strictly reduces costs: less information is preferred.

The ability to judge output is useful for the principal, in the sense that a stronger relationship between $s$ and the final payoff lowers costs:

**Proposition 6.** For any $s$ such that $0 < p(s) < 1$ there exists $\kappa_h, \kappa_l > 0$ with

\[ \sum_{s > \bar{s}} \kappa_h f(s|a) = \sum_{s < \bar{s}} \kappa_l f(s|a) \]

such that $\hat{E}(s,a) = E(s,a) + \kappa_h$ for all $s > \bar{s}$ and $\hat{E}(s,a) = E(s,a) - \kappa_l$ for all $s < \bar{s}$, the principal’s cost is lower.

**Proof.** For small enough $\kappa_h$, only constraints involving $s$ are impacted for small enough $\kappa$. Further, those constraints are both loosened. For reporting type $s + 1$:

\[
p(s)E(s + 1, a) + w(s) + b(s) = p(s + 1)E(s + 1, a) + w(s + 1) + b(s + 1)
\]

\[
< p(s + 1)\hat{E}(s + 1, a) + w(s + 1) + b(s + 1)
\]

and for reporting type $s$:

\[
p(s)E(s, a) + w(s) + b(s) = p(s - 1)E(s, a) + w(s - 1) + b(s - 1)
\]

\[
> p(s + 1)\hat{E}(s - 1, a) + w(s + 1) + b(s + 1)
\]

In the standard model, only the correlation between signal and actions, embodied in $f(s|a)$, matters for incentives; here, taste or judgment about output matters. In particular, stronger taste or better judgment, as reflected in a more informative signal that is more strongly related to value, can lower cost for the principal. It may be especially important in the sense that more information, in terms of distinguishing states, may be detrimental.

### 3 A 2x2 Model

In this section we draw a further connection between $f$ and $E$ by linking them through a common shock. We do this in the context of a simple model with relatively few states and actions. In particular, suppose there is only two actions, normalized to be some fixed $\bar{a}$ and 0, and two signals, 0 and 1 as in the “$2 \times 2$” structure common in moral hazard problems. Actions are linked to values according to $v = a - \lambda \epsilon$, where $0 \leq \lambda \leq 1$. The signal $s$ is a
noisy and coarse signal of value, which takes on the value 1 if \( v - (1 - \lambda)\epsilon > 0 \), and 0 otherwise. In other words the principal observes the signal one if and only if

\[
a \geq \epsilon
\]

where \( \epsilon \) is distributed according to the symmetric, mean zero distribution \( F(\epsilon) \); therefore the signal one is observed with probability \( F(a) \), denoted by shorthand \( F \) in this section, if the agent chooses \( a \), and probability \( 1/2 \) otherwise. The relationship between \( a \) and the signal, therefore, is governed by \( F \).

The case of binary signals is especially interesting in the context of subjective evaluation. First, it is natural to imagine that, especially in the case where the agent is the expert, but more generally with design projects where assessment is subjective, that it may be possible for the principal to decide whether or not the planner likes the design more than the outside option, but much harder to quantify the differences between designs. Second, we know from MacLeod (2003) and the results above that subjective evaluation leads to coarse (often binary) policies even for richer signal spaces. Finally, binary signals generates a tournament-like signal structure that allows for a direct comparison to the tournament solution to non-verifiability in Malcomson (1984) in the next section.

One natural interpretation is that \( a - \epsilon \) measures the principal’s assessment of the agent’s output, relative to an outside option of zero, so that the project appears better than the outside option if \( a > \epsilon \).12 A signal of 0 may or may not be strongly indicative of the value of the project being less than the outside option, conditional on \( a \); \( E(s, a) \) is given by

\[
E(1, a) = a - \lambda \int_{x \leq a} x f(x) \, dx \quad F
E(0, a) = a - \lambda \int_{x > a} x f(x) \, dx \quad 1 - F
\]

For shorthand, let \( E(s, \bar{a}) = E(s) \), since the focus will always be on eliciting effort from the agent. The slope of \( E \) in the signal, conditional on \( \bar{a} \), is governed by \( \lambda \), while \( \lambda \) has no impact on the relationship between signals and

12This could be due either to randomness in evaluation of the project or the outside option.
actions; the fraction $\lambda$ of the shock is relevant to output. Therefore we focus on $\lambda$ as the important new variable that is relevant due to subjectivity. When $\lambda = 0$, the planner truly values projects based on the effort, but evaluates them with a noisy indicator of that true quality. The agent is an expert, in the sense that the subjective evaluation is only erroneous noise. Variance of $\epsilon$ denotes a lack of judgment on the principal’s part: he cannot discern the good from the bad, as the final output is a credence good. In many design projects, the principal must choose a design, but knows that their option is less informative than that of the agent which they hired. When $\lambda = 1$ the noise reflects the actual underlying idiosyncratic taste of the planner, as might be natural in an art project: the noise reflects noise in the true value of the project. In that case variance in $\epsilon$ denotes not a lack of judgment, but rather a strong subjective taste. This assumption is the classical one in principal-agent models where the agent gives effort that leads to stochastic output for the principal, but in the typical model with contractible signals, all that matters is the correlation between $a$ and $s$ (which is determined solely by the distribution of $\epsilon$); true taste (and therefore $\lambda$) plays no role.

Define $\lambda$ such that, given the signal is unfavorable to the project with effort $\bar{a}$, the expected value of the two choices is equal, i.e. $E(0) = 0$. This value is always unique and interior since, for $\lambda = 0$, $E(0) = a > 0$, and for $\lambda = 1$, $E(0) < 0$, and $E(0)$ is strictly decreasing in $\lambda$.

### 3.1 No Reporting Constraints

First, as a benchmark, consider the case with no reporting constraints. In that case the optimal payments for an agent that works $a$ are either the $w$ and $l$ that satisfy the IR and IC constraints. The IC constraint for $a$ reduces to

$$u(w_1) - u(w_0) = c/(2F - 1)$$

and participation

$$Fu(w_1) + (1 - F)u(w_0) = c$$

For the solution to this problem, let $\omega = w_1 - w_0$.

Without reporting constraints for the principal, if $\lambda < \bar{\lambda}$, the planner always takes the project, since it is better than not, and the payoff is

$$\bar{a} - Fw_1 - (1 - F)w_0$$
If, on the other hand, \(\lambda > \bar{\lambda}\), the planner only takes the project if the signal indicates it has value, and the payoff is

\[
\bar{a} - (1 - F)(\bar{a} - \lambda E(\epsilon|\epsilon > \bar{a})) - Fw_1 - (1 - F)w_0
\]

Define \(C^{VS}(\bar{a})\) be defined so that this payoff is \(\bar{a} - C^{VS}(\bar{a})\).

### 3.2 Cost With Reporting Constraints

The cost of implementing effort \(a\) is

\[
C(\bar{a}) = \min_{p, w, \lambda, b_i} \left( \begin{array}{c}
F(w_1 + p_1 E(1) + b_1) + \\
(1 - F)(w_0 + p_0 E(0) + b_0)
\end{array} \right)
\]

s.t.

\[
Fu(w_1) + (1 - F)u(w_0) - c \geq 0 \\
0 \leq Fu(w_1) + (1 - F)u(w_0) - c \\
w_1 + p_1 E(1) + b_1 \leq w_0 + p_0 E(0) + b_0 \\
w_1 + p_1 E(0) + b_1 \geq w_0 + p_0 E(0) + b_0
\]

Note that \(b_1\) is unnecessary, since the planner can always use \(w_1\) at the same cost, and only improve incentives. If \(p_1 = p_0\), then one of the reporting constraints is sufficient for the other. Since \(p\) is weakly increasing, the only other possibility is that \(p_1 > p_0\). It is immediate that in that case, only one of the reporting constraints can bind, and it is the reporting of \(s = 1\). Therefore the last constraint does not bind.

#### 3.2.1 Principal with Strong Idiosyncratic Tastes

Suppose that \(\lambda > \bar{\lambda}\). The principal will always keep a design that looks good, and refuse one that looks bad; the refusal helps generate incentives since it is particularly costly to refuse a design that appeals to the principal. We can characterize the solution to planner’s problem as follows:

**Proposition 7.** Suppose \(\lambda > \bar{\lambda}\). Then \(p_1 = 0\) and \(p_0 = 1\), and \(C(\bar{a}) = C^{VS}(\bar{a}) + \max\{(1 - F)(\omega - E(1), 0)\}\)
Proof. Since $E(1) > 0$, choosing $p_0 = 1$ and $p_1 = 0$ both helps with the reporting constraint and improves the objective. The reporting constraint, if it binds, then says that

$$b_0 = w_1 - w_0 - E(1)$$

So we have that, either $w_1 - w_0 - E(1) < 0$, and $C(\bar{a}) = C^{VS}(\bar{a})$, or the first equation binds and $C(\bar{a}, 0) = C^{VS}(\bar{a}) + (1 - F)(w_1 - w_0 - E(1))$.

When $\lambda > \bar{\lambda}$ the contract uses right of refusal to achieve the incentives: if the planner wants to claim the signal is bad, he cannot enjoy the output. Therefore higher $\lambda$, which means that the signal is more closely associated with the principal’s payoff, improves outcomes.

Large $\lambda$ is consistent with the principal being the “expert” in terms of assessing the principal’s payoff. One reason this would happen would be because the principal’s payoff is, in fact, his subjective affinity for the design. In other words, for truly subjective cases, in other words matters of subjective taste, $\lambda$ is high, right of refusal is a useful part of the contract.

To see how the planner might do better with coarser information, as described generally above, consider the case where $E(1) > \omega$, so that $C(a) = C^{VS}(a)$. Now suppose that the state ”1” is divided into two states, 1 and 2, where $E(2) > \omega > E(1) > 0$. Doing as well as in the coarse case requires that the design always be kept in the two states, since it has net value; however, doing so implies that the extra payment must be at least $\omega$ in both states, which is not incentive compatible (without burning resources) in the state 1. Therefore the planner, in the finer example, will need to employ some combination of burning resources or the project to achieve incentive compatibility, meaning costs are greater than $C^{VS}$. Again coarseness might be viewed as arbitrariness on the principal’s side, and might be beneficial.

### 3.2.2 Principal with Little Knowledge

Now suppose that $0 < \lambda < \bar{\lambda}$, so the principal is sufficiently uninformed that the high effort project is better even when the signal is not favorable to it. In this case the contract is slightly more complicated, but shares many of the qualitative features as $\lambda > \bar{\lambda}$. We characterize the cost function in the following:
Proposition 8. Suppose $\lambda < \bar{\lambda}$. Then

$$C(a, 0) = \begin{cases} C^{VS}(\bar{a}, 0) - (1 - F)\omega_{E(1)} & \text{if } E(1) \geq \omega \\ C^{VS}(\bar{a}, 0) + (1 - F)(\omega - E(1) + E(0)) & \text{if } E(1) < \omega \end{cases}$$

Proof. It is optimal to use $p_0$, rather than $b$, to satisfy incentives whenever $\lambda > 0$. Moreover $p_1 = 0$ so the IC constraint is

$$w_1 \leq w_0 + p_0 E(1) + b_0$$

To satisfy IC the contract needs $p$ to be at least:

$$(w_1 - w_0)/E(1).$$

If $\omega/E(1) \leq 1$, then, the principal can achieve IC by burning the project; when the state is zero this costs $E(0)$ per unit when the state is low, hence the cost in this case is $\omega E(0)/E(1)$.

If $\omega/E(1) > 1$, then $p_0 = 1$ and the principal must burn, in addition to the value of the project $E(0)$, $\omega - E(1)$. \hfill \Box

All of the key results in the prior section hold for the case where the principal has less knowledge. In this case, however, the principal cannot accomplish as much by destroying the project, since even a bad realization of $s$ is not indicative of a truly bad design.$^{13}$

3.3 Substituting taste for ability

In the analysis that follows the key object of interest is the extra cost incurred above and beyond the case with verifiable signals, denoted by $\Gamma = C(\bar{a}, 0) - C^{VS}(\bar{a}, 0)$. Fixing $F(a)$ but increasing $E(1)$ also makes incentives easier to achieve, for instance by a mean preserving spread on the set $(-\infty, -\bar{a}] \cup [\bar{a}, \infty)$. Moreover the benefit of such a change in the distribution of $\epsilon$ is

$^{13}$One special case that has received attention in the literature, and is natural in the design context where the principal hires an agent because of lack of knowledge, is the case of a credence good, where $\lambda = 0$ so the agent is the expert, in the sense that nothing about the principal’s signal is useful in predicting value, i.e. $E(1) = E(0)$. The problem is identical to MacLeod, in the sense that right of refusal ($p_1 < 1$) has no benefit, since it is identical to burning $a$ dollars. The planner must choose $b_0 = w_1 - w_0$ and the cost function is, therefore, $C(a) = u(w_1) = C^{VS}(a) + (1 - F)\omega$. Since in that case $E(0) = E(1)$, this can be seen in Proposition 8.
complementary with $\lambda$. In other words, principal’s with high ability to judge output benefit from having highly variable preference for output. We explore variance explicitly in a parametric example below.

Another feature of having the ability to assess the value of output is that it makes quality of the agent less valuable. Consider the value in raising $\bar{a}$, fixing $c(\bar{a})$. This can be interpreted as a higher quality agent in the sense the agent does more at the same cost to the agent. In the case of verifiable signals this is unambiguously positive. The higher is $\lambda$, the less valuable is raising $\bar{a}$:

Proposition 9. Suppose $\lambda > \bar{\lambda}$. Then $\frac{\partial^2 \Gamma}{\partial \bar{a} \partial \lambda} > 0$

Suppose that higher quality agents had a higher outside option $\bar{u}$. For $\lambda > \bar{\lambda}$ outside option is unrelated to $\Gamma$ (i.e. $\frac{\partial^2 C}{\partial \bar{a} \partial \lambda} = 0$) since $\Gamma$ only depends on $\lambda$ through $E(1)$ which is independent of the outside option. Therefore higher $\lambda$ would be willing to pay less, in delivered utility $\bar{u}$, for quality than would a principal with lower $\lambda$ in this range. In a sense quality of the agent is endogenously a substitute for lack of taste or judgment of the principal.

For $\lambda < \bar{\lambda}$ the same may not be true. While $E(1)$ is rising in $\bar{a}$, so is $E(0)$; there difference is unambiguously falling. As a result, the ability to judge output ($\lambda$) is only a substitute for agent quality when the ability is sufficiently strong, i.e. $\lambda$ is high enough.

4 Tournaments with Multiple Agents

Right of refusal is related to a different form of discipline for a principal with a private signal: a tournament. In the classic tournament ([7, 11]), the planner can either report that one agent wins, or the other; in the symmetric case, the planner’s payoff is identical regardless of the identity of the winner, and therefore reporting honestly is incentive compatible. We therefore modify the 2x2 model to include the possibility of hiring more than one designer. Contracts with competition for designs is common, for instance in architecture, marketing firms, or consulting firms. A relevant question is what the role is of tournaments versus pure right of refusal, for different underlying fundamentals.

Suppose the planner contracts with two agents, 0 and 1 to produce exactly one design. The designer picks a ‘winning’ design and pays accordingly. The
values of the two projects are
\[ v_1 = a_1 - \lambda \epsilon / 2 \]
\[ v_0 = a_0 + \lambda \epsilon / 2 \]
The principal observes the signal one if and only if
\[ a_1 - \epsilon / 2 \geq a_0 + \epsilon / 2 \]
and the signal 1 otherwise. The structure nests the one agent model when \( a_0 = 0 \). Whereas before the model compared the outside option to the project, here the comparison is between designers, but without the outside option. Eliminating the outside option is a useful model because it avoids having multiple agents used because it simply generates more draws; this is certainly a reasonable motivation to use multiple agents, but not one we want to explore here, since its implications are direct. If \( a_0 = 0 \) is interpreted as hiring one agent, the firm still gets the benefits of the same epsilon draws. Here the reason for contracting with multiple agents is purely due to incentive considerations.

In a sense, the model is the polar opposite of a moral hazard in teams problem. In that problem, both inputs are useful (and perhaps even essential); here only one input is useful ex post, since the principal uses one design. In team production, the principal gets a signal of combined inputs, but no measure of the division of inputs across team members, which generates a familiar free riding problem. Here the principal gets a signal that orders relative inputs, but no signal of combined inputs.

First suppose the action profile is \((a, a)\). In that case the contract with verifiable signals is symmetric, and therefore \( C(a, a) = C^{VS}(a, a) \). Therefore the difference between costs with one and two agents can be decomposed into
\[ C(a, a) - C(a, 0) = C^{VS}(a, a) - C^{VS}(a, 0) - \Gamma \]
where \( \Gamma = C(a, 0) - C^{VS}(a, 0) \geq 0 \). This separates out the net benefit of contracting with a second agent, represented by \( \Gamma \), that would only arise because signals are unverifiable, from the difference that would arise even with a contractible signal.

From the previous section it is immediate that

**Corollary 10.** \( \partial \Gamma / \partial \lambda < 0 \).
The incentive benefit of using a second agent declines with $\lambda$. In a sense, to the extent that right of refusal can be used, contracting with one agent acts like a tournament with the outside option; that tournament is only useful if it is credible that the outside option is a real competitor. By contrast, when actions are $a$ for each agent, credibility is never a concern. Therefore principal’s with an ability to judge output don’t need to benchmark output to another agent’s work; this is not because the second agent’s work helps them to assess value, but rather because the second agent’s work is a substitute, in terms of reporting $s$, for taste or judgment.

5 Noise and the Value of Taste: the Laplace Case

This section studies a particular form for the distribution $F(\epsilon)$, so that parameters of that distribution can be considered. Take $F$ to be the Laplace distribution with scale parameter $\sigma$, i.e. the absolute value of $\epsilon$ has standard deviation $\sigma$. This allows the model to address the question of the role of variance in the random component and how it interacts with the ability to judge output. The difference in expected values, given that the signal given that the signal is zero is

$$E(0, a) = E(v_1 - v_0|\epsilon > a_1 - a_0)$$
$$= a_1 - a_0 - \lambda E(\epsilon|\epsilon > a_1 - a_0)$$
$$= (1 - \lambda)(a_1 - a_0) - \lambda \sigma$$

since for the Laplace distribution $E(\epsilon|\epsilon > x) = x + \sigma$ for $x$ above the mean of $\epsilon$. For $x$ below the mean,

$$E(\epsilon|\epsilon < x) = -\frac{1 - F(x)}{F(x)}(x + \sigma)$$

so the difference when the signal is one is

$$E(1, a) = (1 + \lambda \frac{1 - F(a_1 - a_0)}{F(a_1 - a_0)})(a_1 - a_0) + \lambda \frac{1 - F(a_1 - a_0)}{F(a_1 - a_0)} \sigma$$

Using these relationships for the case where $a_1 = \bar{a}$ and $a_0 = 0$:
Proposition 11. Suppose \( \lambda > \bar{\lambda} \) or \( E(1) < \omega \). Then \( \frac{\partial r}{\partial \sigma \partial \lambda} \leq 0 \).

Noise and the ability to assess output are complements, in the sense that the lower cost from being able to assess output is magnified by variance in the shock, whenever right of refusal is fully used (i.e. bad projects are refused with probability one). This includes cases where the the ability to assess output is high (\( \lambda > \bar{\lambda} \)) and cases where refusal is insufficient to generate incentives for reporting success. In intermediate cases, there are offsetting effects: noise both impacts the incentive benefit of right of refusal, but also the amount of refusal (in probability terms) that need to be used to get incentives. In that case the cross partial cannot be signed.

6 Conclusion

This paper introduces a model of a principal agent problem where signals are unverifiable, and the project can be refused as a function of the report of the signal. Such an example corresponds to a contract with right of refusal, a common contract form in some types of design. We show that such a contract is useful to the extent that the principal can forecast his own payoff, and argue that such cases correspond naturally to cases of subjective taste, such as in art. Art contracts are one place where right of refusal is common. On the other hand, right of refusal is not useful for cases where the agent is the expert, and therefore can forecast the principal’s payoff better than the principal can. Such examples, commonly called credence goods, are often cases where right of refusal is not employed, as the model suggests. Further exploration of the empirical usage of this contract form seems like a useful direction for future work.

Appendix: Proofs

Proof of Proposition 1

Proof. Suppose \( b > 0 \). The first order condition for \( w \) is

\[
\mu_0 \left( \frac{df}{da} / f \right) + \mu_1 = 0
\]

where \( \mu_0 \) and \( \mu_1 \) are the Lagrange multipliers on the IC and IR constraints. This can hold for at most one value of \( s \) since the MLRP conditions holds. \( \square \)
Proof of Proposition 2

Proof. Suppose \( p \) is strictly decreasing for some states \( s \) and \( s + 1 \). Then since \( p(s)E(s) + w(s) + b(s) \geq p(s + 1)E(s) + w(s + 1) + b(s + 1) \), it must be the case that \( p(s)E(s + 1) + w(s) + b(s) > p(s + 1)E(s + 1) + w(s + 1) + b(s + 1) \), which violates IC.

Proof of Lemma 3

Proof. For any two contracts with \( p(s) \) the same, it must be the case that \( w(s) + b(s) \) is the same, and therefore we can describe the set of distinct contracts, for the purposes of reporting of the principal, by a strictly increasing set of \( p(s) \) in those contracts; the binding contract is the next distinct \( p(s) \). Suppose that \( p(s) < 1 \), \( E(s, a) > 0 \) and all types \( s' > s \) strictly prefer truthful reporting to reporting \( s \). Then you could raise \( p(s) \) and \( b(s) \), keeping the payoff for \( s \) reporting truthfully the same, and make all lower types \( s'' < s \) strictly prefer to report truthfully to reporting \( s \). But then nothing binds to \( s \) and therefore \( p(s) = 1 \). Note that it can never be the case that, for \( s' > s > s'' \), that both

\[
p(s)E(s', a) + w(s) + b(s) = p(s')E(s', a) + w(s') + b(s')
\]

and

\[
p(s'')E(s', a) + w(s'') + b(s'') = p(s')E(s', a) + w(s') + b(s')
\]

(i.e. \( s' \) binds to both \( s \) and \( s'' \)) since, if that were the case, then

\[
p(s'')E(s, a) + w(s'') + b(s'') > p(s)E(s, a) + w(s) + b(s)
\]

i.e. \( s \) prefers to report \( s'' \). This implies that the contract with the highest \( p(s) \) binds to the next highest \( p(s) \) (since otherwise nothing would bind to that next highest \( p(s) \) from above, as required), and only the next highest \( p(s) \) is a candidate to bind to the third-highest \( p(s) \), and so on.

Proof of Proposition 9

Proof. Since \( \lambda > \bar{\lambda} \)

\[
\frac{d\Gamma}{d\lambda} = -(1 - F)\frac{dE(1)}{d\lambda}
\]
the cross partial is
\[
\frac{d\Gamma}{d\bar{a}d\lambda} = \frac{dE(1)}{d\lambda} \frac{dF}{d\bar{a}} - (1 - F) \frac{dE(1)}{d\lambda d\bar{a}}
\]
Clearly \( E(1) \) is increasing in \( \lambda \) and \( F \) is increasing in \( \bar{a} \). The cross partial of \( E(1) \) is
\[
\frac{dE(1)}{d\lambda d\bar{a}} = -\frac{d}{d\bar{a}} \frac{\int_{x<\bar{a}} xf(x)dx}{F(\bar{a})} - \frac{f(a) \int_{x<\bar{a}} xf(x)dx}{F(\bar{a})^2}
\]
\[
= -\frac{f(\bar{a})}{F(\bar{a})} (\bar{a} - E(x|x<\bar{a})) < 0
\]
Since \( \frac{dE(1)}{d\lambda d\bar{a}} < 0 \), \( \frac{d\Gamma}{d\lambda d\bar{a}} > 0 \)

**Proof of Proposition 11**

*Proof. If \( \lambda > \bar{\lambda} \)*
\[
\frac{\partial \Gamma}{\partial \lambda} = -(1 - F) \frac{\partial E(1)}{\partial \lambda}
\]
but \( 1 - F \) is rising in \( \sigma \) and
\[
\frac{\partial E(1)}{\partial \lambda} = \frac{1 - F}{F} (a + \sigma)
\]
is rising in \( \sigma \).
If \( \lambda < \bar{\lambda} \) and \( E(1) < \omega \) then
\[
\frac{\partial \Gamma}{\partial \lambda} = -(1 - F) \left( \frac{\partial E(1)}{\partial \lambda} - \frac{\partial E(0)}{\partial \lambda} \right)
\]
Since
\[
\frac{\partial E(0)}{\partial \lambda} = -(a + \sigma)
\]
then
\[
\frac{\partial E(1)}{\partial \lambda} - \frac{\partial E(0)}{\partial \lambda} = \frac{1}{F} (a + \sigma)
\]
So again the first term and second term of \( \frac{\partial \Gamma}{\partial \lambda} \) both rise with \( \sigma \).
References


