TrueView Ads

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December 3, 2014

Abstract

We compare two types of advertising in a two-sided media market: (1) traditional ads which have to be shown in their entirety prior to viewer requested contents, and (2) trueview ads which viewers can skip and proceed directly to their requested contents. Our model introduces matching between viewers and ads, and allows advertisers to invest in nuisance-reducing effort to help counter viewers’ ad-skipping behavior. Our results show that trueview ads induce advertisers to invest in nuisance-reducing effort and the equilibrium effort level increases with advertisers’ willingness to pay for viewer access. Relative to traditional ads, trueview ads always raise viewer surplus, but have two opposite impacts on platform profit: the positive demand-enhancing effect and the negative free-riding effect. We also consider the cases of mixed ads (offering a mixture of traditional and trueview ads for advertisers to self-select) and proportion-skipping ads (allowing viewers to skip a fixed proportion of the ads). Focusing on platform profit, we find that optimal mixed ads and optimal proportion-skipping ads strictly dominate optimal traditional ads and weakly dominate pure trueview ads.

Keywords: Trueview ads; Proportion-skipping ads; Two-sided market; Media market; Advertising.

JEL Classification Codes: D42, L12, M37

*We would like to thank Daniel Nedelescu, Markus Reisinger, Régis Renault, Dongsoo Shin, and seminar participants at Indiana University - Purdue University Indianapolis, 2013 IO Theory Workshop (Shandong University), 2014 Western Economic Association International Conference and 2014 Shanghai Economic Theory Workshop for helpful comments and discussions.

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1 Introduction

This paper considers a form of advertising under which content viewers can skip an ad after the ad is shown for a short period of time (e.g., 5 seconds on YouTube). Following the term used by YouTube, we call this form of ads trueview ads. They are in contrast to what we call traditional ads which have to be shown in their entirety before viewer requested contents are delivered. We are interested in the following questions. First, how does the adoption of trueview ads affect the platform’s (e.g., YouTube) profit? Second, what are the impacts of trueview ads on viewer surplus, advertiser surplus and social surplus?

The success and survival of many media markets (e.g., television, radio and online media) rely on ad-generated revenues. Such markets have been extensively studied in the economics and marketing literature (see, for example, Anderson and Coate, 2005 and Gal-Or and Gal-Or, 2005). Media markets are two-sided markets where the content provider (platform) brings together consumers/viewers on one side and advertisers on the other side. Each side (group) generates externality toward the other side in that viewers dislike ads while advertisers value access to viewers. Our model follows this framework but differs in several important aspects which we explain in detail next.

First, most of the existing literature assumes that all ads exert negative externality to all viewers. A straightforward implication of this assumption is that if consumers can skip ads at no cost, then they will always do so. However, YouTube’s internal research shows that when given the option to skip, only 10% of the viewers choose to skip ads all the time.\(^1\) This suggests that not all viewers draw negative values from viewing ads, at least not from all ads. To capture this feature, we allow advertisers to invest in effort (at a cost) which reduces viewers’ nuisance costs of viewing their ads, so that some viewers may choose to watch the ad even when they can skip it at no cost.\(^2\)

Second, it is commonly assumed that an advertiser values all viewers the same and a viewer dislikes all ads the same.\(^3\) In contrast, we introduce matching between a viewer and an ad and we allow the match to be good or bad. It is plausible that not all viewers are the same to advertisers. For example, showing an ad for high school diploma likely will not generate much value for the advertiser if the viewer holds such a diploma already or simply is not interested, i.e., the viewer-ad match is bad.\(^4\) We assume that a viewer generates value for the advertiser when the match is


\(^2\)For example, advertisers can invest in effort to make their ads more fun to watch or provide valuable information which helps the viewers. In fact, this is precisely the advice given to advertisers in the same YouTube TrueView Study: “Make good ads that people want to watch . . . Focus on relevancy with your targeting to increase view rates.”

\(^3\)However, different advertisers can have different valuations for access to the same viewers (see, for example, Anderson and Gans, 2011).

\(^4\)This is confirmed in Google/YouTube’s in-house research findings: “Consumers are more likely to opt out of non-relevant categories and skip ads they have seen repeatedly providing efficiency to advertisers.” Source: YouTube TrueView Study, Google/YouTube/Ipsos, U.S., Jan 2012.
good but not when it is bad.\(^5\) In the same spirit, we assume that the nuisance cost to a viewer of watching ad is (weakly) lower when the match is good than when the match is bad. That is, a viewer may dislike all ads (in the absence of advertisers’ nuisance-reducing efforts) but would dislike more those ads that are bad match to him/her.

It is straightforward that relative to traditional ads, trueview ads make the viewers better off since they can skip the ads they dislike. For the platform, our results show that trueview ads have opposite effects on platform profit. First, by giving the viewers the option to skip any ad they dislike, trueview ads make the platform more attractive to viewers. Therefore, more viewers end up joining the platform in the equilibrium, the demand-enhancing effect. On the other hand, some viewers on the platform may choose to skip ads even when they would generate value for the advertisers and in turn for the platform if they watch the ads (i.e., when the viewer-ad match is good), the free-riding effect. The demand-enhancing effect works to improve the platform’s profit while the free-riding effect does the opposite. The overall impact is thus ambiguous. When advertisers can invest in effort to reduce viewers’ nuisance costs at little cost, advertisers’ equilibrium efforts will largely cancel out viewers’ nuisance costs so few viewers will skip ads in the equilibrium. This minimizes the magnitude of the free-riding effect so trueview ads benefit the platform and advertisers. The results are the opposite if nuisance-reducing effort is sufficiently costly.

Trueview ads also affect advertisers’ incentive to invest in nuisance-reducing effort. While advertisers have no incentive to invest in such effort under traditional ads, the option to skip ads at no cost under trueview ads induces the advertisers to invest in nuisance-reducing effort. Moreover, an advertiser’s optimal effort level increases with its willingness to pay for viewership and decreases with the advertising rate chosen by the platform. The loss from skipped good-match ads and the effort cost have to be weighed against the benefit of more viewers joining the platform. As a result, the overall impact on advertisers is also ambiguous. Following similar logic, trueview ads have ambiguous impacts on social surplus as well.

In practice, a viewer may be shown a trueview ad one time and a traditional ad another time. Or multiple ads may be shown simultaneously and a viewer is required to watch one of them. To analyze these two practices, we consider two extensions (mixed ads and proportion-skipping ads respectively) and focus on how they impact platform profit. In mixed ads, the platform offers a mixture of traditional and trueview ads at different advertising rates. Advertisers self-select into the type of ads depending on their willingness to pay for viewer access. Surprisingly, we find that advertisers with lower willingness to pay for viewer access are more likely to choose trueview ads, even though they have less incentive to invest in nuisance-reducing effort. This may seem to work against the platform. Nevertheless, we find that for platform profit, optimal mixed ads strictly dominates pure traditional ads and weakly dominates pure trueview ads. In proportion-

\(^5\)When an ad reaches a viewer and the match is good, we call the viewer an effective viewer. Bad match generates no value for the advertisers and we define advertisers’ willingness to pay per effective viewer rather than per viewer.
skipping ads, the platform allows the viewers to skip a fixed proportion of the ads shown to them. Here the distinction of good and bad viewer-ad match is critical. When viewers are allowed to skip a small fraction (not exceeding the fraction of bad match) of the ads shown to them, no advertiser will invest in effort in the equilibrium. Only bad match ads are skipped so skipping has no direct impact on advertisers and the platform. However, the option to skip some ads benefits viewers, leading to more viewer participation which indirectly helps the advertisers and platform, a Pareto improvement relative to pure traditional ads. On the other hand, if viewers are allowed to skip a large fraction (exceeding the fraction of bad match) of ads, some good match ads will be skipped. This induces advertisers to compete in nuisance-reducing effort which increases viewer participation and benefits the platform. Our results show that the benefit of advertisers’ effort investment outweighs the cost of having some good match ads skipped. Correspondingly, the platform has an incentive to choose a proportion sufficiently high so that some good match ads are skipped.

1.1 Literature review

Our paper fits in the fastly growing literature on two-sided (or multi-sided) markets where platforms sign up multiple sides (groups) of agents for transaction to take place (see, for example, Caillaud and Jullien, 2001, Rochet and Tirole, 2006 and Armstrong, 2006). In two-sided markets, each group exerts (positive or negative) externalities toward the other group. This in turn gives platform(s) more or less incentive to sign up consumers in either group. The consideration of two-sided markets has generated new results relative to one-sided markets. For example, it has been show that well-known results in one-sided models may not carry through to two-sided models (e.g. Liu and Serfes, 2013).

While group externality can be positive or negative for either group, a subset of the two-sided market literature focuses on the case where the sign of group externality is positive for one group but negative for the other group. A common example is the media market where the content provider (platform) brings two sides (viewers and advertisers) together (e.g., Anderson and Coate, 2005, Anderson and Gabszewicz, 2006, Reisinger, 2012).\footnote{Viewers enjoy the content provided by the platform but need to pay a “price” in the form of watching ads. The platform sells the advertising space to advertisers and the advertising revenue supports the functioning of the platform.} Within this literature, our paper is most closely related to the strand analyzing ad-avoidance (through either exogenous technology or paid membership). Anderson and Gans (2011) analyzes the impacts of exogenous ad-avoidance technologies (AAT) in a media market with a monopoly content provider which relies on ad-generated revenue. They find that AAT reduces the platform’s advertising revenue as well as alters the composition of consumers. These induce the platform to increase advertising level (more
advertising clutter), choose lower content quality and more mass-market content.\textsuperscript{7} Our model follows the setup in Anderson and Gans but differs in “ad avoidance technology”. In Anderson and Gans, viewers make the decision of whether or not to invest in an exogenous ad avoidance technology (e.g., DVR) to avoid all ads. In our benchmark model, the decision is made by the platform (i.e., by offering trueview ads) and no additional technology or cost is needed for viewers to skip ads. In the extensions, either the advertisers self-select into traditional vs. trueview ads (mixed ads) or the viewers decide what ads to skip and what not to (proportion-skipping ads). Our model also allows for more viewer heterogeneity. While Anderson and Gans allow viewers to be heterogeneous in their magnitude of advertising distaste, all viewers in their model generate value for advertisers and a viewer dislikes all ads the same. In contrast, we take viewer heterogeneity one step further and allow viewer-ad match to be good or bad. The match type then determines whether the viewer generates value for a specific advertiser and affects how much the viewer dislikes the ad. We also allow advertisers to invest in effort which reduces viewers’ nuisance costs of viewing its ad. Combined with viewer-ad matching, in the equilibrium a viewer may skip some ads but not others.

Johnson (2013) analyzes the impact of (increasingly) targeted advertising in a setting where consumers can block advertising. He finds that better targeting benefits firms (advertisers) but may hurt consumers because advertising level increases with the accuracy of targeted advertising. Our papers differ in several ways. First, there is an intermediary (platform) in our model (as in Anderson and Gans, 2011). Consumers (viewers) derive value from joining the platform and advertisers pay the platform at a rate chosen by the platform. In his model, there is no platform and consumers gain from successful transaction with advertisers instead and advertising cost is exogenous. The second difference concerns consumer/firm heterogeneity. In particular, benefit parameter ($v_s$) does not vary across firms and the cost/benefit parameters do not vary across consumers ($c_B, v_B$) in his model. In our model, valuation of exposure to consumers varies across advertisers ($t$), and nuisance and benefit parameters also vary across consumers ($\gamma, x$). Third, a consumer blocks either none or all ads in his model. In our setting, a viewer can make different skipping decisions across ads, depending on viewer-ad match (good or bad) and advertisers’ nuisance-reducing efforts.\textsuperscript{8}

T˚ ag (2009) also considers a setup where consumers can skip ads, by purchasing from the platform the option to skip ads (e.g., through paid membership). In this case, the platform directly benefits from ad-skipping in the form of membership fee which gives the platform extra incentive to raise advertising levels to nudge the more ad-sensitive consumers toward paying for membership. The platform chooses optimal membership fee and advertising level to balance between membership revenue and advertising revenue. He finds that the introduction of ad-free membership raises

\textsuperscript{7}Wilbur (2008) finds that AAT tends to increase advertising quantities and decrease content provider’s revenues in a counterfactual experiment.

\textsuperscript{8}Johnson (2013) also analyzes the competition between niche firms vs. mainstream firms which we do not consider.
advertising level, benefits the platform and the advertisers at the cost of viewers. In our model we do not consider ad-free membership. Instead, we allow viewer-ad match to be good or bad and allow advertisers to invest in nuisance-reducing effort.

The rest of the paper is organized as follows. We introduce our model in Section 2. Section 3 contains the analysis where we compare the equilibrium under traditional ads with the equilibrium under trueview ads. In Section 4, we allow the platform to offer a mixture of traditional and trueview ads or offer proportion-skipping ads. We conclude in Section 5. Proofs of Lemmas and Propositions are relegated to the Appendix.

2 The model

Our benchmark model is a modified version of the model in Anderson and Gans. It concerns a two-sided market where a monopolist provider offers a platform to bring viewers and advertisers together. It provides content to viewers and sells advertising space to advertisers. For simplicity, we assume that the platform has constant marginal cost of providing content and selling advertising space which we normalize to zero.

There is a continuum of viewers of measure 1. Viewers are characterized by their location in a two-dimensional space \((x, \gamma)\). We assume that \(x\) and \(\gamma\) are uniformly distributed on \([0, \bar{x}]\) and \([0, \bar{\gamma}]\) respectively and independently. If the viewer joins the platform, her indirect utility before subtracting the nuisance cost of watching ads is

\[
U_{x, \gamma} = \theta + \lambda(1 - x) - s, \tag{1}
\]

where \(\theta \in [-\lambda, \lambda(\bar{x} - 1)]\) is a vertical quality component, \(\lambda\) is a horizontal component, \(s\) is the subscription fee. Similar to Anderson and Gans, we focus on the case of \(s = 0\) so the platform draws revenue from the advertiser side only.\(^9\) Most existing studies assume that each viewer treats all ads the same and all viewers generate positive externality for the advertisers. In contrast, we introduce viewer heterogeneity in their attitudes toward ads and the value they generate for advertisers by introducing viewer-ad matching. With probability \(1 - \beta\), the matching is bad. In this case, the viewer has a disutility of \(\gamma\) from watching this ad and generates zero value for the advertiser.\(^10\) With probability \(\beta \in (0, 1)\), the matching is good in which case the viewer generates value for the advertiser and his/her disutility of watching the ad goes down to \(\delta \gamma\) with \(\delta \in (0, 1]\). Suppose that the advertising level is \(a\) (a measure \(a\) of ads are shown on the platform). The representative

\(^9\)Choi (2006) compare two finances schemes depending on whether or not \(s = 0\).

\(^{10}\)The assumption of zero value is for simplicity. Being forced to see the ad of a product which he/she is not interested, the viewer may go from being uninterested to loathing a product/advertiser. Assuming negative value for the advertiser would make trueview ads even more appealing to the platform, everything else the same.
viewer’s expected utility, net of the nuisance cost, is
\[ U_{x,\gamma} = \theta + \lambda(1 - x) - (\beta \delta \gamma + (1 - \beta)\gamma)a. \]  

Next, we turn to the advertiser side. There is a unit mass of advertisers whose willingness to pay per effective viewer is \( t \), which is assumed to be distributed over the support \([t, \bar{t}]\).\(^{11}\) Let \( F(t) \) denote the distribution function and \( f(t) \) the density function. We assume that \( f(t) \) is continuous and satisfies monotone hazard rate property (\( \frac{f(t)}{1-F(t)} \) increases with \( t \)) on its support. Departing from existing studies, we allow advertisers to exert effort in making their ads more enjoyable for viewers to watch. This is captured by effort \( e \) which reduces viewers’ nuisance costs. For example, consider a good match between a viewer and an ad. The nuisance cost is \( \delta \gamma \) if the advertisers exerts no effort, but goes down to \( \delta \gamma - e \) if the advertisers chooses effort level \( e \). Similarly, the nuisance cost goes down from \( \gamma \) to \( \gamma - e \) for bad matches.\(^{12}\) The cost of effort \( c(e) \) is strictly increasing and convex in \( e \) with \( c(e = 0) = 0 \), \( c'(e = 0) = 0 \). For simplicity, we assume that effort \( e \) cannot turn nuisance into positive utility, so the nuisance costs are \( \max\{\delta \gamma - e, 0\} \) for good matches and \( \max\{\gamma - e, 0\} \) for bad matches respectively.

The stage game we analyze is the following.

- **Stage 1:** The platform chooses the advertising rate \( r \) per effective viewer.
- **Stage 2:** Advertisers simultaneously and independently decide whether or not to advertise on the platform. If they advertise, they also choose the effort level \( e \).
- **Stage 3:** Viewers decide whether or not to join the platform.
- **Stage 4:** Ads are shown to viewers who join the platform and the uncertainty of good/bad match is resolved. Viewers can skip ads under trueview ads but not under traditional ads. When viewers are indifferent between watching or skipping an ad (e.g. \( \delta \gamma - e = 0 \) for a good match), we assume that they will watch the ad.

### 3 Analysis

In this section, we will analyze two types of ads in turn: trueview ads where the viewers are given the option to skip ads, and traditional ads which have to be shown in their entirety. We will

\(^{11}\)In our model, an exogenous fraction \( \beta \) of viewers generate value for advertisers. We call these viewers effective viewers and we model advertisers’ willingness to pay the advertising rate the platform charges as based on per effective viewer. Alternatively, one can model both as based on all viewers, not just effective ones. The two approaches are isomorphic.

\(^{12}\)Alternatively one may assume that advertisers’ efforts affect viewers’ nuisance cost differently for good- and bad-match viewers. An extreme is that advertisers’ efforts reduce nuisance costs for good-match viewers but not for bad-match viewers. This change has no impact on our main results. See section 3.2 for more details.
start with traditional ads.

3.1 Traditional ads

It is intuitive that under traditional ads an advertiser has no incentive to reduce viewers’ nuisance costs of seeing its ad. Viewers cannot skip ads and one advertiser’s effort has near zero impact on viewers’ platform joining decisions, holding other advertisers’ decisions fixed. Correspondingly, investing in effort leads to no benefit to the advertiser but at a cost.\(^{13}\) Let \(r\) denote the advertising rate per effective viewer and \(a\) the corresponding measure of advertisers who advertise on the platform.

Let \(R = r \cdot a\) denote the total advertising revenue per effective viewer. Only advertisers with willingness to pay above \(r\) will advertise on the platform. Therefore,

\[ a = \text{prob}(t \geq r) \Rightarrow a = 1 - F(r). \]

Then \(\frac{da}{dr} = -f(r) < 0\), or alternatively, \(r'(a) = -\frac{1}{f(r)} < 0\).

With no advertiser investing in efforts to reduce nuisance costs, a representative consumer’s expected utility of joining the platform ex-ante is then

\[ U = \theta + \lambda(1 - x) - (\beta\delta\gamma + (1 - \beta)\gamma)a. \]

The marginal consumers line is characterized by

\[ U = 0 \Rightarrow \hat{x}^{\text{trad}}(\gamma) = \frac{\theta + \lambda - a\gamma(1 + \beta\delta - \beta)}{\lambda}. \]

Since \(\theta \in [-\lambda, \lambda(\bar{x} - 1)]\), we have \(\hat{x}^{\text{trad}}(\gamma = 0) \in [0, \bar{x}]\) so the marginal consumers line must cross the bottom horizontal line. However, \(\hat{x}^{\text{trad}}(\gamma = \bar{\gamma})\) may be positive or negative. The marginal consumers line crosses the top horizontal line when \(\hat{x}^{\text{trad}}(\gamma = \bar{\gamma})\) is positive but crosses the left vertical line when it is negative (see Figure 1). It turns out that only the former can be an equilibrium, as summarized in the next Lemma.\(^{14}\)

**Lemma 1** Under traditional ads, marginal consumers line always crosses the top and bottom horizontal lines in the equilibrium.

\(^{13}\)We rule out the possibility that a viewer may choose not to watch the ad while waiting for the requested content, in line with existing literature (e.g., Anderson and Gans).

\(^{14}\)Results in our Lemma 1 and (later) Proposition 1 are in the same spirit as those in Anderson and Gans (2011). In the case of traditional ads, the introduction of viewer-ad match and the ability for advertisers to invest in nuisance-reducing effort have mostly quantitative impacts on the equilibrium.
Proof. See the Appendix.

We have shown that the marginal consumers line must cross the top and bottom horizontal lines. The two intercepts are

\[ \hat{x}^{\text{trad}}(\gamma = 0) = \frac{\theta + \lambda}{\lambda}, \quad \hat{x}^{\text{trad}}(\gamma = \bar{\gamma}) = \frac{\theta + \lambda - \bar{\gamma}a(1 + \beta \delta - \beta)}{\lambda}. \]

The corresponding measure of effective viewers is

\[ N^{\text{trad}} = \beta \cdot \frac{\hat{x}^{\text{trad}}(\gamma = 0) + \hat{x}^{\text{trad}}(\gamma = \bar{\gamma})}{2\bar{x}} = \beta \cdot \frac{2(\theta + \lambda) - \bar{\gamma}a(1 + \beta \delta - \beta)}{2\lambda \bar{x}}. \tag{3} \]

The viewer elasticity is then given by

\[ \varepsilon_N = -\frac{\partial N}{\partial a} \cdot \frac{a}{N} = \frac{\bar{\gamma}a(1 + \beta \delta - \beta)}{2(\theta + \lambda) - \bar{\gamma}a(1 + \beta \delta - \beta)}, \]

which increases with \( a \).

Similarly, the (per effective viewer) elasticity of advertising revenue is

\[ \varepsilon_a = \frac{\partial R}{\partial a} \cdot \frac{a}{R} = \frac{-a/f(r) + r}{r} = 1 - \frac{1 - F(r)}{r f(r)}. \]
Since the distribution of $t$ satisfies the monotone hazard rate property, $f(t)_{1 - F(t)}$ increases with $t$ on its support. Then $\varepsilon_a$ increases with $r$ and decreases with $a$.

Since $\varepsilon_a$ and $\varepsilon_N$ are continuous functions, and $\varepsilon_a(a = 0) = 1 > \varepsilon_N(a = 0)$, $\varepsilon_a(a = \frac{\theta + \lambda}{\gamma(1 + \beta\delta - \beta)}) < 1 = \varepsilon_N(a = \frac{\theta + \lambda}{\gamma(1 + \beta\delta - \beta)})$, there must exist an $a$ such that $\varepsilon_a = \varepsilon_N$. Moreover, the single-crossing condition must hold since $\varepsilon_a$ decreases with $a$ while $\varepsilon_N$ increases with $a$. Combined, we have a unique solution to the platform’s problem, as summarized in the next Proposition.

**Proposition 1** Under traditional ads, there is a unique equilibrium level of advertising. It equates the (per effective viewer) elasticity of advertising revenue and the aggregate viewer demand elasticity with respect to advertising,

$$\varepsilon_a(a^{\text{trad}}) = \varepsilon_N(a^{\text{trad}}) = \frac{\gamma a(1 + \beta\delta - \beta)}{2(\theta + \lambda) - \gamma a^{\text{trad}}(1 + \beta\delta - \beta)} < 1.$$ 

The corresponding platform profit is

$$\pi^{\text{trad}} = N^{\text{trad}}(a) \cdot R(a)|_{a = a^{\text{trad}}}.$$ 

### 3.2 Trueview ads

Under trueview ads, there is no nuisance cost to any of the viewers: either the advertiser’s effort $e$ cancels out the nuisance in which case the viewer will watch the ad, or the viewer will skip the ad. This is equivalent to setting $\gamma = 0$ in the utility function in equation (2). The marginal consumers line is a vertical line represented by

$$\theta + \lambda(1 - x) = 0 \Rightarrow x^{\text{true}} = \frac{\theta + \lambda}{\lambda}.$$ 

Next, we analyze advertisers’ effort choices. When a representative advertiser (with willingness to pay $t$ per effective viewer) chooses effort $e$, good match consumers with $\gamma \leq \frac{e}{\delta}$ will see this particular ad while good match consumers with $\gamma > \frac{e}{\delta}$ will skip the ad (See Figure 2). We focus on good match viewers only since bad match viewers do not generate value for advertisers. Note that while the marginal consumers line remains the same for all advertisers, the threshold $\gamma = \frac{e}{\delta}$ varies across advertisers since $e$ is advertiser-specific.\(^{15}\)

The measure of effective viewers for advertiser $t$ is then

$$N^{\text{true}}(t|r) = \beta x^{\text{true}} \cdot \frac{e/\delta}{\bar{\gamma}} = \frac{\beta(\theta + \lambda)}{\lambda\delta\bar{x}\bar{\gamma}} \cdot e = B \cdot e,$$

\(^{15}\)Our results stay the same if advertisers’ efforts affect viewers’ nuisance costs differently for good- and bad-match viewers. This is because under trueview ads, nuisance cost does not enter into a viewer’s utility and has no impact on his/her decision of whether or not to join the platform. With the same viewers joining the platform, the only difference is that bad-match viewers will choose different skipping decision. But bad-match viewers generate no value for the advertisers so there is no impact on advertisers and in turn the platform.
where \( B = \frac{\beta(\theta + \lambda)}{\lambda x y} \).

Advertiser \( t \)'s profit is

\[
\pi_{ad} = N^{true}(t|r) \cdot (t - r) - c(e) = B \cdot (t - r) \cdot e - c(e).
\]

Let \( e^*(t|r) \) denote the optimal effort level for advertiser \( t \) conditional on adverting rate \( r \). We are interested in how \( e^*(t|r) \) varies with \( t \) and \( r \), which are characterized in the next Lemma.

**Lemma 2** All advertisers with \( t > r \) will advertise and invest in effort to reduce nuisance cost. Moreover, the optimal effort level \( e^*(t|r) \) increases with \( t \) and decreases with \( r \).

**Proof.** See the Appendix. ■

Previously we have explained that under traditional ads, no advertiser has an incentive to invest in \( e \) to reduce viewers’ nuisance cost. Correspondingly, each additional advertiser reduces the attractiveness of the platform to the viewers, exerting negative externality toward other advertisers. In contrast, under trueview ads, viewers have the option to skip the ads so advertisers have an incentive to reduce viewers’ nuisance costs toward their ads. Each advertiser makes its own effort level choice, which in turn impacts only the viewship of its own ad. We can see that trueview ads get rid of the negative externality problem under traditional ads and there is no direct linkage among advertisers. Introducing an additional advertisers has only indirect impact on other advertisers through its impact on the advertising rate \( r \). Moreover, under trueview ads, the equilibrium effort level is not uniform across the advertisers – those with higher willingness to pay per effective viewer.
will exert more effort \( (de^*/dt > 0) \) which boosts the platform’s advertising revenue, although the platform charges a uniform advertising rate \( r \) across advertisers. Trueview ads act as if that the platform can “price discriminate” across advertisers.

The platform’s profit from advertiser \( t \) is \( r \cdot N_{true}(t|\bar{r}) \). Aggregating it over all advertisers who advertise on the platform, the platform’s overall profit is

\[
\pi^{true}(r) = \int_{\bar{r}} r \cdot N_{true}(t|\bar{r})f(t)dt \\
= \frac{\beta(\theta + \lambda)}{\lambda \delta \bar{x} \bar{\gamma}} \cdot r \int_{\bar{r}} e^*(t|\bar{r})f(t)dt. \tag{5}
\]

Note that \( \pi^{true}(r) \) is continuous on the interval \([0, \bar{r}]\) and \( \pi^{true}(r = 0) = \pi^{true}(r = \bar{r}) = 0 \). There must exist an interior \( r_{true} \) which satisfies the FOC and maximizes \( \pi^{true}(r) \).

### 3.3 Comparing traditional ads with trueview ads

In this section, we will compare the welfare results for the two types of ads, starting with viewer surplus.

**Viewer surplus**

Since advertising has no impact on viewer welfare under trueview ads but reduces viewer welfare under traditional ads, trueview ads must make viewers better off.

**Proposition 2** *(Viewer surplus)* Trueview ads always raise viewer surplus.

**Platform profit**

Next, we compare the equilibrium platform profits which in turn determine whether the platform has an incentive to adopt trueview ads.\(^{16}\) It turns out that trueview ads lead to trade-offs relative to traditional ads. First, trueview ads attract more viewers to join the platform in the equilibrium, the demand-enhancing effect. This can be seen by comparing Figure 1 and Figure 2 where the marginal consumers line under traditional ads (\( MCL^{true} \)) rotates clockwise around the point \((x, \gamma) = (\hat{x}^{true}, 0)\) to become \( MCL^{true} \) (the comparison is shown in Figure 3). It is easy to see that the smaller \( \beta \) (the probability of a viewer-ad match being good) is, the flatter \( MCL^{true} \) will be and thus more rotation from \( MCL^{true} \) to \( MCL^{true} \) (larger demand-enhancing effect). Second, under traditional ads, all viewers who join the platform will view the ads. In contrast, under

\(^{16}\)One can think of adding a stage in the beginning of the game where the platform chooses between traditional ads and trueview ads. Later on we will allow the platform to choose a mixture of traditional and trueview ads for advertisers to self-select (see Section 4).
trueview ads, an ad may be skipped by some viewers who would otherwise generate value for the advertiser and in turn for the platform, the free-riding effect. Note that an ad will be skipped by good match viewers with $\gamma > e/\delta$. The threshold $\gamma = e/\delta$ increases with $e$ (which decreases with effort cost) and decreases with $\delta$. Combined, we would expect trueview ads to be more attractive to the platform when $\beta$ is small so the positive effect is large, and when $\delta$ is small or effort is not costly so the negative effect is small.

![Figure 3: demand-enhancing effect vs. free-riding effect.](image)

Next, we focus on the effort cost function $c(e)$ and analyzes how it affects the comparison of $\pi^{\text{trad}}$ and $\pi^{\text{true}}$. The results are summarized in the next Proposition.

**Proposition 3** (Platform profit)

(i) $\pi^{\text{trad}}$ is independent of $c(e)$;

(ii) $\pi^{\text{true}}$ strictly decreases with $c(e)$ in the following sense. If $c_1(e) > c_2(e)$ and $c'_1(e) \geq c'_2(e)$, \( \forall e > 0 \), then platform profit is higher under $c_2(e)$ than that under $c_1(e)$.

(iii) There exists a level curve of $c(e)$ such that $\pi^{\text{true}} = \pi^{\text{trad}}$. $\pi^{\text{true}} > \pi^{\text{trad}}$ if and only if $c(e)$ is below this level curve.

**Proof.** See the Appendix. ■

**Advertiser surplus and social surplus**

We have shown that platform profit $\pi^{\text{true}}$ decreases with effort cost function $c(e)$ and trueview ads raise platform profit if effort cost is sufficiently low. How about advertiser surplus? It turns
out that trueview ads also raise advertiser surplus when \( c(e) \) is sufficiently low. The idea is the following. Under trueview ads, more viewers join the platform in equilibrium relative to under traditional ads. If \( c(e) \) is sufficiently low, then little cost will generate large \( e \) to cancel out the nuisance costs for most views who will then watch ads. Later on we will show that \( r^{true} < r^{trad} \) when \( c(e) \) is sufficiently low (see Lemma 3). Facing lower advertising rate and more viewers, advertisers must be better off under trueview ads. However, advertiser surplus under trueview ads does not necessarily decrease with \( c(e) \) monotonically. While \( r^{true} < r^{trad} \) when \( c(e) \) is sufficiently low, it is unclear how \( r^{true} \) varies with \( c(e) \). Lower effort cost obviously makes advertiser better off (direct effect). But if lower effort cost leads to higher \( r^{true} \) which hurts advertiser surplus (indirect effect), then the overall impact of \( c(e) \) on advertiser surplus may be ambiguous.

**Lemma 3** When effort cost \( c(e) \) is sufficiently low, in equilibrium we have \( r^{true} < r^{trad} \) and \( a^{true} > a^{trad} \).

**Proof.** See the Appendix. ■

We have shown that trueview ads always raise viewer surplus, and raise platform profit and advertiser surplus if effort cost \( c(e) \) is sufficiently low. Combined, trueview ads must raise social surplus when \( c(e) \) is sufficiently low.

**Proposition 4** When effort cost \( c(e) \) is sufficiently low, trueview ads raise advertiser surplus and social surplus.

So far we have allowed fairly general effort cost function \( c(e) \) and distribution of \( t \). While we are able to establish the existence and uniqueness of optimal effort level \( e^* \) and its properties, we cannot obtain the explicit solution of \( e^* \). The platform then chooses advertising rate \( r \) to maximize its profit, which is an integral of its advertiser-specific profit over all advertisers (see equation (5)). The implicit \( e^* \), combined with general distribution of advertisers \( (F(t)) \), prevents us from pinning down the equilibrium advertising rate \( r \) under trueview ads, and to compare it with that under traditional ads. To obtain more concrete results, next, we adopt specific effort cost function and distribution of \( t \). In particular, we assume that \( c(e) = k \cdot e^2 \) so the effort cost function is captured by a single parameter \( k \geq 0 \). We further assume that \( t \) is uniformly distributed on \([0, \bar{t}]\). These two assumptions allow us to compare \( r^{trad} \) and \( r^{true} \) for the full spectrum of \( c(e) \), not just when \( c(e) \) is sufficiently low. The results are summarized in the next Proposition.

**Proposition 5** *(Quadratic effort cost and uniform distribution of \( t \))

\(^{17}\)The cost is that we can only compare welfare results when \( c(e) \) is either sufficiently high or sufficiently low. Intuitively, trueview ads lower advertiser surplus when \( c(e) \) is sufficiently high.
(i) The optimal advertising rate under traditional ads $r^{\text{trad}}$ is independent of $k$;

(ii) The optimal advertising rate under trueview ads $r^{\text{true}}$ weakly decreases with $k$;

(iii) $r^{\text{true}} < r^{\text{trad}}$, $\forall k \geq 0$;

(iv) $a^{\text{true}} > a^{\text{trad}}$, $\forall k \geq 0$.

**Proof.** See the Appendix. ■

**Targeted ads:** Trueview ads have a potential advantage to the platform which we have not considered. A viewer in general has different nuisance costs under good/bad match and would thus make different skipping decisions across different ads. By observing a viewer’s skipping decisions, the platform can infer what is more likely a good match for that consumer and show targeted ads accordingly. Intuitively this would help the viewers, advertisers and the platform. In contrast, under traditional ads, viewers do not have the option to skip ads so the platform cannot infer what would be good/bad match.

4 Extensions

In the baseline model, we have considered two scenarios where the platform chooses either traditional ads or trueview ads alone. Next, we consider two alternative scenarios where the platform uses a combination of the two formats. These two scenarios differ from each other in terms of who (advertisers or viewers) make decision on whether an ad can be skipped or not. Advertisers make the decisions in the first extension (mixed ads) while viewers decide (to some extent) in the second extension (proportion-skipping ads). In practice, there are media platforms (e.g., youtube) where some ads are skippable while others are not. This case fits the mixed ad scenario. In other media platform, multiple ads may be shown to the viewer simultaneously and the viewer has the option to choose which ad to watch but not to skip all ads. This fits the proportion-skipping ads scenario. Next, we analyze these two extensions in turn. We will focus on how the extensions compare to the two pure scenarios (traditional ads and trueview ads) in terms of platform profit.

4.1 Mixed ads

Under mixed ads, the platform offers both traditional and trueview ads and let advertisers self-select. We are interested in how advertisers will self-select between the two types of ads, and how the platform will choose different rates to guide the advertisers’ selection.

We first introduce some notations. Let $r^{\text{mix},j}$ denote the advertising rate per effective viewer of ad type $j \in \{\text{trad, true}\}$ under mixed ads. Let $N^{\text{mix},j}$ denote the number of effective viewers
under ad type \( j \). Let \( \pi_{\text{mix},j}(t) \) denote the optimal profit for advertiser \( t \) if it chooses ad type \( j \). Let \( \Delta \pi(t) \equiv \pi_{\text{mix,true}}(t) - \pi_{\text{mix,trad}}(t) \). Obviously an advertiser prefers trueview ads if \( \Delta \pi(t) \geq 0 \) but favors traditional ads otherwise. We are interested in how \( \Delta \pi(t) \) varies with \( t \). The results are summarized in the next lemma.

**Lemma 4 (Choice of ad type)** Fixing the advertising rates \( r_{\text{mix},j} \), \( j \in \{\text{trad, true}\} \),

(i) Advertisers’ incentive to choose trueview ads over traditional ads decreases with \( t \). In particular, \( \frac{d\Delta \pi(t)}{dt} < 0 \) when \( e^*(t) < \frac{\tilde{e}}{3} \) and \( \frac{d\Delta \pi(t)}{dt} = 0 \) when \( e^*(t) = \frac{\tilde{e}}{3} \).

(ii) If both traditional and trueview ads are chosen by advertisers in the equilibrium, there must exist a threshold \( t_0 \) such that advertisers with \( t \geq t_0 \) \((t < t_0)\) choose traditional \((trueview)\) ads.

(iii) If there is one advertiser choosing trueview ads and investing in full effort level \((e = \delta \tilde{e})\), then in the equilibrium no advertiser will choose traditional ads.

**Proof.** See the Appendix. ■

It is a bit surprising that advertisers with higher willingness to pay for viewer access are less likely to choose trueview ads. One would conjecture the opposite result as advertisers with larger \( t \) will invest more in effort in the equilibrium, so fewer viewers will skip ads, further supporting the advertiser’s choice of trueview ads. This turns out not to be the case. Moving from traditional ads to trueview ads, the advertiser gains on infra-marginal viewers but loses on marginal viewers. By paying lower advertising rate \((r_{\text{mix,true}} < r_{\text{mix,trad}})\), the advertiser saves on infra-marginal viewers who will watch the advertiser’s good match ad under both traditional and trueview ads. The exact savings \((r_{\text{trad}} - r_{\text{true}})\) is independent of \( t \). On the other hand, some marginal viewers watch good match ads under traditional ads but not under trueview ads. For every marginal viewer lost, the advertiser’s monetary loss increases (linearly) with \( t \). When \( t \) increases, the loss from marginal viewers \((\text{increases in } t)\) becomes more important relative to the gain from infra-marginal viewers \((\text{independent of } t)\). Correspondingly, the advertiser is less likely to adopt trueview ads when \( t \) increases.

Lemma 4 shows how a platform may choose different advertising rates to induce advertisers to self-select between traditional ads and trueview ads. When both types of ads are chosen in the equilibrium, the platform is practicing second-degree ‘discrimination’ through two channels. First, it induces different advertisers to self-select different types of ads. Second, it induces advertisers who choose trueview ads to self-select in their optimal effort levels. The second channel works in the platform’s favor as advertisers with more to gain from advertising also put more effort in reducing nuisance cost. The first channel seems counter-productive as it matches advertisers with less incentive to invest in effort with trueview ads. Next, we analyze the platform’s profit under mixed ads and see how it compares with those under the two pure ads scenarios.
Proposition 6 (mixed ads vs. pure ads) For the platform, optimal mixed ads

(1) always strictly dominates optimal pure traditional ads;

(2) always weakly dominates optimal pure trueview ads and strictly dominates when effort cost $c(e)$ is sufficiently high.

Proof. See the Appendix. □

The intuition for (1) is the following. Pick any pure traditional ads setup, we will construct a mixed ads setup with strictly larger platform profit. This is because, under pure traditional ads, advertisers with $t$ immediately below $r^{trad}$ will not advertise on the platform. The platform can choose a mixed ads instead, with $r^{mix,trad} = r^{trad}$ and an appropriate $r^{mix,true}$ below $r^{trad}$ to serve some of the residual advertisers.\footnote{That is, the choice of $r^{mix,true}$ will not induce any advertiser with $t > r^{trad}$ to switch to trueview ads, yet induces some advertisers with $t < r^{trad}$ to choose trueview ads. Such a structure is possible because advertisers with larger $t$'s are more likely to choose traditional ads (Lemma 4).} This has no impact on viewer participation (since the additional ads are all trueview ads) or the advertisers with $t \geq r^{trad}$. However, the additional sales of trueview ads will raise platform profit (see Figure 4). While mixed ads contain pure trueview ads as a special case, optimal mixed ads may take the form of pure trueview ads, so optimal mixed ads weakly dominate pure trueview ads.

The preceding Proposition compares platform profit across the scenarios. How about viewer surplus, advertiser profit and social surplus? Viewer surplus is determined by nuisance cost only, so obviously viewers like truew ads the best, mixed ads the second and traditional ads the least. For advertisers, there are tradeoffs. Introducing trueview ads (mixed or pure) will increase viewer participation which benefits advertisers. On the other hand, viewers may skip ads or the advertising rates may change. One can construct examples to show that it is ambiguous how advertisers surplus and social surplus under mixed ads compare with those under the pure ads scenarios. However, we are unable to calculate or compare the equilibrium $r^{mix,j}$ and $r^j$, $j \in \{\text{trad,true}\}$ in general.

4.2 Proportion-skipping ads

Under proportion-skipping ads, each viewer is free to choose which ads to skip conditional that the proportion of skipped ads does not exceed certain level (denoted by $\tau$).\footnote{In the spirit of the findings in Tucker (2014), giving viewers control over what ads to skip may also make them less antagonized by ads, another advantage over pure traditional ads.} $\tau$ is chosen by the platform. Setting $\tau = 0$ and $\tau = 1$ recovers pure traditional ads and pure trueview ads respectively. Next, we consider two cases depending on how $\tau$ compares to the probability of a viewer-ad match being bad $(1 - \beta)$: $\tau \leq 1 - \beta$ in case 1 but $\tau > 1 - \beta$ in case 2.

Case 1: $\tau \leq 1 - \beta$. 
Advertisers choose traditional ads

Advertisers choose trueview ads

Advertisers choose not to advertise

Pure traditional ads

Mixed ads

Figure 4: Optimal mixed ads strictly dominate optimal pure traditional ads

When the proportion of ads allowed to be skipped is lower than the proportion of bad match ads, naturally we want the viewers to skip bad match ads only. This is achieved when no advertiser invests in nuisance-reducing effort. Turns out that this is the only SPNE if \( \delta < 1 \), i.e., bad match ads cause more nuisance cost to viewers than good match ads.

**Lemma 5** When \( \tau \leq 1 - \beta \), in the equilibrium

(i) All advertisers with \( t \geq r \) advertise but do not invest in nuisance-reducing effort.

(ii) Optimal platform profit strictly increases with \( \tau \).

**Proof.** See the Appendix. □

From Lemma 5, we can see that holding advertising rate \( r \) fixed (so advertising level \( a \) is also fixed), having \( \tau \in (0, 1-\beta] \) is a Pareto improvement relative to \( \tau = 0 \) (pure traditional ads). All skipped ads are bad match ads which have no impact on advertisers, but the option to skip some ads benefits viewers and encourages more viewer participation. This in turn benefits advertisers and platform relative to pure traditional ads.

**Case 2: \( \tau > 1 - \beta \).**
Now a viewer may skip good match ads, if these advertisers do not invest significantly in nuisance-reducing efforts. This introduces “competition” among the advertisers on nuisance-reducing effort which is absent in both traditional ads and trueview ads. We conjecture an equilibrium where there is a threshold advertiser that all advertisers below the threshold will lose in the competition to those above. The threshold advertiser $\tilde{t}$ is such that its good match ad will beat exactly $\tau$ fraction of the total ads. Therefore, good match ad from advertisers $t > \tilde{t}$ is always in the top $1 - \tau$ fraction and never be skipped. Good match ad from advertisers $t < \tilde{t}$ is always below the top $1 - \tau$ fraction and will be skipped unless the viewers’ nuisance cost is already reduced to zero. At the threshold level, advertiser $\tilde{t}$’s good match ad must beat all advertisers’ bad match ads which has a fraction of $1 - \beta$.

Next, advertiser $\tilde{t}$’s good match ad will beat all good match ads from advertisers $t < \tilde{t}$, which has a fraction of $F(\tilde{t}) - F(r)$. The sum of these two fractions must be $\tau$, i.e.,

$$(1 - \beta) + \beta \cdot \frac{F(\tilde{t}) - F(r)}{1 - F(r)} = \tau.$$ 

We want to construct an equilibrium with the following features. Advertisers $t < \tilde{t}$ are not in the top $1 - \tau$ fraction so their good match ads are viewed only by viewers whose nuisance costs are reduced to zero (see Figure 5). These advertisers’ profit maximizing FOCs give the solutions $e_t$—optimal effort level as a function of $t$. For advertisers $t > \tilde{t}$, they bunch by investing in the same effort level ($e^*$) to put their good match ads in the top $1 - \tau$ fraction.\(^{21}\)

The equilibrium is characterized in the following lemma:

**Lemma 6** When $\tau > 1 - \beta$,

(i) Advertiser $t \in [r, \tilde{t})$ chooses effort level $e_t$ which increases in $t$. It only get viewers whose nuisance cost are reduced to zero by its effort. The optimal effort level $e_t$ comes from profit-maximizing FOC directly.

(ii) Advertiser $\tilde{t}$ is indifferent between $e^*$ and $e_{\tilde{t}}$,

$$\pi_{\tilde{t}}(e^*) = \pi_{\tilde{t}}(e_{\tilde{t}}),$$

which then determines $e^*$.

(iii) Advertisers with $t \in (\tilde{t}, \bar{t}]$ chooses effort level $e^*$. Their good-match ads are never skipped.

\(^{20}\)Otherwise some advertisers’ bad matches are in the top $1 - \tau$ fraction and are never skipped. This is not optimal for these advertisers since they can lower their effort slightly but still ensure that their good match ads are never skipped.

\(^{21}\)Note that if $e_t > \delta \bar{\gamma}$ or $e^* > \delta \bar{\gamma}$, then we should replace them by $e_t = \delta \bar{\gamma}$ and $e^* = \delta \bar{\gamma}$ respectively. At effort level $\delta \bar{\gamma}$, all good match viewers’ nuisance costs are already reduced to zero. There is no need to increase effort level further.

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Advertise with fixed effort level $e^*$
Not advertise
Advertise and effort increases with $t$
Proportion skipping ads

Figure 5: Proportion skipping ads vs. pure traditional ads and pure trueview ads

(iv) There is a gap between $e^t$ and $e^*$ ($e^t \neq e^*$) whenever $e^t < \delta\bar{\gamma}$. If $e^t = \delta\bar{\gamma}$, then $e^* = \delta\bar{\gamma}$. If both $e^t$ and $e^*$ are above $\delta\bar{\gamma}$, then they should be replaced by $\delta\bar{\gamma}$.

Proof. See the Appendix.

From Lemma 5, we know the advertiser never has an incentive to choose $\tau < 1 - \beta$. Doing so is dominated by choosing $\tau = 1 - \beta$. Would the platform have an incentive to choose $\tau > 1 - \beta$ in which case some good match ads will be skipped? The answer is yes as illustrated in the next Proposition.

Proposition 7 In terms of platform profit, optimal proportion-skipping ads

(i) strictly dominate optimal pure traditional ads ($\tau = 0$) and optimal $\tau^* > 1 - \beta$;
(ii) weakly dominate optimal trueview ads.

Proof. See the Appendix.

The fact that the platform chooses $\tau > 1 - \beta$ so that some good match ads will be skipped is bad for the advertisers and in turn for
the platform. However, if no good match is skipped, then advertisers will have little incentive to
invest in nuisance-reducing effort. This increases viewers’ nuisance cost and reduces their incentive
to join the platform in the first place, hurting both advertisers and the platform.

5 Conclusion

We analyze and compare two types of ads: traditional ads which have to be shown in their
entirety before viewer requested contents are delivered, and trueview ads where viewers have the
option to skip ads costlessly. We introduce matching between viewers and ads, and allow the match
to be good or bad. Viewing a bad match ad generates no value for the advertiser and a viewer
dislikes a bad-match ad more relative to a good-match ad. Trueview ads allow viewers to skip
bad match ads. This is a Pareto improvement since no value to the advertiser can be generated
from these viewers anyway. Such an improvement induces more viewers to join the platform, the
demand-enhancing effect. The downside is that, viewers with good match may also choose to
skip ads at the loss of advertisers and the platform. To counter this, advertisers need to invest
in nuisance-reducing effort in the equilibrium. Then only viewers who are most sensitive to ads
choose to skip ads in the equilibrium. Intuitively trueview ads always make the viewers better off.
Advertisers and platform are also better off when the effort cost is sufficiently low.

We then consider two extensions where some but not all ads can be skipped. The first extension
allows the platform to offer both traditional and trueview ads at different rates for advertisers to
self-select (mixed ads). We find that advertisers with lower willingness to pay for viewer access
are more likely to choose trueview ads. Those advertisers have less incentive to invest in nuisance-
reducing effort. The second extension allows viewer to skip a fixed proportion of the ads shown
to them. When the proportion is low, in the equilibrium all advertisers make zero investment
in effort and all skipped ads are bad match ads (benefiting viewers at no cost to advertisers and
platform). This is a Pareto improvement to pure traditional ads. However, the platform has an
incentive to choose a sufficiently high proportion so that some good match ads are skipped in
equilibrium. Doing so induces advertisers to compete with each other in nuisance-reducing effort
which increases viewer participation and benefits the platform. We find that optimal mixed ads
and optimal proportion-skipping ads both strictly dominate optimal traditional ads in terms of
platform profit, and weakly dominate optimal trueview ads.

A Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1

Marginal consumers line cannot cross the right vertical line, because $\theta \leq \lambda(\bar{x} - 1)$. Next, we
show that the marginal consumers line cannot cross the left vertical line either. Suppose not and the marginal consumers line crosses the left vertical line at \((0, \hat{\gamma})\) and crosses the bottom horizontal line at \((\hat{x}, 0)\) where

\[
\hat{\gamma} = \frac{\theta + \lambda}{a(1 + \beta\delta - \beta)}, \quad \hat{x} = \frac{\theta + \lambda}{\lambda}.
\]

The fraction of viewers who will join the platform is characterized by \(\frac{\hat{\gamma} \hat{x}}{2 \hat{\gamma} \hat{x}}\). Only those with good match (at probability \(\beta\)) will generate value for the advertisers. Therefore, the measure of effective viewers is

\[
N_F = \beta \frac{\hat{\gamma} \hat{x}}{2 \hat{\gamma} \hat{x}} = \frac{\beta}{2 \hat{\gamma} \hat{x}} \cdot \frac{(\theta + \lambda)^2}{a(1 + \beta\delta - \beta)} = \frac{A}{a},
\]

where \(A = \frac{\beta}{2 \hat{\gamma} \hat{x} \lambda(1 + \beta\delta - \beta)}\).

Define \(\varepsilon_a \equiv \frac{\partial R}{\partial a}\) as the (per effective viewer) elasticity of advertising revenue, and \(\varepsilon_N \equiv -\frac{\partial N_F}{\partial a} \cdot \frac{a}{N_F}\) as the aggregate viewer demand elasticity with respect to advertising. The platform choose \(a\) to solve the problem

\[
\max_a R \cdot N_F,
\]

which is equivalent to maximizing

\[
\max_a \ln R + \ln N_F.
\]

The first-order condition then is

\[
\varepsilon_a(a) = \varepsilon_N(a).
\]

It is easy to verify that \(\varepsilon_N = -\frac{dN_F/da}{N_F/a} = 1\). On the other hand,

\[
\varepsilon_a = \frac{r'(a)a + r(a)}{r(a)} < 1,
\]

for any \(a > 0\) since \(r'(a) < 0\).

Therefore, the first-order condition \(\varepsilon_N = \varepsilon_a\) must be violated and this demand structure cannot be an equilibrium. ■

**Proof of Lemma 2**

It is obvious that advertisers with \(t < r\) will not advertise. The advertiser with \(t = r\) either advertises without effort or does not advertise at all.

Next, consider an advertiser with \(t > r\). Taking derivative of its profit with respect to \(e\) and evaluating the derivative at \(e = 0\), we have

\[
\left.\frac{d\pi_{ad}}{de}\right|_{e=0} = B(t - r) - c'(e)|_{e=0} > 0,
\]

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since $c'(e = 0) = 0$. Note that $\pi_{ad}(e = 0) = 0$. Combined, the optimal advertiser profit $\pi_{ad} > 0$, $\forall t > r$.

Next, we analyze the optimal effort choice for advertisers with $t > r$. FOC implies that

$$B \cdot (t - r) = c'(e). \quad \text{(A.1)}$$

Since $c(e)$ is strictly convex, there exists a unique $e^*(t| r) > 0$ which increases with $t$ ($de^*/dt > 0$) and decreases with $r$ ($de^*/dr < 0$).

Note that when effort cost is sufficiently low, the solution to the FOC will exceed the maximum effort required to cancel out nuisance cost for all good match viewers: $e = \delta \bar{\gamma}$. In this case, the advertiser’s optimal effort level is $e = \delta \bar{\gamma}$ which does not vary with $t$ or $r$. ■

Proof of Proposition 3

(i) Under traditional ads, advertisers never have an incentive to invest in $e$, so $\pi_{trad}$ is independent of $c(e)$.

(ii) Fix the effort cost function at $c_1(e)$. Let $r_1^{true}$ denote the optimal advertising rate. Next, lower the cost function to $c_2(e)$ with $c'_2(e) \leq c'_1(e), \forall e > 0$. Let the platform still choose $r = r_1^{true}$. From equation (A.1), we can see that advertisers will (weakly) increase their effort levels, leading to more effective viewers watching the ads and a higher platform profit. By choosing an optimal $r = r_2^{true}$, platform profit can only go further up. Therefore, $\pi_{true}|_{c_2(e)} > \pi_{true}|_{c_1(e)}$.

(iii) It is intuitive that $\pi_{true} > \pi_{trad}$ must hold when $c(e)$ is sufficiently small ($c(e) \to 0^+$) and the result is reversed when $c(e)$ is sufficiently high. Combined with results in (i) and (ii), we have the results in (iii). ■

Proof of Lemma 3

We will show that $r_{trad} > r_{true}(c(e) = 0)$. By continuity, $r_{trad} > r_{true}$ must hold when $c(e)$ is sufficiently low.

Under traditional ads,

$$\frac{d\pi_{trad}}{dr} = N_{trad}(1 - F(r) - r \cdot f(r)) + \frac{\bar{\gamma}(1 + \beta \delta - \beta)}{2\lambda \bar{\epsilon}} f(r) \cdot (1 - F(r)) \cdot r = 0.$$

$$\Rightarrow 1 - F(r) - r \cdot f(r) < 0$$

$$\Rightarrow \frac{1 - F(r_{\text{trad}})}{r_{\text{trad}} \cdot f(r_{\text{trad}})} < 1.$$
Next, we consider trueview ads. With $c(e) = 0$, we have $e^*(t|r) = \delta \bar{\gamma}$ and $N^{true} = \beta \frac{\theta + \lambda}{\lambda x}$. The platform’s FOC is

$$\frac{d\pi^{true}}{dr} = N^{true}(1 - F(r) - r \cdot f(r)) = 0$$

$$\Rightarrow (1 - F(r) - r \cdot f(r) = 0)$$

$$\Rightarrow \frac{1 - F(r^{true})}{r^{true} \cdot f(r^{true})} = 1.$$

From the monotone hazard rate property, $\frac{1 - F(r)}{r \cdot f(r)}$ decreases with $r$. Then

$$\frac{1 - F(r^{true})}{r^{true} \cdot f(r^{true})} > \frac{1 - F(r^{trad})}{r^{trad} \cdot f(r^{trad})} \iff r^{trad} > r^{true}.$$

Note that $a^i = 1 - F(r^i), i \in \{trad, true\}$. With $r^{trad} > r^{true}$, it must be that $a^{trad} < a^{true}.$

Proof of Proposition 5

(i) Since advertisers have no incentive to invest in $e$ under traditional ads, it must be that $\frac{d\pi^{trad}}{dk} = 0$. 

(ii) With $c(e) = ke^2$, we have $c'(e) = 2ke$. Solving the advertiser’s FOC (equation (A.1)), we can obtain

$$e^* = \frac{B(t - r)}{2k},$$

where $B = \frac{\beta(\theta + \lambda)}{\lambda \delta x \bar{\gamma}}$.

Depending on how $e^*(\bar{t}|r)$ compares with $\delta \bar{\gamma}$ - the maximum effort required to cancel out nuisance cost for all good match viewers, there are two cases.

Case 1: $e^*(\bar{t}|r) \leq \delta \bar{\gamma}$.

This occurs when $k \geq \bar{k} \equiv \frac{\beta(\lambda + \theta)}{\lambda \delta x \bar{\gamma}}$. The platform’s profit is

$$\pi^{true} = \int_r^\bar{t} r \cdot N^{true}(t|r)f(t)dt,$$

where $e = \frac{B(t - r)}{2k}$ and $N^{true}(t|r) = B \cdot e$.

Taking derivative, we have

$$\frac{d\pi^{true}}{dr} = \int_r^\bar{t} \left( N^{true}(t|r) + r \cdot \frac{dN^{true}(t|r)}{dr} \right) f(t)dt$$

$$= \int_r^\bar{t} \left( \frac{B^2(t - r)}{2k} - r \cdot \frac{B^2}{2k} \right) f(t)dt$$

$$= \frac{B^2}{2k} \left( \int_r^\bar{t} (t - 2r)f(t)dt. \right)$$
The optimal $r^{true}$ is defined by $\int_{r}^{t}(t-2r)f(t)dt = 0$ which is independent of $k$, i.e., $\frac{dr^{true}}{dk} = 0$.

**Case 2:** $e^{*}(\bar{t}|r) > \delta \bar{\gamma}$.

This occurs when $k < \bar{k} \equiv \frac{\beta(\lambda+\theta)\bar{t}}{\delta\bar{\gamma}^2\lambda^2}$. Define $\bar{t} < \bar{\tilde{t}}$ such that $e^{*}(\bar{t}|r) = \delta \bar{\gamma} \Rightarrow \bar{\tilde{t}} = 2\frac{\delta \bar{\gamma}k}{B} + r$

All advertisers with $t \leq \bar{\tilde{t}}$ will choose $e = \frac{B(t-r)}{2k}$. Those with $t > \bar{\tilde{t}}$ will choose $e = \delta \bar{\gamma}$. The platform’s profit is

$$\pi^{true} = \int_{\bar{\tilde{t}}}^{\bar{t}} r \cdot N^{true} \cdot f(t)dt + \int_{\bar{\tilde{t}}}^{\bar{t}} r \cdot N^{true} \cdot f(t)dt$$

$$= \int_{r}^{\bar{\tilde{t}}} r \cdot \frac{Be}{t} dt + \int_{\bar{\tilde{t}}}^{\bar{t}} r \cdot \frac{B\delta \bar{\gamma}}{t} dt$$

$$= \frac{r\delta \bar{\gamma}[B(\bar{t} - r) - k\delta \bar{\gamma}]}{t}.$$

Solving the FOC with respect to $r$, we can obtain

$$r^{true} = \frac{B\bar{t} - k\delta \bar{\gamma}}{2B},$$

which leads to $\frac{dr^{true}}{dk} < 0$.

(iii) Previously, we have shown that $r^{true}(c(e) = 0) < r^{trad}$. Since $r^{true}$ weakly decreases with $c(e)$, it must be that $r^{true} < r^{trad}$.

(iv) With $r^{trad} > r^{true}$, it must be that $a^{trad} < a^{true}$ since $a^i = 1 - F(r^i), i \in \{trad, true\}$. ■

**Proof of Lemma 4**

(i) The advertiser $t$’s profits are,

$$\pi^{mix, true}(t) = (t - r^{mix, true})N^{mix, true} - c(e^{*}), \quad \pi^{mix, trad}(t) = (t - r^{mix, trad})N^{mix, trad}.$$

Let

$$\Delta \pi(t) \equiv \pi^{mix, true}(t) - \pi^{mix, trad}(t) = (t - r^{mix, true})N^{mix, true} - c(e^{*}) - (t - r^{mix, trad})N^{mix, trad}.$$

Taking derivative with respect to $t$, we have

$$\frac{d\Delta \pi(t)}{dt} = N^{mix, true} + \left[(t - r^{mix, true})\frac{dN^{mix, true}}{de} - c^{'}(e^{*})\right]_{e=e^*} \cdot \frac{de^{*}}{dt} - N^{mix, trad}$$

$$\leq 0.$$
Note that (1) \((t - r_{\text{mix,true}}) \frac{dN_{\text{mix,true}}}{de} - c'(e) = 0\) by the FOC under trueview ads; (2) \(N_{\text{mix,trad}}\) includes all viewers who join the platform while \(N_{\text{mix,true}}\) excludes those who join the platform but skip advertiser \(t\)'s ad. Therefore, \(N_{\text{mix,true}} \leq N_{\text{mix,trad}}\) always holds and the inequality is strict if and only if the advertiser chooses less than full effort \((e^*(t) < \frac{\bar{\gamma}}{\delta})\).

(ii) is straight from (i).

(iii) Suppose that advertiser \(\tilde{t}\) chooses trueview ads with full effort \((e^*(\tilde{t}) = \frac{\delta \bar{\gamma}}{\delta})\). Then all advertisers with \(t < \tilde{t}\) will choose trueview ads over traditional ads. Moreover, since \(e^*(t)\) (weakly) increases with \(t\), all advertisers with \(t > \tilde{t}\) will choose full effort when choosing trueview ads. From (i), \(\frac{d\Delta\pi(t)}{dt} = 0\) so all advertisers with \(t > \tilde{t}\) will choose trueview ads as well.

A special case is \(\Delta\pi(t) = 0\) for advertisers with \(t \geq \tilde{t}\). These advertisers choose full effort when adopting trueview ads but are indifferent between choosing traditional ads and trueview ads. However, this cannot be optimal for the platform - raising \(r_{\text{mix,trad}}\) will induce these advertisers to strictly prefer trueview ads, leading to more viewer participation and higher platform profit.

Proof of Proposition 6

(1) Let \(r_{\text{trad}}\) denote the optimal advertising rate under pure traditional ads. Note that all advertisers with \(t < r_{\text{trad}}\) will not advertise on the platform (residual demand). Under mixed ads, the platform can choose \(r_{\text{mix,trad}} = r_{\text{trad}}\) to serve the same advertisers under pure traditional ads. The platform can then use trueview ads to serve some of the residual demand (advertisers with \(t < r_{\text{trad}}\)). This will not impact viewers’ participation decisions (all additional ads are trueview ads) or platform profit from advertisers with \(t \geq r_{\text{trad}}\). But now the platform also receives advertising revenue from advertisers \(t < r_{\text{trad}}\). \(r_{\text{mix,true}}\) has to be chosen in a way such that (i) no advertiser with \(t \geq r_{\text{trad}}\) will choose trueview ads; (ii) some advertisers with \(t < r_{\text{trad}}\) will choose trueview ads. This can be done because \(\Delta\pi = \pi_{\text{true}}(t) - \pi_{\text{trad}}(t)\) decreases with \(t\).

(2) Pure trueview ads is a special case of mixed ads (by setting \(r_{\text{mix,trad}}\) sufficiently high so no advertiser will choose traditional ads). Thus optimal mixed ads always weakly dominates pure trueview ads. If effort cost is sufficiently high, then \(\pi_{\text{trad}} > \pi_{\text{true}}\). Combined with \(\pi_{\text{mix}} > \pi_{\text{trad}}\), we have \(\pi_{\text{mix}} > \pi_{\text{true}}\).

Proof of Lemma 5

(i) It is obvious that all advertisers with \(t > r\) advertise but no one invests in nuisance-reducing effort is an equilibrium. An advertiser only needs its good match ad not to be skipped. That is guaranteed when no advertiser invests in effort, since viewers will skip bad match ads first and the proportion of ads allowed to be skipped will be reached before viewers skip good match ads \((\tau \leq 1 - \beta)\).
To show it is the only equilibrium, we need to show that no advertiser will invest effort in equilibrium. Suppose that some advertisers invest in effort. Denote the advertiser with the highest effort by $t_1$ (pick any of the advertisers if there are multiple with the highest effort). Advertiser $t_1$’s bad match beats (lower nuisance cost) all other advertisers’ bad match and will not be skipped. Its good match ad will beat all other advertisers’ bad match ads by a finite margin. That is, it can reduce its effort slightly and its good match ad will still beat all advertisers’ bad match ads, and thus will not be skipped. Thus advertiser $t_1$ always has an incentive to lower its effort unless its effort is already zero.

(ii) When $\tau$ increases, the platform can hold the advertising rate fixed. More viewers will join the platform since they can skip more ads. But the skipped ads are all bad match ads which generate no value for advertisers. Combined, the measure of effective viewers increases with $\tau$ so platform profit increases with $\tau$ as well.■

**Proof of Lemma 6**

We first show that no advertiser has an incentive to deviate from the strategies described in (i)-(iii).

In our setup, each advertiser chooses between two situations:

**Situation 1:** Invest in sufficiently high effort ($e^*$) to put its good match ad among the top $1 - \tau$ proportion of ads. Correspondingly, their good match ads will never be skipped. Advertiser $t$’s expected profit in this situation is

$$\pi^1(t) = (t - r) \cdot N^{prop} - c(e^*),$$

where the superscript ‘1’ denotes situation 1 and $N^{prop}$ is the expected measure of effective viewers (measure of all viewers times $\beta$) joining the platform. We can then obtain

$$\frac{d\pi^1(t)}{dt} = N^{prop}. $$

**Situation 2:** Effort level is not high enough to put their good match ads among the the top $1 - \tau$ proportion of ads. Correspondingly, their good match ads will be skipped unless the viewer’s nuisance cost is reduced to zero by the advertiser’s effort. Advertiser $t$’s expected profit is

$$\pi^2(t) = (t - r) \cdot N^{prop} \cdot prob\left(\gamma \leq \frac{e}{\delta}\right) - c(e).$$

Note that $prob\left(\gamma \leq \frac{e}{\delta}\right) < 1$ and strictly increases with $e$.$^{22}$

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$^{22}$The corner solution case where $\gamma = \frac{e}{\delta}$ is analyzed next in (iv).
Optimal effort level $e_t$ is determined by $\frac{d\pi^2(t)}{de} = 0$. By envelope theorem, we have

$$\left. \frac{d\pi^2(t)}{dt} \right|_{e=e_t} = N^{prop} \cdot \text{prob} \left( \gamma \leq \frac{e}{\delta} \right) \bigg|_{e=e_t}.$$

Then

$$\left. \frac{d\pi^1(t) - d\pi^2(t)}{dt} \right|_{e=e_t} = N^{prop} \cdot \left[ 1 - \text{prob} \left( \gamma \leq \frac{e}{\delta} \right) \right] \bigg|_{e=e_t} > 0.$$

That is, as $t$ increases, advertisers prefer situation 1 more and situation 2 less. Given that advertiser $\tilde{t}$ is indifferent between the two situations, advertisers $t > \tilde{t}$ must strictly prefer situation 1 (invest $e^*$) while advertisers $t < \tilde{t}$ strictly prefer situation 2 (invest $e_t$ which is determined by FOC).

Next, consider advertiser $\tilde{t}$. With a continuum of advertisers, it is unclear whether advertiser $\tilde{t}$’s good match ad is exactly in the top $1 - \tau$ proportion or not. To simplify the analysis, we assume that if it invests in $e^*$, its good match ad will be in the top $1 - \tau$ proportion of ads (holding other advertisers’ strategies fixed). If it invests in level lower than $e^*$ (even when it is above $e_t$), then its good match will not be. This assumption seems counterintuitive and somewhat arbitrary but its idea can be seen better from an example of discrete advertisers in Appendix B.

Given this assumption, then advertiser $\tilde{t}$ will not deviate to any effort level in $(e^*, e_t)$. It obviously has no incentive to choose effort level above $e^*$ or below $e_t$.

(iv) Recall that

$$\pi^1(\tilde{t}) = (\tilde{t} - r) \cdot N^{prop} \cdot [1 - \text{prob} \left( \gamma \leq \frac{e_t}{\delta} \right) - c(e_t)],$$

$$\pi^2(\tilde{t}) = (\tilde{t} - r) \cdot N^{prop} \cdot \text{prob} \left( \gamma \leq \frac{e_t}{\delta} \right) - c(e_t).$$

If $e_t < \delta \tilde{\gamma}$, then $\text{prob} \left( \gamma \leq \frac{e_t}{\delta} \right) < 1$. It must be $e_t \neq e^*$ for $\pi^1(\tilde{t}) = \pi^2(\tilde{t})$ to hold. If $e_t = \delta \tilde{\gamma}$, we have $\text{prob} \left( \gamma \leq \frac{e_t}{\delta} \right) = 1$ then $e^* = \delta \tilde{\gamma}$ must hold as well as it already reduces nuisance cost to zero for all good match viewers. We can see that $e_t$ and $e^*$ reach $\delta \tilde{\gamma}$ simultaneously. When both $e_t$ and $e^*$ are above $\delta \tilde{\gamma}$, they should both be replaced by $\delta \tilde{\gamma}$. ■

Proof of Proposition 7

(i) It is obvious that optimal proportion skipping ads strictly dominates optimal traditional ads. Choosing any $\tau \in (0, 1 - \beta]$ strictly increases platform profit relative to $\tau = 0$ (traditional ads).

Next, we show $\tau^* > 1 - \beta$. Suppose not, then it must be $\tau^* = 1 - \beta$. In the equilibrium all advertisers make zero effort. Now raise $\tau$ infinitesimally. We have shown that the slight increase in $\tau$ leads to a paradigm shift in the equilibrium advertising patterns. All advertisers will now invest in efforts: advertisers $t \in (r, \tilde{t})$ invest $e_t$ and advertisers $t > (\tilde{t}, \bar{t}]$ will invest $e^*$. $\tilde{t}$ will be only slightly above $r$ and $e_t$ will be close to zero. However, there is a finite gap between $e_t$ and $e^*$.
so almost all advertisers \((t > \bar{t})\) will invest a finite amount \(e^*\). That is, an infinitesimal increase in \(\tau\) at \(\tau = 1 - \beta\) leads to a finite increase in effort for almost all the advertisers. This reduces viewers’ nuisance costs, raises viewer participation and in turn platform profit. Therefore, optimal \(\tau^* > 1 - \beta\) must hold.

(ii) Trueview ads is a special case of proportion-skipping ads \((\tau = 1)\) so obviously optimal proportion-skipping ads weakly dominates optimal trueview ads.

\[\blacksquare\]

B Proportion-skipping ads with \(\tau > 1 - \beta\) and discrete \(t\)

Suppose that there are \(\chi\) advertisers who will advertise given advertising rate \(r\). They are ordered as \(t_{\chi} > t_{\chi-1} > \cdots > t_2 > t_1 \geq \tau\).\(^{23}\) Viewers are allowed to skip \(\tau\) fraction of the ads. We first define the threshold advertiser \(M\) such that (i) advertiser \(i > M + 1\)’s good match ads will always be in the top \(1 - \tau\) proportion so never skipped; (ii) advertiser \(i < M\)’s good match ads will never be in the top \(1 - \tau\) proportion of ads; (iii) advertisers \(M\) and \(M + 1\) will play mixed strategies in their efforts and either \(M\) or \(M + 1\)’s good match ads will be in the top \(1 - \tau\) proportion but not both.

Next, we derive the threshold advertiser \(M\). We first use a specific example to illustrate the idea. Suppose that there are \(\chi = 100\) advertisers on the platform. The proportion of good match is \(\beta = \frac{1}{4}\), so the number of bad match ads is \(100 \cdot \left(1 - \frac{1}{4}\right) = 75\). Viewers are allowed to skip \(\tau = \frac{4}{5}\) of the ads, or \(100 \cdot \frac{4}{5} = 80\) ads. The 75 bad match ads will skipped first, and then 80-75=5 good match ads will be skipped. Next, we derive the general formula for \(M\). Advertiser \(M\)’s good match ad must beat all advertisers’ bad match ads.\(^{24}\) Moreover, good match ads from the lowest \(M\) advertisers will not be in the top \(1 - \tau\) proportion. Adding the two fractions together gives the proportion \(\tau\). That is,

\[
\frac{(1 - \beta)\chi + M}{\chi} = \tau \Rightarrow M = (1 - \beta - \tau)\chi.
\]

If \((1 - \beta - \tau)\chi\) is not an integer, we will have \(M = \text{floor}[(1 - \beta - \tau)\chi]\).

Next, we construct the equilibrium as the following:

- Define \(e_i\) as the optimal effort level for advertiser \(i = 1, \cdots, M + 1\) when its good match ad is viewed by only those viewers whose nuisance costs are reduced to zero by its effort.

\(^{23}\)In our model of continuous advertisers, no two advertisers have the same \(t\). Correspondingly, we do not allow two advertisers to have the same \(t\) in the discrete advertisers case either.

\(^{24}\)If not, the advertiser whose bad match ads beat \(M\)’s good match ads is investing too much in effort. It only cares about its own good match ad and it can lower its effort infinitesimally and its good match ad still beats \(M\)’s good match ad.
Advertisers $i = 1, \ldots, M - 1$ chooses effort level $e_i$.

Advertiser $M$ loads some mass (say $\rho_1$) on $e_M$, then mixes continuously on $[e_{M+1}, e^*]$. Advertiser $M + 1$ loads some mass (say $\rho_2$) on $e_{M+1}$, then mixes continuously on $(e_{M+1}, e^*)$.

- For advertiser $M$, it must be that
  \[ \pi_M(e_M) = \pi_M(e_{M+1}) = \pi_M(e^*), \]  
  where
  \[ \pi_M(e_M) = (t_M - r) \cdot N^{\text{prop}} \cdot \text{prob}(\gamma \leq \frac{e_M}{\delta}) - c(e_M), \]
  \[ \pi_M(e_{M+1}) = (t_M - r) \cdot N^{\text{prop}} \cdot \text{prob}(\gamma \leq \frac{e_{M+1}}{\delta}) + c(e_{M+1}), \]
  \[ \pi_M(e^*) = (t_M - r) \cdot N^{\text{prop}} - c(e^*). \]

Note that (i) $N^{\text{prop}}$ is the expected measure of effective viewers on the platform for any given advertiser; (ii) Because advertiser $M + 1$ loads mass at $e_{M+1}$ but advertiser $M$ does not, we assume that when both advertiser $M$ and $M + 1$ choose $e_{M+1}$, advertiser $M$’s good match ad beats advertiser $M + 1$’s good match ad; (iii) $e^*$ is determined by $\pi_M(e_M) = \pi_M(e^*)$, which are independent of the two masses: $\rho_1$ and $\rho_2$.

- For advertiser $M + 1$, it must be that
  \[ \pi_{M+1}(e_{M+1}) = \pi_{M+1}(e^*), \]  
  where
  \[ \pi_{M+1}(e_{M+1}) = (t_{M+1} - r) \cdot N^{\text{prop}} \cdot \text{prob}(\gamma \leq \frac{e_{M+1}}{\delta}) + c(e_{M+1}), \]
  \[ \pi_{M+1}(e^*) = (t_{M+1} - r) \cdot N^{\text{prop}} - c(e^*). \]

Advertiser $i = M + 2, \ldots, \chi$ chooses effort level $e^*$.

Extending the discrete advertisers case to continuous advertisers case (let $\chi \rightarrow +\infty$), the measure of the advertisers around the threshold is zero. The bunching effort level $e^*$ is now determined by the indifference condition of the threshold advertiser $\tilde{t}$, i.e.,

\[ \pi_{\tilde{t}}(e_{\tilde{t}}) = \pi_{\tilde{t}}(e^*). \]

Correspondingly, we make the simplifying assumption that if advertiser $\tilde{t}$’s good match ad will be in the top $1 - \tau$ proportion if it invests $e^*$, but not if it invests less, including $(e \in e_{\tilde{t}}, e^*)$. 

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References


